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ABSTRACT

Computational instability may occur in the mathematical simulation of hydrocarbon reservoirs when small inaccuracies in the calculated pressures cause loss of diagonal dominance in the matrix of coefficients. The problem can be resolved by the use of a more precise technique for computation of pressures. However, this stability problem is still troublesome to users of certain types of reservoir simulators. A computational technique which has been found effective in resolving the problem is presented.

INTRODUCTION

The technology of petroleum reservoir engineering was initially based on empiricism and rule-of-thumb generalizations. Reservoir productivity was first predicted almost entirely on the basis of performance of similar reservoirs. This approach was generally satisfactory for estimating the productivity of new oil fields which did not differ markedly from older reservoirs. However, the empirical approach was not adequate for solving any reservoir engineering problems which were substantially outside the realm of past experience. Thus it was soon recognized that new and more versatile techniques were needed for predicting the performance of hydrocarbon reservoirs.

The next significant development in petroleum reservoir engineering might be considered to be the concept of the material balance and the numerous prediction methods which were derived from this concept. Development of these prediction methods was significant, since they provided a means for solving reservoir engineering problems which could not be handled on the basis of past experience with similar reservoirs. However, since these techniques were devised before the advent of the digital computer, it was necessary that certain assumptions be made in order to limit the amount of computation required. Therefore, all of these early material balance methods were formulated with the assumption that rock and fluid properties would not vary from point to point in the reservoir. Similarly, pressures and saturations were generally handled as average values throughout large segments of the hydrocarbon deposit. Although the use of these simplifying assumptions may give realistic results for some hydrocarbon reservoirs, they can also lead to serious errors in the prediction of reservoir performance. Therefore, these early material balance methods have now been largely supplanted by more advanced techniques.

The digital computer has now become the primary tool for modern reservoir engineering work. This type of computer has the capability not only of performing extremely rapid calculations but also of handling very large sets of data. Thus, it is now feasible to calculate the performance of a petroleum reservoir with rock and fluid properties considered to vary with pressure and position, and with saturations and pressures described by gradients which are functions of position and production history. The calculations are too complex to be performed by hand; therefore, they are incorporated in a computer program which is generally called a mathematical simulation model.

The mathematical relationships which comprise the reservoir simulation model include both algebraic equations and partial differential equations (some of which may have variable coefficients). For most problems which are of practical interest, the set of equations is too complex for solution by any known analytical technique. Therefore, the differential equations are converted to finite difference equations so that numerical solution techniques can be employed. These differential equations are written for each element of a reference grid which is used to describe the system under study. This procedure yields a system of equations which can be solved simultaneously by a suitable numerical technique. The convergence properties of some numerical solution techniques currently in use are such that stability problems can arise during a simulation study. One such problem, which leads to instability when the pressure is near the bubble point, is a reversal in algebraic sign of the total compressibility of the rock-fluid system. This is a computational phenomenon and not an actual field occurrence. However, it should be corrected to prevent the generation of computational instabilities which may cause the model to yield erroneous results. The purpose of this paper is to present a method for correcting this compressibility problem.

PRESSURE EQUATION

The pressure equation is basic to any mathematical model of reservoir behavior. The complexity of this relationship, which describes pressure as a function of position and time, can vary considerably. For a one-dimensional, one-phase slightly compressible system the equation may be written*

$$\frac{\partial^2 P}{\partial x^2} = \frac{\phi \mu c}{k} \frac{\partial P}{\partial t} \qquad (1)$$

* Symbols are defined in the Nomenclature.

The solution of equation (1) causes no stability problems because the equation contains constant coefficients of the pressure derivatives.

A more realistic pressure equation for a hydrocarbon reservoir is given by

$$(B_o - B_g R_s)\nabla\cdot T_o \nabla\Phi_o + B_g \nabla\cdot T_o R_s \nabla\Phi_o + B_g \nabla\cdot T_g \nabla\Phi_g + B_w \nabla\cdot T_w \nabla\Phi_w = h\phi(c_r + S_w c_w - \frac{S_o}{B_o}\frac{dB_o}{dP} - \frac{S_g}{B_g}\frac{dB_g}{dP} + \frac{S_o B_g}{B_o}\frac{dR_s}{dP})\frac{\partial P}{\partial t} + q' \quad (2)$$

This mathematical relationship accounts for variable rock and fluid properties for three mobile fluid phases, and for two- or three-dimensional flow. The nonlinearity of equation (2) requires that special solution techniques be employed in order to maintain stability of the computations. This equation is the basic component of the model in which the compressibility sign reversal problem was studied.

When equation (2) is written as a finite difference, the following result is obtained:

$$(A_{i,j}P_{i-1,j} + B_{i,j}P_{i+1,j} + C_{i,j}P_{i,j} + D_{i,j}P_{i,j-1} + E_{i,j}P_{i,j+1})^{n+1} = F_{i,j} \quad (3)$$

where the superscript denotes the n + 1 time level, and the subscripts i and j identify the x and y coordinate positions, respectively.

The coefficients A, B, D, and E which appear in equation (3) are composite terms containing transmissibilities and material balance terms. These are defined in the Appendix. The term F and the coefficient C are defined below since they contain the overall compressibility term and must be adjusted in order to correct the compressibility problem.

$$F_{i,j} = -G_{i,j}P^n_{i,j} + H_{i,j} \quad (4)$$

$$G_{i,j} = (\frac{h\phi}{\Delta t})_{i,j}(c_r + S_w c_w - \frac{S_o}{B_o}\frac{dB_o}{dP} - \frac{S_g}{B_g}\frac{dB_g}{dP} + \frac{S_o B_g}{B_o}\frac{dR_s}{dP})_{i,j} \quad (5)$$

and

$$C_{i,j} = -A_{i,j} - B_{i,j} - D_{i,j} - E_{i,j} - G_{i,j} \quad (6)$$

The authors customarily describe the fluid properties employed in these equations by means of polynomial expressions. Special attention must be given to maintenance of accuracy of these expressions near the bubble point pressure. Derivatives of the fluid properties are computed by differentiation of the polynomials.

COMPRESSIBILITY REVERSAL PROBLEM

The compressibility sign reversal problem often arises in simulation models which do not use computational techniques which calculate pressures to a high degree of accuracy. Small errors in the pressure computations will be strongly reflected in the saturation calculations and in loss of material balance accuracy. Although these errors may be too small to be of practical importance in predicting reservoir pressure, they may still cause computational instability at pressures near the bubble point.

The compressibility problem will occur when errors in the computation cause the overall compressibility term $G_{i,j}$ to be calculated with an incorrect algebraic sign. This condition causes the loss of diagonal dominance in some rows of the matrix of coefficients which is derived from equation (3).

The requirement for maintaining diagonal dominance in the matrix of coefficients is that for all i and j

$$|C_{i,j}| \geq |A_{i,j}| + |B_{i,j}| + |D_{i,j}| + |E_{i,j}| \quad (7)$$

This relationship is not a necessary criterion for convergence. However, loss of diagonal dominance often requires a substantial reduction in time step size for maintenance of computational stability. Thus the retention of diagonal dominance is desirable from a practical standpoint.

COMPRESSIBILITY REVERSAL CORRECTION

In order to overcome the compressibility reversal problem it is necessary to restore the loss of diagonal dominance which results from a negative value of the compressibility term $G_{i,j}$. This negative term reduces $C_{i,j}$ in accordance with equation (6), so that inequality (7) is not satisfied.

In order to illustrate how the compressibility reversal problem can arise, it is useful to examine the algebraic sign of the various pressure coefficients. The coefficients $A_{i,j}$, $B_{i,j}$, $D_{i,j}$, and $E_{i,j}$ are always positive numbers. Normally, $G_{i,j}$ is also positive and inequality (7) is satisfied. In particular, the quantity $G_{i,j}$ will not become negative above the bubble point pressure since $G_{i,j}$ will contain no negative terms. However, at pressures below the bubble point, the quantity $(-dB_o/dP)$ will always be negative since gas evolution causes the reservoir oil to shrink when the pressure is reduced. This existence of a negative term in $G_{i,j}$ suggests that it can become negative. In particular, $G_{i,j} < 0$

when $\frac{S_o}{B_o}\frac{dB_o}{dP} > (c_r + c_w S_w - \frac{S_g}{B_g}\frac{dB_g}{dP} + \frac{S_o B_g}{B_o}\frac{dR_s}{dP})$ (8)

This condition is especially likely to occur for reservoir fluids that have a high bubble point pressure. Since B_g is small at high pressures, the magnitude of B_o' may exceed the product $B_g R_s'$ for these fluids. Then $S_g B_g'/B_g$ should offset the negative terms in the total compressibility expression, since rock and water compressibilities are relatively insignificant below the bubble point. However, inaccuracy in the material balance may prevent computed gas saturation from increasing as rapidly as it should, and thus allow the negative value of B_o' to assume undue significance in the total compressibility term at pressures slightly below the bubble point.

Occurrence of the compressibility reversal phenomenon frequently coincides with the use of a time step which encompasses the transition from pressures above the bubble point to pressures below the bubble point. The problem seldom persists for more than a few time steps. However, when the problem does occur it may cause the calculation to become unstable.

The problem can be resolved by writing equation (3) in a manner which retains diagonal dominance by precluding the possibility of a negative system compressibility term. Note that equation (3) may be written as

$$W_{i,j} + C_{i,j} P^{n+1}_{i,j} = F_{i,j} \qquad (9)$$

where

$$W_{i,j} = A_{i,j} P^{n+1}_{i-1,j} + B_{i,j} P^{n+1}_{i+1,j} + D_{i,j} P^{n+1}_{i,j-1} + E_{i,j} P^{n+1}_{i,j+1} \qquad (10)$$

Expanding $C_{i,j}$ and $F_{i,j}$ in equation (9) yields

$$W_{i,j} + (-A_{i,j}-B_{i,j}-D_{i,j}-E_{i,j}-G_{i,j}) P^{n+1}_{i,j} = -G_{i,j} P^{n}_{i,j} + H_{i,j} \qquad (11)$$

This equation may be rearranged to obtain

$$W_{i,j} + (-A_{i,j}-B_{i,j}-D_{i,j}-E_{i,j}-G'_{i,j}) P^{n+1}_{i,j} + \left(\frac{h\phi}{\Delta t}\frac{S_o}{B_o}\frac{dB_o}{dP}\right)_{i,j} P^{n+1}_{i,j} = -G'_{i,j} P^{n}_{i,j} + \left(\frac{h\phi}{\Delta t}\frac{S_o}{B_o}\frac{dB_o}{dP}\right)_{i,j} P^{n}_{i,j} + H_{i,j} \qquad (12)$$

where

$$G'_{i,j} = \frac{(h\phi)_{i,j}}{\Delta t} \left(c_r + S_w c_w - \frac{S_g}{B_g}\frac{dB_g}{dP} + \frac{S_o B_g}{B_o}\frac{dR_s}{dP}\right)_{i,j} \qquad (13)$$

Rearranging equation (12)

$$W_{i,j} + (-A_{i,j}-B_{i,j}-D_{i,j}-E_{i,j}-G'_{i,j}) P^{n+1}_{i,j} = -G'_{i,j} P^{n}_{i,j} + H_{i,j} - \left(\frac{h\phi S_o}{B_o}\right)_{i,j} \left(\frac{dB_o}{dP}\frac{P^{n+1} - P^{n}}{\Delta t}\right)_{i,j} \qquad (14)$$

Considering the final term enclosed in parentheses in equation (14), we may note that

$$\frac{dB_o}{dP}\frac{P^{n+1} - P^{n}}{\Delta t} \approx \frac{dB_o}{dP}\frac{\partial P}{\partial t} \qquad (15)$$

and that

$$\frac{dB_o}{dP}\frac{\partial P}{\partial t} = \frac{dB_o}{dt} \qquad (16)$$

Thus we may employ a conventional finite difference approximation of $\frac{dB_o}{dt}$ and write equation (14) as

$$W_{i,j} + (-A_{i,j}-B_{i,j}-D_{i,j}-E_{i,j}-G'_{i,j})P^{n+1}_{i,j} = -G'_{i,j} P^{n}_{i,j} + H_{i,j} - \left(\frac{h\phi S_o}{B_o}(B_o^{n+1} - B_o^{n}) /\Delta t\right)_{i,j} \qquad (17)$$

In order to employ the above relationship when instability might occur, it is convenient to define revised matrix elements $F''_{i,j}$ and $G''_{i,j}$ as follows:

$$G''_{i,j} = \frac{(h\phi)_{i,j}}{\Delta t} \left(c_r + S_w c_w - \frac{\alpha S_o}{B_o}\frac{dB_o}{dP} - \right.$$

$$\frac{S_g}{B_g}\frac{dB_g}{dP} + \frac{S_oB_g}{B_o}\frac{dR_s}{dP})_{i,j} \tag{18}$$

and

$$F''_{i,j} = - G''_{i,j}\, P^n_{i,j} + H_{i,j} +$$

$$(\alpha-1)(h\phi)_{i,j}\left(\frac{S_o}{B_o}(B_o^{n+1} - B_o^n)\right)_{i,j}/\Delta t \tag{19}$$

where $\alpha = 1$ when $G_{i,j} > 0$

$\alpha = 0$ when $G_{i,j} \leq 0$

and where $F''_{i,j}$ replaces $F_{i,j}$ in equation (3) and $G''_{i,j}$ replaces $G_{i,j}$ in equation (5).

The procedure described above introduces a time derivative which is less accurate than the conventional approach, since it requires that

$B^{n+1}_{o_{i,j}}$ be extrapolated from $B^n_{o_{i,j}}$. However,

this approximation is employed only when the more conventional technique would become inadequate because of loss of diagonal dominance.

CONCLUSION

Tests have shown that the procedure presented here will allow a petroleum reservoir simulation to maintain stability through the bubble point pressure without substantial reduction in time step size and without significant loss in accuracy. The technique is recommended for use with simulation models which are sufficiently accurate for solving the problem under study, but which tend to be unstable at the bubble point.

NOMENCLATURE

- B = Formation volume factor
- c = Compressibility
- g = Gravitational acceleration
- h = Thickness
- k = Permeability
- n = Time level, superscript
- P = Pressure
- q = Production rate
- R = Gas/oil ratio
- S = Saturation
- t = Time
- T = Transmissibility
- x = Directional coordinate
- y = Directional coordinate
- z = Depth below datum plane
- μ = Viscosity
- ρ = Density
- ϕ = Porosity
- Φ = Potential

Subscripts:

- c = Capillary
- g = Gas
- gf = Free gas
- i = Index number for x ccordinate
- j = Index number for y coordinate
- o = Oil
- p = Phase
- r = Rock
- s = Solution
- w = Water

APPENDIX

Definition of Matrix Terms

$$A_{i,j} = \frac{2}{\Delta x_i(\Delta x_i + \Delta x_{i-1})}\left((B_{o_{i,j}} - B_{g_{i,j}} R_{s_{i,j}} + B_{g_{i,j}} R_{s_{i-\frac{1}{2},j}})T_{o_{i-\frac{1}{2},j}} + B_{g_{i,j}} T_{g_{i-\frac{1}{2},j}} + B_{w_{i,j}} T_{w_{i-\frac{1}{2},j}}\right)$$

$$B_{i,j} = \frac{2}{\Delta x_i\ (\Delta x_{i+1} + \Delta x_i)}\left((B_{o_{i,j}} - B_{g_{i,j}} R_{s_{i,j}} + B_{g_{i,j}} R_{s_{i+\frac{1}{2},j}})T_{o_{i+\frac{1}{2},j}} + B_{g_{i,j}} T_{g_{i+\frac{1}{2},j}} + B_{w_{i,j}} T_{w_{i+\frac{1}{2},j}}\right)$$

$$H_{i,j} = (B_o - B_gR_s)_{i,j}\Delta_x^2(T_o\rho_o gz)_{i,j} + B_{g_{i,j}}\Delta_x^2(T_oR_s\rho_o gz)_{i,j} + B_{g_{i,j}}\Delta_x^2(T_g\rho_g gz)_{i,j} +$$

$$B_{w_{i,j}}\Delta_x^2(T_w\rho_w gz)+(B_o-B_gR_s)_{i,j}\Delta_y^2(T_o\rho_o gz)_{i,j}\ +$$

$$B_{g_{i,j}}\Delta_y^2(T_oR_s\rho_o gz)_{i,j}\ +\ B_{g_{i,j}}\Delta_y^2(T_g\rho_g gz)_{i,j}\ +$$

$$B_{w_{i,j}}\Delta_y^2(T_w\rho_w gz)_{i,j}\ +\ B_{w_{i,j}}\Delta_x^2(T_wP_{c_{o-w}})_{i,j}\ +$$

$$B_{g_{i,j}}\Delta_x^2(T_gP_{c_{o-g}})_{i,j}+B_{w_{i,j}}\Delta_y^2(T_wP_{c_{o-w}})_{i,j}\ +$$

$$B_{g_{i,j}}\Delta_y^2(T_gP_{c_{o-g}})_{i,j}\ +\ q'$$

where Δ_x^2 and Δ_y^2 are finite difference operators which imply second derivatives with respect to x and y, respectively.

The transmissibilities and potentials for each phase p, and the flow term, q', are defined by

$$T_p = \frac{khk_{rp}}{B_p\mu_p}$$

$$\Phi_p = (P - \rho_p gz + P_{c,o-p})$$

and

$$q' = \frac{q_oB_o + q_wB_w + q_{gf}B_g}{\Delta x\Delta y}$$

The matrix coefficients $D_{i,j}$ and $E_{i,j}$ are identical to $A_{i,j}$ and $C_{i,j}$, respectively, except for the replacement of Δx by Δy and the interchange of the i and j subscripts.