Behavior of Channel and Z-section Beams Braced by Diaphragms

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BEHAVIOR OF CHANNEL AND Z-SECTION

BEAMS BRACED BY DIAPHRAGMS

by

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SCOPE

This paper describes some of the results of current research that is in final stage of completion. The behavior of channels and Z-beams loaded in the plane of the web and braced by shear-rigid diaphragms either along the compression flange or along the tension flange has been investigated. This situation arises, for instance, in channel and Z-purlins in which case compression flange bracing corresponds to the case of downward gravity loading, and tension flange bracing to uplift loading due to wind suction.

The research is aimed at obtaining mathematical solutions for various boundary conditions, test verification and design formulations.

In this paper the physical behavior rather than the details of analytical modeling and solutions will be emphasized. In addition, possibilities for design formulations will be enumerated.

INTRODUCTION

Torsional-flexural behavior of thin-walled prismatic members of open section has been studied by a number of investigators.

The most extensive investigation of the subject was performed by Vlasov (15)[4]. He derived the differential equations for stability of thin-walled sections under general loading conditions and suggested their solution by Galerkin's Method. Vlasov treated also the torsional bending of thin-walled open sections. The concepts of bimoment and flexural twist have been introduced by him. However, he did not consider the coupling of flexural and torsional bending but treated them separately and superposed the resulting stresses. The practical implications of his findings were discussed by K. Z. Koscia(19).

Combined torsional-flexural bending has been investigated at Cornell by Lansing(8), 1949 and McCalley(12), 1952. McCalley has derived pertinent differential equations in non-principal coordinates. He also studied the second order terms in the longitudinal strain expression and found that under non-uniform torsion the cross section does not rotate about the shear center but about the "rotation center" which is defined in Ref. 12. However, he has found that these second order terms can be neglected for engineering purposes.

Torsional-flexural bending of channel beams braced by discrete braces has been investigated by Winter, Lansing and McCalley(18). They presented a simple method to determine the spacing and strength of bracing to counteract the twisting tendency of such members. Zetlin and Winter(20) have given a design method for Z-beams with and without lateral bracing under unsymmetrical bending, when there is no primary torsional load and the amount of twist is restricted so that the secondary torsional moments can be neglected.

Beams and columns are often braced by other elements of the construction. There is a large volume of research dealing with the stability of discretely or continuously braced beams and columns. Goodier studied the stability of a bar attached to a flexible sheet which prevented the displacements in the plane of the sheet. Vlasov presented the differential equations for the stability of thin-walled beams, continuously braced by elastic springs against displacement and rotation.

Winter(17) also studied the stability of braced members. He has developed a method to obtain the lower limits of the strength and rigidity of lateral bracing which provide "full bracing" of beams and columns. The term "full bracing" means that the bracing is equivalent to an immovable lateral support. Larson(9) extended Winter's analysis to shear type lateral supporting media. In this case the restraint is a function of the slope of the member rather than the lateral deflection itself.

The behavior of shear diaphragms has been investigated by Luttrell(10,11) and Pincus(13). The two important characteristics of the bracing are its shear rigidity and shear strength. There is yet no method available to compute these characteristics directly from the geometrical and material properties of the diaphragm and its fasteners. However, proper test procedures have been devised to measure these quantities.

Since 1961 diaphragm-braced columns and beams have been subject to research at Cornell University by G. Pincus(13,14), S. Errera(6,7), and T.V.S.R. Apparao(3,4).

GENERAL THEORY OF DIAPHRAGM-BRACED BEAMS

Diaphragms are frequently used for roofing or wall sheathing of industrial buildings. At the same time they provide bracing to the individual roof beams or columns, thereby increasing their strength and/or stability and reducing their deflections. Fig. 1 illustrates a typical roof assembly involving diaphragm braced Z-section beams.

The main types of loading on such structures are gravity loads and wind suction as indicated in Fig. 2. They are transmitted from diaphragm to the member by bearing for downward loading and through the connectors for uplift loading cases. In Figure 3 notation for point of application of load is illustrated.

Diaphragms usually consist of thin-walled corrugated or stiffened orthotropic steel panels. Due to the orthotropic characteristics, it can be assumed that the axial stiffness is infinite along the corrugations but zero in the direction perpendicular to the corrugations. Hence, along the latter direction no axial force or bending moment (lying in the plane of the diaphragm) can be carried by the diaphragm. Thus, a beam type behavior of

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4 Numerals in parenthesis refer to the corresponding items in Appendix I - References.

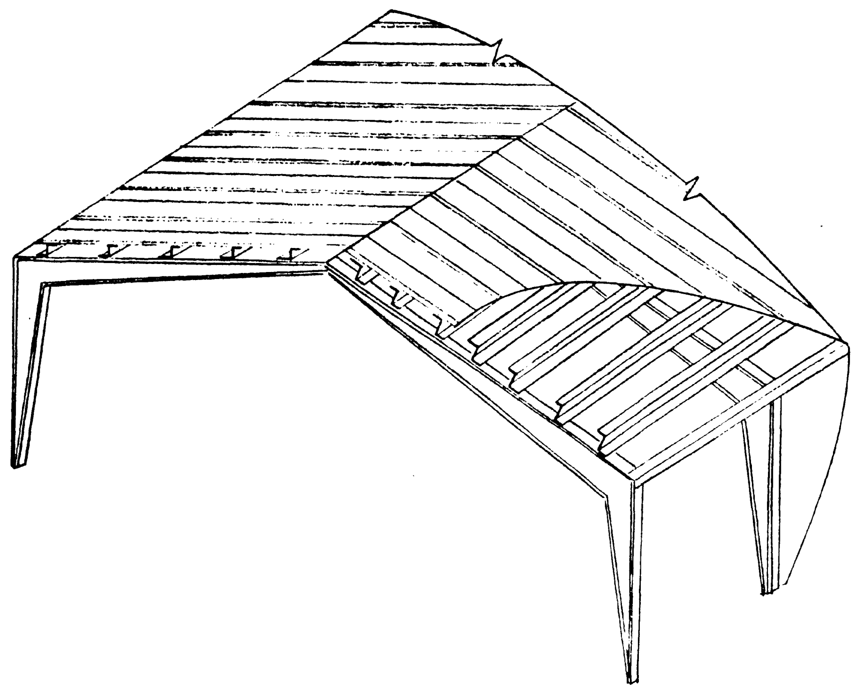

FIG. 1 Roof assembly with diaphragm braced Z-section purlins

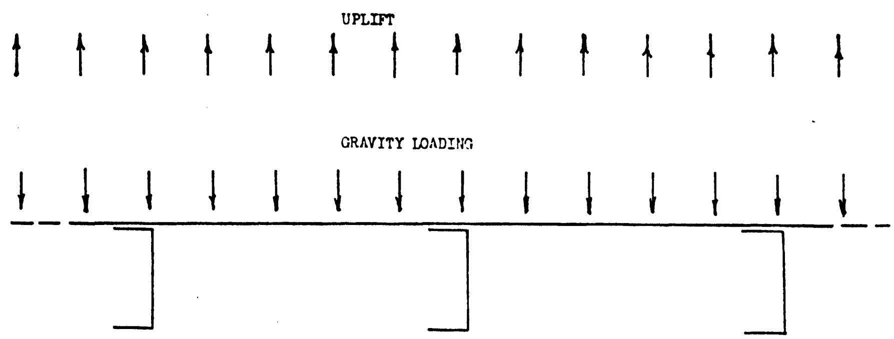

FIG. 2 Possible Loadings

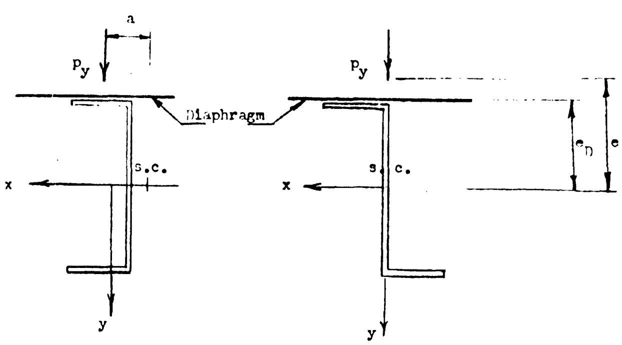

FIG. 3 Notation for the point of application of load

the diaphragm is neglected. The bracing capacity of a diaphragm in its plane is then only due to the shear strength and shear rigidity.

It should be noted that shear type deflection of the diaphragm is not only due to actual shear strains in the material. Cross sectional deformations of the diaphragm and deflections at the fasteners generally contribute the larger portion of the total shear deformations. Hence, shear stiffness depends on several factors, such as cross sectional configuration, the length of the diaphragm along the corrugations, fastener type and spacing, etc.

When the beam deflects sideways, the diaphragm which is connected to it undergoes shear deflections. Consequently, shear forces arise in the diaphragm (Figure 4a). The rate of change of the shear force along the braced member is equivalent to a distributed load (Figure 4b) acting on the beam, restraining its deflection in the plane of the diaphragm bracing. Previous research at Cornell indicates that this idealization is adequate and satisfactory to describe the behavior of diaphragm braced members within engineering accuracy.

The cross-bending rigidity in a direction perpendicular to the corrugations can be neglected. Along the corrugations, however, there is a finite cross bending stiffness denoted F, which may provide rotational-bracing to the member if the corrugations are perpendicular to the member.

The differential equations of the equilibrium of diaphragm braced I, channel and Z-section beams loaded in a plane parallel to the plane of the web have been derived in Ref. 5. These differential equations are solved in the same reference and the displacements u and ϕ (Figure 5) are found in series form. The

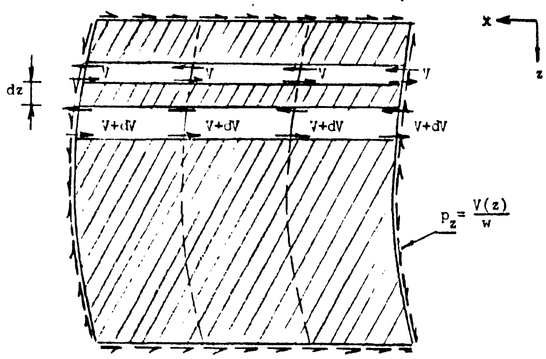

FIG. 4a Shear stresses and forces acting on the diaphragm bracing

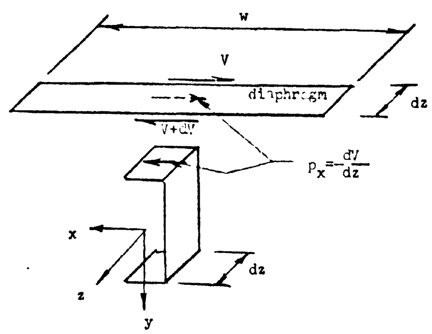

FIG. 4b Lateral force p_x between the beam and the diaphragm bracing

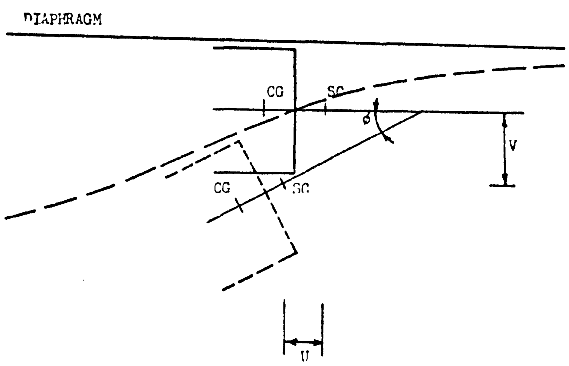

FIG. 5 Deformed configuration

longitudinal stress σ_x is given by the following equation, where the curvatures are determined by taking the second derivatives of the series solution.

$$\sigma_x = \frac{M_x}{I_x} y - \frac{E}{L^2} [(x - \frac{I_{xy}}{I_x} y) u'' + \omega \phi''] \quad (1)$$

where ω = Warping displacement (or Sectorial area)

PHYSICAL BEHAVIOR

Computer programs have been developed to determine the yield loads of diaphragm-braced plain and lipped channel and Z-beams with hinged ends under uniformly distributed downward and uplift loads. The rotational restraint, F, has also been included in the formulation. Neglecting the rotational restraint of the bracing, particularly for beams with large spans, leads to excessively conservative results. For the sake of simplicity the discussion here will be confined to the case F=0. The significance of different values if F will be discussed in detail in a future publication. The shear stiffness of the diaphragm may assume any positive value including the limits Q=0 (no bracing) and Q=∞ (rigid bracing).

The coupling of bending and torsion generally results in a nonlinear relationship between load and stresses. Hence, the yield load cannot be found directly, but through iteration. In the computer examples the yield stress was taken equal to 33 ksi. However, since the maximum stresses generally appear only at corners of the sections, 15% overstressing is allowed. This was first proposed in Ref. 18 and has been incorporated into AISI specifications [1,2,16]. Hence, theoretical failure is defined as the load resulting in a maximum localized stress of 37.95 ksi.

The dimensions of the members involved in the discussion here are given in Figure 6.

Figures 7 and 8 show the lateral deflections and the angles of rotation at mid span for the earlier mentioned sections for uplift loading with L = 60". It is seen that Z-section beams display in general less rotation than channel beams with identical dimensions. However, when the diaphragm stiffness is increased, there is not much difference between them. It is also observed that u_o for channels and ϕ_o for Z-sections are tangential to the load axis at the origin. (Subscripts o and ∞ refer to no and rigid bracing cases, respectively). This is so because there is no load component at the outset to cause these deformations. Slight non-linearity can be observed in these diagrams. The reason is that new components of the loads are created by the deformation u and ϕ of the beam. The effects of these components may add or partly cancel out from case to case.

In general, for uplift loading, both diaphragm-braced or unbraced channel and Z-beams will have pL - ϕ diagrams with decreasing slopes at failure. No examples for the downward case are included here. However, for the unbraced case the pL - ϕ diagrams will be more non-linear and will have a decreasing slope as above. On the other hand for the rigidly braced case a stiffening-type pL - ϕ curve will emerge for downward loading. (This will also be the case for a reasonable stiffness of the diaphragm bracing).

On Figures 9 and 10 the ratio M/M_{bend} versus span length is shown for no and rigid bracing cases. M is the bending moment at midspan due to uniformly distributed load p, i.e., $M = pL^2/8$. It is compared here with the theoretical bending capacity $M_{bend} = \sigma_y I_x/e$. M_{bend} represents the capacity of a beam which is guided or braced such that the only possible deformation is bending in the plane of the load with no rotation or lateral deflection. Thus, M_{bend} for I, channel and Z-section beams of the same cross-sectional dimensions is the

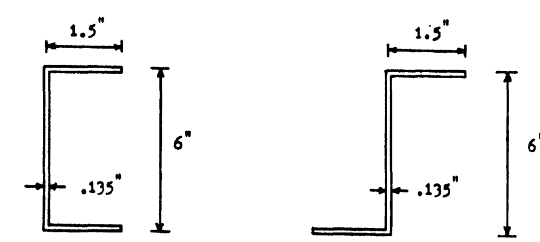

FIG. 6 Dimensions of sections discussed in this paper

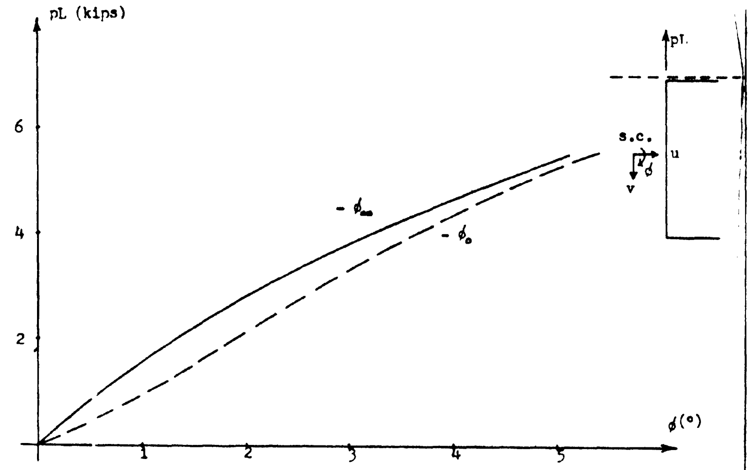

FIG. 7a Mid-span rotation ϕ versus pL upto theoretical failure
Uplift, L = 60"

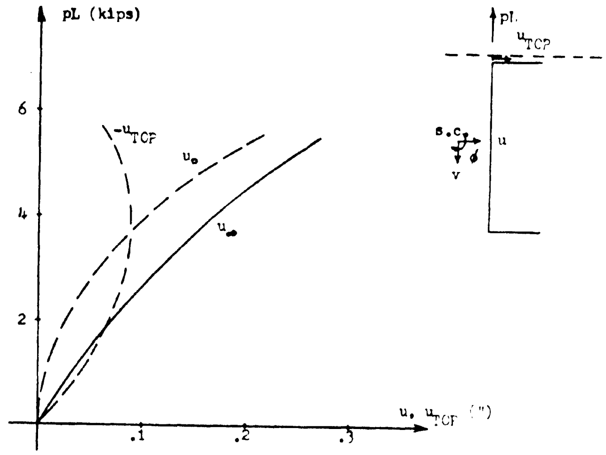

FIG. 7b Lateral deflection u versus pL upto theoretical failure
Uplift, L= 60"

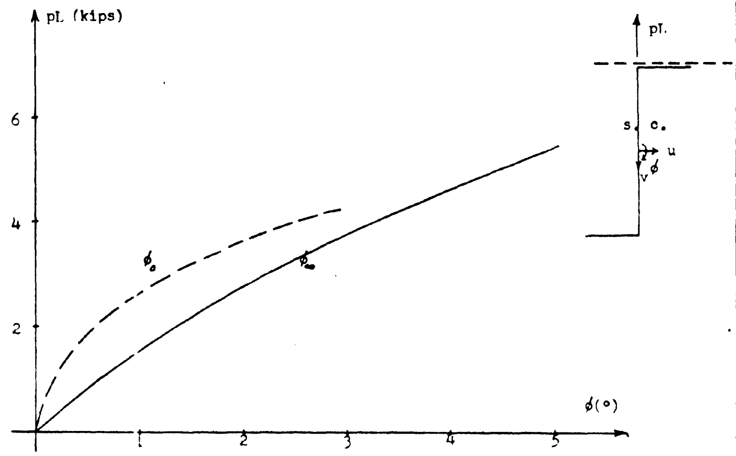

FIG. 8a Midspan rotation ϕ versus pL upto theoretical failure
Uplift, L = 60"

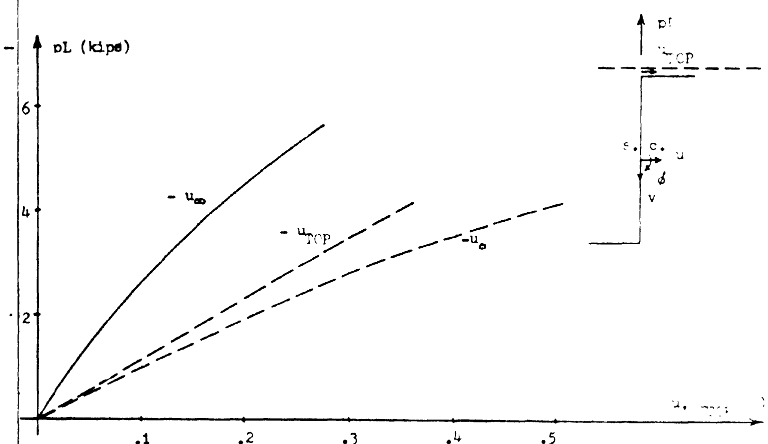

FIG. 8b Lateral deflection u versus pL upto theoretical failure
Uplift, L = 60"

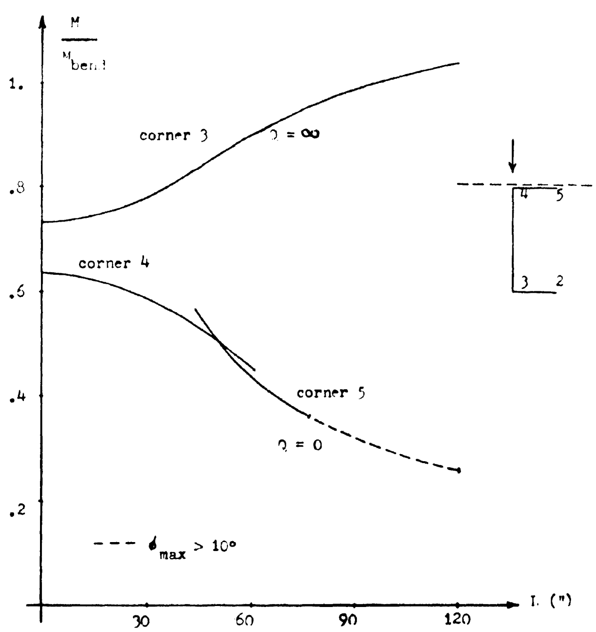

FIG. 9a Comparison of the moment M when stress reaches 1.15 σ_y
with yield moment when twist is restrained M_{bend}
Gravity Load, Channel

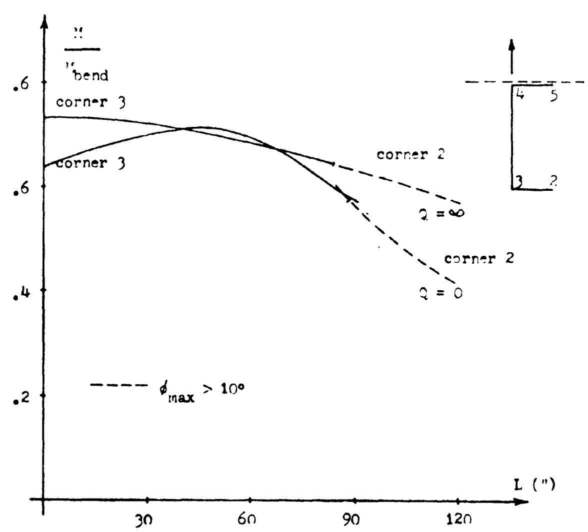

FIG. 9b Comparison of the moment M when stress reaches 1.15 σ_y
with yield moment when twist is restrained M_{bend}
Uplift, Channel

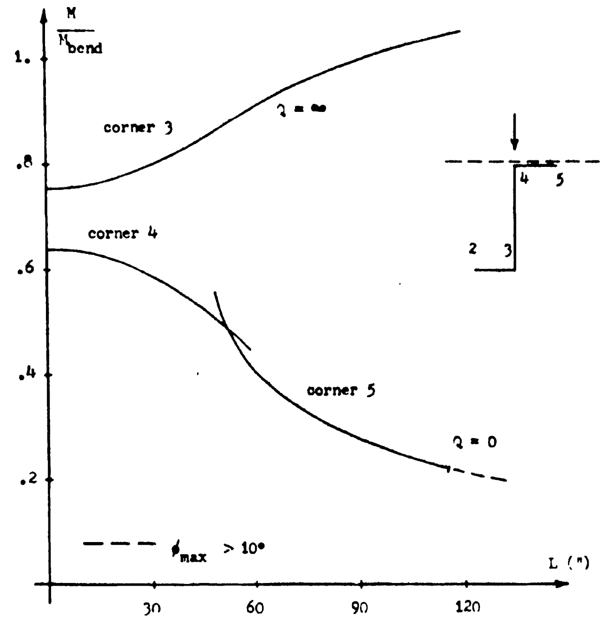

FIG. 10a Comparison of the moment M when stress reaches 1.15 y with yield moment when twist is restrained M_{bend} Gravity Load, Z-section

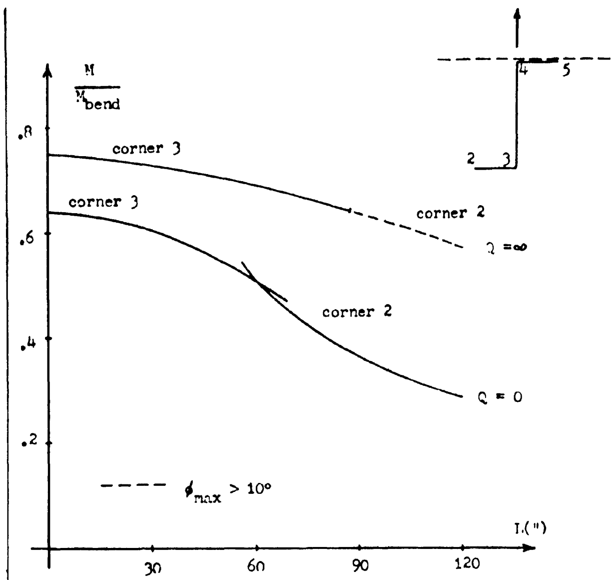

FIG. 10b Comparison of the moment M when stress reaches 1.15 σ_y with yield moment when twist is restrained M_{bend} Uplift, Z-section

same. (Of course for an I beam there is no need for guides or bracing provided that M_{bend} does not cause instability). The numbers in circles show where in the cross section the stress has reached 37.95 ksi. The dotted lines indicate where the maximum angle of rotation is larger than 10°, assumed here as an arbitrary practical limit. Both uplift and downward loading cases are presented.

On these figures, it can also be observed that for downward loading the diaphragm bracing causes a definite increase in yield load capacity of channel and Z-beams. For the uplift case however, only Z-sections show definite improvement due to bracing. For channel beams under uplift loading the yield load capacity may slightly increase or decrease, due to bracing.

This puzzling behavior can be explained qualitatively with the aid of Figures 11a and b. First a channel beam without bracing will be considered. For this case on Figure 11a the uplift load is decomposed into three components with respect to the deformed configuration. The signs of the corresponding component stresses are also indicated in this figure and it can be observed that all component stresses at corner 3 are of the same sign. Thus, it is concluded that for this loading the stress at corner 3 will govern the initiation of yielding. As far as the displacements are concerned, the upper flange has a tendency to move to the right due to component B on Figure 11a and to the left due to component C. If the load is increased continuously from zero until yielding, it would be observed that at first ϕ will be small and component B can be neglected. Therefore, at first, the upper flange will move to the left. The bracing forces corresponding to this displacement if the beam were connected to a diaphragm along its upper flange is illustrated in Figure 11b. The diaphragm force component B' adds to while component C' subtracts from the corresponding components in Figure 11a. (Component A' of the diaphragm forces is of higher order and can be neglected in this discussion). The stress at corner 3 will be affected similarly. If the stress at corner 3 due to the bracing force components B' is larger than the stress due to bracing force component C', then the stress in the braced case is likewise larger than the unbraced case. The reverse is true if component B' is smaller than C'. Thus, if the former case leads to yielding, then the yield load capacity of the braced beam will be smaller than that of unbraced beam. In the latter case the reverse will be true. However, since for small values of ϕ the stresses due to bracing force components B' and C' are of the same order of magnitude, their difference will be small and the net effect of bracing on the yield load capacity will also be small.

If yielding does not occur at relatively small ϕ and the load is increased further, the force component B of Figure 11a will become dominant and eventually the upper flange will move to the right. This in turn changes the sign of the bracing forces. Now component B' of Figure 11a will be reduced while C' is increased. The stress at corner 3 in the braced case may again be larger or smaller than that of the unbraced case. Hence, the yield load capacity again may be increased or decreased by the diaphragm bracing depending on the section geometry, the span length and the magnitude of the yield stress.

The behavior of diaphragm braced channels under downward loading and Z-section beams under both uplift and downward loading cases could be discussed in a similar manner as above. However, intuitively it is clear that for these cases the upper flange of the unbraced beam will move only in one direction with increasing load. Thus, the diaphragm will always restrain this movement, thereby increasing the capacity.

Figures 12 and 13 show the stress distribution in unbraced and rigidly braced channel and Z-beams under uplift or downward loading at failure. Numerical values of failure moments, that

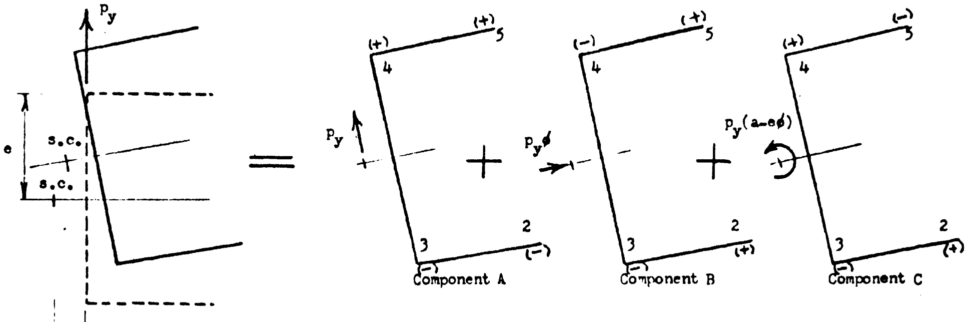

FIG. 11a Components of the vertical uplift load for unbraced channel
(Sign of stress for each component at each corner given in parantheses, + = tension)

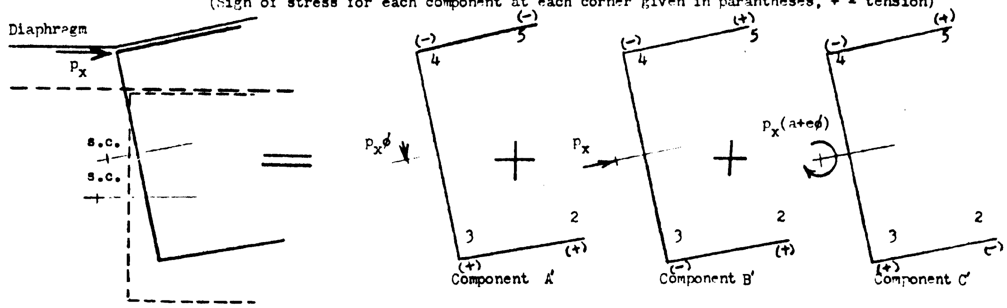

FIG. 11b Components of bracing force
(Sign of stress for each component at each corner given in parantheses, + = tension)

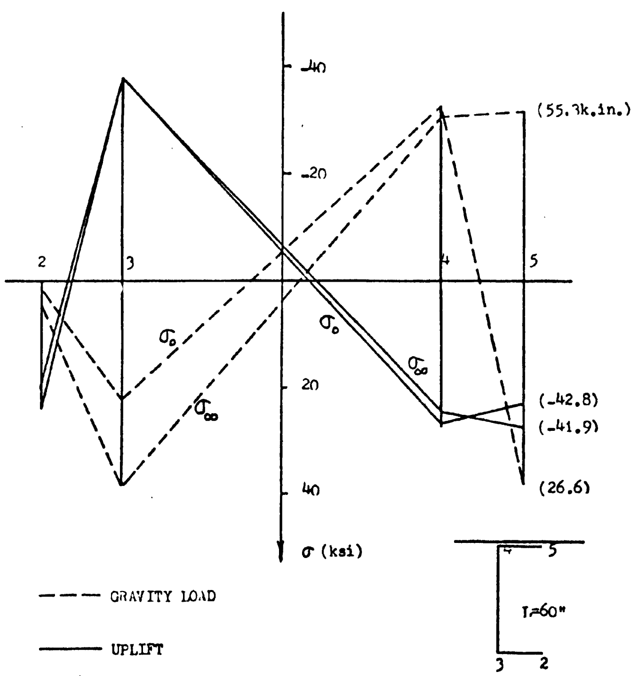

FIG. 12 Stress distribution at failure
(Failure moments in parantheses)

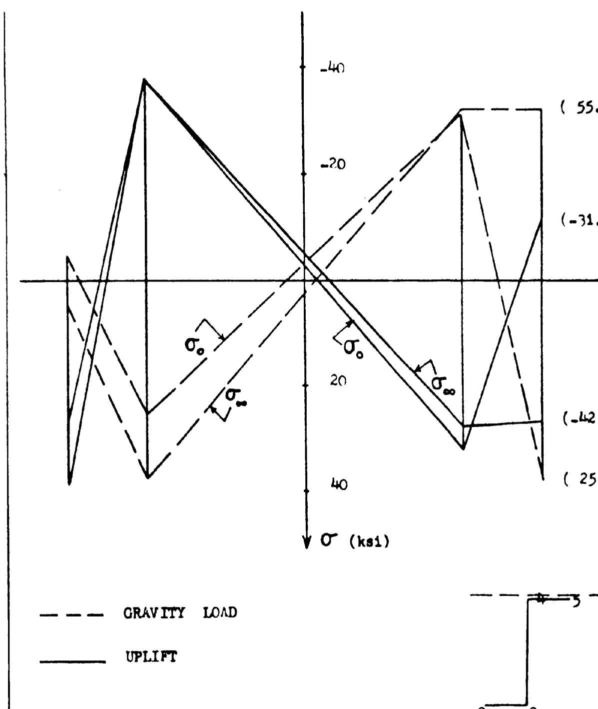

FIG. 13 Stress distribution at failure
(Failure moments in parantheses)

is the moment causing 1.15 times the yield stress, are given in these figures in parentheses. The span length is 60". On Figure 12 we see that bracing has almost no effect on the stress distribution in the channel section for the uplift loading case for this particular span length. This is in accordance with the previous statement on the yield load capacity of channels under uplift load.

Comparing Figures 12 and 13 one sees that rigidly braced channel and Z-beams have similar stress distributions. Since the angle of rotation is almost the same but of opposite sign for channel and Z-beams, their unbraced flange is strained in the same way. In this flange the stress due to twist increases the stress at corner 3 while reducing it at corners 2 and 1. In the braced flange, however, the stress is nearly constant and mainly due to vertical bending.

In Figures 14 and 15, plots of M/M_{bend} versus Q/P_y are given. P_y is the Euler buckling load, that is

$$P_y = \frac{\pi^2 E I_y}{L^2} \qquad (2)$$

These figures consist of several curves each indicating yielding at a particular corner of the cross section. Only the curves determining the yield load are shown.

On these figures it can be seen that, for downward loading yielding initiates at corner 3 for a wide range of values of Q, varying from a relatively small finite value up to infinity. For smaller values of Q, yielding at corner 4 governs. As Q decreases further, yielding at corners 5 or 6 may govern. The value of the diaphragm shear rigidity where the curves for corners 3 and 4 intersect is designated Q_L. It can be observed that there is a rapid increase in the yield load capacity for the range $0 \le Q \le Q_L$. On the other hand, for $Q > Q_L$ the change in the yield load capacity is quite small. Obviously, to provide diaphragm shear rigidities less than Q_L would be uneconomical in practical design. Determination of Q_L and corresponding yield load will be discussed in the section on design simplifications.

POSSIBLE DESIGN FORMULATIONS

A series solution of the differential equations of the diaphragm braced channel and Z-section beams and numerical results have been discussed in the preceding section. On the basis of these solutions, simplifications for design use have been sought. Solutions using only one term of the series have been studied. The objective is to find simple expressions for the optimum shear rigidity of a diaphragm for a given beam as well as yield load and deformations. Consideration of arbitrary values for the shear rigidity Q leads to a cubic equation for the yield load even if only one term of the series is used. However, this cubic equation can be reduced to a quadratic equation for two values of Q for some special cases. These values of Q are Q_∞, that is rigid bracing, and Q_L that is the limiting shear rigidity as defined above. Design formulations on the basis of each of these shear rigidity cases are being studied. The following are the special considerations needed for each case.

When the rigid bracing is used as basis for determining yield loads, then for a finite value of Q the yield load has been over-estimated. A reduction factor must be applied to the yield moment thus obtained and a lower limit for Q such as Q_L should be specified. Such a formulation is valid for channel and Z-sections for both gravity and uplift loads.

As discussed in the preceeding section, Q_L can be used satisfactorily for both channel and Z-sections for gravity loading. The increase in the yield load capacity for the sections is rapid for increasing values of Q for gravity loading if Q is less than Q_L as seen on Figures 14 and 15. However, for values of Q greater than Q_L, the increase in the yield load capacity is insignificant for channel sections and relatively small for Z-sections. A modification for the latter case can be applied to take this increase into account. It should be noted that for Z-sections the simplification obtained by reducing the cubic equation to a quadratic is possible only if the gravity loads act in the plane of the web. For uplift loads, the cubic equation cannot be reduced to a quadratic neither for channel nor Z-sections; thus, formulations for rigid bracing need to be used for uplift loads.

Table I summarizes possible design formulations discussed above. Numeric studies are being carried out on a large number of representative problems to verify the above discussion.

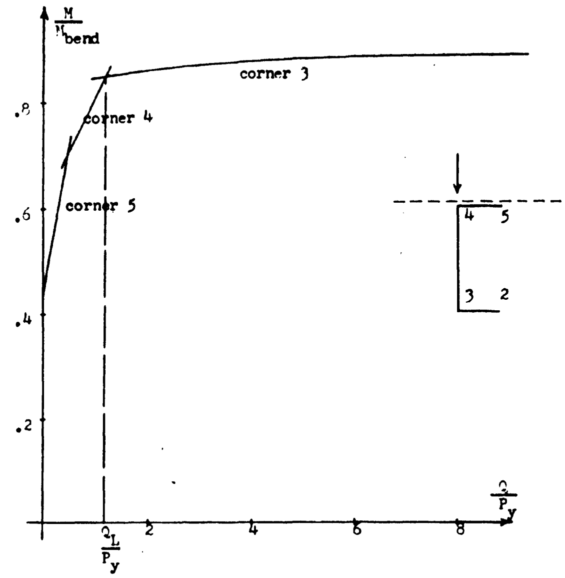

FIG. 14a Comparison of the moment, M, when stress reaches $1.15\sigma_y$ with yield moment when twist is restrained, M_{bend}, versus Q/P_y, gravity load, channel

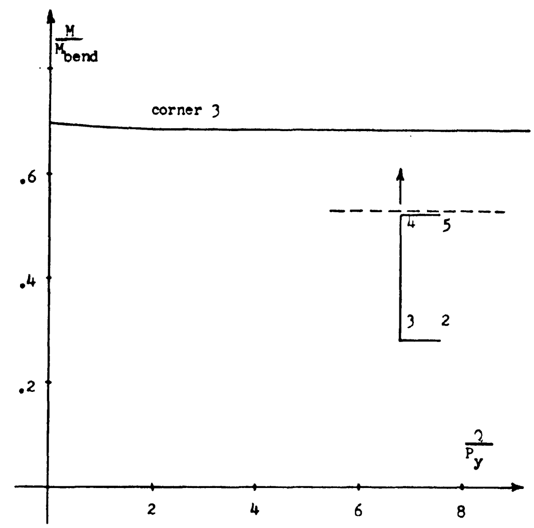

FIG. 14b Comparison of the moment, M, when stress reaches $1.15\sigma_y$ with yield moment when twist is restrained, M_{bend}, versus Q/P_y, uplift, channel

These formulations and studies also include the effect of torsional restraint F briefly mentioned but not discussed in this paper. Charts and simple equations facilitating calculation of the parameters entering into the determination of the yield load and Q_L are being prepared. The results will be reported in a future publication.

TABLE I

Values of Shear Rigidity Q used as Basis for Possible Design Formulations

	Uplift	Gravity
Channel	$Q = \infty$ (1)	$Q = \infty$ (1) or Q_L
Z-section	$Q = \infty$ (1)	$Q = \infty$ (1) or Q_L (2)

(1) with a reduction factor applied to the yield moment

(2) load assumed in the plane of the web

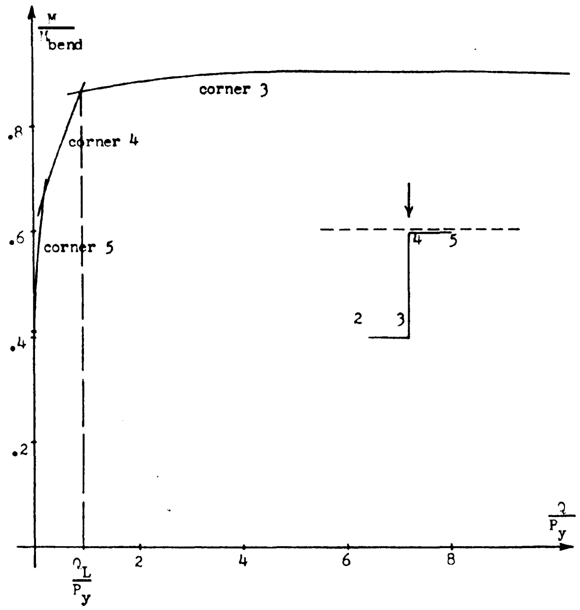

FIG. 15a Comparison of the moment, M, when stress reaches $1.15\sigma_y$ with yield moment when twist is restrained, M_{bend}, versus Q/P_y, gravity, Z-section

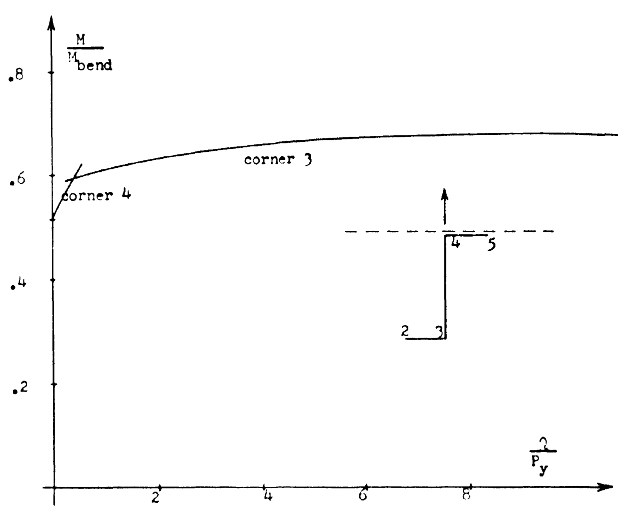

FIG. 15b Comparison of the moment, M, when stress reaches $1.15\sigma_y$ with yield moment when twist is restrained, M_{bend}, versus Q/P_y, uplift, Z-section

EXPERIMENTAL INVESTIGATION

A limited program involving both model and full scale tests was carried out. Its results are in reasonable agreement with analytical results provided that the effect of torsional restraint F is included.

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