Design of Cold-formed Hyperbolic Paraboloid Shells

Peter Gergely

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by

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Introduction

The architectural features of hypar shells are well known. If cold-formed deck elements are used to form the roof, several advantages exist, such as the low dead load to live load ratio and the suitability of cold-formed decks to carry the in-plane shears which exist in hypars. These factors allow relatively large spans. Small structures, such as service stations and drive-in facilities, may be covered using simple standardized warped shell units. Basic hypar elements can be combined in a large variety of ways to produce attractive roofs with skylights and with variable column spacings to cover large areas (Fig. 1).

Three basic, equally important elements make up a cold-formed hypar roof: the orthotropic deck, the edge members, and the connections. The design is controlled either by stresses or by stiffness. If the stiffness is low, excessive deflections or buckling may develop.

The design of deep hypars (i.e. those with large curvatures or span-rise ratios) is not difficult since in such structures the uniform membrane shear stresses control. In the case of shallow roofs, deflections, buckling loads, and bending stresses must usually be investigated, which all depend strongly on the three factors (deck properties, edge members, and connections) mentioned above.

This paper summarizes an extensive analytical and experimental investigation of cold-formed hypar structures, conducted at Cornell University under the sponsorship of the American Iron and Steel Institute. The results of other studies are also discussed. The details of the investigation are reported in References 1, 2, 3, and 7, and are not repeated here. The purpose of this presentation is to give design guidelines to the engineer. Unless otherwise stated, the information reported here stems from the study conducted at Cornell University.

Stiffness Properties of Hypars

The effective shear stiffness of flat cold-formed diaphragms depends strongly on the connections between the panels of the deck and between the deck and the edge members. If two layers of decking are used, the second layer may be connected directly to the edge members or only to the first layer along the perimeter; the latter solution entails a loss of shear stiffness. At present the shear stiffness must be evaluated experimentally.

The effective shear rigidity of plane diaphragms is reduced in the case of hypars due to curvature. Tests indicate variable reduction but a value of about 20% seems to be an acceptable assumption for all cases. The deflections, bending stresses, and the buckling load depend strongly on the value of the shear stiffness.

Frequently, deflections control the design of cold-formed hypar roofs. The deflections at unsupported corners (point A, Fig. 1) or in the middle of shallow decks may be excessive. A number of elements contribute to the stiffness of a hypar structure: a) the shear and bending stiffnesses of the deck, b) the type of connection between the panels of the deck, the deck and the edge members, and between the decks if more than one layer is used, c) the bending and axial stiffnesses of the edge members, d) the geometry (curvature) of the structure, e) the eccentricity of the shear force transmitted from the deck to the edge members. The complex interaction of these factors precludes a simple method of calculation of the deflections of hypars. Finite element of finite difference methods need to be used.

Forces in Hypar Structures

The shape equation of a basic hypar element is

$$z = \frac{c}{ab}xy$$

as illustrated in Figure 2. The cold-formed steel panels are warped individually and placed along the straight lines x = const. or y = const. It is relatively easy to warp the panels if the curvature of the shell is not large.

The basic state of stress in a hypar shell subjected to uniform loading q is that of uniform shear

$$S = \frac{ab}{2c} q \; . \tag{1}$$

where S = internal shear force per unit length.

The force in each of the edge members increases linearly from zero at one end to a maximum at the other. If the shell is deep (say the span to rise ratio is at most 5 or 6), the membrane forces given by Eq. 1 closely represent the actual state of stress in the shell. In such a case the edge members, the connections, and the strength of the deck may be designed for the membrane forces.

When the deck is connected to an edge member in such a way that the latter receives an eccentric axial force, bending is produced in the edge member. This bending and the resulting curvature can be controlled or modified as required by specifying the eccentricity with care.

Finite element analyses and tests showed that the deviation from the uniform membrane state of stress is usually not great and the design of all connections may be based on this force, unless stiffness (deflections or buckling) controls the design.

Obviously, as the curvature (c/ab) of the shell becomes small, an increasing share of the load is carried to the edge members by bending of the deck. While the rise to span ratio is not a correct measure of the curvature, a ratio of c/a = 1/5 is often cited as delineating shallow vs. deep hypars. A better parameter is $\alpha tc/ab$, where α is a non-dimensional measure of the effective shear rigidity of the deck ($\alpha = G_{eff}/G$) and t is the thickness of the deck.

The design of the edge members can usually be based on the axial stresses resulting from the membrane theory. Part of the deck (effective width) participates in carrying the force in the edge member along the strong direction of the orthotropic deck but not in the weak direction (normal to the deformations or

a) presented at the First Specialty Conference on Cold-Formed Steel Structures, August 19-20, 1971, Rolla, Missouri

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corrugations) of the deck. The actual force in the edge members is generally smaller than that given by the membrane theory since there is some bending in the deck and in the edge members. The axial and bending stresses in an umbrella shell from a finite element analysis are shown in Fig. 3. The results are for a 12 ft by 12 ft structure with a rise of 1.2 ft. Thus, the span to rise ratio is 6/1.2 = 5.0. The edge members were made of 1" dia. standard pipes and the decking consisted of two layers of 28 gage standard corrugated sheets.

It is seen that considerable bending exists near the column. Part of the bending stresses are due to the eccentric axial force. The membrane theory gives edge member bending stresses which vary linearly from zero at one end to 14.8 ksi at the other. This value is affected by the contributing resistance of the deck (again depending on the direction of deck deformations) and by the torsional stiffness of the connecting edge members. The rest of the bending is caused by the vertical shears transmitted by the deck to the edge members which, in the present case, adds to the eccentricity effect in the tension members and counteracts it in the compression members. The vertical shear effect (or frame action) is relatively large in the present case since the edge members are small and very flexible.

Bending is significant when partial or concentrated loads act. This is especially serious if only a single layer of decking is used. Tests on 5 ft. by 5 ft. saddle hypars with fully supported edge members and with standard corrugated sheet decking showed that the local deflection under an 8 in. by 12 in. loaded area were nearly three times greater in the case of single decks than with double decks (with the corrugations of the two layers running perpendicular to each other). However, tests at Cornell and by McDermott (Ref. 5) showed that a single layer of deck may be satisfactory under uniform loading.

In addition to membrane forces and bending, part of the load is also carried by in-plane tensile forces along the deformations or corrugations of the deck. Tests and analyses indicate that the stiffening effect of the in-plane direct forces is generally small, especially if the bending stiffness of the edge members in the plane tangent to the deck is relatively small. If the edge member is stiff and bends little inward, in-plane forces can develop under large loads, which reduce the bending stresses. In such a situation the connectors between the deck and the edge member receive this additional force, which is another reason for using strong connectors.

Deflections of Hypar Structures

The design of several common types of hypars is often controlled by deflections rather than by stresses. If the span is large, the edge members are stiff, and the shell is relatively shallow, the center portion of the deck may bend and deflect relative to the edge members. This problem is rare since in such cases the construction of a large deck without intermediate beam members would be difficult.

A more frequent problem is the local deflection of flat corners in some types of structures, such as the inverted umbrella type (Fig. 1a). The flat deck is unable to carry all the load near the corner by in-plane shear and, if the edge members are not stiff enough, the corner bends down. Tests and finite element analyses showed that this corner deflection is due primarily to loads near the corner; in fact, loads near the column of an umbrella structure produce upward corner deflections. The problem of excessive corner deflections can be corrected by increasing the bending stiffness of the edge members, by increasing the curvature of the shell, and by connecting the deck to the edge members with an eccentricity such that the edge members curve upward (e.g. placing the deck below the edge members in umbrella structures). It may also be feasible to design diagonal ties connected to short posts above the corners or by sloping the ties to the upward extension of the central column. Obviously, the simplest solution is to use posts under the corners, if the architectural scheme permits.

The weight of the edge members is carried partly by the deck and partly by cantilever or frame action of the members. If the shell is shallow, the former is small and the deflection due to edge member weight may be noticeable.

The large number of factors, listed above, that influence the behavior of cold-formed hypars does not permit simple evaluation of the deflections. The finite element analysis program developed at Cornell University, which will be made available through the AISI, will enable the designer to predict deflections. For routine applications or where deflections do not control, such complex computer analysis is not necessary.

Buckling

Two types of instability may occur in hypars: the deck may buckle, similarly to a shear diaphragm, or the entire structure may fail due to the beam-column action of the compression edge members. The finite element analysis of a number of hypars indicated that the latter type (called overall buckling) does not occur in otherwise well designed structures. A third type of instability, the local buckling of the flat plate elements of the deck is not considered in this paper since it is similar to that occurring in flat shear diaphragms (Refs. 5 and 6).

Deck buckling is an important design problem for shallow shells and for decks with low effective shear rigidities. The finite element analysis procedure predicts the non-linear load-deflection diagram of the structure from which instability can be estimated. This is a rather lengthy and therefore expensive computer calculation. However, analyses of several structures and also experiments by Leet (Ref. 4) showed that the bending stiffness (i.e. the deflections) of the edge members do not affect the deck buckling load appreciably. In fact, buckling depends on the area of the edge members. An alternative, simple analysis using energy principles was also developed (Ref. 2). The total energy of the orthotropic deck was evaluated using an assumed deflected shape

$$w = \sin \frac{\pi y}{b} \sin \left[\frac{n\pi}{a}(x - sy)\right] \qquad (2)$$

where s is the tangent of the angle of the buckle waves measured from the y axis and n is the number of waves. This analysis resulted in the following critical shear force:

$$S_{cr} = \frac{c}{abs}\left(\frac{\pi}{b}\right)^2 [D_x u^2 + 2D_1(1+u^2s^2) + D_y(1/u^2 + u^2s^4 + 6s^2) + 4D_{xy}(1+u^2) + g/u^2] \tag{3}$$

where

$$u = \frac{nb}{a} \text{ and } g = \frac{4G_{eff}t}{\pi^4}\left(\frac{bc}{a}\right)^2. \tag{4}$$

The minimization of S_{cr} with respect to s and n can be done by trial and error using a computer. The energy approach and the finite element deck buckling analysis both agreed well with tests on 12 ft. by 12 ft. inverted umbrella shells. The buckling load of double-layered decks was found to be about 3 to 4 times greater than that of single decks.

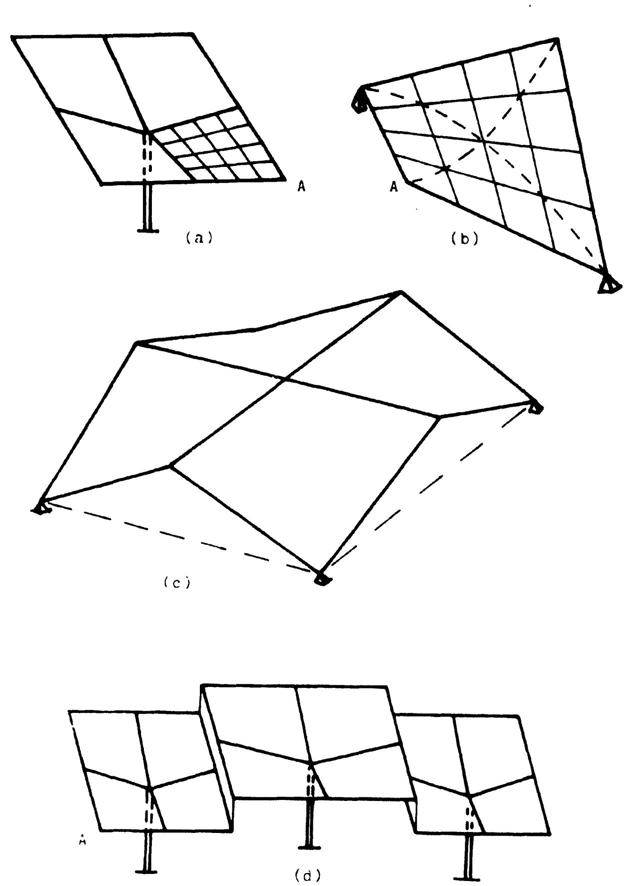

FIG. 1 HYPAR STRUCTURES

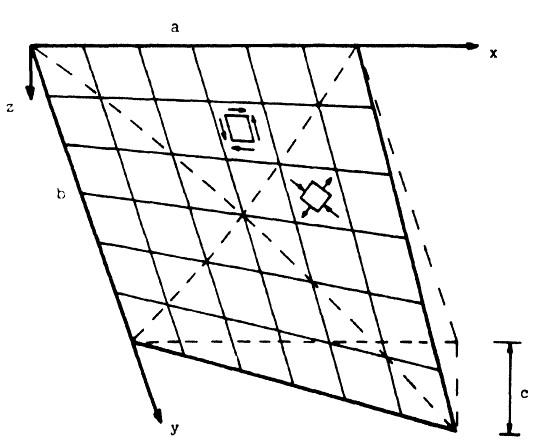

FIG. 2 BASIC HYPAR UNIT

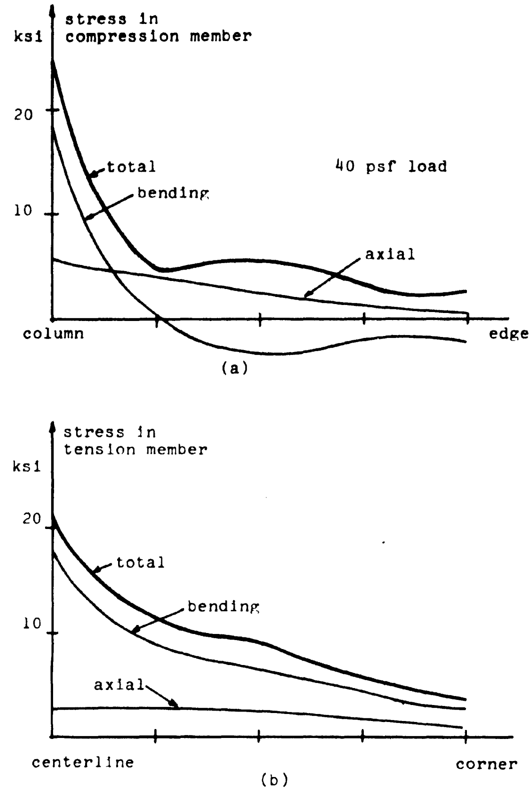

FIG. 3 CALCULATED STRESSES IN THE EDGE MEMBERS OF AN UMBRELLA STRUCTURE

Summary and Conclusions

The design of cold-formed hypar structures is usually controlled by stiffness rather than by stress. The primary factors affecting stiffness are the curvature (rise to span ratio) of the shell and the effective shear rigidity of the deck. The latter depends strongly on the connections between deck panels and between the deck and the edge members. If the stiffness is low, excessive deflections or deck buckling may develop. For certain types of hypars, unsupported flat corners (point A, Fig. 1) may deflect excessively.

The forces in the elements (deck, edge members, connectors) can usually be estimated from the simple membrane theory. However, if the structure is shallow, appreciable bending may occur. In such cases, and when deflections are desired, a finite element analysis program can be used.

An expression, based on an energy approach, provides an estimate of the deck buckling load which can be used in lieu of the finite element buckling analysis.

The guidelines presented in this paper should enable engineers to design attractive, light-weight roofs for many applications.

Acknowledgments

This presentation is based on an extensive investigation conducted at Cornell University with the financial support and technical guidance of the American Iron and Steel Institute. The project has been under the general direction of George Winter. A number of graduate students worked extensively on this program: A. Banerjee, J. A. Parker, P. V. Banavalkar, and R. Muskat. Their help, contribution, and hard work is gratefully acknowledged.

APPENDIX I - REFERENCES

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APPENDIX II - NOTATION

a, b	Plan dimensions of a unit of hypar surface
c	Rise of shell
D_x, D_y, D_{xy}, D_1	Rigidities of orthotropic shell
g	Constant defined in Eq. 4
G	Shear modulus of material
G_{eff}	Effective shear modulus of deck
n	Number of decks or Number of buckling waves
q	Uniform vertical loading on the shell
s	Tangent of the angle between buckles and y axis
S	Membrane shear force in the deck
S_{cr}	Critical membrane shear
t	Thickness of deck
u	nb/a (Eq. 4)
w	Normal deflection of the shell
α	Shear rigidity factor, G_{eff}/G