A valuation model for two-stage contract negotiations over multiple interdependent issues

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A VALUATION MODEL FOR TWO-STAGE CONTRACT NEGOTIATIONS OVER MULTIPLE INTERDEPENDENT ISSUES

by

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Approved by

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ABSTRACT

Most real-world negotiation scenarios involve multiple interdependent issues over which the negotiating parties will seek an agreement. Simultaneous negotiation over multiple interdependent issues is an especially challenging problem because utility functions of negotiating agents are typically nonlinear and difficult to analyze. Also, negotiations often happen under circumstances of incomplete information, where either party has either little or inaccurate information about the preferences or utilities of the other party. Consequently, negotiations may go over multiple rounds until a mutually acceptable agreement is reached. The objective of this research is to create a quantitative model of decision-making that helps one negotiating party (e.g., the supplier) derive efficient solutions in negotiation.

This thesis defines conflicts in negotiation and, accordingly, models the marginal probability of negotiation outcomes on each conflict for the supplier. A probability coupling approach is developed to determine the joint probability distribution of negotiation outcomes. In order to analyze a negotiation over two stages, this thesis formulates a Dynamic Programming (DP) problem to determine the optimal decision for the supplier at each stage of the negotiation. In the DP problem the supplier seeks the best trade-off between the monetary reward of the contract and the successful chance in the negotiation. The valuation model helps the supplier determine a sequentially chained negotiation strategy in an effective and efficient manner. With the model practitioners may gain a scientific understanding of negotiation decision analysis.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>viii</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 BACKGROUND</td>
<td>1</td>
</tr>
<tr>
<td>1.2 CHALLENGES</td>
<td>3</td>
</tr>
<tr>
<td>1.3 PROBLEM STATEMENT</td>
<td>4</td>
</tr>
<tr>
<td>1.4 PROPOSED APPROACH</td>
<td>6</td>
</tr>
<tr>
<td>1.5 EXPECTED CONTRIBUTION</td>
<td>8</td>
</tr>
<tr>
<td>2. LITERATURE REVIEW</td>
<td>9</td>
</tr>
<tr>
<td>2.1 NEGOTIATION</td>
<td>9</td>
</tr>
<tr>
<td>2.2 NEGOTIATION SITUATIONS</td>
<td>12</td>
</tr>
<tr>
<td>2.2.1 Multi-Issue Negotiation.</td>
<td>12</td>
</tr>
<tr>
<td>2.2.2 Multi-Stage Negotiation.</td>
<td>18</td>
</tr>
<tr>
<td>2.2.3 Multi-Item Negotiation.</td>
<td>21</td>
</tr>
<tr>
<td>2.3 NEGOTIATION MODELS</td>
<td>23</td>
</tr>
<tr>
<td>3. MODELING</td>
<td>31</td>
</tr>
<tr>
<td>3.1 CONFLICTS IN CONTRACT NEGOTIATIONS</td>
<td>31</td>
</tr>
<tr>
<td>3.2 MARGINAL PROBABILITY OF REJECTION ON A SINGLE CONFLICT</td>
<td>34</td>
</tr>
<tr>
<td>3.2.1 The Model for Equal Likelihood of Rejection.</td>
<td>36</td>
</tr>
<tr>
<td>3.2.2 Model for Increasing-Decreasing Likelihood of Rejection.</td>
<td>38</td>
</tr>
<tr>
<td>3.2.3 Model for Decreasing Likelihood of Rejection.</td>
<td>39</td>
</tr>
<tr>
<td>3.3 REJECTION PROBABILITY OVER MULTIPLE CONFLICTS</td>
<td>41</td>
</tr>
<tr>
<td>3.3.1 Independent Attributes</td>
<td>41</td>
</tr>
<tr>
<td>3.3.2 Interdependent Attributes</td>
<td>42</td>
</tr>
<tr>
<td>3.3.2.1 Total rejection probability</td>
<td>42</td>
</tr>
<tr>
<td>3.3.2.2 Conditional rejection probability</td>
<td>45</td>
</tr>
</tbody>
</table>
LIST OF ILLUSTRATIONS

Figure 3.1 Marginal Probability of Rejection for a single conflict.................................36
Figure 3.2 Marginal Probability of Rejection as the CDF of a conflict - Uniform Distribution............................................................................................................38
Figure 3.3 Marginal Probability of Rejection as the CDF of a conflict - Triangular Distribution.........................................................................................................................39
Figure 3.4 Marginal Probability of rejection as the CDF of a conflict - Exponential Distribution..........................................................................................................................40
Figure 3.5 Schematic Diagram of Rejection Probability with two conflicts.................45
Figure 3.6 Schematic Diagram of Conditional Rejection Probability.........................46
Figure 3.7 Equivalent Contracts (on party A's Efficient Frontier) with only one conflict.................................................................................................................................48
Figure 3.8 Relationship between Marginal Probability of Rejection and Marginal Probability of Non-acceptance.........................................................................................51
Figure 3.9 Party B's Negotiation Decision at Stage $t = 1, 2, ..., n-1$..........................53
Figure 3.10 Party B's Negotiation Decision at stage $n$..............................................53
Figure 4.1 Marginal Probability of Rejection and Non-Acceptance over conflicts in Price.................................................................................................................................62
Figure 4.2 Marginal Probability of Rejection and Non-Acceptance over conflicts in Quality..............................................................................................................................62
Figure 4.3 Marginal Probability of Rejection and Non-Acceptance over conflicts in Quality and Price (Case 1).................................................................................................74
Figure 4.4 Marginal Probability of Rejection and Non-Acceptance over conflicts in Quality and Price (Case 2).................................................................................................74
Figure 4.5 Marginal Probability of Rejection and Non-Acceptance over conflicts in Quality and Price (Case 3).................................................................................................74
LIST OF TABLES

Table 4.1 Array of possible contracts........................................................................................................59
Table 4.2 Array of possible conflicts...........................................................................................................59
Table 4.3 Marginal probability of rejection over conflicts in price.........................................................61
Table 4.4 Marginal probability of rejection over conflicts in quality......................................................61
Table 4.5 Marginal probability of non-acceptance over conflicts in price..............................................61
Table 4.6 Marginal probability of non-acceptance over conflicts in quality...........................................61
Table 4.7 Probability of Rejection conditioned on Quality....................................................................64
Table 4.8 Probability of Rejection conditioned on Price.........................................................................64
Table 4.9 Joint Probability of Rejection......................................................................................................64
Table 4.10 Probability of Non-acceptance conditioned on Quality......................................................65
Table 4.11 Probability of Non-acceptance conditioned on Price..............................................................65
Table 4.12 Joint Probability of Non-acceptance.........................................................................................65
Table 4.13 Probability of Acceptance.........................................................................................................66
Table 4.14 Probability of Re-negotiation.....................................................................................................66
Table 4.15 Payoff to party B for the corresponding contract.................................................................67
Table 4.16 Contract space considered in benchmark analysis.............................................................67
Table 4.17 Valuation model showing Alternative offers, probabilities and Expected Valuations..........................................................69
Table 4.18 Payoff table for Exponential Price and Triangular Quality........................................73
Table 4.19 Payoff table for Exponential Price and Quality........................................................................73
Table 4.20 Payoff table for triangular Price and Quality...........................................................................73
Table 4.21 Payoff for Marginally Decreasing cost function.......................................................................75
Table 4.22 Payoff for Marginally Increasing cost function........................................75
Table 4.23 Payoff for a more profitable contract.......................................................76
Table 4.24 Payoff for a more challenging contract..................................................76
1. INTRODUCTION

1.1 BACKGROUND

Negotiation is a type of joint decision-making involving two or more parties. Each party expresses their preferences with each other in an attempt to resolve their opposing interests. These parties may verbalize their contradictory demands. Then through a process of concession, they may either move continuously toward a mutually acceptable agreement or search for other alternatives. Typically, the negotiating parties have incompatible preferences among the alternatives under consideration. Thus, they need some degree of concession between one or all parties for a mutual agreement to be reached.

A classic example of the importance of not only negotiation but also a good negotiation strategy is the 1994 Major League Baseball strike. America’s major league baseball contract negotiations had been the subject of constant battle. The conflicts were over contract terms between unions representing the baseball players and the team owners. The troubled negotiations finally led to a work stoppage when the baseball players walked off the field, resulting in the cancellation of the 1994 World Series. Negotiation after negotiation failed until a final agreement was reached. This final agreement, however, was quite similar to the old agreement. Nearly $700 million in losses to team owners were incurred and the average player’s salary dropped by 5%.
Another classic example of negotiation that shows the need for transparency and the need for trust between parties involved in the negotiating process is the Louisiana Purchase by the United States. The United States purchased Louisiana from the French government for a total of $15 million under the term of President Thomas Jefferson in 1803. For the latter part of the 18th century, Louisiana was the subject of many European political debates. France controlled the Louisiana territory from 1699 until it was ceded to Spain in 1762. France later regained control of the region in the 1800s under Napoleon Bonaparte.

The sale of the Louisiana territories to Spain was clouded because the boundaries defined under Spanish deeds differed from those defined under French deeds. Years later, after the seemingly successful negotiation between the United States and France, west Florida was included in the properties sold by France to Spain. This area however wasn’t explicitly included but was only implicitly included in the sale from France to the US. This led to conflict between the United States and the Spanish governments. The heightened war in Europe, however, gave the US the opportunity to acquire this region as France sought assistance from the US to seize Spain as part of its territory.

Negotiation takes place because resources are limited. Thus, there is a need to reach agreements on how best to share value or divide a resource between the negotiating parties. Negotiation is an important part of business, international diplomacy, and organizational management. The study of negotiation becomes increasingly important as
competitiveness continues to grow to determine how faster, efficient resolutions may be achieved in every sphere of both business and human activities.

1.2 CHALLENGES

Most real-world negotiation scenarios involve multiple interdependent issues over which the negotiating parties will seek an agreement. Multi-issue negotiation analysis is an area of growing research interest, and many methods have been suggested to tackle these problems. Research efforts are geared towards having a solution that is easily generalized to accommodate most real-world situations without significant computational effort. The result is that it will help to make for efficient negotiations between individuals, organizations and governments. Negotiation over multiple interdependent issues presents an especially challenging problem because typically, the negotiating agents’ utility functions are typically nonlinear. This situation makes traditional negotiating mechanisms inadequate in solving those problems. Even mechanisms designed for nonlinear utility spaces may fail if the utility space proves to be highly nonlinear. This is often the case in many real-world negotiation scenarios.

A significant number of scientific studies and research into the subjects of negotiation and negotiation theory has been conducted over the last 30 years. Multiple criteria decision problems have been theoretically studied and extensively analyzed, resulting in different methods, tools, to better approach such problems. Many real-life decision problems involve multiple conflicting criteria, multiple decision makers, and
negotiation between different parties aimed at identifying an acceptable solution that is mutually satisfying to both parties.

Two or more parties also tend to negotiate under circumstances in which either party has either little or no information about the other party’s preferences or utilities. The offering party then assumes (or simply guesses) the counterparty’s utility. With every offer and counter-offer made, this guess is updated to reflect the true values of the counter party. Negotiation is conducted over multiple stages throughout this process until either an efficient or a mutually acceptable agreement is reached.

One of the major aims of negotiation research and studies has also been to reduce the time taken to arrive at optimal decisions. The goal is to reduce number of stages required in going back and forth between offers and counter-offers until an acceptable solution is reached. Several have suggested preference-ranking the issues prior to negotiation. This ranking, however, may not always be possible as most negotiating agents would be unwilling to reveal too much information to their opponent. Thus, an important goal in negotiation research is to have either a system or a model that quickly learns the counterparty’s preferences under circumstances of incomplete information so that efficient solutions can be determined.

1.3 PROBLEM STATEMENT

The overall objective of this research was to develop a valuation model that can evaluate the expected consequence of a supplier’s decision at any stage, negotiating
multiple issues of a contract with a customer. More specifically, the research attempts to create a quantitative model of decision-making that will help support efficient solutions in negotiation. The goal therefore, is to come up with a model to efficiently analyze bilateral multi-issue negotiations under multiple stages. This model should help determine optimum negotiation strategy that leads to a mutually satisfying outcome to both parties.

The processes and conditions leading to effective negotiations provide the most joint benefits for all negotiating agents. As a result, they maximize the utility of all parties involved. Therefore, it becomes unlikely that these agreements may be either contested or rejected by any of the parties after the agreements have been reached. Additionally, good relationships are built, and the possibility of further business is increased when the parties provide benefits to one another.

Many of the studies previously conducted on multiple-criteria negotiation involved complex methods, typically utilizing complex algorithms and processes that require superior analytical skills to determine optimal solutions. This research builds from first principles using basic probability theory, with the only few additional ideas being the methods of determining equivalence between offers by comparing with the indifference curve. The objective was to create a simple, scalable approach that yields relatively accurate results in the analysis of bilateral negotiations that involve multiple-issues over multiple-stages.
1.4 PROPOSED APPROACH

A scenario of two parties was assumed to be involved in a bilateral negotiation. It was also assumed that these parties were self-interested agents seeking to maximize either their utility or value gained from the negotiation process. Finally, a situation of incomplete information was assumed to exist in which both parties had incomplete information about the counterparty’s preferences or utility. Negotiation begins when one party makes an offer. The counterparty can accept the offer, reject it, or make a counteroffer to the other party in line with its true utility.

Assuming the position of one party, the conflict in negotiation is defined as the difference between both offers made by the two parties over any given stage of negotiation. The conflict is expected to decrease as a concession is made over several stages of negotiation until a mutually acceptable agreement is reached. Then an attempt is made to characterize the supplier’s decision under uncertainty by modeling the probabilities of different possible negotiation outcomes based on the conflicts between both offers. This research proposes a probability coupling approach to obtain these probabilities in multi-issues negotiations. This is especially useful when considering negotiation over multiple interdependent issues.

An agreement may not be reached in a single round of negotiation in cases of incomplete information. The negotiation that takes place in one stage has a direct impact on that of the next. A dynamic programming model is also proposed in this research to
accommodate for negotiation made over multiple stages. It approach helps to determine a negotiation strategy that yields the most satisfactory agreement towards the end.

In summary, some of the important steps taken in this research paper include:

(i) Introduce the concept of conflicts in negotiation as the difference between the offers, as well as both parties, over any stage of negotiation.

(ii) Model the probabilities of uncertain negotiation outcomes at each stage as functions of the conflict in negotiation

(iii) Use the indifference curve to introduce the idea of equivalence between offers made by either counterparty.

(iv) Incorporate multiple stages of negotiation into the model to account for a true-to-life bilateral negotiation over several negotiation rounds.

(v) Use the method of dynamic programming to derive the optimal decision for the supplier at any stage of negotiation.

To give a feel for this approach, a test case is tried out to test the model and determine the effectiveness of this approach over various conditions. The results obtained are then analyzed and further explained. The limitations of this approach is also investigated to know under what conditions it would work best and what assumptions (if any) that must be made.
Negotiation studies may lead to better organizational effectiveness between work
groups and teams. Organizations as a whole are more likely to succeed if the individual
units find creative ways to reach agreements and reconcile their differences. The
advancement of automated negotiation would also vastly increase the effectiveness of the
business and maximize benefits for both negotiating parties in e-commerce.

1.5 EXPECTED CONTRIBUTION

One value of this research is the fact that its approach to solving a real-world
negotiation problem is based on well-established tools of mathematics and
microeconomic theory. The analysis completed as a part of this research is based on a
two-attribute, two-stage negotiation problem. This analysis can however, be expanded to
include larger dimensions of negotiations while, ultimately, being quite scalable to
accommodate even larger scenarios. This model helps to determine the optimal bilateral
concession the supplier can propose at any stage of the negotiation. It can also help with
the following:

(i) Determine what is the best trade-off (optimal concession) between seeking higher
    payoff or making concessions to the counter-party;
(ii) Update the initial estimate of the counterparty’s utility functions after learning
    information from earlier stages of the negotiation
(iii) Determine contract party’s preferences and behavior changes over time with more
    stages of negotiation

The optimal results were obtained without very challenging computational effort.
2. LITERATURE REVIEW

2.1 NEGOTIATION

Negotiation allows two or more parties to reach mutually beneficial agreements with regard to pre-defined issues. It provides a means of decision-making when the interests between these parties are inherently opposed to each other. Negotiation occurs at every level of human interaction. Whether as individuals or as groups, most attempt to seek out a compromise that is mutually beneficial to all parties. In general, negotiation serves to help develop specific agreements, long-term policies, about roles and mediations.

In a typical negotiation, two or more subjects (or teams) come together to play the role of either buyer or seller, union or management, and so forth. Each agent uses background information on the situation to create a clear understanding on the values or utilities for the options under consideration. The bargainers discuss and exchange notes on the options until either an agreement or an impasse is reached (Pruitt, 2013). Several basic theoretic ideas formed the basis for both current and previously conducted research into the subject of negotiation. The most important of these ideas includes not only the strategic choice model but also the goal (or expectation) hypothesis.

The strategic choice model assumes that a bargainer will most likely deploy one of three basic strategies to arrive at an agreement in a negotiation. He/she may first concede unilaterally. Here, the end goal is in closing the distance between the parties opposing demands to reach an agreement. He/she may also choose to engage in what is
known as a “competitive behavior”. Under this strategy, negotiating agents use pressure tactics (e.g., persuasive arguments or threats) in an attempt to persuade the other party to concede. Finally, he/she may engage in what has been termed “coordinative behavior”. In this case, one negotiating party decides to collaborate with the other party to seek out a mutually beneficial agreement. Examples of this behavior include the involvement of a third-party and the creation of a proposal to compromise. Pruitt (2013) noted that any of these strategies may be combined in any negotiation process. In reality however, the choice of one strategy makes the others less likely to be chosen.

The goal hypothesis is based upon the coordinative behavior; it sets forth the conditions under which a bargainer may decide to engage in such behavior. These conditions include both trust and the ability to coordinate. Pruitt (2013) defined trust as having the belief that the other party is ready for coordination. Trust is a necessary condition because, by default, one-sided willingness to engage in coordinative behavior leaves a bargainer open to the possibility of being exploited by the other party. In general, the conditions that diminish attraction towards unilateral concession and competitive behavior in negotiation strengthen the likelihood for coordinative behavior between negotiating agents. Thus, if either parties recognize that neither will likely concede or succeed in competing, then they are more likely to seek out cooperation or coordination so that an agreement may be reached.

The subject of coordinative behavior in negotiation has been studied extensively as it is of vital importance in negotiation studies. Pruitt (2013) noted that, without the
possibility of coordination, negotiation would likely take a great deal of time. This may end in disagreements and also make future relationships and deals unlikely. Two main types of coordination occur in negotiation. The first is a concession exchange. Here, each party swaps concessions with the other with the hope of arriving at a acceptable agreement. The second approach involves either problem-solving discussions or information sharing. Here, both parties share information about their utilities and priorities on an option. This might lead to an integrative agreement that satisfies the needs of both parties.

A significant amount of scientific study and research has recently been focused on the subjects of negotiation and negotiation theory and particularly in the area of integrative bargaining. The processes and conditions that lead to integrative bargaining are important because they provide the greatest amount of benefit for all negotiating agents involved. Thus, it maximizes the utility of all parties involved, making it unlikely that those agreements may be either contested or rejected Follett, (1940). Additionally, good relationships are established and future interactions become possible when the parties involved provide benefits to one another, Lott & Lott (1965). In general, integrative bargaining within organizations leads to better organizational effectiveness between work groups and teams. The organization as a whole is more likely to succeed if the units involved are able to find creative ways to reach agreements and reconcile their different goals.
2.2 NEGOTIATION SITUATIONS

Important considerations in any bilateral multi-party negotiation include the number of issues involved in the negotiation, the number of stages over which negotiation will be permitted, and whether or not an agreement must be reached before a deadline. The number of items over which a negotiation is to be made and the similarities that exist between those items are also key considerations. These considerations are outlined in the following sections.

2.2.1 Multi-Issue Negotiation. Multi-issue negotiation problems arise in many real world situations. These negotiations are focused on an item that has different attributes or issues. The breadth of literature on multi-issue negotiation covers a large variety of scenarios. The most important issues that affect the outcome in any multi-issue negotiation rest on two critical factors: (i) the negotiation agenda (the set of issues under negotiation) and (ii) the negotiation procedure (whether or not the issues will be discussed independently or separately). Patton and Balakrishnan (2012) explained that the negotiation agenda provides a means of structuring discussions between individuals and groups. It comprises of all the domain of issues under consideration. The agenda specifies what issues are to be considered during the negotiation process. The procedure however specifies how these issues will be negotiated (Abedin et al, 2009). These two factors comprise the basic negotiation parameters for any given two or more self-interested agents.

In general, there are two major approaches to negotiating multiple issues (Fatima et al., 2004). The first approach is used when all of the issues are to be discussed together.
The second approach (known as issue-by-issue negotiation) is used when each issue is to be discussed sequentially and independently from the other. Both procedures yield different results. Therefore, self-interested agents must decide on not only the agenda but also the procedure they will use (Fatima et al., 2004). Another approach that is used occasionally is one in which all of the issues are discussed together as a package deal but they are interdependent with each other (Abedin et al., 2009). The interdependency of these issues to be considered increases the complexity of the negotiation problem. The issue-by-issue approach, however, tends to minimize this problem.

Carraro and Sgobbi (2008) noted that the order in which the issues are discussed in an issue-by-issue negotiation may play a strategic role. It will also affect the final outcome of the negotiation quite often. The order is specified by the negotiation agenda, which can be defined in one of two ways: exogenously or endogenously (Abedin et al., 2009). The agenda (or order) in the first case is determined before actual negotiations begin. In the second case the agents decide what issue will be negotiated next during the process of negotiation (Abedin et al., 2009). It is assumed that each party is interested in arriving at outcomes that maximize their utility from the negotiation. Therefore, the order in which the issues are negotiated is important to arriving at an equilibrium outcome that is mutually agreeable and presents joint gains.

Multi-issue negotiation is helpful if each party has some knowledge of the other party’s preferences. (Preference here is defined as the relative assigned importance of the issues to be negotiated). Multi-issue negotiations often involve very large, complex
search spaces (Abedin et al., 2009). Preference ordering can help reduce the negotiation delay time and search space. Therefore, reaching an optimal outcome is much easier if the preference ordering of both agents is known to the other. Complete information about each agent’s parameters, the time preferences involved, the reservation prices assigned, and other factors that come into to play in the negotiation process, are helpful as well. In reality, however, most agents have incomplete information about their opponents.

Most real world negotiation problems involve issues that are interdependent. These interdependencies add to the complexity of the analysis to be done. This analysis typically involves agent utility functions that are not only non-linear but also have multiple optimum solutions (Fujita et al., 2010). Extensive research has been conducted on interdependent multi-issue negotiations. A number of approaches that use different negotiation protocols have been proposed for the study and analysis of these problems. Regardless of the method used, however, one major problem that still remains is in achieving scalability for use in real world applications. In general, negotiation, when multiple interdependent issues are involved introduces a difficult trade-off between optimality and computational costs.

Fujita et al. (2014) proposed that the analysis of interdependent multiple issue negotiations involves a protocol in which a mediator reorganizes a complex utility space with issue interdependencies into either tractable utility sub-spaces or issue groups. Typically, these approaches consider cases in which there are \( n \) negotiating agents with one mediator. It is also assumed that there are \( m \) number of issues with different levels of
interdependencies. The number of interdependent issues represents the number of dimensions in the utility space. Each issue has a value that is drawn from the domain of integers. A contract is represented as a vector of these values. An agent’s utility for a certain contract is the sum of the utility for all of the constraints the contracts satisfies.

The individual agents, however must first use an algorithm (e.g., the Girvan-Newman algorithm) to develop interdependency graphs that illustrate the relationship between the issues in their individual utility functions. The mediator then uses a nonlinear optimization protocol (e.g., either simulated annealing or an evolutionary algorithm) to determine sub-agreements for each issue group. These sub-agreements are then combined to produce the final agreement. One problem with this approach is the potential for strategic, non-truthful voting in which one agent intentionally exaggerates his/her votes. This exaggeration may introduce biases into the negotiation outcome that favor the agent. This issue-grouping approach, however, yielded higher optimality rates for the same amount of computational effort than it did without issue grouping.

Fujita et al. (2011) also proposed a second approach for a model with complex utility spaces. This approach is based on cone-shaped constraints. The underlying concept was that cone-constraints represent a more realistic constraint shape than either the previously used cube or block-shaped constraints; the agent’s utilities for a contract decline gradually, rather than step-wise with distance from their ideal contract. The utility function for cone-constraints (when compared with cube constraints) is highly complex because its highest point is narrower. The negotiation protocol here also involves a
mediator that gathers private issue interdependency graphs from each negotiating agent, generates a social interdependency graph, identifies the issue sub-groups, and then generates a final agreement using well-known non-linear optimization techniques that satisfy Pareto-optimality.

Robu et al. (2005) used the concept of utility graphs to consider the case of a seller agent negotiating bilaterally with a buyer over multiple issues. The issues under consideration were the bundle or item configuration and price. Utility graphs build on the concept that highly nonlinear utility functions may be decomposable in the sub-utilities of clusters of inter-related items. Graphical models take advantage of the fact that utility functions could be decomposed, thus enabling a more efficient search of the contract space. An important benefit of the concept of utility graphs is its scalability for use in large contract spaces.

The buyer and seller negotiate bilaterally over both a set of binary valued issues and one continuous issue (the price). In this model, and prior to the start of the negotiation process, the seller obtains some prior approximation of the buyer’s utility information, either through a history of past negotiations or from the input of domain experts. At every offer or counter-offer, the seller updates or refines this approximation. This procedure helps the seller make offers that approximate Pareto efficiency within a short time period. The utility assigned to the different negotiation outcomes are represented by monetary values. Thus, the seller’s net utility is the sale price minus the
cost of selling the item. In contrast, the buyer’s net utility is the monetary value of the item minus the price paid to purchase.

Zhang and Klein (2012) also proposed a hierarchical negotiation model for analyzing complex problems with a large number of interdependent issues. This model is based upon the observation that most complex systems can be described hierarchically and decomposed into sub-systems. They used a constraint-based preference model and claimed that it significantly reduced the computational effort required in making complex contracts. Their approach made it no longer necessary to acquire a utility function that specified a numeric value for each possible outcome. This method, they claimed, also generated better negotiation outcomes as the higher level attributes constrain the search space the most. Hence, the search space is very well directed to those attributes that are most important.

The negotiation protocol may consist of a mediator and a set of agents. Abstractions with domain knowledge are used to hierarchically decompose the system in the negotiation into sub-systems. For each system \((S)\), there is a set of constraints \((C)\) that includes a set of attributes. The constraint specifies the allowable combination of values. It may be represented by (i) a set of specified areas in the solution space that are equally good, (ii) logic statement to specify the relationship among multiple attributes, or (iii) mathematical formula that specifies the relationship among multiple attributes. Each constraint can be considered a goal that is to be satisfied. An agent would typically rank, or provide weighting values, for all constraints to be satisfied.
Each agent begins at the first level and submits information about either all issues or all attributes that need to be assigned a value. The mediator may then create a global dependency matrix based on that data. These issues are then clustered into decision groups containing highly interdependent issues. The agents submit their preference about the ranking of these decision groups based on their relative importance. The mediator may create a negotiation agenda that is represented by a directed acyclic graph. Each agent then submits his/her most preferred bids for all attributes in the decision group. The mediator may analyze these bids to generate the top bids. This process is repeated for each decision group until the mediator has a complete set of ordered preferred common choices. These sets of preferred choices will include the top common choices for each decision group. This process may be repeated across all levels of the negotiation.

2.2.2 Multi-Stage Negotiation. Fatima et al. (2004) suggested that time is a very important issue in any bargaining process. They explained that both deadlines and time discount impact all bargaining situations. Two typical scenarios could be given to illustrate this explanation. One case could be where one agent, for example, a buyer prefers a lower price while the other, say, a seller prefers a higher price and both would have to ensure that negotiation ends by a certain deadline. The second scenario could be a case in which the service is to be provided immediately after the negotiation is complete. One agent (typically the buyer) may lose utility with time as a result of not getting the service at the right time. The other agent (typically the seller), however, would gain more utility by providing the service at a later date. An agent that gains utility with time and has the incentive to reach the agreement later (but within the deadline) is said to be a
strong (or patient) player. An agent that loses utility with time and tries to reach an early agreement is said to be a weak (or impatient) player.

Chen et al. (2004) proposed a multiple-stage negotiation architecture to support the negotiation process. Within this architecture, agents are allowed to generate and select effective strategies that lead to high payoffs. The negotiation strategy and negotiation protocol provide negotiating agents with the rules of interaction and communication directives. They also determine when the negotiation process ends. Negotiation begins when one of the agents makes an offer, and the offer includes a set number of issues over which the agents negotiate. The second agent evaluates the offer with its utility function, makes a counter-offer, and replies with a degree of satisfaction with respect to the offer from the first agent. This degree of satisfaction provides useful information for the evaluation of both the offers and the counter-offers. This negotiation strategy is derived from a function that varies the targeted utilities the agent intends to obtain over the sequence of offers. Chen et al. (2004) explained that the negotiation process ends when either of the following is reached:

(i) The maximum number of negotiation stages is obtained, regardless of whether an agreement is reached or not.
(ii) One of the agents makes a final offer, regardless of whether or not an agreement is reached.
(iii) An agreement is reached over a proposal by either of the negotiating agents.
The major contribution of this work was the introduction of a negotiation protocol for agents to carry out cooperative negotiation without revealing their utility functions. A limitation to this work was that the researchers failed to factor in time as the research was primarily related to automated negotiation systems. Time is, however, an important element in bargaining power and, particularly, especially so in automated negotiations.

Hattori et al. (2007) proposed an iterative method in narrowing of the agent’s bids to determine optimal outcomes for the negotiation protocol. This approach didn’t place unrealistic demands regarding how much agents reveal about their utility function. This process is comprised of multiple stages, or rounds, of bidding between both negotiating agents. It also includes a mediator. Agents submit rough bids during the early rounds of negotiation. These bids represent promising contracts space regions to a mediator. In later rounds, the agents submit increasingly narrow bids for the subset of those regions that all agents liked in the previous round. The process is repeated until a deal is reached.

This technique’s strength lies in the fact that, in each round, the agents produce bids that cover high utility subsections of the contract space, cutting out regions with a low utility. The overlap returned by the deal identification step in one round becomes the boundaries within which agents search for bids in the next round. The contract space under consideration thus becomes smaller in each round, until a single winning contract is identified.
Hattori et al. (2007) had a great deal of success with the iterative narrowing protocol to yield near optimal negotiation outcomes for up to 10 negotiating agents. Their protocol also worked well for 4 or fewer shared contract clusters with high correlation. This method’s failure rate increases significantly when scale limits are exceeded. The failure rate also increases in cases of low correlation.

2.2.3 Multi-Item Negotiation. Significant research effort has been made in the areas of multi-issue negotiation. The traditional approaches have been to understand the negotiation partner either by estimating the partner’s preference profile or by predicting its utility or decision function. One method uses different profile modeling techniques to gain a better understanding of the negotiating partner. The second method uses a strategy prediction technique that allows the negotiating agent to maximally exploit his/her negotiating partner and receive the most benefit from the negotiation. Each one of these methods has been either studied or deployed independently of the other in the past. An increasing amount of research, however, is being conducted into how these two methods may be combined to yield better evaluations in different negotiation scenarios. That forms the basis for multi-item negotiations, which studies how results from a multi-attribute or multi-issue negotiation may or replicated across several closely related items, agents or negotiation scenarios.

Hao and Leung (2012) proposed an Adaptive Bilateral Negotiation Strategy (ABiNeS) for automated agents wanting to negotiate across several bilateral multi-issue negotiation environments. Here, the agent’s negotiation strategy and preference profiles
are private information, and the only information available about the negotiating partner lies in his/her previously made negotiation moves. The negotiating partner can adopt any kind of strategy. Thus, predicting which specific strategy the negotiation partner will use next is quite difficult. Hao & Leung (2012) introduced the concept of the non-exploitation point ($\lambda$) to adaptively adjust the degree that an ABiNeS agent exploits his/her negotiating opponent. The value of $\lambda$ is determined by not only the characteristics of the negotiating scenario but also the degree to which the negotiating partner concedes. Also, to make predictions on the preference profile of the negotiating partner, they proposed a reinforcement-learning based approach. This helped to determine the optimal proposal for a negotiating partner based on current negotiation history.

Hao & Leung (2012) focused on bilateral negotiations regulated by the alternating-offers protocol in which the agents are allowed to take turns to exchange proposals. If it is an agent’s turn to make a proposal, he/she is allowed to choose from three options. He/she may choose to accept the offer that is received from his/her negotiating partner. The negotiation ends and an agreement is reached when this option is chosen. He/she may reject the offer and propose a counter-offer. Here, the negotiation continues, and the partner can make a counter-proposal provided the negotiation deadline has not yet been reached. Finally, he/she may terminate the negotiation; ending the negotiation. When this happens, each agent gets the corresponding utility based on its private reservation value. Hao & Leung (2012) also examined the discounting effect of time on the utility value. The negotiation process is terminated when either of the following conditions is satisfied: (i) the deadline is reached, (ii) an agent chooses to terminate the negotiation before the
deadline, or (iii) an agent chooses to accept the negotiation outcome proposed by the negotiating partner.

2.3 NEGOTIATION MODELS

Fatima et al. (2003) noted that a negotiation model should include the following: (i) the negotiation protocol, (ii) the negotiation strategies, (iii) each agent’s information state and (iv) the negotiation equilibrium. Marse-Maestre et al. (2014) stated that a critical challenge facing negotiation research is that no one protocol is the best choice for every possible negotiation scenario. They suggested a collection of design rules to help agents choose the appropriate procedure in any particular negotiation problem.

The negotiation protocol, essentially lays down the rules for each of the agents involved in the negotiation process. It defines both the types of deals that can be made and in what order or sequence they are to be made. As previously noted, the protocol in a multi-stage negotiation may specify for negotiation to be made on all issues together as a whole or for negotiation to be made on the issues one by one. The negotiation strategy also specifies the sequence of actions the agent plans to make during the negotiation. The strategy employed by any agent is crucial in determining what the final outcome of the negotiation process will be.

Each agent’s information state is also critical to the negotiation protocol. Von Neumann and Morgenstern (1944) introduced the classification of information states in negotiation as either complete or incomplete information states. When each agent has
complete information, he/she has all relevant information related to either the negotiation or the rules of the game. The preference of each of the playing agent is represented by the utility function. When an agent possesses incomplete information, he/she may have only probabilistic information or knowledge about a variety of factors in the negotiation problem. Therefore, each agent has some private information about his own specific preferences that are unknown to the other players in the bargaining process.

The negotiation mechanism is comprised of the negotiation protocol and the negotiation strategies for each of the agents involved. Many negotiation mechanisms and models have been developed from principles of game theory. The concepts taken from game theory have been used to study many interactions between self-interested agents. The negotiation strategy must be in equilibrium to have a stable negotiation mechanism. Nash Jr. (1950) explained that two strategies are in Nash equilibrium if each agent’s strategy is a best response to its opponent’s strategy. This condition must be met before system stability can be achieved. Game theory has its limitations in dealing with multi-issue negotiations. These limitations are particularly influential when the participating agents have incomplete information. Chen et al. (2004) noted that, in such cases, when each negotiator’s preferences or utility functions are private information, game theory techniques cannot provide a solution for complex negotiations.

Dang and Huhns (2005) suggested a negotiation model to determine the optimal strategy for multiple-issue negotiation for services between self-interested agents (service requestors and service providers). This model’s approach involved combining the merits
of issue-by-issue negotiation and package deal negotiation to propose a coalition deal negotiation. Here, all negotiation issues are partitioned into disjoint partitions and each partition is negotiated independently of the other partitions. Issues within the same partition are negotiated as a whole package, and an offer includes a value for each issue within this partition. For each partition, the assumption is that agents use the same protocol as for the package deal, but instead of making a set of offers over the issue set, an agent offers a set of offers over issues from this partition. Trade-offs can only be made across issues in the same partition.

This model has several advantages over the other models. One advantage was that it gave better trade-offs between utility optimization and computational efficiency than either the issue-by-issue negotiation or package deal negotiation. It also allows for not only more flexible negotiations but also better management for service-oriented negotiations. Unfortunately, negotiating agents must reveal private information (e.g. deadlines). Thus, this model is best suited for Quality of Service (QoS)-aware service contracting.

Reaching optimal outcomes for bilateral negotiations with multiple issues when the agents preferences are unknown is typically difficult. Many existing multi-agent negotiation frameworks require that agents reveal some aspects of their preferences to a trusted mediator. In real world cases, however, a mediator who is well-trusted by both negotiating agents may not be found. In an attempt to solve this problem without requiring agents to reveal too much of their preferences, Saha and Sen (2006) proposed a Pareto-
Optimal Solution through Equivalence (POSE) negotiating framework. Under this framework, agents alternatively reveal their partial preference information. Previously conducted research into multi-issue bargaining revealed that efficient solutions are more readily reached when negotiating agents are willing to reveal their complete preferences, In & Serranno (2004) and Raiffa (1982). If however, agents are unwilling to reveal their preferences, then, upon the assumption of the rationality of both agents, they may end up with an equal split solution. Unfortunately, this solution may not be optimal. Thus, Saha and Sen (2006) essentially acknowledged the trade-off that exists between the level of information revealed and the solution’s degree of optimality.

Most one-to-one multi-issue negotiations involve parties attempting to reach an agreement through multiple rounds of concession. Although the result of such a negotiation may be deemed acceptable by both parties, it may not be a result that optimizes the utilities of both parties. This is due to the fact that concession typically involves one party yielding on some aspects of the negotiation to gain on some others. Sun et al. (2006) proposed a model that also considered the joint rationality along with the individual rationality of agreements reached in negotiation. They used a simple algorithm to optimize the joint utility of both parties after agreements have already been reached by concession methods.

Bac and Raff (1996) and Rubinstein (1982) note that in the analysis of negotiation between two or more agents, the typical approach has been to treat the agenda and the negotiation procedure as fixed parameters in the analysis. Fatima et al. (2004) attempted
to determine the optimal agenda and procedure for each negotiation by treating both items as variable parameters, all done under conditions of incomplete information. Their approach analyzed the process of bilateral multi-issue negotiation by fixing the protocol while varying both the agenda and the negotiation procedure. In doing so, they determined equilibrium strategies for two negotiation procedures: package-deal and issue-by-issue. They used these strategies to determine the outcomes for all possible agenda-procedure combinations. They then demonstrated the optimal agenda-procedure combination for each agent under different scenarios. The results of this study revealed that, although negotiating agents are self-interested, they may have identical preferences with regard to the optimal agenda and procedure in each scenario. A limitation to their work though, was that under incomplete information settings it is difficult for agents to recognize the optimal scenarios. Also, the work mainly focused on two-issue model and it is not sure how well it may perform for multiple issues greater than two.

Multiple-issue negotiations with interdependent issues are some of the most typical, complex negotiation problems faced. This complexity stems from the fact that they produce utility functions that are highly non-linear. Several search algorithms have been deployed in an attempt to determine the optimal solutions to these problems. Fujita et al. (2009) proposed that utility functions based on cone-constraints could be used to solve such problems because an agent’s utilities for a contract typically decline gradually, rather than step-wise, with distance from their ideal contract. Negotiation protocols well-suited for linear utility spaces also perform poorly when applied to non-linear problems. Fujita et al. (2009) proposed the Secure and Fair Mediator Protocol (SFMP) with
approximate fairness. This protocol first finds the Pareto-optimal agreement points and then selects one-agreement point from the set of Pareto-optimal contracts. The SFMP method considers an agent’s preferences and nearness to the Nash bargaining solution while the approximate fairness selects the agreement point closest to the Nash bargaining solution. This approach may yield optimal results. Unfortunately, it is quite time-consuming. It is also not very scalable for larger problems that may be faced in real-world situations.

Scalability remains a significant problem when attempting to develop non-linear negotiation protocols. Most models perform quite well in either pre-defined or fixed criteria. They begin to have higher failure rates when applied to real-world problems (which are often quite large). Mizutani et al. (2010) proposed a method for decomposing utility space into smaller spaces to reduce computational cost. This method was based on the interdependencies of the issues. Here, the groupings are generated by a mediator according to the examination of the issue interdependencies. The distributed genetic algorithm (DGA) is then used to optimize the solution in each issue-group. The major advantage to this method was the advantage of scalability. The mediator could easily generate issue groupings by analyzing the interdependencies between the different issues. The optimality of the negotiation outcomes, however decrease significantly as the number of issues increases.

Many negotiation scenarios involve complex, non-monotonic preference spaces in which reaching an optimal agreement may be quite computationally challenging. Unless
all clauses are independent (and this is very unlikely), an agents preference regarding the various potential contracts will be non-monotonically distributed throughout the utility space. Various offers may exist that provides the agent the same level of utility. The question then becomes which offer to propose. The similarity criteria has been used to approximate the agent’s preferences in many models Sycara, (1998), Choi et al. (2001); and Faratin et al. (2002). The idea is that, the more similar an offer is to the opponent’s previous offer, the more likely the offer is to be accepted. An extension of this idea for non-monotonic scenarios was the use of the Similarity-Based Negotiation Protocol (SNBP) for bilateral negotiations. This concept is based on an iterative constrained optimization mechanism proposed by Lai and Sycara (2009).

The similarity idea is truly a weak basis for the analysis of non-monotonic negotiation problems due to the lack of information on a negotiating opponents preference structure. Lopez-Carmona et al. (2011) proposed a region-based automated multi-issue negotiation protocol (RBNP) that can operate efficiently in complex utility spaces where the similarity approach may fail. The RBNP was inspired by pattern search optimization methods that were based on previous work done by Lewis et al. (2000). The pattern search methods are, essentially a series of exploratory moves that consider the behavior of the objective function at a pattern of points. These points lie on a rational lattice or mesh around a current reference point. The value of both the mesh size and the poll is considered successful when at least one mesh point which improves the reference point value. Optimization ends when the mesh size is less than a given mesh tolerance. The primary advantage of direct search methods such as this is that they do not need to
compute derivatives. Thus they typically perform quite well with both non-monotonic and non-differentiable functions.

Lopez-Carmona et al. (2011) noted that SBNP and RBNP techniques have been tested under monotonic and non-monotonic scenarios. The results show that RBNP performs better than the SBNP in most cases and also yields better results. The problem with the RBNP method, however, is the length of time it takes to generate optimized results. Therefore, a trade-off exists between the optimality of the results and the negotiation time. The search depth or number of region sizes used in a negotiation encounter plays a significant role in the protocol’s performance. Only under extremely complex spaces do RBNPs fail.
3. MODELING

3.1 CONFLICTS IN CONTRACT NEGOTIATIONS

In any bargaining process between two parties, it is assumed that each party concerns with maximizing its utility through the contract negotiation. The aim of the negotiation is to arrive at a set of values of contract attributes, which are mutually satisfying to both parties and therefore maximize their utilities. In every round of a negotiation, each party suggests an offer or a counteroffer that aims to reflect its true values and also the level of information that it has of the other party. The differences between their offers are termed as conflicts in the contract negotiation.

A contract can be characterized by a set of attributes. Assume a contract has \( m \) different attributes. Let \( i \) be the index of attributes and \( I = \{1, 2, \ldots, m\} \) be the index set for attributes. Let \( X_i \) represent the \( i \)-th attribute, for any \( i \in I \). Therefore, a contract is characterized by a vector of the attributes,

\[
X = \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_I \end{bmatrix}
\]

\( (3.1) \)

Define the two parties in a bilateral negotiation as \( A \) and \( B \). Specifically, party \( A \) is the one initiating a round of negotiation by extending an offer to party \( B \), and party \( B \) is the one responding to the offer. The negotiation may undergo multiple rounds and ends from the
research uses $t$ as the index of negotiation stages, where $t = 1, 2, \ldots$ and $n$ is the final stage of negotiation. Accordingly, the contract offered by $A$ to $B$ at stage $t$ is

$$\mathbf{X}^\perp = \begin{bmatrix} X^1_{t,(1\rightarrow t)} \\ \vdots \\ X^1_{t,(n\rightarrow t)} \\ X^1_{t,(1\rightarrow 1)} \end{bmatrix}$$

which is the decision of party $A$ at stage $t$. Similarly, the counteroffer from $B$ to $A$ is

$$\mathbf{X}^\perp = \begin{bmatrix} X^1_{t,(1\rightarrow t)} \\ \vdots \\ X^1_{t,(n\rightarrow t)} \\ X^1_{t,(1\rightarrow 1)} \end{bmatrix}$$

which is the decision of party $B$ at the same stage.

If $\mathbf{X}^\perp_{i\rightarrow 1}$ is not equal to $\mathbf{X}^\perp_{1\rightarrow i}$, conflicts present in stage $t$, which is denoted by $\Delta \mathbf{X}$.

In this research the conflict on the $i^{th}$ attribute is defined as an undesirable deviation from the attribute value of party $A$’s offer, $X^1_{i,(1\rightarrow 1)}$, for any $i \in I$. Specifically, if a positive deviation from the $i^{th}$ attribute is unfavorable to party $A$, this conflict is measured by

$$[1] - X^1_{i,(1\rightarrow 1)}$$
if a negative deviation is unfavorable to party $A$, the conflict is
\[
\begin{bmatrix}
\Delta X^i \\
( - X_i,_{(\rightarrow 1)} ) \\
( - X^i,_{(1\rightarrow)} )
\end{bmatrix} ;
\]  
\[(3.5)\]

where $\cdot \equiv \max \{\cdot, 0\}$. Therefore, an $m \times 1$ vector, $C\in\mathbb{R}$, is defined to indicate the conflict type for each of the $m$ attributes,
\[
C = \begin{bmatrix}
\Delta X^i \\
\vdots \\
\Delta X^i \\
\end{bmatrix} ;
\]
\[(3.6)\]

where
\[
\begin{cases}
1, & \text{if a positive deviation from } X^i,_{(\rightarrow 1)} \text{ is a conflict to party } A; \\
-1, & \text{if a negative deviation from } X^i \text{ is a conflict to party } A.
\end{cases}
\]  
\[(3.7)\]

Therefore, the conflicts in stage $t$ are,
\[
\Delta X^i = \begin{bmatrix}
\Delta X^i \\
\vdots \\
\Delta X^i \\
\end{bmatrix} = \begin{bmatrix}
\Delta X^i \\
( - X^i,_{(1\rightarrow)} ) \\
( - X^i,_{(1\rightarrow)} ) \\
\end{bmatrix} .
\]  
\[(3.8)\]
After sending the counteroffer to party $A$, party $B$ will be informed the decision that party $A$ made regarding the negotiation at stage $t$, denoted by $O^t$. Party $A$ may accept the counteroffer and the negotiation is end at stage $t$. It may reject the party $B$’s counteroffer and the negotiation is terminated. Another possibility is that party $A$ will propose a modified contract to party $B$, thus moving to the next stage of negotiation. This research uses $o\Diamond o\Diamond$ and $o\Diamond o\Diamond$ to represent the three possible outcomes in the sequence above. That is, $O^t \in \{o\Diamond o\Diamond o\Diamond\}$.

### 3.2 MARGINAL PROBABILITY OF REJECTION ON A SINGLE CONFLICT

Party $B$ does not know the negotiation outcome for stage $t$ when it responds to party $A$, and the negotiation outcome will directly impact the contract value. Therefore, party $B$ wants to guess the chance for each possible outcome in order to assess the expected value of the contract. This thesis describes the modeling of rejection probabilities and skips the details for acceptance probabilities because these are modeled in the same approach.

This research defines the marginal probability of rejection caused solely by the conflict at stage $t$ as:

$$\Delta o\Diamond := P(o\Diamond) = o\Diamond \Delta X_i, \quad \leq \forall j \neq i,$$

(3.9)
wherein $\Delta X_i$ is the parameter of $\phi$. $l_i$ and $u_i$ are the two boundaries for defining $\phi$ as $l_i$ and $u_i$.

$$
\phi = \begin{cases} 
0 & \text{if } \Delta X_i \leq l_i, \\
1 & \text{if } \Delta X_i \geq u_i. 
\end{cases} 
$$

(3.10)

That is, $l_i$ is the minimum amount of conflict in attribute $i$ that may trigger the rejection at stage $t$. $u_i$ is the minimum amount of conflict in attribute $i$ that surely triggers the rejection at stage $t$. Within the range of $[l_i, u_i]$, $\phi$ may incline if $\Delta X_i$ increases, as Figure 3.1 shows.

![Figure 3.1 Marginal Probability of Rejection for a Single Conflict](image)

The shape of marginal probability of rejection is to show a threshold. This research...
defines the likelihood function of rejection as the first order derivative of the corresponding conflict $\Delta X^i$,

$$
L^i := \frac{d}{d\Delta X^i} \phi
$$

Therefore, this research models the likelihood function of rejection, $L^i$, as the probability density function (pdf) of the conflict $\Delta X^i$, $f(\Delta X^i)$, on $[l^i, u^i]$. The marginal probability of rejection, $\phi^i$, is modeled as the corresponding cumulative probability function (cdf) of the conflict $\Delta X^i$, $F(\Delta X^i)$. That is,

$$
L^i := fX^i
$$

$$
\phi^i := F\phi
$$

for any $i \in I$. The following is a set of representative models for $L^i$ and $\phi^i$.

### 3.2.1 The Model for Equal Likelihood of Rejection

Given limited information about the party $A$'s preferences and decision behaviors, and knowing just the two points $l^i$ and $u^i$, party $B$ can only assume an equal likelihood of rejection for any degree of conflict; that is,
and, accordingly, the marginal probability of rejection increase linearly with $\Delta X_i^t$ within the range of $[l_i, u_i]$, 

$$
\Phi^i = \frac{\Delta X_i^t - l_i^t}{u_i^t - l_i^t} 
$$

Clearly, $L_i$ and $\Phi$ are the pdf and cdf of a uniform distribution defined on $[l_i, u_i]$. 

![Graph](image)

Figure 3.2 Marginal Probability of Rejection as the CDF of a Conflict - Uniform Distribution
3.2.2 Model for Increasing-Decreasing Likelihood of Rejection. When party A is willing to negotiate and concede on an attribute, the maximum likelihood of rejection occurs at a level of conflict greater than \( l \). A triangular distribution is an appropriate model for the rejection likelihood, as shown in Figure 3.2. For example, during the party B obtains more information about the preferences of party A and, thus, it is able to update the estimation of the marginal probabilities of rejection by using a triangular distribution for the likelihood of rejection.

![Figure 3.3 Marginal Probability of Rejection as the CDF of a conflict-Triangular Distribution](image)

The likelihood function of rejection in Figure 3.3(a) is expressed as:

\[
L_i^j = \begin{cases} 
\frac{2(ΔX_i^j - l_i^j)}{u_i^j - l_i^j}, & l_i^j \leq ΔX_i^j \leq m_i^j; \\
(u_i^j - l_i^j)(m_i^j - l_i^j), & m_i^j < ΔX_i^j \leq u_i^j, \\
(u_i^j - l_i^j)(u_i^j - m_i), & \text{otherwise.}
\end{cases}
\]
The marginal probability of rejection in Figure 3.3(b) is expressed as,

\[ L_i = \begin{cases} 
\frac{\Delta X_i - l_i}{u_i - l_i} (u_i - m_i - \Delta X_i), & l_i \leq \Delta X_i \leq m_i; \\
1 - \frac{\Delta X_i - l_i}{u_i - l_i} (u_i - m_i), & m_i < \Delta X_i \leq u_i. 
\end{cases} \] (3.17)

That is, \( L_i \) in Equation (3.16) and \( \phi \) in Equation (3.17) are the pdf and cdf of a triangular distribution with parameters \( l_i, m_i, \) and \( u_i \).

### 3.2.3 Model for Decreasing Likelihood of Rejection

If an attribute is very critical to the counterparty and, thus, it is reluctant to concede in the negotiation. The likelihood of rejection may be modeled as a decreasing convex function of the conflict, shown in Figure 3.4.

The likelihood function of rejection in Figure 3.4(a) is modeled as:

\[ L^1 = ae^{-\Delta^1}, \] (18)
where the parameter $\alpha_i$ is the decay speed of the likelihood function at stage $t$. The cumulative distribution function in Figure 3.4(b) is expressed as,

$$
\Delta X_i^t \Rightarrow e^{\frac{\alpha_i \Delta x_i^t}{1}} - e^{\frac{\alpha_i x_i^t}{1}}
$$

(3.19)

That is, $L_i$ in Equation (3.18) is the pdf of an exponential distribution and $\Phi$ (3.19) is the scaled cdf of the exponential distribution on $[l_i, u_i]$. 

Figure 3.4 Marginal Probability of rejection as the CDF of a Conflict - Exponential Distribution

- (a) Likelihood Function of Rejection
- (b) Marginal Probability of Rejection
3.3 REJECTION PROBABILITY OVER MULTIPLE CONFLICTS

This research eventually needs to determine the rejection probability in more general conditions; that is, the rejection probability when multiple conflicts present.

\[
P^1 = P^O^1 = o \Delta X^t.
\]  

(3.20)

The rejection probability defined above has \( m \) parameters. An alternative view is that the rejection probability is a cumulative probability distribution on \( \Delta X^t \). This research proposes a probability coupling approach as follows.

3.3.1 Independent Attributes. When all attributes are independent, the rejection probability can be obtained easily,

\[
\Pi P^1 = 1 - \prod_{1 \in 1} (1 - \Delta),
\]  

(3.21)

For example, the rejection probability for a contract with two attributes is

\[
P^1 = 1 - 1 - \Delta 1 - \Delta = \Delta + 1 - \Delta \Delta
\]  

(3.22)
3.3.2 Interdependent Attributes. In the remainder of this thesis, only two attributes are considered for simplifying the discussion. Work of this thesis can be extended to three or more attributes.

3.3.2.1 Total rejection probability (Equation (3.22)) indicates the probability of contract rejection at each phase. Specifically, party A considers only one attribute of the contract at each phase. Without loss of generality, this research assumes party A makes the rejection decision by assessing the first attribute if the conflict in this attribute does not trigger the rejection at phase-one (i.e., party B survives from this phase). Party A will move to the phase-two decision that assesses the second attribute. Although it is made with respect to the conflict in attribute two, the decision at phase-two may be influenced by the conflict in attribute one. Let $O^{t,1}$ and $O^{t,2}$ respectively denote the outcomes from the phase-one and phase-two decisions that party A makes in stage $t$. The probability of rejection is:

$$P^t := P \{ o \Delta X^1 \neq o \Delta X^1 \} \equiv 1 - P \{ o \Delta X^1 \neq o \Delta X^1 \} = 1 - P \{ o \Delta X^1 \neq o \Delta X^1, \Delta X^1 \leq l_1 \times O^{t,1} \neq R; \Delta X^1 \}.$$  \hspace{1cm} (3.23)

Equation (3.23), the survival probability for phase-one is determined as

$$P_{\text{Surv}}^t = P \{ o \Delta X_1, \Delta X_2 \leq l_2 \} = 1 - \Psi.$$  \hspace{1cm} (3.24)
Since $\Delta X^f$ does not change from phase-one to phase-two, the survival probability for phase-two can be modeled as a function of $\Delta X_1$, which is parameterized by $\Delta X^f$. Accordingly, this research defines the probability of rejection in phase-two as

$$\Pr^f_{\triangleleft} := P \{ o \Delta X^f \mid o \Delta X^f \neq 0 \}.$$  \hspace{1cm} (3.25)

That is, the rejection probability in phase-two is a conditional probability. Clearly, when

$$\Pr^f_{\triangleleft} = \Pr^f$$

the two attributes are independent, $\Pr^f_{\triangleleft} = \Pr^f$.

The rejection probability over multiple conflicts in Equation (3.23) now is rewritten as

$$p^f := 1 - 1 - \Pr^f_{\triangleleft} 1 - \Pr^f_{\triangleleft} = \Pr^f + (1 - \Pr^f) \Pr^f$$  \hspace{1cm} (3.26)

If party $A$ evaluates the second attribute of the contract at phase-one, the rejection probability is

$$p^f := 1 - 1 - \Pr^f_{\triangleleft} 1 - \Pr^f_{\triangleleft} = \Pr^f + (1 - \Pr^f) \Pr^f$$  \hspace{1cm} (3.27)

Theoretically speaking, changing the sequence should not change the probability of rejection.
Figure 3.5 is the schematic diagram of the rejection probability for a contract with two attributes. The rejection probability of the contract is represented by a surface defined within the area of \( l_i, u_i \times l_i, u_i \). This surface has four boundaries as follows:

\[
P^1 = \phi\text{ when } \Delta X_1 = l_1; \tag{3.28}
\]

\[
P^1 = \phi\text{ when } \Delta X_1 = l_1; \tag{3.29}
\]

\[
P^i = 1, \text{ when } \Delta X^i = u^i, \text{ for } i = 1 \text{ and } 2. \tag{3.30}
\]

For any interior points in the defined area of conflicts, the rejection probability is determined by Equation (3.27).
3.3.2.2 Conditional rejection probability. To model the rejection probability when two conflicts present, the conditional rejection probabilities, $P_{2|1}$ and $P_{1|2}$, must be determined. Figure 3.6 is the schematic diagram of $P_{2|1}$ when the two attributes are interdependent. $P_{2|1} = P_2\text{ given } \Delta X_1 = l_1$, and $P_{2|1} \geq P_2\text{ given } \Delta X_1 > l_1$. Accordingly, this research models the “change” of marginal probability of rejection $P_t^2$ caused by the presence of a conflict in attribute one in order to find $P_t$.

![Figure 3.6 Schematic Diagram of Conditional Rejection Probability](image)

The interdependence of the two attributes indicates a relationship between them of contract attributes. The efficient frontier can be expressed as a function of attribute two,
\[ h(!) = h(X^!). \tag{3.31} \]

\[ X_{1,(!\rightarrow !)} = h(!). \tag{3.32} \]

This function \( h(\cdot) \) identifies the most attractive value of attribute two for party \( A \) given any value of attribute one. Alternatively, the efficient frontier can also be expressed as a function of attribute one,

\[ h''(\cdot) \]

where \( h''(\cdot) \) is the inverse function of \( h(\cdot) \) in Equation (3.31). \( h''(\cdot) \) identifies the most attractive value of attribute one for party \( A \) given any value of attribute two. Any offer on the efficient frontier is not dominated by other offers. When it leaves the efficient frontier, party \( A \) thinks it does not make a concession. While it leaves the efficient frontier and moves towards party \( B \)'s counteroffer, party \( A \) thinks it makes a concession in the negotiation.

Assume party \( B \) has an estimate of the counterparty’s efficient frontier. This estimate provides a way of determining the conditional probability of rejection by moving party \( A \)'s contract \( X^{(! \rightarrow !)} \) on the efficient frontier to one with a conflict in just one attribute and then calculate the marginal probability of rejection based on that conflict. That is,
Figure 7 illustrates the idea of finding offers on the efficient frontier, which are equivalent to \( X_{(l\rightarrow l)} \) and conflict with \( X_{(l\rightarrow l)} \) in just one attribute. That is, Figure 7 shows the way of determining \( \Delta X_{l,l'}^{1,1''} \) and \( \Delta X_{l,l''}^{1,1''} \):

\[
\Delta X_i^{1,1''} = \left\{ \begin{array}{c}
\! - h(X_i^l - C \langle \rangle )^1,
\end{array} \right.
\]

\[
\Delta X_i^{1,1''} = \left\{ \begin{array}{c}
\! - h(X_i^l - C \langle \rangle )^1,
\end{array} \right.
\]

Figure 3.7 Equivalent Contracts (on party A’s Efficient Frontier) with only one Conflict
In Figure 3.7, to party A the greater the attribute one the better and the smaller the attribute two the better. Therefore, a negative deviation from $X^1$ is a conflict in attribute one. When its magnitude is greater than $l^1$, the counteroffer made by party B may be rejected by party A. A positive deviation from $X^1$ is a conflict in attribute two. When its magnitude is greater than $l^1$, the counteroffer may be rejected by party A. Figure 3.7 shows that the counteroffer made by party B is inside the efficient frontier of party A; therefore two conflicts, $\Delta X = (\Delta X_1, \Delta X_1)$, present. By moving the offer made by party A, $X_{1 \rightarrow 1}$, to $(X_1, X_1) - l_1$, the conflicts $(\Delta X_1, \Delta X_1)$ are changed to $(\Delta X^{1''}, l^1)$ and the rejection decision is made based on $\Delta X^{1''}$. Similarly, the offer made by party A can be moved to $(X_1, X_1) + l_1, h (X_1, X_1)$ to change the conflicts $(\Delta X_1, \Delta X_1)$ to $l_1, \Delta X_1$.

### 3.4 PROBABILITY OF NON-ACCEPTANCE

To determine the probability of acceptance, this research also models the probability of non-acceptance, $P_{\neg i}$:

$$P_{\neg i}^{1} = P_{\neg i}^{1, (i \rightarrow 1)} \neq o \Delta X_i$$

(3.34)

This research first repeats work in Sections 3.2 to model marginal probabilities of non-acceptance, $\neg i$, for any $i \in I$. 
\[ \Delta \hat{p}_i = p \hat{p}_i \neq o \Delta X_i, \quad \leq \quad \forall j \neq i, \quad (3.35) \]

\[ \hat{p}_i = \begin{cases} 0 & \text{if } \Delta X_i \leq l_i^t, \\ 1 & \Delta X_i \geq u_i^t. \end{cases} \quad (3.36) \]

That is, \( l_i^t \) is the minimum amount of conflict in attribute \( i \) that may trigger the decision of non-acceptance at stage \( t \). \( u_i^t \) is the minimum amount of conflict in attribute \( i \) that party \( A \) surely will not accept party \( B \)'s counteroffer at stage \( t \). Within the range of \([l_i^t, u_i^t]\), \( \hat{p}_i \) may incline if \( \Delta X_i \) increases.

Figure 3.8 illustrates the relationship between the marginal probability of non-acceptance, \( \hat{p}_i \), and the marginal probability of rejection, \( p \). The marginal probability of non-acceptance is greater or equal to the marginal probability of rejection; that is,

\[ \hat{p}_i \geq p \quad (3.37) \]

because the non-acceptance contains the rejection. Moreover,

\[ l_i^t \leq l_i^t \quad (3.38) \]
because the minimum amount of conflict that may trigger a non-acceptance may still be not enough for triggering a rejection. Similarly,

\[ u' \leq u^\perp, \]  

which states the fact that the minimum degree of conflict that definitely triggers the non-acceptance may not be sufficiently high to trigger the rejection.

![Figure 3.8 Relationship between Marginal Probability of Rejection and Marginal Probability of Non-acceptance](image)

With all marginal probabilities of non-acceptance being modeled, the research repeats work in Section 3.3 to determine the overall probability of non-acceptance.
3.5 MULTI-STAGE NEGOTIATIONS

3.5.1 Multi-Stage Negotiation Process. To reach an agreement, the negotiation may undergo multiple rounds. This is modeled as a sequential decision process for party B. The decision made by party A is the state variable in party B’s decision model.

Figure 9 in the following describes the negotiation at stage $t$, for $t = 1, 2, \ldots, n - 1$, and party B’s decision at $t$. At stage $t$, party B receives the offer from party A, $X^t$, and makes the counteroffer, $X_t$. Party B is not able to precisely predict the party A’s response to the counteroffer. Thus, to party B $X^t$ is a random state variable:

$$X^t \sim \begin{cases} X^t \rightarrow \bar{\Phi}, & \text{with probability } 1 - P^t; \\ \Phi, & \text{with probability } P^t; \\ \text{others, with probability } P^t - P^t. \end{cases}$$

(3.40)

That is, with the probability $1 - P^t$ the counteroffer that party B made is accepted and the negotiation is completed at stage $t$. With the probability $P^t$, the counteroffer is rejected (assuming the negotiation protocol uses $\Phi$ to indicate an offer/counteroffer is rejected during the negotiation) and the negotiation is terminated at stage $t$. With the probability $P^t - P^t$, the counterparty proposes a modified contract $X^t$, which is not equal to $X^t \rightarrow \bar{\Phi}$ or $\Phi$, and the negotiation is continued, moving to the next stage, $t + 1$. 

$$\sim \quad \rightarrow$$

$\sim$
Figure 3.9 Party B's Negotiation Decision at Stage $t = 1, 2..., n-1$

If the negotiation moves to the final stage (i.e., $t = n$), the negotiation must be ended there. Therefore, at the final stage party B's decision is to either take the offer $X^*_B$ or reject it, depending on which yields the higher outcome. Figure 3.10 in the following illustrates the negotiation decision at the final stage ($t = n$) as well as at the preceding stage ($t = n - 1$).

Figure 3.10 Party B's Negotiation Decision at stage $n$
3.5.2 Modified Contract. At stage $t$ the modified contract that party $A$ might propose to party $B$ for the next stage of negotiation, $X_{t+1}^{t!}$, is a vector of random variables. If the counteroffer, $X^t$, is not accepted by party $A$, party $B$ hopes the modified offer $X_{t+1}^{t!}$ can move the negotiation towards a desired direction.

If it were not equal to $\Phi$, $X_{t+1}^{t!!}$ must be within the range from “no concession” by party $A$(i.e., $X_{[t]} = X_{[t]}$). Note: no concession should include other contracts equivalently attractive to party $A$ as $X^t$, which are the contracts on party $A$’s efficient frontier. This research does not consider the equivalent contracts for simplification) to “full concession” (i.e., $X_{[t]} = X_{[t]}$). Thus, the range of each element of $X_{[t]}$ is

$$
\min X_{[t]}, X_{[t]} ! ! ! \leq X_{[t]} ! ! ! \leq \max X_{[t]}, X_{[t]}, \quad (3.41)
$$

which indicates conflicts between the two parties will be reduced with the evolvement of the negotiation one or both parties would concede progressively. An agreement might be reached, finally.

The research defines the concession might be made by party $A$ at the next stage, $t+1$ ($t < n$), as
Equation (3.42) further shows that $X_{\rightarrow i}$ can be expressed as the last offer, $X_{\rightarrow i}$, adjusted by the concession, $Y_{\rightarrow i}$, according to (3.41) and (3.42).

$$X_{\rightarrow i} = X_{\rightarrow i} + C \cdot Y_{\rightarrow i},$$

(3.44)

Equation (3.44) suggests the probability distribution of the modified contract $X_{\rightarrow i}$ can be obtained by modeling the probability distribution of the concession $Y_{\rightarrow i}$ to be made party A. To model the probability distribution of $Y_{\rightarrow i}$, the methods presented in Section 3.2 and 3.3 can be employed. Let $gX_{\rightarrow i}$ represent the probability density function of $X_{\rightarrow i}$.
3.5.3 Negotiation Valuation. Figures 3.9 and 3.10 indicate that decisions over the multi-stage negotiation process are interdependent. Party B’s decision at stage $t$ (for $t = 1, 2, \cdots, n-1$) impact the probability distribution of the state variable at stage $t+1$ (i.e., party A’s decision at the next stage). Consequently, party B’s decision at any stage is impacted by its decision in the preceding stage. To ensure the optimality of the sequentially compounded decisions over the multi-stage negotiation process, dynamic programming is applied to determine the optimal decision at each stage of the negotiation in a backward manner.

$v$ denotes the contract value to party $B$, which is a function of the contract attributes. This value varies with the negotiation process. Define $\pi^t$ as the valuation function given the state of stage $t$, (i.e., the decision made by the counterparty at stage $t$). After receiving the offer $X_{t+1}^t$ at stage $t$, Party B will determine the counteroffer $X_{t+1}^t$ based on the valuation function $\pi^t$. Specifically, At $t = n$, party B’s decision is to either accept the offer $X^t$ or reject it, whichever maximizes the contract value:

$$
\pi^t \left( X^t \right) = \max \left\{ V \left( X_{t+1}^t \right), V \left( X^t \right) \right\} = V \left( X_{t+1}^t \right) \cdot (3.45)
$$

That is, $\pi^t \left( X^t \right)$ is the maximized contract value to party $B$ given the offer party A made at the final stage.
At \( t = n - 1, n - 2, \ldots, 1 \), party B, who is given the offer \( X_{t=1} \), would determine \( X_{t=1} \) that maximizes the expectation of the contract value:

\[
X_{t=1} = \max \{ \pi \cdot \phi \cdot V(X_{t=1}) + P \cdot V \cdot \Phi + (P - P) \cdot E \cdot \pi \cdot X_{t=1} \}.
\]

In Equation (3.46), the party B’s decision at stage \( t \) impacts the non-acceptance probability, \( P^t \), the rejection probability, \( P^t \), and the distribution of state variable of the next stage, \( X^{t+1} \). The expectation of \( \pi^{t+1} \) can be calculated as

\[
E \cdot \pi^{t+1} \cdot X^{t+1} = \int \pi^{t+1} \cdot (X^{t+1}) \cdot (t \rightarrow t+1) \cdot (t \rightarrow t+1) \cdot g(X^{t+1}) \cdot dX^{t+1}.
\]

The party B’s optimal strategy of negotiation is defined as

\[
X^{t+1} = \{ \pi^{t+1} \mid t = 1, 2, \ldots, n \}.
\]

where \( X^{t+1} \) is the decision recommended by the negotiation valuation in Equations (3.45) or (3.46).
4. RESULTS & ANALYSIS

4.1 NUMERICAL CASE

Consider two self-interested agents A and B negotiating over a supply contract with price (P) and quality (Q) as the issues under negotiation. Party B is the supplier in this case and will supply one unit of the item after negotiation agreements are made. Price (P) and Quality (Q) are interdependent issues and are measured in scalable units from 1 through 5. Also, party B’s Total Cost (C) to supply the item is linearly dependent on the Quality (Q) of the item to be supplied; as a higher quality (Q) would imply increased cost and vice versa. Negotiation occurs over two stages and a contract \( X_t \) at any stage of the negotiation \( t \) may be defined in terms of price and quality as:

\[
X_t = \begin{bmatrix} P_t \\ Q_t \end{bmatrix}
\]

Correspondingly a contract from party A to party B and from party B to party A would be defined respectively as:

\[
\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix}
\]

The possible contract offerings are shown in table 4.1 below:
Table 4.5 Array of possible contracts

<table>
<thead>
<tr>
<th>Price</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>(5,5)</td>
<td>(5,4)</td>
<td>(5,3)</td>
<td>(5,2)</td>
<td>(5,1)</td>
</tr>
<tr>
<td>4</td>
<td>(4,5)</td>
<td>(4,4)</td>
<td>(4,3)</td>
<td>(4,2)</td>
<td>(4,1)</td>
</tr>
<tr>
<td>3</td>
<td>(3,5)</td>
<td>(3,4)</td>
<td>(3,3)</td>
<td>(3,2)</td>
<td>(3,1)</td>
</tr>
<tr>
<td>2</td>
<td>(2,5)</td>
<td>(2,4)</td>
<td>(2,3)</td>
<td>(2,2)</td>
<td>(2,1)</td>
</tr>
<tr>
<td>1</td>
<td>(1,5)</td>
<td>(1,4)</td>
<td>(1,3)</td>
<td>(1,2)</td>
<td>(1,1)</td>
</tr>
</tbody>
</table>

Given the possible contract offerings, the corresponding conflict between offers by party A and B therefore would be defined as:

\[
\Delta X' = \begin{bmatrix} \Delta P' \\ \Delta Q' \end{bmatrix} = \begin{bmatrix} & ! & ! \\ \Delta P & - & - \\ ! & \Delta Q_{A\rightarrow B} & Q_{B\rightarrow A} \end{bmatrix}
\]

And the number of possible conflicts for this contract is shown as an array in table 4.2 below:

Table 4.6 Array of possible conflicts

<table>
<thead>
<tr>
<th>delta P</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>(4,4)</td>
<td>(4,3)</td>
<td>(4,2)</td>
<td>(4,1)</td>
<td>(4,0)</td>
</tr>
<tr>
<td>3</td>
<td>(3,4)</td>
<td>(3,3)</td>
<td>(3,2)</td>
<td>(3,1)</td>
<td>(3,0)</td>
</tr>
<tr>
<td>2</td>
<td>(2,4)</td>
<td>(2,3)</td>
<td>(2,2)</td>
<td>(2,1)</td>
<td>(2,0)</td>
</tr>
<tr>
<td>1</td>
<td>(1,4)</td>
<td>(1,3)</td>
<td>(1,2)</td>
<td>(1,1)</td>
<td>(1,0)</td>
</tr>
<tr>
<td>0</td>
<td>(0,4)</td>
<td>(0,3)</td>
<td>(0,2)</td>
<td>(0,1)</td>
<td>(0,0)</td>
</tr>
</tbody>
</table>
Consider that on both price (P) and quality (Q), a conflict level of zero (0) is the minimum amount of conflict that might trigger both non-acceptance and rejection at any stage of the negotiation. Also, a conflict of 3 is the minimum level of conflict that surely triggers non-acceptance while a conflict of 4 surely triggers rejection on both price and quality.

To determine the marginal probabilities of rejection for conflicts in quality, it is modeled as a decreasing convex exponential function while the marginal probability of rejection for conflicts in price is modeled as a triangular probability distribution. One reason for this is that quality is typically modeled as an exponential function and it is expected that the counterparty A is increasingly more likely to reject the contract for larger levels of conflict in quality up to conflict level $\Delta P = 3$ after which the curve flattens out for a certain rejection with even higher levels of conflict. Similarly, the triangular distribution is typically used in corporate finance and business decision models where not very much is known about the distribution of an outcome. In this case, not very much is known about the counterparty’s preference on price but it is assumed that he will negotiate and concede on price right up to a certain limit.

The marginal probability of rejection for a triangular distribution and an exponential distribution is given from equation 3.17 and 3.19 earlier:

$$
\mathcal{P} = \begin{cases} 
\frac{(\Delta X^l_i - l_i)^{l_i}}{u_i - l_i}, & l_i \leq \Delta X^l_i \leq m_i; \\
\frac{(u_i - \frac{l_i}{\Delta X^r_i})^{m_i}}{1 - \frac{u_i}{\Delta X^r_i}}, & m_i < \Delta X^r_i \leq u_i; \\
(u_i - l_i)(u_i - m_i) 
\end{cases}
$$
\[ e^{\alpha t} - e^{\alpha \Delta t} \]

Using decay speed values of \( \alpha = 0.5 \) and modal values \( m = 2 \) for rejection and non-acceptance respectively and proceeding to determine the marginal probabilities of rejection and non-acceptance from conflicts on a single attribute table 4.3 through 4.6 is shown below:

**Rejection:**

Table 4.7 Marginal probability of rejection over conflicts in price

<table>
<thead>
<tr>
<th>Probability</th>
<th>( 0 )</th>
<th>0.125</th>
<th>0.500</th>
<th>0.875</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>delta ( P )</td>
<td>( 0 )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 8.4 Marginal probability of rejection over conflicts in quality

<table>
<thead>
<tr>
<th>Probability</th>
<th>( 0 )</th>
<th>0.455</th>
<th>0.731</th>
<th>0.898</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>delta ( Q )</td>
<td>( 0 )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

**Non-Acceptance:**

Table 4.5 Marginal probability of non-acceptance over conflicts in price

<table>
<thead>
<tr>
<th>Probability</th>
<th>( 0 )</th>
<th>0.540</th>
<th>0.885</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>delta ( P )</td>
<td>( 0 )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 4.6 Marginal probability of non-acceptance over conflicts in quality

<table>
<thead>
<tr>
<th>Probability</th>
<th>( 0 )</th>
<th>0.867</th>
<th>0.984</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>delta ( Q )</td>
<td>( 0 )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>
Corresponding graphs of marginal probability for rejection and non-acceptance for conflicts in both price and quality are indicated in figure 4.1 and 4.2 below:

![Figure 4.1 Marginal Probability of Rejection and Non-Acceptance over Conflicts in Price](image1)

![Figure 4.2 Marginal Probability of Rejection and Non-Acceptance over Conflicts in Quality](image2)
The combined probability of rejection over multiple conflicts has been determined in equation 3.26 and 3.27 as:

\[
P^i \equiv 1 - 1 - \pi_1 - \pi_2 = \pi_1 + (1 - \pi_2)\]

Or:

\[
P^i \equiv 1 - 1 - \pi_1 - \pi_2 = \pi_1 + (1 - \pi_2)\]

depending on which attribute is first evaluated at phase-one. And in this specific case these equations become:

\[
P^i \equiv 1 - 1 - \pi_1 - \pi_2 = \pi_1 + (1 - \pi_2)\]

And:

\[
P^i \equiv 1 - 1 - \pi_1 - \pi_2 = \pi_1 + (1 - \pi_2)\]

Theoretically speaking, changing the sequence should not change the probability of rejection. But since this is not the case, taking the average of the two values to get an optimized value for our probability of rejection and non-acceptance. Table 4.7 through 4.12 shows the combined values of rejection probability for the case under consideration.
Rejection:

Table 4.7 Probability of Rejection conditioned on Quality

<table>
<thead>
<tr>
<th>Price</th>
<th>Quality</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.455</td>
<td>0.731</td>
<td>0.898</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.125</td>
<td>0.765</td>
<td>0.911</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.500</td>
<td>0.949</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.875</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.8 Probability of Rejection conditioned on Price

<table>
<thead>
<tr>
<th>Price</th>
<th>Quality</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.455</td>
<td>0.731</td>
<td>0.898</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.125</td>
<td>0.728</td>
<td>0.966</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.500</td>
<td>0.932</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.875</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.9 Joint Probability of Rejection

<table>
<thead>
<tr>
<th>Price</th>
<th>Quality</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.455</td>
</tr>
<tr>
<td>1</td>
<td>0.125</td>
<td>0.746</td>
</tr>
<tr>
<td>2</td>
<td>0.500</td>
<td>0.941</td>
</tr>
<tr>
<td>3</td>
<td>0.875</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Non-acceptance:

Table 4.10 Probability of Non-acceptance conditioned on Quality

| \( P | Q \) | Quality |
|--------|--------|
| Price | 0 | 1 | 2 | 3 | 4 |
| 0     | 0 | 0.867 | 0.984 | 1 | 1 |
| 1     | 0.540 | 0.993 | 1 | 1 | 1 |
| 2     | 0.885 | 1 | 1 | 1 | 1 |
| 3     | 1 | 1 | 1 | 1 | 1 |
| 4     | 1 | 1 | 1 | 1 | 1 |

Table 4.11 Probability of Non-acceptance conditioned on Price

| \( Q | P \) | Quality |
|--------|--------|
| Price | 0 | 1 | 2 | 3 | 4 |
| 0     | 0 | 0.867 | 0.984 | 1 | 1 |
| 1     | 0.540 | 0.989 | 1 | 1 | 1 |
| 2     | 0.885 | 1 | 1 | 1 | 1 |
| 3     | 1 | 1 | 1 | 1 | 1 |
| 4     | 1 | 1 | 1 | 1 | 1 |

Table 4.12 Joint Probability of Non-acceptance

<table>
<thead>
<tr>
<th>( Average )</th>
<th>Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.540</td>
</tr>
<tr>
<td>2</td>
<td>0.885</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

The probability of non-acceptance \( P^l \) has been determined in table 4.12 above therefore the probability of acceptance is easily determined in table 4.13 as \( 1 - P^l \). Similarly, the probability for a modified contract or for a re-negotiation by the counterparty is easily
determined as \( P^1 - P^1 \). Table 4.13 and 4.14 show the values for the probability of acceptance and re-negotiation respectively.

**Acceptance:**

Table 4.13 Probability of Acceptance

<table>
<thead>
<tr>
<th>Price</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.133</td>
<td>0.016</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.460</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.115</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Re-negotiation (modified contract):**

Table 4.14 Probability of Re-negotiation

<table>
<thead>
<tr>
<th>Price</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.412</td>
<td>0.253</td>
<td>0.102</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.415</td>
<td>0.243</td>
<td>0.061</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.385</td>
<td>0.059</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.125</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Total Cost \((C)\) is related to Quality \((Q)\) by a constant marginal cost function shown below:

\[
TC = 0.75 \times Q
\]

The final Payoff \((Z)\) to party B would be determined as the price less the total cost:
\[ Pa \text{off} Z = \text{Price} P - \text{Cost} C \]

The corresponding pay-off table for party B is then shown in table 4.15 below:

<table>
<thead>
<tr>
<th>Quality</th>
<th>Price</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.25</td>
<td>2.00</td>
<td>2.75</td>
<td>3.50</td>
<td>4.25</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>1.00</td>
<td>1.75</td>
<td>2.50</td>
<td>3.25</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.75</td>
<td>0.00</td>
<td>0.75</td>
<td>1.50</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-1.75</td>
<td>-1.00</td>
<td>-0.25</td>
<td>0.50</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-2.75</td>
<td>-2.00</td>
<td>-1.25</td>
<td>-0.50</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

### 4.2 ANALYSIS

Considering the shaded region in the space of contract possibilities from table 4.15 to simplify analysis of the valuation model for optimum decision analysis during negotiation, the maximum pay-off table to party B is indicated in table 4.16 below:

<table>
<thead>
<tr>
<th>Quality</th>
<th>Price</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.25</td>
<td>1.50</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
<td>0.50</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

This is a two-stage negotiation process so assume that at stage \( t = 1 \), party A makes an offer \([1,3]\) to party B (for the most quality at the least price) i.e;
\[ X^1 \rightarrow 1 \]

From Table 4.16 Party B can make any number of counteroffers except \([1,3]\) because that would imply full concession and end the negotiation at the first stage. Therefore, the likely counteroffers from Party B at stage \(t=1\) would be any of the alternatives: \([1,1], [2,1], [2,2], [2,3], [3,1], [3,2]\) or \([3,3]\). For each of the alternative counter offers \(X^1 \rightarrow 1\) that Party B may propose, Table 4.17 below shows the corresponding acceptance, rejection and re-negotiation probabilities along with the payoff or value, \(V\) to Party B. The grayed portions indicate unlikely alternatives for \(X^1 \rightarrow 1\) as the value \(V(X^1 \rightarrow 1)\) of these alternatives to Party B is zero.

At stage \(t=2\) for any of the likely alternatives for \(X^1\), the corresponding counter offers from Party A is determined along with the corresponding probability of acceptance, the likelihood of that offer being selected, value to Party B and the expected value, \(\mathbb{E}[V]\) given that selection. The likelihood is determined as a measure of the preference of Party A for that specific alternative \(X^1 \rightarrow 1\) based on its acceptance probability.

The final stage \(t=2\) of the negotiation process is assumed to maximize the expected value of the contract value for Party B is determined from equation 3.46 as the sum:

\[
\pi^1 = \max_{X^1} \left[ \pi^1 X^1_1 \rightarrow 1 \right] (1 - P^1) \pi^1_1 \rightarrow 1 \rightarrow 1 V X^1_1 \rightarrow 1 + P^1 \mathbb{V} + (P^1 - P^1) \mathbb{E} \pi^1_1 \rightarrow 1 \rightarrow 1
\]
and is determined from this example to be alternative contract 8 or contract [3,3]

At the terminal stage \( t = 2 \), given any of the values for offer \( \Diamond \rightarrow \land \) party B may arrive at offer \( X_{\land \rightarrow \land} \) and will accept or reject \( \land \), depending on which maximizes the contract value calculated from equation 3.45 as:

\[
\pi(\land \rightarrow \land) = m_a \left( X_{\land \rightarrow \land} \right) = \left\{ \left( V_{\land \rightarrow \land} \right), V_\Phi \right\} = V X_{\land \rightarrow \land} = m_a \left( X_{\land \rightarrow \land} \right)
\]

Table 4.17 shows contract values for each of the non-zero payoff alternatives at the terminal stage with contracts yielding the highest payoffs to party B highlighted. The valuation model shown in table 4.17 indicates offer [3,3] as the optimal offer party B may proposes. This result is understandable considering that party A is a quite quality-conscious agent and will more than likely concede on conflicts on price rather than concede on conflicts on quality as determined earlier.
## Valuations

Table 4.17 Valuation model showing alternative offers' probabilities and expected values.
4.3 FACTORS AFFECTING OPTIMAL DECISION

The results obtained may be influenced by a few factors and some assumptions that have been made about the negotiating parties. Some of the factors that may affect the results obtained for the optimal decision include:

(i) The nature of the probability distribution for the issues under negotiation.
(ii) The nature of the cost function
(iii) The payoff to the negotiating agent (i.e. the supplier)

4.3.1 Nature of the Probability Distribution Function. The probability distribution function models the preference of the counterparty over any given attribute that makes up the contract. It is important that the probability distribution function for each attribute is modeled appropriately to correctly predict or approximate the counterparty’s true values. In the benchmark case, the preference of the counterparty over Quality (Q) was modeled as an exponential probability distribution while preference over Price (P) was modeled as a triangular probability distribution.

It is useful to observe how the nature of the probability distribution affects the results obtained. Three different cases are reviewed; in the first case, the order of things is reversed to assume a triangular distribution for Quality (Q) and an exponential distribution for Price (P). Second, an exponential distribution is assumed for both attributes and finally, a triangular distribution is assumed for both attributes, keeping all other parameters constant.
The respective payoff tables and graphs of marginal probabilities of rejection and non-acceptance for conflicts in Price (P) and Quality (Q) are shown in table 4.18 through 4.20 and figure 4.3 through 4.5 below. A similar valuation model yields corresponding optimal results of [1,1], [1,1] and [3,3] indicating how the results obtained change with the assumption of the probability distribution for any attribute that characterizes the contract.

Table 4.18 Payoff table for Exponential Price and Triangular Quality

<table>
<thead>
<tr>
<th>Price</th>
<th>Quality 1</th>
<th>Quality 2</th>
<th>Quality 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.25</td>
<td>1.50</td>
<td>0.75</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
<td>0.50</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 4.19 Payoff table for Exponential Price and Quality

<table>
<thead>
<tr>
<th>Price</th>
<th>Quality 1</th>
<th>Quality 2</th>
<th>Quality 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.25</td>
<td>1.50</td>
<td>0.75</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
<td>0.50</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 4.20 Payoff table for triangular Price and Quality

<table>
<thead>
<tr>
<th>Price</th>
<th>Quality 1</th>
<th>Quality 2</th>
<th>Quality 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.25</td>
<td>1.50</td>
<td>0.75</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
<td>0.50</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Figure 4.3 Marginal Probability of Rejection and Non-Acceptance over Conflicts in Quality and Price (Case 1)

Figure 4.4 Marginal Probability of Rejection and Non-Acceptance over Conflicts in Quality and Price (Case 2)

Figure 4.5 Marginal Probability of Rejection and Non-Acceptance over Conflicts in Quality and Price (Case 3)
4.3.2 Nature of the Cost Function. The nature of the cost function determines to a great degree the optimal negotiation outcome. In the benchmark case, a linear cost function \( C = 0.75*Q \) with constant marginal cost yielded results [3,3]. To investigate how the nature of the cost function impacts results obtained, increasing concave and convex cost functions with marginally decreasing and increasing costs are assumed instead, while keeping all other parameters the same. For cost functions: \( C = 0.75*Q^{0.5} \) and \( C = 0.75*Q^{1.5} \) the respective payoff functions are shown in table 4.21 and table 4.22 below. Similar valuation model yields corresponding optimal results as [2,3] and [1,1] indicating just how the nature of the cost function determines final optimal results. The cost function also determines the payoff to the supplier, therefore it is important to know exactly the relationships between these parameters in a negotiation.

Table 4.21 Payoff for Marginally Decreasing cost function

<table>
<thead>
<tr>
<th>Price</th>
<th>Quality</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.70</td>
<td>1.94</td>
<td>2.25</td>
</tr>
<tr>
<td>2</td>
<td>0.70</td>
<td>0.94</td>
<td>1.25</td>
</tr>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 4.22 Payoff for Marginally Increasing cost function

<table>
<thead>
<tr>
<th>Price</th>
<th>Quality</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.00</td>
<td>0.00</td>
<td>2.25</td>
</tr>
<tr>
<td>2</td>
<td>0.00</td>
<td>0.00</td>
<td>1.25</td>
</tr>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.25</td>
</tr>
</tbody>
</table>
4.3.3 Payoff to the Negotiating Party. Considering changes to the payoff table that make the negotiation either more profitable or more challenging to the supplier. In the first case, the contract terms are profitable to the supplier regardless of the choice he/she makes while in the second case, the supplier only has limited options in which the contract may be profitable. Considering cost functions, \( C = 0.25*Q \) and \( C = 1.75*Q \) with payoff tables indicated below, similar valuation model yields corresponding optimal results as [2,3] and [3,1], indicating how profitability affects the optimal results obtained.

Table 4.23 Payoff for a more profitable contract

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Price</strong></td>
<td>3</td>
<td>2.75</td>
<td>2.50</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.75</td>
<td>1.50</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.75</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Table 4.24 Payoff for a more challenging contract

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Price</strong></td>
<td>3</td>
<td>1.25</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.25</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Finally, through an examination of payoff tables 4.23 and 4.24 and also payoff in the benchmark case as in table 4.16 an attempt might be made to determine optimal offers using a simple heuristic. However, such methods may not always yield offers that are optimal as it generally may fail to take into account the underlying preferences of the counterparty over contract attributes and other factors that also affect optimal results.
5. CONCLUSION & FUTURE RESEARCH

5.1 SUMMARY AND CONTRIBUTIONS

An important objective of negotiation decision analysis is to reduce the time required to convert negotiation into a win-win agreement. Therefore, it is important to have a system that helps determine an optimal concession to make at each time from simultaneous negotiation of multiple interdependent issues. This is especially under circumstances of having incomplete information about the counterparty. To help the supplier effectively and efficiently derive the optimal decision for negotiating the supplier contract, this thesis has developed a valuation framework for the supplier. This valuation model helps to simultaneously negotiate multiple interdependent issues of a contract with a customer.

A probability coupling approach was developed. This approach can be used to estimate the probability distribution of negotiation outcomes (accepted, rejected, continued). The probability of each possible outcome is a function of conflicts between negotiation parties. The probability function characterizes the customer’ preference to each of the contract attributes. Also, a valuation function was formed. This function can be used to determine the expected contract value to the supplier under each possible negotiation outcome. It can also be used to determine the suppliers overall expected. Although making a concession reduces the reward of the supply contract, they reduce conflict and increase the chance that the supplier will succeed from the negotiation.
Therefore, the optimal concession at each time of negotiation is the one that strikes the best trade-off between the contract reward and the probability of success.

A dynamic program (DP) was created in an attempt to optimize the supplier’s two-stage negotiation. The customer’s offer in the DP is the state variable, and the supplier’s counteroffer is the decision variable. The supplier’s decision in the first stage may impact the distribution of the state variable in the next stage. The DP assumes the supplier’s decision is optimal at each stage of negotiation. Finally, a numerical example was developed that illustrates the implementation of the proposed valuation framework.

5.2 FUTURE RESEARCH

This research has filled out some gaps in the current literature of simultaneous negotiation of multiple interdependent issues, as well as that of decision supports for iterated negotiations. Important extensions from this research, however, will bring more important changes to the areas of negotiation decisions.

This research was limited to the case of simultaneous negotiation of two interdependent issues negotiated over two-stages. An immediate extension to this work would be to consider cases that involve more than two issues and undergo more than two-stages, as are commonly seen in real-world applications. The extension of this study to more general cases is relatively straightforward. It would, however, substantially increase the computational complexity, which would need to be addressed appropriately.
An important area of future work from this research would be to consider negotiation over multiple items with the same counterparty. It will be interesting to investigate the behavior of the supplier when multiple items are presented. Of particular interest would be how that will impact the reward from the contract as well as the likelihood of success in the negotiation.

Assumptions were made about the counterparty’s preferences when faced with conflicts over different attributes. The preference (or attitude) of the negotiating party over any given attribute determines the nature of the indifference curve. It also helps with determining what the efficient frontier will be. The sources of information and data that were the basis for these assumptions were not, however, included in this work. Information is an important part in decision analysis. Therefore, future work is needed to fill this gap.


VITA

Nnaemeka Amaeshi was born on March 7th, 1986 in Lagos, Nigeria. In 2008, he received his bachelor’s degree in mechanical engineering from the Federal University of Science and Technology in Owerri, Nigeria.

Nnaemeka’s work experience includes work as project engineer working on large-scale energy projects for multinational oil and energy companies such as Shell.

In 2013, Nnaemeka began attending Missouri University of Science and Technology (Missouri S&T) in Rolla, USA. During the spring semester of 2015, he worked as a graduate research assistant and also as a teaching assistant for the graduate course on Advanced Financial Management. In the spring semester of 2015 he won the 2014/2015 outstanding MS graduate student research award. In August 2015, Nnaemeka will receive a Masters degree in engineering management.