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Behavior of Compressor Foundation – Predictions and Observations

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SYNOPSIS In a Compressor foundation undergoing excessive vibrations, its amplitudes at operating frequency and natural frequency in free vibrations were monitored. Also in-situ dynamic properties were determined to check design and predict its response. Since the soil constants are strain dependent, two sets of computations were done (1) from the known soil constants and permissible amplitudes and (2) from the known soil constants and the observed amplitudes. The soil constants were corrected for confining pressure and relative density of the non-cohesive soil also.

Both weightless spring theory (Barkans' Method) and elastic half space theory were used in predicting the response. A critical evaluation of these two design approaches has been made and necessity to monitor the performance of machine foundations is highlighted.

INTRODUCTION

A reciprocating compressor foundation was vibrating excessively. Its performance was monitored and in-situ soil properties were determined to check its design and compute its response.

Figure 1 shows a dimensional plan and section of the foundation. The pertinent machine and foundation data are as follows:

Operating speed	405 RPM
Weight of compressor and motor	11 · 0t
Horizontal unbalanced force	= 0
Vertical unbalanced force P_z	= 0.205 t
Horizontal moment M^z_{xy}	= 0.185 t-m
Vertical moment M^x_{yz}	= 2.2 t-m
Permissible vibration amplitude (peak to peak)	= 0.025 mm
Area of the foundation	A = 7.103 m^2
Weight of the foundation	W = 49.79 t
Depth of foundation	= 2.4 m

Subsequent to the monitoring of the foundation, performance, (Arya et al 1978) and in-situ dynamic properties determination, the design of foundation by (1) the Barkan's approach and (2) elastic half-space approach for 2-cases have been discussed in this paper; one as for usual design stage as if the monitored performance of the machine is not known; two, after knowing the monitored performance. The two sets of computation are similar except that strains in the soil in two cases are different which affect the relevant soil properties considerably.

The computations by two methods and their comparison with the monitored performance throw light on the applicability of one method to the analysis of such problems better than the other. Remedial measures are described elsewhere (Arya et al 1978)

OBSERVATIONS ON THE FOUNDATIONS

Amplitudes Vertical and horizontal amplitudes of vibration were measured at a number of points on the foundation, with the pertinent data as follows:

Maximum amplitude of horizontal vibrations at top of block in Y-direction 0.3156 and in z-direction = 0.1085 mm. Foundation was excited in free vibrations along χ-drictions and the natural frequency of free vibrations was observed to be 17.5Hz. (Fig. 2)

In-situ Dynamic Properties The dynamic properties of the soil used in the analysis of machine foundation may be determined by a number of laboratory or in-situ tests.

The most important parameters which affect these properties are (1) the mean effective confining pressure (2) the shear strain amplitude and (3) density in the soil. A good discussion on these corrections has been presented by (Nandakumaran and Puri (1977), Prakash and Puri (1977), Nandakumaran et al (1977), Prakash and Puri (1981) and Prakash (1981) and Indian Standard Code (IS 5249 - 1977).

In-situ Soil investigations consisted of (1) block vibration tests, (2) cyclic plate load tests and (3) standard penetration tests (Prakash et al 1975). Figure 3 shows a typical borelog of the area. From the cyclic plate load test data, values of dynamic shear modulus "G" were computed. From the uncorrected standard penetration (N) values shear wave velocity "Vs" at a particular depth was determined from equation 1 Ima (1977) and dynamic shear modulus "G" was computed from equation (2)

$$V_s = 91.0\, N^{0.337} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

$$G = V_s^2 \times \rho \quad \ldots\ldots\ldots\ldots\ldots (2)$$

in which $\rho = \frac{\gamma}{g}$ = mass density of soil. Values of "G" from different tests were corrected for (1) effective confinement in each case computed for an effective overburden pressure of 1 kg/cm^2 using equation 3.

$$\frac{G_1}{G_2} = \left(\frac{\bar{\sigma}_{v1}}{\bar{\sigma}_{v2}}\right)^{0.5} \qquad (3)$$

in which G_1 = shear modulus at an effective overburden pressure of $\bar{\sigma}_{v1}$

and G_2 = shear modulus at an effective overburden pressure of $\bar{\sigma}_{v2}$

The variation of G with strains from these tests is shown in Fig. 4 curve A. Fig. 4 also shows a plot of

$\frac{G}{G_{max}}$ vs. γ_θ obtained by dividing the ordinates of

G vs γ_θ plot at 4.0 m depth at different strains by the value of G_{max} curve B.

Selection of Soil Parameters Since the foundation is located at a depth of 2.4 m below ground level whereas the earlier tests had been conducted at depth of 4.0 m, following procedure was adopted to determine the values of G at this depth using the data at 4.0 m depth-

(1) Value of G_{max} at 2.4 m depth was computed using equations 1 and 2 and the N-value observed at that depth. (2) The value of G_{max} was corrected for effective overburden pressure and the value of G at an effective overburden pressure of 1 kg/cm^2 computed using equation 3.

The Values of G vs γ_θ (curve C) were obrained by multiplying G_{max} with ordinates of curve B (Fig. 4). This plot was subsequently used to determine the values of "G" at 2.4 m depth at the desired strain level for analysis of the foundation response.

The strain for two sets of computation (1) for design stage and (2) after monitoring the performance, and the corresponding soil properties were picked up as follows:

1. Design Stage.

Permissible amplitude in any mode $= 0.0125$ mm

Average width of the foundation $= 2104.5$ mm

Shear strain (γ_θ) (Prakash and Puri 1981) $= \frac{0.0125}{2104.5} = 5.94 \times 10^{-6}$

G at $\gamma_\theta = 5.94 \times 10^{-6}$ and $\bar{\sigma}_v = 1 kg/cm^2$ is $885\ kg/cm^2$ (Fig. 4 curve C)

Effective overburden pressure $\bar{\sigma}_v$ at a depth equal to one half of the width of the foundation given by

$$\bar{\sigma}_v = \bar{\sigma}_{v1} + \bar{\sigma}_{v2} \qquad (4)$$

in which $\bar{\sigma}_{v1}$ = Overburden due to weight of soil

and $\bar{\sigma}_{v2}$ = Vertical stress intensity at a depth equal to 1/2 width due to superimposed load of machine and foundation and may be computed using Boussinesq theory.

The value $\bar{\sigma}_v$ in this case was computed to be 0.8381 kg/cm^2. The effective value of $G = 885(\frac{0.9381}{1.0})^{1/2}$

$= 810\ kg/cm^2$

Value of the coefficient of elastic uniform compression "C_u" was computed from equations.

$$C_u = \frac{1.13 \times 2G(1+\nu)}{(1-\nu^2)} \cdot \frac{1}{\sqrt{A}} \qquad (5)$$

in which ν = Poissons ration (assumed 0.337).

The value of C_u is computed to be 10.25 kg/cm^3

(2) After monitoring performance.

Measured amplitude in y direction $= 0.3156$ mm

and in z direction $= 0.1089$ mm

Shear strain γ_θ induced in the soil $= \frac{0.3156 + 0.1089}{2104.5}$

$= 2.01 \times 10^{-4}$

Value of "G" corresponding to $\gamma_\theta = 2.01 \times 10^{-4}$ (Fig. 4) and effective confinement below the foundation

$= 415\ (\frac{0.8381}{1})\ 0.5$

$= 400\ kg/cm^2$

The corresponding value of C_u from eqn. 5 is computed t be 5.130 kg/cm^3.

PREDICTED RESPONSE OF THE FOUNDATION

The methods commonly used for the analysis and design o foundations for machines are (1) Barkan's approach and (2) Elastic half space approach. In the Barkan's appro (Barkan, 1962) the foundation soil system is represente as a springmass system, the spring stiffness due to the soil and mass of the foundation and supported equipment only are considered and intertia of the soil and dampin are neglected. In the elastic half space approach the vibrating footing is treated as resting on the surface of an elastic, semi-infinite, homogenous, istropic half space (Richart 1962). The elasticity of the soil and t energy carried into the half space by waves travelling away from the vibrating footing (geometric damping) are thus accounted for and the response of such a system ma be predicted using a mass-spring-dashpot model (Richart and Whitman 1967a Richart, Hall and Woods (1970).

The dynamic response of the foundation was computed usi both the above methods of analysis.

Barkan's Method

(a) Vertical vibrations: - Natural frequency of vertica vibrations ω_{nz} is given by

$$\omega_{nz} = \sqrt{\frac{Cu.A}{m}} = \sqrt{\frac{K_z}{m}} \qquad (6)$$

and amplitude of vertical vibration Az is given by

$$Az = \frac{P_z}{m(\omega^2 nz - \omega^2)} \qquad (7)$$

in which m = mass of the foundation and Kz = stiffness vertical soil spring.

ω = operating frequency. The computated values of natu frequency of vertical vibrations and amplitudes of vibr tions are listed in Table 1 line 1 and 7 respectively.

(b) Simultaenous rocking and sliding: Limiting natural frequency of the foundation in sliding ω_{nx} is given by

$$\omega_{n\phi} = \sqrt{\frac{C\tau\ A}{m}} = \sqrt{\frac{Kx}{m}} \qquad (8)$$

$C\tau$ = Coefficient of elastic uniform shear = 1/2 Cu

and limiting natural frequency in rocking $\omega_{n\phi}$ is given by

$$\omega_{n\phi} = \sqrt{\frac{C\phi.I}{M_{mo}}} = \sqrt{\frac{K_\phi}{M_{mo}}} \qquad (9)$$

in which
$C\phi$=Coefficient of elastic non-uniform compression = $2.C_u$

I = moment of inertia of the foundation contact area about the axis of rotation and
M_{mo} = Mass moment of inertia of the machine and foundation about the area of rotation.

$$M_{mo} = M_m + mz^2 \qquad (10)$$

in which M_m = Mass moment of inertia of the system about an axis through its centre of gravity for the appropriate direction of vibration. The two natural frequencies of the system ω_{n1} and ω_{n2} due to combined rocking and sliding are obtained in terms of ω_{nx} and $\omega_{n\phi}$ using equation 11.

$$\omega_n^4 - \frac{\omega_{nx}^2 + \omega_{n\phi}^2}{r}\omega_n^2 + \frac{\omega_{nx}^2 \omega_{n\phi}^2}{r} = 0 \qquad (11)$$

in where
$r = M_m/M_{mo}$

The amplitudes of vibration due to combined rocking and sliding due to an exciting moment are given by equations 12 and 13 Horizontal displacement.

$$Ax = \frac{C_\tau A.Z.\ M^X_{yz}}{\Delta(\omega^2)} \qquad (12)$$

Rotation. $$A_\phi = \frac{C_\tau A - m\omega^2}{\Delta(\omega^2)} M^X_{yz} \qquad (13)$$

where $\Delta(\omega^2) = m.M_m \ (\omega_{n_1}^2 - \omega^2)\ (\omega_{n_2}^2 - \omega^2) \qquad (14)$

Total horizontal displacement due to combined rocking and sliding A_x^* is given by equation (15).

$$A_x^* = A_x + h.A_\phi \qquad (15)$$

where h = height of block above the combined centre of gravity system.

Total vertical displacement due to vertical vibrations and rocking is given by equation (16).

$$A_z^* = A_z + L.A_\phi \qquad (16)$$

where L = distance of the point under consideration from the axis of rotation.

The natural frequency and amplitude of motion in yawing were also computed and all the values computed for different modes of vibration for 2-values of the shear strain in the soil are listed in Table I.

ELASTIC HALF SPACE METHOD
(a) Natural Frequencies: The natural frequencies of the foundation may be computed using equations 6,8,9, and 11. The soil spring stiffness for different modes of vibration may be computed as follows (Richart and Whitman (1967), Richart, Hall and Woods (1970)).

$$K_z = \frac{4Gr_o}{1-\nu} \qquad (17)$$

$$K_x = \frac{32(1-\nu)Gr_o}{7-8_\nu} \qquad (18)$$

$$K_\phi = \frac{8Gr_o^3}{3(1-\nu)} \qquad (19)$$

and $$K_\psi = \frac{16}{3}\ Gr_o^3 \qquad (20)$$

r_o = equivalent radius of the foundation and is given by eqns 21-23.

For vertical vibrations or sliding

$$r_o = \sqrt{\frac{A}{\pi}} \qquad (21)$$

For rocking

$$r_o = 4\sqrt{\frac{4I}{\pi}} \qquad (22)$$

For yawing

$$r_o = 4\sqrt{\frac{2\cdot I_{zz}}{\pi}} \qquad (23)$$

Computed values of the natural frequency for different modes of vibrations are listed in table 1.

(b) Amplitudes of Vibration:

$$A_z = \frac{P_z}{K_z\sqrt{(1-(\frac{\omega}{\omega_{nz}})^2)^2 + (2D_z\frac{\omega}{\omega_{n}2})^2}} \qquad (24)$$

where D_z = Damping ratio = $\frac{0.425}{\sqrt{B_z}}$ (25)

B_z = Modified mass ratio = $\frac{1-\nu}{4} \cdot \frac{W}{pr_o^3}$ (26)

Amplitudes in Rocking and Sliding

Damped amplitudes in sliding and rocking due to the exciting moment M^y_{xz} are given by equation 27 and 28 respectively

$$A_x = \frac{M^y_{xz}}{Mm} \cdot Z \cdot \sqrt{\frac{(\omega_{nx}^2)^2 + (2D_x\ \omega_{nx})^2}{\Delta\ (\omega^2)}} \qquad (27)$$

$$A_\phi = \frac{M^y_{xz}}{Mm} \cdot \sqrt{\frac{(\omega_{nx}^2)^2 + (2D_x \cdot \omega_{nx}\cdot\omega)^2}{\Delta\ (\omega^2)}} \qquad (28)$$

where

$$\Delta(\omega^2) = \left\{\left[\omega^4 - \omega^2\left(\frac{\omega^2_{n\phi} + \omega^2_{nx}}{r} - \frac{4D_x \cdot D_\phi\ \omega_{nx}\omega_{n\phi}}{\sqrt{r}}\right) + \frac{\omega_{nx}\omega_{n\phi}}{r}\right]^2 + \left[\frac{D_x\cdot\omega_{nx}\cdot\omega}{r}(\omega^2_{n\phi}-\omega^2) + \frac{D_\phi\cdot\omega_{n\phi}\cdot\omega(\omega^2{}_{nx}-\omega^2)}{\sqrt{r}}\right]^2\right\}^{\frac{1}{2}} \qquad (29)$$

D_x = Damping ratio in sliding

and D_ϕ = Damping ratio in rocking

$$D_x = \frac{0.288}{\sqrt{B_x}} \tag{30}$$

Where B_x = modified mass ratio = $\frac{7 - 8\nu}{32(1-\nu)} \cdot \frac{m}{\rho r o^3}$ (31)

$$D_\phi = \frac{0.15}{(1+B_\phi)\sqrt{B_\phi}} \tag{32}$$

in which B_ϕ = inertia ration = $\frac{3(1-\nu)}{8} \cdot \frac{I}{\rho r o^5}$

Total horizontal and vertical displacements may then be obtained using equations (15) and (16) respectively.

Amplitude in yawing may then similarly be computed (line 12 Table 1)

FREE VIBRATIONS

The values natural frequencies in x-direction for a sheer strain of 1×10^{-6} (free vibration condition) are as follows:

	Barkan's method	Elastic Half Space	Observed
f_{n2} (Hz)	14.75	13.01	
f_{n1} (Hz)	36.77	58.07	(17.5)

DISCUSSION AND CONCLUSIONS

1. The computed amplitude Ax* (line 10, cols. 3 and 4 for case I ($\gamma_\theta = 5.94 \times 10^{-6}$) is 0.1998 mm and 0.1091 mm respectively by Barkan's method and elastic half space approach against permissible amplitude of 0.0125 mm. Therefore, the foundation design needed revision. This highlights the necessity of proper design using realistic soil parameters in ensuring satisfactory performance of machine foundations.

2. The computed values of lower natural frequency "f_{n2}" in combined rocking and sliding (Line 5, cols. 5 & 6) for case II ($\gamma_\theta = 2.01 \times 10^{-4}$) are 5.93 and 7.03 Hz respectively, (frequency ratios of 0.88 and 1.09) respectively by Barkan's and elastic half space approach. These are too close to the operating frequency of 6.75. Hence large amplitudes should be anticipated, which actually have been observed to occur.

3. Vertical natural frequencies (line 1, cols. 3 & 4 and 5 & 6) show a remarkable agreement with each other. However, the natural frequencies in sliding and rocking (lines 4 & 5) differ from 19% to 31% from each other. The lowest natural frequency in horizontal free vibrations ($\gamma_\theta = 10^{-6}$) as computed by the Barkan's and elastic half space approach is 14.75 and 13.01 Hz. The percent error with respect to the smaller natural frequency is 16% and 26% respectively. Thus the computed natural frequency by Barkan's approach is closer to the observed frequency and the error by the elastic half space approach is large. No other published data is available on actual phototypes for comparison.

4. Amplitudes of vibration in the vertical direction A_z^* by Barkan's and elastic half space approach are 0.135 mm and 0.3031 respectively against the measured value of 0.1089 mm. Similarly in the case of horizontal vibrations the values of A_x^* are 0.4542 and 0.785 respectively against the measured value of 0.3156 mm.

The amplitudes computed using elastic half space model take into account the geometric damping. Even then these are higher than the undamped amplitudes by Barkan's approach and also much higher than the observed amplitudes. The observed amplitudes represent the overall effect of geometrical as well as material damping. Translational modes have a much higher geometrical damping associated with them compared to material damping but in rotational modes material damping may be significant since geometric damping is usually small. Therefore predicted amplitudes using half space model with geometric damping alone may be expected to be somewhat higher than observed amplitudes In the present case the difference is much larger than wha may be explained by inclusion of material damping in the system also. It was observed earlier (Richart and Whitman (1967b), that the half space approach generally do not agree with observed amplitudes and may be higher or lower than observed amplitudes and in some cases the difference may be as large as 100%. However, in this case, Barkan's approach predicts the amplitudes which are reasonably closer to observed amplitudes as compared with elastic half space approach. However a single set of data does not warrant a general conclusion.

5. There is an urgent necessity to monitor data on performance of machine foundations so that it may be possible to establish conclusively the superiority of one approach over the other in design of machine foundations. Such a data will be meaningful only if sufficient information on dynamic soil properties is also obtained.

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TABLE I. Computed Natural Frequencies and Amplitudes

S.No.	Quantity $\frac{f\ (Hz)}{A\ (mm)}$	$\gamma_\theta = 5.94 \times 10^{-6}$		$\gamma_\theta = 2.01 \times 10^{-4}$	
		Barkan's Method	Elastic Half Space Method	Barkan's Method	Elastic Half Space Method
1	2	3	4	5	6
1	f_{nz} Hz	19.06	19.05	13.48	13.43
2	f_{nx} Hz	13.74	17.28	9.53	12.18
3	$f_{n\phi}$ Hz	9.95	11.51	7.05	8.12
4	f_{n1} Hz	26.96	33.26	19.07	23.65
5	f_{n2} Hz	8.38	10.96	5.93	7.05
6	$f_{n\phi}$ Hz	16.28	24.30	11.53	17.12
7	A_z mm	0.0032	0.0031	0.0075	0.00686
8	A_n mm	0.133	0.0071	0.340	0.523
9	A_ϕ rad	6.16×10^{-5}	3.47×10^{-5}	1.049×10^{-4}	2.42×10^{-4}
10	A_x^* mm	0.1998	0.1091	0.4552	0.785
11	A_z^* mm	0.0787	0.06156	0.135	0.3031
12	A_ψ rad	2.78×10^{-6}	1.5×10^{-6}	7×10^{-6}	2.396×10^{-4}

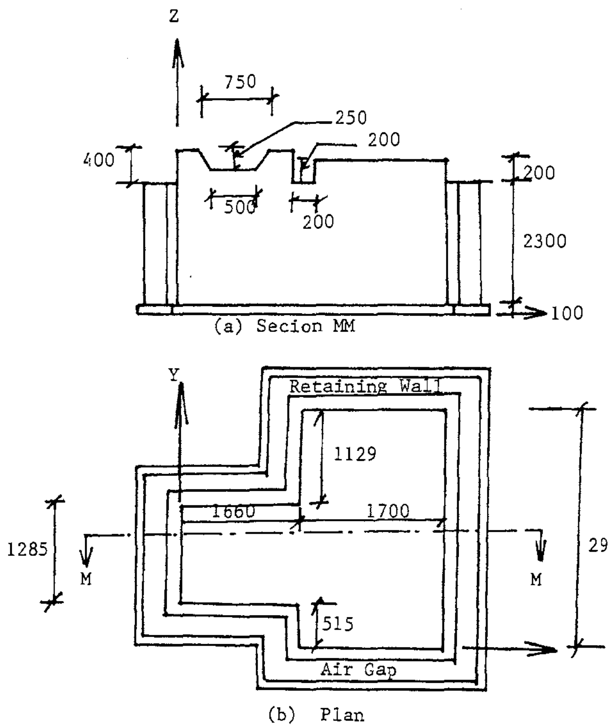

Fig. 1 Foundation Details

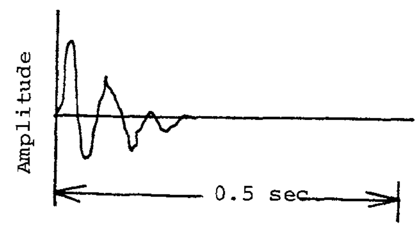

Fig. 2 Typical Free Vibration Reco:

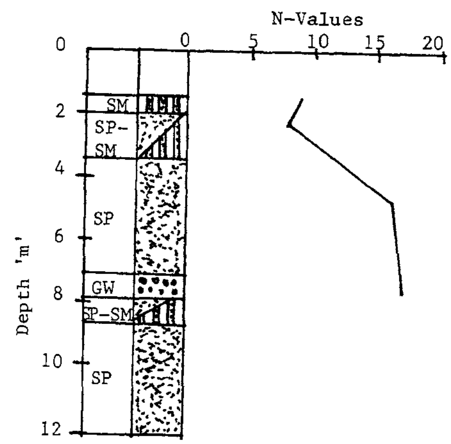

Fig. 3 Typical Borelog

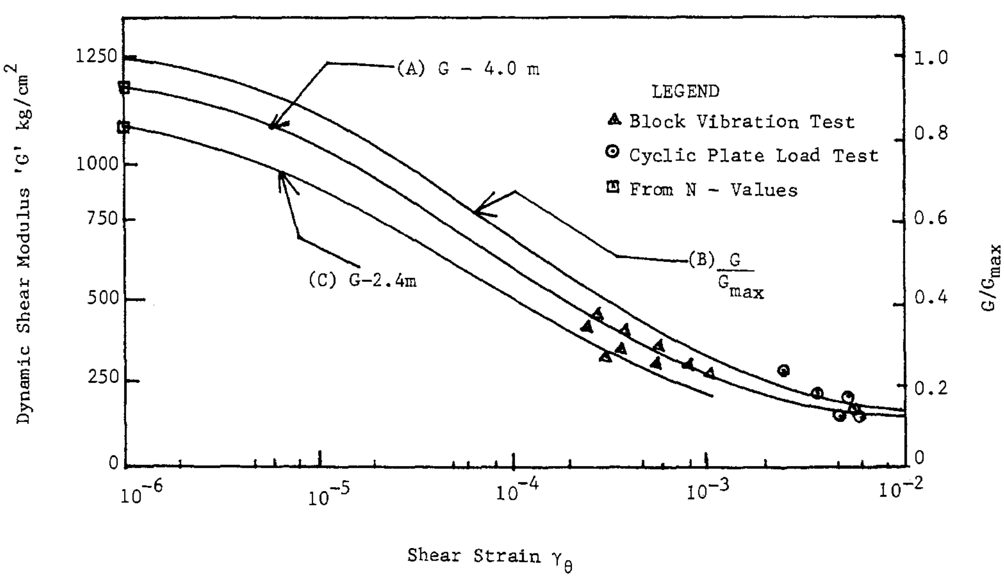

Fig. 4 G vs γ_θ and $\frac{G}{G_{max}}$ vs γ_θ