Investigation of cased wellbore integrity in the Wabamun area sequestration project

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INVESTIGATION OF CASED WELLBORE INTEGRITY IN THE WABAMUN
AREA SEQUESTRATION PROJECT

by

BENJAMIN LEE WEIDEMAN

A THESIS

Presented to the Faculty of the Graduate School of the
MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY
In Partial Fulfillment of the Requirements for the Degree

MASTER OF SCIENCE IN PETROLEUM ENGINEERING

2014

Approved by

Runar Nygaard, Advisor
Shari Dunn-Norman
Ralph Flori
ABSTRACT

Loss of cased wellbore integrity can cause many significant problems, including lost production, aquifer contamination, blowouts, and other environmental damage. The objective of this study is to conduct an analysis of the Wabamun Area Sequestration Project (WASP) to determine if wells in that area are suitable for CO₂ sequestration. This was done by conducting laboratory experiments on the common cement compositions found in the area, simulating the various conditions possible in a wellbore, and by analyzing three case studies from the WASP. The cases include two existing production wells and a proposed injection well specifically designed for CO₂ sequestration. This study investigated multiple wellbore construction scenarios using a variety of parametric analyses and three case studies. The conditions that can affect leakage risk were simulated, including cement composition, experimentally derived mechanical and thermal properties, heat of hydration, shrinkage/expansion, pore pressure variations, injection thermal loading, and injection mechanical loading. Transient chemo-poro-thermo-elastic staged analytical models and staged finite element simulations were conducted for each of the scenarios and case studies. The results of the case studies found that existing wells can be repurposed to become CO₂ sequestration wells as long as the well is structurally sound before injection. There is a significant risk to the loss of cased wellbore integrity if the effects of a well’s life are not taken into account. The model developed in this paper is useful to investigate the integrity for the wells in the WASP as well as other onshore and offshore wells.
ACKNOWLEDGEMENTS

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\[ \tau_{rz} \quad \text{Shear Stress, MPa} \]
\[ \tau_{r0} \quad \text{Shear Stress, MPa} \]
\[ \tau_{r\theta z} \quad \text{Formation or Cylinder } z \text{ Shear Stress, MPa} \]
\[ \tau_{\theta z} \quad \text{Shear Stress, MPa} \]
\[ r \quad \text{Radius, m} \]
\[ r_i \quad \text{Radius of Interest, m} \]
\[ r_{i-1} \quad \text{Interface Outside Radius of Interest, m} \]
\[ r_{i+1} \quad \text{Interface Inside Radius of Interest, m} \]
\[ z \quad \text{Axial Position, m} \]
\[ \alpha \quad \text{Linear Thermal Expansion Coefficient, } \mu \varepsilon/°C \]
\[ E \quad \text{Young’s Modulus, MPa} \]
\[ \nu \quad \text{Poisson’s Ratio} \]
\[ \nu_s \quad \text{Steel/Casing Poisson’s Ratio} \]
\[ \nu_c \quad \text{Cement Poisson’s Ratio} \]
\[ \nu_f \quad \text{Formation Poisson’s Ratio} \]
\[ K \quad \text{Bulk Modulus, MPa} \]
\[ G \quad \text{Shear or Rigidity Modulus, MPa} \]
\[ G_s \quad \text{Steel/Casing Shear Modulus, MPa} \]
\[ G_c \quad \text{Cement Shear Modulus, MPa} \]
\[ G_r \quad \text{Formation Shear Modulus, MPa} \]
\[ \rho \quad \text{Density, kg/m}^3 \]
\[ k \quad \text{Thermal Conductivity, W-m/K} \]
\[ \{c\} \quad \text{Specific Heat Capacity, J/kg-K} \]
\[ \Delta \quad \text{Change in} \]
\[ \sigma' \quad \text{Effective Stress} \]
\[ T \quad \text{Temperature Change, } °C \]
\[ \Delta S \quad \text{Shrinkage Strain} \]
\[ P_p \quad \text{Pore Pressure, MPa} \]
\[ P_{p_x} \quad \text{Casing or Cylinder Pore Pressure, MPa} \]
Pp_y  Cement or Cylinder x Pore Pressure, MPa
Pp_z  Formation or Cylinder z Pore Pressure, MPa
β    Biot’s Coefficient
  t    Time, seconds
  a    Inner/Casing Radius, m
  b    Inner Cement Radius, m
  c    Inner Formation Radius, m
θ    Angle, degrees
σ_h  Minimum Principal Horizontal Stress, MPa
σ_H  Maximum Principal Horizontal Stress, MPa
Pm   Mud Pressure, MPa
  u    Displacement, m
  p    Pressure, MPa
  P_a  Pressure at a, MPa
  P_b  Pressure at b, MPa
  P_c  Pressure at c, MPa
C_{1x}  Casing or Cylinder x 1^{st} Integration Constant
C_{2x}  Casing or Cylinder x 2^{nd} Integration Constant
C_{1y}  Cement or Cylinder y 1^{st} Integration Constant
C_{2y}  Cement or Cylinder y 2^{nd} Integration Constant
C_{1z}  Formation or Cylinder z 1^{st} Integration Constant
C_{2z}  Formation or Cylinder z 2^{nd} Integration Constant
  L    Length, mm
  C    Thermal Expansion Experimental Correction Factor
  V_p  P-Wave or Compressional Wave Velocity, m/s
  V_s  S-Wave or Shear Wave Velocity, m/s
  q    heat flow rate, W
  x    Length, m
  Q    Heat, J
m  Mass, kg
\( T_0 \)  Tensile Strength, Pa
\( P \)  Brazilian Load Strength, N
\( D \)  Diameter of Sample, m
\( t \)  Thickness of Sample, m
\( C_o \)  Unconfined Compressive Strength, Pa
\( C_m \)  Measured Compressive Strength, Pa
\( F \)  Force on Sample, N
\( A \)  Area of Sample, m²
UCS  Uniaxial Compressive Strength, MPa
1. INTRODUCTION

1.1. OVERVIEW

The casing and cement sheath are the main barriers of leakage and wellbore integrity in cased wellbores. The primary goals of the cement sheath, the cement placed between the casing and formation, are to isolate the wellbore from the formation fluids and to support the casing. The cement is designed to prevent fluid flow into adjacent formations and to the surface by creating an impermeable barrier in which formation fluid cannot flow past. In addition, the cement sheath also prevents the wellbore fluids from entering the formation during the drilling operation, better known as lost circulation. For further description see Nelson and Guillot (2006).

The cement sheath can be damaged by completing, pressure testing, stimulating, and producing which can cause loss of zonal isolation (Ravi et al., 2002). Zonal isolation is the prevention of fluid communication between drilled permeable formations (Le Roy-Delage et al., 2000). The failure of the cement sheath can cause many damaging and expensive problems, especially due to the high costs of cement repair operations which can be upwards of $500,000 to $1,500,000 (Rusch et al., 2004). In addition, the remedial cementing operation is considered to be dangerous. It runs the risk of personnel being injured or killed, equipment being damaged or destroyed, and the risk of blowouts or spills which pose a significant environmental risk (Rusch et al., 2004). Fluid flow can either pass into another permeable formation or if the seal failure is large enough the leakage could reach to the liner top or even to the surface. Studies have shown that leakage is a reoccurring problem for cased wells (Watson and Bachu, 2007). For example, in Alberta Watson and Bachu (2007) investigated the leakage of 316,439 wells and found that 0.6% had gas migration (GM) and 3.9% has surface casing vent flow (SCVF), of these wells 98% of the leaking wells were cased wells.

If the fluid passes into another permeable formation it could lead to severe environmental damage such as if the fluid flows into a fresh water aquifer and contaminates the drinking water. Also, if the flowing formation is the reservoir formation then it could release valuable hydrocarbons into the annulus which would cause a loss in production and profit from the well. In addition if the fluid is a highly pressurized gas,
when the gas reaches the liner top or the surface it will expand rapidly due to the lower pressure and could cause damage to the wellbore, surface facilities, or cause fatal injuries (Watson and Bachu, 2007). Under any of these conditions if the issues could not be fixed then they could result in the loss of the well. The well would then have to be plugged and abandoned causing a loss of the entire investment.

The various leakage paths that can occur in the near wellbore region are presented in Figure 1.1. The leakage paths are divided into two categories: primary and secondary. Primary leakage paths occur due to events and conditions during the primary cementing job. These leakage paths include: 1) incomplete annular cementing job does not reach seal layer, 2) lack of cement plug or permanent packer to prevent flow after abandonment, 3) failure of the casing, 4) poor bonding caused by the mudcake, 5) channeling in the cement, and 6) primary permeability in the cement sheath or cement plug. The secondary leakage paths are those that occur after the primary cementing is complete. Secondary leakage paths include; 7) de-bonding due to tensile stress on casing-cement-formation boundaries, 8) Fractures induced into the cement and formation, 9) cement dissolution and carbonation, and 10) wear or corrosion of the casing. All of these leakage paths compromise the cement sheath integrity and can allow fluid to flow into the annulus or the wellbore.

Figure 1.1: Leakage Paths in the Near Wellbore Region
<table>
<thead>
<tr>
<th># from Figure 1</th>
<th>Causes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>High well inclination, Poor hole quality, Poor centralization, Incorrect calculation of cement needed, Cement loss into the formation (Bois et al., 2011)</td>
</tr>
<tr>
<td>2</td>
<td>Packer or cement plug fails, Packer or cement plug were never set (Watson and Bachu, 2007)</td>
</tr>
<tr>
<td>3</td>
<td>Formation fluid invasion during cement setting, Flow Induced by Loss in Annular Pressure (FILAP) (Cooke et al., 1983)</td>
</tr>
<tr>
<td>4</td>
<td>Poor selection of chemical agents for mud removal, Poor matching between volume/rheologies/displacement rate of the cement train (Bois et al., 2011)</td>
</tr>
<tr>
<td>5</td>
<td>Formation fluid invasion during cement setting, Poor mud removal (Nelson and Guillot, 2006)</td>
</tr>
<tr>
<td>6</td>
<td>High water content, Free water or air entrained in the cement, Flow Induced by Loss in Annular Pressure (FILAP) (Cooke et al., 1983)</td>
</tr>
<tr>
<td>7</td>
<td>Exceed bond strength, Large wellbore temperature and/or pressure reductions, Increase in cement pore pressure, Cement hydration/shrinkage (Bois et al., 2012)</td>
</tr>
<tr>
<td>8</td>
<td>Large changes in wellbore temperature, Large changes in wellbore pressure, Changes in pore pressure, Shear failure, Tensile failure (Bois et al., 2012)</td>
</tr>
<tr>
<td>9</td>
<td>Corrosive formation fluids, CO₂ injection (Nygaard et al., 2011)</td>
</tr>
<tr>
<td>10</td>
<td>Casing wear, Sand production, Corrosive formation fluids and CO₂ injection (Watson and Bachu, 2007, Nygaard et al., 2011)</td>
</tr>
</tbody>
</table>

There are many causes of the various leakage paths, but a majority of them are caused by operational conditions under the operator’s control. Some of these operational conditions include poor hole centralization, incorrect calculation of cement, poor
selection of chemicals to remove mud. Table 1.1 gives examples of various causes of the leakage paths occurring in Figure 1.1.

The causes of the leakage paths that occur after the primary cementing are mainly due to pressure, temperature, and chemical changes. The pressure changes can occur from changing mud weight during drilling, stimulation treatments, production flow, and injection. The temperature changes occur from pumping a fluid into the well such as from drilling, stimulation, CO₂ injection, water injection, and steam injection, or from the formation fluid being produced through the tubing. Chemical changes to the cement composition can greatly affect the permeability and mechanical properties of the cement sheath. Besides shrinkage and expansion of the cement, all other chemical changes are outside the projects scope.

Several approaches have been used to investigate the effects on the cement sheath from changes in fluid pressure and temperature. The most common method is thermo-elastic modeling which has been expanded to more advanced models including chemo-thermo-poro-elastic approaches. The use of system response curves is another approach that has been used for evaluating cement sheaths. The next section outlines these methods in more detail.

1.2. THERMO-ELASTIC MODELING

Modeling of the cement sheath to determine the risk of leakage has been done using different constitutive equations (Thiercelin et al., 1998a). The most widely used method is assuming a thermo-elastic constitutive model. The thermo-elastic model takes into account the temperature changes as well as surface stress changes that contribute to the state of stress in the cement sheath, by treating it as a linear elastic material. Timoshenko and Goodier (1951) were one of the first authors to present the constitutive relations for thermo-elasticity. Thiercelin et al. (1998a) solved analytical thermo-elasticity in terms of stress, Eqns. 1-4, where Eqn. 1, Eqn. 2, Eqn. 3, and Eqn. 4 are the radial, the circumferential, axial, and shear stress-strain relationships with temperature, respectively. Thiercelin et al (1998a) also used a finite element approach to determine the radial temperature distribution with time, using the equation found in Eqn. 5. Eqns. 1-4
represent Hooke’s Law in cylindrical coordinates for isotropic materials, constant Young’s Modulus and Poisson’s Ratio in all directions, with the addition of a term for temperature influence which relates the change in temperature and the linear thermal expansion coefficient to strain. Eqn. 5 is the partial differential equation for radial temperature variation with respect to depth and temperature. Due to the complexity of Eqn. 5, Thiercelin et al. (1998a) used the finite element method to solve for change in temperature.

\[ \varepsilon_r - \alpha T = \frac{\sigma_r}{E} - \frac{\nu}{E} (\sigma_\theta + \sigma_z) \]  

(1)

\[ \varepsilon_\theta - \alpha T = \frac{\sigma_\theta}{E} - \frac{\nu}{E} (\sigma_r + \sigma_z) \]  

(2)

\[ \varepsilon_z - \alpha T = \frac{\sigma_z}{E} - \frac{\nu}{E} (\sigma_r + \sigma_\theta) \]  

(3)

\[ \gamma_{rz} = \frac{1}{G} \tau_{rz} \quad \gamma_{r\theta} = \frac{1}{G} \tau_{r\theta} \quad \gamma_{\theta z} = \frac{1}{G} \tau_{\theta z} \]  

(4)

\[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{\rho[c]}{k} \frac{\partial T}{\partial t} \]  

(5)

After Thiercelin et al. (1998a) the primary method to determine state of stress using thermo-elastic constitutive equations was with the use of the finite element method. A finite element analysis is used due to the complexity of the calculations when multiple materials are added into the model and the complexity of the temperature distribution. Philppacopoulos et al. (2002), Gray et al. (2009), Bosma et al. (1999), Takase et al. (2010), Ravi et al. (2002) all used finite element software packages to run thermo-elastic models. All authors concluded that shrinkage will increase the risk of de-bonding. Also, that decreases in temperature induce tensile de-bonding and temperature increases can induce shear failure in the cement sheath.
1.3. CHEMO-THERMO-PORO-ELASTIC MODELING

Fourmaintraux et al. (2005) emphasized the need for models to encompass additional phenomena observed in cements, for instance pore pressure and shrinkage. To do this the authors suggested that a chemo-thermo-poro-mechanical model is necessary to accurately model the casing, cement, formation, and their interfaces. Fourmaintraux et al. (2005) used a system response curve (SRC) methodology to analyze the various loading conditions. These curves were used to identify interface failure. These models are complex and require much background knowledge and calculation to define the SRC. Bois et al. (2011) and Bois et al. (2012) emphasized the chemo-thermo-poro-mechanical model as necessary to model the cement sheath, due to stress changes caused by the pressure pore fluid acting on the casing and cement and cement shrinkage. The analytical equations for chemo-thermo-poro-elastic modeling in terms of strain are given in Eqns. 6-8 (Jaeger et al., 2007). Adding the pore pressure and shrinkage terms to Eqns. 1-4 gives stress strain relationships for the chemo-thermo-poro-elastic model (Eqns. 6-9 where Eqn. 6, Eqn. 7, Eqn. 8, and Eqn. 9 are the radial, the circumferential, axial, and shear stress-strain relationships, respectively). The pore pressure term when the equations are described in terms of strain and pore pressure can be directly applied as a stress which is the Biot’s Coefficient multiplied with the pore pressure which gives the contribution to the state of stress in the material (Terzaghi, 1936). The chemical shrinkage of the cement is applied as an isotropic volumetric strain in which each direction has an equal strain applied.

\[
\sigma_r - \beta P p = \frac{(1-v)E}{(1-2v)(1+v)} \left[ \varepsilon_r + \frac{v}{(1-v)} \varepsilon_\theta \right] - \frac{E(a T + \Delta S)}{(1-2v)} \tag{6}
\]

\[
\sigma_\theta - \beta P p = \frac{(1-v)E}{(1-2v)(1+v)} \left[ \varepsilon_\theta + \frac{v}{(1-v)} \varepsilon_r \right] - \frac{E(a T + \Delta S)}{(1-2v)} \tag{7}
\]

\[
\sigma_z - \beta P p = \nu(\sigma_r + \sigma_\theta) - a E T - E \Delta S \tag{8}
\]

\[
\gamma_{rz} = \frac{1}{G} \tau_{rz} \quad \gamma_{r\theta} = \frac{1}{G} \tau_{r\theta} \quad \gamma_{\theta z} = \frac{1}{G} \tau_{\theta z} \tag{9}
\]
Bois et al. (2012) found that if the pore pressure in the formation is greater than
the pore pressure in the cement a compressional force is applied to the cement sheath, but
if the pore pressure in the cement is greater than or equal to the formation pore pressure
the pore fluid will exert a force on both the casing and the formation putting the cement
in tension. The loading conditions that occur in the wellbore can be described as pressure
and/or temperature changes. When drilling, the drilling mud is pumped from the surface
down the hole. The drilling mud has a hydrostatic pressure, the pump applies pressure to
flow the mud, and the drilling mud has a temperature less than that of the formation
because it comes from the surface equipment. Other operations such as hydraulic
fracturing, fluid injection, acidizing, etc. exert pressure and temperature changes on the
inside of the casing similar to that of the drilling mud. The pressure changes including
pore pressure changes can be described using the equations for hoop stress, radial stress,
and shear stress, Eqns. 10, 11, and 12, respectively (Jaeger et al., 2007).

\[
\sigma_{rr}^\prime = \frac{1}{2}(\sigma_H + \sigma_h - 2Pp) \left[ 1 - \left(\frac{a}{r}\right)^2 \right] + \frac{1}{2}(\sigma_H - \sigma_h) \left[ 1 - 4\left(\frac{a}{r}\right)^2 + 3\left(\frac{a}{r}\right)^4 \right] \cos 2\theta + (Pm - Pp) \left(\frac{a}{r}\right)^2 \quad (10)
\]

\[
\sigma_{\theta\theta}^\prime = \frac{1}{2}(\sigma_H + \sigma_h - 2Pp) \left[ 1 + \left(\frac{a}{r}\right)^2 \right] - \frac{1}{2}(\sigma_H - \sigma_h) \left[ 1 + 3\left(\frac{a}{r}\right)^4 \right] \cos 2\theta - (Pm - Pp) \left(\frac{a}{r}\right)^2 \quad (11)
\]

\[
\tau_{r\theta}^\prime = -\frac{1}{2}(\sigma_h - \sigma_H) \left[ 1 + 2\left(\frac{a}{r}\right)^2 - 3\left(\frac{a}{r}\right)^4 \right] \sin 2\theta \quad (12)
\]

1.4. SYSTEM RESPONSE CURVES

System response curves (SRC) were presented by Fourmaintraux et al. (2005) as a
method to decompose the well into its individual components and use equilibrium
between components to solve them. The method of SRC is derived from the
convergence-confinement method used in geomechanical tunnel design (Oreste, 2009).
The convergence-confinement method used in tunnel design consists of two materials:
support and rock. A circular hole is made in the rock and a support material such as steel
or concrete is placed inside the hole to support the opening. The curves for each material
are developed from their respective material properties and the changes in conditions. An
example convergence-confinement curve is presented in Figure 1.2, where the convergence-confinement curve represents the rock material and the support curve represents the supporting material. The elastic regions of each material are described by the sloped lines and the plastic regions of each material are described by the curved portion of the convergence-confinement curve and the horizontal line portion of the support curve. As used in tunnel design only pressure or stress changes are accounted for (Oreste, 2009).

Figure 1.2: Convergence-Confinement Method (Oreste, 2009)

Bois et al. (2011) and Bois et al. (2012) applied this method to wellbores by introducing three materials to the method: casing, cement, and rock. In addition the curves are updated to include internal pressure changes, temperature changes, pore pressure changes, shrinkage/expansion, and external pressure changes. A summary of the equations presented in Bois et al. (2012) are presented in Eqns. 13-14 and Figure 1.3 to give a visual representation of the various parameters.
The equations and method described by Bois et al. (2012) for the inner casing cement interface are presented in Eqn. 13 and Eqn. 14. Eqn. 13 gives the casing support curve. The equations assume that the change in pressure inside the casing is zero. Eqn. 14 gives the convergence-confinement curve for the cement and rock for the casing-cement interface. The factors that Bois et al. (2012) presents that affect the convergence-confinement curve are the temperature, pore pressure, and shrinkage/expansion. The equations are presented as a line where $\Delta p_i$ is on the $y$-axis and $\Delta u_i$ is on the $x$-axis. The convergence-confinement curve is dependent on two materials, the cement and the rock, the relationship between these two materials is described in the $V_c$ and $U_c$ terms. This method assumes that pressure inside the inner material is known and that the combination of the two outer materials goes to infinity. The greatest limitation of the system response curve method applied to the wellbore as an elastic, analytical method is that the stresses are total stresses not including pore pressure. Additionally, the loading of temperature and pore pressure changes are assumed to be uniform across the entire material, which is not the case in real world applications. This means that the results of the model will not be accurate. Another limitation is that the method only covers the radial stress. The axial and hoop stresses found in the materials are not accounted for in this method. The result is that there are limitations on the failure the modes this model can predict.
1.5. STAGED FINITE ELEMENT MODELS

Ravi et al. (2002) introduced staged finite element models to more realistically model the conditions that occur within the wellbore. A staged finite element model applies loads and materials in various steps replicating the actual drilling process, and not all at once as previously done. The advantage of this method is that by loading the model in steps it provides greater control over the material interactions. The typical stages used to model the cement sheath are listed below.

1. Drill Hole
2. Run Casing and Pump Cement Slurry
3. Cement Hydration
4. Pressure and Temperature Loading

Gray et al. (2009) proposes a detailed 7 stage finite element model. The loads that go into cement hydration were described in 3 stages and the last 3 stages describe various operational loading conditions. While the model used by Gray et al. (2009) gives highly detailed stage definitions, the stages only describe a chemo-thermo-elastic model. Bois et al. (2012) describes the various changes in pore pressure that occur during hydration. This suggests that even though Gray et al. (2009) has a detailed chemo-thermo-elastic description of cement hydration in may not be enough to accurately model the state of stress. This is why a thermo-poro-elastic model is required.
1.6. CEMENT HYDRATION

Cement hydration has been described differently by many different authors including Bosma et al. (1999) who described cement as having three possible hydration scenarios: shrinking, non-shrinking, and expanding. Bosma et al. (1999) observed the effects of these different hydration scenarios and described the results of the modeling. Bois et al. (2012) clarified the effect of shrinking on de-bonding by explaining the effect shrinkage would have on the cement sheath. Their main conclusions are described in Table 1.2.

<table>
<thead>
<tr>
<th>Hydration Scenario</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shrinking</td>
<td>Cements failed in tension, and ratio of Young’s Modulus of rock to cement was critical in determining which cement will fail (Bosma et al., 1999)</td>
</tr>
<tr>
<td></td>
<td>Shrinkage causes outer de-bonding not inner de-bonding (Bois et al., 2012)</td>
</tr>
<tr>
<td></td>
<td>4% Shrinkage causes de-bonding (Ravi et al., 2002)</td>
</tr>
<tr>
<td>Non-Shrinking</td>
<td>Cement with low Young’s Modulus failed in shear (Bosma et al., 1999)</td>
</tr>
<tr>
<td>Expansion</td>
<td>Cement experienced drastic failure when cement was stiffer than rock and minor shear failure when cement was less stiff than rock (Bosma et al., 1999)</td>
</tr>
</tbody>
</table>

The state of stress within the cement once it sets is the key to determining cement sheath integrity. Thiercelin et al. (1998a), Bosma et al. (1999), Ravi et al. (2002), Nelson and Guillot (2006), and James and Boukhelifa (2008) assumed that the initial effective state of stress before shrinkage or expansion in the cement was equal to 0 MPa. Nelson and Guillot (2006) used the observations from Cooke et al. (1983) and Morgan (1989) to determine that the total stress in the cement unloads as the gel strength develops and drops to at least formation pore pressure. Therefore Nelson and Guillot (2006) predicted
there is zero-effective stress in the cement if the total stress equals the pore pressure. The pressures calculated by Bosma et al. (1999) also confirm the ideas of Nelson and Guillot (2006), but under these conditions the risk for a shrinking cement to fail under tension is great if the formation pore pressure recharges the cement column, because the effective stress could put the cement sheath under tension. However, as pointed out in Gray et al. (2009) and Bois et al. (2012), assuming zero effective stress with no shrinkage is inaccurate. Gray et al. (2009) and Bois et al. (2012) both stated that the total stresses within the cement sheath would be equal to the hydrostatic column. Gray et al. (2009) notes that the casing and formation are subjected to cement slurry hydrostatic pressure and according to Newton’s Third Law, if a body exerts a force on a second body then the second body exerts an equal and opposite force. Bois et al. (2012) says that as the casing expands and contracts during cement setting the cement is in a plastic fluid state which means it is unable to restrain these movements so no stress is developed.

Another component of the state of stress within the cement sheath is the pore pressure. Cooke et al. (1983) and Morgan (1989) experimented with placing strain gages and temperature sensors on the casing and measuring the pressure and temperature changes that occurred during the drilling and cementing operations. What was found in the literature was that the pressure pushing against the casing changes from slurry hydrostatic column pressure as the cement sets. This change can be due to many factors including shrinkage (Bois et al. 2012). Cooke et al. (1983) observed a well where the measurements were taken opposite a permeable formation the pressure against the casing decreased to formation pore pressure. In another well with an impermeable formation opposite the gages the pore pressure dropped to 2.5 lbm/gal, well below formation pressure. Morgan (1989) saw that once the cement began to set the pressure against the casing decreased when the cement sets, but the experiment was interrupted when fluid began to flow into the well. Combining these observations with Bois et al. (2012) who said that the cement cannot restrain casing dilation it is apparent that the casing feels the pore pressure within the cement and not the cement itself. The variations in pore pressure are described in Bois et al. (2012) in the modeling of the cement hydration. The authors state that due to the low permeability of the cement the pore pressure will drop to 0 MPa as the cement hydration consumes the water. The permeability of the formation
determines the recharge rate of the fluid into the cement. This can span from days to recharge to formation pore pressure for a high permeability or years for a formation with low permeability. Bosma et al. (1999) confirms the idea of cement hydration affecting the pore pressure using Shell’s in house software which predicts that a shrinking cement uses all the water present (API 10TR2). Non-shrinking cements have a pore pressure equal to that of the formation pore pressure (Bosma et al. 1999), which is due to the excess water that fills the cement column either from the formation or in place for the operation (API 10TR2). For expanding cements Bosma et al. (1999) explains that the pressure is formation hydrostatic plus the restrained expansion, which uses the same concept as the non-shrinking cement with the restrained stress increase from the cement expansion.

Shrinkage and expansion are a risk to cement integrity as stated in Ravi et al. (2002) who found that 4% shrinkage in the cement sheath results in failure in the hydration stage of cement development. Shrinkage occurs due to lack of free water within the cement causing a reduction in internal pore pressure (Thiercelin et al. 1998b). Ravi et al. (2002) and Gray et al. (2009) attempted to describe the modeling of shrinkage as the percent shrinkage observed by lab experiments. The lab experiment shrinkages are unrestrained which means that the outer boundary of the cement is not bonded or kept in contact with the outer material. As pointed out by Thiercelin et al. (1998b) the experimental shrinkage using a flexible membrane test is due mainly to the decrease in the pore pressure. Thiercelin et al. (1998b) also states that the true shrinkage under downhole conditions depends on the boundary conditions and initial conditions such as pressure and temperature, and that the results of a particular experiment to predict downhole behavior must be used with caution. The cement is a plastic and ductile material during setting and may not shrink significantly if the cement is bonded to the casing and formation which are able to restrain and deform the cement during setting (Thiercelin et al. 1998b). Sabins and Sutton (1991) additionally did experiments to determine the shrinkage that occurs during the plastic phase of cement setting and found that 95% of the shrinkage occurs after the cement has set and 90% of the shrinkage occurs after a strength equivalent to the ASTM final strength is reached.
1.7. **SCOPE OF WORK**

An investigation of the integrity and risk of leakage in Wabamun Area CO\textsubscript{2} Sequestration project site wells was conducted to evaluate if existing wells are suitable to be repurposed for injection. The cement compositions were collected for various wells throughout the Wabamun area. The mechanical and thermal properties of these cements were determined through experimental investigation and were used to construct models for common wells in the area. The various conditions that the wells encountered throughout their lives up to injection were then simulated. These conditions are cement hydration and cement setting which can induce stresses caused by heat of hydration, shrinkage/expansion, and pore pressure variations. Additionally thermal and mechanical stresses induced by injection of cold CO\textsubscript{2} or previous production of hot fluids through the subsurface can change near wellbore hoop and radial stresses. To investigate the stresses, staged finite element simulations were developed and verified with staged analytical models. The advantage of this methodology is that it allows for the implementation and operation of the wells to be simulated step by step. This allows for the investigation of every part of the well's life for leakage risk but also ensures proper development of in-situ stresses. The simulations were done using a chemo-thermo-poro-elastic model which gives considerably more accurate and realistic results compared to the thermo-elastic models presented by many of the previous authors. Previous models primarily focused on the stress felt by the casing without considering the cement in-situ stresses, typically assuming it was 0 MPa. The modeling approach used was a very detailed staged finite element model following the approach by Gray et al. (2009), with the exception that pore pressure variations were considered during the cement hydration stage.
2. WABAMUN AREA SEQESTRATION PROJECT

2.1. PROJECT DESCRIPTION

In central Alberta, Canada there is a series of four large coal-fired power plants in the Wabamun Lake area, southwest of Edmonton, with emissions between 3 to 6 Mt/year. Collectively they have a cumulative annual emission on the order of 30 Mt CO$_2$. Because of these large emissions in a small geographical area the Wabamun area was identified as a potential site for future large-scale CO$_2$ injection (Michael et al., 2006). Although significant CO$_2$ storage capacity exists in depleted oil and gas reservoirs, these may not be available in the near future because most of these reservoirs in the area are still producing. A comprehensive characterization of large-scale CO$_2$ storage opportunities in the Wabamun Lake area (Figure 2.1) was conducted in the Wabamun Area CO$_2$ Sequestration Project (WASP) which identified the possibility of storing 0.25 to 0.40 Gt of CO$_2$ (Ghaderi and Leonenko, 2009). Additionally, the possible injection formations within the study area were assessed based on storage capacity, ease of injectivity, leakage likelihood, and interference with current petroleum production (Lavoie and Keith, 2010). In Figure 2.1 the study area is outlined in red and the locations of four large power plants are marked by grey boxes with their respective names. Black dots show wells that penetrate the targeted geologic interval. Purple lines mark important depositional boundaries of the Upper Devonian. The study area has an aerial extent of approximately 5000 km$^2$ (Nygaard and Lavoie, 2010).
2.2. **FIELD DESCRIPTION**

The stratigraphy in the Wabamun Lake area above and below the Nisku Formation is shown in Figure 2.2 (modified from Bachu and Bennion, 2008). The Nisku Formation is capped by the overlying Calmar Formation. The Nisku Formation was identified as the most likely injection zone with storage capacity of 0.25 to 0.4 Gt CO$_2$ which could be increased two to three fold with brine removal (Ghaderi and Leonenko, 2009). In addition the Nisku Formation in this area does not contain oil or gas resources. Some oil, however, is being produced from Nisku pinnacles in the lower Nisku basin region in the northwest portion of the study area labeled as the “Moon Lake Reef Play” in Figure 2.1. The Nisku platform to the southeast has adequate geographical space to ensure contamination of oil and gas reserves is not a concern (Nygaard and Lavoie, 2010).
The study area contains more than 1000 wells, but only a small fraction of the wells penetrate the Calmar shale formation above the Nisku injection horizon. It was assumed that the Calmar seal would hold and only the wells penetrating Calmar and/or into Nisku are at risk and included in this study. In the area there are 95 wells that penetrate the Nisku formation (Figure 2.3). Figure 2.3 presents the age distribution of when these wells were drilled. The wells are classified as either D&A - drilled and abandoned (grey colored in Figure 2.3) or DC - drilled and cased (white colored in Figure 2.3). Well age spans from the earliest well drilled in 1949 until 2008 (Nygaard and Lavoie, 2010).
2.3. STATE OF STRESS

Regional-scale studies of the stress regime indicate that in southern and central Alberta the vertical stress ($\sigma_v$) is the largest principal stress, being greater than the maximum horizontal stress ($\sigma_H$) (Bell and Bachu, 2003). The state of stress in the Wabamun area can then be described as an Andersonian Stress State in which one of the principal stress directions is the $\sigma_v$ is the vertical principal stress, $\sigma_H$ is the maximum horizontal principal stress, and $\sigma_h$ is the minimum horizontal principal stress (Jaeger et al. 2007).

A pressure vs. depth plot of $\sigma_v$, $\sigma_h$, and pore pressure is presented in Figure 2.4. The $\sigma_v$ magnitude is calculated by integrating the bulk density log from ground surface to the depth of interest (Haug et al., 2007). The $\sigma_v$ gradient is approximately 23 kPa/m (Michael et al., 2006). The minimum horizontal stress can be evaluated using a variety of tests. The method used for estimating the magnitude of the $\sigma_h$ is through micro-fracture testing, but mini-fracturing, leak-off tests and fracture breakdown pressures are also used (Bell, 2003; Bell & Bachu, 2003). The average gradient $\sigma_h$ in the Wabamun Lake study area is approximately 20 kPa/m (Michael et al., 2006).
Figure 2.4: Vertical Principal Gradient, Minimum Horizontal Gradient, and Formation Pore Pressure Gradient (Michael et al., 2006)
3. THEORY & ANALYTICAL MODELING METHODOLOGY

Staged three-dimensional finite-element models and staged analytical models were built to study the cased wellbore integrity using chemo-thermo-poro-elastic equations. Fourmaintraux et al. (2005), Bois et al. (2011), and Bois et al. (2012) emphasized the need for models to encompass all possible conditions. The mentioned authors suggest that a chemo-thermo-poro-mechanical model is necessary to accurately model the casing, cement, formation, and their interfaces.

3.1. GENERAL EQUATIONS

Eqn. 15 describes the radial stress-strain relationship with factors included to account for temperature variations, shrinkage or expansion, and pore pressure changes (Timoshenko and Goodier, 1951 and Bois et al., 2012).

\[ \varepsilon_r - \alpha T - \Delta S = \frac{\sigma_r}{E} - \frac{\nu}{E} (\sigma_\theta - \beta Pp + \sigma_z - \beta Pp) \]  

(15)

Eqn. 16 describes the hoop stress-strain relationship with factors included to account for temperature variations, shrinkage or expansion, and pore pressure changes (Timoshenko and Goodier, 1951 and Bois et al., 2012).

\[ \varepsilon_\theta - \alpha T - \Delta S = \frac{\sigma_\theta}{E} - \frac{\nu}{E} (\sigma_r - \beta Pp + \sigma_z - \beta Pp) \]  

(16)

Eqn. 17 describes the axial stress-strain relationship with factors included to account for temperature variations, shrinkage or expansion, and pore pressure changes. The assumption of plane strain is used throughout the derivation. Plane strain assumes that \( \varepsilon_z = 0 \) (Timoshenko and Goodier, 1951 and Bois et al., 2012).

\[ \varepsilon_z - \alpha T - \Delta S = \frac{\sigma_z}{E} - \frac{\nu}{E} (\sigma_r - \beta Pp + \sigma_\theta - \beta Pp) \]  

(17)

Eqn. 18 describes the shear stress-strain relationship. The shear relationships are not affected by temperature changes, shrinkage, or pore pressure changes.
Eqn. 19 describes the radial temperature distribution with time. Transient temperature distributions are used to avoid a steady state temperature distribution being applied to an infinite rock mass (Thiercelin 1998a).

\[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{\rho c}{k} \frac{\partial T}{\partial t} \]  (19)

3.2. INITIAL CONDITION MODELING

Using the solutions for infinite cylinders with Cartesian far-field stresses and the equations for thick walled cylinders under applied wellbore pressures \((Pm)\) and pore pressures \((Pp)\) the initial conditions of a cased wellbore model can be developed. Figure 3.1 shows a model sketch for a cased wellbore in terms of generic cylinders \(x\), \(y\), and \(z\), and radii \(a\), \(b\), and \(c\). Starting from the formation conditions, assuming the wellbore trajectory is aligned with a principal stress. Assuming a vertical trajectory the maximum horizontal principal stress \((\sigma_H)\), the minimum principal horizontal stress \((\sigma_h)\), and the vertical principal stress \((\sigma_v)\). The staged analytical model uses the equations of hoop and radial stress for far field stress distributions to obtain the initial conditions for the formation, Eqns. 20, 21, and 23. The inputs are the maximum and minimum horizontal principal stresses assuming an Andersonian stress distribution.

![Figure 3.1: Wellbore Sketch with Casing (Cylinder x), Cement (Cylinder y), and Formation (Cylinder z)](image)
Eqn. 20, Eqn. 21, Eqn. 22, and Eqn. 23 describe the radial, circumferential, axial, and shear stress, respectively, relationship between far field stresses, initial pore pressure, wellbore pressure, and stress distributions in cylinder $z$. The radial, hoop, axial, and shear stresses vary around the wellbore and at different radii (Jaeger et al. 2007).

\[
\begin{align*}
\sigma_{rz} - Pp &= \frac{1}{2} (\sigma_H + \sigma_h - 2Pp) \left[ 1 - \left( \frac{c}{r} \right)^2 \right] + \frac{1}{2} (\sigma_H - \sigma_h) \left[ 1 - 4 \left( \frac{c}{r} \right)^2 + 3 \left( \frac{c}{r} \right)^4 \right] \cos 2\theta + \\
(Pm - Pp_z) \left( \frac{c}{r} \right)^2 \tag{20}
\end{align*}
\]

\[
\begin{align*}
\sigma_{\theta z} - Pp &= \frac{1}{2} (\sigma_H + \sigma_h - 2Pp) \left[ 1 + \left( \frac{c}{r} \right)^2 \right] - \frac{1}{2} (\sigma_H - \sigma_h) \left[ 1 + 3 \left( \frac{c}{r} \right)^4 \right] \cos 2\theta - \\
(Pm - Pp_z) \left( \frac{c}{r} \right)^2 \tag{21}
\end{align*}
\]

\[
\begin{align*}
\sigma_{zz} - Pp &= \sigma_v - Pp - 2v(\sigma_H - \sigma_h) \left( \frac{c^2}{r^2} \right) \cos(2\theta) \tag{22}
\end{align*}
\]

\[
\begin{align*}
\tau_{r\theta} &= -\frac{1}{2} (\sigma_H - \sigma_h) \left[ 1 + 2 \left( \frac{a}{r} \right)^2 - 3 \left( \frac{a}{r} \right)^4 \right] \sin 2\theta \tag{23}
\end{align*}
\]

To solve for the casing component initial conditions the solution to the single thick walled cylinder with no change in temperature is used. The solved solution for radial stress (Eqn. 24), hoop stress (Eqn. 25) and axial stress (Eqn. 26), are present below. $P_a$ is the inner casing pressure exerted by the wellbore fluids, $P_b$ is the outer casing pressure which is typically the hydrostatic pressure exerted by the cement column, and in this case of casing, $Pp_x$ would be zero since steel does not have pore pressure. (Jaeger et al. 2007)

\[
\begin{align*}
\sigma'_{rx} &= \left( \frac{b^2 P_b - a^2 P_a}{b^2 - a^2} \right) + \frac{a^2 b^2 (P_a - P_b)}{(b^2 - a^2) r^2} - Pp_x \tag{24}
\end{align*}
\]

\[
\begin{align*}
\sigma'_{\theta x} &= \left( \frac{b^2 P_b - a^2 P_a}{b^2 - a^2} \right) - \frac{a^2 b^2 (P_a - P_b)}{(b^2 - a^2) r^2} - Pp_x \tag{25}
\end{align*}
\]

\[
\begin{align*}
\sigma'_{zx} &= \sigma_v - Pp_x \tag{26}
\end{align*}
\]

For the cement sheath the same procedure is applied as with the casing. $P_b$ and $P_c$ are the hydrostatic pressures exerted by the cement column, and in the cement pore
pressure $P_{Py}$ would be typically be equal to that of the formation pore pressure. This solution is found in the following equations, radial stress (Eqn. 27), hoop stress (Eqn. 28) and axial stress (Eqn. 29).

\[
\sigma_{ry} = \frac{(c^2 p_c - b^2 p_b)}{(c^2 - b^2)} + \frac{b^2 c^2 (p_b - p_c)}{(c^2 - b^2) r^2} - P_{Py} \tag{27}
\]

\[
\sigma_{\theta y} = \frac{(c^2 p_c - b^2 p_b)}{(c^2 - b^2)} - \frac{b^2 c^2 (p_b - p_c)}{(c^2 - b^2) r^2} - P_{Py} \tag{28}
\]

\[
\sigma_{zy} = \sigma_{vy} - P_{Py} \tag{29}
\]

3.3. LOADING CONDITION MODELING

The derived general thick walled model will be used for each of the cylinders of casing, cement, and rock. Cylinder $x$ will represent the casing. The equations are given for radial stress (Eqn. 30), hoop stress (Eqn. 31) and axial stress (Eqn. 32) with the temperature, shrinkage, and pore pressure distributions applied to the casing are input as functions of radius if the applied loading is uniform then the values remain constant and will not vary with radius. The radius in this section is limited to the range between $a$ and $b$. The constants of $C_{1x}$ and $C_{2x}$ are solved for using the two thick walled cylinders with a 3rd infinite walled cylinder solution above.

\[
\sigma_{rx} = -\frac{E}{1 - \nu} \left( \frac{1}{r^2} \int_a^r \alpha T r + \Delta S r \ dr + \frac{1 - 2\nu}{1 - \nu} \frac{1}{r^2} \int_a^r P r r \ dr + \frac{E}{1 + \nu} \left( \frac{C_{1x}}{1 - 2\nu} - \frac{C_{2x}}{r^2} \right) \right) \tag{30}
\]

\[
\sigma_{\theta x} = \frac{E}{1 - \nu} \left( \frac{\int_a^r \alpha T r + \Delta S r \ dr - \alpha T - \Delta S}{r^2} \right) - \frac{1 - 2\nu}{1 - \nu} \beta \left( \frac{1}{r^2} \int_a^r P r r \ dr - P_P \right) + \frac{E}{1 + \nu} \left( \frac{C_{1x}}{1 - 2\nu} + \frac{C_{2x}}{r^2} \right) \tag{31}
\]

\[
\sigma_{zx} = -\frac{E (\alpha T + \Delta S)}{1 - \nu} + \frac{1 - 2\nu^2}{1 - \nu} \beta P_P + \frac{2\nu E C_{1x}}{(1 + \nu)(1 - 2\nu)} \tag{32}
\]
Cylinder $y$ represents the cement. The equations are given for radial stress (Eqn. 33), hoop stress (Eqn. 34) and axial stress (Eqn. 35) with the temperature, shrinkage, and pore pressure distributions applied to the cement are input as functions of radius if the applied loading is uniform then the values remain constant and will not vary with radius. The radius in this section is limited to the range between $b$ and $c$. The constants of $C_{1y}$ and $C_{2y}$ are solved for using the two thick walled cylinders with a 3rd infinite walled cylinder solution above.

$$\sigma_{ry} = -\frac{E}{1-v} \int_b^r aTr + \Delta Sr \ dr + \frac{1-v}{1-v} \frac{1}{r^2} \beta \int_b^r Ppr \ dr + \frac{E}{1+v} \left(\frac{C_{1y}}{1-2v} - \frac{C_{2y}}{r^2}\right)$$ (33)

$$\sigma_{\theta y} = \frac{E}{1-v} \left(\frac{1}{r^2} \int_b^r aTr + \Delta Sr \ dr - \alpha T - \Delta S\right) - \frac{1-v}{1-v} \beta \left(\frac{1}{r^2} \int_b^r Ppr \ dr - Pp\right) + \frac{E}{1+v} \left(\frac{C_{1y}}{1-2v} + \frac{C_{2y}}{r^2}\right)$$ (34)

$$\sigma_{z y} = -\frac{E(aT + \Delta S)}{1-v} + \frac{1-v}{1-v} \beta Pp + \frac{2vE C_{1y}}{(1+v)(1-2v)}$$ (35)

Cylinder $z$ represents the formation. The equations are given for radial stress (Eqn. 36), hoop stress (Eqn. 37) and axial stress (Eqn. 38) with the temperature, shrinkage, and pore pressure distributions applied to the formation are input as functions of radius if the applied loading is uniform then the values remain constant and will not vary with radius. The radius in this section is limited to the range between $c$ and $\infty$. The constants of $C_{1z}$ and $C_{2z}$ are solved for using the two thick walled cylinders with a 3rd infinite walled cylinder solution above.

$$\sigma_{rz} = -\frac{E}{1-v} \int_c^r aTr + \Delta Sr \ dr + \frac{1-v}{1-v} \frac{1}{r^2} \beta \int_c^r Ppr \ dr + \frac{E}{1+v} \left(\frac{C_{1z}}{1-2v} - \frac{C_{2z}}{r^2}\right)$$ (36)

$$\sigma_{\theta z} = \frac{E}{1-v} \left(\frac{1}{r^2} \int_c^r aTr + \Delta Sr \ dr - \alpha T - \Delta S\right) - \frac{1-v}{1-v} \beta \left(\frac{1}{r^2} \int_c^r Ppr \ dr - Pp\right) + \frac{E}{1+v} \left(\frac{C_{1z}}{1-2v} + \frac{C_{2z}}{r^2}\right)$$ (37)
\[
\sigma_{zz} = -\frac{E(\alpha T + \Delta S)}{1-\nu} + \frac{1-2\nu^2}{1-\nu} \beta Pp + \frac{2\nu E C_{1z}}{(1+\nu)(1-2\nu)}
\]  \hspace{2cm} (38)

The distributions for pore pressure and shrinkage will be treated as uniform changes that do not vary with radial position, i.e. \( \Delta S = \text{constant}, Pp = \text{constant} \).
4. **FINITE ELEMENT MODELING METHODOLOGY**

4.1. **STAGED FINITE ELEMENT MODELING**

The approach followed in this study is based on replicating the life of the well by including all loading steps occurring before mechanical and thermal loads caused by the injection. Figure 4.1 shows a schematic of the borehole cross section and the loads (mechanical and thermal) that are applied on them. Details of the steps followed in numerical simulations are also illustrated on the right side of the Figure 4.1. The mesh for finite element study was constructed in HyperMesh™ (Altair, 2010) and the actual finite element simulations were conducted in Abaqus™ software (SIMULIA, 2013). The three-dimensional mesh has one and five meter length in $z$ and $(x,y)$ directions respectively and is composed of first order elements. The decision on model geometry was based on Kirsch Analytical solution of the disturbed stress field around a circular opening where the ratio of the model size and wellbore diameter is kept greater than three to present a good approximation to the infinite case (Fjaer et al., 2008). The size will be increased above a ratio of three in order to prevent unintentional boundary effects affecting the model as a result of the temperature distribution reaching the boundary of the model. The numerical model also assumes homogeneity in all materials including casing, cement and formation. Although heterogeneous considerations would be more realistic it is outside the scope of this study. The main goal from the numerical models is to propose a robust multi-stage modeling approach for near wellbore integrity situations and to capture effect of dynamic loads on the near wellbore area.
The staged finite-element model uses the property of superposition to build the model initial conditions before the next step of loading is implemented. The advantage of building the model in several steps is to observe and record stress and deformation changes after each loading (Gray et al., 2007). Figure 4.1 shows a schematic of the borehole cross section and the loads (mechanical and thermal) that are applied on them.
Details of the steps modified from Nygaard et al. (2012) to include a cement hydration step are followed in the numerical simulations and are as follows:

Step 1. Loading the model with in-situ stress: In this step two horizontal stresses (minimum and maximum) and overburden stress are applied to the all elements in the model. Additionally, an initial temperature for reservoir temperature was applied to all the nodes.

Step 2. Drilling step: In this step wellbore elements are removed from the model, and mud pressure is applied to the wellbore face. Stress equilibration was achieved at the end of the step and near wellbore state of stress was imposed. This step simulates the drilling process of the borehole.

Step 3. Running Casing: Casing elements were introduced to the model at this step with mud pressure applied to the inside of the casing. The cement column pressure was also applied to the outside of the casing and the inside of the formation. Linear-elastic behavior was assumed for the casing elements.

Step 4. Cementing: At this stage cement elements were introduced to the model. The cement elements were fully bonded to the formation. These elements were also activated by zero deformation but under initial hydrostatic slurry pressure. The cement is not yet hardened and its internal stress will be equal to the cement hydrostatic pressure. This status is defined as initial conditions for the cement elements before loading step starts. Mohr-Coulomb softening material model is applied for cement elements, which is essential for predicting plastic failure in cement when thermal and mechanical loads are applied to the model.

Step 5. Hydration: After the cement is in place hydration of the cement begins. The hydration includes a decrease in temperature from the elevated temperature of hydration that develops while the cement is in slurry state and develops no strain. Shrinkage of the cement occurs as a volumetric strain, according to Sabins and Sutton (1991) who found that 95% of shrinkage occurs after the initial set. Pore pressure is reduced in this step.

Step 6. Applying pore pressure, thermal, and mechanical loads: After cement and casing were set, the final stage is to apply thermal and mechanical loads for the cased wellbore. Mechanical loads were applied by using distributed load on
casing surface and thermal load was defined by putting thermal boundary conditions on casing nodes. One day’s worth of temperature change using a transient model was simulated to allow the boundaries to remain at their initial temperature. The elements used for casing, cement and formation have features for coupled thermal-displacement analysis with the options to define thermal conductivity, thermal expansion and specific heat values.

4.2. EFFECT OF STAGE FINITE ELEMENT MODEL VS. A NON-STAGED FINITE ELEMENT MODEL

Using a finite element model to develop the initial conditions requires a staged finite element model. This can be shown by understanding the in-situ stresses in the wellbore and how they develop. According to Bois et al. (2012) and Gray et al. (2009) the stress in the cement, assuming no shrinkage, is equal to the hydrostatic pressure exerted by the cement column. Using this information it would be expected that after the cement sets to see a radial total stress at the interfaces between casing-cement and cement-formation as well as through the entire cement sheath equal to the cement hydrostatic pressure of the cement. Figure 4.2 shows two model wellbores. The model on the left is a non-staged finite element model and the model on the right is a staged finite element model. The non-staged model has all the conditions applied in a single step. This includes the far field stresses, the cement column pressure in the cement and applied to the interfaces, and the mud pressure against the casing. Figure 4.2 shows that when all of the loading and initial conditions are put in a single step the interaction of these conditions result in an undesirable stress state, such as the cement sheath and interfaces have a radial stress less than the applied cement pressure. In addition, in the non-staged model the cement-formation interface radial stresses do not match between the cement and formation sides of the interface. This would violate Newton’s third law because no external force is applied to the cement or formation, so the radial stress cannot change drastically between the two but instead must be the same on both sides of the interface. The staged model in Figure 4.2 (right) follows the procedure and steps outlined by Steps 1-4. The radial stresses are seen as expected in the staged model, the radial stress
throughout the cement is equal to the cement column pressure, 30 MPa, and the pressure at the interfaces is also equal to the cement column pressure of 30 MPa. This shows that a finite element model can be constructed that coincides with predetermined theories of cement in-situ stress.

Figure 4.2: Non-Staged Finite Element Model (left) compared to Staged Finite Element Model (right)
5. EXPERIMENTAL REVIEW

The main objective for the laboratory tests was to provide cement properties to use in the analytical and numerical models. The lab tests included experiments to determine thermal properties and rock mechanical properties of cement samples. These properties will be used to simulate the cement sheath as described in Section 4. The thermal properties of specific heat capacity and thermal conductivity are used to determine the temperature distribution and heat flow through the wellbore. The linear thermal expansion coefficient is used to determine the strain caused by the associated temperature changes which allow the model to be thermo-elastic. The mechanical properties of Young’s Modulus and Poisson’s Ratio help to determine the strain and stress when pressure changes are applied to the model. Finally, the tensile strength and uniaxial compression strength tests are used to determine the failure conditions of the cement sheath to determine when failure of the wellbore occurs.

5.1. LINEAR THERMAL EXPANSION COEFFICIENT

The linear thermal expansion coefficient, \( \alpha \), is defined as strain per change in temperature. This value is required to describe the effect of a temperature changes in a material. When a material is constrained by other materials additional stresses can form as temperature changes.

The coefficient is tested for using a direct LVDT measurement. The apparatus, Figure 5.1, consists of ¾ in steel frame, a temperature probe (Oakton Ion 6 Acorn Series), an LVDT (Omega LD630-10), a glass beaker filled with water, and the sample. The test begins with heating the sample, 2 in diameter by 4 in length, and water in a beaker until a constant temperature is reached. Next, the beaker is placed under the LVDT and once stabilized the initial measurement is taken from the display and the temperature probe. The LVDT measures in millimeters with accuracy of ± 0.0001 mm and the temperature probe measure in Celsius with ± 0.1°C. As the sample and water cool, the sample will shrink proportionally to its thermal expansion coefficient. As this
occurs the LVDT measures this change in length. Using a plot of temperature versus displacement and calibration using a 316 stainless steel cylinder, a linear regression can be used to determine the thermal expansion coefficient.

![Figure 5.1: Linear Thermal Expansion Experiment Set-Up](image)

The results of the experiments described are presented in Appendix A. The results of the Linear Thermal Expansion Coefficient are presented in Table A.1 with the sample type being given along with the associated linear thermal expansion coefficient. The equation below, Eqn. 39, is used to describe alpha in the experiment used, with C as a correction factor per temperature change.

\[
\alpha = \frac{\Delta L - C \cdot \Delta T}{L \cdot \Delta T}
\]  

(39)

5.2. **SONIC VELOCITY DYNAMIC MECHANICAL PROPERTIES**

The parameters determined from the sonic velocity tests are P and S wave velocities. These values are then substituted into equations to find dynamic elastic properties. These are important mechanical properties that can be correlated to the static
properties for quick determination of static properties without the use strain gages or other devices.

The sonic velocity apparatus, Figure 5.2, consists of the signal generator, the emitter and receiver sensors, and Ultrasonic software (GCTS ULT-100). The P and S wave velocities are measured by measuring the transit time of each wave through the test sample. The sample is placed between the two sensors and the software will measure the transit time by measuring the first peak of the emitted wave. Detailed procedure can be found in GCTS user manual (GCTS 2004).

The results of the Sonic Velocity tests are presented in Table A.2. Table A.2 presents the sample composition, the average sample thickness, the density of the sample, the P-wave travel time, and the S-wave travel time. From the travel times and the average thickness the velocity of the P and S waves can be calculated. Once these values were determined the Dynamic Poisson’s Ratio (Eqn. 40), Dynamic Young’s Modulus (Eqn. 41), Dynamic Bulk Modulus (Eqn. 42), and Dynamic Shear Modulus (Eqn. 43) can be found using the equations provided below which require the P and S wave velocities and the sample density.

\[ \nu = \frac{v_p^2 - 2v_s^2}{2(v_p^2 - v_s^2)} \quad (40) \]

\[ E = \frac{\rho v_p^2 (3v_p^2 - 4v_s^2)}{v_p^2 - v_s^2} \quad (41) \]
\[ K = \frac{\rho(3V_p^2 - 4V_s^2)}{3} \]  \hspace{1cm} (42)

\[ G = \rho V_s^2 \]  \hspace{1cm} (43)

5.3. DIVIDED BAR THERMAL CONDUCTIVITY

Thermal conductivity is a measure of the how readily heat flows through a material for a certain length and change in temperature (Pribnow and Sass, 1995). The divided bar apparatus consist of highly heat conductive material, a lower conductivity material with known conductivity, and the test sample. The apparatus, Figure 5.3, has a heat source which transfers heat to the aluminum to the first high conductivity aluminum and the cooler ambient room temperature is on the other side which transfers heat away from the aluminum cylinder. The aluminum is attached to the known material by silicone adhesive which is chosen due to silicone’s high thermal conductivity. The test sample, which is not permanently attached, is kept in full contact with the aluminum by silicone grease, which was also chosen for silicone’s high thermal conductivity. The thermal conductivity can only be found when the apparatus reaches steady-state, which could take upwards of eight hours. In order to get results in a more reasonable time frame, the thermal conductivity will be measured continuously before steady-state and using a translated power regression the thermal conductivity can be projected to steady-state.

Figure 5.3: Divided Bar Thermal Conductivity Experiment Set-Up with 1inch Aluminum and Acrylic Cylinders
Thermal conductivity depends on the heat flow, sample length, and temperature change. Heat flow across each of the known acrylic plastics is calculated from the change in temperature measured at each aluminum section multiplied with the thermal conductivity of the acrylic divided by the length of the acrylic, Eqn. 44. By averaging the heat flows across the acrylics we can predict the heat flow across the cement sample. With the heat flow across the acrylics we can predict the heat flow across the cement sample.

\[ k = \frac{q \times x}{\Delta T} \]  

(44)

5.4. CALORIMETER SPECIFIC HEAT CAPACITY

The specific heat capacity apparatus, Figure 5.4, is a calorimeter. The apparatus consists of a known mass of water and a known mass sample and known temperatures. The water is insulated so that it operates as a closed system. The water temperature is recorded and then the sample of its own recorded temperature is placed into the water. The final temperature is measured from the water temperature when it becomes constant.

Specific heat capacity is defined as the heat needed to raise a mass of one gram one Kelvin degree temperature. In the equation below, Eqn. 45, heat will be defined in Joules, mass in kilograms, and temperature in Kelvin. This value is need in time.
dependent models because as heat flows heat capacity will define the temperature of the material.

\[ [c] = \frac{\Delta Q}{\Delta T \cdot m} \]  

(45)

5.5. BRAZILIAN TENSILE TEST

To characterize the failure envelope of the cement splitting tensile strength tests (Brazilian tests) were conducted. The Brazilian test set up, Figure 5.5, follows the ASTM protocol for rock specimen (ASTM D3967 – 08).

Figure 5.5: Brazilian Test Set-Up

The cement sample is cut into a 2 in diameter by 1 in thickness and placed between two curved pieces of steel to center the sample. A hydraulic piston is lower as a loading rate of 50 psi/min, and the force is recorded by the load sensor above the curved steel. The splitting tensile strength is given by Eqn. 46.

\[ T_o = \frac{2 \cdot P}{\pi \cdot D \cdot t} \]  

(46)
5.6. **UNIAXIAL COMPRESSION TEST**

The compressive strength of the material is an important failure criterion. This value is much greater than the tensile strength in rock materials. The experiment, Figure 5.6, uses a hydraulic piston to compress the cement sample of L/D=2. The force is measured by a load sensor placed above the cement sample and the strain of the sample during loading is measured by an LVDT which measures the displacement of the top of the sample. The equation to determine compressive strength is Eqn. 47 below.

![Uniaxial Compression Experiment Set-Up](image)

The equation of compressive strength is corrected for samples that may have dimension other than L/D=2. Another parameter that can be determined from the uniaxial compression test is the static Young’s Modulus. The Young’s Modulus is determined by dividing stress by strain, Eqn. 48.

\[
C_o = \frac{C_m}{0.88 + 0.24(D/L)} \quad (47)
\]

\[
E = \sigma / \varepsilon = \frac{F/A}{\Delta L/L} \quad (48)
\]
5.7. STATISTICAL METHODS FOR LABORATORY RESULTS ANALYSIS

The statistical tests were conducted on the cement property results to determine trends and characteristics of the various properties. The two tests that were used were the Spearman’s Rho test and the Mann-Whitney test.

The Spearman’s Rho test is a measure of how much two random variables change together (MiniTab™, 2012). The test looks at whether the data points along the x-axis increase or decrease as the values progress along the axis. The test is nonparametric which means that it does not matter how large the difference in two values is just the progression of values matters and whether they were greater or less than the points before it. The significance of the tests is measured by the p-value. The p-value is the probability of obtaining a test statistic that meets the null hypothesis. In the case of the Spearman’s Rho test the null hypothesis is that there is a correlated change between the two variables, so the closer to 1 or -1 the p-value is the greater the correlation of the variables. The test is used to look for a positive or negative correlation between the densities of the cement compositions and the physical property of interest. This test shows if the modification of density is what affects the physical properties or if there is another factor caused by the various additives in each of the cement samples.

The Mann-Whitney test is a non-parametric test used to identify if two sample populations are statistically equal or to say that the null hypothesis is that the two populations are the same (MiniTab™, 2012). The p-value represents the probability of the two samples being from the same distribution; in this case if the p-value is greater than 0.15 or 15% then the two populations come from the same distribution. Visualization of this can be done using a box plot of each of the sample populations. The box plot shows the median of the population by the line in the middle of the box. The top and bottom of the box represent the first and third quartiles or the probabilities of 25% and 75%. The lines outside the box represent the data outside the first and third quartiles. If there is overlap between the two sample population box plots then there is a higher probability they are of the same distribution, and if there is no overlap then the distributions are more statistically different. This will show whether the addition of an additive has a significant effect on the physical property being investigated.
6. EXPERIMENTAL RESULTS & INTERPRETATION

6.1. TEST MATRIX

The test matrix, Table 6.1, for the various cement compositions to be tested includes a variety of different additives. The additives selected are those which are present in the cements in the WASP area (Nygaard et al. 2011). These additives include Bentonite Gel, CaCl₂, a mixture of Bentonite Gel and CaCl₂, Barite, and Sand. Bentonite Gel is a density reducer in cement commonly used for its high water requirements as to not take any water away from the hydration process when the clay swells (Bourgoyn et al., 1986). CaCl₂, calcium chloride, is a set time accelerator. CaCl₂ adds chloride to the cement hydration which accelerates the building of the cement structure. This effect aids in the hydration of the cement which reduces the time needed for the cement to harden (Nelson and Guillot, 2006). Barite is a weight material in cement. The barite which has a very high density and a low water requirement increases the overall weight of the cement (Bourgoyn et al., 1986). Sand is used as a weight material. Even though the sand has a lower density than the cement grains, the sand has no water requirement so no water will be added with the addition of sand which will help increase density overall (Bourgoyn et al., 1986). The test matrix chooses has additive percentages a 2%, 5%, 8%, and 10% for Bentonite Gel, CaCl₂, and Barite. Only one Bentonite and CaCl₂ mixture sample and one sand sample were made because the effect of these two was to be investigated. Additionally, a 1.1% Bentonite Gel sample was made to add an additional data point to help define the effects of Bentonite Gel on the cement.

Table 6.1: Cement Test Matrix

<table>
<thead>
<tr>
<th>Additive</th>
<th>Bentonite</th>
<th>CaCl₂</th>
<th>Bentonite + CaCl₂</th>
<th>Barite</th>
<th>Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1%</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>5%</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>8%</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>10%</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>
6.2. LABORATORY RESULTS ANALYSIS

Figure 6.1 (left) shows the plot of tensile strength vs. density. The plot shows that visually there may be a correlation between density and tensile strength. The Spearman’s Rho test, Figure 6.1 (bottom), gives a p-value for the statistical test of 0.92 which is greater than the 15% confidence interval which means there is a significant positive correlation between tensile strength and density. Figure 6.1 (right) shows the plot of heat capacity vs. density. The plot shows that visually there may be a declining correlation between density and heat capacity, but from the Spearman’s Rho test, Figure 6.1 (bottom), gives a p-value for the statistical test of -0.82 which is lower than the 15% confidence interval which means there is no significant correlation between heat capacity and density. While the plot looks as if there may be a correlation, the higher density data is not consistently lower than the density data before it which causes there to be no significant correlation.

Figure 6.2 (left) shows the plot of static Young’s Modulus vs. density. The plot shows that visually there may be a positive correlation, positive sloping trend, between density and static Young’s Modulus. The Spearman’s Rho test, Figure 6.2 (bottom), gives

![Figure 6.1: Scatter Plot of Tensile Strength (left) and Heat Capacity (right) with density and Spearman’s Rho w/ density (bottom)](image-url)
a p-value for the statistical test of 0.65 which is less than the 15% confidence interval which means there is no significant correlation between static Young’s Modulus and density. Figure 6.2 (right) shows the plot of dynamic Young’s Modulus vs. density. The plot shows that visually there may be a positive correlation between density and dynamic Young’s Modulus. The Spearman’s Rho test, Figure 6.2 (bottom), gives a p-value for the statistical test of 0.99 which is greater than the 15% confidence interval which means there is a significant correlation between dynamics Young’s Modulus and density and since the p-value is positive the correlation is positive as well.

Figure 6.2: Scatter Plot of Static Young’s Modulus (left) and Dynamic Poisson’s Ratio (right) with density and Spearman’s Rho w/ density (bottom)

Figure 6.3 (left) shows the plot of dynamic Poisson’s Ratio vs. density. The plot shows that visually there is no obvious correlation between density and dynamic Poisson’s Ratio. The Spearman’s Rho test, Figure 6.3 (bottom), gives a p-value for the statistical test of -0.75 which is lower than the 15% confidence interval which means there is no significant correlation between dynamic Poisson’s Ratio and density. Figure 6.3 (right) shows the plot of thermal expansion coefficient vs. density. The plot shows that visually there may be an increasing correlation between density and thermal
expansion coefficient, but from the Spearman’s Rho test, Figure 6.3 (bottom), gives a p-value for the statistical test of less than 0.50 which is lower than the 15% confidence interval which means there is no significant correlation between thermal expansion coefficient and density. While the plot looks as if there may be a correlation, the higher density data is not consistently greater than the density before it which causes there to be no significant correlation.

Figure 6.3: Scatter Plot of Thermal Expansion Coefficient (left) and Dynamic Young’s Modulus (right) with density and Spearman’s Rho w/ density (bottom)

Figure 6.4 (top) shows the plot of uniaxial compressive strength (UCS) vs. density. The plot shows that visually there may be a correlation between density and tensile strength, but from the Spearman’s Rho test, Figure 6.4 (bottom), gives a p-value for the statistical test of 0.93 which is greater than the 15% confidence interval which means there is a significant positive correlation between UCS and density.
Figure 6.4: Scatter Plot of Uniaxial Compressive Strength (UCS) with density and Spearman’s Rho w/ density (bottom)

Correlation w/ Density
15% Confidence Interval
UCS  
\[ p = 0.93 \]  Positive Correlation

6.3. MANN-WHITNEY TEST

Figure 6.5 shows the distribution of tensile strength measurement for five different cement compositions. From the box plot it is visually evident that in comparison to the neat cement composition, the bentonite samples have a lower tensile strength, the CaCl\(_2\) samples have a higher tensile strength, the barite samples have the same tensile Strength, and the sand samples have the same tensile strength. This visual inspection can be confirmed using the Mann-Whitney test statistics. In comparison to the neat cement composition the bentonite samples’ tensile strengths have a p-value of 0.0304 which is less than the 15% confidence interval which means the samples are statically different. The CaCl\(_2\) samples’ tensile strengths have a p-value of 0.0518 which is less than the 15% confidence interval which means the samples are statically different. The barite samples’ tensile strengths have a p-value of 0.5959 which is greater than the 15% confidence interval which means the samples are statically equal.
Figure 6.6 shows the distribution of heat capacity measurements for five different cement compositions. From the box plot it is visually evident that in comparison to the neat cement composition, the bentonite samples have a higher heat capacity, the CaCl₂ samples have a higher heat capacity, the barite samples have the same heat capacity, and the sand samples have the same heat capacity. This visual inspection can be confirmed using the Mann-Whitney test statistics. In comparison to the neat cement composition the bentonite samples’ heat capacities have a p-value of 0.0736 which is less than the 15% confidence interval which means the samples are statically different. The CaCl₂ samples’ heat capacities have a p-value of 0.136 which is less than the 15% confidence interval which means the samples are statically different. The barite samples’ heat capacities have a p-value of 0.7656 which is greater than the 15% confidence interval which means the samples are statically equal.
Figure 6.7 shows the distribution of static Young’s Modulus measurements for five different cement compositions. From the box plot it is visually evident that in comparison to the neat cement composition, the bentonite samples have a lower static Young’s Modulus, the CaCl₂ samples have the same static Young’s Modulus, the barite samples have the same static Young’s Modulus, and the sand samples have a lower static Young’s Modulus. This visual inspection can be confirmed using the Mann-Whitney test statistics. In comparison to the neat cement composition the bentonite samples’ static Young’s Modulus have a p-value of 0.1052 which is less than the 15% confidence interval which means the samples are statically different. The CaCl₂ samples’ static Young’s Modulus have a p-value of 0.3770 which is greater than the 15% confidence interval which means the samples are statically equal. The barite samples’ static Young’s Modulus have a p-value of 0.3768 which is greater than the 15% confidence interval which means the samples are statically equal.
Figure 6.7: Static Young’s Modulus vs. Composition Box Plot (left) and Mann-Whitney Test Results (right)

Figure 6.8 shows the distribution of dynamic Young’s Modulus measurements for five different cement compositions. From the box plot it is visually evident that in comparison to the neat cement composition, the bentonite samples have a lower dynamic Young’s Modulus, the CaCl\textsubscript{2} samples have a lower dynamic Young’s Modulus, the barite samples have the same dynamic Young’s Modulus, and the sand samples have a lower dynamic Young’s Modulus. This visual inspection can be confirmed using the Mann-Whitney test statistics. In comparison to the neat cement composition the bentonite samples’ dynamic Young’s Modulus have a p-value of 0.0304 which is less than the 15% confidence interval which means the samples are statically different. The CaCl\textsubscript{2} samples’ dynamic Young’s Modulus have a p-value of 0.1116 which is less than the 15% confidence interval which means the samples are statically different. The barite samples’ dynamic Young’s Modulus have a p-value of 1.0000 which is greater than the 15% confidence interval which means the samples are statically equal.
Figure 6.9 shows the distribution of dynamic Poisson’s Ratio measurements for five different cement compositions. From the box plot it is visually evident that in comparison to the neat cement composition, the bentonite samples have a higher dynamic Poisson’s Ratio, the CaCl$_2$ samples have the same dynamic Poisson’s Ratio, the barite samples have the same dynamic Poisson’s Ratio, and the sand samples have a higher dynamic Poisson’s Ratio. This visual inspection can be confirmed using the Mann-Whitney test statistics. In comparison to the neat cement composition the bentonite samples’ dynamic Poisson’s Ratio have a p-value of 0.0304 which is less than the 15% confidence interval which means the samples are statically different. The CaCl$_2$ samples’ dynamic Poisson’s Ratio have a p-value of 0.3768 which is greater than the 15% confidence interval which means the samples are statically equal. The barite samples’ dynamic Poisson’s Ratio have a p-value of 0.8597 which is greater than the 15% confidence interval which means the samples are statically equal.
Figure 6.10 shows the distribution of Thermal Expansion Coefficient measurements for five different cement compositions. From the box plot it is visually evident that in comparison to the neat cement composition, the bentonite samples have a lower Thermal Expansion Coefficient, the CaCl₂ samples have the same Thermal Expansion Coefficient, the barite samples have a lower Thermal Expansion Coefficient, and the sand samples have a Thermal Expansion Coefficient. This visual inspection can be confirmed using the Mann-Whitney test statistics. In comparison to the neat cement composition the bentonite samples’ Thermal Expansion Coefficient have a p-value of 0.1052 which is less than the 25% confidence interval which means the samples are statically different. The CaCl₂ samples’ Thermal Expansion Coefficient have a p-value of 1.000 which is greater than the 25% confidence interval which means the samples are statically equal. The barite samples’ Thermal Expansion Coefficient have a p-value of 0.2453 which is less than the 25% confidence interval which means the samples are statically different.
Figure 6.11 shows the distribution of UCS measurements for five different cement compositions. From the box plot it is visually evident that in comparison to the neat cement composition, the bentonite samples have a lower UCS, the CaCl₂ samples have the same UCS, and the barite samples have the same UCS. This visual inspection can be confirmed using the Mann-Whitney test statistics. In comparison to the neat cement composition the bentonite samples’ UCS have a p-value of 0.0085 which is less than the 15% confidence interval which means the samples are statically different. The CaCl₂ samples’ UCS have a p-value of 0.6711 which is greater than the 15% confidence interval which means the samples are statically equal. The barite samples’ UCS have a p-value of 0.3502 which is greater than the 15% confidence interval which means the samples are statically equal.
6.4. NORMALIZED DATA SET

Due to the scatter in the data of the different percent concentration of each additive, the properties will be trended linearly based on the additive percentage. The results of the normalization are found in Table 6.2. The linear fitting ensures that if the additive percentage is increased that there is a uniform change in the mechanical and thermal properties. This is done so that there will never be inconsistencies in the changes in the properties as the additive percentage increases. For example, as percent bentonite gel increases the Young’s Modulus decreases. The linear trend ensures that a higher percentage of bentonite will never have a higher Young’s Modulus than any lower concentrations of bentonite gel before it, even if the raw data reflects that for a single case.

Figure 6.11: UCS vs. Composition Box Plot (left) and Mann-Whitney Test Results (right)
### Table 6.2: Normalized Cement Properties

<table>
<thead>
<tr>
<th>Cement</th>
<th>ρ slurry (g/cc)</th>
<th>E, GPa</th>
<th>ν</th>
<th>α, με/°C</th>
<th>[c], J/kg·K</th>
<th>To, MPa</th>
<th>UCS, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neat</td>
<td>1.951</td>
<td>9.67</td>
<td>0.214</td>
<td>9.71</td>
<td>1.02</td>
<td>1.49</td>
<td>45.97</td>
</tr>
<tr>
<td>1.1% Gel</td>
<td>1.884</td>
<td>8.50</td>
<td>0.231</td>
<td>9.41</td>
<td>1.00</td>
<td>1.35</td>
<td>31.33</td>
</tr>
<tr>
<td>2% Gel</td>
<td>1.827</td>
<td>8.06</td>
<td>0.233</td>
<td>9.22</td>
<td>1.03</td>
<td>1.27</td>
<td>28.47</td>
</tr>
<tr>
<td>5% Gel</td>
<td>1.696</td>
<td>6.57</td>
<td>0.238</td>
<td>8.57</td>
<td>1.10</td>
<td>1.01</td>
<td>18.92</td>
</tr>
<tr>
<td>8% Gel</td>
<td>1.605</td>
<td>5.08</td>
<td>0.243</td>
<td>7.92</td>
<td>1.17</td>
<td>0.74</td>
<td>9.38</td>
</tr>
<tr>
<td>10% Gel</td>
<td>1.559</td>
<td>4.09</td>
<td>0.247</td>
<td>7.49</td>
<td>1.22</td>
<td>0.57</td>
<td>3.01</td>
</tr>
<tr>
<td>2%+2%</td>
<td>1.829</td>
<td>6.66</td>
<td>0.274</td>
<td>8.52</td>
<td>0.99</td>
<td>1.59</td>
<td>-</td>
</tr>
<tr>
<td>2% CaCl₂</td>
<td>1.951</td>
<td>9.32</td>
<td>0.220</td>
<td>9.79</td>
<td>1.04</td>
<td>1.83</td>
<td>53.10</td>
</tr>
<tr>
<td>5% CaCl₂</td>
<td>1.952</td>
<td>8.58</td>
<td>0.237</td>
<td>9.57</td>
<td>1.08</td>
<td>2.17</td>
<td>50.34</td>
</tr>
<tr>
<td>8% CaCl₂</td>
<td>1.952</td>
<td>7.84</td>
<td>0.253</td>
<td>9.35</td>
<td>1.13</td>
<td>2.51</td>
<td>47.58</td>
</tr>
<tr>
<td>10% CaCl₂</td>
<td>1.952</td>
<td>7.35</td>
<td>0.264</td>
<td>9.20</td>
<td>1.15</td>
<td>2.74</td>
<td>45.74</td>
</tr>
<tr>
<td>2% Barite</td>
<td>1.966</td>
<td>9.78</td>
<td>0.207</td>
<td>9.08</td>
<td>1.05</td>
<td>1.51</td>
<td>43.95</td>
</tr>
<tr>
<td>5% Barite</td>
<td>1.988</td>
<td>9.24</td>
<td>0.212</td>
<td>8.38</td>
<td>1.04</td>
<td>1.64</td>
<td>37.65</td>
</tr>
<tr>
<td>8% Barite</td>
<td>2.009</td>
<td>8.71</td>
<td>0.217</td>
<td>7.69</td>
<td>1.03</td>
<td>1.78</td>
<td>31.34</td>
</tr>
<tr>
<td>10% Barite</td>
<td>2.024</td>
<td>8.35</td>
<td>0.220</td>
<td>7.23</td>
<td>1.02</td>
<td>1.87</td>
<td>27.14</td>
</tr>
<tr>
<td>5% Sand</td>
<td>1.969</td>
<td>9.50</td>
<td>0.270</td>
<td>5.83</td>
<td>1.06</td>
<td>1.71</td>
<td>-</td>
</tr>
</tbody>
</table>
7. ANALYTICAL/FINITE ELEMENT MODELING RESULTS

7.1. SIMULATION RESULTS

To model the integrity of a CO\textsubscript{2} injection well a finite element simulation was conducted. This begins with creating the model sketch found in Figure 4.1, and then a finite element mesh must be created. Figure 7.1 shows a cut out of the 3D-finite element mesh built for the simulations when all the materials (casing, cement, and formation) are present in the model. The mesh was verified with Kirsch analytical solution for accuracy and also tested for convergence rate resulting in a finer mesh in near-wellbore region to increase results accuracy in this region.

![Figure 7.1: The Three-Dimensional Mesh Built for Simulations of Wellbore: casing (black), cement (gray) and formation elements (green)](image)

<table>
<thead>
<tr>
<th>Prod Casing ID</th>
<th>Prod Casing OD</th>
<th>Hole D</th>
</tr>
</thead>
<tbody>
<tr>
<td>147 mm</td>
<td>168.3 mm</td>
<td>215.9 mm</td>
</tr>
</tbody>
</table>

Figure 7.1: The Three-Dimensional Mesh Built for Simulations of Wellbore: casing (black), cement (gray) and formation elements (green)
The results of the laboratory experiments for neat cement, no additives, are provided below in Table 7.1. These values are combined with the casing and Calmar shale properties that are also used in the finite-element simulations (Nygaard and Lavoie 2010). The cohesion and frictional angle are determined from the unconfined compressive strength, UCS. The other values are determined from the experimental methodologies and apparatuses provided in Sections 5 and 6.

<table>
<thead>
<tr>
<th>Material</th>
<th>Casing</th>
<th>Cement</th>
<th>Calmar-shale</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (GPa)</td>
<td>200</td>
<td>9.67</td>
<td>24.8</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.3</td>
<td>0.214</td>
<td>0.27</td>
</tr>
<tr>
<td>UCS (MPa)</td>
<td>-</td>
<td>40</td>
<td>-</td>
</tr>
<tr>
<td>Friction Angle (°)</td>
<td>-</td>
<td>30</td>
<td>35</td>
</tr>
<tr>
<td>Cohesion (MPa)</td>
<td>-</td>
<td>15</td>
<td>25.9</td>
</tr>
<tr>
<td>Thermal Expansion (10^6/K)</td>
<td>11.433</td>
<td>9.71</td>
<td>10</td>
</tr>
<tr>
<td>Thermal Conductivity (W/m-K)</td>
<td>43</td>
<td>0.29</td>
<td>2.4</td>
</tr>
<tr>
<td>Specific Heat (J/kg-K)</td>
<td>490</td>
<td>900</td>
<td>900</td>
</tr>
<tr>
<td>Tensile Strength (MPa)</td>
<td>-</td>
<td>3</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 7.2 shows stress results when wellbore elements are removed, cement column pressure is applied to the model, and the near-wellbore stresses (hoop and radial stress) are imposed. The finite element model results are given as material effective stresses. Such that the effective stress shown is the total stress minus the material pore pressure. The radial stress in the model is seen to be equal to the cement column pressure applied minus the formation pore pressure. This simulation step replicates the drilling process and the near wellbore stresses that are generated by drilling.
The next step in the simulations was to add casing (Figure 7.3) and cement (Figure 7.4) elements to the model. As mentioned before, the cement will have slurry hydrostatic pressure before the start of hardening. The casing will have the 20 MPa wellbore pressure is applied to the inside and the 30 MPa cement hydrostatic pressure applied to the outside. In the formation the 30 MPa cement hydrostatic pressure is applied. Since the initial condition of 30 MPa hydrostatic pressure was defined for the cement with a 20 MPa pore pressure throughout the cement and the formation. The stress distribution after adding cement and casing is illustrated in the Figure 7.4. The radial stress is changing from 20 MPa inside the casing to 30 MPa outside the casing to 10 MPa in cement and 10 MPa inside formation which increases to far field value. The results still indicate compressive stresses in all the elements (casing, cement and the formation).

No shrinkage from hydration is given in this model so the final step in the model was to apply the injection pressure as a mechanical load (40 MPa) and injection cooling as thermal load (−40°C) at the inside of the casing. Figure 7.5 illustrates the stress results after the mechanical loading of 40 MPa is applied. Figure 7.6 illustrates the stress after both mechanical and thermal loading of 40°C decrease is applied to the inner casing.
Figure 7.3: Radial (left) and Hoop (right) Stress distributions in the model after casing is added and mud weight is applied to the casing.

Figure 7.4: Radial (left) and Hoop (right) Stress distributions in the model after cement elements are added.

Figure 7.5: Radial (left) and Hoop (right) Stress distribution after injection pressure of 40 MPa applied to the inside of the casing.
Figure 7.6: Radial (left) and Hoop (right) Stress distribution after injection temperature 1 day of 20°C is applied to the inside of the casing

Detailed presentation of radial and hoop stresses in maximum horizontal stress orientation before and after loading are given in Figure 7.7 and Figure 7.8. For all the figures, the inside of the casing is at 0.073546 m (5.971” ID), casing cement boundary is at 0.08414 m (6 5/8” diameter casing), and cement formation boundary is at 0.10795 m (8 1/2” hole diameter measured from the wellbore center. The stress data is taken from the element centroid instead of averaging at the exact wellbore interfaces (element nodes) to obtain higher accuracy. Figure 7.7 shows the radial stress results based on the distance from cased borehole before any loading, after mechanical loading, and after thermal loading. Radial stresses after mechanical loading are increased in all three materials and most significantly in the casing. After applying thermal load, radial stress is reduced for all materials but still it is higher than before loading status in the casing. Figure 7.8 shows the hoop stress results based on distance from cased borehole before any loading, after mechanical loading and after thermal loading. Hoop stresses have dropped in the casing and cement materials after each loading. Most significantly hoop stress has dropped in the casing after thermal loadings because of high thermal conductivity on casing and also because CO₂ injection pressure is added. Although, results show that no radial fracture is created in cement, the stress reduction may be higher if additional loads are applied to the cased wellbore through life of the well. There is not any indication of tensile fracture in the formation after thermal and mechanical loads. Similar to the radial stress results, there is no indication of tension after the thermal and pressure loads are added, however, the
hoop stress profile inside the cement (being close to tensile) indicates potential for tensile fractures to form.

Figure 7.7: Radial stress in maximum horizontal stress orientation before mechanical loading, after mechanical loading and after thermal loading

Figure 7.8: Hoop stress in maximum horizontal stress orientation before mechanical loading, after mechanical loading and after thermal loading
8. SENSITIVITY ANALYSIS

The results of the finite element model and the analytical model were confirmed using simple loading conditions. The in-situ stresses in the formation were assumed to be 43.7 MPa, 41.8 MPa and 38 MPa for overburden, maximum and minimum horizontal stresses respectively, the cement pressure was 30 MPa, wellbore pressure was 20 MPa, 20 MPa pore pressure, 0% shrinkage, 0 MPa pore pressure change, the neat cement properties were used, a 40 MPa pressure was applied as the injection load, and 1 day at 40°C temperature drop was used as the thermal loading. The results of each model are plotted in Figure 8.1. The casing and cement radial and hoop stress data are in good agreement between the numerical and analytical models. The radial and hoop stress in the formation matches between the two models as well. It was observed by Jo (2008) that the analytical hoop stress in the casing did not match the finite element hoop stress and this was due to the method of using an additional integration constant to maintain uniform axial stress, which is an incorrect assumption because in the cased wellbore a change in axial strain of the wellbore should cause a change in stress. The analytical model and FE model were compared with the work of Bois et al. (2012) which calculates the interface displacements and stresses and it was found that they match. A weakness of the Bois et al. (2012) equations is that they assume total stresses and uniform temperature distributions throughout a material which means that it is unable to accurately predict the hoop stress, but can closely match the radial stress. The analytical and FE models presented assume a time and position dependent distribution.
8.1. VARIATION OF CEMENT COMPOSITION

The base loading conditions for each study will be the same as the one provided in the previously described model, where the in-situ stresses in the formation were assumed to be 43.7 MPa, 41.8 MPa and 38 MPa for overburden, maximum and minimum horizontal stresses respectively, the cement pressure was 30 MPa, wellbore pressure was 20 MPa, 20 MPa pore pressure, 0% shrinkage, 0 MPa pore pressure change, the neat cement properties were used, a 40 MPa pressure was applied as the injection load, and 1 day at 40°C temperature drop was used as the thermal loading. Since the value of mechanical properties varies depending on the composition of the cement it was decided to run a parametric study using the extreme variations of cement compositions. These compositions included neat cement, 10% Bentonite Gel cement, 10% CaCl₂ cement, 10% Barite cement, 5% Sand cement, and 2% Gel + 2% CaCl₂ cement. The results of these cement compositions are presented in Figures 8.2 and 8.3. Figure 8.2 and 8.3 represent the radial stress and hoop stress distributions, respectively, through the casing, cement, and formation. The change from casing and cement is denoted by the left most vertical line on both the radial and hoop stress plots which are a result of the data points jumping. This is because there is no pore pressure in the casing which makes the radial stress in the cement and formation separate from that of the casing. In the hoop direction the separation between casing-cement and cement-formation is because the hoop stress is not continuous between materials of different properties.
What can be observed is that the various cement mechanical and thermal properties cause little difference in the stress distributions. Given that the hydrostatic pressure varies between the different cement compositions, which is not taken into account in these models, a lower density cement would have a greater risk of failure in tension than a higher density cement because the hydrostatic pressure in the cement column would be less, see Section 8.8.

8.2. VARIATION IN INJECTION PRESSURE

In order to investigate the effect of the different injection loads a parametric study was done using different hydrostatic wellbore pressures. The loading pressures selected were the base condition 40 MPa injection pressure, 10 MPa injection pressure, 20 MPa injection pressure, 30 MPa injection pressure, and 50 MPa injection pressure. Figures 8.4 and 8.5 represent the radial and hoop stress distributions, respectively. These results show that as injection pressure decreases the risk of radial de-bonding increases. Under the conditions used in the parametric study, the wellbore is not at risk of failure. The hoop stress varies in the cement and formation layers show a greater risk in wellbore fracturing as injection pressure increases. A notable effect is that as the injection pressure changes,
the hoop and radial stress in the casing change significantly. This shows an increased risk of casing failure.

**8.3. VARIATION IN TEMPERATURE LOADING**

Investigation of the effect of thermal loading was conducted using the base condition of 40°C decrease for 1 day, 80°C decrease for 1 day, 40°C increase for 1 day, 80°C increase for 1 day, and no temperature change. Figures 8.6 and 8.7 represent the radial and hoop stress distributions, respectively. The results of varying temperature loading show that with a greater temperature decrease the risk of radial de-bonding and cement fracturing will increase. Under the conditions described, failure was not reached in any of the parametric temperature variations. With decreasing temperature, the hoop stress in the casing becomes greater increasing the risk of casing failure.
Another factor in temperature loading is the duration of injection. The baseline injection duration was set at 1 day, but if the injection were to continue for a month (30 days) then there would be a greater thermal load. The temperature profiles for 1 day and 30 days is presented in Figure 8.8. Figure 8.8 shows that while the high conductivity casing remains at a constant temperature, the cement and formation have a greater temperature decrease as the distance from the center of the wellbore increases. This is as would be expected to happen in the case of greater exposure to cool injection fluids. The results of increasing the injection duration are shown in Figure 8.9 and Figure 8.10. Figure 8.9 gives the radial stress for the baseline of 1 day of injection and the 30 days of injection. The difference 1 day to 30 days line is shown on the scale on the right. Here it is apparent that the increasing the duration of injection does not significantly increase the risk of failure in the cement sheath slightly. The radial stress drops 0.1 MPa at the casing interface and 0.2 MPa at the formation interface. Figure 8.10 gives the hoop stress for the baseline of 1 day of injection and the 30 days of injection. The difference 1 day to 30 days line is shown on the scale on the right. Here it is apparent that the increasing the duration of injection also increases the risk of failure in the cement sheath. The hoop stress in the cement sheath drops 0.1 MPa at the casing interface and 1 MPa at the formation interface.
8.4. VARIATION IN SHRINKAGE/EXPANSION

Cement shrinkage is a result of the hydration of cement as it consumes water (Thiercelin, 1998b). Expansion is a result of expanding agents added to the cement slurry, which are typically used to counteract the shrinkage during hydration. Shrinkage commonly occurs in cement slurries and may cause cement failure (Bosma et al. 1999,
Bois et al. 2011, Bois et al. 2012). The parameters of shrinkage used were the base case 0% shrinkage, 0.1% shrinkage, 0.5% shrinkage, -0.1% shrinkage (0.1% expansion), and -0.5% shrinkage (0.5% expansion). Figures 8.11 and 8.12 represent the radial and hoop stress distributions, respectively, for the various shrinkage amounts. The results show that with 0.5% shrinkage the cement and formation are greatly induced into radial and hoop tension. The result of this high tension would be radial fractures forming at the casing-cement boundary which is at the greatest tensile stress and which fails at 0.1% shrinkage. Additionally, at 0.1% shrinkage radial tensile fracture will develop in the cement sheath due the tensile hoop stress. Another observation is that under 0.5% expansion there is no radial de-bonding but the radial and hoop stresses increase greatly. Under the conditions of greater than 0.5% expansion, the cement may experience shear failure under Mohr-Coulomb failure criteria due to the large differential stress.

![Figure 8.11: Radial stress in maximum horizontal stress orientation with varying cement shrinkage](image1.png)

![Figure 8.12: Hoop stress in maximum horizontal stress orientation with varying cement shrinkage](image2.png)

8.5. **VARIATION IN PORE PRESSURE**

Pore pressure variations in the cement can have various effects on the cement and its bonds with the casing and formation. The pore pressure variations studied were the base case 0 MPa, 10 MPa increase, 20 MPa increase, 10 MPa decrease, and 20 MPa
decrease. Figures 8.13 and 8.14 represent the radial and hoop stress distributions, respectively, for the variations in pore pressure. The stress distribution resulting from the 20 MPa increase in pore pressure results in radial tension at the cement interfaces which results in de-bonding. As the pore pressure increases, the risk of de-bonding increases at both interfaces, but the greatest risk is at the casing-cement boundary. The effective stress distribution results in the cement moving into tension. The effective stress will govern the fracturing of the cement and under the 20 MPa increase in cement pore pressure tensile fractures would occur in the circumferential direction.

![](image)

**Figure 8.13**: Radial stress maximum horizontal stress orientation with increases and decreases in pore pressure

**Figure 8.14**: Hoop stress in maximum horizontal stress orientation with increases and decreases in pore pressure

### 8.6. BOUNDARY CONDITIONS

Another parameter to investigate is the integrity of various initial boundary conditions. The base case boundary condition assumed a cement slurry pressure of 30 MPa and a pore pressure of 20 MPa throughout the cement and formation. The stress distribution of the base case is shown in Figure 8.15 and Figure 8.16 by the preloading stress distribution in which the effective stress in the cement is 10 MPa. Another boundary condition which was analyzed in Figure 8.15 and Figure 8.16 was the total stress boundary condition. The total stress assumption neglects the effects of pore
pressure. The preloading stress distribution of the total stress conditions has the stress throughout the cement at the cement slurry pressure, 30 MPa. The third boundary condition was the zero effective stress boundary condition. The zero effective stress assumption assumes that the pressure of the cement column reduces to hydrostatic pressure or the pore pressure in the formation (Nelson and Guillot, 2006). The zero effective stress preloading stress distribution has an effective stress of 0 MPa which puts the cement sheath at high risk for tensile fracture because the stress is so close to tension. Figure 8.15 and Figure 8.16 show the before loading stress state and the after mechanical and thermal loading stress state for each of the in-situ stress assumptions. The effect of the mechanical and thermal loading on the radial stress, Figure 8.15, is a reduction in the radial stress throughout the cement and formation. At a maximum the radial stress decreases 2 MPa. The conditions of the base case effective stress assumption and the total stress assumption remain intact in the radial direction while the zero effective stress assumption’s radial stress becomes tension resulting in de-bonding at the casing-cement boundary because as the temperature is decreased the inner boundary will have the greatest change in temperature during the early steps of thermal loading. The effects on the hoop stress are shown in Figure 8.16. The base case effective stress assumption and the total stress conditions remain intact while the zero effective stress conditions have reduced to tension. This results in tensile fractures forming in the cement sheath and de-bonding at the cement interface boundaries. It is shown is that the zero effective stress assumption is infeasible because the failure of the cement sheath would occur under almost any type of loading condition.
In a scenario in which injection is carried out through tubing in the cased wellbore and the fluid behind the tubing does not allow the transfer of stress, the only loading that would occur is the thermal loading. This is also representative of when the injection pressure is reduced and the thermal loading is still present in the wellbore. In this case the pressure acting against the casing is held at the hydrostatic pressure of 20 MPa. Figure 8.17 shows the radial stresses while Figure 8.18 shows the hoop stresses as a result of thermal loading and no mechanical loading. In these simulations the boundary conditions from Figure 8.15 and 8.16 were used, the preloading stress distributions in Figure 8.17 and 8.18 are the same as those in Figure 8.15 and 8.16. In the radial direction Figure 8.17 shows that the total stress and base case effective stress conditions remain intact, while the zero effective stress condition results in de-bonding of the casing-cement boundary. The de-bonding is apparent in the maximum tensile stress developing in the inside of the cement sheath which would then pull away from the casing. In the hoop direction the Figure 8.18 shows that the total stress and base case conditions remain intact, while the zero effective stress condition results in tensile fractures occurring in the cement at the casing-cement boundary. The maximum tensile stress there exceeds the given tensile strength of the cement. Again it is shown is that the zero effective stress assumption is
infeasible because the failure of the cement sheath would occur under almost any magnitude of temperature reduction.

8.7. INITIAL CONDITIONS

In terms of initial conditions there is primarily one parameter that can be easily changed and that is wellbore pressure. All other aspects of the wellbore are constant, including the formation stresses and the hydrostatic density of the cement. Figure 8.19 depicts the radial stress of a wellbore after the loading described in the base case simulation, except that the initial wellbore pressure applied to the inner casing is varied. When a higher pressure is applied to the casing during cement hydration there is a less dramatic change in wellbore pressure to reach the injection pressure after setting. In this case it is apparent that with less increase in wellbore pressure, which is a change from the cement set conditions after its plastic behavior is complete, there is lower radial stress in the cement and greater risk of de-bonding. In Figure 8.20 which depicts the hoop stress we see that with less increase in wellbore pressure there is less tensile stress built in the hoop direction which decreases the risk of fracturing the cement sheath.
Another initial condition study to test is what happens when the base case scenario is released from the high pressure setting wellbore pressure to a lower pressure of 20 MPa with no temperature change, this is what would happen if injection was stopped. In Figure 8.21, the set pressures shown in Figures 8.19 and 8.20 are decreased to 20 MPa once injection has stopped. The 30 MPa, 20 MPa, and 10 MPa cases all remain in compression, but it is seen that the radial stress of the cement set at 40 MPa which was decreased to 20 MPa after injection of CO₂ was stopped developed a tensile radial stress at the casing-cement interface. This tension would result in de-bonding of cement sheath and failure of the wellbore. In Figure 8.22, the hoop stress remains in compression for all the samples, but the hoop stress of the 10 MPa setting cement the nearest to forming fractures in the cement as the increase in wellbore pressure induced a tensile stress into the cement as a decrease in wellbore pressure would add compression to the cement sheath.
The effect of having different cements in the wellbore has the result of different material properties, as described in Section 8.1, and a different cement hydrostatic pressure. Figure 8.23 and 8.24 show the radial and hoop stresses for the cased wellbore loaded through the base case loading procedure. The differences in the radial stresses are that neat cement at cement pressure 30 MPa, 24 MPa, 36 MPa, 42 MPa, and 48 MPa. These cement column pressures were chosen to give a range for low density and high density high pressure cements. Figure 8.23 and Figure 8.24 show that the lower cement pressure has a much lower effective radial and hoop stresses. This would indicate that lower cement pressures result in having a greater risk of de-bonding and tensile fracturing than higher density cements.
8.9. CONCENTRIC CASINGS

In the lower portions of a well the single casing model used in the previous models is appropriate. But in the upper portions of the well the surface casing and the intermediate or production casing will overlap. These situations result in concentric casings which can be described by the formation on the outside, then the outer cement which cements the outer surface casing in place. The surface casing is followed by the inner cement which cements the intermediate or production casing in place. Figure 8.25 is the sketch of the wellbore and the finite element mesh that was used to model the concentric wellbore. The casing and hole diameters are described below the sketch which is a 244.5 mm or a 9-5/8” surface casing and a 177.8 mm or a 7” production casing.
The results of the finite element modeling and the analytical modeling of the concentric wellbore under the base case loading conditions of neat cement with 30 MPa cement pressure, 20 MPa hydrostatic/pore pressure, 40 MPa injection pressure, and 40°C decrease in temperature of 1 day are presented in Figure 8.26. In Figure 8.26 it is apparent that the introduction of the surface casing between the two cement layers has effect on the stress distributions. One effect is that the radial and hoop stresses between the two cement sheaths would not be continuations of each other. This indicates that different forces are acting on the surface casing cement than the production casing cement. Additionally, the opportunity to have two cement sheaths means the cement may have different compositions and cement pressures which add to the complexity of the
concentric casing system. Concentric casings cannot be ignored or assumed to be resilient in an analysis of a wellbores integrity or leakage risk.

Figure 8.26: Concentric Casing Finite Element and analytical matching results verify the accuracy of the simulations
9. WASP CASE STUDIES

One potential method to reduce injection cost of CO$_2$ sequestration is to repurpose existing wells as CO$_2$ injection wells. To investigate if this method is feasible wells from the WASP, presented in Section 2, were investigated. To identify potential candidates, wells must be cased past the Calmar shale and into the Nisku formation. The average properties of the in-situ stress field the WASP area was presented in Section 2. The stress field is determined to be an Andersonian stress state where the vertical stress gradient is 23 kPa/m, the maximum horizontal stress gradient is 22 kPa/m, and the minimum horizontal stress gradient is 20 kPa/m. The pore pressure in the field is hydrostatic which gives a pore pressure gradient of 9.81 kPa/m (Michael et al., 2006). The lithology and formation properties found for the field are found in Table 9.1 (Nygaard, 2010). The temperature profile recorded in previous well reports is shown in Figure 9.2.

The wellbore model was based on actual cased and cemented wellbores in the area. The drilling fluid used to displace the cement is assumed to be equal to a gradient of 9.81 kPa/m. Further it is assumed that during the cementing of the casing, cement, and formation will all be at a uniform temperature equal to the formation temperature. The injection temperature was based off of Ruan et al. (2013). Ruan et al. (2013) ran numerical simulations to determine what temperatures would be encounter in the wellbore as CO$_2$ is injected into the well. The modelling has the CO$_2$ injected through tubing into the well and the temperature is radiated out to the casing through water in the annulus. The result of the modeling is presented in Figure 9.1 with the points to the right being equal to the casing injection temperature. Finally the injection pressure in the wellbore will be equal to the hydrostatic pressure of the supercritical CO$_2$, which about equal to water, with a gradient of 9.81 kPa/m plus a 20 MPa pump pressure applied at the surface to keep the CO$_2$ under pressure.
The thermal loading of the CO\(_2\) injection is dependent on the change in temperature. By using the data for casing temperatures from Figure 9.1 and plotting temperature and depth data from wells in the Wabamun area results in Figure 9.2. Figure
9.2 shows these two data sets on a single plot. The difference between the two data sets is the change in temperature to use for the loading conditions.

Figure 9.1: Wellbore injection temperature (Ruan et al., 2013)

Figure 9.2: Formation and casing temperature variations with depth

The cases chosen for this study are the proposed injection well for WASP project (Nygaard and Lavoie, 2010). This well is specifically design for the purposed of injecting CO₂ in the Nisku formation. The other two cases are existing wells that would be repurposed as injection wells. These wells were chosen based off the criteria of sufficient well data available and being cased through the Calmar shale into the Nisku formation.
The wells are the 100/02-01-046-01W5/00 and the 100/09-10-047-01W5/00. These wells have sufficient well data including the cement compositions, casing depths, and formation depths. They also have the correct well design to inject into the Nisku formation.

9.1. **CASE 1: PROPOSED INJECTION WELL**

This case study looks at the proposed injection well described in the University of Calgary WASP report (Nygaard and Lavoie, 2010). This well is to be drilled and completed in the Nisku formation to be used as an injection well to inject CO₂. This well would be a new well designed for only this purpose as opposed to existing wells designed to be production wells. The proposed well is sketched in Figure 9.3. The points of interests to test for leakage and integrity are at the surface casing shoe and the production casing shoe. These points are chosen because it is at these points that loss of integrity or leakage would cause leakage into the fresh water aquifer or from the sealing formation, respectively. The integrity of the design will be evaluated based on an injection pressure 20 MPa greater than hydrostatic and a thermal loading of 1 day at the change in temperature described in Figure 9.2. A temperature of 1 day was chosen because as seen in Section 8.3 the majority of the thermal loading takes place in the first day.

The proposed vertical injection well sketch, Figure 9.3, is described in total vertical depth. The 9-5/8” or 244.5 mm surface casing, 36 lb/ft or 53.57 kg/m, is set at a TVD of 550 m in the Belly River formation in a 13-3/4” or 349 mm hole. The casing was cemented to surface using neat cement. The Belly River is just below the fresh water aquifer so this will be the first point of interest for the proposed injection well. The 7” or 177.8mm production casing, 20 lb/ft or 29.76 kg/m, is set at a TVD of 1960 m in the Nisku formation in a 8-3/4” or 222 mm hole. The casing was cemented to surface using neat cement. The production casing exits the sealing formation, the Calmar Shale, at TVD 1850m, making this the second point of interest for the proposed injection well.
The simulation loading for the concentric casing at 550 m is described in Table 9.2. Table 9.2 gives the production casing, surface casing, and hole diameters used in the model. The surface casing cement and production casing cement are both given as neat cement for the proposed well. The cement pressure will be determined from the cement type at its associated density. Since the two cements are cementsed to the surface the cement pressures will be based on the depth and density of the cement found in Table 6.2. The injection temperature and formation temperature are given as 10.8°C and 27.1°C, respectively. These values are determined from Figure 9.2 at a depth of 550m. The hydrostatic and pore pressure of 5.4 MPa is found by using a fresh water density of 1000 kg/m³ and the depth of interest. The injection pressure assumes a 20 MPa pump pressure at the surface to compress the CO₂ gas to a supercritical state which would result in a CO₂ density near that of fresh water, 1000 kg/m³.
Table 9.2: Proposed Injection Well Model Data at 550 m with Concentric Casings

<table>
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<tr>
<th>Prod Casing ID</th>
<th>Prod Casing OD</th>
<th>Surf Casing ID</th>
<th>Surf Casing OD</th>
<th>Hole D</th>
</tr>
</thead>
<tbody>
<tr>
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<td>177.8 mm</td>
<td>226.6 mm</td>
<td>244.5 mm</td>
<td>349 mm</td>
</tr>
<tr>
<td>Surface casing cement</td>
<td>Neat Cement</td>
<td>Production casing cement</td>
<td>Neat Cement</td>
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</tr>
<tr>
<td>Formation temperature</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Hydrostatic/Pore Pressure</td>
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<tr>
<td>Injection Pressure</td>
<td>25.4 MPa</td>
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</tbody>
</table>

The results of the concentric casing simulation for the proposed injection well are given in Figure 9.4. Figure 9.4 shows the results for the finite element (FE) model by the points and the results of the analytical model by the continuous line. It can be seen that these two models, the analytical and the FE, give the exact same results which verifies the model’s accuracy. The radial stress in Figure 9.4 shows that there is no identified risk in radial de-bonding of the cement sheath from the casing as the cement radial stress does not approach zero. The hoop stress in Figure 9.4 shows that in the production cement sheath there is a low risk of cement fracturing as the cement hoop stress approaches zero even if the injection were to be continuous for 30 days. While the production cement sheath is at low risk for fracturing the security of the barrier protecting the fresh water aquifer, the surface casing and surface cement sheath, remains intact. This shows that this barrier point will not be at risk for leakage.
Figure 9.4: Proposed injection well radial and hoop stress in maximum horizontal stress orientation simulation results confirmed with analytical and FE model

The simulation loading for the single casing at 1850m is described in Table 9.3. Table 9.3 gives the production casing and hole diameters used in the model. The production casing cement is given as neat cement for the proposed well. The cement pressure will be determined from the cement type at its associated density. Since the cement is cemented to the surface the cement pressure will be based on the depth and density of the neat cement found in Table 6.2. The injection temperature and formation temperature are given as 25.6°C and 71.2°C, respectively. These values are determined from Figure 9.2 at a depth of 1850 m. The hydrostatic and pore pressure of 18.1 MPa is found by using a fresh water density of 1000 kg/m³ and the depth of interest. The injection pressure assumes a 20 MPa pump pressure at the surface to compress the CO₂ gas to a supercritical state which would result in a CO₂ density near that of fresh water, 1000kg/m³.

The results of the single casing simulation for the proposed injection well are given in Figure 9.5. Figure 9.5 shows the results for the finite element (FE) model by the points and the results of the analytical model by the continuous line. It can be seen that these two models, the analytical and the FE, give the exact same results which verifies the model’s accuracy. The radial stress in Figure 9.5 shows that there is a small risk in radial de-bonding of the cement sheath from the casing as the cement radial stress does not approach zero. The hoop stress in Figure 9.5 shows that in the production cement
sheath there is a low risk of cement fracturing, as well, as the cement hoop stress does not approaches zero. This shows that this barrier point will not be at risk for leakage. These simulations show that the proposed injection well would be sufficient in as a CO$_2$ sequestration injection well.

Table 9.3: Proposed Injection Well Model Data at 1850 m with Single Casing

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<th>Surf Casing OD</th>
<th>Hole D</th>
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<td>Prod Casing OD</td>
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<td>Surface casing cement</td>
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<td></td>
</tr>
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</tr>
<tr>
<td>Formation temperature</td>
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<tr>
<td>Hydrostatic/Pore Pressure</td>
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<tr>
<td>Injection Pressure</td>
<td>38.1 MPa</td>
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</table>

Figure 9.5: Proposed injection well radial and hoop stress in maximum horizontal stress orientation simulation results confirmed with analytical and FE model
9.2. **CASE 2: EXISTING WELL 100/02-01-046-01W5/00**

This case study looks at the well 100/02-01-046-01W5/00 as it is described in the Tour Reports (CRC, 1984) and Well Tickets (GeoWebworks, 2009a). This well was drilled in 1987 and abandoned in 2003. The well has been drilled into the Nisku formation and is a candidate to be used as an injection well to inject CO₂. This well would be an existing well designed which would be repurposed from its original design as a production well. When a well is to be used for another purpose other than what it was designed to do, careful consideration must be made of the well’s current integrity as well as the effectiveness of the older well design to meet new design criteria. The integrity of the design will be evaluated based on an injection pressure 20 MPa greater than hydrostatic and a thermal loading of 1 day at the change in temperature described in Figure 9.2. The existing well is sketched in Figure 9.3. The points of interests to test of leakage and integrity are at the surface casing shoe and the production casing shoe. These points are chosen because it is at these points that loss of integrity or leakage would cause leakage into the fresh water aquifer or from the sealing formation.

The 100/02-01-046-01W5/00 well sketch, Figure 9.6, is described in total vertical depth. The 9-5/8” or 244.5 mm surface casing, 36 lb/ft or 53.57 kg/m, is set at a TVD of 247 m in the Belly River formation in a 13-3/4” or 349 mm hole. The casing was cemented to surface using neat cement. The Belly River is just below the fresh water aquifer so this will be the first point of interest for the existing well. The 7” or 177.8 mm production casing, 26 lb/ft or 38.69 kg/m, is set at a TVD of 2284 m in the Nisku formation in an 8-3/4” or 222 mm hole. The casing was cemented to TVD of 630 m using neat cement. The production casing exits the sealing formation, the Calmar Shale, at TVD of 2031 m, making this the second point of interest for the existing well.
The simulation loading for the concentric casing at 247 m is described in Table 9.4. Table 9.4 gives the production casing, surface casing, and hole diameters used in the model. The surface casing cement and production casing cement are given as 2% CaCl$_2$ and water, respectively. Water is in the production section due to the cement top being located at 630 m therefore not reaching the depth of interest. The surface casing cement pressure will be determined from the cement type at its associated density from Table 6.2 and the depth of 247 m since the casing is cemented to the surface. Since the production cement top is at 630 m a fluid, equal that of water, will fill the annulus between the production and surface casings. The injection temperature and formation temperature are given as 7.34°C and 19.4°C, respectively. These values are determined from Figure 9.2 at a depth of 247 m. The hydrostatic and pore pressure of 2.42 MPa is found by using a fresh water density of 1000 kg/m$^3$ and the depth of interest. The injection pressure assumes a 20 MPa pump pressure at the surface to compress the CO$_2$ gas to a supercritical state which would result in a CO$_2$ density near that of fresh water, 1000 kg/m$^3$. 
Table 9.4: 100/02-01-046-01W5/00 Model Data at 247 m with Concentric Casings

<table>
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<th>Prod Casing OD</th>
<th>Surf Casing ID</th>
<th>Surf Casing OD</th>
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<tbody>
<tr>
<td>159.4 mm</td>
<td>177.8 mm</td>
<td>226.6 mm</td>
<td>244.5 mm</td>
<td>349 mm</td>
</tr>
</tbody>
</table>

- Surface casing cement: 2% CaCl₂ Cement
- Production casing cement: Water
- Injection temperature: 7.34°C
- Formation temperature: 19.4°C
- Hydrostatic/Pore Pressure: 2.42 MPa
- Injection Pressure: 22.4 MPa

The results of the concentric casing simulation for the 100/02-01-046-01W5/00 well are given in Figure 9.7. The concentric casing model has been reduced to a single casing model because there would be no pressure transfer through the production casing annulus fluid. The temperature profile was created using a concentrically cased wellbore where the production cement was replaced with water. Figure 9.7 shows the results of the analytical model by the continuous lines. Two possible outcomes could occur at this point so two scenarios were considered and simulated. The first scenario is that there is no leakage of wellbore fluids to the annulus of the production casing, and the second scenario is that there is leakage of wellbore fluids to the annulus of the production casing. The radial stress in Figure 9.7 shows that there is a small risk in radial de-bonding of the cement sheath from the casing in both scenarios as the radial stress does not approach zero. The hoop stress in Figure 9.7 shows that in the case of leakage behind the casing the surface cement sheath will have tensile fracturing due to high tensile hoop stress. There would be no risk of cement fracturing in the no leakage scenario because the cement hoop stress does not approach zero. These scenarios show that the integrity of the production casing must be confirmed in order to protect the fresh water aquifers from wellbore leakage. In situations with highly corrosive fluids such as CO₂, a special corrosion resistant casing would need to be in place to prevent leakage and ultimately wellbore failure. An older well designed as a production well may not have a proper casing selected for a corrosive fluid to be in the wellbore, indicating that if this is true this well may not be a good candidate for CO₂ sequestration.
Figure 9.7: 100/02-01-046-01W5/00 well radial and hoop stress in maximum horizontal stress orientation concentric casing simulation results simplified to single casing

Table 9.5: 100/02-01-046-01W5/00 Well Model Data at 2031 m with Single Casing

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<td>-</td>
<td>-</td>
<td>222 mm</td>
</tr>
<tr>
<td>Surface casing cement</td>
<td>-</td>
<td>Production casing cement</td>
<td>Neat Cement</td>
<td></td>
</tr>
<tr>
<td>Injection temperature</td>
<td>27.7°C</td>
<td>Formation temperature</td>
<td>78.7°C</td>
<td></td>
</tr>
<tr>
<td>Hydrostatic/Pore Pressure</td>
<td>19.9 MPa</td>
<td>Injection Pressure</td>
<td>39.9 MPa</td>
<td></td>
</tr>
</tbody>
</table>

The simulation loading for the single casing at 2031 m is described in Table 9.5. Table 9.5 gives the production casing and hole diameters used in the model. The production casing cement is given as neat cement for the existing well. The cement pressure will be determined from the cement type at its associated density. The production cement is cemented to TVD of 630 m, so the cement pressure will be based on 630 m of fresh water at 1000 kg/m³ and the interval of cement from 630 m to 2031 m and density of the neat cement found in Table 6.2. The injection temperature and formation temperature are given as 27.7°C and 78.7°C, respectively. These values are
determined from Figure 9.2 at a depth of 2031 m. The hydrostatic and pore pressure of 19.9 MPa is found by using a fresh water density of 1000 kg/m$^3$ and the depth of interest. The injection pressure assumes a 20 MPa pump pressure at the surface to compress the CO$_2$ gas to a supercritical state which would result in a CO$_2$ density near that of fresh water, 1000 kg/m$^3$.

The results of the single casing simulation for the repurposed injection well are given in Figure 9.8. Figure 9.8 shows the results of the analytical model by the continuous line. The radial stress in Figure 9.8 shows that there is a small risk in radial de-bonding of the cement sheath from the casing as the cement radial stress does not approach zero. The hoop stress in Figure 9.8 shows that in the production cement sheath there is a low risk of cement fracturing, as well, as the cement hoop stress does not approaches zero. This shows that this barrier point will not be at risk for leakage. These simulations show that if the production casing is not cemented to the casing shoe that the integrity of the production casing is of great concern of whether or not the wellbore will fail at the surface casing shoe or not. The production casing must be free of leaks and be corrosion resistant in order to be used for CO$_2$ sequestration.

Figure 9.8: 100/02-01-046-01W5/00 well radial and hoop stress in maximum horizontal stress orientation simulation result
9.3. **CASE 3: EXISTING WELL 100/09-10-047-01W5/00**

This case study looks at the well 100/09-10-047-01W5/00 as it is described in the Tour Reports (CRC, 1987) and Well Tickets (GeoWebworks, 2009b). This well was drilled and abandoned in 1987. The well has been drilled and cased in the Nisku formation and is a candidate to be used as an injection well to inject CO₂. This well would be an existing well designed which would be repurposed from its original design as a production well. The integrity of the design will be evaluated based on an injection pressure 20 MPa greater than hydrostatic and a thermal loading of 1 day at the change in temperature described in Figure 9.2. The existing well is sketched in Figure 9.9. The points of interests to test of leakage and integrity are at the surface casing shoe and the production casing shoe. These points are chosen because it is at these points that loss of integrity or leakage would cause leakage into the fresh water aquifer or from the sealing formation.

The 100/09-10-047-01W5/00 well sketch, Figure 9.9, is described in total vertical depth. The 13-3/8” or 339.6 mm surface casing, 48 lb/ft or 71.43 kg/m, is set at a TVD of 455 m in the Belly River formation in a 17-1/2” or 444 mm hole. The casing was cemented to surface using 1.5% CaCl₂ cement. The Belly River is just below the fresh water aquifer so this will be the first point of interest for the existing well. The 9-5/8” or 244.5 mm production casing, 40 lb/ft or 59.53 kg/m, is set at a TVD of 2050 m in the Nisku formation in a 12-1/4” or 311.15 mm hole. The casing was cemented to surface using 2%CaCl₂ from production shoe to at TVD 500 m and 2%Gel + 2%CaCl₂ to surface. The production casing exits the sealing formation, the Calmar Shale, at TVD of 1982.3 m, making this the second point of interest for the existing well.
The simulation loading for the concentric casing at 455 m is described in Table 9.6. Table 9.6 gives the production casing, surface casing, and hole diameters used in the model. The surface casing cement and production casing cement are given as 1.5% CaCl$_2$ cement and 2% Gel + 2% CaCl$_2$ for the 100/09-10-047-01W5/00 well. The cement pressure will be determined from the cement type at its associated density. Since the two cements are cemented to the surface the cement pressures will be based on the depth and density of the cement found in Table 6.2. The injection temperature and formation temperature are given as 9.71°C and 24.6°C, respectively. These values are determined from Figure 9.2 at a depth of 455 m. The hydrostatic and pore pressure of 4.46 MPa is found by using a fresh water density of 1000 kg/m$^3$ and the depth of interest. The injection pressure assumes a 20 MPa pump pressure at the surface to compress the CO$_2$ gas to a supercritical state which would result in a CO$_2$ density near that of fresh water, 1000 kg/m$^3$. 
Table 9.6: 100/09-10-047-01W5/00 Model Data at 455 m with Concentric Casings

<table>
<thead>
<tr>
<th>Prod Casing ID</th>
<th>Prod Casing OD</th>
<th>Surf Casing ID</th>
<th>Surf Casing OD</th>
<th>Hole D</th>
</tr>
</thead>
<tbody>
<tr>
<td>224.4 mm</td>
<td>244.5 mm</td>
<td>323 mm</td>
<td>339.6 mm</td>
<td>444 mm</td>
</tr>
</tbody>
</table>

Surface casing cement: 1.5% CaCl₂
Production casing cement: 2% Gel + 2% CaCl₂
Injection temperature: 9.71°C
Formation temperature: 24.6°C
Hydrostatic/Pore Pressure: 4.46 MPa
Injection Pressure: 24.5 MPa

Figure 9.10: 100/09-10-047-01W5/00 well radial and hoop stress in maximum horizontal stress orientation concentric casing simulation results

The results of the concentric casing simulation for the 100/09-10-047-01W5/00 well are given in Figure 9.10. Figure 9.10 shows the results of the analytical model by the continuous lines. The radial stress in Figure 9.10 shows that there is no identified risk in radial de-bonding of the cement sheath from the casing as the cement radial stress does not approach zero. The hoop stress in Figure 9.10 shows that in the production cement sheath there is a low risk of cement fracturing as the cement hoop stress approaches zero even if injection is continuous for 30 days. While the production cement sheath is at risk for fracturing the security of the barrier protecting the fresh water aquifer, the surface
casing and surface cement sheath, remains intact. This shows that this barrier point will not be at risk for leakage.

The simulation loading for the single casing at 1982.3 m is described in Table 9.7. Table 9.7 gives the production casing and hole diameters used in the model. The production casing cement is given as a 2%\(\text{CaCl}_2\) cement with an expansion agent which will be modeled as a 0.1% expansion. The cement pressure will be determined from the cement type at its associated density. Since the cement is cemented to the surface with two different cement compositions, the cement pressure will be based on the pressure exerted by the cement column using the densities from Table 6.2. The injection temperature and formation temperature are given as 27.1°C and 76.7°C, respectively. These values are determined from Figure 9.2 at a depth of 1982.3 m. The hydrostatic and pore pressure of 19.4 MPa is found by using a fresh water density of 1000 kg/m\(^3\) and the depth of interest. The injection pressure assumes a 20 MPa pump pressure at the surface to compress the CO\(_2\) gas to a supercritical state which would result in a CO\(_2\) density near that of fresh water, 1000 kg/m\(^3\).

<table>
<thead>
<tr>
<th>Prod Casing ID</th>
<th>Prod Casing OD</th>
<th>Surf Casing ID</th>
<th>Surf Casing OD</th>
<th>Hole D</th>
</tr>
</thead>
<tbody>
<tr>
<td>224.4 mm</td>
<td>244.5 mm</td>
<td>-</td>
<td>-</td>
<td>311.15 mm</td>
</tr>
</tbody>
</table>

Surface casing cement

Production casing cement 2%\(\text{CaCl}_2\) w/ 0.1% Expansion

Injection temperature 27.1°C

Formation temperature 76.7°C

Hydrostatic/Pore Pressure 19.4 MPa

Injection Pressure 39.4 MPa

The results of the single casing simulation for the 100/09-10-047-01W5/00 well are given in Figure 9.11. Figure 9.11 shows the results of the analytical model by the continuous lines. The radial stress in Figure 9.11 shows that there is a small risk in radial de-bonding of the cement sheath from the casing as the cement radial stress does not
approach zero. The hoop stress in Figure 9.11 shows that in the production cement sheath there is a low risk of cement fracturing, as well, as the cement hoop stress does not approaches zero. This shows that this barrier point will not be at risk for leakage. These simulations show that the existing design of the well for 100/09-10-047-01W5/00 would be sufficient to be repurposed as a CO₂ sequestration injection well.

![Figure 9.11: 100/09-10-047-01W5/00 well radial and hoop stress in maximum horizontal stress orientation simulation results](image)
10. DISCUSSION

10.1. EXPERIMENTAL FINDINGS

The findings of the mechanical and thermal property experimentation are divided into two categories, correlation with density and the effect of additives. The heat capacity, static Young’s Modulus, Poisson’s Ratio, and linear thermal expansion have no correlation with density, while dynamic Young’s Moduli, UCS, and tensile strength have a positive relationship with density. The lack of correlation of various properties is to be expected. This is due the effect of additives other than density modification. Bentonite gel is used as density and the reducer bentonite cement compared to neat cement using the Mann-Whitney test has a lower tensile strength, lower UCS, higher heat capacity, lower Young’s Modulus, higher Poisson’s Ratio, and lower thermal expansion. The lower Young’s Modulus, UCS, and tensile strength are a result of the ductility and weakness of the bentonite clay which never becomes an integral part of the cement but only creates large voids which reduces the overall density but has no structural strength (Nelson and Guillot, 2006). Barite cement compared to neat cement using the Mann-Whitney test has an equal tensile strength, equal UCS, equal heat capacity, equal Young’s Modulus, equal Poisson’s Ratio, and lower thermal expansion. Additives such as barite and sand which are also density modifiers do not change volume like bentonite, so they do not leave voids but they do not have a great effect on mechanical properties because they are such a small component of the cement that they do not interfere with the structure of the cement which is the dominant component so the properties do not change. CaCl$_2$ cement compared to neat cement using the Mann-Whitney test has a higher tensile strength, slightly higher UCS, higher heat capacity, equal Static Young’s Modulus, lower Dynamic Young’s Modulus, equal Poisson’s Ratio, and equal thermal expansion. These properties and the lack of correlation with density can be explained by the set time accelerating effects of CaCl$_2$. When CaCl$_2$ is dissolved in water the chloride ions enhance the formation of the cement structure, thereby accelerating the curing process which can result in cement that has cured more in a shorter amount of time compared to neat cement. This advanced curing results in a stronger crystalline structure in the cement giving higher tensile strength (Nelson and Guillot, 2006).
Dynamic Young’s Modulus was found to vary positively with density. This is expected because the equation to determine dynamic Young’s Modulus from P and S wave velocities has density in the equation. While the P and S waves have an effect on the resulting Dynamic Young’s Modulus the value is significantly affected by the density, so higher density cements like barite cements have higher dynamic Young’s Modulus and lower density cements like bentonite cements have lower dynamic Young’s Modulus. In addition to dynamic Young’s Modulus varying with density, the static Young’s Modulus also varied positively with density. This can be explained by the fact that the bentonite cements because of the voids created by the swelled clay create a lower cross sectional area that is measured by the outside of the sample, thereby requiring less force to compress a sample with porosity. The correlation in static Young’s Modulus is not as significant as dynamic Young’s Modulus and even though it was found that there was a positive correlation with static Young’s Modulus the change with higher density samples is small.

The thermal expansion coefficient while not dependent on density is dependent the additives used. Specifically the additives which replace cement with another material. These cements are the bentonite, barite, and sand cements which replace cement with their respective additive, but CaCl$_2$ is dissolved in the water added to the cement which reduces the volume that the CaCl$_2$ can occupy. When other materials occupy space where cement would occupy in a neat cement sample the cement expands into the voids in bentonite samples but is held together by the more rigid and lower thermal expanding grains of the sand and barite cements. The result is that the thermal expansion coefficient is reduced.

This correlation data shows that the density of the cement cannot be used to predict its mechanical and thermal properties. The Mann-Whitney data shows that bentonite cement is more ductile cement compared to neat cement having a lower Young’s Modulus, lower tensile strength, and higher Poisson’s Ratio. CaCl$_2$ cement is a higher strength cement compared to neat cement having higher tensile strength. Finally, barite cement is equal to neat cement in most mechanical and thermal properties.
10.2. SENSITIVITY ANALYSIS FINDINGS

The findings of the parametric studies are that the various cement mechanical and thermal properties cause little difference in the stress distributions. This finding was somewhat contradictory to many of the findings by Bosma et al. (1999) and Ravi et al. (2002) who saw that a higher Young’s Modulus has greater likelihood of failing by fracturing the cement. Bosma et al. (1999) and Ravi et al. (2002) are correct that there is a greater likelihood but the on the scale of reasonable numbers, unless a high stiffness cement is placed in the well their effect is small. The reason for the greater conclusion in Bosma et al. (1999) and Ravi et al. (2002) is that there is a zero effective stress in-situ state of stress in place in the cement, so the differences are large when changing from zero, but are small when coming down from the effective stress state of stress in the cement.

As the injection pressure decreases the risk of radial de-bonding increases. Also, as injection pressure increases the risk of cement fracturing increases. This was seen by Bosma et al. (1999), Thiercelin et al. (1998a), and Ravi et al. (2002), and is explained by solid mechanics. As the internal pressure of a hollow cylinder increases the hoop stress decreases increasing the risk of fracturing the cement. When the internal pressure decreases the radial stress will decrease which increases the risk of radial de-bonding. This confirms the results of these studies.

When the temperature decreases the risk of radial de-bonding and cement fracturing will increase. This was also observed by Bosma et al. (1999), Thiercelin et al. (1998a), Ravi et al. (2002), and Bois et al. (2011), because as the cement shrinks around the casing the circumference of the cement sheath tries to decrease but is restrained by the stiff casing causing tension to develop in the hoop stress. Also as the cement shrinks between the stiff casing and the stiff formation the cement develops radial tension to counter the inflexibility of the other two materials.

As shrinkage increases the risk of tensile fracturing at the casing-cement interface and de-bonding at the cement-formation boundary increases, with fracturing being the most likely to occur first. This result was similar to that of Ravi et al. (2002) who found that all models run using 4% volume shrinkage sustained large failures. This is to be expected because the 4% volume shrinkage is that of the unrestrained lab sample and not
representative of a high pressure restrained wellbore cement (Thiercelin et al., 1998b). In the modeling of this study the shrinkage was restricted to 0.1% and 0.5% because these values resulted in an intact wellbore and wellbore failure, respectively. The value of cement shrinkage is for modeling of downhole condition is different than that of the shrinkage value obtained in the laboratory tests under atmospheric conditions. When a cement sheath shrinks, the 95% of shrinkage occurs after the cement sets (Sabins and Sutton, 1991). If this is the case then the elasticity of the casing and formation is a factor which reduces the actual value of cement shrinkage to a less severe number. Additionally, much of the shrinkage comes from the reduction of the column height (Nishikawa and Wojtanowicz, 2002). This would say that some of the shrinkage that happens in the cement sheath happens vertically or along the wellbore instead of all being perpendicular to the wellbore trajectory.

As pore pressure increases the risk of de-bonding and tensile failure increases, because the effective stress takes into account pore pressure as a reduction of the total stress of the material (Terzaghi, 1936). The case of the cement sheath as pore pressure increases the effective stress will decrease until ultimately the radial stress reaches tension and de-bonds or the hoop stress reaches the tensile strength of the cement.

A boundary condition where the cement stress is equal to the slurry hydrostatic pressure and effective stress is taken into account is the most reasonable boundary condition assumption, because a zero effective in-situ state of stress would result in persistent failure and a total stress assumption would result in an overly resilient wellbore. This finding was made by Gray et al. (2009), but others such as Bosma et al. (1999), Ravi et al. (2002), Nelson and Guillot (2006), and Thiercelin et al. (1998a) believed that the zero effective stress boundary condition was correct, but that results in a wellbore which would be unstable for most conditions. Fourmaintraux et al. (2005), Bois et al. (2011), and Bois et al. (2012), while stressing the importance of a poro-elastic model did include effective stress in the system response curves equations. This leaves out a very important factor because without effective stress the wellbore would be overly strong. A effective stress boundary condition is the middle ground between the two extremes and more accurate to what is observed in the field by Cooke et al. (1993) and Morgan (1989).
The higher the inner casing setting pressure the greater the risk of de-bonding during injection, but the lower the inner casing setting pressure the greater risk of cement fracturing during injection. Additionally, the higher the inner casing setting pressure resulted in a greater risk of de-bonding after injection stops, but a lower inner casing setting pressure resulted in a greater risk of cement fracturing after the injection stops. As well, the lower the cement hydrostatic pressure the greater the risk of de-bonding and cement fracturing. These results about the setting conditions of the cement sheath have not been previously investigated. The initial conditions of the cased wellbore is rarely discussed in its entirety in much of the literature, which leaves many open ended and general results which cannot be applied to an actual field case.

Concentric casings add another level of complexity to the wellbore system and are something that cannot be ignored or assumed to be resilient in an analysis. Commonly the concentric casings are assumed to be stable because of the large amount of cement and steel in that section, but when leakage is a concern concentric casings can become a leakage path to the surface due to the stress contrasts between the cement sheaths.

10.3. CASE STUDIES FINDINGS

The findings of the case studies are that for Case 1, the proposed injection well, that at the surface casing shoe the production cement sheath has is a low risk of cement fracturing as the cement hoop stress approaches zero, but the barriers protecting the fresh water aquifer, the surface casing and surface cement sheath, remains intact. At the sealing formation, the simulation shows that this barrier point will not be at risk for leakage. These simulations show that the proposed injection well would be sufficient in as a CO₂ sequestration injection well assuming no deterioration of the cement sheath or casing due to chemical effects. For Case 2, the existing well 100/02-01-046-01W5/00 at the surface casing shoe the simulations shows that in the case of leakage behind the casing the surface cement sheath will have tensile fracturing due to high tensile hoop stress, but there would be no risk of cement fracturing in the no leakage scenario because the cement hoop stress does not approach zero. At the sealing formation the simulation shows that this barrier point will not be at risk for leakage. These simulations together
show that if the production casing is not cemented to the surface casing shoe that the integrity of the production casing is of great concern of whether or not the wellbore will fail at the casing shoe or not. The production casing must be free of leaks and be corrosion resistant in order to be used for CO\textsubscript{2} sequestration. For Case 3, the existing well 100/09-10-047-01W5/00 at the surface casing shoe simulations show that the production cement sheath is at a low to no risk for fracturing the security of the barrier protecting the fresh water aquifer, the surface casing and surface cement sheath, remains intact. At the sealing formation there is a low risk of cement fracturing. Overall the 100/09-10-047-01W5/00 would be at low risk for leakage or loss of integrity.
11. CONCLUSIONS

11.1. SUMMARY

The objective of this study was to investigate the integrity and risk of leakage in Wabamun Area CO₂ Sequestration project site wells. This was done to evaluate if existing wells and proposed injection wells were suitable to be re-used for injection. The mechanical and thermal properties of 16 different cements were determined through experimental investigation. This cement property database was used to construct models for common wells in the Wabamun area. The simulations were conducted using staged finite element models as well as staged analytical models. The combinations of these models were used to verify the accuracy of the modeling procedures. The conditions of cement hydration and cement setting, as well as the thermal and mechanical loading induced by injection and production of different fluids through the subsurface were simulated using the finite element and analytical models. These conditions were simulated through parametric studies which included 22 simulations of the effects of variations in cement composition, injection pressure, injection temperature, shrinkage, and pore pressure. Also, 6 simulations were conducted to address the different theories on cement boundary and in-situ stress conditions. Additionally, 8 simulations were conducted to look at the different conditions surrounding the setting of the cement, including initial conditions and cement hydrostatic pressure. Finally, 1 simulation was conducted to discuss the effects of concentric casings. Three case studies were conducted in order to apply the modeling to real world wells in the WASP project site. The first case study was a proposed injection well in the Wabamun area and the second and third case studies were existing wells in the area. The points of greatest risk for leakage and loss of integrity were modeled with the expected CO₂ sequestration conditions, and recommendations were made on whether these wells would be appropriate candidates for CO₂ sequestration injection wells.
11.2. CONCLUSIONS AND RECOMMENDATIONS

The main conclusions and recommendations of this research are listed below.

- A 6-step staged finite element procedure and a matching staged analytical solution has been developed.
- Boundary conditions and in-situ stresses in the cement have the greatest effect on the simulation of the cased wellbore. An effective stress model has the best accuracy and should be used in future modeling of cased wellbores.
- Initial condition of the setting cement can have a significant effect on the long term integrity of the cased wellbore.
- Loading conditions of the life of the cased wellbore must be considered in the evaluation of the cement and initial condition selections.
- Shrinkage has a significant effect on the integrity of the cased wellbore and requires greater investigation in order to be properly simulated.
- The change in material properties of various cement compositions has less of an effect than the change of density and the associated cement hydrostatic pressure.
- The injection temperature and pressure has a significant effect on the integrity of the wellbore, with small changes to the prescribed CO₂ injection conditions failure could occur.
- The proposed injection well and the two existing wells could be used for CO₂ sequestration as long as the existing wells are structurally sound before repurposing.
APPENDIX A
EXPERIMENTAL DATA
## A. EXPERIMENTAL DATA

Table A.1: Linear Thermal Expansion Results

<table>
<thead>
<tr>
<th>Sample</th>
<th>Linear Thermal Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neat</td>
<td>9.4</td>
</tr>
<tr>
<td>1.1% Gel</td>
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<td>2% Gel</td>
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<td>8.135</td>
</tr>
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<tr>
<td>10% Gel</td>
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<tr>
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<tr>
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<tr>
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<td></td>
</tr>
<tr>
<td>10% CaCl₂</td>
<td></td>
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<tr>
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</tr>
<tr>
<td>2% Barite</td>
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</tr>
<tr>
<td>5% Barite</td>
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</tr>
<tr>
<td>8% Barite</td>
<td>7.46</td>
</tr>
<tr>
<td>10% Barite</td>
<td></td>
</tr>
<tr>
<td>5% Sand</td>
<td>5.83</td>
</tr>
<tr>
<td>Sample</td>
<td>tavg</td>
</tr>
<tr>
<td>-----------------</td>
<td>------</td>
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<tr>
<td>2% CaCl₂</td>
<td>99.57</td>
</tr>
<tr>
<td>2% Gel</td>
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<td>5% Sand</td>
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<tr>
<td>5% Sand</td>
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<td>99.18</td>
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<tr>
<td>10% CaCl₂</td>
<td>99.26</td>
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</table>
Table A.3: Specific Heat Capacity Results

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<tr>
<th>Sample</th>
<th>#</th>
<th>m</th>
<th>Tci</th>
<th>Twi</th>
<th>Twf</th>
<th>Q</th>
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<td>Neat</td>
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<td>28.52</td>
<td>89.0</td>
<td>20.9</td>
<td>24.1</td>
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<tr>
<td>Neat</td>
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<td>27.32</td>
<td>88.0</td>
<td>21.0</td>
<td>24</td>
<td>1883</td>
<td>1.08</td>
</tr>
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<td>Neat</td>
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<td>25.05</td>
<td>87.0</td>
<td>20.8</td>
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<td>1632</td>
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<tr>
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<td>4</td>
<td>43.62</td>
<td>88.0</td>
<td>20.0</td>
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<td>2887</td>
<td>1.04</td>
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*Recured means that the sample was not cured initially but was after the initial setting

**Cured means that the sample was cured initially after being set

***Precracked means the sample had pre-existing cracks
Table A.5: UCS & Young’s Modulus Data

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<td>52.35</td>
<td>1644</td>
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*R means recurred or that the sample was not cured initially but was after the initial setting

**C means cured or that the sample was cured initially after being set

***U means uncured or that the sample was never cured

****Cracked means the sample had pre-existing cracks
APPENDIX B
ANALYTICAL DERIVATION
B. ANALYTICAL DERIVATION

DERIVATION OF THE EQUATION OF EQUILIBRIUM IN CYLINDRICAL COORDINATES

The equation of equilibrium represents the force balance required to keep a body in static conditions. The derivation begins with the sketching of a cylindrical element and the labeling of all the normal and shear stresses felt by the cylindrical element.

![Stress Element in Cylindrical Coordinates](image)

Figure B.1: Stress Element in Cylindrical Coordinates

A summation of forces is performed in the radial direction, because the loading conditions of shrinkage, pore pressure, temperature, and pressure changes will all be axisymmetric so there is no need to sum forces in the hoop direction. The forces will be equated to zero to represent static conditions. Each of the stresses on the element must be multiplied by its effecting area in order to determine the equivalent forces, those forces must then be broken into their radial and hoop components with the radial component being used in the sum of forces.

\[ \sum F_r = 0 \]
\[ \sum F_r = \left( \sigma_{rr} + \frac{\partial \sigma_{rr}}{\partial r} \right) (r + dr) d\theta dz - \sigma_{rr} rd\theta dz + \left( \sigma_{\theta r} + \frac{\partial \sigma_{\theta r}}{\partial \theta} d\theta \right) rdz \cos \left( \frac{d\theta}{2} \right) \\
- \sigma_{\theta r} rdz \cos \left( \frac{d\theta}{2} \right) - \left( \sigma_{\theta \theta} + \frac{\partial \sigma_{\theta \theta}}{\partial \theta} d\theta \right) rdz \sin \left( \frac{d\theta}{2} \right) \\
- \sigma_{\theta \theta} rdz \sin \left( \frac{d\theta}{2} \right) = 0 \]

Simplifying \( \cos \left( \frac{d\theta}{2} \right) \approx 1 \) & \( \sin \left( \frac{d\theta}{2} \right) \approx \frac{d\theta}{2} \), where the cosine of a very small value is nearly one and the sine of a very small value is equal to that value. The previous equation can be written as;

\[ \sum F_r = \left( \frac{\partial \sigma_{rr}}{\partial r} dr \right) drd\theta dz + \sigma_{rr} rd\theta dz + \left( \frac{\partial \sigma_{rr}}{\partial r} \right) rd\theta dz + \left( \frac{\partial \sigma_{\theta r}}{\partial \theta} \right) rd\theta dz \]
\[ - \left( \frac{\partial \sigma_{\theta \theta}}{\partial \theta} d\theta \right) rdz \frac{d\theta}{2} - \sigma_{\theta \theta} rd\theta dz = 0 \]

Further simplification can be done by neglecting very small values using Perturbation Theory, in which terms that are multiplied by terms that are infinitesimally small, \( dr \approx 0 \) & \( d\theta \approx 0 \), are equal to 0.

\[ \sum F_r = \left( \frac{\partial \sigma_{rr}}{\partial r} dr \right) + \sigma_{rr} + \left( \frac{\partial \sigma_{rr}}{\partial r} \right) + \left( \frac{\partial \sigma_{\theta r}}{\partial \theta} \right) - \left( \frac{\partial \sigma_{\theta \theta}}{\partial \theta} d\theta \right) \frac{1}{2} - \sigma_{\theta \theta} = 0 \]

As stated above the problem is axisymmetric meaning there is no variation in the hoop directions. This makes the final Equation of Equilibrium what is found below in equation below.
Using Hooke’s Law for stress-strain relationships the following equations can be obtained. In these equations the effect of temperature is represented by $\alpha T$, which is the thermal strain which acts in all directions. The shrinkage effect is represented as $\Delta S$, which is shrinkage strain which acts similarly to the thermal strain. For simplicity pore pressure will be lumped into the stress until a further step (Timoshenko and Goodier, 1951 & Barron and Barron, 2011). The equation below describes the radial stress-strain relationship with factors included to account for temperature variations, shrinkage or expansion, and pore pressure changes (Timoshenko and Goodier 1951 and Bois et al. 2012).

\[
 \frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad a
\]

The equation below describes the hoop stress-strain relationship with factors included to account for temperature variations, shrinkage or expansion, and pore pressure changes can be described as (Timoshenko and Goodier 1951 and Bois et al. 2012);

\[
 \varepsilon_r - \alpha T - \Delta S = \frac{1}{E} (\sigma_r - \nu(\sigma_\theta + \sigma_z)) \quad b
\]

The equation below describes the axial stress-strain relationship with factors included to account for temperature variations, shrinkage or expansion, and pore pressure changes. (Timoshenko and Goodier 1951 and Bois et al. 2012).

\[
 \varepsilon_z - \alpha T - \Delta S = \frac{1}{E} (\sigma_z - \nu(\sigma_r + \sigma_\theta)) \quad c
\]

In order to find a solution for the relationships an assumption about the stress-strain relationship must be made. The two assumptions are either plane strain or plane stress.
The assumption of plane strain is that the cylinder will have zero axial strain no matter the loading, which represents a situation where the ends are fixed. The assumption of plane stress is that the cylinder will have zero axial strain no matter the loading, which represents a situation where the ends are free to move. For the case of a cased wellbore the assumption of plane strain was chosen because the ends of the wellbore are fixed at the bottom of the hole and at the wellhead or hanger. To satisfy this assumption, $\varepsilon_z = 0$ will be substituted into equation d;

$$\sigma_z = v(\sigma_r + \sigma_\theta) - \alpha ET - E\Delta S \ldots e$$

Substitute equation e the into radial and hoop strain equations b and c, respectively to apply the plane strain assumption to the radial and hoop equations. The equations below represent the derivation of the substitution to a simplified form for the radial conditions. The process would be repeated for the hoop conditions as well.

$$\varepsilon_r - \alpha T - \Delta S = \frac{1}{E} \left( \sigma_r - v(\sigma_\theta + \sigma_r + \sigma_\theta) - \alpha ET - E\Delta S \right)$$

$$\varepsilon_r - \alpha T - \Delta S = \left( \frac{\sigma_r - \frac{\nu^2 \sigma_r}{E} - \frac{v \sigma_\theta}{E} - \frac{\nu \sigma_\theta}{E} + \frac{\nu \alpha ET}{E} + \frac{\nu E\Delta S}{E} }{E} \right)$$

$$\varepsilon_r - (1 + \nu)(\alpha T + \Delta S) = \frac{1 - \nu^2}{E} \left( \sigma_r - \frac{\nu + \nu^2}{1 - \nu^2} \sigma_\theta \right)$$

$$\varepsilon_r - (1 + \nu)(\alpha T + \Delta S) = \frac{1 - \nu^2}{E} \left( \sigma_r - \frac{\nu}{1 - \nu} \sigma_\theta \right) \ldots f$$

$$\varepsilon_\theta - (1 + \nu)(\alpha T + \Delta S) = \frac{1 - \nu^2}{E} \left( \sigma_\theta - \frac{\nu}{1 - \nu} \sigma_r \right) \ldots g$$

Solve f and g for $\sigma_r$ and $\sigma_\theta$ including pore pressure (Pp). Pore pressure is brought into the equation here as part of effective stress using Terzaghi’s formulation for effective
stress in which effective stress equals the total stress minus the pore pressure times the Biot's coefficient, as can be seen below. (Bois 2012)

\[
\sigma'_r = \sigma_r - \beta Pp = \frac{(1 - v)E}{(1 - 2v)(1 + v)} \left[ \varepsilon_r + \frac{v}{(1 - v)} \varepsilon_\theta \right] - \frac{E(\alpha T + \Delta S)}{(1 - 2v)} = \ldots h
\]

\[
\sigma'_\theta = \sigma_\theta - \beta Pp = \frac{(1 - v)E}{(1 - 2v)(1 + v)} \left[ \varepsilon_\theta + \frac{v}{(1 - v)} \varepsilon_r \right] - \frac{E(\alpha T + \Delta S)}{(1 - 2v)} = \ldots i
\]

With equation for radial and hoop stress these equations can be substituted into the equation of equilibrium in order to solve for the static relationship in cylindrical coordinates. This is done by substituting \( h \) and \( i \) into \( a \).

\[
\frac{d}{dr} \left[ \frac{(1 - v)E}{(1 - 2v)(1 + v)} \left[ \varepsilon_r + \frac{v}{(1 - v)} \varepsilon_\theta \right] - \frac{E(\alpha T + \Delta S)}{(1 - 2v)} + \beta Pp \right] 
+ \frac{(1 - v)E}{(1 - 2v)(1 + v)} \left[ \varepsilon_r + \frac{v}{(1 - v)} \varepsilon_\theta \right] - \frac{E(\alpha T + \Delta S)}{(1 - 2v)} + \beta Pp 
= 0
\]

\[
\frac{d}{dr} \left[ \varepsilon_r + \frac{v}{(1 - v)} \varepsilon_\theta \right] - \frac{(1 + v)}{(1 - v)} \left( \alpha \frac{dT}{dr} + \frac{d\Delta S}{dr} \right) + \frac{(1 - 2v)(1 + v)}{(1 - v)E} \beta \frac{dPp}{dr} 
+ \left[ \varepsilon_r + \frac{v}{(1 - v)} \varepsilon_\theta \right] - \left[ \varepsilon_\theta + \frac{v}{(1 - v)} \varepsilon_r \right] 
= 0
\]

\[
\frac{d}{dr} \left[ \varepsilon_r + \frac{v}{(1 - v)} \varepsilon_\theta \right] - \frac{(1 + v)}{(1 - v)} \left( \alpha \frac{dT}{dr} + \frac{d\Delta S}{dr} \right) + \frac{(1 - 2v)(1 + v)}{(1 - v)E} \beta \frac{dPp}{dr} 
+ \frac{(1 - 2v)}{(1 - v)} \frac{\varepsilon_r - \varepsilon_\theta}{r} = 0 \ldots j
\]
A relationship for radial and hoop strain in terms of radius, \( r \), and displacement, \( u \), must be substituted into equation \( j \), the equation of equilibrium. This can be done using the following relationships

\[
\frac{d}{dr} \left[ \frac{du}{dr} + \frac{v}{1-v} \frac{u}{r} \right] - \left( \frac{1+v}{1-v} \right) \left( \frac{dT}{dr} + \frac{dS}{dr} \right) + \frac{(1-2v)(1+v)}{(1-v)E} \frac{dPp}{dr} + \frac{(1-2v)}{1-v} \frac{du}{dr} \frac{u}{r} = 0
\]

\[
\frac{d^2u}{dr^2} + \frac{(v)}{(1-v)} \frac{d}{dr} \left[ \frac{du}{dr} + \frac{1-2v}{1-v} \frac{u}{r} \right] - \frac{(1-2v)}{(1-v)} \frac{du}{dr} \left( \frac{1}{r^2} \right) - \frac{(1-2v)}{(1-v)} \frac{u}{r^2} = \left( \frac{1+v}{1-v} \right) \left( \frac{dT}{dr} + \frac{dS}{dr} \right) - \frac{(1-2v)(1+v)}{(1-v)E} \frac{dPp}{dr}
\]

\[
\frac{d^2u}{dr^2} + \frac{(v)}{(1-v)} \frac{d}{dr} \left[ \frac{du}{dr} + \frac{1-2v}{1-v} \frac{u}{r} \right] \left( \frac{1}{r} \right) - \frac{(1-2v)}{(1-v)} \frac{du}{dr} \left( \frac{1}{r^2} \right) = \left( \frac{1+v}{1-v} \right) \left( \frac{dT}{dr} + \frac{dS}{dr} \right) - \frac{(1-2v)(1+v)}{(1-v)E} \frac{dPp}{dr}
\]

\[
\frac{d^2}{dr^2} \frac{du}{dr} + \frac{1}{r} \frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} \left( ru \right) \right] = \frac{1}{r} \left( \frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} \left( ru \right) \right] \right) = \left( \frac{1+v}{1-v} \right) \left( \frac{dT}{dr} + \frac{dS}{dr} \right) - \frac{(1-2v)(1+v)}{(1-v)E} \frac{dPp}{dr}
\]

Separation of variables and integration with respect to \( r \)

\[
\int \frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} \left( ru \right) \right] \frac{dr}{r} = \int \left( \frac{1+v}{1-v} \right) \left( \frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} \left( ru \right) \right] \right) dr - \int \frac{(1-2v)(1+v)}{(1-v)E} \frac{dPp}{dr} \frac{dr}{r^2}
\]
\[
\frac{1}{r} \frac{d(ru)}{dr} = \left( \frac{1 + \nu}{1 - \nu} \right) (\alpha T + \Delta S) - \frac{(1 - 2\nu)(1 + \nu)}{(1 - \nu)E} \beta Pp + 2C_1
\]

\[
\frac{d(ru)}{dr} = \left( \frac{1 + \nu}{1 - \nu} \right) (\alpha Tr + \Delta Sr) - \frac{(1 - 2\nu)(1 + \nu)}{(1 - \nu)E} \beta Ppr + 2C_1 r
\]

Separation of variables and integration with respect to ru

\[
\int d(ru) = \int \left[ \left( \frac{1 + \nu}{1 - \nu} \right) (\alpha Tr + \Delta Sr) - \frac{(1 - 2\nu)(1 + \nu)}{(1 - \nu)E} \beta Ppr + 2C_1 r \right] dr
\]

\[
ru = \left( \frac{1 + \nu}{1 - \nu} \right) \left( \alpha \int_a^r Tr \, dr + \int_a^r \Delta Sr \, dr \right) - \frac{(1 - 2\nu)(1 + \nu)}{(1 - \nu)E} \beta \int_a^r Ppr \, dr + C_1 r^2 + C_2
\]

\[
u = \left( \frac{1 + \nu}{1 - \nu} \right) \frac{1}{r} \left( \alpha \int_a^r Tr \, dr + \int_a^r \Delta Sr \, dr \right) - \frac{(1 - 2\nu)(1 + \nu)}{(1 - \nu)E} \beta \frac{1}{r} \int_a^r Ppr \, dr + C_1 r + \frac{C_2}{r} \ldots k
\]

Solve for the radial and hoop strains using the equations \( \varepsilon_r = \frac{du}{dr}, \varepsilon_\theta = \frac{u}{r} \)

\[
\varepsilon_r = \frac{du}{dr} = \frac{1 + \nu}{1 - \nu} \left( \alpha T - \alpha \frac{1}{r^2} \int_a^r Tr \, dr + \Delta S - \frac{1}{r^2} \int_a^r \Delta Sr \, dr \right)
- \frac{(1 - 2\nu)(1 + \nu)}{(1 - \nu)E} \beta \left( Pp - \frac{1}{r^2} \int_a^r Ppr \, dr \right) + C_1 - \frac{C_2}{r^2} \ldots l
\]

\[
\varepsilon_\theta = \frac{u}{r} = \left( \frac{1 + \nu}{1 - \nu} \right) \frac{1}{r^2} \left( \alpha \int_a^r Tr \, dr + \int_a^r \Delta Sr \, dr \right) - \frac{(1 - 2\nu)(1 + \nu)}{(1 - \nu)E} \beta \frac{1}{r^2} \int_a^r Ppr \, dr
+ C_1 + \frac{C_2}{r^2} \ldots m
\]
Equations k, l, and m are the resulting solutions for displacement and the solutions for radial and hoop strain, respectively. Substitute l and m into h to solve for the radial stress equation

\[
\sigma_r = \frac{(1 - v)E}{(1 - 2v)(1 + v)} \left[ \varepsilon_r + \frac{v}{(1 - v)} \varepsilon_\theta \right] - \frac{aET}{(1 - 2v)} \cdots h
\]

\[
\sigma_r = -\frac{E}{1 - v} \frac{1}{r^2} \int_a^r \alpha Tr + \Delta Sr \, dr + \frac{1 - 2v}{1 - v} \frac{1}{r^2} \beta \int_a^r Ppr \, dr + \frac{E}{1 + v} \left( \frac{C_1}{1 - 2v} - \frac{C_2}{r^2} \right) \cdots n
\]

Substitute l and m into I to solve for the hoop stress equation

\[
\sigma_\theta = \frac{(1 - v)E}{(1 - 2v)(1 + v)} \left[ \varepsilon_\theta + \frac{v}{(1 - v)} \varepsilon_r \right] - \frac{aET}{(1 - 2v)} \cdots i
\]

\[
\sigma_\theta = \frac{E}{1 - v} \left( \frac{1}{r^2} \int_a^r \alpha Tr + \Delta Sr \, dr - \alpha T - \Delta S \right) - \frac{1 - 2v}{1 - v} \beta \left( \frac{1}{r^2} \int_a^r Ppr \, dr - Pp \right)
\]

\[+ \frac{E}{1 + v} \left( \frac{C_1}{1 - 2v} - \frac{C_2}{r^2} \right) \cdots o
\]

Substitute n and o into e to solve for the axial stress equation

\[
\sigma_z = v(\sigma_r + \sigma_\theta) - \alpha ET - E\Delta S + \beta Pp \cdots e
\]

\[
\sigma_z = -\frac{E(\alpha T + \Delta S)}{1 - v} + \frac{1 - 2v^2}{1 - v} \beta Pp + \frac{2vEC_1}{(1 + v)(1 - 2v)} \cdots p
\]

**SOLUTION TO THE CYLINDRICAL EQUATIONS**

Equations k, n, o, and p are the equations for a thick walled cylinder. To solve a single thick walled cylinder boundary conditions of the equations must be identified. The most common boundary conditions are a constant radial stress at the outside and inside, \( P_b \) and
$P_a$ at $b$ and $a$, respectively. With the boundary conditions identified as below there are two equations, $\sigma_{rx@a}$ and $\sigma_{rx@b}$, with two unknowns, $C_1$ and $C_2$, which can be solved using a system of equations (Jo, 2008).

In order to solve a composite cylinder such as a one with two thick walled cylinders, they must be linked together. They can be linked by the displacement, $k$, and the radial stress, $n$. These equations must be equal at the interface of both cylinders, $b$. Similar boundary conditions are then used as in the single thick walled cylinder in which the outer radial stress is zero.
stresses are identified as $P_c$ and $P_a$ at $c$ and $a$. With the identified boundary conditions below there are 4 equations with 4 unknowns, $C_{1x}$, $C_{2x}$, $C_{1y}$, and $C_{2y}$, which can be solved using a system of equations (Jo, 2008).

BC1

$$\sigma_{rx@a} = P_a$$

$$\sigma_{rx@a} = P_a = -\frac{E}{1-v}\frac{1}{a^2} \int_a^a \alpha Tr + \Delta Sr \ dr + \frac{1-2v}{1-v} \frac{1}{2} \int_a^a Pr \ dr$$

$$+ \frac{E}{1+v} \left( \frac{C_{1x}}{1-2v} - \frac{C_{2x}}{r^2} \right)$$

BC2

$$u_{x@b} = u_{y@b}$$

$$u_{x@b} = \left( \frac{1+v}{1-v} \right) \frac{1}{b} \left( \alpha \int_a^b Tr \ dr + \int_a^b \Delta Sr \ dr \right) - \frac{(1-2v)(1+v)}{(1-v)E} \beta \frac{1}{b} \int_a^b Pr \ dr + C_{1x} b$$

$$+ \frac{C_{2x}}{b}$$
In the case of a wellbore where there are two thick walled cylinders and an infinitely thick cylinder the boundary conditions must be set up to include two interactions between cylinders \( x \) and \( y \) and cylinders \( y \) and \( z \). In addition there must also be boundary conditions similar to a single thick walled cylinder where at \( a \) the stress is \( P_a \) and at \( \infty \) the stress is 0 (Jo 2008).

\[
\begin{align*}
\sigma_{rx@b} &= \sigma_{ry@b} \\
\sigma_{rx@b} &= -\frac{E}{1 - \nu} \frac{1}{b^2} \int_a^b \alpha Tr + \Delta Sr \ dr + \frac{1 - 2\nu}{1 - \nu} \frac{1}{r_{a-b}} \beta \int_a^b Ppr \ dr \\
&\quad + \frac{E}{1 + \nu} \left( \frac{C_{1x}}{1 - 2\nu} - \frac{C_{2x}}{b^2} \right) \\
\sigma_{ry@b} &= -\frac{E}{1 - \nu} \frac{1}{b^2} \int_b^b \alpha Tr + \Delta Sr \ dr + \frac{1 - 2\nu}{1 - \nu} \frac{1}{b^2} \beta \int_b^b Ppr \ dr \\
&\quad + \frac{E}{1 + \nu} \left( \frac{C_{1y}}{1 - 2\nu} - \frac{C_{2y}}{b^2} \right) \\
\sigma_{ry@c} &= P_c \\
\sigma_{ry@c} &= -\frac{E}{1 - \nu} \frac{1}{c^2} \int_b^c \alpha Tr + \Delta Sr \ dr + \frac{1 - 2\nu}{1 - \nu} \frac{1}{c^2} \beta \int_b^c Ppr \ dr \\
&\quad + \frac{E}{1 + \nu} \left( \frac{C_{1y}}{1 - 2\nu} - \frac{C_{2y}}{c^2} \right)
\end{align*}
\]
BC1

\[ \sigma_{rx@a} = P_a \]

\[ \sigma_{rx@a} = P_a = -\frac{E}{1 - \nu} \frac{1}{a^2} \int_a^a \alpha Tr + \Delta Sr \, dr + \frac{1 - 2\nu}{1 - \nu} \frac{1}{r^2} \beta \int_a^a Ppr \, dr \]

\[ + \frac{E}{1 + \nu} \left( \frac{C_{1x}}{1 - 2\nu} - \frac{C_{2x}}{r^2} \right) \]

BC2

\[ u_{xb} = u_{yb} \]

\[ u_{xb} = \left( \frac{1 + \nu}{1 - \nu} \right) \frac{1}{b} \left( \alpha \int_a^b Tr \, dr + \int_a^b \Delta Sr \, dr \right) - \frac{(1 - 2\nu)(1 + \nu)}{(1 - \nu)E} \beta \frac{1}{b} \int_a^b Ppr \, dr + C_{1x}b \]

\[ + \frac{C_{2x}}{b} \]

\[ u_{yb} = \left( \frac{1 + \nu}{1 - \nu} \right) \frac{1}{b} \left( \alpha \int_b^b Tr \, dr + \int_b^b \Delta Sr \, dr \right) - \frac{(1 - 2\nu)(1 + \nu)}{(1 - \nu)E} \beta \frac{1}{b} \int_b^b Ppr \, dr + C_{1y}b \]

\[ + \frac{C_{2y}}{b} \]

BC3

\[ \sigma_{rx@b} = \sigma_{ry@b} \]
\[
\begin{align*}
\sigma_{rx@b} &= -\frac{E}{1 - v} \frac{1}{\beta} \int_a^b \alpha Tr + \Delta Sr \, dr + \frac{1 - 2v}{1 - v} \frac{1}{b^2} \int_a^b Ppr \, dr \\
&\quad + \frac{E}{1 + v} \left( \frac{C_{1x}}{1 - 2v} - \frac{C_{2x}}{b^2} \right) \\
\sigma_{ry@b} &= -\frac{E}{1 - v} \frac{1}{\beta} \int_b^c \alpha Tr + \Delta Sr \, dr + \frac{1 - 2v}{1 - v} \frac{1}{b^2} \int_b^c Ppr \, dr \\
&\quad + \frac{E}{1 + v} \left( \frac{C_{1y}}{1 - 2v} - \frac{C_{2y}}{b^2} \right)
\end{align*}
\]

BC4

\[
u_{y@c} = u_{z@c}
\]

\[
u_{y@c} = \left( \frac{1 + v}{1 - v} \right) \frac{1}{c} \left( \alpha \int_b^c Tr \, dr + \int_b^c \Delta Sr \, dr \right) - \frac{(1 - 2v)(1 + v)}{(1 - v)E} \beta \frac{1}{c} \int_b^c Ppr \, dr + C_{1yc}
\]

\[
u_{z@c} = \left( \frac{1 + v}{1 - v} \right) \frac{1}{c} \left( \alpha \int_c^\alpha Tr \, dr + \int_c^\alpha \Delta Sr \, dr \right) - \frac{(1 - 2v)(1 + v)}{(1 - v)E} \beta \frac{1}{c} \int_c^\alpha Ppr \, dr + C_{1zc}
\]

BC5

\[
\begin{align*}
\sigma_{rx@c} &= -\frac{E}{1 - v} \frac{1}{c^2} \int_b^c \alpha Tr + \Delta Sr \, dr + \frac{1 - 2v}{1 - v} \frac{1}{c^2} \beta \int_b^c Ppr \, dr + \frac{E}{1 + v} \left( \frac{C_{1y}}{1 - 2v} - \frac{C_{2y}}{c^2} \right) \\
\sigma_{ry@c} &= -\frac{E}{1 - v} \frac{1}{c^2} \int_c^\alpha Tr + \Delta Sr \, dr + \frac{1 - 2v}{1 - v} \frac{1}{c^2} \beta \int_c^\alpha Ppr \, dr + \frac{E}{1 + v} \left( \frac{C_{1x}}{1 - 2v} - \frac{C_{2x}}{c^2} \right)
\end{align*}
\]

BC6
\[ \sigma_{rz@\infty} = 0 \]

\[ \sigma_{rz@\infty} = 0 = -\frac{E}{1-v} \frac{1}{\infty^2} \int_{r}^{\infty} \alpha Tr + \Delta Sr \, dr + \frac{1 - 2v}{1-v} \frac{1}{\infty^2} \beta \int_{r}^{\infty} Ppr \, dr \]

\[ + \frac{E}{1+v} \left( \frac{C_{1z}}{1-2v} - \frac{C_{2z}}{\infty^2} \right) \]

\[ C_{1z} = 0 \]

**INITIAL CONDITION MODELING**

Using the Kirsch Equations and the equations for thick walled cylinders under applied wellbore pressures \( (P_m) \) and pore pressures \( (P_{pz}) \) the initial conditions of a cased wellbore model can be developed. Starting from the formation conditions, assuming the wellbore trajectory is aligned with a principal stress. Assuming a vertical trajectory the maximum horizontal principal stress \( (\sigma_H) \), the minimum principal horizontal stress \( (\sigma_h) \), and the vertical principal stress \( (\sigma_{vz}) \) (Fjaer et al. 2008)

The equation below describes the relationship between far field stresses, initial pore pressure, wellbore pressure, and radial stress distribution. The radial stress varies around the wellbore and at different radii (Jaeger et al. 2007).
The relationship between far field stresses, initial pore pressure, wellbore pressure, and hoop stress distribution can be described as below. The hoop stress varies around the wellbore and at different radii (Jaeger et al. 2007).

\[
\sigma'_{rz} = \frac{1}{2} (\sigma_H + \sigma_h - 2Pp) \left[ 1 - \left( \frac{c}{r} \right)^2 \right] + \frac{1}{2} (\sigma_H - \sigma_h) \left[ 1 - 4 \left( \frac{c}{r} \right)^2 + 3 \left( \frac{c}{r} \right)^4 \right] \cos 2\theta
\]

\[
+ (Pm - Pp_x) \left( \frac{c}{r} \right)^2
\]

The equation below describes the relationship between far field stresses, initial pore pressure, wellbore pressure, and axial stress distribution. The axial stress varies around the wellbore and at different radii (Fjaer et al. 2008).

\[
\sigma'_{xz} = \frac{1}{2} (\sigma_H + \sigma_h - 2Pp) \left[ 1 + \left( \frac{c}{r} \right)^2 \right] - \frac{1}{2} (\sigma_H - \sigma_h) \left[ 1 + 3 \left( \frac{c}{r} \right)^4 \right] \cos 2\theta
\]

\[
- (Pm - Pp_x) \left( \frac{c}{r} \right)^2
\]

The equation below describes the relationship between far field stresses, initial pore pressure, wellbore pressure, and axial stress distribution. The axial stress varies around the wellbore and at different radii (Fjaer et al. 2008).

\[
\sigma'_{xz} = \sigma_{xz} - Pp - 2v(\sigma_H - \sigma_h) \left( \frac{c^2}{r^2} \right) \cos(2\theta)
\]

To solve for the casing component initial conditions the solution to the single thick walled cylinder with no change in Pore pressure, no change in shrinkage, and no change in temperature. The solved solution is present below. \( P_a \) is the inner casing pressure exerted by the wellbore fluids, \( P_b \) is the outer casing pressure which is typically the hydrostatic pressure exerted by the cement column, and in this case of casing, \( Pp_x \) would be zero since steel does not have pores. (Jaeger et al. 2007)
For the cement sheath the same procedure is applied as with the casing. $P_b$ and $P_c$ are the hydrostatic pressure exerted by the cement column, and in the cement pore pressure $Pp_y$ would be typically be equal to that of the formation pore pressure.

\[
\sigma'_{rx} = \frac{(b^2P_b - a^2P_a)}{(b^2 - a^2)} + \frac{a^2b^2(P_a - P_b)}{(b^2 - a^2)r^2} - Pp_x
\]

\[
\sigma'_{\theta x} = \frac{(b^2P_b - a^2P_a)}{(b^2 - a^2)} - \frac{a^2b^2(P_a - P_b)}{(b^2 - a^2)r^2} - Pp_x
\]

\[
\sigma'_{zx} = \sigma_{vx} - Pp_x
\]

\[
\sigma'_{ry} = \frac{(c^2P_c - b^2P_b)}{(c^2 - b^2)} + \frac{b^2c^2(P_b - P_c)}{(c^2 - b^2)r^2} - Pp_y
\]

\[
\sigma'_{\theta y} = \frac{(c^2P_c - b^2P_b)}{(c^2 - b^2)} - \frac{b^2c^2(P_b - P_c)}{(c^2 - b^2)r^2} - Pp_y
\]

\[
\sigma'_{zy} = \sigma_{vy} - Pp_y
\]
LOADING CONDITION MODELING

The derived general thick walled will be used for each of the cylinders of casing, cement, and rock. Cylinder x will represent the casing. The temperature, shrinkage, and pore pressure distributions applied to the casing are input as functions of radius if the applied loading is uniform then the values remain constant and will not vary with radius. The radius in this section is limited to the range between a and b. The constants of $C_{1x}$ and $C_{2x}$ are solved for using the 2 thick walled cylinder with 3rd infinite walled cylinder solution above.

\[
\sigma_{rx} = -\frac{E}{1-v}\left(\frac{1}{r^2}\int_{a}^{r} \alpha Tr + \Delta Sr \, dr + \frac{1-2v}{1-v} \frac{1}{r^2} \beta \int_{a}^{r} Ppr \, dr + \frac{E}{1+v} \left(\frac{C_{1x}}{1-2v} - \frac{C_{2x}}{r^2}\right)\right)
\]

\[
\sigma_{\theta x} = \frac{E}{1-v} \left(\frac{1}{r^2}\int_{a}^{r} \alpha Tr + \Delta Sr \, dr - \alpha T - \Delta S\right) - \frac{1-2v}{1-v} \beta \left(\frac{1}{r^2}\int_{a}^{r} Ppr \, dr - Pp\right)
\]
\[
+ \frac{E}{1+v} \left(\frac{C_{1x}}{1-2v} + \frac{C_{2x}}{r^2}\right)
\]

\[
\sigma_{zx} = -\frac{E(\alpha T + \Delta S)}{1-v} + \frac{1-2v^2}{1-v} \beta Pp + \frac{2vEC_{1x}}{(1+v)(1-2v)}
\]

Cylinder y will represent the cement. The temperature, shrinkage, and pore pressure distributions applied to the cement are input as functions of radius if the applied loading is uniform then the values remain constant and will not vary with radius. The radius in this section is limited to the range between b and c. The constants of $C_{1y}$ and $C_{2y}$ are
solved for using the 2 thick walled cylinder with 3\textsuperscript{rd} infinite walled cylinder solution above.

\[
\sigma_{ry} = -\frac{E}{1 - v} \frac{1}{r^2} \int_b^r \alpha T r + \Delta Sr \, dr + \frac{1 - 2v}{1 - v} \frac{1}{r^2} \beta \int_b^r Ppr \, dr + \frac{E}{1 + v} \left(\frac{C_{1y}}{1 - 2v} - \frac{C_{2y}}{r^2}\right)
\]

\[
\sigma_{\theta y} = \frac{E}{1 - v} \left(\frac{1}{r^2} \int_b^r \alpha T r + \Delta Sr \, dr - \alpha T - \Delta S\right) - \frac{1 - 2v}{1 - v} \beta \left(\frac{1}{r^2} \int_b^r Ppr \, dr - Pp\right)
\]

\[
+ \frac{E}{1 + v} \left(\frac{C_{1y}}{1 - 2v} + \frac{C_{2y}}{r^2}\right)
\]

\[
\sigma_{zy} = -\frac{E(\alpha T + \Delta S)}{1 - v} + \frac{1 - 2v^2}{1 - v} \beta Pp + \frac{2vEC_{1y}}{(1 + v)(1 - 2v)}
\]

Cylinder z will represent the formation. The temperature, shrinkage, and pore pressure distributions applied to the formation are input as functions of radius if the applied loading is uniform then the values remain constant and will not vary with radius. The radius in this section is limited to the range between \(c\) and \(\infty\). The constants of \(C_{1z}\) and \(C_{2z}\) are solved for using the 2 thick walled cylinder with 3\textsuperscript{rd} infinite walled cylinder solution above.
The distribution of temperature with radius and time is described by the equation below. Transient temperature distributions are used to avoid a steady state temperature distribution being applied to an infinite rock mass (Thiercelin 1998).

\[
\sigma_{rz} = -\frac{E}{1-v} \frac{1}{r^2} \int_{0}^{r} \alpha T r + \Delta S r \, dr + \frac{1-2v}{1-v} \frac{1}{r^2} \beta \int_{0}^{r} P r r \, dr + \frac{E}{1+v} \left( \frac{C_{1z}}{1-2v} - \frac{C_{2z}}{r^2} \right)
\]

\[
\sigma_{\theta z} = \frac{E}{1-v} \left( \frac{1}{r^2} \int_{0}^{r} \alpha T r + \Delta S r \, dr - \alpha T - \Delta S \right) - \frac{1-2v}{1-v} \beta \left( \frac{1}{r^2} \int_{0}^{r} P r r \, dr - P p \right)
+ \frac{E}{1+v} \left( \frac{C_{1z}}{1-2v} + \frac{C_{2z}}{r^2} \right)
\]

\[
\sigma_{zz} = -\frac{E(\alpha T + \Delta S)}{1-v} + \frac{1-2v^2}{1-v} \beta PP + \frac{2vEC_{1z}}{(1+v)(1-2v)}
\]

The distribution of temperature with radius and time is described by the equation below. Transient temperature distributions are used to avoid a steady state temperature distribution being applied to an infinite rock mass (Thiercelin 1998).

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{\rho C \partial T}{K \partial t}
\]

The distributions for pore pressure and shrinkage will be treated as uniform changes that do not vary with radial position.

\[
\Delta S = \text{constant}, \, Pp = \text{constant}
\]
APPENDIX C
EXPERIMENTAL PROCEDURES
C. EXPERIMENTAL PROCEDURES
Sample Molds

The cement samples must be cured in a mold that makes the correct dimensions for the variety of tests to be performed. The UCS test requires that the sample have the dimensions of the Length divided by Diameter equal to 2 or L/D=2. In this case a diameter of 2” and a length of 4” will be chosen as to have a sample that is not too small but also fits well into the testing equipment.

Materials
2” diameter steel pipe
Hose clamps that have a range containing the OD of the pipe and less than 2”
Caulk or window sealant
Sheet metal pieces (at least 3” by 3”

1. Take 2” ID steel pipe, steel so that it can withstand higher temperature ranges, cut into 5 ½” lengths.
   a. Extra length in order to cut a UCS and a tensile test specimen, and to cut imperfections out of the ends of the sample
2. Using a water jet or a band saw with steel cutting teeth cut a slit along the length of the steel pipe on one side of the pipe, clean the steel mold
   a. This will be used to pry open the mold to release the sample once set
3. Apply a small bead of caulk or window sealant along the slit on the outside of the pipe, smooth caulk along length of slit
4. Slide the hose clamp around the outside, center of the pipe and tighten to close the slit, spread caulk on inside so that there is none intruding into the mold
5. Place the pipe on top of the sheet metal piece and apply a bead of sealant along the bottom of the pipe in contact with the sheet metal, spread bead to prevent any leaks.

6. Allow mold to sit for 24hrs in open air or 4hrs in an oven at 180°F.
Mixing Cement

Materials

Mass balance or scale
Automated stirrer or hand stirrer
Mixing Container
Class H Cement (S.G. 3.15)
Bentonite Gel (S.G. 2.65)
CaCl$_2$ solution (S.G. 1.033)
Barite (S.G. 4.20)
Sand (S.G. 2.65)
Water (S.G. 1.00)
Cement mold
Plastic Wrap

1. Determine the mix of cement and additives required (for example: 2% Bentonite cement, 5% CaCl$_2$ cement, etc.)
2. Select the mass of cement to be in the sample slurry
3. Select the percentage of each additive, the percentage will be the % of mass of cement to be added to the mix
4. Determine the water requirements
   a. Cement powder requires 40% by mass of cement
   b. Bentonite requires 11.4% by mass of cement for every 2% of bentonite
   c. CaCl$_2$ has no water requirement
   d. Barite has negligible water requirements for lab testing
   e. Sand has no water requirement
5. Set up design chart
<table>
<thead>
<tr>
<th>Component</th>
<th>% Composition</th>
<th>Mass of Component</th>
<th>Water Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>-</td>
<td>$m_c$</td>
<td>$40% \times m_c$</td>
</tr>
<tr>
<td>Bentonite Gel</td>
<td>%BG</td>
<td>%BG $\times m_c$</td>
<td>$11.4%/2 \times %BG \times m_c$</td>
</tr>
<tr>
<td>CaCl$_2$</td>
<td>%C</td>
<td>%C $\times m_c$</td>
<td>-</td>
</tr>
<tr>
<td>Barite</td>
<td>%B</td>
<td>%B $\times m_c$</td>
<td>-</td>
</tr>
<tr>
<td>Sand</td>
<td>%S</td>
<td>%S $\times m_c$</td>
<td>-</td>
</tr>
</tbody>
</table>

6. Measure out all components before beginning to mix
7. Add half of the water to the mixing container
8. Add the cement and additives to the container
9. Add the second half of the water to the container
10. Mix well using an automated stirrer or by hand
   a. Make sure all cement and additives are hydrated
11. Vibrate the slurry to remove all air bubbles
12. Pour slurry into the cement mold and cover with plastic wrap
Linear Thermal Expansion Coefficient

Materials
¾in steel frame w/ ¾in diameter opening and ring clamp
Temperature probe
LVDT with Data Acquisition box
Laptop with Labview and Omega 2.06 programs
Glass beaker filled with water
Small rubber pads
Hot plate
Wood block
Cement sample
Caliper

1. Measure the height of the sample using a caliper
2. Turn on hot plate to approximately 150 C
3. Set three small rubber pads evenly spaced at the bottom of the beaker
   a. These will be used to keep the sample level when the bottom of the beaker is not
4. Place the cement sample in the beaker and then fill with hot water from tap to reduce time on hot plate
   a. Adjust the cement sample and rubber pads until sample is level
   b. Let the tap run over the sample for a few minutes to help heat the sample
5. Heat the sample and water in a beaker on the hot plate until a constant temperature of 50 C to 80 C is reached or for about 1 hour to heat the sample
6. Set up the apparatus by placing the LVDT into the clamp, checking the temperature probe and LVDT to see if they are calibrated
7. Remove beaker from the hot plate and remove water from above the top of the cement sample
   a. The water level should be as close to the top of the sample as possible without going over
8. Place the beaker with the sample under the LVDT using a wood block to raise the height of the beaker to reach the LVDT
   a. The LVDT should read near 5mm, the middle of the LVDT’s range, for the greatest accuracy
   b. Wood is used due to its low thermal conductivity and low thermal expansion coefficient so it will not expand with increased temperature
9. Start the programs, Labview program ‘LVDT’ and Thermocouple program ‘Omega 2.06’
   a. Labview ‘LVDT’
      i. Ctrl+E to open file schematic
      ii. Double click the file writer
      iii. Select the location and name to save the output file
      iv. On dashboard, select recording increment to match ‘Omega 2.06’
      v. Click the arrow on the upper left to start
   b. ‘Omega 2.06’
      i. Click ‘Device’ drop down menu
      ii. Select ‘Start Device’
      iii. Select recording increment to match that of ‘LVDT’
      iv. Select ‘Start’ to begin
10. Continuously measure and record height and temperature data using the LabView program and the Omega software, respectively, or record them by hand
    a. The LVDT measures in millimeters to the 4th decimal and the probe measure in Celsius to the 2nd decimal. As the sample and water cool the sample will shrink proportionally to its thermal expansion coefficient, and as this occurs the LVDT measures this change in length
11. Allow the test to run for at least 3 hour
12. Stop the programs and export the csv file from ‘Omega 2.06’
   a. Labview ‘LVDT’
      i. Click ‘Stop’ button on dashboard
   b. ‘Omega 2.06’
      i. Click ‘Device’ drop down menu
ii. Select ‘Stop Device’ to stop recording

13. Using a plot of temperature versus displacement and calibration using 316 Stainless Steel a linear regression can be used to determine the thermal expansion coefficient.

a. \[ \alpha = \frac{\Delta L - C \Delta T}{L \Delta T} \], where C is the correction factor
Uniaxial Compression Test

Materials
L/D=2 dimensioned sample
Load sensor
Hydraulic Piston
Isco Pump
LVDT
Large hose clamps to attach LVDT to load frame
Bent steel plate to attach to load sensor
Computer with
Labview program “Load Log with LVDT”
2 serial ports

1. Test specimens should be cylinders having a height to diameter ration of 2.
2. The ends of the specimen should be grinded flat
3. The sides of the sample should be smoothed
4. Measure the diameter at the top, middle, and bottom of the sample and average the diameters
5. Measure the height of the sample in three locations around the sample and average the heights
6. Place the cement sample under the load sensor and lower the piston until the sample is tight in place
   a. Make sure not to load the sample over 50lbs
7. Attached the LVDT to the load frame and put in contact with the load sensor
8. Start the Labview program “Load Log with LVDT” ensuring that the LVDT and load measurements are being made
9. Begin loading the sample at a rate so that failure occurs within 5-10 minutes, 5MPa/min-10MPa/min
10. Once failure occurs stop the experiment
11. Go into the data and determine the UCS and the Young’s modulus of the sample
Brazilian Indirect Tensile Test

Materials
D/t=2 dimensioned sample
Load frame
Load sensor
Hydraulic Piston
Isco Pump
Steel Brazilian test sample holder
Computer with
   Labview program “Load Log”
   1 serial ports

1. Measure the Length 3 times and the Diameter 3 times to find the average sample dimensions
2. Place the sample in the sample holder, using the steel blocks raise or lower the sample and holder until it fits below the piston and the load sensor
3. Turn on the pump and adjust the piston height to allow sample holder to fit if necessary.
4. Using the Gradient Program option, set the load rate to 50psi/min and the initial psi to 50psi
5. Zero out the load sensor by running through all the menu options until reset is reached or hold the reset button
6. Lower the piston until the load sensor reads approximately 50lbs
   a. This will be significant enough load to prevent overloading due to the program’s calculation of flow rate when the sample and the piston are not in contact
7. Start the Labview program “Load Log”
8. Begin loading the sample to 50psi using the Gradient Program then stop the pumps
   a. This prevents rapid loading of sample
9. Restart the pumps and allow the test to run until the sample fails
10. Once failure occurs stop the pumps and stop the Labview program
11. Within the Labview output identify the maximum load applied to the sample
12. Use the following equation to calculate the tensile strength of the sample
   
   \[ T_o = \frac{2P}{\pi D t} \], where \( P \) is the maximum load, \( D \) is the diameter, and \( t \) is the thickness
Thermal Conductivity Test

Materials
Divided Bar

2x (Aluminum glued to acrylic glued to aluminum)

Silicon grease
Sample 2” Diameter by approximately 1” thickness
2x right angle aluminum approximately 6” long

Hot Plate

Beaker

Water
Long adjustable clamp (8” length required)

Temperature probe

Wood blocks

Thermocouples

Laptop with “Omega 2.06”

Tape (Relatively weak strength)

Fiberglass pipe insulation

1. Fill the beaker with water and place on the hot plate at 200 C
2. Allow water to heat to near 80 C and to remain constant
3. Place the angled aluminum in the beaker
4. Attach 4 thermocouples to the 4 pieces of aluminum
5. Spread the silicon grease on both sides of the sample
6. Place the sample between the two halves of the divided bar
7. Wrap the complete divided bar in fiberglass insulation
8. Place the divided bar and insulation on the wood blocks adjusting height until the divided bar is above the top of the beaker on the hot plate
9. Clamp the two angled aluminum to the divided bar
   a. On one end the angled aluminum will be in the water resting on the bottom of the beaker
b. On the other end the angled aluminum will be used to balance the divided bar by resting on a wood block or the table

10. Begin recording thermocouple data

11. Monitor the water bath so that the water level does not drop too low and that the temperature of the water remains constant

12. Let the experiment run for at least 4 hrs

13. Once the experiment is finished export the data for the thermocouple

14. Calculate the thermal conductivity at each time interval
   a. Calculate \( q = \frac{k \Delta T}{x} \), across acrylic samples. Average \( q \) and solve for thermal conductivity of test sample using \( k = \frac{q x}{\Delta T} \)

15. Plot thermal conductivity vs. time and curve fit using MATLAB, cftool, using a power equation in the format, \( a x^b + c \), where \( c \) will be the thermal conductivity of the material at steady state
BIBLIOGRAPHY


CRC (1984), 100/09-10-047-01W5/00, Operator: Gulf Canada Resources LTD, Contractor: Nabors Drilling LTD.

CRC (1987), 100/02-01-046-01W5/00, Operator: Gulf Canada Resources LTD, Contractor: Nabors Drilling LTD.


VITA

Benjamin Lee Weideman received his Bachelors of Science in Petroleum Engineering from Missouri University of Science and Technology in May 2012. During his time as an undergraduate, Benjamin interned with Continental Resources and Chevron. Benjamin also conducted undergraduate research beginning in August 2009. In August of 2012 Benjamin enrolled in the Petroleum Engineering Master’s degree at the same university. During his time as a Master’s student he held positions of graduate research assistant and graduate teaching assistant in the Department of Geological Science and Engineering. Benjamin completed his Masters of Science in Petroleum Engineering in May 2014.