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Stiffness Change Effects on Structure Control Systems

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ABSTRACT

This paper contains the results of a computer analysis performed on a two-dimensional model structure, in which the structure response was found under dynamic loading conditions. A structural dynamics program (BASIC) was modified to perform its analysis on a two-dimensional structure with a control system modelled into it. The modified program was then used to determine whether a change in the stiffness of the structure would cause the control algorithm to be ineffective for the structure. In light of the limited scope of the analysis (two-dimensional model with constant applied force on one story), the results indicate that a control algorithm, which dampens the adverse response of the structure under dynamic loading, will continue to be effective if there is a measurable (25%) increase or decrease in the stiffness of the structure.

INTRODUCTION

The basic objective of this research project was to determine whether the control algorithm used to dampen the response of a structure under dynamic loading conditions would be effective if the stiffness of the structure was drastically changed. This problem was simplified by using a two-dimensional model structure and a BASIC program in performing the analysis.

In progressing from the conception of the project to its finish, three major tasks had to be undertaken. First, a working understanding of structural dynamics and the modelling of structural control systems had to be gained. This subject is not in the current curriculum. Second, the BASIC program written by Mario Paz had to be decoded and adapted to perform its analysis using a control algorithm, modelling a structure with a control system [1]. Hand calculations were performed to check the original and adapted programs. Third, a model structure was chosen and the stiffness analysis was performed.

The analysis was performed on a model structure of two stories, thus the scope of the project is limited. A two-dimensional analysis was performed because it displayed the concepts behind the analysis, without incurring the vast complexity involved in the analysis of higher dimensional structures. Simplifying conditions were established such that the results would be representative, but not past the scope of the time constraints.

THEORY

The analysis was based on structural dynamics with the structure modeled as a mass system with stiffness resisting the displacement of the stories. Viscous damping was neglected for this analysis (C=0). The time dependent force equations for the two-dimensional system are functions of the mass, m, stiffness, k, acceleration, $\ddot{x}$, and displacement, x, and are represented in Figure 1.

$$F_1 = m_1\ddot{x}_1 + k_1x_1 - k_2(x_2 - x_1) = 0$$

$$F_2 = m_2\ddot{x}_2 + k_2(x_2 - x_1) = 0$$

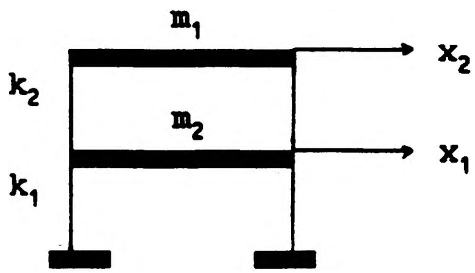

Figure 1. Model two-dimensional structure.

The control forces, U are added to the above force equations, at a short time increment, dt, after the initial forces are applied, as follows:

$$F_1'(t+dt) = F_1(t+dt) + U_1(t)$$

$$F_2'(t+dt) = F_2(t+dt) + U_2(t)$$

The control forces at one time increment are a function of the velocity, $\dot{x}$ caused by the forces at the previous time increment. They are also a function of the mass, m, time increment, dt, and the control algorithm inputs, Q and R. All Q's were set at unity so that only a scaled R value had to be input into the equation. All R's were set equal for this same reason. The control forces are as follows:

$$U_1(t) = (dt / 2)[Q / (R * m_1)] * x_1(t)$$

$$U_2(t) = (dt / 2)[Q / (R * m_2)] * x_2(t) - U_1(t)$$

A force applied to the second story adds another component to the force equation and causes the accelerations, velocities, and displacements, of which the control forces work to counteract.

METHODOLOGY

The project was divided into three phases. The first phase consisted of learning enough about structural dynamics and structural control systems to perform the project. This was accomplished by the study of references on the subject [1,2,3,4]. Reading and problem assignments were given by the faculty advisor. The faculty advisor monitored the progress and decided when the next phase of the project could be engaged.

The second phase of the project began with the task of decoding the BASIC structural analysis program[1]. This was accomplished by checking the results of a text problem performed using the program with hand calculations [1]. After an understanding of the program was gained, the source code had to be adapted to accept the use of a control algorithm for a two story mass structure with control on both floors. This too was checked upon completion by hand calculations.

The third phase of the project involved using a model structure, finding a control algorithm input value to maximize the dampening effects of the control, checking if the control worked for other forcing functions, and finally, performing the stiffness parameter study. An example problem was used in which the mass, stiffness, and damping matrices were known [1].

PROBLEM 18.1. Determine the response of a two-degree-of-freedom system excited by a constant force of magnitude 10 Kips and duration 0.3 seconds acting on the second coordinate. The stiffness, mass and damping matrices of the system are:

$$[K] = \begin{bmatrix} 75 & -44.3 \\ -44.3 & 44.3 \end{bmatrix} \text{(k/in)}$$

$$[M] = \begin{bmatrix} 0.136 & 0 \\ 0 & 0.066 \end{bmatrix} (\text{k} \cdot \text{sec}^2/\text{in}),$$

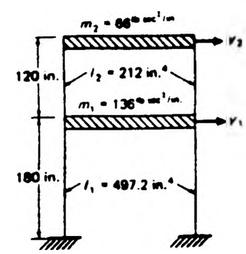

Figure 2. Example Problem.

A constant applied force was used for the forcing function in the determination of the control algorithm input value and the final stiffness parameter study. An optimum control algorithm input value was found by iteration. Increasing and decreasing forcing functions were used to make sure that the input value was optimum for forcing functions that were not constant. The stiffness parameter study involved the increasing and decreasing of the stiffness of the model structure by 5% increments up to 125% and down to 75% of the original stiffness.

RESULTS

The computer analysis results are in the form of a time history printout, showing for each time step and each story, the incremental force, control force, displacement, velocity, and acceleration response of the structure to the applied dynamic force. To find the optimum control algorithm input value, all five of the data outputs were compared to the output for the structure without control. Of the data, the forces developed at the different stories and the displacement of each story are the most important analysis results. The data is best presented by plotting, on the same graph, the output from the structure without control and the output from the structure with control versus time.

Figures 3. and 4. display the response forces, as a function of time, for story 1 and 2, respectively. These are the response forces developed in the structure under a constant applied force to story 2. Notice that the control dampens the adverse effects of the applied force and that story 2 developed larger forces than did story 1.

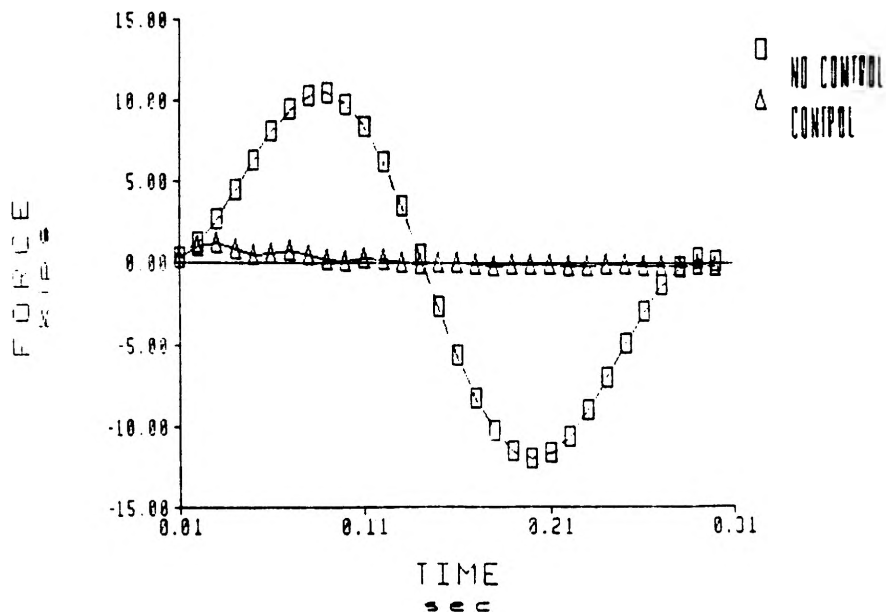

Figure 3. Response Forces of Story 1 Under a Constant Applied Force to Story 2, With and Without Control.

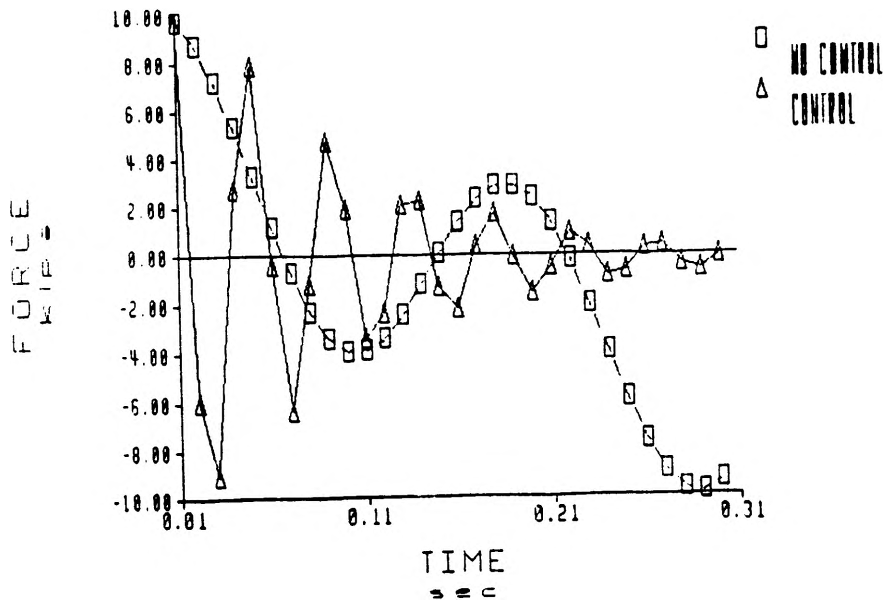

Figure 4. Response Forces of Story 2 Under a Constant Applied Force to Story 2, With and Without Control.

Figures 5. and 6. show the displacements, as a function of time, for story 1 and 2, respectively. These are the displacements corresponding to the application of a constant force to story 2. In comparison with Figure 3. and 4., the response forces and displacements follow the same shaped function of time. This is not true for the velocity and acceleration responses, however.

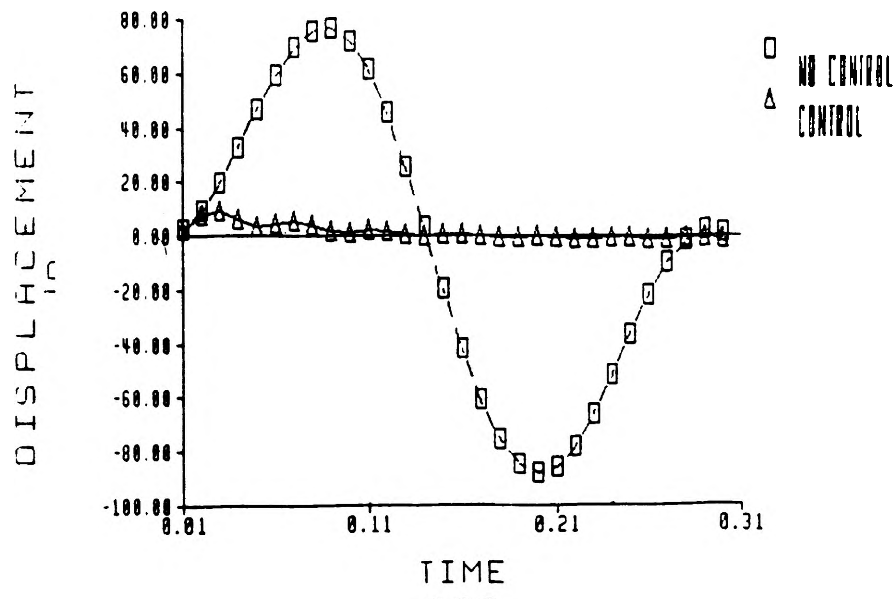

Figure 5. Displacement of Story 1 Under a Constant Applied Force to Story 2, With and Without Control.

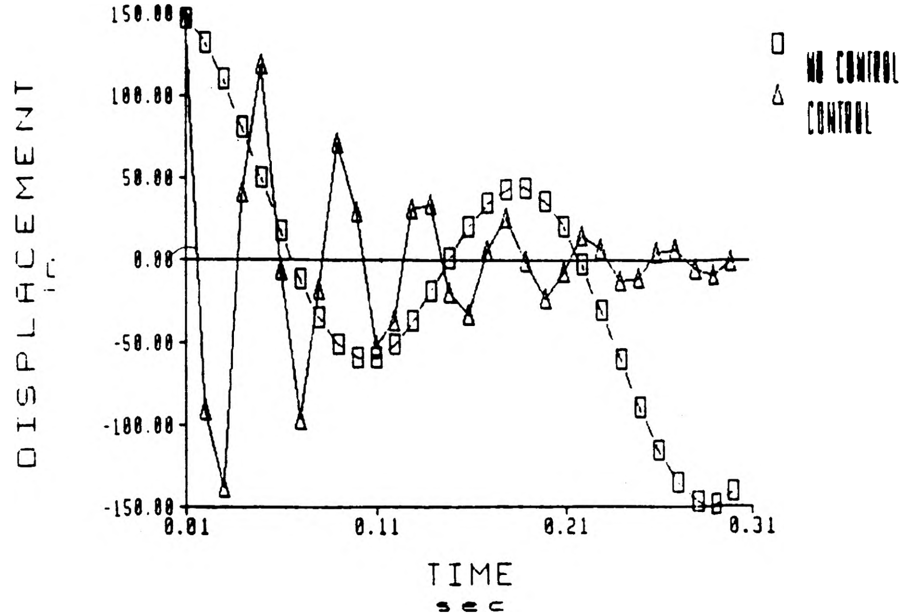

Figure 6. Displacement of Story 2 Under a Constant Applied Force to Story 2, With and Without Control.

Figure 7. displays the control forces of stories 1 and 2, propagated under a constant applied force on story 2. It can be seen that the control forces develop opposite to the response forces, as they should by definition.

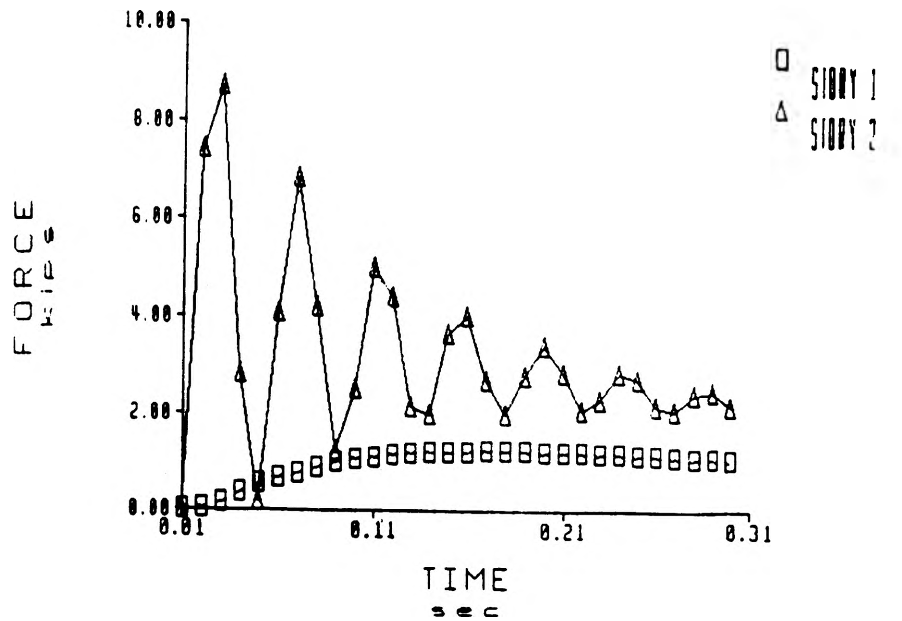

Figure 7. Control Forces of Stories 1 and 2 Under a Constant Applied Force.

The stiffness parameter study produced results of which the worst cases are presented in Figure 8. and 9. The response forces for a constant applied force on story 2, at 75, 100, and 125 percent of the original stiffness, are plotted as functions of time for story 1 and 2, respectfully. In that the displacements follow the same shaped curve, they have not been presented here.

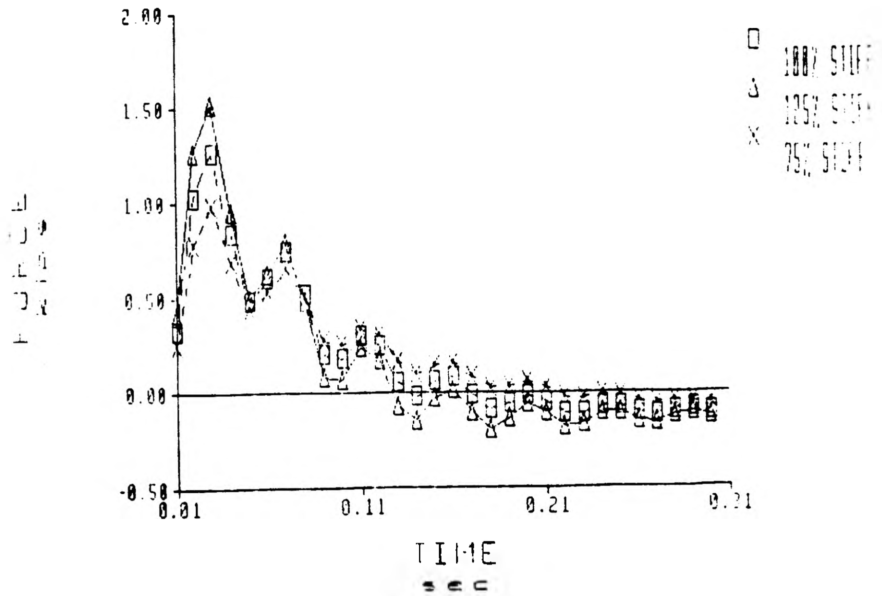

Figure 8. Response Forces of Story 1 Under a Constant Applied Force to Story 2, for 75, 100, and 125 Percent of the Original Stiffness.

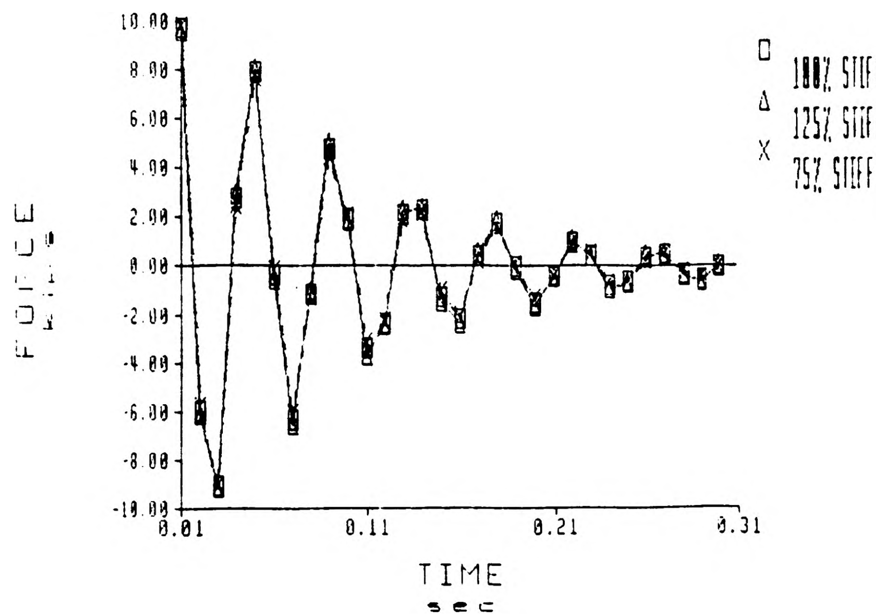

Figure 9. Response Forces of Story 2 Under a Constant Applied Force to Story 2, for 75, 100, and 125 Percent of the Original Stiffness.

CONCLUSIONS

The results of this project show that for a modelled structure, the control algorithm, once set for optimum control, will not be measurably effected by a change in the stiffness of the structure. Whether or not this holds true for real structures is beyond the scope of this project, but the fact that there are structures with control systems in operation today is an indication that it could be applicable. If the results of this project are pertinent to real structures, then those structures with control systems are not in danger of failure due to a change in stiffness causing the control system to break down. This is both legally and ethically important to the engineers designing and installing control systems in structures as well as the owners and occupants of those structures.

ACKNOWLEDGEMENTS

The faculty advisor for this project was former University of Missouri-Rolla professor of Civil Engineering, Dr. C.P. Pantelides. Dr. Pantelides has published papers on the subject of structures with control systems, and is knowledgeable in the field of structural dynamics [4]. Without his knowledge, ideas, and guidance this project would not have been made possible.

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