01 Jan 1969

A Simple Observational Technique for the Investigation of Boundary Layer Stability and Turbulence

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A SIMPLE OBSERVATIONAL TECHNIQUE FOR THE INVESTIGATION OF BOUNDARY-LAYER STABILITY AND TURBULENCE

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ABSTRACT

A dense suspension of aluminum flakes in a fluid has many advantages as a flow-visualization method, and has been used to study circular Couette flow and Benard instability, among others. In the present contribution the scientific and pedagogical merits of the technique are examined further for three different cases: (a) Stability of Poiseuille flow; (b) Stability of the side-wall boundary layers created when a fluid-filled, rotating container is suddenly brought to rest; (b) Stability of the boundary layer beneath a concentrated vortex.

The technique is also useful in other cases, especially when fundamentals of fluid flow are to be demonstrated in the classroom. Several of these further possibilities are briefly discussed.

INTRODUCTION

Schultz-Grunow and Hein[1] and Coles,[2] in their studies of circular Couette flow[+], suspended small aluminum flakes in oil and used them to observe the conditions at which the flow became unstable. The technique has also been used[3] to investigate Benard instability* and in fact forms the basis of a toy[4] which produces aesthetically pleasing patterns when the fluid is agitated thermally, or mechanically.

It is ideal when the flow is unstable in a stationary sense, i.e., instability leads to a new, steady laminar flow; although it has been extended beyond this point by Coles[2] to study wave motions and spiral turbulence in Couette flow.

The present contribution examines both the scientific and pedagogical merits of the method by considering three more cases in detail, and by suggesting flow systems which would benefit from using aluminum-flakes to observe the flow.

The physical basis for the technique is somewhat obscure; it is usually stated that the particles** align themselves in a shear flow so that light reflected from them makes them look different from their, non-oriented, neighbors. Study of the available literature[5] shows that this is only partly true and that the real mechanism is considerably more complicated.

When the fluid is at rest, as a whole, the particles are in Brownian motion with random orientations. Since they are relatively massive their "rotational diffusion rate" is low,[5] and the fluid has the appearance of a flat, matte, silver-painted surface. If the flow is in steady, laminar motion its appearance is similar except that particles in laminar shear, e.g. close to a solid wall, are translating at the local convective velocity and rotating. Solutions to ideal models of such flows show there is a high probability that the particles take up a preferred orientation in the shear flow. They always rotate but spend a longer time aligned with the shear than with a long dimension perpendicular to the shear. For the probability of being aligned with the shear to be large, the ratio of velocity gradient to rotational-diffusion coefficient must be large. Since for massive particles in Brownian motion the denominator of this ratio is small, the ratio is large even for small velocity gradients. Thus even a small localized shear will preferentially orient the particles and cause that region to appear different from its surroundings. Flows which form stationary instabilities[2] are particularly well suited to this method. Flows with instabilities in the form of travelling waves, e.g. Tollmein-Schlichting waves, can also be observed (as shown in Experiment III) because the shear perturbations associated with the turbulent eddies (or interacting wave fields) orient particles the most where shear is highest.

During the preceding discussion it has been tacitly assumed that the particles can always follow the local, translational motions of the fluid. Numerous studies show that this is a very good assumption in the present cases where the accelerations and the particle inertia are small.

Three cases, which illustrate each type of flow, are studied in the following pages. We start with a flow which is familiar to everyone and which illustrates the power of the technique in a well understood situation. We then progress to flows which are less well understood and on which the method can shed some new light. The final section contains suggestions for future use by those interested in experiments for classroom demonstration.

EXPERIMENT I: THE STABILITY OF POISEUILLE FLOW

Since Reynolds first performed his famous stability experiment in a circular pipe there have been few improvements in the methods used to observe the flow. Pressure gradient measurements have been made in large numbers and some effort using hot wires has proved useful. Lindgren[9] has used a method similar to the present one, using a fluid which exhibits streaming birefrigence; which, of course, requires that long-chain molecules become aligned with the shear. However, an apparatus to produce polarized light is required and, if a long test section is to be viewed, considerable expense results. The present method has all of the advantages and few of the disadvantages of Lindgren's technique. An existing pipe flow experiment was modified by incorporating a rotating test section which was used to observe a variety of flows. The fluid used was water; the particles were first mixed with Kodak Photo-Flo solution so that they would readily form a suspension.*

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[+]The flow between rotating, co-axial, circular cylinders.

*The instability which results when a thin, horizontal layer of fluid is heated from below and cooled from above.

**Particles, supplied by paint manufacturers, are flake-like, i.e., have a thickness small compared to length and breadth.

*It has recently come to the author's attention that the aluminum flake technique was also used, independently, by Cannon & Kays[7] to study pipeflow with and without pipe rotation.

Non-rotating, laminar flow, at low Reynolds number (R), exhibited a typical mattesilver appearance which, because of a poorly designed rotating seal at the upstream end of the test section, was occasionally disturbed by a decaying patch of turbulence. This disturbance, although initially viewed with horror, was actually very useful. As the transition R was approached, one of two things happened to it. It either decayed, if below a certain amplitude, or it grew into a turbulent slug, if above a certain amplitude. One could readily conclude that such a flow was only unstable to finite-amplitude disturbances. The R at which this growth first occurred was 2800.

The turbulent slugs which were formed were quite fascinating to watch and contained the expected features. Turbulence was rapidly formed within a central core at the rear of the slug. It progressed slowly towards the front of the slug, where it was dissipated and the flow became laminar again through a very long cone-shaped region of small included angle (typically 7 or 8 degrees). Since Lindgren[9] has discussed slug-growth rates, shape etc. as they vary with R, the same measurements were not repeated here.

Fully turbulent flows showed the randomly fluctuating motions typical of such flows. Photographic information is of little use under such circumstances and one is forced to use more sophisticated techniques either viewing light fluctuations with photocells[2] using hot wires[10] or a Laser-Doppler type of velocity measuring device.[11]

EXPERIMENT II: THE STABILITY OF SIDE-WALL BOUNDARY LAYERS IN NON-LINEAR SPINDOWN

Linear spin-up and spin-down*, in containers of many shapes, have been studied in profuse detail. For example, if the angular velocity of a rotating, cylindrical container filled with a viscous fluid is slightly changed, interior fluid is spun up or down by the convection of angular momentum caused by Ekman layer suction. At the same time a side-wall boundary layer is formed (since there must be a region to match the difference in angular velocity between the interior and the container). Vorticity diffuses into the interior at a rate proportional to $(\nu t)^{\frac{1}{2}}$, so that during the spin-up time, $E^{-\frac{1}{2}}\Omega^{-1}$, the layer comes to have a thickness $E^{+\frac{1}{4}}$ a. Non-linear spin-up and down have also been treated when the velocity increment is not too large. On the other hand, if the change in velocity is very large, so that $\Delta\Omega/\Omega \sim O(1)$, then new phenomena appear. Spin-down exhibits violent instabilities within the side wall layer and Ekman layers. Spin-up is better behaved and only the Ekman layers become unstable. Ekman layer instability has been actively studied[14] but stability of the side-wall layer has only received cursory treatment.[12]

In spin-down the interior fluid is rotating faster than the container; negative relative vorticity diffuses inwards and the square of the circulation decreases outwards. This, of course, is just the requirement for Rayleigh's stability criterior to be satisfied. Thus the instability is of the type studied many years ago by Taylor[15].

In this case it is more appropriate to look at the related problem formulated by Gortler[16] who considered the stability of boundary-layer flow over a concave wall. The stability criterion found was that:

$$G = [U_o\theta/\nu][\theta/r]^{\frac{1}{2}} > G_{crit.} \quad \text{for instability.} \qquad (1)$$

In the present case $U_o = \Omega a$, $r = z$ and $\theta = C\,(\nu t)^{\frac{1}{2}}$*, where we have assumed that the container is brought to rest from an initial angular velocity Ω. We can show, a posteriori, that instability occurs so quickly that the interior fluid does not have time to spin-down by the usual mechanism and that the velocity exterior to the layer is Ωa during this time. Substitution into (1) gives an estimate of the time taken for the instability to first appear:

$$t_i = C^2 G_{crit.}^{2/3} \nu^{1/3} a^{-2/3} \Omega^{-4/3} = k\Omega^{-4/3} \qquad (2)$$

Experiments were performed by mounting a laboratory beaker on a turntable; rotating it, at a given Ω, until the fluid was in solid body rotation; suddenly stopping the turntable and timing the interval to the first appearance of instability. The results, shown in figure 1, agree with (2) for small angular velocities but not at large values. Unfortunately turntable inertia prevented an instantaneous stopping of the beaker. For large Ω, the stopping time and t_i were of the same order which explains the discrepancy in this range.

From the experiment k = 4.5 so that $G_{crit.}$ = 25.8. Liepmann[17] obtained a value of 9 in a wind tunnel experiment. At instability the boundary layer has a thickness

$$\theta = C(\nu t_i)^{\frac{1}{2}}$$

or (3)

$$\theta = G_{crit.}^{2/3} \nu^{2/3} a^{-1/3} \Omega^{-2/3} = K\Omega^{-2/3}$$

If we assumed that the wavelength of the most unstable disturbance is given by $\lambda = \alpha\theta$ then

$$\lambda = \alpha K\Omega^{-2/3} \qquad (4)$$

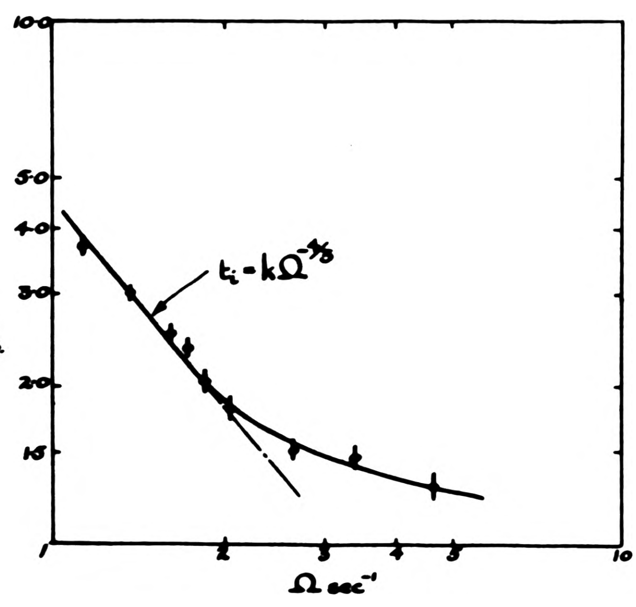

Fig. 1 Time elapsed from stopping of the rotating tank to appearance of instability (t_i) versus initial rotation rate (Ω).

*Obviously in spin-up the container velocity increases, in spin-down it decreases.

*$C = 2/\pi^{\frac{1}{2}}$ for a Rayleigh type of boundary layer.

From photographs (fig. 2) we can estimate λ. The measurements are plotted in fig. 3, whence $\lambda = 1.76\ \Omega^{-2/3}$. Note that the cell size is less affected by the inability of the turntable to stop suddenly. Since $G_{crit.}$ is known we find that $\alpha = 15.1$, and the cell size is much greater than the boundary-layer momentum thickness.

Development of the instability beyond its initial stages can also be observed (fig. 4). Velocities within the original cells increase in intensity until suddenly the wavelength of the cells increases. This process is repeated several times until the final wavelength is several times the original one, and the disturbance has penetrated far into the interior of the fluid. In figure 5 is plotted the penetration distance as a function of initial rotation rate, and it can be seen that it is almost one order of magnitude larger than predicted by linear theory and increases with increasing Ω instead of decreasing. An explanation of the phenomena which contains the basic physics - but is incorrect in details - is as follows: Once the initial instability has begun the layer near the wall is very effectively mixed and the value of θ for this region rapidly increases. Soon the value of θ approaches the value of λ and the flow adjusts itself to a new wavelength which is again large compared to θ.* This process is repeated several times until the layer has become so thick and the outer velocity so low that the effective value of

*One useful way of thinking about this process is to imagine ordinary Couette flow between circular cylinders and allowing the radius of the inner cylinder to decrease with time and fluid to pass through the inner wall at the same time.

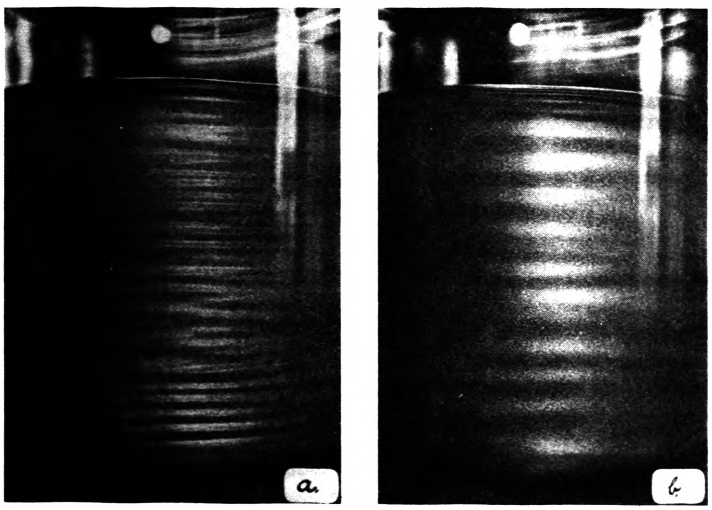

Fig. 2 Photographs of the initial instability for (a)Ω = 4.65 sec^{-1}, (b)Ω = 1.13 sec^{-1}.

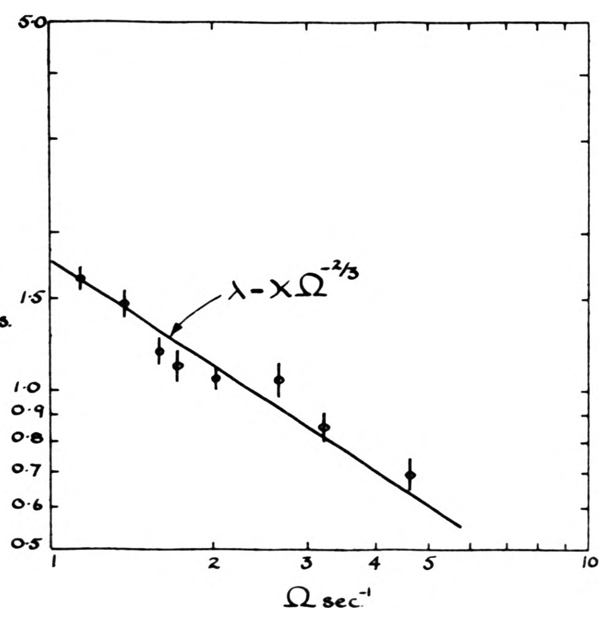

Fig. 3 Wavelength of initial instability (λ) versus Ω.

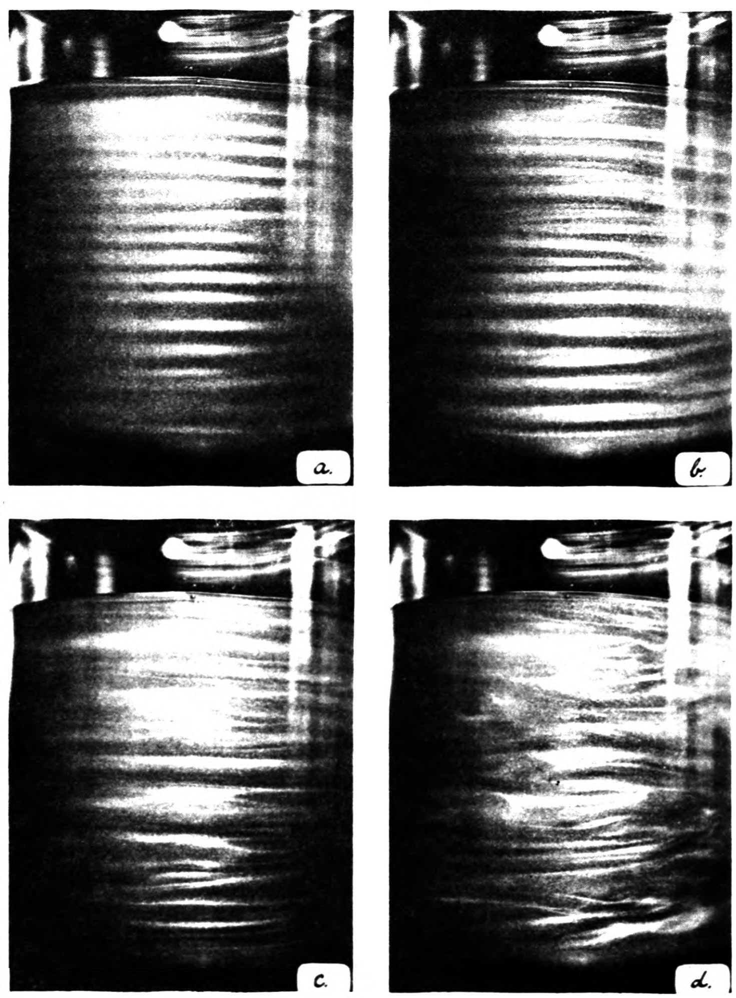

Fig. 4 Photographs of the development of the instability for Ω = 2.64 sec^{-1}.

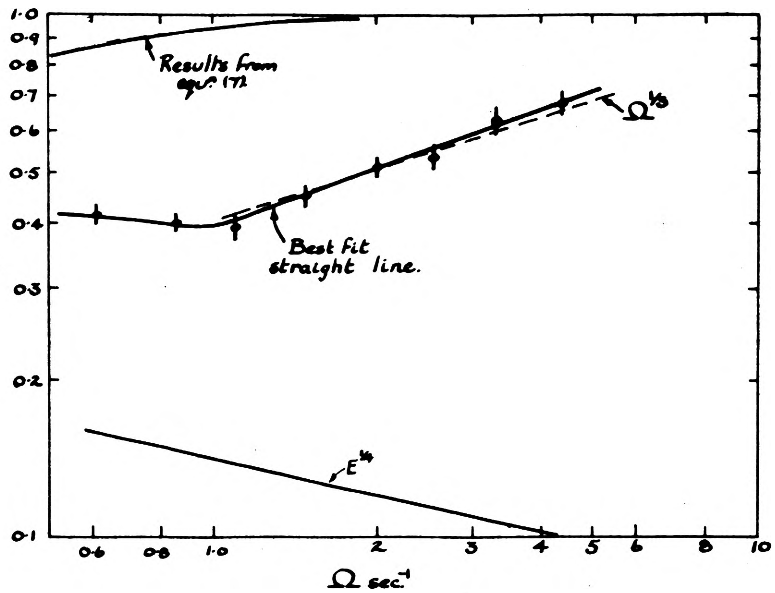

Fig. 5 Penetration distance of final unstable flow into interior of the container $(a - r_p)/a$ versus Ω.

G is less than $G_{crit.}$ and the flow remains at its final wavelength. The outer velocity, given by Ωr_p, is decreasing for two reasons, one; the interior fluid is being spun-down by vortex line contraction, i.e. Ω is decreasing, and two; r_p is decreasing as the layer penetrates further into the interior.

Estimated values resulting from this process can be calculated in the following way:

Assume the cells to be approximately square so that

$$(a-r_p) = \lambda/2 = \alpha\ \theta/2 \qquad (5)$$

At the neutral stability boundary

$$[\Omega r_p \theta/\nu][\theta/a]^{\frac{1}{2}} = G_{crit.} \qquad (6)$$

Substituting for θ, from (5), results in an equation for $(a-r_p)$ which is:

$$a(a-r_p)^3 + (a-r_p)^5/a - 2(a-r_p)^4 = [G_{crit.}\nu^2\alpha^3][8\Omega^2]^{-1} \qquad (7)$$

Solution to this equation are also shown in figure 5, where it is assumed that Ω remains constant. It is obvious that the simple theory does not describe the finite-amplitude stability of layers which are of the same order of thickness as the radius of curvature of the wall. Even allowing Ω to decrease considerably does not help. It seems as though the flow does in fact recross a stability boundary and become stable again but the details are impossible to predict.

EXPERIMENT III: THE STABILITY OF THE BOUNDARY LAYER BENEATH A CONCENTRATED VORTEX

The last experiment represents a small part of a continuing attack on the problem of the flows within concentrated vortices. A similarity to hurricane and tornado-like processes will, hopefully, become evident as the discussion proceeds.

The apparatus consisted of a stationary, 3-ft. diameter, Lucite tank, within which a vortex flow was produced by injecting fluid tangentially at the periphery and withdrawing it at the center. Based on exploratory experiments the central suction device was made in the form of an annulus; 9 in. in outside diameter and 1 in. wide. This, in essence, simulated the form of interior, verticle flow known to occur in hurricanes and gave some clues to the dynamics of formation of the "eye" region of a mature hurricane. The top surface of the fluid was free and the aluminum flakes were viewed from below through the thick, Lucite bottom plate. The only independent variable in the present experiment was the flow rate into (and out of) the device.

Turner[18] has, independently, performed an experiment with a similar, annular type of driving mechanism and presents comments on the current knowledge of hurricaneprocesses and the need to perform laboratory experiments under controlled conditions.

Details of the overall flow field are similar to those which have been presented before for tornado-like flows[19] so that only the essential features will be presented here. Five distinct regions of flow can be seen (fig. 6). A region (I) within which the flow is almost circular and free vortex-like

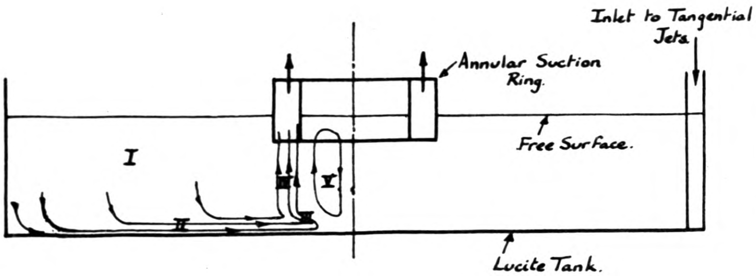

Fig. 6 Sketch of azimuthal-flow streamlines for vortex flow into an annular suction tube.

with weak radial and axial flows to feed fluid into a boundary layer region (II) on the bottom plate. Since the flow exterior to the boundary layer is moving in circles, there is a radial pressure gradient impressed on the layer and an intense radial inflow is formed. Because the outer flow is like a free vortex the total mass flow in the layer increases with decreasing radius.[20] The increase in flux comes from axial flow into the layer. This flux continues to increase until at some radius it rapidly turns and forms an annular vortex jump (III) which matches the boundary layer flow to the vertical flux (IV) into the suction annulus. In the present case the vortex jump leaves a circular region in the center within which there is little motion(V). This region is bounded by the annular region (IV) which carries the majority of the upward, vertical mass flux and across which there is a rapid change in swirl velocity.

In the present case it is the stability of the lower boundary layer which is of primary interest. Figure 7 shows the form of the waves of instability. The exterior flow is moving counter-clockwise in this photograph. Two features are immediately obvious; namely the waves, which are propagating inwards, and the central stagnant region.

From their orientation to the basic flow and general appearance the waves are evidently the Class I type of Faller (Class B of Greenspan).[12] Measured angles between the waves and a circle vary from 16° to 18°.* Previous measurements in rotating tanks gave values of approximately $14\frac{1}{2}^{\circ}$.[21] By conventional definition, the Rossby number** of the present experiment is infinite, so that detailed comparison to previous work is difficult and has not yet been attempted. A major disadvantage of the present method is that it gives little detail about the exterior, free-vortex flow and must be coupled with other techniques to yield useful information. Efforts are currently underway to measure the nature of these flows in more detail using more quantitative techniques.

One further observation is useful in deciding the possible role of this type of wave in atmospheric flows. Close viewing of the motion of individual particles, as the wave travels past them, shows that some are ejected vertically upwards into the exterior flow producing considerable disturbance there. Detailed measurements of this situation should show how moisture laden air in the boundary layer beneath a hurricane could be ejected into the exterior flow to produce active cumulus convection on a spiral path.

Fig. 7 Photograph of the unstable boundary layer beneath a concentrated vortex. Note spiral waves of instability and the central, non-rotating region.

FURTHER USES FOR THE ALUMINUM-FLAKE TECHNIQUE

Many more uses for the method jump to mind. In a two-dimensional channel one could observe the stability of the flat-plate boundary layer, demonstrating the development of Tollmien-Schlichting waves and the production of turbulent spots. If the channel is narrow enough, the stability of plane Poiseuille flow to infinitesimal disturbances could be shown. By varying wall angles, stability of boundary layer flows with favorable and adverse pressure gradients could be presented with great clarity in a classroom situation. If the channel were curved, to be concave to the flow, then spatially varying longitudinal vortices, analogous to the temporally varying ones of experiment III, could be observed.

*Significantly, the angles of spiral rain bands in some hurricanes are of approximately these magnitudes.

**Rossby number is a dimensionless quantity which measures the importance of inertia forces to Coriolis forces in rotating fluid systems.

Other processes which might be interesting include the stability of wall jets, of natural convection on a vertical flat or curved plate, and of stagnation point flow.

SYMBOLS

a	Radius of the beaker (Expt. II).
c	Constant in boundary layer growth rate formula (II).
d	Diameter of tube (I).
$E = \nu/\Omega a^2$	Ekman number (II).
G and $G_{crit.}$	Görtler number defined by eq. (1) (II).
R	Reynolds number $U_m d/\nu$ (I).
r	Radial distance (II).
r_p	Radial distance to point of penetration of sidewall layers (II).
t	Time from stopping of rotating tank (II).
t_i	Time from stopping of tank to appearance of instability (II).
U_o	Velocity exterior to boundary layer (II).
U_m	Mean velocity in pipe (I).
α	Constant of proportionality between θ and λ (II).
λ	Wavelength of unstable waves (II).
ν	Kinematic viscosity.
Ω	Rotation rate (II).
$\Delta\Omega$	Change in rotation rate from its initial value Ω (II).
θ	Boundary layer momentum thickness (II).
k and K	Constants of proportionality (II).

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