01 Oct 1974

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Research Report No. 32 Structural Division October 1974

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St. Louis, Mo.

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This report presents results of research work sponsored by the American Iron and Steel Institute under AISI Project 163 "Load Factor Design of Steel Buildings".

ABSTRACT

Nominal design equations and resistance factors are developed for steel beam-columns as part of Load and Resistance Factor Design criteria for steel buildings. The resistance factors are derived from principles of first-order probability theory using calibration to present designs.

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1. INTRODUCTION

This report will deal with Load and Resistance Factor Design (LRFD) criteria for steel beam-columns. Previous reports have presented the general background of the first-order probabilistic theory underlying the LRFD criteria (1,2). Load factors γ for various important load combinations were developed (1,2), and resistance factors ϕ were derived for compact beams and simple columns (1,2), for beams (3) and for plate girders (4).

The LRFD criterion can be expressed by the formula (1,2)

$$\phi R_n \geq \gamma_o (\gamma_D c_D D_m + \gamma_L c_L L_m + \gamma_W c_W W_m + \dots) \tag{1}$$

where the right side represents the factored load effects (γ_o is the load factor accounting for the uncertainties of structural analysis, γ_D, γ_L, γ_W, etc., are the dead, live and wind load factors, respectively, D_m, L_m and W_m are the corresponding mean load intensities, and c_D, c_L and c_W are the deterministic influence coefficients which translate load intensity into load effect) and the left side represents the factored capacity of the structural member, where ϕ is the resistance factor and R_n is the nominal resistance for the desired limit state. The resistance factor ϕ accounts for the uncertainties underlying the determination of the nominal resistance, and it is equal to (1,2)

$$\phi = (R_m/R_n) \exp(-\alpha \beta V_R) \tag{2}$$

In this equation R_m is the mean resistance, α is a numerical factor equal to 0.55 (1), β is the "safety index" and V_R is the coefficient of variation of the resistance. The safety index β was obtained by calibration with the 1969 AISC Specification and its numerical value was found to be $\beta = 3.0$ (1,2).

This report is concerned with the development of the resistance factor ϕ and the nominal resistance R_n for steel beam-columns. The derivation will be made for laterally braced wide-flange beam-columns bent about their major axis. Subsequently, extensions will be presented for laterally unbraced beam-columns, for biaxially loaded beam-columns and for beam-columns in frames.

2. LATERALLY BRACED WIDE-FLANGE BEAM-COLUMNS

2.1 Assumptions

1) Members are prismatic rolled steel wide-flange shapes bent by end moments about their major axis (Fig. 1)

2) Member failure is by inelastic instability in the plane of the applied moments, as illustrated in Fig. 2 by a schematic load-deformation curve. The maximum force P_F is the limit state defining the resistance of the beam-column. Failure involving lateral-torsional buckling or biaxial flexure will not be considered in the present development, although these limit states will be discussed later in the report.

3) The in-plane capacity of beam-columns of wide-flange shape is known from previous work where the interaction curves relating axial force P, maximum end moment M_o, moment ratio κ, and member geometry were determined by numerical integration*. This analytical development assumed that P and κ remained constant while M_o was monotomically increased until it reached its maximum value. There is enough evidence in the literature to demonstrate, both from an experimental as well as a theoretical point of view, that proportional loading would give essentially the same interaction curves.

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* There is no need for summarizing the extensive literature on the behavior of beam-columns here; reference can be made to Chapter 5 in Ref. 5, Chapter 8 in Ref. 11, or Chapter 7 in Ref. 6 for such a review.

4) In order to establish a unique relationship between the limit state as characterized by the interaction curve, and the axial load and end moments as determined by an analysis of the structure with the factored loads (Fig. 3), it will be assumed that the ratio of the end moment to the axial load, $e = M_o/P$, and the end moment ratio $\varkappa$ is the same at failure as at the design level. By this assumption it is possible to establish a comparison between a point on the interaction curve and a point in the P-M_o domain representing the design condition (Fig. 4). Without this assumption the uniqueness of the relationship between the right and the left side of the design inequality (Eq. 1) is lost.

5) Overall frame instability is not considered at this time, and only the member capacity of the beam-column for the forces P and M_o is involved. The top of the beam-column is assumed not to move laterally with respect to its bottom.

2.2 Nominal Beam-Column Resistance

For the purposes of LRFD criteria interaction equations will be used to define beam-column resistance rather than sets of interaction curves. The following familiar interaction equations will be used (Fig. 5)

$$\frac{P}{P_u} + \frac{C_m M_o}{M_p(1 - P/P_E)} = 1.0 \tag{3}$$

$$\frac{M_o}{M_p} = 1.18\ (1 - \frac{P}{P_y}) \le 1.0 \tag{4}$$

In any given situation both equations must be checked, and the smaller value of either M_o or P, whichever is computed as the dependent variable, controls. These two equations approximate the numerically obtained interaction equations rather well (5,6). The terms in these equations are defined as follows:

P, M_o : a point on the limit state interaction curve

P_u : the limit state axial load which can be supported by the member in the absence of bending moment,

P_E : Euler buckling load

where

$$P_u = P_y(1 - 0.25\,\lambda^2) \qquad \text{for} \quad \lambda \leq \sqrt{2} \tag{5}$$

$$P_u = P_y/\lambda^2 \qquad \text{for} \quad \lambda \geq \sqrt{2} \tag{6}$$

$$P_E = P_y/\lambda^2 \tag{7}$$

$$P_y = A\,F_y \tag{8}$$

$$\lambda = \frac{L}{r_x}\left(\frac{1}{\pi}\right)\sqrt{\frac{F_y}{E}} \tag{9}$$

The remaining terms in Eqs. 3 and 4 are

$$M_p = F_y\,Z_x \tag{10}$$

$$C_m = 0.6 + 0.4\,\varkappa \geq 0.4 \tag{11}$$

where Z_x and A are the plastic section modulus and the area of the member, respectively, $\varkappa$ is the moment ratio (Fig. 1) and F_y and E are, respectively, the yield stress and the modulus of elasticity of the material.

The two equations (Eqs. 3 and 4) will be used herein as the nominal resistance equations.

2.3 The Resistance Factor ϕ

The interaction curves represent the limiting strength of beam-columns. These curves were determined by numerical integration, and this operation involves a certain number of inherent computational idealizations. Further idealizations were made in the derivations of the moment-curvature relations which were integrated. The material properties also introduce uncertainties. Thus the interaction curves are random variables, and in the first-order probabilistic theory used, the characteristic statistical properties are the mean value and the coefficient of variation of the interaction curves. These are the properties which need to be estimated.

The curve R_m in Fig. 6 is a representation of a mean interaction curve. Conceptionally, one could decompose the numerical integration process into all of its constituent pieces and perform an analysis to obtain mean values and coefficients of variation. This would be a formidable computational task which was avoided by correlating the ideal interaction curves with the many existing beam-column tests.

Since the final outcome will be a design rule involving the empirical interaction equations (Eqs. 3 and 4) as the nominal strength equations R_n (curve R_n in Fig. 6), the task is to correlate R_n with the ideal interaction curves. The variations in material properties must also be taken into account. Thus the mean resistance is determined through a series of three transformations, going from the tests to the predictions by the interaction curves, to the empirical interaction equations with the mean material properties, finally to the nominal interaction equations which use the nominal material properties. Symbolically this can be written as follows:

$$R_m = \left[\frac{\text{Test strength}}{\text{Prediction by theory}} \right]_m \times$$

$$\left[\frac{\text{Prediction by theory}}{\text{Prediction by interaction equation}} \right]_m \times$$

$$\left[\frac{\text{Prediction by interaction equation}}{R_n} \right]_m \times R_n$$

In abbreviated form this equation can be written as

$$R_m = B_{Ex} B_{Th} B_{Mat} R_n \tag{12}$$

where B stands for "bias", "Ex", "Th" and "Mat" signify "experiment", "theory" and "material", respectively.

The question now arises: along which axis should the ratios indicated by Eq. 12 be measured? If either the ordinate P or the abscissa M_o (Fig. 6) is used, then undue bias is introduced at the ends of the curves (near P_o or M_p), and, therefore, it is necessary to determine the ratios along rays OABC from the origin. The angle δ of these rays is determined by the proportionality which is assumed to exist between P and M_o.

The Test strength-to-Prediction by theory ratio was obtained from Fig. 5.23 in Ref. 5, where a histogram of this ratio was given for 83 beam-column tests. From this source

$$B_{Ex} = 1.005 \quad \text{and} \quad V_{Ex} = 0.093$$

The symbol V is the coefficient of variation.

An analysis using tabulated theoretical interaction curve data (from Sec. 7 in Ref. 7) and predictions of R_n from the interaction curves (Eqs. 3 and 4) was made to determine B_{Th} in Eq. 12. This analysis included curves for six values of λ (corresponding to L/r_x = 20, 40, 60, 80, 100, 120 for A36 steel) and five values of $\varkappa$(1, 0.6, 0, - 0.6, - 1) for a total of 30 curves. For each curve six to nine ray angles δ were used. Thus data covering the whole practical domain of parameters was included. The resulting values are

$$B_{Th} = 1.01 \quad \text{and} \quad V_{Th} = 0.04$$

The determination of the material bias (B_{Mat} in Eq. 12) was done in the following manner. Because of proportionality,

$$M_o = Pe \tag{13}$$

and if Eq. 13 is substituted into Eqs. 3 and 4, the following expressions are obtained for P:

From Eq. 3:

$$P = \frac{1}{2}\left\{ (P_u + P_E + \frac{C_m\, e\, P_u\, P_E}{M_p}) - \sqrt{(P_u + P_E + \frac{C_m\, e\, P_u\, P_E}{M_p})^2 - 4\, P_u\, P_E} \right\} \tag{14}$$

From Eq. 4:

$$P = \frac{P_y}{\frac{P_y\, e}{1.18\, M_p} + 1} = \frac{P_y}{\frac{A\, F_y\, e}{1.18\, F_y\, Z} + 1} = \frac{A\, F_y}{\frac{A\, e}{1.18\, Z} + 1} \tag{15}$$

The first of these equations is dependent on F_y and E (see Eqs. 5 through 10) and, therefore, the standard deviation σ is determined by the formula (8)

$$\sigma^2_{Mat} = \left(\frac{\partial P}{\partial F_y}\right)^2_m \sigma_{F_y}^{2} + \left(\frac{\partial P}{\partial E}\right)^2_m \sigma_E^{2} \tag{16}$$

where the partial derivatives were evaluated with the mean values of the yield stress (F_{ym} = 1.05 F_y, where F_y is the nominal value) and the modulus elasticity (E_m = 29,000 Ksi). The standard deviations $\sigma_{F_y} = F_{ym} V_{F_y}$ = $(1.05\, F_y)(0.1) = 0.105\, F_y$ and $\sigma_E = E_m V_E = 29{,}000 \times 0.06 = 1740$ Ksi were used in Eq. 16. The second equation (Eq. 15) is dependent only on F_y, and from this equation

$$B_{Mat} = F_{ym}/F_y = 1.05 \text{ and } V_{Mat} = 0.1$$

The corresponding values from Eqs. 14 and 16 turned out to be somewhat smaller, and so, for the sake of consistency over the whole domain, the larger numbers above will be used. The material properties statistics $F_{ym} = 1.05\ F_y$, $V_{F_y} = 0.1$, $E_m = 29{,}000$ Ksi and $V_E = 0.06$ were taken from previous work (1,2,9).

From the information presented above, then, the mean resistance is equal to (Eq. 12)

$$R_m = 1.005 \times 1.01 \times 1.05\ R_n = 1.07\ R_n$$

The corresponding coefficient of variation is equal to (1,2)

$$V_R = \sqrt{V_{Ex}^2 + V_{Th}^2 + V_{Mat}^2 + V_F^2} \qquad (17)$$

$$V_R = \sqrt{0.093^2 + 0.04^2 + 0.10^2 + 0.05^2} = 0.15$$

The fourth coefficient of variation above, $V_F = 0.05$, is the coefficient of variation of fabrication, and it is an assumed value used throughout all previous reports (Refs. 1 through 4).

The resistance factor ϕ can now be determined from Eq. 2.

$$\phi = 1.07 \exp(-\ 0.55 \times 3.0 \times 0.15) = 0.84$$

Thus $\phi = 0.84$ is the resistance factor consistent with the interaction equations for wide-flange beam-columns failing by in-plane inelastic instability.

2.4 The Design Criterion

The resistance factor ϕ is applied to the nominal resistance R_n (OB in Fig. 6), that, is to the straight line from the origin to the interaction curve R_n. The design condition is, thus, from Eq. 1,

$$\phi R_n \geq \sqrt{P_D^2 + M_{oD}^2} \tag{18}$$

where P_D and M_{oD} are the design axial force and the design endmoment determined by structural analysis from the factored loads. (Fig. 3, with M_{oD} being the absolutely larger value of the two end moments M_{UD} or M_{LD}). Since proportionality is assumed between the ratio of P and M, at the design level and the failure level, that is,

$$\frac{M_{oD}}{P_D} = \frac{M_o}{P_u} = e \tag{19}$$

Eq. 18 can be written as

$$\phi \sqrt{P_u^2 + M_o^2} \geq \sqrt{P_D^2 + M_{oD}^2} \tag{20}$$

or

$$\phi P_u \sqrt{1 + e^2} \geq P_D \sqrt{1 + e^2} \tag{21}$$

The square root terms cancel out, and so the design criterion can be written as

$$\phi P_u \geq P_D \tag{22}$$

In this equation, P is the smaller of the two loads determined by Eqs. 23 and 24:

$$P = \frac{1}{2}\left\{(P_u + P_E + \frac{C_m\, e\, P_u\, P_E}{M_p}) - \sqrt{(P_u + P_E + \frac{C_m\, e\, P_u\, P_E}{M_p})^2 - 4\, P_u\, P_E}\right\} \tag{23}$$

$$P = \frac{P_y}{\frac{A\ e}{1.18\ Z} + 1} \tag{24}$$

Both Eqs. 23 and 24 are subject to the restriction that if

$$P \geq \frac{M_p}{e} \tag{25}$$

the plastic moment is exceeded and so in this case the design criterion becomes

$$\phi\ M_p \geq M_{oD} \tag{26}$$

regardless of the value of P_D.

The scheme presented above is straightforward and is relatively easy to apply. However, the advantage of the traditional ease with which ratios are summed to check if their sum is less than or more than unity is lost in this procedure. This advantage can be recaptured by using the following factored interaction equations:

$$\frac{P_D}{\phi\ P_u} + \frac{C_m\ M_{oD}}{\phi\ M_p\ (1 - \frac{P_D}{\phi\ P_E})} \leq 1.0 \tag{27}$$

$$\frac{P_D}{\phi\ P_y} + \frac{M_{oD}}{1.18\ \phi\ M_p} \leq 1.0 \tag{28}$$

In any design situation both equations must be checked. A further check against exceeding the plastic moment must be made if the design axial load is less than 0.153 $\phi\ P_y$ by assuring that Eq. 26 is also satisfied.

The validity of Eqs. 27 and 28 can be checked by noting that at the limit where $M_{oD} = \phi\ M_o$ and $P_D = \phi$ P the left side of each equation equals unity. If M_o = Pe is then substituted and each equation is solved for P, the resulting expressions will be, indeed, identical to Eqs. 23 and 24, respectively, as they should.

In the previous section of this report it was shown that the resistance factor ϕ for steel wide-flange beam-columns bent about the major axis and failing by inelastic instability in the plane of the applied end-moments is equal to 0.84 in accordance with the statistics which were used. This value of ϕ must now be compared to the resistance factors from the previous studies (1,2) for the limiting cases where P = 0 (i.e., the member is a beam) and $M_o = 0$ (i.e., the member is a column). From Ref. 1, the following ϕ-factors were obtained:

$$\phi_b = 0.86 \quad \text{for beams} \tag{29}$$

$$\left.\begin{aligned} \phi_c &= 0.86 && \text{for } \lambda \le 0.16 \\ \phi_c &= 0.9 - 0.25\,\lambda && \text{for } 0.16 \le \lambda \le 1.0 \\ \phi_c &= 0.65 && \text{for } \lambda \ge 1.0 \end{aligned}\right\} \text{for columns} \tag{30}$$

With $\phi = 0.84$ for beam-columns as developed in this report, discontinuities will exist at the limits of the beam-column domain. Such inconsistencies could hardly be avoided since the three resistance factors were derived from different data sets. The extent of the effect of the discontinuities is shown in Fig. 7, where the interaction curves (Eqs. 27 and 28) are plotted for $\lambda = 0.7$. This λ corresponds to slenderness ratios L/r_x of about 62, 53 and 46, respectively, for F_y = 36, 50 and 65 ksi. The columns are thus relatively slender; for shorter members the effect would be relatively less.

The effect when $P_D = 0$ (the member is a beam) is not very significant (see Fig. 7), but it is pronounced when $M_{oD} = 0$ (the member is a column). Since the beam-column tests on which the derivation of ϕ, as presented here, was based were tests with significant bending, the results of this analysis really do not apply to the case where flexure is slight or even absent, so in this region the column ϕ-values (Eq. 30) must supersede the ϕ-value for beam-columns.

Two possible courses of action are suggested in Fig. 8. The solid lines represent the interaction Equations (Eqs. 27 and 28) with $\phi = 0.86$ (this slight adjustment of ϕ from 0.84 to 0.86 reconciles the beam-column resistance with beam resistance) with the proviso that

$$P_D \leq \phi_c \, P_u \qquad (31)$$

where ϕ_c are the ϕ-factors for axially loaded columns (Eq. 30). Thus a cut-off plateau is provided when the beam-column axial load becomes equal to the factored resistance of the member acting as a column. This implies that at the factored column ultimate load a slight amount of bending can be tolerated. This is reasonable, since the column theory on which the column ϕ-factors were based assumed an initial crookedness, which is equivalent to saying that flexure is present.

The other scheme, represented in Fig. 8 by dashed lines is a more conservative approach whereby the factored interaction equations are adjusted to end where the columns are, i.e.,

$$\frac{P_D}{\phi_c \, P_u} + \frac{C_m \, M_{oD}}{\phi_b \, M_p \, (1 - P_D/\phi_b \, P_E)} = 1.0 \qquad (32)$$

$$\frac{P_D}{\phi_b \, P_y} + \frac{M_{oD}}{1.18 \, \phi_b \, M_p} = 1.0 \qquad (33)$$

In these equations $\phi_b = 0.86$ (Eq. 29) and ϕ_c is the resistance factor for columns (Eqs. 30). In this scheme effectively a variable ϕ-factor is used; when flexure predominates the effective resistance factor approaches $\phi = 0.86$, which is the value obtained for beams; when axial load predominates the effective ϕ-factor approaches ϕ_c, the value for columns. In this case the uncertainties introduced by the variations in residual stress and shape begin to become more significant, while for beams these effects have no significance.

Thus both schemes presented in Fig. 8 are physically reasonable. The final choice will be made when further extensions are examined in the next section for beam-columns failing by lateral-torsional buckling and for biaxially loaded members.

3. LATERALLY UNBRACED BEAM-COLUMNS AND BIAXIALLY LOADED MEMBERS

The previous section dealt with wide-flange beam-columns bent about the major axis of the section and failing by inelastic instability in the plane of the applied moments. In this section the behavior of beam-columns under biaxial bending, as well as the behavior of beam-columns bent about the major axis but failing by lateral-torsional buckling, will be discussed.

For both the case of beam-columns failing by lateral-torsional buckling and for biaxially loaded members adequate theory exists to predict the maximum capacity accurately, and comparison of test capacities and theoretical predictions of the resistance are excellent. Compared to the vastly expanded domain of parameters (as contrasted to the relatively few parameters involved in the in-plane resistance of beam-columns) there are few tests. However, an examination of these tests indicates that theory can predict test capacity with about the same accuracy as was noted for the in-plane beam columns in the previous section (i.e., the mean is essentially equal to the theoretical prediction, and the coefficient of variation is about 10%).

3.1 Beam-Columns Failing By Lateral-Torsional Buckling

The latest and most comprehensive research work on the lateral-torsional buckling of wide-flange beam-columns is contained in the report of Lim and Lu (10). This report reviews all previous work and it examines the effects of end-restraint (in-plane restraint, end warping restraint and lateral restraint) on the lateral-torsional buckling strength and on the ultimate capacity of beam-columns with column-type cross sections (i.e., 8 in x 8 in, 10 in x 10 in, etc., sections). The report concludes that for relatively short members ($L/r_x \leq 40$, $P/P_y \leq 0.4$ and $J/Ad^2 \leq 925 \times 10^{-6}$, where J is the torsion constant, A is the cross-sectional area and d is the depth of the section) the occurrence of lateral-torsional buckling does not reduce the in-plane capacity because of post-buckling strength.

Unfortunately the required computational effort is large and so not enough curves are given in Ref. 10 to construct a set interaction curves which include the various parameters. The report shows for two sections (W8 x 31 and W14 x 142) and for $P = 0.4\ P_y$ that for $\kappa = +1$ (equal end moments causing single curvature bending) and for $F_y = 36$ Ksi the following interaction equation gives predictions of the ultimate moment which are conservative by about 5%:

$$\frac{P}{P_{uy}} + \frac{M_o}{M'(1 - P/P_{Ex})} = 1.0 \qquad (34)$$

This equation is the same as Eq. 3 presented previously, except that P_{uy} is determined for the weak axis slenderness ratio L/r_y and M' is the maximum moment capacity of the member when the axial force vanishes.

This interaction equation (Eq. 34) has been suggested for use by various publications (5, 6 and 11 are but a few) as being conservative over the whole domain of all parameters. This fact, however, is based on an incomplete comparison with theory (10, 12, 13), and on but a small number of tests (10, 12, 13, 14, 15). Because of the large numerical effort which would be required in performing the necessary theoretical calculations it is thus not possible to move from test-to-theory-to-interaction equation, as was done previously for the case of beam-columns failing by in-plane inelastic instability. Furthermore, a large number of the available experiments (14, 16) were tested in a manner that the ends of the columns were fixed-ended about the weak axis, and most of the test ultimate loads, although failure was definitely by lateral-torsional buckling, were quite adequately predicted by in-plane theory (16).

Even though the information is incomplete, it is possible to make the following statements with a reasonable degree of certainty:

1) For the cases where comparisons could be made with the most complete theory, the test capacities can be predicted with about the same accuracy as this was possible for in-plane failure.

2) For relatively short beam-columns ($L/r_x \leq 40$)* it is possible to reach the in-plane capacity because of post-buckling strength and because of weak-axis end restraint which is usually always present. For such members the interaction equation

$$\frac{P}{P_{uy}} + \frac{C_m M_o}{M'(1 - P/P_{Ex})} = 1.0 \qquad (35)$$

*Note that most beam-columns in buildings are usually not more slender than $L/r_x = 40$

is slightly conservative.

3) As beam-column slenderness increases, Eq. 35 becomes more and more conservative.

Since the test statistics are about the same, and the interaction equation is always conservative in comparison to theory, the resistance factor value of ϕ = 0.86, which appeared to be satisfactory for the in-plane case, will be conservative in the case of lateral-torsional buckling, Since not enough information is available to develop a more refined value, it is suggested that ϕ = 0.86 be used, conservatively, with the lateral-torsional buckling interaction equation.

3.2 Biaxially Loaded Beam-Columns

The biaxially loaded beam-columns represent essentially the same picture as the beam-columns failing by lateral-torsional buckling. An adequate theory exists to predict test results* and the usual interaction equation

$$\frac{P}{P_{uy}} + \frac{C_{mx} M_{ox}}{M'_x (1 - P/P_{Ex})} + \frac{C_{my} M_{oy}}{M_{py} (1 - P/P_{Ey})} = 1.0 \qquad (36)$$

is quite conservative with respect to the theory, especially for column type sections. For such sections a more realistic interaction equation has been suggested by Chen (11). Unfortunately more work needs to be done to expand Chen's interaction equation to beam-type wide-flange sections.

Springfield and Hegan (18), as well as Pillai (19), have evaluated 123 biaxially loaded beam-column tests as regards their comparison with the two interaction equations (Eq. 36 and Chen's interaction equation).

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* For a series of 12 tests the mean test-to-theory ratio is 0.99, and the coefficient of variation is 0.04 (17,18).

The resulting statistics are as follows:

1) Comparison with Eq. 36:

Mean Test-to-Prediction ratio:	1.11 (Ref. 18)
	1.16 (Ref. 19)
Coefficient of variation	0.12 (Ref. 18)
	0.11 (Ref. 19)

2) Comparison with Chen's interaction equation:

Mean: 0.97 ; $V = 0.13$

With $\beta = 3.0$, $\alpha = 0.55$ (Eq. 2) and

$V_R = \sqrt{V_{Mat}^2 + V_{Test}^2 + V_{Fab}^2}$, assuming the material bias as 1.05, $V_{Mat} = 0.1$, and $V_{Fab} = 0.05$, the various values of the resistance factor are as follows:

For Eq. 36, using data from Ref. 18:

$$\phi = 1.05 \times 1.11 \exp(-0.55 \times 3 \times 0.16) = 0.90$$

For Eq. 36, using data from Ref. 19:

$$\phi = 1.05 \times 1.16 \exp(-0.55 \times 3 \times 0.16) = 0.94$$

For Chen's interaction equation:

$$\phi = 1.05 \times 0.97 \exp(-0.55 \times 3 \times 0.17) = 0.77$$

It appears that for the more precise interaction equation (Chen's) a smaller ϕ is required than for the more conservative interaction equation, Eq. 36. However, it should be noted that the tests for which these comparisons were made included tests for which Chen's equation does not strictly apply, and, furthermore, the data for some of the tests is probably not reliably interpreted (translation from the Russian). Thus the ϕ-values above can be considered to have only a comparative significance. If a value of $\phi = 0.86$ is adopted, it is surely conservative when used with Eq. 36; if Chen's interaction equation is adopted a lower value of ϕ, say $\phi = 0.8$, should probably be used.

For the sake of simplicity and consistency it is tentatively recommended that $\phi = 0.86$ and Eq. 36 be used for the design criterion of beam-columns under biaxial loading.

4. DESIGN CRITERIA FOR BEAM-COLUMNS

4.1 General Criteria

In view of the fact that the interaction equations (Eq. 35 and 36) with $\phi_b = 0.86$ will be conservative for beam-columns failing by lateral-torsional buckling and for beam-columns under biaxial bending, and considering that $\phi_b = 0.86$ is satisfactory for in-plane beam-columns, it is suggested that the following design criteria be used for beam-columns:

$$\frac{P_D}{\phi_b\ P_u} + \frac{C_{mx}\ (M_{oD})_x}{\phi_b\ M_{ux}\left(1 - \frac{P_D}{\phi_b\ P_{Ex}}\right)} + \frac{C_{my}\ (M_{oD})_y}{\phi_b\ M_{uy}\left(1 - \frac{P_D}{\phi_b\ P_{Ey}}\right)} \leq 1.0 \qquad (37)$$

and

$$\frac{P_D}{\phi_b\ P_y} + \frac{(M_{oD})_x}{\phi_b\ M_{px}} + \frac{(M_{oD})_y}{\phi_b\ M_{py}} \leq 1.0 \qquad (38)$$

For any beam-column both equations must be satisfied. In addition it must be assured that

$$P_D \leq \phi_c\ P_u \qquad (39)$$

$$(M_{oD})_x \leq \phi_b\ M_{px} \qquad (40)$$

$$(M_{oD})_y \leq \phi_b\ M_{py} \qquad (41)$$

The various terms in Eqs. 37 through 41 are defined as follows:

P_D : Factored design axial load

$(M_{OD})_x$: Numerically largest factored design end-moment about the x-axis

$(M_{oD})_y$: Numerically largest factored design end-moment about the y-axis.

These design forces are obtained from structural analysis for the factored loads (right side of Eq. 1)

C_m : Equivalent moment factor for the moments about the x-and y-axes, respectively, where

$$C_m = 0.6 + 0.4\,\varkappa \le 0.4 \tag{42}$$

P_u : Limit state axial capacity in the absence of flexure, determined for the largest effective slenderness ratio about the x-or the y-axis of the section, where

$$P_u = P_y\,(1 - 0.25\,\lambda^2) \qquad \text{for} \quad \lambda \le \sqrt{2} \tag{43}$$

$$P_u = P_y/\lambda^2 \qquad \text{for} \quad \lambda \ge \sqrt{2} \tag{44}$$

$$\lambda = \frac{Kh}{r}\left(\frac{1}{\pi}\right)\sqrt{\frac{F_y}{E}} \tag{45}$$

where K is the effective length factor, h is the member length, F_y is the yield stress, E = 29,000 Ksi and r is the radius of gyration. Both K and r refer to either the x-or the y-axis, as appropriate.

P_{Ex} ; P_{Ey} : Euler buckling load about the x-and y-axis, respectively, where

$$P_E = P_y/\lambda^2 \tag{46}$$

$$P_y = A F_y \qquad (47)$$

A : Cross-sectional area of member

M_{ux} ; M_{uy} : Limit state bending capacity in the absence of axial force for equal bending moments causing single curvature deformation (κ = + 1 in Fig. 1 or Fig. 3). M_{ux} and M_{uy} are computed by the formulas given for beams in Ref. 3.

M_{px} ; M_{py} : Plastic moment capacities about the x-or y-axes, respectively

$\phi_b = 0.86$: the resistance factor for beams and beam-columns

ϕ_c : resistance factor for columns, where

$$\left.\begin{array}{lll} \phi_c = 0.86 & \text{for} & \lambda \le 0.16 \\ \phi_c = 0.9 - 25\lambda & \text{for} & 0.16 \le \lambda \le 1.0 \\ \phi_c = 0.65 & \text{for} & \lambda \le 1.0 \end{array}\right\} \qquad (48)$$

The interaction equations, as presented in the Eqs. 37 and 38, assume that the factored end-moments and the axial force are the actual computed values, including, if appropriate, also the secondary P - Δ moments. In the case of unbraced planar frames (i.e., $(M_{oD})_y = 0$), where the forces are determined by first-order analysis, an effective length factor $K > 1$ is to be used and $C_m = 0.85$, as in the 1969 AISC Specification. In this case it is necessary to determine bending capacity M_{ux} and M_{uy} with due consideration of the end-moment ratio κ by using the appropriate value of C_b from Ref. 3.

4.2 Special Cases

a) Flexure about one of the principal axes only

When flexure is only about one of the principal axes, i.e., $(M_{oD})_x$ or $(M_{oD})_y$ are equal to zero, the second or the third ratio in Eq. 37 vanishes.

In lieu of Eq. 38 more accurate interaction equations can be used in this case (Ref. 6):

Flexure about the major axis of a W-shape:

$$\frac{P_D}{\phi_b P_y} + \frac{M_{oD}}{1.18\ \phi_b M_{px}} \leq 1.0 \tag{49}$$

except that M_{oD} may not exceed $\phi_b M_{px}$

Flexure about the minor axis of a W-shape:

$$\left(\frac{P_D}{\phi_b P_y}\right)^2 + \frac{M_{oD}}{1.19\ \phi_b M_{py}} \leq 1.0 \tag{50}$$

except that M_{oD} may not exceed $\phi_b M_{py}$.

b) Members in flexure and tension

The previous formulas apply to the usual case of beam-columns under flexure and axial compression. For the case of flexure and tension, only strength considerations, but not stability criteria, apply. Therefore, only Eq. 38, or if flexure is about one of the principal axes only, Eq. 49 or 50 as appropriate, need be used.

c) Tapered beam-columns

For tapered wide-flange beam-columns with a single web-taper under flexure about the major axis, P_u and P_{Ex} are determined for the properties of the smaller end, using the effective length factors from Appendix D of the Commentary to the AISC Specification (Supplement 3, effective June 12, 1974), and M_{ux}, M_D and M_{px} are determined for the larger end. The value of M_{ux} is determined from the formula

$$M_{ux} = \left(\frac{5}{3}\right) S_x F_{b\gamma} \tag{51}$$

where 5/3 is the AISC factor safety which underlies the allowable flexural stress $F_{b\gamma}$, and S_x is the major axis elastic section modulus. The value of $F_{b\gamma}$ is to be determined from Appendix D of the AISC Specification (Supplement 3), where methods of determining C_m are also presented.

d) Beam-columns with transverse forces between ends

If transverse forces are present between the ends of the beam-column, M_D in Eq. 37 is the maximum moment between the ends and C_m must be determined by a separate analysis. Examples of such an analysis are presented in Ch. 8 of Ref. 11 and in Sec. 1.6 of the Commentary to the AISC Specification. Conservatively C_m can be taken as unity.

e) Beam-columns of W-shape under biaxial loading

For wide-flange beam-columns under biaxial bending it is possible to utilize the more liberal interaction equations resulting from recent research (Chap. 8 in Ref. 11 and Refs. 20 and 21). These interaction equations must be used, however, with a more severe resistance factor $\phi_b = 0.80$, as discussed earlier. Since these interaction equations predict a considerably higher capacity than the straight-line equations (e.g. Eqs. 37 and 38), it is possible that member yielding under service loads may occur. This condition must, therefore, also be checked by assuring that Eqs. 37 and 38 are also fulfilled in addition to the equations given below. However, these equations (i.e., Eqs. 37 and 38) are to be checked for serviceability loads only and with a larger ϕ-factor, as outlined in Sec. C.1.2.2 of the proposed Criteria for "Load and Resistance Factor Design of Steel Building Structures" (under preparation in draft form, February 1976). According to these criteria $\phi = 0.94$, $\gamma_o = 1.05$, $\gamma_D = 1.05$ (dead loads), $\gamma_{L_I} = 1.50$ (instantaneous live load), $\gamma_{W_A} = 1.30$ (annual wind load) and $\gamma_{S_A} = 1.65$ (annual snow load) under serviceability limit states. The factored ultimate strength interaction equations for biaxial bending are (from Ref. 11, 20 or 21):

$$\left(\frac{(M_{oD})_x}{\phi_b M_{pcx}} \right)^{e_1} + \left(\frac{(M_{oD})_y}{\phi_b M_{pcy}} \right)^{e_1} \leq 1.0 \tag{52}$$

and

$$\left\{ \frac{C_{mx} (M_{oD})_x}{\phi_b M_{ux} \left(1 - \frac{P_D}{\phi_b P_y}\right)\left(1 - \frac{P_D}{\phi_b P_{Ex}}\right)} \right\}^{e_2} + \left\{ \frac{C_{my} (M_{oD})_y}{\phi_b M_{uy} \left(1 - \frac{P_D}{\phi_b P_y}\right)\left(1 - \frac{P_D}{\phi_b P_{Ey}}\right)} \right\}^{e_2} \leq 1.0 \tag{53}$$

where $\phi_b = 0.80$, all other terms are as defined previously, except that

$$e_1 = \frac{1.6 \dfrac{P_D}{\phi_b P_y}}{2 \ln \left(\dfrac{P_D}{\phi_b P_y} \right)} \tag{54}$$

$$e_2 = 1.4 + \frac{P_D}{\phi_b P_y} \tag{55}$$

Equations 52 and 54 apply strictly only if the flange width-to-beam depth (b_f/d) ratio is larger than 0.8. In a recent paper, Ross and Chen have tentatively removed this restriction, and they recommended that Eqs. 52, 53 and 54 are valid, but Eq. 55 is to be replaced by the formula

$$e_2^* = 0.4 + \frac{P_D}{\phi_b P_y} + \frac{b_f}{d} \tag{56}$$

except that e_2^* may not be less than unity.

f) Concrete-filled pipe-columns

While concrete-filled circular or square tubular beam-columns are not covered in the AISC Specification, it is possible to formulate LRFD criteria for such columns also. The resistance factor $\emptyset$ was determined by Ganguly (Ref. 22) for an interaction equation proposed by Furlong (Refs. 23 and 24)

$$\left(\frac{P_D}{\emptyset_b \, P_u} \right)^2 + \left(\frac{M_{oD}}{\emptyset_b \, M_u} \right)^2 \leq 1.0 \tag{57}$$

except that M_{oD} may not exceed $\emptyset_b \, M_u$. In this interaction equation P_D and M_{oD} are defined as previously in this report, $\emptyset_b = 0.75$, P_u is the axial load capacity in the absence of end moment and M_u is the plastic moment capacity of the steel tube alone when the axial force is zero.

$$P_u = A_s \, F_y + A_c \, f'_c \sqrt{\frac{F_y}{0.0018 \, E_s}} \tag{58}$$

where A_s = area of steel tube

A_c = area of encased concrete

F_y = specified yield stress of steel

f'_c = specified compressive strength of concrete

The value of P_u may not exceed $A_s \, F_y + A_c \, f'_c$. The ultimate moment M_u is the plastic moment of the steel tube alone, i.e.,

$$M_u = \frac{F_y}{6} \left(D_o^3 - D_i^3 \right) \tag{59}$$

for a circular tube (D_o = outside diameter, D_i = inside diameter) and

$$M_u = \frac{F_y}{4} \left(D_o^3 - D_i^3 \right) \tag{60}$$

for a square tube (D_o = outside dimension, D_i = inside dimension of tube).

The interaction equation applies when the slenderness ratio of the beam-column, as determined for the steel section alone, is less than or equal to 50.

5. SUMMARY

This report has presented Load and Resistance Factor Design Criteria, including resistance factors ϕ and nominal resistance interaction equations, for steel beam-columns. The design criteria are summarized in Sec. 4 above.

6. ACKNOWLEDGMENTS

The research work reported here is sponsored by the Committee of Structural Steel Producers and Committee of Steel Plate Producers of the American Iron and Steel Institute as AISI Project 163 "Load Factor Design of Steel Buildings." The members of the Advisory Task Force are Messrs. I. M. Viest (Chairman), L. S. Beedle, C. A. Cornell, E. H. Gaylord, J. A. Gilligan, W. C. Hansell, I. M. Hooper, W. A. Milek, Jr., C. W. Pinkham and G. Winter. Their discussion of earlier drafts of this report and their suggestions are greatly appreciated by the authors.

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ASCE Journal of the Structural Division, Vol. 94, ST1, Jan. 1968.

8. NOMENCLATURE

A_c : Concrete area in a concrete-filled beam-column

A_s : Steel area in a concrete-filled beam-column

B : Bias factor

C_m : Equivalent moment factor

c : Influence factor

D_o, D_i : Outside and inside pipe dimension

D_m : Mean dead load intensity

E : Modulus of elasticity

e_1, e_2 : Exponents in the biaxial interaction equation

e : Eccentricity

$F_{b\gamma}$: Allowable flexure stress of tapered beams

F_y : Yield stress

f'_c : Specified compressive strength of concrete

h : Story height

K : Effective length factor

L : Member length

L_m : Mean live load intensity

M_o : End moment

M_{oD} : Factored design moment

M_p : Plastic moment

M_{pc} : Plastic moment in the presence of axial force

M' : Maximum moment capacity

P : Axial load

P_D : Factored design axial load

P_E : Elastic buckling load

P_u : Ultimate column load

P_y : Yield load

R_m : Mean resistance

R_n : Nominal resistance

S_x : Elastic section modulus

V : Coefficient of variation

α : Numerical factor

β : Safety index

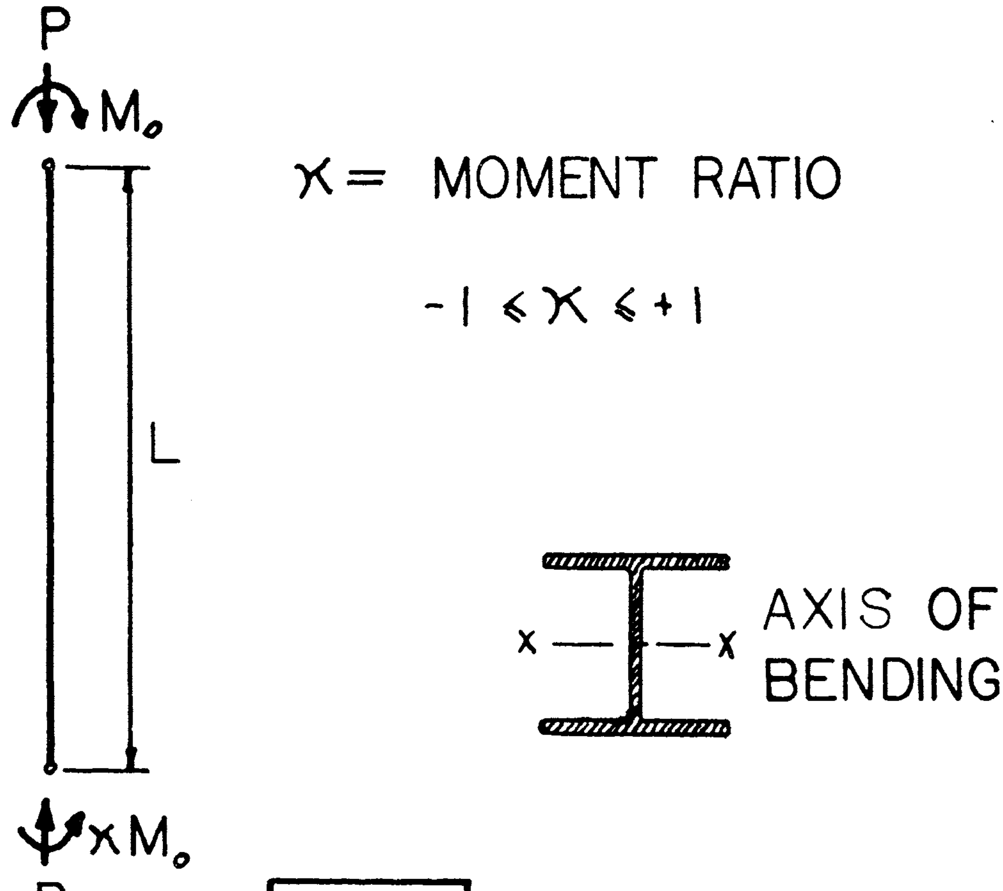
γ : Load factor

ϕ : Resistance factor

λ : Slenderness parameter

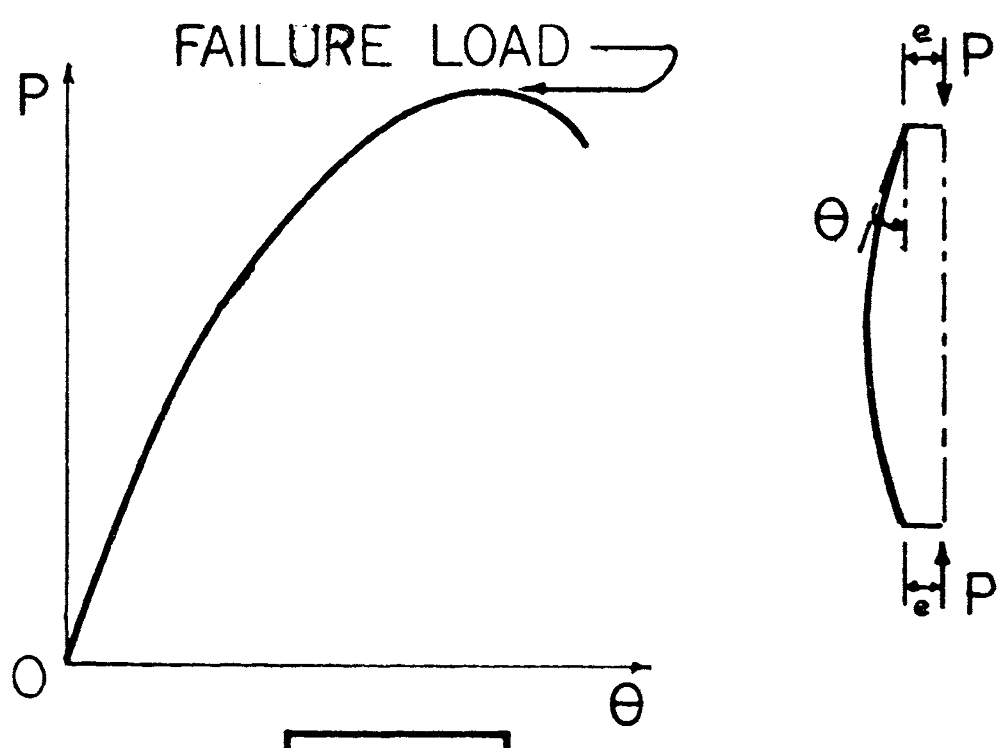
κ : Moment ratio

σ : Standard deviation



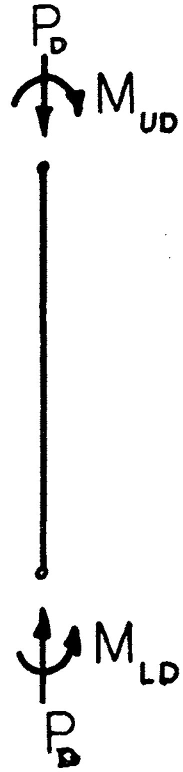
P
M₀
χ = MOMENT RATIO
-1 ≤ χ ≤ +1
L
x
x
AXIS OF BENDING
χM₀
P

FIG. 1



FAILURE LOAD
P
O
θ
e
P
θ
P
e

FIG. 2



$$e = \frac{M_{UD}}{P_D} \; ; \quad \chi = \frac{M_{LD}}{M_{UD}} \; ; \text{IF } |M_{UD}| > |M_{LD}|$$

$$e = \frac{M_{LD}}{P_D} \; ; \quad \chi = \frac{M_{UD}}{M_{LD}} \; ; \text{IF } |M_{UD}| < |M_{LD}|$$

$P_D; M_{UD}; M_{LD}$ = FACTORED DESIGN LOADS

FIG. 3

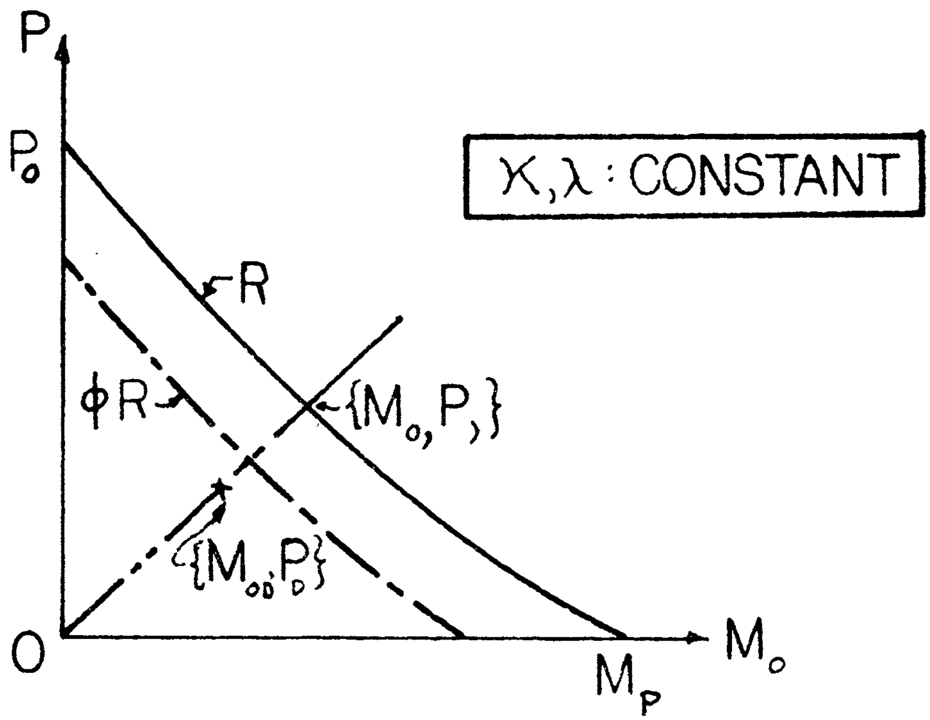


FIG. 4

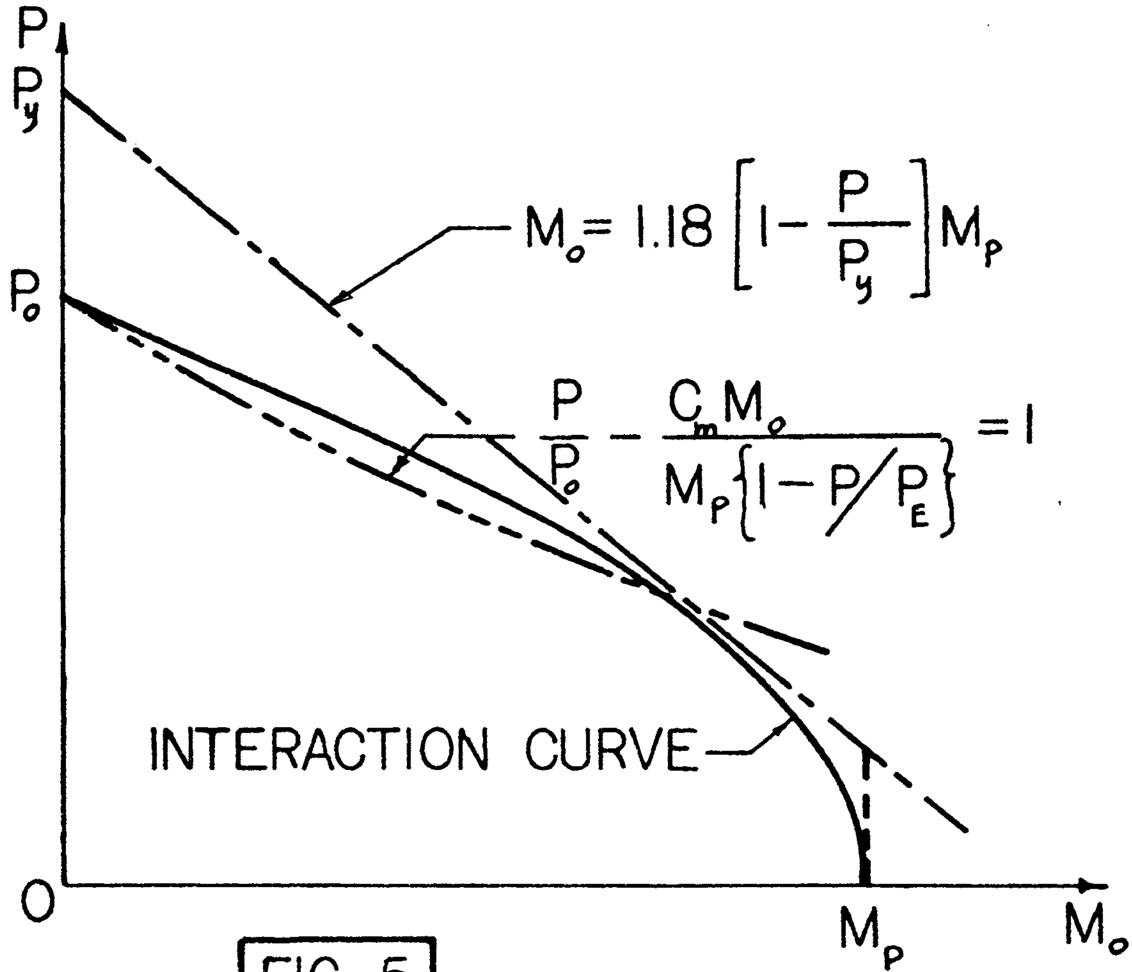

P
P_y
P_o
$M_o = 1.18\left[1 - \frac{P}{P_y}\right]M_p$
$\frac{P}{P_o} + \frac{C_m M_o}{M_p\{1 - P/P_E\}} = 1$
INTERACTION CURVE
O
M_p
M_o

FIG. 5

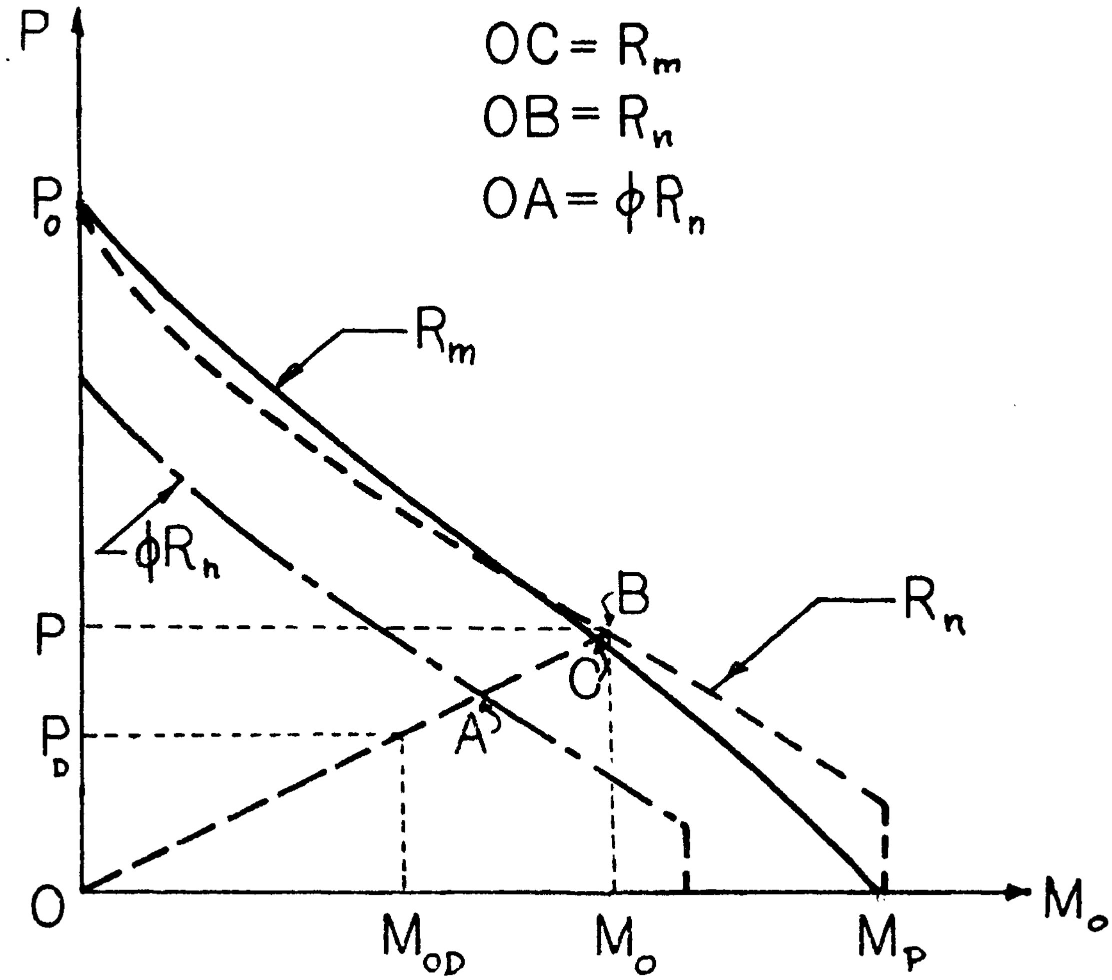


FIG. 6

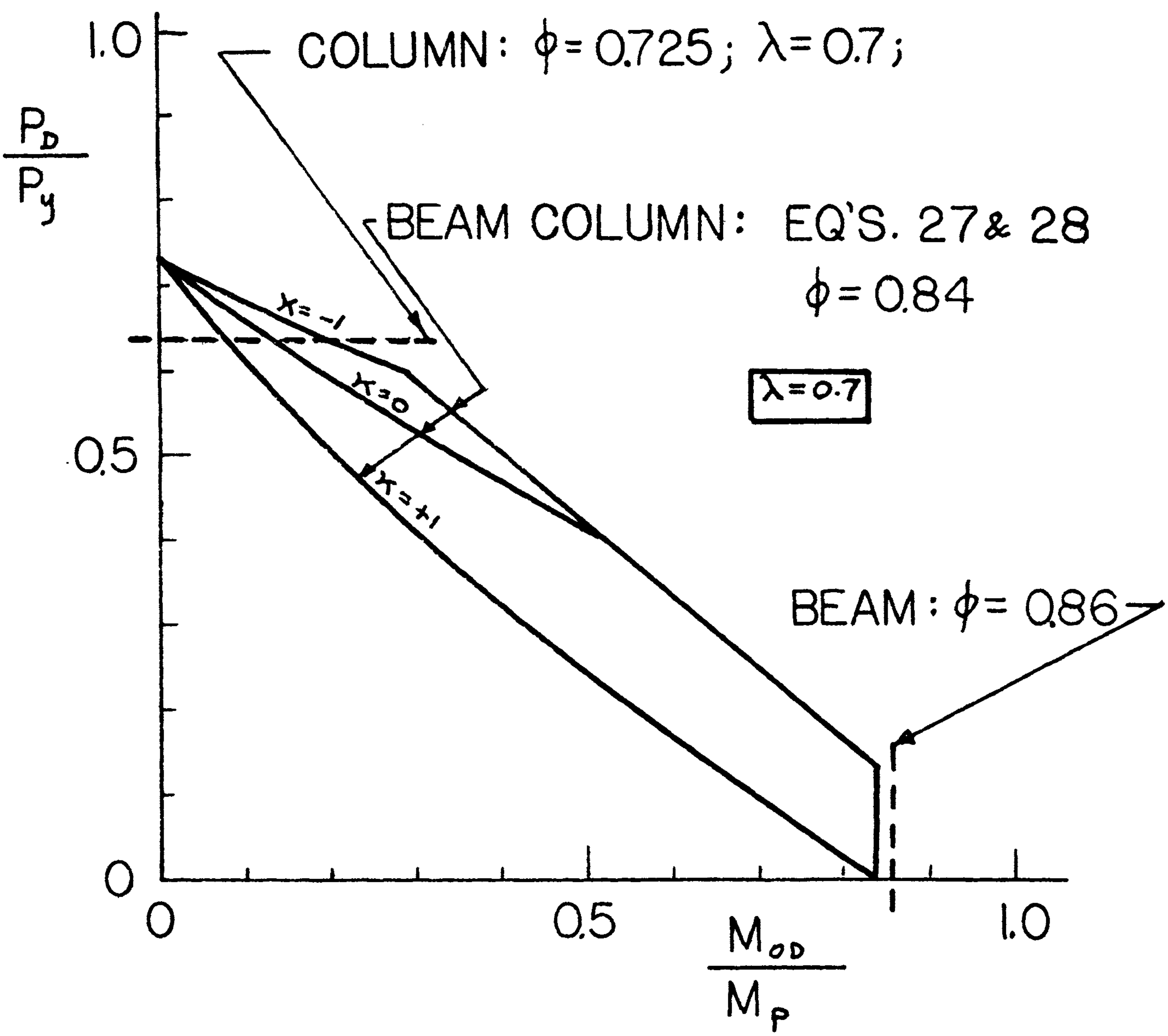

COLUMN: φ = 0.725; λ = 0.7;
BEAM COLUMN: EQ'S. 27 & 28
φ = 0.84
λ = 0.7
BEAM: φ = 0.86
x = -1
x = 0
x = +1
1.0
0.5
0
0
0.5
1.0
P_D / P_y
M_OD / M_P

FIG. 7

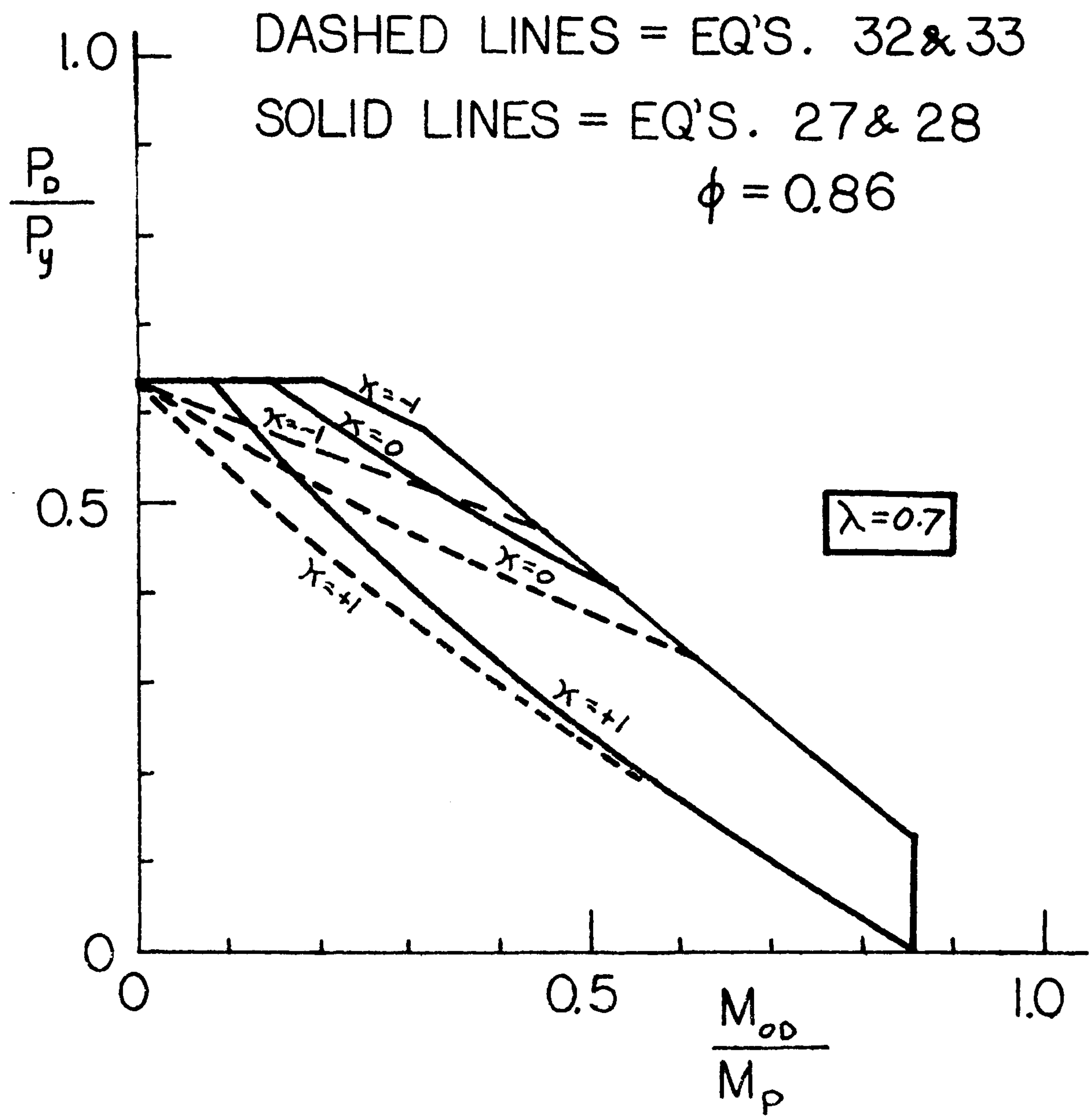

DASHED LINES = EQ'S. 32 & 33
SOLID LINES = EQ'S. 27 & 28
φ = 0.86
λ = 0.7
1.0
0.5
0
0
0.5
1.0
Pp/Py
Mop/Mp
x = -1
x = 0
x = -1
x = 0
x = +1
x = +1

FIG. 8