An investigation of the adjustable element concept for design of automotive exhaust mufflers

Phillip Scott Gegesky

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AN INVESTIGATION OF THE ADJUSTABLE ELEMENT CONCEPT
FOR DESIGN OF AUTOMOTIVE EXHAUST MUFFLERS

BY

PHILLIP SCOTT GEGESKY, 1946 -

A

THESIS

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Approved By

[Signatures]

[Names]
ABSTRACT

The concept of the adjustable muffler is felt to be a step in the direction of easing the present tedium of constructing and testing prototype mufflers. The evaluation of such a muffler also furthers the knowledge of the interaction of acoustic elements.

A review of past and present efforts in the design and evaluation of acoustic filters and filter elements is herein presented. Some consideration is given to phenomena, such as tube attenuation and transverse modes of vibration, which are encountered in muffler evaluation techniques.

The characteristics of wave guides are discussed at length. The solution of the plane wave equation is presented and its applicability to standing wave tube measurements is scrutinized. The derivation of reflection factor and transmission factor equations is reviewed, and these equations are related to standing wave tube measurements. Furthermore, the standing wave tube which was used in this investigation is described.

Methods of sound filtration are presented, and a theoretical foundation is constructed with a discussion of acoustic elements and their analogous mechanical and electrical components. A lumped parameter means of predicting muffler performance is also delineated, and the filtering characteristics of simple elements such as orifices and chambers are discussed.

Also described are the design and evaluation of an adjustable muffler. The experimental means of evaluating the reflection and transmission characteristics are presented, and tables and graphs are
incorporated to portray the effect of acoustic element adjustment on muffler performance. Furthermore, the effect of adjustment on automotive exhaust spectrums is presented and comparison of pure tone and spectral analysis is given.

Inclusive in the conclusions of this investigation is the conviction that construction and evaluation of adjustable element mufflers lead to a greater understanding of the action and interaction of acoustic elements, and that this knowledge, in turn, will greatly simplify the design of silencers for specific applications.
ACKNOWLEDGEMENTS

I would like to take this opportunity to express my heartfelt gratitude to a number of people who have been instrumental in the preparation of this dissertation. The obvious recipients of my indebtedness are my mother and father, whose continual encouragement and faith greatly aided me when progress was negligible and results disappointing. Also prominent in my feeling of appreciation is Dr. William S. Gatley whose concept of an integrally adjustable, automotive muffler started my investigations. Furthermore, his advice and guidance during the course of my studies was invaluable.

My thanks must also be extended to Lacleade Metal Products Company in Lebanon, Missouri. A modification of one of their Car Care muffler designs was utilized for the adjustable muffler in these investigations. The supply of basic muffler components and the initial machining of the adjustable muffler provided by Lacleade was also instrumental in the successful completion of this treatise.

I am also indebted to the University of Missouri - Rolla and particularly the Mechanical Engineering Department, whose facilities made my explorations both interesting and enjoyable.

Mr. Lee Anderson and his machinists are to be particularly recognized for their prompt and accurate aid in the modification of the adjustable muffler prototype. I would also like to thank Mr. Thomas V. Huber for his help in solving many miscellaneous problems which were encountered during the course of this investigation.

My appreciation must also be extended to Miss Janet Kidwell whose dexterity with the typewriter greatly aided me in the preparation of this manuscript.
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CHAPTER I
INTRODUCTION

During the past decade, the general public has become more cognizant of the noise created by, and in their surroundings. Manufacturers of noisy products have found their sales declining and their service complaints mounting. One of the most prominent offenders of peace and quiet in today's society is the internal combustion engine. As of late, many state governments have found it necessary to enact laws governing the maximum permissible noise level of automobiles. And, almost every police force in the nation issues summonses for inadequate or defective mufflers.

Thus, public opinion and law enforcement have put a great deal of pressure on the automotive muffler manufacturer for a better and more efficient product. It is, therefore, imperative that a system of muffler design be developed, which is more of an exact science and less of an art. At the present time the method of design of reactive filters for specific applications may be termed "trial and error." In general, the automotive muffler industry, while able to pinpoint its problems by simple acoustic measurements, apparently has no method at hand to rapidly and simply design acoustic filters to alleviate its problems. The industry, on the whole, utilizes established muffler configurations for design initiation and randomly adds and subtracts elements until the desired effect is obtained. This is not only time consuming, but also monetarily wasteful since each time the design is changed a new prototype must be constructed and evaluated. Hence, if
a basic prototype was built on the form of a firm's standard production muffler, with adjustable and/or readily additive or removable elements, a great deal of time and money might be saved.

Furthermore, W. S. Gatley [10] and other authors have proposed a muffler design system based on a "library" of acoustic elements. Very little is presently known about the effects of actual acoustic elements on the filtration of sound. If suitable corrections to plane wave theory can be made to account for the influence of effects not included in the plane wave equation, then even the need to construct muffler prototypes could be greatly reduced.

Gatley, in his investigations, states:

"Methods for selection and rapid analysis of possible muffler designs should be investigated, specifically:
A.) Automation of information gathering and processing.
B.) Use of the digital computer for solution of equations and for design optimization by variation of parameters.
C.) Development of a scheme for rapid assembly of muffler prototypes from a "library" of acoustic elements."

The objective of this investigation falls under the third category above. It is the intent of this dissertation to evaluate the efficacy of the adjustable muffler concept, and in so doing, to gain knowledge of the action and interaction of acoustic elements.

As previously stated, an adjustable prototype based on a standard production muffler might be useful in alleviating a firm's muffler development problems. Thus, using this concept as a starting point for this analysis, a standard Car Care* production muffler design was modified to be adjustable in certain respects.

It should be noted that the firm which fabricates Car Care mufflers readily admits that this muffler was not initially designed from any basic

*Brand name for Lacleade Metal Products Company mufflers.
acoustical criteria. Rather, the design is a random combination of elements, found to be more or less acceptable in alleviating the noise problem encountered.

Hence, from these unscientific beginnings, the prototype adjustable muffler was conceived. The greater percentage of this report is concerned with the evaluation of this muffler. The remainder of the thesis summarizes the current state of the art in both muffler design and in the measurement of acoustic filter characteristics; develops the plane wave equation for use in muffler analysis; and discusses the theoretical characteristics of several basic acoustic elements.
CHAPTER II
A SURVEY OF ACOUSTIC THEORY PERTAINING TO MUFFLER DESIGN AND TESTING

A. The Effect of Attenuation in Waveguides

When a sound wave propagates through free space, a certain amount of energy present in the sound wave is dissipated. This loss of energy is primarily a result of viscosity, heat transfer, and conservation of momentum between colliding molecules in the medium. The latter mechanism of energy dissipation involving exchange of translational, rotational, and vibrational energy is termed "molecular absorption". When a sound wave propagates down an acoustical waveguide, however, it encounters much more attenuation than when traveling through free space. This phenomenon is an effect of viscous and thermal boundary layers at the tube wall. In fact Beranek [2]* states that the attenuation due to viscosity and heat conduction in free space is negligible when compared to the attenuation caused by the boundary layers encountered in small diameter tubes at frequencies below 20,000 Hertz.

The ultimate effect of attenuation on a sound wave propagating down a waveguide is to decrease the sound pressure, particle velocity, and the propagation velocity of the wave. Helmholtz and Kundt were actually the first two investigators to scrutinize the action of tube attenuation. However, Kirchoff in 1868 [35] presented the complete acoustic theory for the decrease in propagation velocity and sound pressure due to the effects of viscous and thermal boundary layers at the tube wall.

*Numbers in brackets refer to references listed at end of paper.
Observations indicate that the phenomenon of attenuation is exponential in nature and may be expressed in the form:

\[ P = P_0 e^{-\alpha x} \]  

(1)

where: \( P \) = sound pressure at any \( x \) location.

\( P_0 \) = sound pressure at \( x = 0 \).

\( e \) = base of the natural logarithms.

\( \alpha \) = attenuation constant.

Kirchoff found that, in general, the attenuation constant for circular waveguides may be expressed as:

\[ \alpha = \frac{\omega}{2Cr} [L_v + (\gamma-1)L_h] \]  

(2)

where: \( C \) = propagation velocity of sound in free space (Cm/sec)

\( r \) = inside radius of the waveguide (Cm.)

\( \omega \) = circular frequency (Rad./sec.)

\( \gamma \) = ratio of specific heats \( (C_p/C_v) \)

\( L_v = [\frac{2\nu}{\omega}]^{1/2} \) = viscous boundary layer thickness (Cm.)

\( L_h = [\frac{2K/\omega\rho_0 C_p}{\omega}]^{1/2} \) = thermal boundary layer thickness (Cm.)

\( \nu \) = kinematic viscosity (Cm\(^2\)/sec)

\( K \) = thermal conductivity (Cal/sec-Cm-°C)

\( \rho_0 \) = mass density (g/m/cm\(^3\))

\( C_p \) = specific heat at constant pressure (Cal./g/m.-°C)

Kirchoff based his theoretical expression of the attenuation constant on the following assumptions [35]:

1.) Only plane sound waves exist in the tube.

2.) The medium is homogeneous and only the effects of viscosity and heat conduction are present.
3.) Only "wide" tubes* are utilized.
4.) The sound wave amplitude is small enough such that no turbulence occurs in the medium.
5.) The sound wave is harmonic in nature.
6.) The tube is infinitely long such that no end effects are encountered, and the velocity profile is stable.
7.) The inside surface of the tube is smooth, rigid, symmetrically and impenetrable.
8.) There is no fluid motion at the tube wall.
9.) The gas temperature at the tube wall is constant.
10.) The sound wavelength is much greater than the molecular mean free path.†

Tube attenuation has been the subject of much investigation since Kirchoff presented his hypothesis. Fay [8] studied rigid, "wide" tubes and suggested that a closer agreement between theoretical and experimental values could be obtained if a linear frequency term were added to Kirchoff's equation. He also concluded that variations of humidity could cause large inconsistencies in the results obtained from attenuation measurements.

The effects of water vapor and temperature on sound absorption in air and other gas mixtures was investigated by Knudsen [19],

*This term is applicable when only viscous and thermal boundary layers cause attenuation. A "narrow" tube has a radius less than the thickness of the boundary layer. While, in a "very wide" tube the sound energy is assumed concentrated at the walls and is not constant over the tube cross-section.[35]
†This limit is $10^9$ Hertz at standard temperature and pressure.
Chrisler and Miller [7], Harlow and Kitching [12], and Harris [13,14]. In short, these investigators concluded that molecular absorption depends characteristically on the amount of water vapor present in the medium as an impurity. A more rigorous consideration of the molecular absorption of sound in tubes was presented by Shields, Lee, and Wiley [32] in which they stated that molecular absorption is also a function of the ratio of the driving frequency to the static pressure of the gas.

Beranek [2] compared the results of previous investigations and concluded that the value of the attenuation constant is best described by Kirchoff's equation increased by 15 percent. Beranek's modified numerical evaluation of the Kirchoff attenuation constant for thinwall, "wide" tubes in undried air is:

\[ \alpha = 3.18 \times 10^{-5} \frac{f}{r^{1/2}} \]  

(3)

where:  
- \( f \) = driving frequency (Hertz)  
- \( r \) = inside radius of the tube (Cm.)

Simon [33] in his experimentation found that "experimental values of the attenuation constant tend to group around the theoretical values (from Kirchoff's formula) of the attenuation constant". The modified formula is utilized for all attenuation calculations in this thesis. The numerical values of the attenuation constant as a function of frequency for the two inch inside diameter standing wave tube are presented in Appendix I.

Kirchoff, using the same assumptions as in his attenuation derivation, further found that for a "wide" tube; the speed of sound may be represented by the relationship:
\[
C' = C\left[1 - \frac{L_{vh}}{2r(\pi f)^{1/2}}\right]
\]

where: 
\[L_{vh} = \sqrt[1/2]{\gamma-1} \left[K/\rho_0 C_p\right]^{1/2}\]

\[C = \text{adiabatic speed of sound in free space} = [\gamma RT]^{1/2}\]

\[R = \text{ideal gas constant}\]

\[T = \text{static temperature (°R)}\]

\[\nu = \text{kinematic viscosity (Cm}^2/\text{sec)}\]

\[\gamma = \text{ratio of specific heats} = 1.4 \text{ for air}\]

\[K = \text{thermal conductivity (cal./sec.-Cm.-°C)}\]

\[\rho_0 = \text{mass density (gm/Cm}^3)\]

\[C_p = \text{specific heat at constant pressure (cal./gm.-°C)}\]

\[r = \text{inside radius of the tube (Cm.)}\]

\[f = \text{driving frequency (Hz.)}\]

The equation for the corrected speed of sound in waveguides was experimentally verified by Scott [31]. The numerical values of the theoretical speed of sound in tubes, as a function of frequency for the two inch inside diameter standing wave tube, are presented in Appendix II. Also included in this appendix are the values of the wavelength corresponding to the corrected speed of sound in the tube. These values are obtained from the relation:

\[
\lambda = \frac{C'}{f}
\]

where: 
\[\lambda = \text{wavelength (Cm.)}\]

\[C' = \text{corrected speed of sound (Cm/sec.)}\]

\[f = \text{driving frequency (Hz.)}\]

The values of the attenuation constants are used in the calculation of reflection and transmission factors of the adjustable muffler. The theoretical wavelength is necessary for the calculation of the
reflection phase angle when two minimums cannot be found in the standing wave tube.

B. Utilization of the Standing Wave Tube

The measurement of acoustic impedance by means of a standing wave tube has received considerable attention since the turn of the century. However, the use of such apparatus in conjunction with muffler evaluation has not received as much consideration. However, most of the experimental procedures and guidelines developed in the investigation of absorption characteristics of materials may be applied to the evaluation of reactive acoustic filters. This is possible because both the absorption of a material and the reflection characteristics of a muffler are obtained from the measurement of the magnitude and location of sound pressure nodes and anti-nodes in the standing wave tube.

Beranek [2,5] lists nine conditions, applicable to muffler evaluation, on which accurate experimental analysis with the use of the standing wave tube depends:

1.) The bore of the tube must be uniform and the walls smooth and substantially rigid and leak free.
2.) The sound wave in the tube must consist of plane, uniform, axially directed waves.
3.) The microphone used for the exploration of the sound field must not in itself appreciably affect the field and must be sensitive and stable.
4.) The measurement of the position of the microphone orifice must be accurate to about 0.1 millimeter.
5.) The sound pressure measured with the microphone must arise only from a single frequency.

6.) The frequency and ambient temperature must be stable.

7.) The frequency must be accurately known.

8.) Residual noise in the system must be held to a minimum.

9.) The method should be absolute and not dependent on a reference or standard impedance.

Sabine [30], in discussing standing wave tube measurements, stated that rapid measurements should be made to minimize the effect of drift errors. He also stressed the importance of using well defined pressure minimums for phase angle calculations, rather than broad pressure maximums. Sabine also suggested that all electronic signals should be filtered to eliminate harmonics.

Scott [31] presented a list of measurement requirements for accurate results that is similar to Beranek's. However, one requirement not mentioned by Beranek was that symmetric readings should be taken on both sides of a pressure minimum to accurately determine its location.

The American Society of Testing Materials published a report [1] standardizing the measurement of absorption and impedance of acoustic materials. Listed in the report are specifications for the construction of standing wave tubes as well as the procedures for measurement. The most pertinent of these specifications are:

1.) The microphone output shall be filtered.

2.) The sound source shall be mounted directly on the end of the tube or to a 45 or 90 degree elbow.

3.) A pressure minimum shall not be measured any closer to the source than one tube diameter.
4.) The electronic equipment shall be stable.

5.) The measuring system shall be capable of determining the microphone location to within \( \pm 0.2 \) millimeter.

6.) The seal between the sample and the end of the tube should be as airtight as possible.

The preceding principles and considerations were employed in the evaluation of the adjustable muffler in an attempt to make the analysis as exact and repeatable as possible.

C. Application of Plane Wave Theory to Muffler Design

Attempts to predict the performance of mufflers in automotive engine exhaust systems have been limited to a few simple muffler designs because of the involved mathematics encountered. Several investigators have analyzed simple acoustic filters such as side branch resonators and expansion chambers by utilizing electrical and mechanical analogies with the lumped parameter approach.

Stewart [34] was one of the forerunners in the field of theoretical analysis of acoustic filters by means of lumped parameter methods. Stewart constructed low frequency, high frequency, and band-pass filters and found that agreement between theoretical and experimental data was "fairly satisfactory, in view of assumptions made". He also observed that high attenuation could be achieved with relatively few acoustic sections, and concluded that the attenuation of sound by such filters was due to interference rather than dissipation.

One investigation of note employed the continuous or distributed parameter approach for analysis of simple systems. Davis, Stokes,
Moore, and Stevens [8] completed an investigation for the government in which "equations are presented for the attenuation characteristics of single-chamber and multiple-chamber mufflers of both the expansion-chamber and resonator types, for tuned side-branch tubes, and for the combination of an expansion chamber with a resonator". Furthermore, experimental curves of attenuation versus frequency were presented for 77 different anechoically terminated mufflers. The results of these tests, when compared to theoretical data, showed reasonably good agreement. The effects of temperature, exhaust-gas velocity, and sound pressure were also considered in the report.

Igarashi and Toyama [17] applied four terminal network theory to the analysis of various filter configurations; employing inlet and outlet sound pressure and particle velocity as variables. The experimental measurements were taken under anechoic conditions and the results, when compared to theoretical data, were found to be in good agreement.

Franken [4] lists three methods of experimental and theoretical muffler evaluation: "insertion loss", "transmission loss", and "noise reduction". In his derivations, Franken relates the fundamental "equations of motion" governing linear acoustics to electrical circuit theory to obtain theoretical performance data. Some discussion of mechanical performance is also included in the treatise.

Gatley [10] presents a variety of experimental methods for the evaluation of small acoustic filters used in refrigeration systems. In his dissertation he also proposes a method of design which combines experimental measurements with acoustic theory, and successfully predicts attenuation resulting from the insertion of a given muffler. The method requires that the reflection and transmission characteristics of the filter's acoustic elements be known. Once these characteristics
have been determined, Gatley suggests that these components may then be combined mathematically in a sequential manner to determine a filter's overall transmission characteristics. From his success with this method, Gatley concludes that "a logical scheme for construction of many possible prototypes from a 'library' of basic elements would simplify the search for an optimum design".

Simon [33], following Gatley's initial investigations, describes a method for determining the reflection and transmission characteristics of acoustic filters under static conditions. In his treatise, Simon also presents digital computer programs suited for standing wave evaluation of acoustic filters. These programs, with certain essential modifications, were utilized in the evaluation of the adjustable muffler and are presented in Appendix III. Simon also describes the design of a two inch inside diameter standing wave tube. This tube was employed in the analysis of the adjustable muffler and the evaluation of this tube at selected frequencies is presented in Appendix IV.

Buckley [6] presents a method, similar to those of Gatley and Simon, for the evaluation of the attenuation characteristics of a muffler. In his thesis is also presented the design and evaluation of a static (no flow) anechoic termination. This termination, after undergoing certain modifications, was utilized in this investigation. Its characteristics at selected frequencies are presented in Appendix V.

Dyer [27] discusses the variables with which an engineer is concerned in designing a muffler:

1.) Attenuation of the muffler as a function of frequency.
2.) Size of the muffler.
   a.) Length
   b.) Cross-sectional area
3.) Effect of gas flow on the muffler.
   a.) velocity erosion
   b.) resistance to high temperatures
   c.) corrosion and contamination
   d.) pressure drop
   e.) noise generated by the muffler as a result of gas flow
4.) Initial cost of the muffler.
5.) Ease of inspection and maintenance once the muffler is installed.

The May, 1956, Noise Control [27] is almost entirely devoted to vehicular noise. Included in this issue are specifications for heavy duty, truck muffler design and performance. Also presented is a staff report on exhaust silencing progress over the years, and a summation of vehicular noise legislation.

D. Transverse Modes of Vibration

The use of an acoustical waveguide, such as in this investigation, implies the existance of plane waves. However, discontinuities or vibrating sources create transverse modes of vibration in the tube. When such vibrations are present, they produce effects which necessarily cannot be predicted by simple plane wave theory. It has been observed that in a tube, transverse modes are excited if the wavelength of the driving frequency is less than a specific value relative to the tube radius. This value, therefore, establishes a critical frequency which, when exceeded, violates the plane wave conditions.
Lord Rayleigh [29] predicted that a wave propagating in a tube can be considered plane if its frequency is lower than the critical frequency corresponding to the first transverse mode of vibration. He also predicted that, for a symmetrical source over a tube cross-section of radius \( r \), a wave could be considered plane if the wavelength was approximately greater than \( 1.57r \).

Hartig and Swanson [15], and Miles [25], in separate investigations of the effect of transverse modes in tubing, arrived at the same conclusion that the first transverse mode is excited and propagated when the wavelength \( \lambda \) of the driving frequency is approximately equal to \( 1.64r \).*

Several other investigators have experimentally established critical wavelengths. Beranek [5] reports that the first transverse mode will not be excited until the wavelength is approximately \( 1.7r \). Scott [31] found good agreement in data with plane wave theory until a limit of \( \lambda = 1.73r \) was reached. Davis, Stokes, Moore, and Stevens [8], in investigating the acoustic characteristics of a wide variety of cylindrical muffler configurations, found the results predicted by plane wave theory were in close agreement within the muffler. They observed the limiting wavelength to be \( 1.64r \). This limiting value corresponds to Miles, and Hartig and Swanson's findings.

For discontinuities in general, the most noticeable effect produced by the propagation of transverse modes is phase distortion of the

* This is approximately 8000 Hz. for a 2-inch inside diameter tube, and 3000 Hz. for the 5 3/8 inch inside diameter muffler.
transmitted and reflected waves [23,24,25]. Thus, it may be concluded that the effects produced by higher order transverse modes of vibration become more significant as the frequency and/or the tube diameter increases.

E. Conclusions

One fact is outstanding at the completion of a literature survey on muffler design and analysis: though general design methods and varying procedures of acoustic filter analysis have been published, the design of a muffler is not yet an established science. A designer cannot assemble a filter from acoustic components with assurance that his end product will function within certain close tolerances. Until a catalog of the action and interaction of acoustic elements as a function of flow, temperature, sound pressure level, and frequency has been compiled; the design of acoustic filtering systems will be a tedious art.
CHAPTER III
INHERENT PROPERTIES OF WAVEGUIDES

A. The Plane Wave Equation

Plane waves are the simplest type of wave motion propagated through a fluid medium. The characteristic property of such waves is that the acoustic pressures, particle displacements, and particle velocities have common phases, and amplitudes at all points on any plane perpendicular to the direction of wave propagation. It is this property that simplifies the mathematics of acoustical investigations.

Plane waves may readily be produced in a fluid confined in a rigid tube, through the action of a vibrating diaphragm located at one end of the tube. The vibrating diaphragm accelerates adjacent air particles, and compresses that part of the fluid nearest to it, as it moves forward. These closely crowded air particles have, in addition to their random velocities, a forward momentum gained from the diaphragm. They collide with their neighbors and during the collision transfer forward momentum to these particles, which were at rest. Progressively more and more remote parts of the medium are set into motion and, thus, the plane wave travels down the tube.

The propagation of the sound in the tube is described by the plane wave equation:

\[ \frac{c^2}{x^2} \frac{\partial^2 P}{\partial x^2} = \frac{\partial^2 P}{\partial t^2} \]  

where:
- \( c \) = speed of sound in the medium (cm./sec.)
- \( P \) = sound pressure (v.)
- \( x \) = propagation direction (cm.)
- \( t \) = time (sec.)
However, for this equation to hold true, the following restrictions are placed on the fluid [2]:

1.) The fluid obeys the perfect gas laws, and no dissipative forces such as viscosity are present.
2.) The fluid is homogeneous and isotropic.
3.) The fluid has adiabatic and reversible compression and expansion.
4.) The acoustic pressure is much less than the ambient pressure of the fluid.
5.) The incremental density is much less than the ambient density of the fluid.
6.) The steady flow velocity of the medium is less than or equal to the particle velocity.

The assumption of an ideal fluid disregards the effect of shear forces produced by viscosity. However, as can be seen from the discussion of attenuation in tubes, viscosity does become an important parameter, and can be accounted for by an attenuation factor. Furthermore, for low pressures and moderate temperatures, the behavior of air is predicted, to a fair degree of accuracy, by the perfect gas laws [21].

The assumption that the fluid is homogeneous and isotropic simplifies the calculations, and is valid for air at room temperature and pressure. However, if the medium was ocean water, where the salinity, pressure, and temperature vary with position, then this assumption would be invalid.

Since a sound wave traveling down a tube produces rapid expansions and compressions of the medium about the ambient pressure; fluctuations are created in the ambient temperature. However, the propagation of the thermal diffusion wave is much slower than the propagation of the sound
wave. Therefore, there is insufficient time for significant heat exchange to take place and the propagation may be considered adiabatic. Also, for small pressure fluctuations about the ambient pressure, the expansions and compressions may also be considered reversible. Thus, since these rarifications and compressions can be assumed adiabatic and reversible, they are isentropic [21].

Now consider a long tube with a vibrating diaphragm at one end and a rigid plug at the other: the pressure at any point in the tube, as given by the plane wave equation, is a summation of the incident sound pressure from the diaphragm and the reflected sound pressure from the rigid plug. These two waves traveling in opposite directions create a condition, in steady state, known as a standing wave.

Since the pressure at any point in the tube is a function only of distance and time; the plane wave equation may be solved by the separation of variables method, or:

\[ P(x,t) = X(x) T(t) \]  

where: \( X(x) \) is a function only of location

\( T(t) \) is a function only of time

Substituting this assumption (7) into the plane wave equation (6) yields:

\[ C^2 X'' = XT'' \]  

Separating variables:

\[ C^2 \frac{X''}{X} = \frac{T''}{T} \]

For this relationship to be valid, each side of the equation must be equal to some constant. Assuming this constant to be \(-\omega^2\) yields the equations:
The solution to these equations, from elementary differential equations, is:

\[ X(x) = A \sin \left( \frac{\omega}{C} x \right) + B \cos \left( \frac{\omega}{C} x \right) \]

\[ T(t) = C \sin (\omega t) + D \cos (\omega t) \] (10)

Substituting these solutions (10) into the original assumption (7) yields:

\[ P(x,t) = [A \sin \left( \frac{\omega}{C} x \right) + B \cos \left( \frac{\omega}{C} x \right)] [C \sin (\omega t) + D \cos (\omega t)] \] (11)

The constants A and B can be found from boundary conditions and the constants C and D can be found from initial conditions. The equivalent form of the above equation (11) in complex notation is:

\[ P(x,t) = [A e^{\frac{\omega}{C} x} + B e^{-\frac{\omega}{C} x}] e^{j \omega t} \] (12)

or:

\[ P(x,t) = [A_m e^{\frac{j\omega x}{C}} + B_m e^{-\frac{j\omega x}{C}}] e^{j \omega t} \] (13)

where:

- \( A_m \) = The complex value of the incident sound pressure wave traveling in the negative x direction.

- \( B_m \) = The complex value of the reflected sound pressure wave traveling in the positive x direction.

- \( A_m \) = The magnitude of the complex value of the incident wave.

- \( B_m \) = The magnitude of the complex value of the reflected wave.

- \( \theta \) = The phase angle between the incident and reflected pressure waves.

- \( \omega \) = circular frequency (Rad/sec.)

- \( C \) = speed of sound (cm/sec.)

- \( t \) = time (sec.)

- \( x \) = location relative to the end of the waveguide (cm.)
B. The Standing Wave Concept

As mentioned previously, when there are reflections in a waveguide the waves traveling in opposite directions, in the steady state, create "standing waves". Beranek states that:

"Standing waves constitute a wave system resulting from the interferance of progressive waves of the same frequency and kind. They are characterized by the existence of nodes or partial nodes in the interferance pattern. In order to obtain standing waves, the interfering waves must have components traveling in opposite direction."

When attempting to evaluate a filter or acoustic material with the use of a waveguide, such a situation is encountered. The summation of the incident and reflected waves creates a phenomenon such that a specific sound pressure no longer shifts to a position to the right or left of the original point as time varies. Rather, in a standing wave, a regular pattern of pressure maximums and minimums is created in the waveguide at fixed locations and the wave, in effect, "stands still". Hence, when a waveguide is utilized in this manner, it is termed a "standing wave tube", and the pressure maximums and minimums created by reflections from an object under investigation may be measured to determine reflection factors and subsequently transmission factors (subjects which are covered in detail in successive material).

It may be noted here that reflections occur at the end of a finite waveguide even with no obstruction in the wave path. This phenomenon is created by the change in volume at the end of the tube to the surrounding room and will be discussed later. However, it is helpful when evaluating acoustic filters to be able to assume that there are no reflections other than those created by the subject of the investigation. For this reason an "anechoic termination" is utilized. Anechoic is an adjective meaning "echo-free", and a chamber or termination is said to
anechoic if a sound field can be established in it without reflection. In this investigation anechoic conditions are assumed when the reflection factor is less than 0.10. This is an effective number in that considering filters with low transmission factors (below 0.5) then, of a sound wave incident on an anechoically terminated filter using this limit, an upper limit of 50 percent of the incident wave will be transmitted through the filter. From the transmission sound, a maximum of 5 percent of the original sound energy will be reflected back into the filter. Finally, of the reflected energy, 2.5 percent of the original sound wave will be transmitted back through the muffler.

C. The Reflection Factor

When a sound wave is traveling down a length of tube, terminated by an acoustic filter or muffler, three effects may be observed:

1.) Part of the incident wave is transmitted out of the filter.
2.) Part of the incident wave is absorbed by the termination.
3.) Part of the incident wave is reflected into the tube.

If a microphone probe were introduced into such a sound field, it would measure a summation of the incident and reflected waves produced by the sound source and termination. The "Reflection Factor" of the termination is then defined as the complex ratio of the reflected wave to the incident wave. From equation (13) it may be remembered that the incident pressure wave is denoted as:

$$A = A_m e^{\frac{j\omega t}{C}} e^{\frac{j\omega x}{C}}$$

and the reflected pressure wave is described by:
\[ B = B_m e^{J \omega t} \]

Thus, the reflection factor is equivalent to:

\[ R = \frac{B}{A} = \frac{B_m e^{J \omega x}}{A_m e^{J \omega t}} \]

Simplifying yields the complex reflection factor:

\[ R = \frac{B_m}{A_m} e^{J(\theta - \frac{\omega}{C} x)} \]

Gatley [10], in his research, found that the standing wave tube method was the best method of experimentally measuring reflection factors. He states:

"After advantages and disadvantages [of the various methods] were compared, the standing wave method was adopted, primarily because it provides, with reasonable effort, useful information that was previously unavailable." [Obtainable data such as magnitude and phase angle of the reflected sound, the propagation velocity, the wavelength, and the attenuation]

From this and review of other work done in this area, the standing wave method was adopted for evaluation of the adjustable muffler.

On the following pages is presented an adaptation of equation (14) to standing wave measurements.

As previously noted, a microphone probe, placed in the sound field of an acoustical waveguide, measures the summation of the incident and reflected waves in the tube, or:

\[ P(x, t) = A + B \]

where:

\[ A + B = [A_m e^{J \omega x} + B_m e^{J(\theta - \frac{\omega}{C} x)}] e^{J \omega t} \]
However, instruments, such as the RMS voltmeter, indicate values which are proportional to only the magnitude or the microphone measurement. Rearranging equation (13) gives:

\[ A + B = [A_m + B_m e^{J(\theta - \frac{2\omega}{C}x)}] e^{J(\frac{\omega}{C}x + \omega t)} \]

Taking the magnitude of this quantity and substituting for the exponential term, yields:

\[ |A + B| = |A_m + B_m [\cos(\theta - \frac{2\omega}{C}x) + J \sin(\theta - \frac{2\omega}{C}x)]| \] (15)

Then squaring the real and imaginary parts of the equation, summing and taking the square root to find the absolute value of equation (15) renders:

\[ |A + B| = [A_m^2 + 2A_m B_m \cos(\theta - \frac{2\omega}{C}x) + B_m^2]^{1/2} \] (16)

From the above equation it may be observed that a maximum occurs when \( \cos(\theta - \frac{2\omega}{C}x) = 1 \). Thus, a maximum pressure summation can be observed in a waveguide equal to:

\[ P_{\text{max}} = |A + B|_{\text{max}} = A_m + B_m \] (17)

when: \( \theta - \frac{2\omega}{C}x = \pm N\pi \)

where: \( N = 0,2,4,6 \ldots \)

It also may be noted that a minimum pressure is observed in equation (16) when \( \cos(\theta - \frac{2\omega}{C}x) = -1 \). Thus, a minimum value for the incident and reflected pressure wave summation can be measured in a waveguide:

\[ P_{\text{min}} = |A + B|_{\text{min}} = A_m - B_m \] (18)
Combining equations (17) and (18) by addition and subtraction to obtain the ratio of the reflected wave magnitude to the incident wave magnitude yields the reflection factor:

\[ \frac{(A_m + B_m) - (A_m - B_m)}{(A_m + B_m) + (A_m - B_m)} = \frac{2B_m}{2A_m} = |R| \]

which, by substitution, is the same as:

\[ |R| = \frac{p_{\text{max}} - p_{\text{min}}}{p_{\text{max}} + p_{\text{min}}} \]  

(19)

It may be shown by the above methods that, in cases where tube attenuation must be considered, the magnitude of the reflection factor, \( R_o \), is given by the relationship [10]:

\[ |R_o| = \frac{p_{\text{max}} e^{\alpha x_{\text{min}}} - p_{\text{min}} e^{\alpha x_{\text{max}}}}{p_{\text{max}} e^{-\alpha x_{\text{min}}} + p_{\text{min}} e^{-\alpha x_{\text{max}}}} \]  

(20)

The phase angle, \( \theta \), between the incident and reflected waves is defined to be positive and between the limits of 0 and 360 degrees. Thus, \( N \) is chosen such that this criteria is fulfilled. For the case of a minimum pressure:

\[ \theta = \frac{2\omega}{C x_{\text{min}}} + N\pi \]  

(21)

where:  
\( N = 1, 3, 5, 7, \ldots \)

\( \omega = \) circular frequency (Rad/sec.)

\( C = \) speed of sound in free space (cm./sec.)

However, it may be noted:

\[ \frac{2\omega}{C x_{\text{min}}} = 2 \cdot \frac{2\pi f}{C} x_{\text{min}} = \frac{2\pi x_{\text{min}}}{\lambda / 2} \]
where:  \( \lambda \) = wavelength of sound in the medium (cm.)

\( f \) = driving frequency. (cycles/sec.)

\( C \) = speed of sound in free space (cm/sec.)

Since half of the wavelength of the driving frequency is the distance between adjacent standing wave minima, then:

\[
\theta = \frac{2\pi x_{\text{min}_1}}{x_{\text{min}_2} - x_{\text{min}_1}} + N\pi
\]  \hspace{1cm} (22)

where:  \( x_{\text{min}_1} \) = the location of the first sound pressure minimum encountered in the waveguide.

\( x_{\text{min}_2} \) = the location of the second sound pressure minimum encountered in the waveguide.

It can also be shown that under circumstances where tube attenuation must be considered:

\[
\theta = \sin^{-1} \left[ \frac{\alpha(|R|) e^{-2\alpha x}}{\omega/C'} - \frac{e^{2\alpha x}}{|R|} \right] + \frac{2\omega x_{\text{min}}}{C'}  \hspace{1cm} [10] \hspace{1cm} (23)
\]

where:  \( \alpha \) = attenuation constant (1/cm)

\( R \) = reflection factor

\( \omega \) = circular frequency

\( C' \) = speed of sound corrected for tube effects (cm/sec.)

Gatley [10] states that equation (23) need be used only in extreme cases where low values of reflection factors are encountered and high values of attenuation are present. Thus, equation (22) is valid in most cases for phase angle calculations.

As can be seen from the preceding equations, it is necessary to measure two adjacent minimums and the included maximum in the standing wave tube to calculate the magnitude of the reflection factor and phase angle for a given frequency. However, at low frequencies,
where the wavelength is long compared to the tube length, normally only one maximum or minimum pressure may be observed in the measuring length of a standing wave tube. Thus, an alternate method of calculation, involving only one maximum or minimum pressure and some other arbitrary pressure must be employed. Gatley [10], in his research, has found that the method utilizing a maximum pressure has a tendency to introduce excessive error. Hence, only the method employing a sound pressure minimum and some other arbitrary pressure will be derived here.

Since the square of the sum of the incident and reflected pressure waves, considering tube attenuation, is:

\[ p(x)^2 = A_m^2 e^{2\alpha x} + B_m^2 e^{-2\alpha x} + 2A_mB_m \cos (\theta - \frac{2\omega}{C}x) \]  \hspace{1cm} (24)

where:
- \( C' \) = speed of sound in the tube considering attenuation (cm/sec.)
- \( \cos (\theta - \frac{2\omega}{C}x) = +1 \) at a pressure minimum
- \( \alpha \) = attenuation constant (1/cm.)

Then expressing the distance \( x \) as a relative location:

\[ x = x_{\text{min}} + x_{rx} \] \hspace{1cm} (25)

where
- \( x \) = arbitrary pressure location relative to the end of the tube.
- \( x_{\text{min}} \) = minimum pressure location relative to the end of the tube.
- \( x_{rx} \) = pressure location relative to the minimum pressure location.

The substitution of equation (25) into the last term of equation (24) yields:

\[ 2A_mB_m \cos (\theta - \frac{2\omega}{C}x_{\text{min}} - \frac{2\omega}{C}x_{rx}) \] \hspace{1cm} (26)

Since at a minimum pressure location \( \cos (\theta - \frac{2\omega}{C}x_{\text{min}}) = -1 \), then:

\[ \theta - \frac{2\omega}{C}x_{\text{min}} = \pi \]
Then substituting this result into equation (26):

$$2A_m B_m \cos \left( \pi - \frac{\omega}{c} x \right)$$  \hspace{1cm} (27)

Now defining $\phi = \frac{2\omega}{c} x r_x$ and substituting into equation (24):

$$P(x)^2 = A_m^2 e^{2ax} + B_m^2 e^{-2ax} + 2A_m B_m \cos \phi$$  \hspace{1cm} (28)

The expression for the minimum pressure, considering attenuation is:

$$P_{\min} = A_m e^{\alpha x_{\min}} - B_m e^{-\alpha x_{\min}}$$

Rearranging and solving for the reflected pressure magnitude:

$$B_m = A_m e^{\alpha x_{\min}} - P_{\min} e^{\alpha x_{\min}}$$  \hspace{1cm} (29)

Substituting this into equation (28) yields:

$$P(x)^2 = A_m^2 \left[ e^{2ax} + e^{2\alpha(2x_{\min} - x)} + 2e^{\alpha x_{\min}} \cos \phi \right]$$

$$+ A_m [-2P_{\min}] \left[ e^{\alpha x_{\min}} \cos \phi + e^{\alpha(3x_{\min} - x)} \right]$$

$$+ P_{\min}^2 \left[ e^{2\alpha(x_{\min} - x)} \right]$$  \hspace{1cm} (30)

which can be solved for the incident pressure magnitude by applying the quadratic formula:

$$A_m = \frac{-e \pm \sqrt{e^2 - 4df}}{2d}$$  \hspace{1cm} (31)

where: $d = [e + e^{2\alpha(2x_{\min} - x)} + 2e^{\alpha x_{\min}} \cos \phi]$}

$$e = [-2P_{\min}] \left[ e^{\alpha x_{\min}} \cos \phi + e^{\alpha(3x_{\min} - 2x)} \right]$$

$$f = P_{\min}^2 e^{2\alpha(x_{\min} - x)} - P(x)^2$$
Since $A_m$ is the magnitude of the incident sound wave, it cannot be negative and, thus, only the positive root has any physical meaning. Assuming the value of the attenuation in the tube is small, then the exponential terms approach unity and the incident pressure magnitude becomes:

$$A_m = \frac{1}{2} [P_{\min}^2 + P_{\min}^2 - 2P_{\min}^2 - 2(P(x))^2 1/2]$$  \hspace{1cm} (32)

Now using the expressions for the incident and reflected pressure waves (equations (29) and (32), respectively) to find the reflection factor:

$$|R| = \frac{B_m}{A_m}$$

or:

$$R = \frac{A_m e^{2\alpha x_{\min}} - P_{\min} e^{\alpha x_{\min}}}{A_m}$$  \hspace{1cm} (33)

Where: $A_m$ is determined from equation (32) and the phase angle is found as previously discussed.

D. The Transmission Factor

Another important measurement which can be made with the use of waveguides is the determination of the amount of sound which is transmitted through a specific substance, filter, or such. This representation of sound transmission is normally referred to as the transmission factor, and is calculated with the use of the reflection factor for a given acoustic element.

The transmission factor is defined as the ratio of the amplitude of the transmitted wave ($A_2$) to that of the incident wave, ($A_1$) or:
The amplitude of the incident and transmitted waves must be calculated, since a microphone probe introduced into the tube measures the summation of reflected and incident waves. This is accomplished by taking measurements of the pressure amplitudes on the upstream and downstream sides of the element being investigated, and the relative phase angle between the upstream and downstream pressures. The incident wave \( A_1 \) is then found by using the previously determined complex reflection factor, and the pressure amplitude on the upstream side of the element. Once the incident wave amplitude has been found then the transmission factor, \( A_2/A_1 \), can be calculated using the amplitude and relative phase of the measurement on the downstream side of the subject if the element is terminated anechoically.

Since all of these calculations involve complex numbers, a convenient method of handling the mathematics is by graphical methods or through use of a digital computer.

The procedure for graphically determining the transmission factors at given frequencies is outlined below (refer to figure (3-1)):

1.) Determine the anechoic reflection factor for the subject under investigation.

2.) Assume that the incident wave, \( A_1 \), has a value of unity at a zero degree phase angle.

3.) Obtain the magnitude and phase of the reflected wave, \( B_1 \), from the complex product of the reflection factor and the amplitude of the assumed incident wave, \( A_1 \).
Figure 3-1. Graphical Determination of the Transmission Factor for an Arbitrary Filter.
4.) Graph the phasors representing the incident and reflected waves, and rotate the incident wave positively and the reflected wave negatively through the phase angle, \( \frac{\pi L_1}{\lambda/2} \), where \( L_1 \) is the distance to the upstream measuring station from the element. Note that this rotation is necessary, since the incident wave has a positive phase angle and the reflected wave has a negative phase angle with respect to the upstream measuring station.

5.) After correcting the magnitudes of the waves for the effect of attenuation, combine the two phasors, \( A'_1 \) and \( B'_1 \), vectorially to obtain the total pressure at the upstream measuring station, \( C_1 \).

6.) From the experimentally measured pressure amplitude ratio and the relative phase angle between the upstream and downstream measuring stations, the total sound pressure vector, \( C_2 \), at the downstream measuring station may be found. This result is obtained by multiplying the total pressure at the upstream station, \( C_1 \), by the amplitude ratio, \( P_2/P_1 \), and rotating the result through the relative phase angle between \( P_2 \) and \( P_1 \). Note that this total pressure, \( C_2 \), consists only of the transmitted wave since an anechoic termination is used.

7.) The value of the transmitted wave, \( A_2 \), is then obtained by rotating the total pressure, \( C_2 \), through a positive phase angle, \( \frac{\pi L_2}{\lambda/2} \), where \( L_2 \) is the distance to the downstream measuring station.
8.) The anechoic transmission factor may then be obtained from the graph by measuring the amplitude ratio (corrected for attenuation) of the transmitted and incident waves and the phase angle between them:

\[ T = \frac{A_2}{A_1} \]  \hspace{1cm} (34)

E. Summation

In this chapter the theory necessary for the evaluation of acoustical filters has been established. There are numerous other measurements that can be made with waveguides, and several different methods are available for obtaining each measurement. However, the object here has been to establish only the necessary theoretical background for the methods used in the evaluation of the adjustable muffler.
CHAPTER IV

THE FILTRATION OF SOUND

A. Acoustic Elements

A treatment of physical problems in terms of their basic components or "elements" is often of value, particularly in the dynamic analysis of physical phenomena. It is reasonable to expect, therefore, that a study of the filtration of sound be facilitated by the study of basic acoustic elements which frequently comprise a filtration system.

In acoustical studies the quantity most readily measurable, without modification of the object under investigation, is the sound pressure. As in the measurement of reflection and transmission factors, a sound pressure measurement is made by inserting a small microphone probe into the sound field at a desired point for an electrical representation of the pressure. The microphone records the difference between atmospheric pressure and the pressure created within the sound field, and this difference is termed the "sound pressure."

The dimensions of the various elements of an acoustic system are often small in comparison with the wavelength of the sound encountered. Under these circumstances, the motion of the medium in the system is analogous to that of a mechanical system having lumped parameters of mass, stiffness, and damping.

The acoustic mass is a quantity proportional to physical mass, yet having the dimensions of kilograms per meter. It is associated with a mass of air accelerated by a net force which acts to displace the gas without appreciably compressing it. A tube, open at both ends and with rigid walls, behaves as an acoustic mass if it is short enough so that
the air in it moves as a whole, without compression. An example of such an element would be the probe tube used with the microphone in standing wave measurements.

The physical law governing the motion of a mass acted on by a force is Newton's second law, $F = MA$. This law may be expressed in acoustical terms as:

$$P(t) = M_A \frac{\partial U(t)}{\partial t}$$

(35)

Where: $P(t)$ = instantaneous difference between pressures in Newtons per square meter at each end of the air column.

$M_A$ = the mass of the medium, $M_m$, divided by the cross sectional area of the tube, $S$, and corrected for end conditions.

$U(t)$ = instantaneous volume velocity of the air in cubic meters per second across any cross sectional plane in the tube. The volume velocity $U(t)$ is equal to the linear velocity, $u(t)$, multiplied by the cross sectional area, $S$.

In the steady state with angular frequency $\omega$, we have:

$$P = J \omega M_A U$$

(36)

The acoustic mass is also termed the acoustic inertness of a system.

Any device in which the flow of gas occurs in phase with and directly proportional to the applied pressure may be represented as a pure acoustic resistance; that is, there is no stored energy associated with the flow. Four principal forms of acoustic resistance are commonly employed in acoustic filters; fine meshed screens, small bore tubes, narrow slits or louvers and porous acoustical materials.
Screens are used in transducers because of low cost, ease of selection, control of manufacture, satisfactory stability, and relative freedom from inductive reactance. Slits and louvers are often used where an adjustable resistance is desired. This adjustment is accomplished by changing the width of the slit or amount of louver opening. Tubes have the disadvantage that; unless their diameter is very small, which in turn results in a high resistance, there is usually appreciable inductive reactance associated with them. However, this is sometimes very desirable. Fibrous and porous acoustic materials, such as fiberglass, porous ceramics, and sintered metals, are often utilized as mixtures of mass and resistance. In all cases the frictional effects producing the resistance occur in the same manner.

In general, the acoustic impedance, $Z$, of a fluid medium acting on, or through a given surface area, $S$, is the complex quotient of the acoustic pressure at the surface divided by the volume velocity at the surface:

$$Z = \frac{P(x,t)}{U(x,t)} \quad (37)$$

However, when concentrated, rather than distributed, impedances are considered, the impedance of a portion of the acoustic system is defined as the complex ratio of the effective pressure difference to the volume velocity. The acoustic impedance of any system is readily expressed in terms of mechanical impedance; that is, the mechanical impedance of the system, divided by the square of the area of the surface being considered. The unit of acoustic impedance is the acoustic ohm, with the dimensions:
\[
\frac{\text{Pressure}}{\text{Volume Velocity}} = \frac{\text{dynes/cm}^2}{\text{cm}^3/\text{sec}} = \frac{\text{gm.}}{\text{cm}^4\text{ sec}}.
\]

It may be noted that a complex acoustic impedance may be separated into its real and imaginary parts:

\[Z = R + JX\]

(38)

Where:  
- \(R\) = acoustic resistance of the sound medium. The term is associated with the dissipation of energy.  
- \(X\) = acoustic reactance of the sound medium. The term is the result of the effective mass and stiffness of the medium.

The compliance, \(C\), of an acoustic element is defined as the volume displacement that is produced by the application of a unit pressure. It is a constant quantity having the dimensions of meter per Newton, and it may be said that compression without acceleration identifies acoustic compliance. In other words, a volume of air that is compressed by a net force without appreciable average displacement of the center of gravity of the air in the volume constitutes an acoustical compliance.

The physical law governing the compression of a volume of fluid in a chamber being acted on by a net force is:

\[f(t) = \frac{1}{C_m} \int u(t) \, dt.\]

or, in acoustical terms:

\[P(t) = \frac{1}{C_A} \int U(t) \, dt\]

(39)

Where:  
- \(P(t)\) = the instantaneous pressure in Newtons per square meter acting to compress the volume, \(V\), of the air.  
- \(C_A\) = the acoustic compliance in meters per Newton or \(\text{cm}^4\text{ sec}^{-2}/\text{gm}\). For an acoustic element having an enclosed
volume the acoustic compliance is closely approximated
by the volume of the medium divided by the product of
the density of the medium \( \rho \) and the squared speed of
sound in the medium \( C \), or:

\[
C_A = \frac{V}{\rho C^2}
\]  

(40)

\( U(t) \) = the instantaneous volume velocity in cubic meters per
second flowing into a volume that is undergoing com-
pression. The volume velocity is the product of the
linear velocity, and the cross sectional area.

\( C_m \) = the mechanical compliance.

In the steady state with an angular frequency, \( \omega \), we have:

\[
P = \frac{U}{J \omega C_A}
\]  

(41)

Where: the pressure, \( P \), and the volume velocity, \( U \), are taken to be
RMS complex quantities.

B. The Helmholtz Resonator

The simple Helmholtz resonator is an important acoustic system.
It consists of a rigid enclosure of volume \( V \), communicating with the
external medium through an opening of small area which can either be a
hole or in the form of a neck. In either case the opening may be
considered to have a radius "a" and depth "L".

It is a simple matter to separate the acoustic element in this
system: the air in the opening of the resonator acts as an acoustic
mass; the volume of gas inside the resonator, which is alternately
compressed and expanded by the acoustic mass, provides the compliance
of the system. At the opening, furthermore, there is a radiation of sound into the surrounding medium which dissipates acoustic energy. It will suffice here to state that the equation of motion, considering the mathematical expressions for each element, is

\[
P = \frac{\rho_o L}{S} \ddot{x} + \frac{\rho_o \omega}{\lambda} \dot{x} + \frac{\rho_o C^2}{V} x
\]

(42)

Where:
- acoustic mass term = \( \frac{\rho_o L}{S} \ddot{x} \)
- acoustic dissipation term = \( \frac{\rho_o \omega}{\lambda} \dot{x} \)
- acoustic compliance term = \( \frac{\rho_o C^2}{V} x \)

\( S \) = area of opening.
\( L \) = depth of opening.
\( \rho_o \) = density of the medium.
\( \omega \) = circular frequency.
\( \lambda \) = wavelength of sound.
\( C \) = speed of sound in the medium.
\( V \) = volume of resonator.
\( X = S\xi \), where \( \xi \) is the displacement producing the volume change of the fluid in the resonator.

Solving this equation (42), the resonant frequency of the chamber under consideration may be obtained:

\[
\omega_R = \frac{C^2}{LV} \frac{S}{1 + \frac{S^2}{2L^2\lambda}}
\]

(43)

The second term in the denominator may normally be neglected, since it is usually much less than 1. Stewart [35] states that these theoretical conclusions are confirmed by experimental results.
Helmholtz resonators are observed to have additional resonant frequencies which are higher than the fundamental frequency given by the preceding equations (43). These higher modes result from the formation of standing waves in the cavity, rather than the oscillatory motion of the acoustic mass in the orifice. The overtone frequencies are not harmonically related to the fundamental since they depend on the shape of the cavity rather than its volume. As a rule, the frequency of the first overtone is several times that of the fundamental. This fact makes the Helmholtz resonator a good acoustic measuring device, since the fundamental frequency is easily distinguished from the overtone response.

C. The Effect of Change in Cross-Section in a Wave Guide

In the previous discussion of the plane wave equation it was noted that the propagation of plane waves in waveguides could be handled by the assumption that the wavefront was normal to the direction of propagation. In systems where a change or changes in cross-section occur, the wavefront deviates from its plane configuration in the immediate vicinity of the discontinuities. However, it has been found that the wave returns to its plane configuration within a small distance on either side of the discontinuity. It is, therefore, expedient to assume that planes exist throughout the system, with each discontinuity being treated simply as an abrupt change in acoustic impedance.

In the case of discontinuities of tube area, the pressure and volume velocity must be continuous across the plane at the junction of the two tubes of different cross section. Assuming that \( x = 0 \) at the junction (Fig. 4-1) and writing a balance for the pressure and volume velocity:
Figure 4-1. Transmission and reflection of plane wave at a junction between two pipes.
\[ P_1 = P_i + P_r = P_t \]
\[ U_1 = u_i + u_r = U_t \]

The volume velocity equals
\[ U_1 = Su = \frac{S}{\rho C} P \]

Where:
- \( S \) = cross-sectional area
- \( u \) = linear velocity
- \( \rho \) = density of the medium
- \( C \) = speed of sound in the medium
- \( P \) = sound pressure.

Thus:
\[ U_1 = S_1(u_i + u_r) = \frac{S_1}{\rho C} (P_i - P_r) \]

Dividing the pressure by the volume velocity to find the acoustic impedance:
\[ \frac{P_1}{U_1} = \frac{\rho C}{S_1} \cdot \frac{P_i + P_r}{P_i - P_r} = \frac{P_t}{U_t} \]  \hspace{1cm} (44)

The ratio \( P_t/U_t \) is the acoustic impedance looking to the right at the junction. If anechoic conditions exist the acoustic impedance is \( \rho C/S_2 \).

Therefore,
\[ \frac{\rho C}{S_1} \cdot \frac{P_i + P_e}{P_i - P_r} = \frac{\rho C}{S_2} \]  \hspace{1cm} (45)

The reflection factor, \( R \), is the ratio of the reflected wave to the incident wave. Thus,
\[ \frac{1 + R}{1 - R} = \frac{S_1}{S_2} \]  \hspace{1cm} (46)

and
\[ R = \frac{S_1 - S_2}{S_1 + S_2} \]  \hspace{1cm} (47)
Since the transmission factor is the ratio of the transmitted wave to that of the incident wave:

\[ T = \frac{4 S_1 S_2}{(S_1 + S_2)^2} \]  

[18] \hspace{1cm} (48)

It can also be shown that the acoustic impedance at any point in a tube, assuming anechoic conditions, is:

\[ Z_A = \left( \frac{\rho C}{S_1} \right) \frac{-j \omega}{C} + \frac{(S_1 - S_2)/(S_1 + S_2)}{e^{j \omega x}} \]  

[18] \hspace{1cm} (49)

It is important to note that this theory holds for both expanded or contracted sections in tubes.

From the above equation (49), if there were two changes in cross section (Fig. 4-2) then the acoustic impedance at the entry to tube II would be:

\[ Z = \frac{\rho C}{S_2} \left( \frac{J_{WL}^L}{C} + \frac{R'e}{C} \right) - \frac{J_{WL}^L}{C} \right) e^{-j \omega L} \]  

\[ e^{-j \omega L} - \frac{R'e}{C} \right) e^{-j \omega L} \]  

[50]

Where: \( R' \) = reflection factor at the junction between Tube II and Tube III

\( L \) = length of Tube II

and thus, the complex reflection factor at the junction between pipe I and pipe II is:

\[ R = \frac{Z - \rho C/S_1}{Z + \rho C/S_1} \]  

[51]
Figure 4-2 Transmission and reflection of a plane wave at two changes in cross-section
Similarly, the transmission factor is:

\[
T = \frac{1}{1 + \frac{1}{4}(S_{21}^2 - 2) \sin^2 KL}
\]  

(52)

Where:  \( S_{21} \) = the ratio of the area of Tube II to Tube I

\( L \) = length of Tube II

A simple form of acoustic filter may be constructed by inserting a short tube of larger or smaller cross-sectional area into a waveguide. Very often, for short lengths of intermediate tubing, the major frequencies of the transmitted sound are below the frequency at which the first minimum of the transmission factor occurs. The element is then termed a "low-pass filter", with the attenuation being roughly proportional to frequency. The major factor determining the cutoff frequency of the filter is the length of the expanded or contracted section. Low pass filters will be discussed in more detail later in the discussion of acoustic filters.

D. General Theory of a Side Branch

If open or closed tubes are attached as branches to an acoustic conduit, there is an observable effect on the transmission of sound through the conduit. The presence of such a branch causes the acoustic impedance at the junction to vary from the characteristic value for plane waves in tubing. Not only is sound energy transmitted up the branch tube and dissipated, but a reflected wave is also produced. Both of those factors contribute to a transmission loss in energy traveling down the conduit.

Considering a tube of uniform cross section, \( S \), with an arbitrary side branch which may be open or closed at the end and in which anechoic
conditions exist, then the incident pressure wave may be represented
by:

\[ J(\omega t - \frac{\omega x}{C}) \]
\[ p_i = A_1 e^{Jt} \]  
(53)

This wave will, in general, produce a reflected wave:

\[ J(\omega t + \frac{\omega x}{C}) \]
\[ p_r = B_1 e^{Jt} \]  
(54)

and a transmitted wave:

\[ J(\omega t - \frac{\omega x}{C}) \]
\[ p_t = A_2 e^{Jt} \]  
(55)

If the branch junction is chosen as the origin of the x coordinate, then the pressures at this point may be represented by:

\[ J\omega t \]
\[ p_i = A_1 e^{J\omega t} \]  
(56)

\[ J\omega t \]
\[ p_r = B_1 e^{J\omega t} \]  
(57)

\[ J\omega t \]
\[ p_t = A_2 e^{J\omega t} \]  
(58)

The presence at the entrance to the branch may be similarly represented by:

\[ J\omega t \]
\[ p_b = A_b e^{J\omega t} \]  
(59)

It may be noted that this analysis assumes that the diameters of all tubes considered are small in comparison with the wavelength of the sound. Thus, the conditions of continuity of pressure and volume velocity can be applied to obtain:

\[ p_i + p_r = p_t = p_b \]  
(60)

\[ u_i - u_r = u_t + u_b \]  
(61)
Then dividing equation (61) by equation (60):

\[
\frac{u_i - u_r}{P_i + P_r} = \frac{u_t + u_b}{P_t + P_b}
\]  

(62)

Since the volume velocity \( U = \frac{PS}{\rho C} \), then:

\[
\frac{S}{\rho C} \frac{P_i - P_r}{P_i + P_r} = \frac{S}{\rho C} \frac{P_t}{P_t} + \frac{1}{Z_b}
\]

or:

\[
\frac{S}{\rho C} \frac{A_i - B_i}{A_i + B_i} = \frac{S}{\rho C} \frac{1}{Z_b}
\]

(63)

It may be observed that this is the inverse of the impedance terms, or:

\[
\frac{1}{Z} = \frac{1}{Z_t} + \frac{1}{Z_b}
\]

(64)

Solving equation (63) for the reflection factor at the junction, \( R \):

\[
R = \frac{B_i}{A_i} = \frac{1}{1 + \frac{2S}{\rho C} Z_b} = \frac{1}{1 + \frac{2S}{\rho C} Z_b}
\]

(65)

And, the transmission factor, \( T \), is:

\[
T = \frac{A_2}{A_1} = \frac{Z_b}{\frac{\rho C}{2S} + Z_b}
\]

(66)

Where: \( A_i \) = incident pressure wave.
\( A_2 \) = transmitted pressure wave.
\( B_i \) = reflected pressure wave.
\( Z_b \) = impedance of the branch.
\( e \) = density of the medium.
\( C \) = speed of sound in the medium.
\( S \) = tube cross section.
It is to be noted that when there is no sound energy transmission past the branch tube, the transmission factor is zero. However, for the transmission factor to be zero in equation (66), the impedance of the branch tube must be zero. And, in turn, if the impedance of the branch is zero, then the reflection factor is unity. A filter for which this is true does not absorb all the sound energy that reaches the junction, rather it absorbs no energy and reflects all of the incident waves back through the tube towards the sound source. At the opposite extreme, as $Z_b$ becomes very large as compared to $\rho C/S$ then almost all of the sound energy is transmitted past the branch.

If a Helmholtz resonator was employed as a side branch in an acoustic conduit, it would be observed that there is no net dissipation of sound energy in the resonator (neglecting viscosity losses) [35]. Thus, the real part of the impedance of the branch is zero. However, the acoustic reactance of the branch [35], the imaginary part of the impedance, is:

$$X_b = \rho \frac{\omega L'}{S_b} - \frac{C^2}{\omega V}$$  \hspace{1cm} (67)

Where:

- $\rho$ = density of the medium
- $\omega$ = circular frequency
- $L$ = length of the resonator's neck
- $L' = L + 1.7a$, correction for end conditions on the neck length
- $a$ = radius of the neck
- $S_b$ = area of the neck opening, $\pi a^2$
- $C$ = speed of sound in the medium
- $V$ = volume of the resonator

A substitution of equation (67) into equation (66) for the complex transmission factor yields:
\[ T = \frac{1}{1 + \frac{\rho C}{2S} \left( \frac{1}{\rho} \right) \left[ \frac{1}{\omega L} \frac{1}{C^2} \right] S_b \omega V - \omega^2 L'V \} - S_b C^2 \] 

or:

\[ T = \frac{1}{1 + \frac{S_b}{2S} \left[ \frac{C\omega V}{\omega^2 L'V - S_b C^2} \right]} \]  

(68)

It may be noted that the transmission factor becomes zero when:

\[ \omega = C \frac{S_b}{L'V} \]  

(69)

Where: \( \omega \) = the resonant frequency of the Helmholtz resonator.

At this frequency large volume velocities exist at the neck of the resonator, and all of the acoustic energy that is transmitted into the resonator from the incident wave is returned to the conduit with such a phase relationship as to be reflected back to the source. Both Stewart and Lindsay [35], and Kinsler and Frey [18] noted that experimental measurement was in close agreement to the values predicted by equation (67) for large necks. However, for long narrow necks the experimental values deviate from theoretical, and equation (67) must be modified for viscosity effects. One significant characteristic of the Helmholtz resonator, when used as a side branch is that it creates a significant reduction in transmission over a two octave bandwidth: the center frequency being the resonant frequency of the device. This filtering response has a much broader band-width than that of a Helmholtz resonator used in series with a waveguide.

Another configuration commonly employed as a side branch is the orifice. This branch is merely a hole drilled or stamped in an acoustical conduit. The length of the orifice side branch (the wall thickness
of the conduit) is essentially zero, and the radius of the opening is assumed to be small when compared to the incident wavelength. Taking these factors into account, the transmission factor can be expressed as:

$$T = \frac{1}{1 + \frac{a}{J\pi ac/3.45\omega}} \quad [35]$$

where:  
- $$a$$ = radius of the opening.  
- $$c$$ = speed of sound in the conduit.  
- $$\omega$$ = circular frequency.  
- 3.45 = end correction for the orifice.

As can be observed from equation (70), the transmission factor is very small at low frequencies and approaches unity at high frequencies. Thus, the presence of a single orifice converts a tube into a high-pass filter. Furthermore, the filtering action of an orifice is a direct result of the reflection of acoustic energy toward the source rather than transmission of the sound energy out of the orifice. This is to be expected in that a high resistance is characteristic of such an orifice.

As the radius of the orifice is increased, relative to the dimensions of the conduit, the attenuation of low frequency sound is increased. However, in general, it has been found that the low-frequency attenuation of a number of suitably spaced orifices can be made much greater than that of a single orifice of equal total area. [18,35]

E. The Consideration of Acoustic Filters Utilizing a Combination of Elements

In the previous sections, the filtration characteristics of singular acoustic elements have been illustrated, and their limitations noted. However, by employing combinations of resonators, orifices, branch pipes, and enlargements or constrictions of the main pipe, it is possible to
construct a multitude of acoustic filtering systems. The design of these systems is greatly simplified by use of electrical analogies. The theory of acoustic filters has progressed on the basis of such electric comparisons as the generalizations that an inertance in an acoustic system (constriction in a conduit) will have an effect similar to an inductance in an electrical circuit and a compliance (a resonant cavity) have an effect similar to an electrical capacitance. For example, an elementary type low-pass, electrical filter can be produced by shunting a capacitor across a transmission line; whereas, the analogous acoustic filter is constructed by inserting a short expanded section of tubing in the main conduit. Furthermore, the greater the length of the enlarged section of tubing, the lower the frequency at which the filter begins to attenuate sound waves. Similarly, increasing the capacitance in an electrical circuit has the effect of lowering the cut-off frequency of an electrical filter.

It has been observed that the sharpness of the cut-off frequencies of an acoustical filter can be increased by constructing reactance networks [35]. Such networks are constructed of sections which are connected in tandem to form a "ladder network". Each section consists of a series impedance and a shunt impedance, and most of the important characteristics of such networks can be given in terms of these two impedances. Furthermore, Kinsler and Frey [18] state that a non-dissipative recurrent structure composed of series and shunt impedances will only pass frequencies that cause the ratio of the series impedance to the shunt impedance to lie between 0 and -4.

Three common, ladder-type, acoustical filters - low pass, high pass, and bandpass filters, are shown in figures (4-4,4-5,4-6) with
their electrical analogies. The theory behind such networks is not presented here, although it may be stated that the cut-off frequencies for both low- and high-pass filters are given by the electrical analog:

\[ f_c = \frac{1}{\pi L_e C_e} \]  \hspace{1cm} (71)

where:  \( L_e \) = electrical impedance

\( C_e \) = electrical capacitance

G. W. Stewart [35], in his work with lumped parameters, found that for a low-pass filter utilizing Helmholtz resonators, such as in figure 4-3, the cut-off frequency per chamber is:

\[ f_{c_L} = \frac{C}{\pi} \left[ \frac{S_1}{L_1 V_2} \left( \frac{1}{1 + \frac{4S_1}{L_1 C_0}} \right) \right]^{1/2} \]  \hspace{1cm} (72)

where:  \( C \) = speed of sound in the medium.

\( S_1 \) = area of cross-section of the main line.

\( L_1 \) = length of one main line section.

\( V_2 \) = volume of resonator.

\( C_0 = \frac{\pi a^2}{L + \frac{\pi a}{2}} \) = conductivity of orifice (must be multiplied by number of orifices per resonator).

\( a \) = radius of orifice.

\( L \) = length of channel opening.

Since this is the cut-off frequency per chamber and conductivities in parallel are additive, the frequency obtained by equation (72) must be multiplied by the number of chambers to obtain the overall cut-off frequency for the filter. Similarly, the cut-off frequency for a high-pass filter, such as in figure 4-4, has been found to be:
Figure 4-3 Low-pass, ladder-type acoustic filter
Physical Filter Configuration

Electrical Circuit Analogy

Figure 4-4  High-pass, ladder-type acoustic filter
\[ f_{CH} = \frac{C}{2\pi} \cdot \frac{C_0}{V_1} \]  

(73)

where:  \( V_1 \) = volume of one section of the line.

\( C_0 \) = orifice conductivity.

As in the low-pass filter, this cut-off frequency applies only to one filter section, and must be multiplied by the number of sections.

Stewart has also found that for band-pass filters of the type in figure 4-5, the cut-off frequencies per chamber are:

\[ f_L = \frac{C}{2\pi} \left[ \frac{C_0 S_2}{V_2 (L_2 C_0 + S_2)} \right]^{1/2} \]  

(74)

\[ f_H = \frac{C}{2\pi} \left[ \frac{S_2}{L_2 V_2} \left( \frac{4 L_2 S_1}{L_1 S_2} \right) \right]^{1/2} \]  

(75)

where:  \( C_0 \) = conductivity of orifice.

\( S_1 \) = cross-sectional area of main conduit.

\( L_1 \) = section length of main conduit.

\( S_2 \) = cross-sectional area of side branch.

\( L_2 \) = length of side branch.

\( V_2 \) = volume of side chamber.

\( \gamma \) = speed of sound in the medium.

It should be noted that the values obtained from equations (74) and (75) must be multiplied by the number of filter sections.

F. Considerations for the Application of Filter Theory

It must be remembered that the theory presented in this section applies to infinite filters (filters terminated by the characteristic
Figure 4-5 Band-pass, ladder-type acoustic filter
impedance of the medium); whereas, all filters constructed are finite. However, the only modification necessary for application of this theory to finite filters is an allowance for the impedance of the conduit at the two ends of a finite number of filter sections. If possible the impedance of the conduit should match that which is theoretically observed in infinite filters. However, this matching of impedances is possible only for a selected frequency and, consequently, the characteristics of an infinite filter cannot be obtained for an entire frequency range, in actual practice.

It should also be mentioned that filters can be divided into two categories: dissipative types and reactive types. A dissipative type is one whose acoustical characteristics are determined predominantly by flow-resistive, acoustically absorbent material. On the other hand, reactive mufflers are filters whose acoustical performance depends primarily on the interaction of reflective acoustical elements. This is not to say, however, that some such reactive elements are not used with dissipative material in order to enhance their acoustical value.

This thesis specifically is concerned with reactive filters and filter elements and, therefore, a discussion of dissipative constructions is not included. It is also important to realize that many simplifying assumptions are implied in the foregoing discussion of acoustic elements. Below are itemized the more important assumptions:

1.) Steady flow through a system does not affect acoustic performance.
2.) Temperature variations in the system are negligible and do not affect sound propagation.
3.) Sound pressures are small compared with absolute pressures, such that non-linear effects are negligible.

4.) Filter wall surfaces do not conduct or transmit sound.

5.) Sound is propagated in plane waves, unattenuated by viscosity or heat transfer.

It is intuitively obvious that each qualification above has some effect in an actual filtering system, and a combination of these effects might produce a large amount of error in a theoretical analysis. However, it is felt that a significant preliminary estimate of muffler performance can be obtained from the theory presented in this section, when combined with experimentally measured reflection and transmission factors.
CHAPTER V
THE DESIGN AND EVALUATION OF AN
ACoustically AdjUSTABLE AUTOMOTIVE MUFFLER

A. Introduction

In present-day terminology, a muffler is a combination of acoustical elements which impedes the transmission of sound through it, while permitting the free flow of gas or liquid. This definition implies three criteria which must be satisfied for successful muffler design [4]:

1.) The "Acoustical Criterion" specifies the minimum noise reduction required for the muffler as a function of frequency.
2.) The "Aerodynamic Criterion" specifies the maximum average permissible pressure drop through the muffler at a given temperature and mass flow.
3.) The "Geometrical Criterion" specifies the maximum allowable volume and restrictions on shape.

This section is primarily concerned with the "Acoustical Criterion". Herein is presented the design and evaluation of an adjustable automotive muffler in which the interaction of several reactive acoustic elements can be varied. Also included are the descriptions of analytical methods used, and the experimental apparatus employed.

B. The Physical Design of the Adjustable Muffler

The idea of an adjustable automotive muffler was conceived by W. S. Gatley through his work with mufflers for small refrigeration systems. As previously stated, the actual design was a modification of a standard production muffler. Necessarily, the design incorporated
adjustment of two basic acoustical elements: acoustical compliance and acoustical inertance. It was felt that adjustment of these two elements should, in theory, change the filtering characteristics of the muffler.

The casing of the adjustable muffler consists of an 18 inch length of 5 5/8 inch outside diameter aluminum tubing. The wall thickness of the tubing is 1/8 inch, and there are 14 holes drilled and tapped for 3/16 inch cap screws to hold the internal chamber dividers in place. (See figures 5-1, 5-2). The muffler has four chamber dividers and two end caps machined from 3/8 inch thick steel plate. The outside diameter of the dividers and end caps is slightly smaller than the 5 3/8 inch inside diameter of the casing to allow clearance for ease of movement of the dividers. In each divider and end cap a reciprocating type O-ring groove is machined to accommodate high temperature, 1/4 inch, silicon O-rings. (See figure 5-3). Only one chamber divider is readily adjustable from outside the muffler. This adjustment creates a difference in chamber size and is, thus, a variation of acoustic compliance. (See figures 5-4, 5-5). However, the design enables the internal dimensions of the muffler to be changed if desired. For ease of assembly and discussion, the dividers are numbered in sequence from outlet to inlet. As can be seen in figure 5-4 the outlet end cap is number 1 and the inlet end cap is number 6.

End caps numbered 1 and 2 are held in place by six 3/16 inch cap screws each (figure 5-2). Dividers numbered 3, 4, and 5 are fixed in relation to one another by a 3/8 inch threaded rod (figure 5-4). Dividers numbered 3 and 4 are fixed by design to remain in one position; however, divider number 5 is free to be adjusted to create a difference.
Figure 5-1. External view of adjustable muffler showing outlet end and adjustments.

Figure 5-2. Internal view of muffler from the inlet end.
outlet louver adjustment

outlet louver

center-section louvers

muffler inlet

center-section louver adjustment

outlet chamber size adjustment

chambered attenuator

silicon 0-rings

Figure 5-3 Schematic of the internal construction of the adjustable muffler
Figure 5-4. Disassembled view of the adjustable muffler.
Figure 5-5 Internal dimensions of the adjustable muffler
in inlet chamber size by dismantling the muffler and adjusting its position on the threaded rod. This is, however, somewhat clumsy and time consuming, so divider number 5 was left in one position throughout the tests. Chamber divider number 2 is the only externally adjustable partition. The amount of travel of this partition and other pertinent dimensions of the muffler are shown in figure 5-6.

In all, there are three external adjustments readily available to the investigator: outlet chamber size adjustment and two side branch louver adjustments (figures 5-4, 5-5). The adjustment of the amount of louver opening is, in reality, an adjustment of an acoustical inertance in combination with a resonant chamber.

The muffler is constructed and lubricated such that it can be utilized in simulated exhaust environments with appropriate temperatures and gas flows.

C. Methods of Evaluating Muffler Performance

There are many methods of evaluation available to the prospective investigator of muffler performance. "Insertion loss", "transmission loss", "noise reduction", "sound pressure level differences", "end differences", and "attenuation" are but a few of the terms used to describe the effectiveness of acoustic filters. It is, therefore, difficult to determine which evaluation method best describes the characteristics of a given muffler configuration. Since standardization of muffler evaluation criteria has not yet occurred in the field of noise control, this section will describe some of the more popular means of muffler appraisal.

"Insertion loss" is defined as the difference between two sound pressure levels which are measured at the same point in space,
Figure 5-6. View of the acoustic driver and standing wave tube inlet plenum chamber.
before and after a muffler is inserted between the measurement point and the noise source.

"Transmission Loss" is defined as the ratio expressed in decibels of the sound power incident on the muffler to the sound power transmitted by the muffler. It is a useful analytic concept; however, calculations are hampered by the fact that the acoustic power cannot be measured directly, and must be calculated from pressure measurements. Also, the incident pressure amplitude must first be separated from the reflected amplitude.

"Sound pressure level difference," "noise reduction", and "end differences" refer to the difference between the sound pressure level measured on the upstream side of a muffler and the sound pressure level measured on the downstream side.

"Attenuation" is the decrease of sound power in decibels between two points in an acoustical system. It is a useful concept for describing the loss of power of a sound wave traveling through a muffler, but it does not in itself tell how effective a muffler is in a given system.

It is also important to note that "insertion loss", "transmission loss", and "noise reduction" are not uniquely related to the physical properties of a muffler. Since the acoustical effectiveness of any muffler depends on source and termination impedances, each of these parameters must be considered when measuring actual measures of the muffler performance. Beranek [4] states that, in noise control engineering, "insertion loss is perhaps the most useful measure of acoustical effectiveness".
All of the above evaluation methods are essentially designed to quickly determine the overall effectiveness of an acoustic filter. That is to say, these methods are created to evaluate mufflers on a "go no-go" basis. They lend essentially nothing to the investigator's knowledge of the interaction of the acoustic element; only the overall performance picture is presented. Thus, a method of evaluation was devised which incorporates the fundamentals of several of the above-mentioned methods. The first phase of the investigation was a pure tone analysis of the reflection and transmission characteristics of the adjustable muffler. This analysis closely relates to the "transmission loss" evaluation criteria. In addition, engine exhaust recordings were played through the muffler and the input spectrum was then compared to the output to investigate the effect of the adjustable muffler on spectral noise. This portion of the investigation is similar to "noise reduction" methods of evaluation.

D. Experimental Determination of Reflection Factors

When utilizing the two inch inside diameter standing wave tube developed by Simon [33], reflection factor measurements require location of a sound pressure minimum, and either a sound pressure maximum or a reference pressure. For frequencies from 0 to 200 Hertz, when no maximum pressure can be found in the tube measuring length to coincide with a minimum pressure, the reflection factor magnitude is given by equation (76):

$$|R| = \frac{A_m e^{2\alpha x_{\text{min}} - P_{\text{min}}} e^{\alpha x_{\text{min}}}}{A_m}$$

(76)

where:

$$A_m = \frac{1}{2} \left[ P_{\text{min}} + \left[ P_{\text{min}}^2 - \frac{2(P_{\text{min}}^2 - P(x)^2)^{1/2}}{1 - \cos \phi} \right] \right]$$
\( P_{min} = \) minimum sound pressure (v.)

\( P(x) = \) reference pressure (v.)

\( x_{min} = \) location of the minimum sound pressure relative to the filter element (cm.)

\( \alpha = \) attenuation constant (1/cm.)

\( e = \) base of the natural logarithms

\( \phi = 2 \frac{\omega}{C'} x_{Rx} \)

\( X_{rx} = \) arbitrary pressure location relative to the minimum pressure location (cm.)

\( C' = \) corrected speed of sound in the tube (cm/sec)

\( \omega = \) circular frequency (rad/sec)

For frequencies about 200 Hertz, when both a maximum pressure and a minimum pressure can be found in the measuring length of the standing wave tube, the reflection factor is given by equation (77)

\[
|R| = \frac{p_{max} e^{\alpha x_{min}} - p_{min} e^{\alpha x_{max}}}{p_{max} e^{-\alpha x_{min}} + p_{min} e^{\alpha x_{max}}}
\]

where:

\( p_{max} = \) maximum sound pressure (v.)

\( p_{min} = \) minimum sound pressure (v.)

\( x_{max} = \) location of the maximum sound pressure relative to the filter element. (cm.)

\( x_{min} = \) location of the minimum sound pressure relative to the filter element (cm.)

\( \alpha = \) attenuation constant (1/cm.)

\( e = \) base of the natural logarithms

Also, the reflection factor phase angle can be determined from equation (78):
\[ \theta = \frac{2\pi x_{\text{min}_1}}{x_{\text{min}_2} - x_{\text{min}_1}} - N\pi \]  

(78)

where: \( N = 1, 3, 5, \ldots \)

\[ \lambda/2 = x_{\text{min}_2} - x_{\text{min}_1} \]

\( x_{\text{min}_2} \) = second minimum pressure measured in the tube.

Experimentally, the determination of the reflection factor and its related phase angle involves measurement of two sound pressure minimums and a sound pressure maximum for frequencies above 200 Hertz, and, in the case of low frequencies, a sound pressure minimum and a reference pressure. The pressure measurements for high frequencies and taken such that the first minimum is located closest to the muffler and the measured maximum is in between the first and second pressure minimums. This method of measurement minimizes the effect of tube attenuation. For low frequencies, extensions are added to the standing wave tube until a pressure minimum can be found in the measuring length. Reference pressures are then measured as far away from the minimum pressure as possible in the direction of the filter.

In general, it has been observed that the location of sound pressure minimums can be measured with much greater accuracy than sound pressure maximums, since the minimum pressures are generally sharp and well defined. Furthermore, the most exact location of sound pressure minimums is found by locating equal sound pressures on either side of the minimum and then taking the mean of the two distance measurements. This procedure was suggested by several other investigators.\([10,33,31]\)

As can be observed from equation (78), the calculation of reflection phase angles requires that the distance from the muffler to each minimum location must be accurately measured to determine the half-
wavelength of the impinging sound. For low frequencies the wavelength must be theoretically calculated. (See Appendix II). It is possible, at higher frequencies, to observe several wavelengths in the standing wave tube within 30 inches of the muffler. If the sound pressure minimum nearest the filter is used in the phase angle calculations only a small error in measurement of the wavelength is encountered. However, if a sound pressure minimum farther away from the muffler is utilized, additional error is incorporated in each integral wavelength existing between the muffler and the measured sound pressure minimum. Thus, in this investigation, sound pressure minimums were measured as close as possible to the muffler. In general, the error in phase angle calculations decreases in magnitude as the wavelength increases.

E. The Standing Wave Tube

In this analysis the reflection factor measurements were made with a 2-inch inside diameter standing wave tube constructed by V. H. Simon[33]. All measurements were made with the adjustable muffler terminated anechoically. The standing wave tube consists of a 6 foot length of 2 1/2 inch outside diameter aluminum tubing coupled at one end to an acoustic driver and plenum chamber and mounted on an 8 foot length of 8 inch wide steel channel (figure 5-7). The sound field is explored by a microphone probe inserted through the wall of the tube. The top of the tube is milled flat, and a 1/8 inch slot is machined down the center of the flat area for insertion of the microphone probe (figure 5-8). The slot is sealed by a 1 inch wide continuous spring steel band. The probe and microphone are mounted in an aluminum block that is attached to the steel band by two dowel pins, four small machine screws,
Figure 5-7. The standing wave tube and related apparatus.
Figure 5-8 Schematic of the two inch inside diameter aluminum tube used for standing wave measurements
and gasket cement. The relationship between the tube, steel band, and microphone block assembly is shown in Figure 5-9.

Sound pressure measurements are made by sliding the microphone block assembly and steel strip along the length of the slot. The steel band is pressed firmly against the flat area of the tube by two lengths of surgical tubing placed along the inside faces of the guide rails and pressurized by compressed air at approximately 15 psig, (figure 5-10). The locations of the sound pressures are measured by a system that incorporates a cursor mounted on the movable microphone block which reads the location on a fixed two-meter scale attached to the tube cradles. The hairline of the cursor is accurately aligned with the center of the microphone probe and the zero end of the meter stick is accurately aligned with the filter end of the aluminum tube. The configuration of the apparatus allows measurements to be made within 31 centimeters of the termination (figure 5-11).

The pure tone sound in the standing wave tube is generated by a beat-frequency oscillator, amplified by a 40 watt amplifier, and introduced into the tube through a high intensity driver. A frequency counter is also incorporated into the circuit so that the frequencies investigated are always exact. The output from the microphone is passed through a sound and vibration analyzer with a 1/10 octave band filter before being measured by an Rms voltmeter.

All measurements were made as rapidly as possible to minimize drift errors, and the environment kept as stable as possible to minimize temperature and humidity effects. Also, before the muffler was evaluated, the standing wave tube and the anechoic termination were examined for the frequencies of interest (Appendix IV, V).
Figure 5-9 Exploded view of two inch inside diameter standing wave tube measuring station
Figure 5-10 Cross-sectional view of the two inch inside diameter standing wave tube apparatus
Figure 5-11. View of the standing wave tube pressure location measuring system.
As previously mentioned, the low frequencies of sound investigated require the use of tube extensions to obtain a sound pressure minimum in the measuring length of the standing wave tube. Also, it is necessary to employ Kirchoff's equation for the speed of sound in tubes [4] in conjunction with the relationship:

$$\frac{\lambda}{2} = \frac{C'}{2f}$$

(5)

to obtain the half-wavelength of sound for transmission calculations. The calculated values for the speed of sound and the wavelength are given in Appendix II. The tube attenuation factors found from the modified Kirchoff formula (3), which are used in reflection factor calculations, are presented in Appendix I. All of the reflection factor calculations were performed by digital computer, and the programs used for these calculations are given in Appendix III.

F. Experimental Determination of Transmission Factors

As will be recalled from Chapter III the transmission factor is the ratio of the transmitted wave to the incident wave:

$$T = \frac{A_2}{A_1}$$

(1)

This result is obtained by combining the results of the reflection factor measurements with data acquired from transmission measurements. The transmission measurements consist of the determination of the magnitude of the sound pressures that exist at a known distance on either side of the muffler, and the relative phase angle between these pressures. These measurements are accomplished with the use of "transmission tubes". The tubes consist of two, 2 inch outside diameter, brass conduits with microphone stations soldered into the sidewalls. The distance from the
microphone stations to the end of the brass transmission tubes is 18 inches. The only restriction placed on this distance is that it have a value of at least one tube diameter to avoid the possibility of transverse modes and near field distortion.

The sound pressure magnitudes on the upstream and downstream sides of the muffler were measured with two matched microphones.* The microphones were connected to a switchbox which was in turn connected to the 1/10 octave band frequency analyzer and the Rms voltmeter. The switchbox allowed measurements to be taken on the upstream and downstream sides of the muffler without connecting and disconnecting microphone leads. The switchbox was also connected to a phase meter which measured the relative phase angle between the two microphone stations. The frequency counter was also employed to insure exact input frequencies. The details of the experimental setup may be seen in figures 5-12 and 5-13.

All transmission factors were calculated by digital computer, by utilizing an adaptation of the graphical calculation method presented in Chapter III. The program for these calculations is presented in Appendix III. A list of equipment employed in this analysis may be found in Appendix IX.

G. The Results of the Experimental Pure Tone Reflection and Transmission Factor Measurements

As presented previously in this chapter, the adjustable muffler has three integral adjustments. Thus, to evaluate the muffler it was necessary to develop a sequence of adjustments which would be amenable

*An error analysis is presented in Appendix VIII.
Figure 5-12. Experimental system for reflection factor analysis.
Figure 5-13. Experimental system for transmission factor analysis
to scientific investigation. It was decided that the change of 
compliance (movement of the Piston to change chamber size) would be 
the main parameter of adjustment, with the two side branch adjustments 
being secondary. The total possible movement of the piston is 2 5/8 
inches (figure 5-6); therefore, four chamber settings were selected: 
the first being full chamber; the second, one inch smaller; the third, 
two inches smaller; and the fourth, a fully closed chamber. For each 
chamber size the internal louvers were either fully open or fully 
closed. By means of this system, sixteen adjustments were made for each 
pure tone frequency. The muffler was evaluated at seven different 
frequencies: 50, 100, 200, 500, 750, 1000, and 1550 Hertz. Therefore, 
a total of 112 different data points were obtained from the adjustable 
muffler.

To simplify matters, each adjustment is designated by a run number. 
The terminology for each adjustment at a specific frequency is given 
below (refer to figures 5-4 and 5-14):

1.) Roman numeral - piston settings
2.) Alphameric numeral - internal louver setting
3.) Cardinal numeral - outlet louver setting

where: I - Chamber fully open.
II - Piston setting one inch toward outlet endcap.
III - Piston setting two inches toward outlet endcap.
IV - Chamber fully closed.
A - Internal louvers closed.
B - Internal louvers open.
1 - Outlet louvers closed.
2 - Outlet louvers open.
Figure 5-14. Electronic equipment for acoustic analysis.
Thus, for example, a setting designated by the run number IIA2 would be: The outlet chamber piston one inch out from fully open; the internal louvers closed, such that a straight flow path with no side branch may be assumed; and the outlet louvers open, such that a side branch effect exists.

On the following pages, the reflection and transmission factors and their respective phase angles are presented in both tabular and graphical form, with observations on muffler performance.
<table>
<thead>
<tr>
<th>RUN</th>
<th>REFLECTION FACTOR</th>
<th>TRANSMISSION FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude</td>
<td>Phase Angle</td>
</tr>
<tr>
<td></td>
<td>(Dimensionless)</td>
<td>(Degrees)</td>
</tr>
<tr>
<td>IA1</td>
<td>0.312</td>
<td>173.17</td>
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<tr>
<td>IA2</td>
<td>0.334</td>
<td>168.21</td>
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<td>172.91</td>
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<td>IIA1</td>
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</tr>
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Figure 5-15 Reflection Characteristics of Adjustable Muffler at 50 Hertz
Figure 5-16 Reflection Phase Angle of Adjustable Muffler at 50 Hertz
Figure 5-17 Transmission Characteristics of Adjustable Muffler at 50 Hertz
Figure 5-18 Transmission Phase Angle for Adjustable Muffler at 50 Hertz
Observations on Muffler Adjustment at 50 Hertz

From figures 5-15 through 5-18 it may be observed that the adjustments at 50 Hertz have more effect on phase angle than on the magnitudes of the reflection and transmission factors. This is to be expected from simple acoustic theory since the wavelength at 50 Hertz is much greater than any dimension of the muffler.

A general tendency which may be observed in the reflection factor curve (figure 5-15) is that no matter what the chamber size; the reflection factor is higher when the center section louvers are open. This is indicative that the dimensions of this orifice, side branch and resonant chamber configuration are effective in partial cancellation of low frequency noise. Also observable in the reflection factor curve is the effect of the change of compliance by use of the movable chamber divider. The reflection factor reaches a maximum as the divider moves through its central position and then drops off slightly. The reflection factor phase angle (figure 5-16) also has a general tendency to increase as the chamber divider moves toward the endcap.

The effect of the muffler adjustments on the transmission factor (figure 5-17) are less obvious than on the reflection factor. However, in general, when the outlet louvers are covered, the transmission factors are higher. This is no doubt due to the lack of side branch dissipation in the outlet chamber. Furthermore, the adjustment of chamber size has no observable effect on the transmission factor. The transmission phase angle remains fairly constant for all adjustments except at the last piston setting; it is, however, generally higher when the outlet louvers are open.
The low transmission of the prototype, adjustable muffler (less than 50 percent transmission) is fairly good at this low frequency. However, this is an overall effect of muffler dimensions rather than the product of a single adjustable element as may be observed from the lack of response in the transmission factor to adjustment. Furthermore, the reflection factor of the anechoic termination at this frequency is 0.279; the observed reflection factor of the adjustable muffler is only slightly higher than this figure. Thus, the results of this frequency study are somewhat questionable; the observed characteristics may be due more to the effects of the termination rather than the muffler.
### TABLE 5-2

**REFLECTION AND TRANSMISSION CHARACTERISTICS**

**OF THE ADJUSTABLE MUFFLER AT 100 Hertz**

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<th>TRANSMISSION FACTOR</th>
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<td></td>
<td>Magnitude</td>
<td>Phase Angle</td>
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<tr>
<td></td>
<td>(Dimensionless)</td>
<td>(Degrees)</td>
</tr>
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<td>0.484</td>
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<td>IIA2</td>
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</tr>
<tr>
<td>IIB1</td>
<td>0.530</td>
<td>188.03</td>
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<tr>
<td>IIB2</td>
<td>0.512</td>
<td>189.06</td>
</tr>
<tr>
<td>IIIA1</td>
<td>0.532</td>
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</tr>
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</tr>
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<td>IIIB2</td>
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<td>178.17</td>
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<td>IVB1</td>
<td>0.635</td>
<td>181.26</td>
</tr>
<tr>
<td>IVB2</td>
<td>0.625</td>
<td>178.65</td>
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Figure 5-19  Reflection Characteristics for the Adjustable Muffler at 100 Hertz
Figure 5-20  Reflection Factor Phase Angle for the Adjustable Muffler at 100 Hertz
Figure 5-21 Transmission Characteristics for the Adjustable Muffler at 100 Hertz
Figure 5-22  Transmission Phase Angle for the Adjustable Muffler at 100 Hertz
Observations on Muffler Adjustment at 100 Hertz

As can be observed from figures 5-19 through 5-22, the response of the muffler to adjustment is greater at 100 Hertz than at 50 Hertz. In general, the reflection factor (figure 5-19) can be seen to increase as the volume of the chamber into which the outlet louvers open decreases.

Also, for each chamber setting, the reflection factors are higher when the center section louvers are opened, and the outlet louvers closed. The reflection factor phase angle (figure 5-20) has a tendency to decrease as the outlet chamber size decreases.

The adjustments have a less observable effect on the transmission factor than on reflection factor (figure 5-21). However, as the outlet chamber size decreases, the transmission factor decreases. Also, as can be expected from the reflection factor curves the transmission factors are the lowest when the center-section louvers are open and the outlet louvers are closed. The transmission phase angle (figure 5-22) remains essentially the same for all adjustments; the maximum variation being 45 degrees. The greater phase angles are observed when the outlet louvers are closed.
<table>
<thead>
<tr>
<th>RUN</th>
<th>REFLECTION FACTOR</th>
<th>TRANSMISSION FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude</td>
<td>Phase Angle</td>
</tr>
<tr>
<td></td>
<td>(Dimensionless)</td>
<td>(Degrees)</td>
</tr>
<tr>
<td>IA1</td>
<td>0.446</td>
<td>221.00</td>
</tr>
<tr>
<td>IA2</td>
<td>0.394</td>
<td>206.82</td>
</tr>
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<td>IB1</td>
<td>0.276</td>
<td>199.85</td>
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<td>IB2</td>
<td>0.260</td>
<td>170.86</td>
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<tr>
<td>IIA1</td>
<td>0.490</td>
<td>220.71</td>
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<td>IIA2</td>
<td>0.432</td>
<td>217.60</td>
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<td>IIB1</td>
<td>0.286</td>
<td>225.61</td>
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<td>IIB2</td>
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<td>204.30</td>
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<td>IIIA1</td>
<td>0.515</td>
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<td>IIIA2</td>
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<td>IIIB1</td>
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<td>228.84</td>
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<tr>
<td>IIIB2</td>
<td>0.337</td>
<td>231.61</td>
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<tr>
<td>IVA1</td>
<td>0.573</td>
<td>229.94</td>
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<td>IVA2</td>
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<td>IVB1</td>
<td>0.422</td>
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<td>IVB2</td>
<td>0.394</td>
<td>233.17</td>
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Figure 5-23 Reflection Characteristics for the Adjustable Muffler at 200 Hertz
Figure 5-24 Reflection Phase Angle for the Adjustable Muffler at 200 Hertz
Figure 5-25 Transmission Characteristics for the Adjustable Muffler at 200 Hertz
Figure 5-26 Transmission Phase Angle for the Adjustable Muffler at 200 Hertz
Observations on Muffler Adjustment at 200 Hertz

The effect of acoustic element adjustment at 200 Hertz is even more readily observable than at previous frequencies in figures 5-23 through 5-26. As the outlet chamber volume of the muffler decreases, the reflection factor (figure 5-23) can be seen to increase. The reflection factor also shows a marked increase when the center-section louvers are covered no matter what the outlet chamber volume. In general, the closing of the outlet louvers has the effect of raising the reflection factor; such that the highest amount of reflection occurs when all side branch orifices are covered. The reflection factor phase angle (figure 5-24) has an overall tendency to increase as the outlet chamber volume decreases.

The transmission factor (figure 5-25) also has an inclination to increase as the outlet chamber size diminishes. A very regular pattern with respect to adjustment is also observed: the sound transmission is the highest for each chamber setting when the outlet chamber louvers are open; the center-section louvers seemingly have little effect on the overall performance at this frequency.

Likewise the transmission phase angle (figure 5-26) has a general tendency to increase as the outlet chamber dimensions decrease. Furthermore, adjustment of the louvers has a pronounced effect on the phase angle. The angle is observed to be the highest at each chamber setting when the center-section louvers are closed and the outlet chamber louvers are open. The lowest phase angle for each chamber setting are noticed when the internal louvers are open and the outlet louvers covered.
TABLE 5-4

REFLECTION AND TRANSMISSION CHARACTERISTICS
OF THE ADJUSTABLE MUFFLER AT 500 HERTZ

<table>
<thead>
<tr>
<th>RUN</th>
<th>REFLECTION FACTOR</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude (Dimensionless)</td>
<td>Phase Angle (Degrees)</td>
</tr>
<tr>
<td></td>
<td>Magnitude (Dimensionless)</td>
<td>Phase Angle (Degrees)</td>
</tr>
<tr>
<td>IA1</td>
<td>0.958</td>
<td>84.33</td>
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<td>IA2</td>
<td>0.961</td>
<td>178.11</td>
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<td>IB1</td>
<td>0.968</td>
<td>138.69</td>
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<tr>
<td>IB2</td>
<td>0.967</td>
<td>162.42</td>
</tr>
<tr>
<td>IIA1</td>
<td>0.964</td>
<td>175.17</td>
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<tr>
<td>IIA2</td>
<td>0.960</td>
<td>141.44</td>
</tr>
<tr>
<td>IIB1</td>
<td>0.963</td>
<td>147.92</td>
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<tr>
<td>IIB2</td>
<td>0.967</td>
<td>161.57</td>
</tr>
<tr>
<td>IIIA1</td>
<td>0.968</td>
<td>161.27</td>
</tr>
<tr>
<td>IIIA2</td>
<td>0.968</td>
<td>141.44</td>
</tr>
<tr>
<td>IIIB1</td>
<td>0.962</td>
<td>157.65</td>
</tr>
<tr>
<td>IIIB2</td>
<td>0.965</td>
<td>151.85</td>
</tr>
<tr>
<td>IVA1</td>
<td>0.964</td>
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<tr>
<td>IVA2</td>
<td>0.962</td>
<td>176.85</td>
</tr>
<tr>
<td>IVB1</td>
<td>0.954</td>
<td>176.85</td>
</tr>
<tr>
<td>IVB2</td>
<td>0.965</td>
<td>150.04</td>
</tr>
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</table>
Figure 5-27 Reflection Characteristics for the Adjustable Muffler at 500 Hertz
Figure 5-28 Reflection Phase Angle for the Adjustable Muffler at 500 Hertz
Figure 5-29 Transmission Characteristics for the Adjustable Muffler at 500 Hertz
Figure 5-30 Transmission Phase Angle for the Adjustable Muffler at 500 Hertz
Observations on Muffler Adjustment at 500 Hertz

Acoustical adjustment of the prototype muffler has little or no effect on the reflection and transmission characteristics (figures 5-27 and 5-29) at 500 Hertz. In fact, the muffler is over 95 percent reflective at all adjustments. Furthermore, almost all of the incident wave, which is not reflected, is dissipated in the muffler and very little sound is transmitted. There are two adjustments where the transmission factor is zero to three decimal places. This transmission characteristic occurs at the third chamber divider setting (i.e., the outlet chamber is approximately cut in half) when the outlet louvers are open. There are several other adjustments that have what can be considered negligible transmission characteristics also.

The phase angles at this frequency do show the effect of acoustical adjustment. The reflection phase angle plot (figure 5-28) shows no general pattern with respect to adjustment. However the transmission phase angle (figure 5-30) may be observed to decrease slightly as the outlet chamber volume diminishes. Furthermore, the highest transmission phase angles are observed when the outlet louvers are closed.
TABLE 5-5

REFLECTION AND TRANSMISSION CHARACTERISTICS
OF THE ADJUSTABLE MUFFLER AT 750 HERTZ

<table>
<thead>
<tr>
<th>RUN</th>
<th>REFLECTION FACTOR</th>
<th>TRANSMISSION FACTOR</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude (Dimensionless)</td>
<td>Phase Angle (Degrees)</td>
</tr>
<tr>
<td>IA1</td>
<td>0.971</td>
<td>104.63</td>
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<tr>
<td>IA2</td>
<td>0.991</td>
<td>137.19</td>
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<tr>
<td>IB1</td>
<td>0.967</td>
<td>142.43</td>
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<td>IB2</td>
<td>0.981</td>
<td>155.29</td>
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<td>IIIA1</td>
<td>0.989</td>
<td>162.77</td>
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<tr>
<td>IIIA2</td>
<td>0.982</td>
<td>157.46</td>
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<tr>
<td>IIIB1</td>
<td>0.971</td>
<td>146.26</td>
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<td>IIIB2</td>
<td>0.982</td>
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<td>IIIIA1</td>
<td>0.978</td>
<td>136.33</td>
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<tr>
<td>IIIIA2</td>
<td>0.984</td>
<td>155.58</td>
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<tr>
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<td>151.51</td>
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<td>IIIB2</td>
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<td>IVA1</td>
<td>1.003</td>
<td>308.74</td>
</tr>
<tr>
<td>IVA2</td>
<td>1.013</td>
<td>294.28</td>
</tr>
<tr>
<td>IVB1</td>
<td>1.009</td>
<td>143.84</td>
</tr>
<tr>
<td>IVB2</td>
<td>0.989</td>
<td>81.39</td>
</tr>
</tbody>
</table>
Figure 5-31 Reflection Characteristics for the Adjustable Muffler at 750 Hertz
Figure 5-32 Reflection Phase Angle for the Adjustable Muffler at 750 Hertz
Figure 5-33  Transmission Characteristics for the Adjustable Muffler at 750 Hertz
Figure 5-34 Transmission Phase Angle for the Adjustable Muffler at 750 Hertz
Observations on Muffler Adjustment at 750 Hertz

At 750 Hertz there is little effect of acoustical adjustment on the reflection factor (figure 5-31). In fact, the muffler is almost totally reflective. There is, however, an observable pattern of adjustment with respect to the transmission factor (figure 5-33) at this frequency. It may be noted that, though the transmission factor is decidedly small for all adjustments, the sound transmission is greatest when the center-section louvers are open. Furthermore, there is negligible dissipation within the muffler.

Both the reflection and transmission phase angles (figures 5-32 and 5-34) are significantly affected by the acoustical adjustment of the muffler, however no definitive trends are observable.

It can also be seen that the square of the reflection factor and transmission factor sum to more than unity at several data points. This may be due to a deviance of the actual tube attenuation from the theoretical value due to temperature and humidity effects. These three points, however, are the only data points where this discrepancy occurs in all of the data.
<table>
<thead>
<tr>
<th>RUN</th>
<th>REFLECTION FACTOR</th>
<th>TRANSMISSION FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude (Dimensionless)</td>
<td>Phase Angle (Degrees)</td>
</tr>
<tr>
<td>IA1</td>
<td>0.888</td>
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<td>IA2</td>
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<td>IB1</td>
<td>0.870</td>
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<td>IB2</td>
<td>0.821</td>
<td>36.74</td>
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<tr>
<td>IIA1</td>
<td>0.764</td>
<td>58.42</td>
</tr>
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<td>IIA2</td>
<td>0.773</td>
<td>100.72</td>
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<td>IIB1</td>
<td>0.913</td>
<td>103.32</td>
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<td>IIB2</td>
<td>0.908</td>
<td>5.28</td>
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<td>IIIA2</td>
<td>0.683</td>
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<td>IIIB1</td>
<td>0.906</td>
<td>164.47</td>
</tr>
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<td>IIIB2</td>
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<td>102.61</td>
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<td>IVA1</td>
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<td>IVA2</td>
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<td>IVB1</td>
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<td>53.19</td>
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<tr>
<td>IVB2</td>
<td>0.934</td>
<td>30.39</td>
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</table>
Figure 5-35 Reflection Characteristics for the Adjustable Muffler at 1000 Hertz
Figure 5-36 Reflection Phase Angle for the Adjustable Muffler at 1000 Hertz
Figure 5-37 Transmission Characteristics for the Adjustable Muffler at 1000 Hertz
Figure 5-38  Transmission Phase Angle for the Adjustable Muffler at 1000 Hertz
Observations on Muffler Adjustment at 1000 Hertz

Acoustical adjustment of the prototype muffler is effective at a puretone driving frequency of 1000 Hertz. After the first chamber settings, a definitive pattern in the reflection factor (figure 5-35) with respect to adjustment is established; such that, when the center-section louvers are open, the reflection of the muffler is much higher than when they are closed. Furthermore, the reflection factor with the center-section louvers closed is lowered as the outlet chamber becomes smaller, while the reflection factors with open center-section louvers remain essentially constant. Adjustment of the outlet chamber louvers affects the reflection only slightly. The reflection factor phase angle (figure 5-36) is greatly affected by adjustment. Its maximum variation is approximately 240 degrees; however, no regular pattern with respect to adjustment is apparent.

The transmission factor (figure 5-37) at this frequency also shows a pattern with respect to adjustment. The lowest sound transmission can be observed to occur when the outlet chamber louvers are open. In fact, at muffler setting IIA2, (i.e., the piston is one inch from the full chamber position, the center-section louvers are covered, and the outlet louvers open) the transmission of the muffler is zero. The maximum variance of the transmission phase angle (figure 5-38) with respect to adjustment is almost 280 degrees.
### Table 5-7

**Reflection and Transmission Characteristics of the Adjustable Muffler at 1550 Hertz**

<table>
<thead>
<tr>
<th>Run</th>
<th>Reflection Factor</th>
<th>Transmission Factor</th>
</tr>
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<tbody>
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<td></td>
<td>Magnitude (Dimensionless)</td>
<td>Phase Angle (Degrees)</td>
</tr>
<tr>
<td>IA1</td>
<td>0.717</td>
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<td>IA2</td>
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<td>3.89</td>
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<td>0.836</td>
<td>58.38</td>
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<td>54.54</td>
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<td>IIIA1</td>
<td>0.732</td>
<td>345.03</td>
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<td>IVA1</td>
<td>0.721</td>
<td>343.81</td>
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<td>0.682</td>
<td>22.92</td>
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<td>0.831</td>
<td>58.70</td>
</tr>
<tr>
<td>IVB2</td>
<td>0.829</td>
<td>48.80</td>
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</table>
Figure 5-39 Reflection Characteristics for the Adjustable Muffler at 1550 Hertz
Figure 5-40 Reflection Phase Angle for the Adjustable Muffler at 1550 Hertz
Figure 5-41  Transmission Characteristics for the Adjustable Muffler at 1550 Hertz
Figure 5-42 Transmission Phase Angle for the Adjustable Muffler at 1550 Hertz
Observations on Muffler Adjustment at 1550 Hertz

The acoustical adjustment of the prototype muffler has an appreciable effect on the reflection factor at 1550 Hertz (figure 5-39). There is a very definite and characteristic pattern established with respect to adjustment, which is remarkably similar to those patterns observed at lower frequencies. The open, center-section louvers have the effect of significantly raising the reflection factor at each outlet chamber setting, a notable effect of the open, outlet louver adjustment is to slightly lower the reflection factor at each setting. Also, the reduction of outlet chamber volume has a tendency to somewhat lower the overall reflection characteristics of the muffler. The reflection factor phase angle (figure 5-40) is again greatly affected by the adjustments and a fairly regular pattern is established with the largest phase angles being observed when the center section louvers are open.

There is a negligible effect of adjustment on the transmission factor at this frequency (figure 5-41). Approximately 80 percent of the incident wave is reflected at all muffler settings and it is evident that the muffler dissipates most of the rest of the incident energy. However, the transmission factor phase angle (figure 5-42) is greatly affected by acoustical adjustment, with the maximum variation being approximately 150 degrees. The phase angle also shows a fairly regular pattern with respect to adjustment; with the greatest phase differences observed when both sets of louvers are covered.
H. General Observations on the Pure Tone Analysis of the Adjustable Muffler.

From the preceding tables and figures it is readily observable that acoustical adjustment of the prototype muffler does not effect its overall pure-tone filtering characteristics. In general, the effect of adjustment is more apparent with regard to reflection factor than transmission factor at the frequencies selected.

Furthermore, the adjustment of the side branch orifices (or louvers) created the greatest observable effect on the pure tone transmission and reflection factors of the muffler. Using the equation: [8]

\[ T = \frac{1}{1 + \frac{1}{4}(m-1/m)^2 \sin^2 K L_e} \]

where: 
\( T \) = theoretical transmission factor
\( m \) = ratio of chamber area to exhaust pipe area
\( K = 2\pi f/c \)
\( f \) = frequency (Hz.)
\( c \) = speed of sound (in/sec)
\( L_e \) = length of expansion chamber (in.)

the theoretical transmission factor for an expansion chamber can be calculated. With the aid of this equation, the theoretical transmission factors for the outlet chamber were calculated for the fully open position (chamber length 5.25 inches), and are listed in table 5-8 (page 129). From this table it may be observed that the effective frequency at which the theoretical transmission factor goes to zero is somewhere between 500 and 1000 Hertz. It may be shown that the effective chamber length for maximum attenuation is \( \lambda/4 \) [8]. Thus, the optimum frequency for the outlet chamber (5.25 inch position) is found to be
Table 5-8

Expansion Chamber Transmission Factors

<table>
<thead>
<tr>
<th>Frequency (Hz.)</th>
<th>Theoretical Transmission Factor</th>
<th>Experimental Transmission Factor*</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.839</td>
<td>0.477</td>
</tr>
<tr>
<td>100</td>
<td>0.615</td>
<td>0.388</td>
</tr>
<tr>
<td>200</td>
<td>0.373</td>
<td>0.265</td>
</tr>
<tr>
<td>500</td>
<td>0.197</td>
<td>0.029</td>
</tr>
<tr>
<td>750</td>
<td>0.194</td>
<td>0.004</td>
</tr>
<tr>
<td>1000</td>
<td>0.278</td>
<td>0.165</td>
</tr>
<tr>
<td>1550</td>
<td>0.306</td>
<td>0.010</td>
</tr>
</tbody>
</table>

*The experimental transmission factor is the overall measured transmission factor for the adjustable muffler at setting IAL.

645 Hertz. Furthermore, the smaller the outlet chamber becomes, the higher the optimum cancellation frequency. With the chamber adjusted to its smallest volume ($L_e = 2.625$ in.), the critical frequency is 1291 Hertz. Thus, the chamber adjustment should be most effective in the 500 - 1400 Hz. frequency range.

To observe the effects of chamber adjustment, plots of chamber adjustment versus reflection and transmission factor were made while holding louver adjustment constant. The isolation of the effects of this adjustment are shown in figures 5-43 through 5-50.

At 100 Hertz the general effect of chamber adjustment is to decrease the transmission factor of the muffler while significantly increasing the reflection factor. At 200 Hertz, in all cases, both the reflection and transmission factor are increased by chamber adjustment. The adjustment of the outlet chamber at 500 and 750 Hertz has almost no effect
on the transmission and reflection factors. These frequencies are near
the optimum frequency for the fully open outlet chamber; however, the
overall combination of acoustic elements of the muffler are apparently
so effective that there is almost no transmission at these frequencies.

At 1000 Hertz the characteristics of the chamber adjustment changes
significantly. At the louver adjustment Al, where both louver sections
are closed and the effect of the chamber adjustment theoretically
should be the most significant, the results of the chamber piston
location are graphically obvious. At piston adjustment II the trans­
mission factor is significantly lower than in other positions. The
chamber dimension (4.25 inches) should theoretically be significant in
the area of 800 Hertz. The critical dimension for 1000 Hertz is 3.39
inches; however, the shift is no doubt due to the interaction of
acoustic elements. This observation is also apparent at other louver
adjustments at 1000 Hertz.

The only observable effect of chamber adjustment at 1550 Hertz is
on the reflection factor. There is negligible variance of the trans­
mission factor with respect to adjustment.

As previously stated the adjustments of the louvers are most sig­
nificant in affecting the characteristics of the prototype muffler. In
fact, the effects of the unique combination of acoustical elements in
the muffler apparently explain the relative insensitivity of muffler per­
fomance to outlet chamber adjustment. In general, the center section
louver adjustment, when open, has the effect of raising the reflection
factor significantly. The outlet louvers, while affecting the reflection
factor slightly, seemingly have the greatest effect on the transmission
characteristics of the muffler. On the average, when the outlet louvers
are open, the transmission factor of the prototype muffler is lowered somewhat. This lowering of transmission may be explained in that these louvers have the effect of many small side branches which create acoustical dissipation in the muffler.

The prototype adjustable muffler, as an entity, is a fairly respectable pure tone filter. As can be noted in figure 5-51, and from comparisons of the preceding tables; the sound transmission of the muffler with respect to frequency never exceeds fifty percent of the incident sound pressure at any of the selected frequencies.

Figure 5-51, the reflection and transmission characteristics of the adjustment IIAl versus frequency, is characteristic of all muffler data taken. Other plots were made but were not included because of the similarity between plots. It can be seen from this figure that the sound transmission is the highest at low frequencies. This fact can be theoretically predicted from the dimensions of the muffler with respect to wavelength at these frequencies. As an illustration, the highest transmission of the muffler is observed at 50 Hertz. The calculated wavelength of a 50 Hertz tone in free space is 690.58 centimeters, while the overall length of the muffler is only 45.7 centimeters.
Figure 5-43 The Effect of Chamber Adjustment at a Constant Al Louver Position
Figure 5-44 The Effect of Chamber Adjustment at a Constant Al Louver Position

Note: □ = Transmission Factor, ○ = Reflection Factor
Note: □ = Transmission Factor, O = Reflection Factor

Figure 5-45 The Effect of Chamber Adjustment at a Constant A2 Louver Position
Note: □ = Transmission Factor, ○ = Reflection Factor

Figure 5-46 The Effect of Chamber Adjustment at a Constant A2 Louver Position
Note: □ = Transmission Factor, 0 = Reflection Factor

Figure 5-47 The Effect of Chamber Adjustment at a Constant Bl Louver Position
Figure 5-48 The Effect of Chamber Adjustment at a Constant Bl Louver Position

Note: □ = Transmission Factor, ○ = Reflection Factor
Figure 5-49 The Effect of Chamber Adjustment at a Constant B2 Louver Position

Note: □ = Transmission Factor, ○ = Reflection Factor
Figure 5-50 The Effect of Chamber Adjustment at a Constant B2 Louver Position

Note: □ = Transmission Factor, O = Reflection Factor
Figure 5-51 Reflection and Transmission Characteristics of the Adjustable Muffler as Compared to Frequency at Adjustment IIAl
I. Experimental Automotive Exhaust Analysis of the Adjustable Muffler

The foregoing pure tone analysis is both interesting and informative. However, pure tones are seldom encountered in nature; speech, music, automotive exhaust noise, and explosive sounds are complex phenomena which vary in time, frequency composition, and intensity. No single descriptive number adequately portrays all of these variables. Thus, a spectral analysis of the prototype adjustable muffler is desirable to determine the relevancy of the pure tone study.

A test was developed which was both compatible to indoor laboratory environment and closely comparable to the pure tone investigation of the muffler. An actual "on the engine" test was not run because of the desirability of creating a simulated exhaust environment in the laboratory. Since a spectrum shaper was not available, it was necessary to record automotive exhaust noise on magnetic recording tape. Current information (Appendix VII) revealed that the average American automobile engine has a displacement of 331 cubic inches. Therefore, a 330 cubic inch V-8 Oldsmobile engine was utilized for test data. The engine was a low compression regular fuel type, with a two barrel carburetor and a single exhaust pipe. Recordings were taken at 1500 and 2000 rpm. These engine speeds were selected to simulate in town and on the highway exhaust conditions.

The recordings were taken in a paved lot on an Ampex tape recorder with a Shure microphone. The standard muffler and tailpipe were removed from the automobile and the recordings were taken at the end of a four foot exhaust pipe. The microphone was placed on vibration deadening material, six inches away from the end of the exhaust pipe at an angle of 45 degrees. The microphone was placed at this angle so that exhaust gases would not impinge on the microphone diaphragm. The engine rpm was
measured by means of a dwell tachometer and a constant speed was main­
tained by using the automatic choke adjustment screw. One difficulty
encountered in taking recordings was the fouling of the spark plugs under
no load and subsequent missing in engine firing sequence.

Approximately two minutes of recording at a tape speed of 7.5 inches
per second was taken for each engine speed. A section of tape free from
abnormalities at each engine speed was then selected and tape loops were
made. This method assumes that the engine exhaust noise is "steady-state
sound". That is, the assumption is made that the sound can be analyzed
leisurely so that; although fluctuating in amplitude and frequency during
short intervals of time; the sound has the same average spectrum at one
time as another when integrated over sufficiently long periods of time.

The tape loops were then played back directly through a heterodyne­
type analyzer with a one-tenth octave band-pass filter, whose output was
connected to a graphic level recorder. The results are shown in figures
5-52 and 5-53. Attempts were then made to reproduce these spectrums
played through an acoustical driver and a section of tubing (using the same
microphone position for the comparative reading as in recording). It was
found that the length of the tubing between the microphone measuring
station and the driver greatly affects the spectrum at the measuring
station. Through trial and error, an optimum length of 18 inches was
selected. This is not to say that the entire spectrum was perfectly
reproduced, but that the frequencies of interest (the fundamental and
several harmonics were closely approximated).

So that some basis of experimental correspondence could be initially
assumed, side wall measuring stations and anechoic conditions were utilized
for experimental spectrum measurements. Also, the same decibel level
(120 - 140 db) was maintained in the playback simulation as was observed
Figure 5-52 Automatic Exhaust Spectrum at 1500 rpm
Figure 5-53 Automotive Exhaust Spectrum at 2000 rpm

Writing Speed: 3 in/sec
Chart Speed: 5 in/min
Bandwidth: 1/10 Octave
in the recording session. A Lansing high frequency driver was used as an input to this system, where the input of the driver was the output of an Ampex tape recorder amplified through an Eico amplifier. Initially, a length of brass tubing with a side-wall measuring station was coupled with the anechoic termination (figure 5-54) to investigate the correspondence of the actual exhaust spectrum to that which was to be used as an input to the adjustable muffler. As can be seen in comparing figure 5-55 to figure 5-52 and figure 5-56 to figure 5-53, the center frequencies of the exhaust spectrum were almost perfectly reproduced in shape and amplitude. However, at low frequencies the shape of the experimental input in practically the same as the actual exhaust spectrum but the amplitude is greatly reduced. This discrepancy is no doubt due to the frequency response of the driver, since the Eico amplifier has a frequency response of 8 to 60,000 Hertz ±0.5 decibels and a power band width of 10 to 40,000 Hertz at one percent distortion.

It may also be noted that at higher frequencies, the amplitude of the spectrum is magnified somewhat with a large discrepancy in amplitude around 1500 Hertz. It is felt that this discrepancy in spectrum shape is due to a resonance of the input tube. This may be verified by calculating the normal frequencies of the tube:

\[ f_N = \frac{NC}{2L} \]  

where \( N = 1, 2, 3, 4, \ldots \)

\( f_N \) = Nth resonance frequency of the tube
\( L \) = length of tube

Since we assume that 1500 Hertz is a resonant frequency, and the length of input tubing is 27 inches:
Figure 5-54. Experimental System for the Investigation of the Correspondence Between Actual Automotive Exhaust and the Experimental Muffler Input.
Figure 5-55 Simulated Automotive Exhaust Spectrum at 1500 Rpm Used for the Experimental Input to the Adjustable Muffler
Figure 5-56 Simulated Automotive Exhaust Spectrum at 2000 rpm Used for the Experimental Input to the Adjustable Muffler
\[ N = \frac{(1500)(2)(2.25)}{1120} \]

\[ N = 6.03 \]

Thus, it can be seen that approximately 1500 Hertz is a resonant frequency of the input tube and the discrepancy in amplitude simulation is no doubt due to this fact. The magnification of the spectrum amplitude may be due to the frequency response of the driver. It may be assumed that the majority frequencies observed in the automotive exhaust are not in the flat portion of the response curve of the driver. It is, therefore, possible that the area of near perfect reproduction is a function of input tube length and is somewhat attenuated by not being in the flat frequency response area of the driver; the high frequency amplitudes would then appear magnified. In any case the fundamental frequency and at least one of its harmonics is adequately reproduced for experimental investigation.

After it had been ascertained that the automotive frequency spectrums could be adequately reproduced for laboratory experimentation the adjustable muffler and another measuring station were added between the first measuring station and the anechoic termination (figure 5-57). This experimental set-up allows the input spectrum to be compared to the output spectrum of the muffler under the same conditions that pure tone transmission factors were measured.

One noteworthy point is the fact that the spectrum at the input measuring station is affected by the insertion of the muffler into the system. This is not to say that the input is changed in any way; the microphone at the input station is merely recording both incident and reflected waves and this is what appears in the spectrum. This approximates the actual
Figure 5-57. Experimental System for the Comparative Measurement of the Output Spectrum with Respect to the Input of the Adjustable Muffler.
spectrum at a similar location in the complete exhaust system.

Several spectrum comparisons of the input and output spectrums of the muffler at various adjustments were then recorded on the graphic level recorder. In general, when observing figures 5-58 through 5-63, it is immediately obvious that the adjustable muffler is effective at all adjustments above 150 Hertz. However, the main objective of this phase of the investigation was to ascertain the effectiveness of the acoustical adjustments on an automotive spectrum.

It is graphically obvious in comparing figures 5-58 and 5-59 of the comparative input and output spectrums at 1500 rpm that adjustment does affect the output spectrum of the muffler. In figure 5-58 of adjustment IAl, the attenuation of the muffler at 120 Hertz is 4.5 db; while figure 5-59 of adjustment IB2 shows an attenuation of only 3 db. This relative difference of 1.5 db is barely audible to the human ear. But, at 165 Hertz the relative difference in attenuation becomes 3 db and at 245 Hertz the relative difference is 5 db; both of these disparities are significant to the human ear.

In the recordings at the engine speed of 2000 rpm, the effect of adjustment is even more obvious. For example the attenuation in the adjustments in figures 5-60, 5-61, 5-62, and 5-63 are 14 db for adjustment IAl, 13 db for adjustment IB2, 7 db for adjustment IIAl, and 2 db for adjustment IIIB2. This accounts for a relative difference of 12 db between the first and last adjustment which is very evident to the human ear. Furthermore, when reviewing the four comparative spectrums at 2000 rpm, it can be seen that the frequency content is greatly affected by adjustment. For example, adjustments IAl and IIIAl show frequency content
Figure 5-58 Comparative Exhaust Spectrums for Muffler Adjustment IAI at 1500 rpm
Figure 5-59  Comparative Exhaust Spectrum for Muffler Adjustment IB2 at 1500 rpm
Writing Speed: 3 in/sec  
Chart Speed: 5 in/min  
Bandwidth: 1/10 Octave

Figure 5-60  Comparative Exhaust Spectrum for Muffler Adjustment IAl at 2000 rpm
Writing Speed: 3 in/sec  
Chart Speed: 5 in/min  
Bandwidth: 1/10 Octave

Figure 5-61 Comparative Exhaust Spectrums for Muffler Adjustment IB2 at 2000 rpm
Figure 5-62 Comparative Exhaust Spectrums for Muffler Adjustment IIIIAI at 2000 rpm
Figure 5-63 Comparative Automotive Exhaust Spectrums for Muffler Adjustment IIIB2 at 2000 rpm
above 300 Hertz at 110 decibels; while the spectrums for adjustments IB2 and IIIB2 do not. Furthermore, there is an 11 decibel spike at 380 Hertz in the output specimen for adjustment IIIA1 that does not appear in the other adjustment spectrums. Adjustment of the muffler also affects the higher frequencies as can be seen in the spectrums in the area of 1500 Hertz, where alteration of specimen shape and decibel level from input to output may easily be observed. It was stated that noise in this frequency area was due to input tubing resonance; however, the adjustable muffler effectively controls this problem at the output.

J. Comparison of Testing Methods.

In comparing the aforementioned experimental methods it is well to remember that the frequency spectrums recorded are one-tenth octave-band recordings, i.e.; the amplitude observed at a specified frequency is actually an average amplitude taken over a band-width of one-tenth of an octave rather than the exact amplitude at that frequency. Keeping this in mind results of the two methods shall now be compared.

Since the two testing methods involve output displayed in two different forms, the first requisite in correlating the two sets of data is transforming both data sets into the same output form. The initial step in this process is to convert the decibel attenuation between the input to the muffler and the output on the respective sound spectrums (figures 5-58 through 5-63) to a pressure ratio at a given frequency. This is accomplished using the relationship:

\[
ATT = 20 \log_{10} \frac{P_{\text{output}}}{P_{\text{input}}} \quad (78)
\]
where: \( \text{ATT} \) = attenuation in decibels.

\[ P_{\text{output}} = \text{sound pressure on the downstream side of the muffler.} \]

\[ P_{\text{input}} = \text{total sound pressure on the upstream side of the muffler.} \]

Next solving for the pressure ratio for a specific frequency at the measuring station 9 inches from the muffler:

\[
\frac{P_{\text{out}}}{P_{\text{in}}}_{9''} = -\frac{1}{\text{ATT}} \log_{10} 20
\]

Then using graphical transmission factor methods (figure 3-1) the theoretical total pressure at a distance of 9 inches (spectral input measuring station) and 18 inches (pure tone input measuring station) can be calculated. From these theoretical calculations and the total pressure ratio at the 18 inch location found from transmission measurements the total sound pressure ratio at the 9 inch location may be found. In other words:

\[
\frac{(P_{\text{in}})_{9''}}{(P_{\text{in}})_{18''}} \cdot \frac{P_{\text{in}}}{P_{\text{out}}}_{18''} = \frac{(P_{\text{in}})_{9''}}{(P_{\text{out}})_{18''}}
\]

But \( (P_{\text{out}})_{18''} = (P_{\text{out}})_{9''} \) for zero attenuation; thus, the total pressure ratio at 9 inches can be found:

\[
\frac{(P_{\text{in}})_{9''}}{(P_{\text{out}})_{18''}} = \frac{P_{\text{in}}}{P_{\text{out}}}_{9''}
\]

Finally, the pressure ratio obtained from formula (79) may be compared with the theoretical sound pressure ratio obtained from equation (81). This comparison was made in decibels utilizing the ratio of the experimental pure tone pressure ratio to the spectrally obtained pressure ratio, or:

\[
\text{Deviation} = 20 \log_{10} \frac{p_r}{s_r}
\]
where: \( P_r \) = pure tone pressure ratio at 9 inches. (eq. 81)

\( S_r \) = spectral pressure ratio. (eq. 79)

The results of this comparison are presented in tables 5-9 and 5-10.
Table 5-9 Correlation of Testing Methods at 2000 RPM Engine Speed

<table>
<thead>
<tr>
<th>Frequency (Hz.)</th>
<th>Run Number</th>
<th>Transmission Factor Pressure Ratio</th>
<th>Spectral Pressure Ratio</th>
<th>Pure Tone Pressure Ratio</th>
<th>Deviation (db.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>IA1</td>
<td>0.388</td>
<td>8.223-</td>
<td>1.000</td>
<td>1.121</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>100</td>
<td>IB2</td>
<td>0.385</td>
<td>8.291-</td>
<td>1.000</td>
<td>1.220</td>
</tr>
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<td>1.727</td>
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<tr>
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<td>11.535-</td>
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<td>30.752-</td>
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<td>20.466</td>
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<td>*</td>
<td>154.906</td>
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<tr>
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<td>40.457</td>
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<td>0.001</td>
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<td>40.000-</td>
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<td></td>
<td></td>
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</tr>
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</table>

*NOTE: These points were not obtained, since the output from the muffler was below the baseline of the spectrums.
Table 5-10  Correlation of Testing Methods at 1500 RPM Engine Speed

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Run Number</th>
<th>Transmission Factor Pressure Ratio</th>
<th>Spectral Pressure Ratio</th>
<th>Pure Tone Pressure Ratio</th>
<th>Deviation (db.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>IA1</td>
<td>0.388</td>
<td>8.223</td>
<td>1.496</td>
<td>1.121</td>
</tr>
<tr>
<td>100</td>
<td>IB2</td>
<td>0.385</td>
<td>8.291</td>
<td>1.334</td>
<td>1.220</td>
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<tr>
<td>200</td>
<td>IA1</td>
<td>0.265</td>
<td>11.535</td>
<td>3.548</td>
<td>2.270</td>
</tr>
<tr>
<td>200</td>
<td>IB2</td>
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<td>2.661</td>
<td>2.961</td>
</tr>
<tr>
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<td>IA1</td>
<td>0.029</td>
<td>30.752</td>
<td>*</td>
<td>20.466</td>
</tr>
<tr>
<td>500</td>
<td>IB2</td>
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<td>154.906</td>
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<tr>
<td>1550</td>
<td>IA1</td>
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</tr>
<tr>
<td>1550</td>
<td>IB2</td>
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<td>*</td>
<td>93.697</td>
</tr>
</tbody>
</table>

*Note: These points were not obtained, since the output from the muffler was below the baseline of the spectrum.
As may be seen from the preceding tables there is reasonable agreement between the two methods, except at 1550 Hertz. As previously discussed in this chapter, when reviewing the simulation of automotive exhaust noise in the experimental system, the area of 1500 Hertz on the simulated spectrum was found to be incompatible with the originating spectral noise. This deviation was attributed to a natural resonance of the tubing in the system. The large incongruity in the comparison of the two testing methods is probably related to this resonance phenomenon and the comparison is thus invalid at this frequency. The rest of the data points show relatively small decibel deviations. A difference of 3 decibels is barely detectable to the human ear under normal circumstances.

The observed deviation between the two testing methods may be anticipated from three sources. First, the spectrums were obtained through tenth octave analysis and this can produce errors when one wishes an exact amplitude at a specific frequency. Second, experimental measurements of reflection and transmission factors at frequencies corresponding to tube resonances (where sound pressures theoretically become infinite) can significantly be in error. Finally, a coupling effect, wherein sound pressures at one frequency can influence sound amplitudes at a different frequency may have influenced the sound spectra presented earlier.

However, it may be observed that the overall effect of the muffler and its adjustments as indicated from the comparative spectrums could be predicted as a general trend from the pure tone analysis. It is believed that the difficulty in obtaining a predictable relationship between the two testing methods at some frequencies is attributable to measurement techniques rather than to failure of the pure-tone method to predict muffler performance.
CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

The principal findings of this thesis are summarized below:

1.) A prototype automotive muffler whose internal acoustic elements are easily removed or attached and whose components are adjustable offers considerable promise in the laboratory solution of filtering problems. This statement has been evaluated and the results show a definite pattern of acoustic characteristics at selected frequencies with respect to adjustment.

2.) Determination of muffler performance by pure tone analysis provides a reasonable indication of its performance in an automotive exhaust system when:
   a.) Pure tone sound pressure levels approximate those present in an automotive exhaust system;
   b.) Neither steady flow nor temperature gradients are present;
   c.) Strong tube resonances are not present at frequencies selected for analysis.

Effects of sound pressure levels and steady flow on muffler performance were not considered in this thesis. However, the expectation is that their influence in an actual exhaust system can be determined by laboratory simulation and pure tone analysis.

3.) The length of input and output tubing in a muffling system has a great deal of effect on system performance. This fact
was observed while attempting to duplicate an actual exhaust spectrum in the experimental system.

4.) A "library" of acoustic elements formed from a combination of plane wave acoustic theory and experimental data would be useful for the simplified design of automotive mufflers.

5.) Modifications of methods and equipment are desirable for:

a.) Evaluating automotive mufflers at appropriate gas flows and temperatures so that pure tone analysis can be directly applied to the systematic design of mufflers under actual operating conditions.

b.) Automating the measurement of reflection and transmission factors such that the methods are not so tedious and time consuming.

In considering automotive mufflers, acoustical performance is not the only parameter necessary for satisfactory design. It is also important to be able to assess the changes in mechanical performance of an engine created by its exhaust system. Present understanding of the factors influencing this performance is inadequate. With the development of a system incorporating pulsating gas flow and temperature effects, would be introduced a method for evaluating the effects of acoustic elements on engine performance. Thus, with the innovation, an optimum automotive muffling system could be designed from both the acoustical and the mechanical standpoints. Furthermore, the introduction of such a system, simulating temperature, gas flow, and possibly saturation effects would be a significant step in the direction of making a more exact science of muffler design.
In future investigations it is felt that attempts to correlate spectral and pure tone data would be greatly enhanced by utilizing a narrow band frequency analyzer and as high as possible a pen response on the data recorder. Furthermore, the effect of coupling between frequencies in spectral noise, and tube resonance effects, should be thoroughly examined.

The adjustable muffler designed and used for this investigation was a modification of a standard production model. It is felt that a muffler could be constructed from simple acoustic theory whose adjustments would be even more radical in their effects. Furthermore, the construction of such a muffler would provide increased knowledge of the interaction of acoustic elements.

The standing wave tube measurement system should be modified by mechanizing the microphone traverse and a micrometer adjustment should be incorporated into the system. These modifications would facilitate experimental measurements and reduce the tedium presently required for analysis of acoustic filters.
APPENDIX I

CALCULATED ATTENUATION CONSTANTS

Since an acoustical waveguide is utilized to validate the plane wave assumptions of this investigation, allowances must be made for energy loss due to viscous and thermal boundary layers at the tube wall.

As noted previously, the modified Kirchoff equation has been found to be the best theoretical estimation of tube attenuation. With the use of a desk calculator and mathematical tables, the equation:

$$\alpha = 3.18 \times 10^{-5} \frac{f^{1/2}}{r}$$

(3)

where: \( f \) = driving frequency in Hertz.
\( r \) = tube radius in centimeters.

was evaluated and the following table of attenuation constants was obtained:

<table>
<thead>
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<th>FREQUENCY (Hz)</th>
<th>ATTENUATION CONSTANT (x 10^4 1/Cm.)</th>
</tr>
</thead>
<tbody>
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<td>0.88</td>
</tr>
<tr>
<td>100</td>
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<tr>
<td>1000</td>
<td>3.96</td>
</tr>
<tr>
<td>1550</td>
<td>4.93</td>
</tr>
</tbody>
</table>

The above constants were employed in the calculation of the reflection and transmission characteristics of the prototype, adjustable, automotive muffler.
APPENDIX II

THE SPEED AND WAVELENGTH OF SOUND IN A WAVEGUIDE

At low frequencies (50-200 Hz) when only one pressure minimum can be observed in the measuring length of the two inch inside diameter, standing wave tube; it becomes necessary to use the speed of sound in the calculation of the reflection factor phase angle for the adjustable muffler. Because of energy losses, the speed of sound in the tube is generally less than that of sound in free space. The corrected values for the speed of sound can then be obtained from Kirchoff's equation:

\[
C' = C \left[ 1 - \frac{0.579}{2r(\pi f)^{1/2}} \right] \tag{4}
\]

where:  
\( C = \) speed of sound in free space.  
\( r = \) radius of the tube.  
\( f = \) driving frequency.

The temperature in the acoustics laboratory during the collection of experimental data ranged from 71.5°F to 76.0°F. The adiabatic speed of sound in free space for dry air at the average temperature of 74.0°F and 760 mm. of mercury is 34,529 cm/sec as evaluated from the equation [3]:

\[
C^2 = \gamma \frac{P_0}{\rho_0}
\]

where:  
\( C = \) speed of sound in free space  
\( \gamma = \) ratio of the specific heat at constant pressure to that at constant volume; \( C_p/C_v = 1.4 \) for air.  
\( P_0 = \) ambient pressure.  
\( \rho_0 = \) mass density at the medium.
The wavelength corresponding to the value of the speed of sound at a given frequency and tube radius is found from the relation [3]:

\[ \lambda = \frac{C'}{f} \]

where:  \( \lambda = \) wavelength of sound

\( C' = \) corrected speed of sound

\( f = \) driving frequency

The equations for the acoustic speed and wavelength of sound at selected frequencies were evaluated by means of an IBM 360/50 digital computer. The following table is a presentation of the calculated data.

<table>
<thead>
<tr>
<th>FREQUENCY (Hz)</th>
<th>SPEED OF SOUND (cm/sec)</th>
<th>WAVELENGTH (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>34117</td>
<td>684.30</td>
</tr>
<tr>
<td>100</td>
<td>34238</td>
<td>343.07</td>
</tr>
<tr>
<td>200</td>
<td>34323</td>
<td>171.86</td>
</tr>
</tbody>
</table>

The above corrected speeds of sound and wavelengths were utilized in the calculation of low frequency reflection factor phase angles.
APPENDIX III

COMPUTER PROGRAMS UTILIZED IN THE PURE TONE ANALYSIS
OF THE ADJUSTABLE MUFFLER

A. Low Frequency Reflection Factor Calculations

As previously mentioned, the calculation of low frequency reflection factors involves only one pressure minimum measured in the standing wave tube and some reference pressure. The basic formula utilized in these calculations is equation (33):

\[ |R| = \frac{A_m e^{2\alpha x_{\text{min}}}}{A_m} - \frac{P_{\text{min}} e^{\alpha x_{\text{min}}}}{A_m} \]  

(33)

where:

\[ A_m = \frac{1}{2} \left[ P_{\text{min}} + \left( P_{\text{min}}^2 - \frac{2(P_{\text{min}}^2 - P(x)^2)}{1 - \cos \phi} \right)^{1/2} \right] \]

\[ \phi = \frac{2\omega}{C'_c} x_{rx} \]

The phase angle for low frequencies is given by equation (22):

\[ \theta = \frac{2\pi x_{\text{min}}}{\lambda/2} - N\pi \]

However, \( \lambda/2 \) cannot be measured at these low frequencies, and must be theoretically approximated from the formula:

\[ \frac{\lambda}{2} = \frac{C'_c}{2f} \]

The calculated wavelengths for low frequencies are given in appendix II.

The calculations were performed on an IBM 360/50 digital computer. Below is presented the nomenclature used to adapt experimental data to the computer language.

**Input Data**

\( F \) = driving frequency in the tube in Hertz
ATT = the theoretical tube attenuation from calculations presented in Appendix I
C = the corrected speed of sound in the tube from Appendix II
PREF = the measured reference pressure in volts (Rms).
PMIN = the minimum sound pressure at this frequency
XREF = the location of the reference pressure in the tube corrected for only extensions used at the end of the standing wave tube
XMIN = the location of the minimum pressure in the tube, corrected for any extensions used at the end of the standing wave tube
RUN = the number utilized to indicate a particular adjustment of the muffler at a given frequency

Output Data

RFO = reflection factor not considering attenuation
RFATT = reflection factor considering attenuation
PHASE = the reflection factor phase angle in degrees

On the following page is presented the computer program developed for these calculations in WATFOR language.
PROGRAM TO SOLVE FOR LOW FREQUENCY REFLECTION FACTORS
AND REFLECTION ANGLES 50-200 CPS
DATA TAKEN MARCH 29, 1969 TEMP. 74.8 F VOLTAGE 0.5
CORRECTION 13.2 CM.

INTEGER RUN
PI=3.1416
WRITE(3,200)
DO 10 J=1, 16
READ(1,100)F,ATT,C,PREF,PMIN,XREF,XMIN,RUN
B=(4.0*PI*F)/C
Z=B*XMIN
DO 2 N=1, 99, 2
PH=Z-N*PI
PHASE=PH*(180.0/PI)
IF(ABS(PHASE)-360.0)1,1,2
CONTINUE
ETA=B*ABS(XMIN-XREF)
DEM=1.0-COS(ETA)
AO=(PMIN+SQRT(PMIN**2-(2.0*(PMIN**2-PREF**2)/DEM)))/2.0
RO=(AO-PMIN)/AO
RA=((AO*EXP(2.0*ATT*XMIN))-(PMIN*EXP(ATT*XMIN)))/AO
WRITE(3,201)F,PREF,XREF,PMIN,XMIN,RO,RA,PHASE, RUN
CONTINUE
CALL EXIT
FORHAT(7F10.0,I2)
FORNAT(3X,'FREQUENCY',2X,'PREF',8X,'XREF',8X,'PMIN',8X,'XMIN',8X,'RFATT',6X,'PHASE',6X,'RUN')
END
<table>
<thead>
<tr>
<th>FREQUENCY DATA</th>
<th>PREF</th>
<th>XREF</th>
<th>PMIN</th>
<th>XMIN</th>
<th>RFO</th>
<th>RFATT</th>
<th>PHASE</th>
<th>RUN</th>
</tr>
</thead>
<tbody>
<tr>
<td>200.0</td>
<td>2.430</td>
<td>138.11</td>
<td>0.990</td>
<td>95.58</td>
<td>0.421</td>
<td>0.446</td>
<td>221.00</td>
<td>1</td>
</tr>
<tr>
<td>200.0</td>
<td>2.200</td>
<td>141.19</td>
<td>1.030</td>
<td>92.20</td>
<td>0.371</td>
<td>0.394</td>
<td>206.82</td>
<td>2</td>
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</tr>
<tr>
<td>200.0</td>
<td>2.280</td>
<td>143.14</td>
<td>1.050</td>
<td>98.48</td>
<td>0.370</td>
<td>0.394</td>
<td>233.17</td>
<td>16</td>
</tr>
</tbody>
</table>
B. High Frequency Reflection Factor Calculations

In general, the calculation of reflection factors involves two sound pressure minimums and an intermediate sound pressure maximum measured in the standing wave tube. The expression for the general reflection factor is given in equation (20):

\[
|R| = \frac{p_{\text{max}} e^{-\alpha x_{\text{min}}} - p_{\text{min}} e^{-\alpha x_{\text{max}}}}{p_{\text{max}} e^{-\alpha x_{\text{min}}} + p_{\text{min}} e^{-\alpha x_{\text{max}}}}
\]

Furthermore, the reflection factor phase angle can be determined from equation (22):

\[
\theta = \frac{2\pi x_{\text{min}1}}{x_{\text{min}2} - x_{\text{min}1}} - N\pi
\]

The above equations were evaluated by means of an IBM 360/50 digital computer. Below is presented the terminology utilized for the adaptation of the experimental data to computer language.

Input Data

RUN = the number utilized to indicate a particular adjustment of the muffler at a given frequency

PMAX = the maximum sound pressure observed in the tube

PMIN = the first minimum sound pressure observed in the tube

X1,X2 = symmetric pressure locations on either side of the first minimum

X3,X4 = symmetric pressure locations on either side of the second minimum

ATT = the theoretical tube attenuation from Appendix I

DELTA = the location correction for any extensions added to the standing wave tube
Output Data

\[ \text{DELTAX} = \frac{\lambda}{2}, \text{half the wavelength at this frequency} \]

\[ \text{RFO} = \text{the reflection factor not considering attenuation} \]

\[ \text{RFATT} = \text{the reflection factor considering attenuation} \]

\[ X_{\text{MAX}} = \frac{X_{\text{2MIN}} - X_{\text{1MIN}}}{2} + X_{\text{1MIN}} \quad (76) \]

On the following page is presented the computer program developed for the above calculations in WATFOR language.
GENERAL PROGRAM FOR REFLECTION FACTORS AND PHASE ANGLES
DATA TAKEN APRIL 9, 1969 TEMP. 75.0 VOLTAGE 0.5 500
OUTPUT IS IN THE FOLLOWING ORDER
FREQUENCY IN CYCLES PER SECOND
SOUND PRESSURE IN VOLTS
X IN CENTIMETERS
PHASE ANGLE IN DEGREES
ATTENUATION FACTOR PER CENTIMETER

INTEGER RUN

WRITE(3,200)

DO 10 J=1,16
READ(1,100)RUN,F,PMAX,PMIN,X1,X2,X3,X4,ATT,DELTA
X1MIN=X2+X1+DELTA
X2MIN=X4+X3+DELTA
DELTB=X2MIN-X1MIN
XMAX=(DELTB/2.0)+X1MIN
R1=(PMAX-PMIN)/(PMAX+PMIN)
A=ATT*X1MIN
B=ATT*XMAX
C=EXP(A)
D=EXP(B)
R2=((PMAX*C)-(PMIN*D))/((PMAX/C)+(PMIN/D))
Z=X1MIN/DELTB
DO 2 N=1,99,2
PI=3.1416
PH=(2.0*PI*Z)-N*PI
PHASE=PH*(180.0/PI)
IF(ABS(PHASE)-360.0)1,1,2
WRITE(3,201)F,PMAX,PMIN,X1MIN,X2MIN,DELTB,XMAX,R1,ATT,R2,PHASE,RUN
10 CONTINUE
CALL EXIT
100 FORMAT(12,F6.0,8F8.0)
200 FORMAT(3X,'FREQUENCY',5X,'PMAX',5X,'PMIN',5X,'X1MIN',6X,'X2MIN',6X
5,'DELTAX',5X,'XMAX',7X,'RFO',7X,'ATTENIATION',4X,'RFATT',5X,'PHASE
6 ANGLE',3X,'RUN')
7F5.3,5X,E10.3,5X,F5.3,5X,F7.2,5X,F2)

END
<table>
<thead>
<tr>
<th>FREQUENCY</th>
<th>PMAX</th>
<th>PMIN</th>
<th>X1MIN</th>
<th>X2MIN</th>
<th>DELTAX</th>
<th>XMAX</th>
<th>RFO</th>
<th>ATTENUATION</th>
<th>RFATT</th>
<th>PHASE ANGLE</th>
<th>RUN</th>
</tr>
</thead>
<tbody>
<tr>
<td>500.0</td>
<td>0.505</td>
<td>0.030</td>
<td>136.30</td>
<td>172.80</td>
<td>36.50</td>
<td>154.55</td>
<td>0.888</td>
<td>0.280E-03</td>
<td>0.958</td>
<td>84.33</td>
<td>1</td>
</tr>
<tr>
<td>500.0</td>
<td>0.485</td>
<td>0.028</td>
<td>136.70</td>
<td>170.92</td>
<td>34.22</td>
<td>153.81</td>
<td>0.890</td>
<td>0.280E-03</td>
<td>0.961</td>
<td>178.11</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500.0</td>
<td>0.431</td>
<td>0.025</td>
<td>136.90</td>
<td>171.20</td>
<td>34.30</td>
<td>154.05</td>
<td>0.891</td>
<td>0.280E-03</td>
<td>0.962</td>
<td>176.85</td>
<td>14</td>
</tr>
<tr>
<td>500.0</td>
<td>0.421</td>
<td>0.026</td>
<td>136.90</td>
<td>171.20</td>
<td>34.30</td>
<td>154.05</td>
<td>0.883</td>
<td>0.280E-03</td>
<td>0.954</td>
<td>176.85</td>
<td>15</td>
</tr>
<tr>
<td>500.0</td>
<td>0.438</td>
<td>0.025</td>
<td>136.50</td>
<td>171.35</td>
<td>34.85</td>
<td>153.92</td>
<td>0.894</td>
<td>0.280E-03</td>
<td>0.965</td>
<td>150.04</td>
<td>16</td>
</tr>
</tbody>
</table>
C. Transmission Factor Calculations

The calculation of transmission characteristics from equation (34):

\[ T = \frac{A_2}{A_1} \]  

(34)

involves the separation of the incident and transmitted waves from total sound pressure readings. A graphical method of accomplishing these calculations was previously presented in Chapter III, Figure 3-1. For expediency, these calculations were adapted to computer solution using phasor notation. The calculations were performed on an IBM 360/50 digital computer. Below is presented the nomenclature used to adapt experimental and calculated data to computer language.

Input Data

- \( F \) = driving frequency in the tube
- \( ATT \) = tube attenuation from Appendix I
- \( R \) = reflection factor (RFATT)
- \( RANG \) = reflection factor phase angle (PHASE)
- \( DELTA \) = half the wavelength of the driving frequency (DELTA)
- \( W1 \) = distance from the upstream measuring station to the muffler
- \( W2 \) = distance from the downstream measuring station to the muffler
- \( P1 \) = sound pressure reading at the upstream measuring station
- \( P2 \) = sound pressure reading at the downstream measuring station.
- \( TANG \) = the transmission factor phase angle as read from the phase angle meter (+ lead, - lag)
- \( RUN \) = the number utilized to indicate a particular adjustment of the muffler at a given frequency

On the following page is presented the WATFOR language computer program developed for these calculations.
GENERAL PROGRAM TO SOLVE FOR TRANSMISSION FACTORS
AND TRANSMISSION ANGLES

TRANSIM DATA TAKEN MARCH 22, 1969   TEMP. 76.2   VOLTAGE 0.5   500

INTEGER RUN

COMPLEX RO, BO, Y, A1, V, B1, C1, Z1, C2, Y1, A2, CMPLX, CEXP

WRITE(3, 200)

PI = 3.1416

DO 10 J = 1, 16

READ(1, 100) F, ATT, R, RANG, DELTA, W1, W2, P1, P2, TANG, RUN

A0 = 1.0

ANGLE = RANG / 57.3

F1 = R * COS(ANGLE)

F2 = R * SIN(ANGLE)

RO = CMPLX(F1, F2)

BO = RO * A0

Z = (P1 * W1) / DELTA

Y = CMPLX(0.0, Z)

D1 = ATT * W1

D1P = EXP(D1)

A1 = D1P * A0 * CEXP(Y)

A = -1.0 * Z

V = CMPLX(0.0, A)

D2 = -D1

D2P = EXP(D2)

B1 = D2P * BO * CEXP(V)

C1 = A1 + B1

RP = P2 / F1

CX = TANG / 57.3

Z1 = CMPLX(0.0, CX)

C2 = C1 * RP * CEXP(Z1)

D3 = ATT * W2

D3P = EXP(D3)

Z2 = (P1 * W2) / DELTA

Y1 = CMPLX(0.0, Z2)

A2 = D3P * C2 * CEXP(Y1)

G = REAL(A2)
H=AIMAG(A2)
VAL=SQRT(G**2+H**2)

IF(G)12,13,12
IF(H)14,14,15

P=-90.0
GO TO 20

P=90.0
GO TO 20

P=(ATAN2(ABS(H),ABS(G)))*57.3

IF(G)91,17,17
IF(H)18,18,19
IF(H)21,21,20

P=P-180.0
GO TO 20

P=180.0-P
GO TO 20

P=-1.0*P

WRITE(3,201) F, R, RANG, VAL, P, RUN
CONTINUE

CALL EXIT

FORMAT(1OF7.0,I2)

FORMAT(3X,'FREQUENCY',3X,'REFLECTION FACTOR',3X,'REFLECTION ANGLE'
1,3X,'TRANSMISSION FACTOR',3X,'TRANSMISSION ANGLE',3X,'RUN')

END
<table>
<thead>
<tr>
<th>FREQUENCY DATA</th>
<th>REFLECTION FACTOR</th>
<th>REFLECTION ANGLE</th>
<th>TRANSMISSION FACTOR</th>
<th>TRANSMISSION ANGLE</th>
<th>RUN</th>
</tr>
</thead>
<tbody>
<tr>
<td>500.0</td>
<td>0.958</td>
<td>84.33</td>
<td>0.012</td>
<td>291.34</td>
<td>1</td>
</tr>
<tr>
<td>500.0</td>
<td>0.961</td>
<td>178.11</td>
<td>0.005</td>
<td>153.92</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0.012</td>
<td>291.34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.005</td>
<td>153.92</td>
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<td></td>
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<tr>
<td></td>
<td>0.001</td>
<td>134.15</td>
<td></td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>500.0</td>
<td>0.965</td>
<td>150.04</td>
<td>0.001</td>
<td>134.15</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX IV

DISCUSSION OF THE THEORETICAL TRANSMISSION AND REFLECTION FACTORS OF TUBE COUPLING METHODS

During the course of experimental measurements, varied lengths of thinwall brass and aluminum tubing were utilized in conjunction with the standing wave tube (e.g. transmission measurements). The outside diameter of this tubing was 2 inches. Since the inside diameter of the standing wave tube was 2 inches, the additional tubes were coupled by inserting the thinwall tubing into the standing wave tube approximately 2 inches. The effect of this change in cross section was neglected in the calculations. Herein is discussed the theoretical magnitude of the transmission and reflection factor encountered by this change in cross section.

The wall thickness of the thinwall tubing was approximately 1/16 inch. From previous information the reflection factor at such a junction is:

\[ R = \frac{S_1 - S_2}{S_1 + S_2} \]  

(47)

Where:

- \( S_1 \) = inside cross sectional area of the standing wave tube
- \( S_2 \) = inside cross sectional area of the thinwall tubing.

Thus:

\[ S_1 = \pi R_1^2 = \pi (1)^2 = 3.142 \]
\[ S_2 = \pi R_2^2 = \pi \left( \frac{15}{16} \right)^2 = 2.761 \]

Therefore:

\[ R = \frac{S_1 - S_2}{S_1 + S_2} = \frac{3.142 - 2.761}{3.142 - 2.761} \]
\[ R = \frac{0.381}{0.381} = 0.0645 \]
Furthermore, as previously presented, the transmission factor is:

\[ T = \frac{4 S_1 S_2}{(S_1 + S_2)^2} \]  \hspace{1cm} (48)

The cross sectional areas being the same as above:

\[ T = \frac{4(3.142)(2.761)}{(5.903)^2} = \frac{34.70}{34.85} \]

\[ T = 0.9957 \]

Thus it may be seen that the neglect of the effect of the coupling had little or no effect on the calculations since fairly high reflection factors and low transmission factors were encountered.
APPENDIX V

THE EVALUATION OF THE ANECHOIC TERMINATION

As previously mentioned in the thesis, anechoic conditions were assumed on the outlet side of the adjustable muffler. Thus, in the calculation of the reflection factor all reflection was assumed to emanate from the muffler and not from end conditions. Similarly, in the calculation of the transmission factor, the downstream sound pressure measurement was assumed to be composed only of sound transmitted through the muffler and no reflected sound contributed to the pressure amplitude at that station. In both cases, an assumption is made that the anechoic termination is completely non-reflective.

The termination was evaluated by means of the standing wave tube to check the validity of the anechoic assumptions. The calculated data is presented below.

<table>
<thead>
<tr>
<th>FREQUENCY (Hz.)</th>
<th>REFLECTION FACTOR (considering attenuation)</th>
<th>PHASE ANGLE (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.279</td>
<td>315.10</td>
</tr>
<tr>
<td>100</td>
<td>0.223</td>
<td>223.57</td>
</tr>
<tr>
<td>200</td>
<td>0.084</td>
<td>92.91</td>
</tr>
<tr>
<td>500</td>
<td>0.032</td>
<td>88.08</td>
</tr>
<tr>
<td>750</td>
<td>0.065</td>
<td>47.69</td>
</tr>
<tr>
<td>1000</td>
<td>0.050</td>
<td>94.89</td>
</tr>
<tr>
<td>1550</td>
<td>0.046</td>
<td>247.73</td>
</tr>
</tbody>
</table>

It can be observed that the termination at low frequencies is fairly poor. The reflection factor of the adjustable muffler at 50 Hertz is mostly between 0.3 and 0.4. Thus, it may be assumed that the
termination plays an important part in the observed characteristics of the muffler. The observed reflection characteristics of the muffler at 100 Hertz is from two to three times greater than the reflection factor of the termination. Thus, the anechoic assumption is fairly valid at this frequency.

For the remainder of the selected frequencies the reflection of the termination is never above 10% and the reflection characteristics of the adjustable muffler are always much larger in comparison.

It may, therefore, be stated that the anechoic outlet assumptions are valid above 100 Hertz.
APPENDIX VI

CONSIDERATION OF FREQUENCY RANGE FOR AUTOMOTIVE EXHAUST NOISE ANALYSIS

In examining automotive exhaust sound spectrums, it may be observed that the noise is mostly low frequency in content. Simon [33] indicates that the two inch inside diameter standing wave tube is capable of accurate acoustic measurements down to a frequency of 50 Hertz. Thus, this frequency limits the low end evaluation of the adjustable muffler.

The engine speed corresponding to this frequency for an eight cylinder engine is:

\[
\text{RPM} = \frac{\text{pulses}}{\text{sec}} \cdot \frac{2 \text{ rev}}{1 \text{ cycle}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{1 \text{ cycle}}{8 \text{ exhaust pulses}}
\]

Therefore:

\[
\text{RPM} = \frac{(50)(2)(60)}{8} = 750 \text{ RPM}
\]

This low limit frequency is slightly above the normal idle speed of approximately 500 RPM.

Maximum brake horsepower for today's eight cylinder engines is rated anywhere between 4000 and 5000 RPM. Therefore, for an upper frequency limit, assuming that the maximum brake horsepower for an eight cylinder engine is rated at an engine speed of 4500 RPM, then:

\[
f_1 = \frac{\text{REV}}{\text{min}} \cdot \frac{1 \text{ cycle}}{2 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{8 \text{ exhaust pulses}}{\text{cycle}}
\]

Therefore:

\[
f_1 = \frac{(4500)(8)}{(2)(60)} = 300 \text{ Hertz}
\]

W. S. Gatley in his research has found that five harmonics are significant in automotive exhaust noise. This is borne out by the exhaust
spectrums recorded off of an 8 cylinder engine (figures 5-45 and 5-46). Therefore, the upper limit of experimental investigation was set at:

\[ f_u = 5f_1 = 1500 \text{ Hertz} \]
APPENDIX VII
CONSIDERATIONS OF ENGINE SIZE FOR AUTOMOTIVE EXHAUST SPECTRUM RECORDINGS

According to the seventh edition of *Marks' Standard Handbook for Mechanical Engineers*, the displacement of American automotive engines is substantially in the range of 250 to 330 cubic inches. The average cylinder bore in 1965 was 3.91 inches and the average stroke was 3.45 inches. Then, calculating the average displacement for an eight cylinder engine from these figures:

\[ D_{\text{avg.}} = N_c A_c S \]

Where:  
\( N_c \) = number of cylinders  
\( A_c \) = cross sectional area of one cylinder  
\( S \) = length of stroke

Therefore:

\[ D_{\text{avg.}} = (8)[\pi(3.91/a)^2](3.45) = 331 \text{ in}^3 \]

Using this criteria, a 330 cubic inch Oldsmobile engine was selected for test data.
APPENDIX VIII
A DISCUSSION OF SOURCES OF ERROR

As in the case of almost all experimental evaluations, one may presuppose the existence of a certain amount of error introduced by empirical methods and limitations of the investigative systems. However, since there are no theoretical comparisons available, the type and amount of error present in these tests may only be hypothesized.

A. Reflection Factor Measurements

The sound pressure in the standing wave tube in this segment of the pure tone analysis was measured to three significant digits. Since the measured voltage in the required decibel level range was always less than unity. This implies an absolute error in sound pressure voltage of less than 0.001 volts. Thus, the magnitude of the reflection factor;

\[ |R| = \frac{p_{\text{max}} - p_{\text{min}}}{p_{\text{max}} + p_{\text{min}}} \]

may be assumed correct to third decimal place.

The locations of the pressure maximums and minimums were measured to two decimal places (i.e., hundredths of a centimeter). This requires interpolation of the hundredths column numeral since the standing wave measuring apparatus is divided into millimeters. This infers an absolute error in measurement of less than 0.01 cm. Since the wavelength of the incident sound should remain essentially constant, then the average deviation from the mean of the wavelength at a given frequency should be a good indication of accuracy of the measurement. The average deviation of the mean wavelength at 500 Hertz was found to be -0.072% while at 1550 Hertz the deviation was -0.158%. These figures are also
indicative of the fact that error in calculation of the phase angle;

\[ \theta = \frac{2\pi x_{\text{min}}}{\lambda/2} \]

increases as the wavelength shortens. Another comparison which might be relevant here is the collation between the mean and theoretical wavelengths. The mean wavelength at 500 Hertz was found to be 69.56 cm. while the theoretical wavelength considering attenuation is 68.86; a deviation of 1.02%.

Thus, it may be assumed that error due to the measuring apparatus for experimental reflection factors is well within reason and the pure tone reflection analysis is valid.

B. Transmission Factor Measurements

The sound pressure measurement system for transmission factors utilizes the same voltmeter as in reflection measurements. Therefore, the accuracy may be assumed the same. However, the microphones on the upstream and downstream side of the adjustable muffler were assumed to be matched in response. In actuality the microphones were not exactly twins. However, the microphones were always utilized in the same positions such that the error was systematic. Below is tabularized the relative error incurred with the use of these microphones.

<table>
<thead>
<tr>
<th>FREQUENCY (Hz)</th>
<th>RELATIVE ERROR UPSTREAM SIDE (V.)</th>
<th>RELATIVE ERROR DOWNSTREAM SIDE (V.)</th>
<th>RELATIVE PHASE ANGLE ERROR (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.096</td>
<td>0.036</td>
<td>3.0</td>
</tr>
<tr>
<td>200</td>
<td>0.036</td>
<td>0.000</td>
<td>4.0</td>
</tr>
<tr>
<td>750</td>
<td>0.038</td>
<td>0.0004</td>
<td>5.0</td>
</tr>
<tr>
<td>1550</td>
<td>0.067</td>
<td>0.001</td>
<td>11.5</td>
</tr>
</tbody>
</table>
As may be noted from the above table the transmission phase incurs more error as the frequency increases. Also the relative error in low sound pressure measurements is much less than at high input levels. In general the microphone used on the upstream side of the muffler transduced higher sound pressure level readings than the output microphone. Thus the transmission factors would be slightly lower than if two exactly matched microphones were utilized.
APPENDIX IX

LIST OF EQUIPMENT

1. General Radio Co., Beat-Frequency Audio Generator, Type 1304-B, Serial No. 3276.
10. General Radio Co., Data Recorder, Type 1525-A, Serial No. 119.
12. AD YU Electronics, Inc., Precision Phase Meter, Type 406L, Serial No. 3399L.
13. Dynakit, MARK IV 40 watt amplifier, Serial No. 8741024.
15. J. B. Lansing Sound, Inc., high frequency driver, Model 375 HP, Serial No. 8257.
16. Modern Engineering Co., Welding Oxygen Pressure Regulator, Type P.
BIBLIOGRAPHY


VITA

Phillip Scott Gegesky was born in Joplin, Missouri, on June 28, 1946. At the age of three he traveled to Yokahama, Japan, where his father was stationed with the Army for three years. He attended St. Peter's Parochial Grade School in Joplin, Missouri, and Christ The King Parochial Grade School in Haddon Field, New Jersey. At the age of eleven he traveled to Izmir, Turkey, where he attended the military dependent's school. During his father's two year tour of duty he visited Italy, Greece, Spain, and Libya. Upon his return to the United States in 1960 he attended and subsequently graduated, in June 1964, from Arundel High School in Gambrills, Maryland.

In September of 1964 Mr. Gegesky began working toward a Bachelor of Science Degree in Mechanical Engineering at the University of Missouri - Rolla, and in May of 1968 he received his degree. In June of the same year he began a program leading to a Master of Science Degree in Mechanical Engineering.

Mr. Gegesky held a fellowship from the Officer's Wives Club of Fort Meade, Maryland.

He is a citizen of the United States of America.