Initial operation and calibration of the UMR supersonic axisymmetric wind tunnel

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INITIAL OPERATION AND CALIBRATION
OF THE UMR SUPERSONIC
AXISYMMETRIC WIND TUNNEL

BY

JAMES RILEY MURPHY, 1945-

A

THESIS
submitted to the faculty of
UNIVERSITY OF MISSOURI - ROLLA
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Approved by

[Signatures]

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ABSTRACT

Initial testing and a preliminary calibration were conducted in the UMR supersonic variable Mach number axisymmetric blowdown wind tunnel facility.

During initial operations problems were encountered with control valve seat failure, control valve response and stability, and automatic operation. After having brittle fracture failures with seats made of nylon, Teflon and Telfon-copper composite, a copper seat was found to perform satisfactorily with minimal valve leakage. Control valve response and stability were greatly influenced by the setting of the needle valve located between the total pressure probe and the controller. The needle valve setting was observed to depend upon the stagnation pressure, the supply pressure, and the rate at which the valve was stroked. The result of each of the various methods of automatic operation tried was a pressure overshoot, bursting bypass diaphragms. Through manual operation excellent repeatability of stagnation pressure was obtained.

The tunnel calibration was performed at a stagnation pressure of 180 psig. By means of schlieren flow visualization the free jet was found to be completely expanded at this pressure. From cone-shock angle measurements the test section Mach number was approximately 2.8. A more discreetly defined nozzle contour and improved machining and polishing techniques are needed to rid the flow field of Mach waves emanating from the nozzle. An estimate of the run time by an empirical quasi-steady isentropic analysis was found to be in good agreement with the experimentally determined value.
ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation to Bruce P. Selberg, Assistant Professor of Aerospace Engineering, for his guidance and assistance in this endeavor.

Thanks are also due to: Mr. Lee Anderson, Machine Shop Supervisor, for his assistance in designing and fabricating much of the equipment used, Dr. Richard T. Johnson, Assistant Professor of Mechanical Engineering, for his efforts in solving the problems inherent in the control valve operation, Mr. Ken Mirly, Laboratory Manager, for his help in designing and operating the completed UMR supersonic wind tunnel, and last, but certainly not least, my wife, Lin, for typing the final manuscript and for her encouragement during this project.
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I. INTRODUCTION

The University of Missouri - Rolla Supersonic Axisymmetric Wind Tunnel was designed and constructed with funds provided under Title II of the Department of Health, Education and Welfare and the State of Missouri. Design of the facility was begun in the fall of 1968 by Professors Bruce P. Selberg and Ronald H. Howell. Construction was completed in the summer of 1969. The facility is housed in the Gas Dynamics Laboratory of the Department of Mechanical and Aerospace Engineering. The first successful supersonic operation occurred on June 13, 1970.

The UMR supersonic wind tunnel is intended for research in the field of high speed gas dynamics at the senior and graduate level. Experimental studies in jet mixing, shock formation, and aerodynamics can be conducted provided knowledge of the operating characteristics is at hand.

This thesis presents the findings and results of the initial operation and preliminary calibration.
II. UMR SUPERSONIC AXISYMMETRIC WIND TUNNEL

A. General Description

The University of Missouri - Rolla Supersonic Axisymmetric Wind Tunnel is of the intermittent blowdown type producing a supersonic axisymmetric enclosed free jet. High pressure air (2000 psig) is supplied to five storage tanks, 268 cubic feet capacity, by a three-stage compressor-dryer system as shown in Fig. 1. The high pressure air is then regulated to a desired stagnation pressure, \(0 \leq P_0 \leq 400\) psig, by means of a control valve-settling chamber stagnation pressure feedback loop. Stagnation pressure is sensed by a total pressure probe; stagnation temperature by a fast response platinum resistance thermometer.

Test section Mach number is achieved by means of a two-piece converging-diverging nozzle, providing easy interchangeability of nozzles for a desired test section Mach number. The present nozzle produces a Mach 2.8, five-inch diameter axisymmetric jet.

Model visibility and schlieren optical photography are accomplished through two-inch thick, ten-inch diameter, plate glass windows of fair optical quality as shown in Fig. 2. Flowfield stagnation pressure and static pressure, enabling the calculation of test section velocity and Mach number, are measured by means of total and static pressure probes respectively. Pressure output is displayed by connecting the pressure taps to a 30-tube back-lighted mercury manometer bank through bulkhead fittings in an air tight instrumentation port. Total pressure differential is 60 inches of mercury; for other pressures a known reference pressure
may be input into the mercury reservoir wells.

Models and pressure probes are held in position during a test by means of a model support, the position of which may be varied during a run. The support consists of an aluminum block, 60° wedge front, with one 1/4 inch and one 3/8 inch hole for sting insertion. The model support position may be varied 5.0 inches axially, 2.3 inches vertically, and ± 20° angle-of-attack. Support (model) movement is currently done manually. In the future positioning will be done automatically.

Aft of the test section sufficient space is provided for the insertion of a diffuser, enabling some pressure recovery. Finally, atop the building a fiberglass-lined muffler provides relatively noise-free operation outside the facility.

Tunnel operation conditions are controlled and observed at a control panel as shown in Fig. 3. Currently displayed are: control pressure to control valve, control valve operation pressures, supply pressure, stagnation pressure, stagnation temperature, test section window pressure, manometer reference pressure, and test duration time. In addition, panel mounted switches control the main power, manometer lights, schlieren light source (xenon), manometer reference pressure inlet and exhaust, and emergency control valve shut down. Above the control panel an 11 x 14 piece of ground plate glass serves as a viewing screen for the schlieren image.
B. Method of Operation and Instrumentation

1. High Pressure Air Supply

High pressure air is supplied to the storage tanks by means of a three-stage compressor-dryer system. The first stage compressor consists of a Davey Model 150-PDA Industrial Permavane Rotary Air Compressor requiring a Marathon Electric 150 hp motor to drive it. This stage takes atmospheric air and compresses it to approximately 110 psig. The second and third stage consist of Gardner-Denver Model LAO and LAQ Compressors respectively, each requiring a General Electric 40 hp motor to drive it. The second stage compressor takes the 110 psig air and compresses it to approximately 450 psig; the third stage takes the 450 psig air and compresses it to 2000 psig. The air then passes through a Lectrodryer Model BAC-40-AS-SP Dehumidifier which dries the air to -100 °F dew point relative to atmospheric pressure.

The high pressure air is delivered to five storage tanks outside the building at a rate of 280 scfm from the compressors. Total tank capacity is 268 cubic feet. Safety pressure switches provide some redundancy in shutting off the second and third stage compressors in the event the tank pressure should reach 2000 psig or 2050 psig.

Pumping time versus tank pressure is presented in Fig. 4. Since most runs will be conducted at a tank pressure near 2000 psig, for maximum run time, the minimum time between runs is limited by the approximate 2 1/2 hours of pumping time.
Pressure is maintained in the storage tanks merely by the control valve. However, in the event the control valve should leak some amount or be desired to be removed for inspection, etc., a manually operated valve lies between the storage tanks and control valve (Fig. 1), allowing no loss in tank pressure in such circumstances.

The leakage of the present valve seat is such that the maximum attainable supply pressure is 1250 psig. However, by closing the manual valve the maximum attainable pressure becomes 1970 psig. A discussion of the valve seat leakage, as well as the seat materials considered, will be given later.
2. Control Valve

The control valve's purpose is to take the changing (decreasing) upstream pressure and deliver it to the stagnation (settling) chamber at a constant stagnation pressure. The UMR supersonic wind tunnel utilizes a Fisher Governor Type HSC-667 four-inch Guided Plug Throttling Valve, a Fisher Governor Type 667 reverse acting Diaphragm Actuator, a Fisher Governor Type 4160 Wizard II Pressure Controller, a Fisher Governor Series 3560 Valve Positioner, and two Fisher Governor Type 67FR Supply Pressure Regulators.

As shown in Fig. 5 supply air from the Davey compressor is used to pneumatically operate the control valve. Since the diaphragm actuator is a normally closed reverse acting mechanism (air pressure lifting the valve and spring force closing the valve) the control valve can be shut down simply by cutting off the supply air from the Davey. However, the primary purpose of the supply pressure cut-off is in preventing operation of the system unless it is in the "on" position. Since the output from the Davey is not a constant value and relatively high, pressure regulators regulate the output to fixed quantities for use in the controller and valve positioner.

Two adjustments within the controller can drastically affect the response and stability of the feedback loop: the Proportional Band adjustment and the Reset adjustment, as pictured in Fig. 6. The proportional band is defined as the change in controlled pressure required to stroke the control valve. The ratio of the proportional band to the bourdon tube rating, which measures the pressure to be controlled, is termed the percent proportional band. For good response it would thus be desirable to have a minimum change in
controlled pressure stroke the valve. However, at very low settings instabilities arise and some trade-off must occur for an optimum setting. Percent proportional band may be varied from 0 to 100%.

The second adjustment which may be made is analogous to fine tuning some mechanism. The reset adjustment is a precision made needle valve that is calibrated from 0.005 to 1 minute per repeat. This is the time in minutes required for the reset action to produce a correction which is equal in magnitude to the correction produced by proportional control action. Obviously it would be desirable to have the valve respond in the shortest possible time, but here again instabilities arise and some optimum setting must be obtained experimentally.

Another adjustment in using the controller lies in the needle valve position. The needle valve lies between the pressure sensor, the total pressure probe, and the controller and serves to dampen out any pulsations in the controlled pressure. Care must be exercised in setting this valve in that when closing this valve to eliminate pulsations it is possible to restrict the pressure to the controller and thereby indicate and deliver to the controller a pressure lagging and considerably less than the actual controlled pressure. The true pressure would eventually reach the controller but only after wasting run time and often creating a dynamic control problem. This setting can be easily checked by first closing the valve until all pulsations appear to be removed and simultaneously observing the lag between the stagnation pressure gauge and the controller pressure gauge. To set this valve without operating the wind tunnel the total pressure probe can be disconnected and the line connected to a known reference pressure, e.g. regulated compressed air. Then by observing
the gauge readings to changes in the known pressure the valve can be set. Once set this valve should cause no further problems.

The valve positioner, as shown in Fig. 7, lying between the controller and the diaphragm actuator serves to more accurately position and provide additional force in positioning the valve. Originally installed in the valve positioner was a linear cam. This cam varied the valve position linearly for a given input signal. For example, for 30% of the instrument pressure span the valve stem traveled 30% of the complete stroke. At the beginning of a given run the pressure difference across the control valve is a maximum and a short valve stem travel can lead to large changes in downstream pressure. Thus in order to make the system less sensitive at this high supply pressure - short valve stem travel operation, a non-linear cam was installed in the valve positioner. Now for 30% of the instrument pressure span the valve stem travels only 16% of the complete stroke, and so on.

In order to quickly shut down the control valve for changes in model position, operating conditions, etc., or as an emergency shut down, a solenoid switch has been installed in the control valve as shown in Fig. 7. When energized the solenoid switch vents the pressure on the diaphragm actuator to the atmosphere allowing the spring to close the valve. When the valve is full open a shut down time of two seconds is obtained.
3. Stagnation (Settling) Chamber

Before the air passes through the converging-diverging nozzle it assumes near stagnation conditions in the settling chamber. The settling chamber consists of 3/8 inch steel pipe, 15 1/4 inches in diameter, 52 inches long. Six stainless steel wire mesh screens serve to straighten the flow and reduce the turbulence level. As shown in Fig. 8 the total pressure probe and temperature probe are installed in a spacer after the first four screens. Here the flow is essentially uniform and two screens remain to negate any disturbances caused by the probes.

The total pressure probe consists of a 16 gauge hypodermic needle bent 90° facing upstream. The probe extends into the settling chamber 2 5/8 inches, thereby away from the wall boundary layer. Stem effects have been made negligible by locating the stem a distance downstream six times the external diameter of the tube. The fitting holding the total pressure probe into the Swagelock compression fitting in the tunnel wall is keyed such that the probe may be easily removed, inspected, and returned to its original position with no change in yaw.

The temperature sensor consists of a Rosemount Engineering Co. Model 104MN Fast Response Platinum Resistance Thermometer and Model 414L Linear Bridge. The linear bridge converts the resistance of the platinum resistance thermometer to a millivolt output signal. The slope of the output is one millivolt per degree Fahrenheit within the non-linearity tolerance provided (maximum tolerance 0.42°F.) Thus, when the linear bridge output is input to a
millivolt reading device, e.g., a recording potentiometer, the device will read temperature in degrees Fahrenheit with no conversion necessary.
4. Nozzle

The contour of the nozzle currently in use was obtained from CLIPPINGER (1951). He solved the method of characteristics for axisymmetric flow numerically, neglecting the effects of viscosity. His results are presented in tabular form for various exit Mach numbers and nozzle lengths. The contour selected for the UMR supersonic wind tunnel corresponds to the shortest nozzle contour computed, having a sharp edge in the sonic plane, for an exit Mach number of 3.041 (parallel flow.)

The nozzle has been machined from aluminum, with the final finish obtained using grit no. 600 water sandpaper. The nozzle throat has a diameter of 2.384 inches and the exit a diameter of 5.000 inches for an area ratio of 4.41. The (diverging) nozzle is 10.292 inches long.

The converging portion of the nozzle lies solely in the stagnation (settling) chamber. The diverging portion then screws into the converging section. By sliding the test section downstream the diverging section of the nozzle may be easily removed with a spanner wrench.

The test section coordinate system is referenced from the center of the nozzle exit plane. Defining X as the axial direction, Y as the vertical direction, and Z as the horizontal direction the center of the nozzle exit plane has the coordinates (0,0,0). The positive axial direction is then downstream, the positive vertical direction is up, and the positive horizontal direction is right as given by a
conventional right-hand coordinate system looking downstream.

Work on a nominal Mach 4.0 nozzle is currently in progress.
5. Test Section

The five-inch diameter jet of air from the axisymmetric nozzle is completely enclosed (enclosed free jet) by the test section, as shown in Fig. 9. The test section consists of 3/8 inch steel pipe, 15 1/4 inches in diameter, 49 inches long. The effective test section is limited by the ten-inch diameter viewing windows. However, if model visibility is not desired the effective test section can be extended downstream into the area currently reserved for the diffuser.

The viewing windows currently in use are two-inch thick plate glass of fair optical quality. Small bubbles, scratches, and ream are present but for purposes of calibration and preliminary testing these windows are sufficiently adequate. Available for future use are viewing windows optically ground and polished flat to within 1/4-wavelength of light.

Downstream of the viewing windows a ten-inch diameter instrumentation port as shown in Fig. 9 permits internal pressure taps to be externally connected to a mercury u-tube manometer. The port cover consists of a two-inch thick, 13 1/2 inch diameter aluminum plate with 30 holes drilled though. The interior surface is tapped for 1/8 inch Poly-Flo tubing connectors. The exterior surface is tapped for 1/4 inch Poly-Flo tubing connectors. Taps not being used can be sealed merely by interconnecting the interior Poly-Flo fittings with 1/8 inch tubing.

Test section stagnation pressure is obtained by measuring the stagnation pressure behind a normal shock. Then by finding the
free stream Mach number at this location, by stagnation-static pressure readings (to be shown later) and/or shock angle off a cone, the free stream stagnation pressure can be obtained from

\[
\frac{p_{oy}}{p_{ox}} = \left[ \frac{\frac{k+1}{2} M_x^2}{k-1} \right]^{\frac{k}{k-1}} \left[ \frac{2k}{k+1} M_x^2 - \frac{k-1}{k+1} \right]^{\frac{1}{1-k}}
\]

where

- \( p_{oy} \) = stagnation pressure behind normal shock
- \( p_{ox} \) = free stream stagnation pressure
- \( M_x \) = free stream Mach number
- \( k \) = ratio of specific heats, 1.4

With this data at hand and the settling chamber stagnation pressure, the nozzle efficiency can be found.

The stagnation pressure behind a normal shock is measured by a stagnation pressure probe as shown in Fig. 10. The probe consists of a 16 gauge hypodermic needle, 0.065 inch outside diameter, 1 3/4 inches long, which has been squared at the end. The probe is then fastened in the end of an 8° half-angle, half-inch base diameter, cone which has been drilled through. For connection to 1/8 inch tubing, 0.042 inch inside diameter, a length of 19 gauge hypodermic tubing, 0.049 outside diameter, is fastened in the base of the cone. In addition to the snug fit of the hypodermic tubing in the 1/8 inch tubing, epoxy strengthens and seals the juncture. In order to be held firmly in position by the model support, the cone is attached to a four-inch piece of 3/8 inch thick wall stainless steel tubing.
Test section static pressure is measured by a static pressure probe as shown in Fig. 10. From PETERSON and PANKHORST and HOLDER (1965) for conical tipped static pressure probes, with orifices placed between 12 and 18 body diameters downstream of the nose section, the measured static pressure is very close to the true free stream static pressure. The probe currently in use consists of a 16 gauge hypodermic needle with a 6° half-angle conical tip. The distance from the nose section to the diametrically opposed orifices has been placed as 15 body diameters, approximately one inch. The probe is then fastened in the end of an 8° half-angle, 3/8 inch base diameter, cone a distance downstream of 1 3/4 inches. The cone is drilled through and again a length of 19 gauge hypodermic tubing is fastened in the base of the cone for connection to 1/8 inch tubing, epoxy being applied at the juncture. The cone is then attached to a 4 1/2 inch piece of 1/4 inch thick wall stainless steel tubing.

In order to determine Mach number from stagnation and static pressure readings the location of the orifices of the static pressure probe, not the tip, must coincide with the tip of the stagnation pressure probe. Thus the pressure probes may be simultaneously mounted in the model support. Having taken readings for a given position the model support must then be displaced vertically one-inch, the distance between probe centerlines, in order to have stagnation and static pressures at one location.

The mercury u-tube manometer has a total pressure differential of 60 inches of mercury. For other pressures a reference pressure
may be input into the mercury reservoir wells. For example, when measuring high stagnation pressures supply air from the Davey compressor is input, by means of a valve, into the reservoir wells. By placing a gauge in the line between the valve and the reservoir wells the reference pressure is always known. So as not to "blow" the mercury during rapid valve closing, a solenoid switch has been placed between the valve and the gauge for quick exhaust of the reference pressure.
6. Control Panel

Wind tunnel operating conditions are controlled and observed at the control panel shown in Fig. 3. The "stagnation pressure" gauge is the stagnation pressure in the settling chamber as sensed by the total pressure probe. The "controller pressure" gauge was installed as a result of the previously mentioned problem of control valve stability and response. It is the stagnation pressure, less pulsations removed by the needle valve, actually delivered to the controller. The controller is shown in detail in Fig. 6. The "test duration timer" serves as an indication of the run time remaining in a given run. The estimated total run time must be supplied as an input. The "supply pressure" is the storage tank supply pressure as measured upstream of the valve. The "control pressure" is the output from the first stage Davey compressor. The "control valve" serves as a supply pressure ("control pressure") cut-off. Since the system is pneumatically operated this valve regulates supply air from the Davey and thus presently serves as a master control.

The "test section pressure" gauge was originally intended to indicate the pressure on the test section windows. When it was found that the window pressure was negligible this gauge was then used to indicate the manometer reference pressure. Switches activate the "stagnation temperature" measuring apparatus, the schlieren light source ("xenon power"), the "manometer," and the "emergency control valve shut down." The "regeneration light" indicates that the dryers are regenerating. The valve below the "main power" was
used in attempting to operate the system automatically. In the future this space will be occupied by a key switch which will serve as the main control.
III. MODEL SUPPORT

Models and instrumentation probes are held in position during testing by a model support as shown in Fig. 11. The model support is housed in a flanged container, 10 inches in diameter, 17 inches long, aligned with the viewing window.

The model (support) positioning mechanism consists of two horizontal axial screws, 40 threads per inch, which must be turned simultaneously for axial travel; and two vertical screws, 40 threads per inch which are reduced (5:1) through a worm-gear arrangement to two horizontal axial slotted shafts. Currently the axial positioning of the model support is performed by simultaneously turning cranks affixed to the horizontal screws. Vertical positioning is performed by simultaneously turning, or alternately turning, being careful not to exceed the maximum positive or negative angle-of-attack, cranks affixed to the slotted shafts. The horizontal screws and horizontal shafts rotate in solid aluminum bushings. The vertical screws rotate in ball bearings. Each rotation of the horizontal screws results in a model support axial travel of 0.025 inches. Each rotation of the horizontal shafts (which in turn rotate the vertical screws) results in a vertical travel of 0.005 inches.

With the nozzle-wind tunnel centerline as a reference, the maximum vertical travel is + 0.925 inches and -1.375 inches. Maximum axial travel is 5.00 inches. By rotating the two vertical screws in opposite directions a maximum angle-of-attack of $\pm 20^\circ$ can be
achieved. Axial travel can be supplemented by adjusting the model or probe sting location.

Currently in work is a modification to the model support which would allow automatic operation. Controls and revolution counters as readout will be panel mounted. Motors mounted exterior to the model support container will enable axial or vertical positioning of the model at a rate of 6.8 seconds per inch.

That portion of the model support which extends into the free jet was machined from aluminum and has a 60° wedge front. Two holes, one 3/8 inch and one 1/4 inch, centerline spaced one inch apart, allow stings of either of these diameters to be inserted into the model support. Forces sufficient to hold the stings firmly in place are provided by seven set screws which hold the two pieces of the upper portion of the support together.
IV. SCHLIEREN SYSTEM

For flow visualization the UMR supersonic wind tunnel employs a J. Unertl double parabolic mirror schlieren system as shown in Fig. 12. Flow visualization is extremely valuable in providing information about the flow without disturbing it. In addition to providing qualitative information about the flow, a properly sensitized flow visualization system can provide quantitative checks such as free stream Mach number and location and state of the boundary layer.

The basic principle of flow visualization is that a variation in the density in the flow implies a variation in the index of refraction. The variation can be detected with any of three different optical methods: the shadowgraph, the schlieren system, and the interferometer. The shadowgraph indicates the second derivative of the density and is particularly useful for showing rapid changes in the density, e.g., shock waves and turbulence. Any data obtained by the shadowgraph must be interpreted qualitatively only. The schlieren system indicates the first derivative (gradient) of the density and is at least an order of magnitude more sensitive than the shadowgraph. It is useful in showing near sonic shocks, expansion fans and weak shock waves (Mach lines.) The image obtained by the schlieren system can be described as a relief map of the density gradient illuminated from the side. The interferometer indicates the density directly and may thus be quantitatively interpreted and evaluated to provide a density distribution. The image obtained by an interferometer can be
described as a contour map of the density concentration. Of the last two methods the schlieren system is the least expensive in providing useful data which may be interpreted qualitatively and quantitatively.

The double-mirror schlieren system shown in Fig. 12 consists of a PEK-75 xenon light source, a front silvered plane mirror, two front silvered parabolic mirrors (parabolic within 1/10 wavelength of light), a 90° prism, a knife edge, a 50-50 beam splitter, an 11 x 14 inch ground plate glass, and a Graflex camera. The light source, mirrors, prism, and knife edge were custom designed by the J. Unertl Optical Company. The beam splitter was installed so that the image (with approximately one-half the illumination) could be photographed apart from the image which is displayed above the control panel.

Within the light source a condenser lens serves to uniformly illuminate a portion of the light given off by the xenon lamp. This functional light then passes through an opening which may be varied from a point to a slit and becomes the functional light source. A plane mirror then redirects this light to the first parabolic mirror.

Since the test section windows are ten inches in diameter, the diameter of the parabolic mirrors was chosen to be ten inches. Selecting an f-number, ratio of focal length to diameter, of ten then results in a focal length of 100 inches. Thus 100 inches was maintained as the distance from the light source to the first parabolic mirror and from the second parabolic mirror to the knife edge. Like the plane mirror, the prism serves to redirect the beam for space limitation reasons.
The presence of the knife edge in the system is an example of the Toepler Method of detecting the displacement of the image of the light source. In the Toepler Method the displacement of the image of the source corresponding to the deflection of the light passing through a particular point in the field results in a change of illumination of the image of this point on the screen. The knife edge is placed at the focal point of the second parabolic mirror and is adjusted so that, in the absence of any optical disturbances, part of the light from the image of the source is cut off in a way that the illumination on the screen is uniformly reduced. Then when optical disturbances are introduced part of the image of the source is displaced and the illumination of the corresponding part of the image on the screen will decrease or increase depending on whether the deflection is towards or away from the knife edge. Displacement of the image of the source parallel to the knife edge produces no effect at the screen. Therefore, the knife edge must be set perpendicular to the direction in which the density gradients are to be observed. For shock wave visualization the knife edge must thus be set in a direction roughly parallel to the shock front. For boundary layer observation it must be set parallel to the surface of the body.
V. RESULTS

A. Operation

1. Control Valve Seating

Whenever it is desired to seal and control large pressure differentials across a valve an area of great interest lies in the valve seat. During initial operations the UMR supersonic wind tunnel experienced such problems.

The nylon seat originally installed in the Fisher Governor control valve was apparently not intended for use in a system of large pressure differential, accompanied by high velocities and low temperatures. As shown in Fig. 13 the original nylon seat developed cracks and fractured at the point of seating. The resulting leakage was appreciable and a new nylon seat was inserted. Again failure occurred and the resulting leakage was the same.

Assuming that extreme cold temperatures at the valve seat caused the nylon to become brittle, a softer Teflon seat was used. Teflon, however, has the property of "flowing" under pressure and, in fact, did just that. The Teflon seat also fractured to some degree as shown in Fig. 13 and leakage was still appreciable.

In an attempt to retain the soft seating qualities of Teflon and provide additional strength, a copper seat was machined so as to retain the Teflon and thus prevent any flowing. Each component of this composite seat was one-half the thickness of the original seat. Though the copper prevented most of the cracking and fracture
at the point of seating, the Teflon still flowed and leakage remained a problem.

The valve seat currently in use is solid copper. The required degree of flatness was obtained by hand lapping with lapping compound of grit no. 600. Though leakage was greatly reduced it became necessary to close the manual valve between the storage tanks and the control valve to decrease the necessary pumping time and increase the maximum supply pressure.

As previously mentioned and shown (Fig. 4) with the manual valve closed a pumping time of 2 1/2 hours is needed to obtain 2000 psig supply air. Pumping time and tank supply pressure data were taken with the manual valve open and the solid copper seat installed. The leak rate and maximum supply pressure are presented in Fig. 4 for comparison.
2. Controller Sensitivity

When operating the control valve it was found that controller sensitivity was an important factor for accurate, responsive control. The already mentioned non-linear cam installed in the valve positioner helped make the system somewhat less sensitive, particularly for high supply pressures.

A consequence of the uncontrolled sensitivity remaining in the controller is shown in Fig. 14. The bypass burst diaphragms are located in a bypass line downstream of the control valve, in parallel with the stagnation (settling) chamber. The bypass diaphragms have a burst pressure of approximately 400 psig at room temperature. It was at first believed that too rapid an opening of the control valve caused a large pressure rise and a normal shock to propagate into the bypass line, thereby bursting the diaphragm. This might well have been what happened. However, diaphragms were burst even for relatively slow openings of the valve.

Upon disconnecting the total pressure probe and connecting the line to regulated compressed air, it was found that the stagnation pressure gauge responded very slowly to rapid changes in the reference pressure. This indicated that the needle valve was closed too much offering too small an opening for the pressure to pass through to fill the volume, even though all pulsations in controlled pressure were removed when operating the tunnel (cautiously). Hence, it appeared that some trade-off must occur for operation.

Resuming operations it was found that the needle valve position which gave responsive readings to changes in the reference pressure
was not sufficient to dampen controlled pressure pulsations. To remedy the situation a controller pressure gauge (Fig. 5) was installed between the needle valve and the controller. By varying the needle valve position a setting was reached where the controller pressure lagged the stagnation pressure by approximately 10 psi when opening the valve. (When varying the position of the needle valve a rotation of only 1/8 of a complete turn was found to change the stability.) At this setting all pulsations appear to be removed for supply pressures below 1200 psig. To operate the system at higher pressures the procedure would have to be repeated. For this reason calibration was conducted at a supply pressure of 1400 psig. This allowed sufficient pressure (and time) to raise the valve and begin stable operation at or near 1200 psig.

It was also found that by replacing the 3/32 inch orifice solenoid between the valve positioner and the diaphragm actuator with a solenoid switch having a 5/8 inch orifice, operation of the control valve was made more stable and responsive. In so doing, the valve closure time, by venting the pressure on the diaphragm actuator, was reduced from ten seconds to two seconds.
3. Control Valve Automatic Operation

An attempt was made to make the valve operate automatically in order to save run time and assure repeatability of operating conditions. By setting the pressure setting within the controller the valve was operated by means of the solenoid switch, between the valve positioner and the diaphragm actuator. The result was a too rapid excursion of the valve when opening and a pressure overshoot, often bursting bypass diaphragms.

Another attempt at automatic operation was made by using the control air from the Davey compressor to start operation. Again a pressure overshoot occurred.

While other methods of adapting the control valve for automatic operation are being investigated, a device which provided repeatability of operating conditions was placed on the pressure setting dial as shown in Fig. 6. The tunnel was then operated by slowly moving the circular knob clockwise, in turn slowly raising the valve, until the stop was reached. As will be seen later, repeatability of stagnation pressure was excellent.
B. Calibration

As previously mentioned, calibration was conducted at a storage tank supply pressure of approximately 1400 psig. This then allowed sufficient pressure (and time) to raise the valve and begin stable operation at approximately 1200 psig. Defining the run time as the time from stable operation, constant stagnation pressure, \( (p_0 = 180 \text{ psig}) \) to the time at which the stagnation pressure begins to fall, \( (p_0 < 180) \) an experimental value of 46 seconds was obtained.

This value can be compared with theoretical values obtained assuming the flow process to be 1) isothermal and 2) isentropic. As shown in Appendix B the isothermal solution gives a run time of 68 seconds, the isentropic solution gives a run time of 54 seconds.

In the Isothermal Analysis the stagnation temperature, and consequently the mass flow rate, is assumed to remain constant. In reality the stagnation temperature was observed to decrease from 80°F (540°R) to -22°F (438°R.) As expected, the isothermal analysis resulted in a longer run time. In order to maintain constant stagnation temperature, heat would have to be added to the process. The additional thermal energy thus increases the run time. However, the isothermal analysis does provide a rough approximation of the actual run time.

A more rigorous solution is given by the Isentropic Analysis. Here the expansion from the supply pressure storage tanks is assumed to be an adiabatic process. Frictional losses from the storage tanks to the stagnation chamber are neglected (i.e. reversible).
From the quasi-steady solution the theoretical run time compares favorably with the actual run time. A refinement to this solution would be to include frictional losses and solve the set of equations numerically for smaller time increments, e.g., one second. In assuming the mass flow rate over a given time interval to be constant and equal to the initial mass flow rate the run time is expectedly shorter.

Another approach in determining the theoretical run time is empirical in nature. When determining the run time experimentally the observed pressure upstream of the valve, the tank pressure, at which the stagnation pressure began to decrease was found to be approximately 400 psig. This is substantiated by the control valve operation, which requires a pressure drop of approximately 200 psig for proper operation. Then if in the Isothermal Analysis we define the final mass remaining in the tank as that corresponding to a tank pressure of 400 psig the run time becomes 53 seconds. In a similar manner, if in the Isentropic Analysis we find the time for the tank pressure to reach 400 psig the solution becomes 41 seconds.

These results are summarized in Appendix B, Table I.

During the calibration the settling chamber stagnation pressure was controlled at a constant nominal value of 180 psig. This value was selected by varying the stagnation pressure until a completely expanded, parallel jet was observed on the schlieren viewing screen. Repeatability of operating conditions, namely stagnation pressure,
was provided by the stop on the pressure setting dial previously mentioned.

The Mach number distribution in the free jet was determined by flow visualization through schlieren photography. By measuring the shock angle off a cone in supersonic flow the Mach number can be found. Thus a cone-cylinder was positioned at nine different axial and vertical locations. The cone-cylinder consists of a 3/8 inch diameter steel rod with an 8° half-angle cone taper.

The points at which the Mach number was determined, as well as the stagnation pressure, are given in Appendix C, Table II. Note the excellent repeatability of the stagnation pressure. Schlieren photographs at each position are presented in Fig. 15.

The axial stations \(x = 2.0, x = 5.0, x = 7.5\) were selected as being representative of possible model locations. The symmetry of the free jet is established at the rearward position. At each of the other locations equidistant points give the Mach number distribution within the current model support position limitations.

In all the schlieren photographs Mach lines emanating from slight imperfections in the nozzle are clearly visible. These have little effect on the flow. However, improvements in final polishing and possibly additional points in the contour would minimize the number of Mach lines. In addition the boundaries of the free jet are clearly visible in each photograph. This free boundary layer growth could possibly affect testing downstream of the viewing windows.
An interesting observation can be made from Fig. 151. The reflection of the shock wave from the free boundary layer is shown. Thus a compression wave is seen to reflect from a constant pressure boundary in unlike sense, i.e., as an expansion wave.

The structure of the flowfield is given in more detail in Fig. 16, an enlarged detail of Fig. 15b. In a manner similar to this each schlieren was enlarged and the shock wave angle was measured. The Mach number for the given cone angle and shock angle was then obtained from NACA 1135. These results are given in Appendix C, Table II. The greatest deviation in Mach number occurs in the $x = 5.0$ plane. This deviation is due in part to a slight overexpansion of the free jet.

A more accurate method of determining the Mach number is through total and static pressure measurements. When placed in a supersonic stream a total pressure probe measures the stagnation pressure behind a normal shock. A static pressure probe measures the free stream static pressures. As shown in Appendix C the Mach number can then be obtained from

$$\frac{p_{oy}}{p_x} = \left[ \frac{k+1}{2} \frac{M^2}{x} \right]^{\frac{k}{k-1}} \left[ \frac{2k}{k+1} \frac{M^2}{x} - \frac{k-1}{k+1} \right]^{\frac{1}{1-k}}$$

where

- $p_{oy}$ = stagnation pressure behind normal shock
- $p_x$ = free stream static pressure
- $M_x$ = free stream Mach number
The point selected for the check calibration was \( x = 7.5, y = -0.925 \).

High pressure 1/8 inch plastic tubing was connected to each probe and taped along the test section wall up to the point of connection to the instrumentation port. However, due to the expansion of the free boundary layer, as well as the low temperatures of expansion, along the test section wall the tubing was not able to survive the test. Failure of the tubing occurred both at the probes and at the instrumentation port. In the future some provision must be made for running pressure leads to the instrumentation port.

However, a schlieren photograph was obtained during the test and is presented in Fig. 17. Note that the normal shock standing in front of the stagnation pressure probe is visible.

The previously mentioned variation in stagnation temperature for a given run is given in Fig. 18. The effects of control valve leakage (after opening the manual valve between the supply pressure storage tanks and the control valve) and manual operation of the control valve in attaining the desired stagnation pressure are clearly shown. After reaching an equilibrium condition data was taken and the control valve was immediately closed by energizing the solenoid switch.
VI. CONCLUSIONS

1. Control valve seating was found to be a problem. A copper seat was found to be a temporary solution in minimizing leakage but necessitated pumping with the manual valve between the supply pressure storage tanks and the control valve being closed.

2. Approximately 2 1/2 hours of pumping time is required to obtain 2000 psig supply air.

3. Controller sensitivity was found to be greatly affected by the setting of the needle valve lying between the total pressure probe and the controller. For a stagnation pressure setting of 180 psig the control valve operation was stable only for supply pressures below 1200 psig. It thus appears that the needle valve setting is dependent on the supply pressure as well as the stagnation pressure. It was also found that the controller sensitivity varied from run to run. The actual supply pressure at which a constant stagnation pressure was reached seemed to vary with the rate at which the control valve was manually operated.

4. By venting the pressure on the diaphragm actuator a control valve shutdown time of two seconds was obtained.

5. Automatic operation of the UMR supersonic wind tunnel may not be possible. Various methods were tried, the result of each being an initial pressure overshoot (bursting bypass diaphragm). Manual operation resulted in excellent repeatability of stagnation pressure. However, through automatic operation, stagnation temper-
nature repeatability and run time can be improved and made more predictable.

6. For a stagnation pressure of 180 psig and an initial supply pressure of 1200 psig a run time (t) of 46 seconds was obtained. Favorable comparison was obtained from an empirical quasi-steady isentropic analysis (t = 41 seconds).

7. The free jet was found to be completely expanded at a stagnation pressure of approximately 180 psig.

8. The test section Mach number was found to be approximately 2.8. Mach waves emanating from the diverging portion of the nozzle can be eliminated by the fabrication of a nozzle with a much more discreetly defined contour and improved final polishing techniques.

9. An attempt was made to verify the schlieren flow visualization results by means of stagnation and static pressure probes. This is not feasible until some provision is made for running pressure leads to the instrumentation port. An inherent advantage in the installation of a diffuser downstream of the test section would be in protecting the pressure leads by swallowing the free jet before it has expanded to the test section walls.
VII. APPENDICES
APPENDIX A.

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APPENDIX B.

Theoretical Run Time Determination
B. Theoretical Run Time Determination

1. Isothermal Analysis

The mass flow rate is governed by the critical conditions at the nozzle throat, that is

\[ \dot{m} = \rho_c A_c a_c \]  

(eqn. 1)

Setting as our initial stagnation conditions

\[ p_o = 180 \text{ psig} = 195 \text{ psia} \]
\[ T_o = 80^\circ F = 540^\circ R \]

The stagnation pressure is held constant by the control valve. The stagnation temperature is assumed to remain constant under the assumption of an isothermal process occurring between the storage tanks and the stagnation (settling) chamber.

At the throat

\[ \rho_c = 0.634 \rho_o \]
\[ = 0.634 \frac{p_o}{RT_o} \]
\[ = 0.634 \left( \frac{195 \cdot 144}{53.35 \cdot 540 \cdot 32.2} \right) \]
\[ = 0.0192 \text{ slugs/ft}^3 \]

* ideal gas assumed \( (p = \rho RT) \)

\[ A_c = \frac{\pi d^2}{4} \]
\[ = \frac{(2.384)^2}{4 \cdot 144} \]
\[ = 0.0311 \text{ ft}^2 \]

\[ a_c = \sqrt{\frac{k}{\rho_c}} \]
\[ p_c = 0.528 \rho_o \]
\[ = 0.528 \ (195) \]
\[ = 103 \text{ lb/in}^2 \]

\[ a_c = \sqrt{\frac{1.4 \ (103) \ (144)}{0.0192}} \]
\[ = 1040 \text{ ft/sec} \]

Substituting back into eqn. 1, we find for the mass flow rate

\[ \dot{m} = 0.0192 \ (0.0311) \ (1040) \]
\[ = 0.621 \text{ slugs/sec} \]

The five storage tanks have a volume of 268 ft\(^3\), then the mass in the tanks is

\[ m = \rho_T V_T \]

where \[ V_T = 268 \text{ ft}^3 \]

and \[ \rho_T = \frac{p_T}{RT_T} \]

Setting as our initial storage tank conditions

\[ p_T = 1200 \text{ psig} = 1215 \text{ psia} \]
\[ T_T = 80^\circ \text{F} = 540^\circ \text{R} \]

then

\[ \rho_T = \frac{1215 \ (144)}{53.35 \ (540) \ (32.2)} \]
\[ = 0.1885 \text{ slugs/ft}^3 \]

The initial mass is then

\[ m_I = 0.1885 \ (268) \]
\[ = 50.5 \text{ slugs} \]
The stagnation pressure in the settling chamber will begin to decrease when the tank supply pressure reaches 180 psig (195 psia.) This then gives us our final mass in the tanks

\[
m_f = \frac{195 \times 144 \times 268}{53.35 \times 540 \times 32.2}
\]

\[
= 8.1 \text{ slugs}
\]

The total mass to flow through the system is then

\[
50.5 - 8.1 = 42.4 \text{ slugs}
\]

Therefore the run time is

\[
t = \frac{m}{\dot{m}}
\]

\[
= \frac{42.4}{0.621}
\]

\[
= 68 \text{ sec}
\]
2. Isentropic Analysis (Quasi-Steady)

The mass flow rate is governed by the critical conditions at the nozzle throat

\[
\dot{m} = \rho_c A_c s_c
\]

Since

\[
\rho_c = 0.634 \rho_0 = 0.634 \frac{p_0}{RT_0}
\]

\[
A_c = \frac{\pi d^2}{4}
\]

\[
s_c = \sqrt{\frac{k}{\rho_c}} \left(\frac{p_c}{\rho_c}\right)
\]

\[
p_c = 0.528 p_0
\]

\[
s_c = \sqrt{\frac{k (0.528 \rho_0)}{0.634 \rho_0/RT_0}}
\]

the mass flow rate can be expressed in terms of the stagnation pressure and temperature

\[
\dot{m} = 0.634 \frac{p_0}{RT_0} \frac{\pi d^2}{4} \sqrt{\frac{k (0.528 \rho_0)}{0.634 \rho_0/RT_0}}
\]

(eqn. 3)

In this expression

\[
p_0 = \text{constant stagnation pressure}
\]

\[
T_0 = T_0(t) = \text{time dependent stagnation temperature}
\]

The flow through the control valve can be approximated by a throttling process, i.e.,

\[
h = \text{constant}
\]

and

\[
h = h(T)
\]
the temperature downstream of the valve is the same as the temperature upstream of the valve. Neglecting friction between the supply pressure storage tanks and the stagnation (settling) chamber, this means

\[ T_o = T_T \]  
\[ (eqn. 4) \]

Assuming that the expansion from the supply pressure tanks occurs as an isentropic process of an ideal gas with constant specific heats we can then relate the pressures and temperatures of some time increment by

\[ \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} \]  
\[ (eqn. 5) \]

Combining eqns. 4 and 5 we find that for our analysis

\[ \frac{T_{02}}{T_{01}} = \frac{T_{T2}}{T_{T1}} = \left( \frac{p_{T2}}{p_{T1}} \right)^{\frac{k-1}{k}} \]  
\[ (eqn. 6) \]

Using the ideal gas equation of state

\[ pV = mRT \]  
\[ (eqn. 7) \]

in eqn. 6 the solution for the stagnation temperature is

\[ T_{02} = T_{T2} = T_{T1} \left( \frac{mRT_{T2}}{Vp_{T1}} \right)^{\frac{k-1}{k}} \]

which becomes

\[ T_{02} = T_{T1} \left( \frac{mR}{Vp_{T1}} \right)^{\frac{k-1}{k}} \]  
\[ (eqn. 8) \]

From eqn. 8 a step-by-step (quasi-steady) solution for the stagnation temperature can be obtained provided initial conditions and the mass in the storage tanks are known.
With the initial storage tank conditions

\[ P_T = 1200 \text{ psig} = 1215 \text{ psia} \]
\[ T_T = 80^\circ F = 540^\circ R \]

(whence \( T_o = 80^\circ F = 540^\circ R \))

and the constant stagnation pressure

\[ P_o = 180 \text{ psig} = 195 \text{ psia} \]

the initial mass and initial mass flow rate are, from the Isothermal Analysis

\[ m = 50.5 \text{ slugs} \]
\[ \dot{m} = 0.621 \text{ slugs/sec} \]

Choosing a ten-second time increment, after the first ten seconds the mass remaining in the tanks is

\[ 50.5 - 10 (0.621) = 44.29 \text{ slugs} \]

With the initial tank pressure and temperature this is then substituted into eqn. 8 to find the new stagnation (or tank) temperature

\[ T_{02} = (540)^{1.4} \left( \frac{44.29 \times (53.35) \times (32.2)}{268 \times (1215) \times (144)} \right)^{0.4} \]
\[ = 512^\circ R \]

The new tank pressure is given by eqn. 7

\[ P_{T2} = \frac{mRT_{T2}}{V} \]
\[ = \frac{44.29 \times (53.35) \times (512) \times (32.2)}{268 \times (144)} \]
\[ = 1007 \text{ psia} \]

The new mass flow rate is obtained from eqn. 3 and the new stagnation temperature, rewriting eqn. 3
\[ \dot{m} = \frac{\pi d^2 p_o}{4} \sqrt{\frac{0.634 (0.528)k}{RT_o}} \]  

(eqn. 9)

then

\[ \dot{m} = 0.0311 (195) (144) \sqrt{\frac{0.634 (0.528) (1.4)}{53.35 (512) (32.2)}} \]

\[ = 0.638 \text{ slugs/sec} \]

Repeating this procedure for increments of ten seconds (the last increment being four seconds) we find that the time for the tank pressure to reach 180 psig is approximately 54 seconds.
3. Theoretical Run Time Summary

<table>
<thead>
<tr>
<th>Method</th>
<th>Run Time (seconds)</th>
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<tr>
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<td>68</td>
</tr>
<tr>
<td>Empirical Isothermal</td>
<td>53</td>
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<tr>
<td>Theoretical Isentropic</td>
<td>54</td>
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<td>Empirical Isentropic</td>
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<td>Experimental (Actual)</td>
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</table>
APPENDIX C.

Mach Number Determination
C. Mach Number Determination

1. Mach Number Determination from Schlieren Flow Visualization

<table>
<thead>
<tr>
<th>X (in.)</th>
<th>Y (in.)</th>
<th>p₀ (psig)</th>
<th>M</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5</td>
<td>+0.925</td>
<td>179</td>
<td>2.8</td>
<td>15a.</td>
</tr>
<tr>
<td>7.5</td>
<td>0.0</td>
<td>182</td>
<td>2.8</td>
<td>15b.</td>
</tr>
<tr>
<td>7.5</td>
<td>-0.925</td>
<td>180</td>
<td>2.7</td>
<td>15c.</td>
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<td>15d.</td>
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<td>2.8</td>
<td>15h.</td>
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<tr>
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<td>-1.250</td>
<td>179</td>
<td>2.8</td>
<td>15i.</td>
</tr>
</tbody>
</table>
2. Determination of Mach Number from Total and Static Pressure Measurements

The stagnation pressure drop across a normal shock, given by

\[
\frac{p_{oy}}{p_{ox}} = \left[ \frac{k+\frac{1}{2} M_x^2}{1 + k-\frac{1}{2} M_x^2} \right]^{\frac{k}{k-1}} \left[ \frac{2k}{k+1} M_x^2 - \frac{k-1}{k+1} \right]^{\frac{1}{1-k}}
\]  
(eqn. 10)

where

- \( p_{ox} \) = free stream stagnation pressure
- \( p_{oy} \) = stagnation pressure behind shock
- \( M_x \) = free stream Mach number

and the ratio of total to static pressure

\[
\frac{p_{ox}}{p_x} = \left[ 1 + k-\frac{1}{2} M_x^2 \right]^{\frac{k}{k-1}}
\]  
(eqn. 11)

where

- \( p_x \) = free stream static pressure

may be combined to give

\[
\frac{p_{oy}}{p_{ox}} = \frac{p_{oy}}{p_x} = \left[ \frac{k+\frac{1}{2} M_x^2}{1 + k-\frac{1}{2} M_x^2} \right]^{\frac{k}{k-1}} \left[ \frac{2k}{k+1} M_x^2 - \frac{k-1}{k+1} \right]^{\frac{1}{1-k}}
\]  
(eqn. 12)

Thus a solution for the Mach number may be iterated upon provided values for \( p_{oy} \) (obtained from total pressure probe) and \( p_x \) (obtained from static pressure probe) are available.

Equation 12 is also known as the "Rayleigh pitot-tube formula."

It is given in tabular form in NACA 1135 and by SHAPIRO (1953, Table B.3).
VIII. BIBLIOGRAPHY


IX. VITA

James Riley Murphy was born on July 14, 1945, in St. Louis, Missouri. He received his primary and secondary education in Bridgeton, Missouri. He has received his college education from the University of Missouri - St. Louis in St. Louis, Missouri and the University of Missouri - Rolla in Rolla, Missouri. He received a Bachelor of Science degree in Mechanical Engineering from the University of Missouri - Rolla in Rolla, Missouri, in June, 1968.

He has been enrolled in the Graduate School of the University of Missouri - Rolla since September, 1968, and has held the position of Graduate Assistant for the period September 1968 to June 1969 and September 1969 to June 1970.