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Determination of reservoir characteristics from capillary pressure curves and electrical properties of rocks

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DETERMINATION OF RESERVOIR CHARACTERISTICS FROM CAPILLARY PRESSURE CURVES AND ELECTRICAL PROPERTIES OF ROCKS

By

PRASANTA KUMAR GUHAROY, 1943

A Thesis submitted to the faculty of THE UNIVERSITY OF MISSOURI-ROLLA in partial fulfillment of the requirements for the Degree of

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Approved by

M.D. Arnold (Advisor)

J.S. Stover

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ABSTRACT

Based upon the assumptions that a porous medium is analogous to a bundle of capillary tubes and that there is equivalence of electrical and hydraulic tortuosity in a porous medium, empirical expressions are presented which relate capillary pressure data and electrical properties of the rock to the porosity, permeability and relative-water permeability, respectively, of completely water-wet rocks. Experimental data are presented for comparison in order to verify their validity.

Three principal models are presented, each of which is characterized by a particular expression for estimating tortuosity. The method of determining this parameter was varied in order to find the form best matching experimental results.
ACKNOWLEDGEMENT

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I. INTRODUCTION

Capillary pressure is defined as the pressure difference that exists across the curved interface of a wetting phase in contact with a non-wetting phase, as a result of surface forces between the two phases. Capillary pressure in a porous medium is known to be a function of the saturation distribution of the contained fluids, of the saturation history and of the wetting characteristics of the rock-fluid system.

In 1941, Leverett\textsuperscript{1} presented a paper which gave the essential concepts of capillary behavior in a porous medium. He dealt with the subject both theoretically and experimentally and derived a dimensionless function, correlating porosity, absolute permeability, capillary pressure and surface tension at the fluid interface. Subsequently, Bruce and Welge\textsuperscript{2}, Hassler and Brunner\textsuperscript{3}, Purcell\textsuperscript{4} and others\textsuperscript{5,6,7} have presented various experimental techniques to determine capillary pressure characteristics of small core samples.

In recent years, attention has also been paid to the problem of measuring reservoir parameters from capillary pressure data and electrical properties of the reservoir rock\textsuperscript{4,5,6,8,9,10,11,12,13,16} Since relative permeabilities of cores are difficult to measure, special
attention has been paid to correlations between relative permeability and some easily measured properties of porous media as proposed by a number of authors.\textsuperscript{5,9,10,11,12,13,16}

In this work, based upon capillary pressure equations, Darcy's and Poiseuille's laws for fluid flow, and electrical properties of the rock, theoretical equations have been developed which correlate capillary pressure data and electrical properties of the rock to the porosity, absolute permeability, and the wetting phase relative permeability of a water-wet porous medium. Available experimental data are presented to support the validity of the correlations.

In order to extend the applicability of the developed correlations, tortuosity characteristics of three porous models, in particular case of a completely water-wet rock, have been examined and modified for better agreement with available data.
II. LITERATURE REVIEW

It is only recently that serious attempts have been made to apply and extend the capillary phenomena in porous media in cognizance of its importance in oil reservoirs and oil production.

However, due to the complex nature of porous structures, theoretical developments pertaining to capillary pressure in porous media have been semi-empirical.

One of the early works on capillary behavior in porous media is that of Leverett (1941). He defined the term "capillary pressure" and applied it to an ideal porous system. He also developed a semi-empirical relationship, supported by dimensionless analysis and experimental data. This relationship, the J-function, is given as follows:

\[
J(sw) = \frac{\Delta \rho \cdot g \cdot h}{\nu \cdot \sqrt{\frac{K}{\rho}}} = \frac{P_c}{\nu} \sqrt{\frac{K}{\gamma}}
\]  

(1)

where

\( g \) = gravitational constant

\( h \) = height of the capillary interface above free wetting phase surface

\( \Delta \rho \) = density difference between two fluid phases

\( \nu \) = surface tension between two fluid phases
\( K = \text{permeability of porous medium to a homogeneous fluid} \)

\( \phi = \text{porosity of the medium} \)

\( P_c = \text{capillary pressure of the porous medium at any particular saturation of the wetting phase} \)

Equation (1) is known as the Leverett "J-function" or "the capillary pressure function". Leverett assumed \( J(sw) \) to be a dimensionless function only of the wetting phase saturation, but it has, also, been found to be a function of the type of the rock formation. Calhoun et al.\(^6\) states that to extend its applicability to all liquids, Equation (1) can be written as:

\[
J(sw) = \frac{P_c}{\nu \cos \theta} \sqrt{\frac{K}{\phi}}
\]  

(2)

where \( \theta \) is the contact angle of the fluid interface with the rock. Leverett also established that the interfacial boundary between two immiscible fluids is curved due to interfacial tension between fluids and he defined the pressure differential across the interface as "capillary pressure". The capillary pressure is related to the curvature of the interface by the following well known expression:

\[
P_c = \nu \left( \frac{1}{R_1} + \frac{1}{R_2} \right)
\]  

(3)
where

\[ P_c = \text{capillary pressure} \]
\[ \gamma = \text{interfacial tension} \]
\[ R_1 \text{ and } R_2 = \text{principal radii of curvature of the interface} \]

The expression \( \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \) is defined as the mean curvature of the interface. Figure 1 shows the approximate shape of an oil-water interface when only a small amount of water is present between two spherical sand grains. \( P_c \) in Equation (2) is proportional to \( J(sw) \), to interfacial tension, \( \gamma \), and is proportional to the square root of permeability-porosity radio, \( \sqrt{\frac{K}{\phi}} \), which has the basic unit of length. Hence, there is theoretical justification of Equation (1) since it confirms the basic concepts of capillarity as defined by Equation (2).

Leverett concluded that a plot of the capillary function \( J(sw) \), against the wetting phase saturation, \( sw \), yields an unique curve for clean unconsolidated sands. Rose and Bruce compared Leverett's average data with measurements, obtained on a number of consolidated systems and showed that consolidated systems are not all uniquely represented by a single \( J \)-function curve. These results are shown in Figure 2.

In another earlier attempt Carman showed that the height of capillary rise is calculable from grain size and porosity of a sand. He derived the following equation for
Figure 1: Idealized Contact for Wetting Fluid and Spherical Grains (from Leverett)
Figure 2: Capillary Retention Curves (from Rose and Bruce)
capillary pressure:

$$P_c = \frac{\gamma S}{\varnothing}$$  \hspace{1cm} (4)

where $S$ is the surface area per unit of bulk volume of the medium. Experimental data support his theoretical argument. The close connection between the work of Leverett\textsuperscript{1,14} and Carman\textsuperscript{15} is evident by considering the Kozeny equation which describes streamline flow through porous media. This equation will be derived in the next section of this paper and is given here as Equation 5.

$$K = \frac{\varnothing}{k_z S_p^2}$$  \hspace{1cm} (5)

where

- $K$ = permeability
- $k_z$ = a numerical (rock textural) constant (which makes Kozeny equation describe streamline motion through granular beds)
- $S_p$ = surface area per unit volume of pore space

Based on the definition of the term $S$ in Carman's equation, it can be further concluded that:

$$S_p = \frac{S}{\varnothing}$$  \hspace{1cm} (6)

Rose and Bruce\textsuperscript{5} combined Equation (4), (5) and (6) and found that for wetting phase saturation, $Sw$, equal to unity.
\[
\frac{P_d}{v} \left( \frac{k}{\phi} \right)^5 = \left( \frac{1}{k_z} \right)^5 \tag{7}
\]

where \( P_d \) is the displacement pressure, defined as

\[
\lim_{Sw \to 1} P_c = P_d \tag{8}
\]

Comparing Equation (7) and (1), a possible minimum value of \( J(Sw) \) is obtained as follows:

\[
\lim_{Sw \to 1} J(Sw) = \left( \frac{1}{k_z} \right)^5 \text{ minimum} \tag{9}
\]

It is also noted that the relationship \( (1/k_z)^5 \) minimum is applicable only to media of uniform pore size.

Rose and Bruce\(^5\) also derived the universal capillary pressure-porosity relationship as:

\[
\phi = Sw \left[ 1 + k_z \right]^{1/2} \int_{Sw}^{1} J(Sw) dSw \tag{10}
\]

and for the special case of a capillary tube

\[
\phi = \frac{Sw}{(2-Sw)^2} \tag{11}
\]

It is readily shown that the slope of the above \( \phi \) versus \( Sw \), as \( Sw \) approaches unity, is numerically equal to three (3) in both cases of Equations (10) and (11).
Rose and Bruce\textsuperscript{5} and Rose\textsuperscript{10} were the first to apply the (concept of) Kozeny equation and capillary pressure data to the case of partially desaturated porous media to describe wetting phase relative permeability, $K_{rw}$.

Rose and Bruce\textsuperscript{5} wrote the Kozeny equation in the form:

$$K = \left(\frac{\phi}{k}\right) \left(\frac{\nu}{P_c}\right)^2$$  \hspace{1cm} (12)

at 100 percent water saturation and as:

$$K_{ew} = \left(\frac{\phi \cdot S_w}{k}\right) \left(\frac{\nu}{P_c}\right)^2$$  \hspace{1cm} (13)

for a fractional wetting phase saturation, $S_w$, and the corresponding capillary pressure, $P_c$. They assumed that $k_z$ is constant at all saturations and arrived at:

$$K_{rw} = S_w \left(\frac{P_d}{P_c}\right)^2$$  \hspace{1cm} (14)

where $K_{rw}$ is the relative permeability for the wetting phase.

This expression was found to show serious discrepancies when compared with available data for $K_{rw}$.

Thornton\textsuperscript{11} modified Equation (14) by considering the Kozeny constant to be a function of saturation and derived $K_{rw}$ as:

$$K_{rw} = \left(\frac{1}{2} S_w\right) \left(\frac{P_d}{P_c}\right)^2$$  \hspace{1cm} (15)
where I is the resistivity index, defined as the ratio of resistivity of a partially water-saturated rock, $R_t$ and resistivity of the rock at 100 per cent water saturation, $R_o$.

The above argument was found to be substantially validated by experimental measurements. A further expression of relative wetting phase permeability has been suggested by Rose and Wyllie$^{12}$, which has the extreme simplicity:

\[ K_{rw} = \frac{1}{I^2} \]  

(16)

However, this equation is only applicable when $P_c = P_d$ throughout the saturation range. This equation was subsequently improved to:

\[ K_{rw} = \frac{S_w}{I^2} \]  

(17)

at any fractional saturation, $S_w$.

Purcell$^4$ derived the following expression for absolute permeability for a porous medium from Poiseuille’s equation:

\[ K = .66 \int_0^{Sw=100} \frac{dSw}{(P_c)^2} \]  

(18)

where

\[ K = \text{permeability at 100 percent saturation to wetting phase, milli-darcies} \]
\[ \phi = \text{percent porosity} \]

\[ Sw = \text{percent of total pore space occupied by the liquid} \]

\[ P_c = \text{capillary pressure, atmospheres} \]

\[ F = \text{lithology factor} \]

He showed that the value of the integral is the area under the curve \( \frac{1}{P_c^2} \) versus saturation.

Fatt and Dykstra\textsuperscript{13} followed the basic method of Purcell for calculating permeability. They developed an expression for relative permeability to the wetting phase considering that the lithology factor, \( F \), in Equation (18) is a function of saturation and lithology factor is essentially a correction for deviation of path length from the length of the porous medium. They assumed that

\[
F = \frac{a}{r^b} \quad (19)
\]

where

- \( a, b = \text{constants for rock materials} \)
- \( r = \text{radius of a pore} \)

They further assumed that \( b=1/2 \) and derived an equation for relative permeability of wetting phase as:

\[
K_{rw} = \frac{\int_0^{Sw} \frac{dSw}{P_c^3}}{\int_0^1 \frac{dSw}{P_c^3}} \quad (20)
\]
Burdine also reported equations for computing relative permeability curves for both wetting and non-wetting phases.

Wyllie and Spangler derived the following expression for relative permeability to the wetting phase:

\[
K_{rw} = \frac{1}{I^2Sw^2} \int_{Sw=0}^{Sw=Sw/Pc} \frac{dSw}{Sw} \left( \frac{Sw}{1.0} \right)^2 \int_{Sw=0}^{Sw=Sw/Pc} \frac{dSw}{Sw} \right.
\]

This equation was derived by summing the individual flows for all the infinitesimal portions of the pore space. Each of the individual flow rates was calculated from Kozeny-Carman equation.

After reviewing the literature, it was concluded that existing techniques are inadequate for accurately calculating reservoir parameters from capillary pressure data and electrical properties of porous media. Also, previous work appears inadequate in that the validity of different porous models characterized by tortuosity of the porous media has not been well established. It was the purpose of this study to extend existing methods, develop new techniques, and improve the accuracy of permeability, porosity and relative permeability calculations based on capillary pressure and resistivity measurements by developing more meaningful characterization parameters.
III. DEVELOPMENT OF THEORY

Consider a porous medium composed of a bundle of capillary conduits of cross sectional area, $\bar{A}$ and average effective length, $L_a$. In developing the theory, the following assumptions are made:

1. The porous medium is conceived to be a bundle of capillary tubes of complex shapes.

2. Capillaries are oriented in a direction parallel to the macroscopic flow.

3. The shape factor, $k_o$, in Poiseuille's law, does not depend on the saturation of the wetting phase and it varies between 2.0 to 3.0 depending upon the shape of a pore's cross sectional area (circular, elliptical, square, etc.).

4. The equivalence of electrical and hydraulic tortuosity ratios is substantially valid.
5. The contact angle between water and the rock surface in the porous medium is zero.

6. The tortuous path, through which the fluid flows in the porous medium, is a length $L_a$, which is greater than the external length $L$.

7. The porous medium is considered to be completely water-wet.

The following three basic equations are considered in the development:

1. Poiseuille's Law
2. Darcy's Law
3. Capillary pressure equation

Poiseuille's Law for laminar flow through a tube of circular cross section is given by:

$$v = \frac{d^4 \Delta p}{32 \mu L}$$  \hspace{1cm} (22)

where

$v$ = average velocity inside the tube
$L$ = length of the tube
$\mu$ = viscosity
$\Delta p$ = pressure difference
d = diameter of the tube

in consistent units.
Previous investigations of fluid flow in conduits with noncircular cross section have revealed that a quantity, \( m \), the mean hydraulic radius, should be introduced in Equation (22).

\[
m = \frac{\text{Volume of Conduit}}{\text{Area of Wetted Surface}}
\]

for a circular pipe,

\[
m = \frac{\pi r^2 L}{2\pi r L} = \frac{d}{4}
\]

when substituted into Equation (22), this relation yields:

\[
v = \frac{m^2}{2} \frac{\Delta p}{\mu L}
\]

Poiseuille's law in this form approximates flow in non-circular conduits quite well.

Wyllie and Spangler\(^9\) suggested that the factor 2.0 in Equation (24) should be replaced by a parameter, \( k_0 \), called the shape factor. In general, \( k_0 \) falls within the range of 2 to 3 for the majority of complex shapes.

In applying Poiseuille's law to flow in a porous medium it is necessary to correct the length of the system, \( L \), to the length of the actual tortuous flow path through which fluid is flowing (\( L_a \)). In straight pipes of conduits \( L = L_a \). However, in porous media \( L_a > L \). The actual average flow velocity, \( v_a \), within the porous medium must be greater than the superficial velocity \( v \) (equal to \( Q/A \)).
The increase in velocity is due to both a decrease in cross-sectional area available for flow (\( \mathcal{A} < \mathcal{A} \)) and also to the increase in length of the actual tortuous flow path, \( L_a (L_a > L) \) within the porous medium. Thus, the actual velocity, \( v_a \), is related to the superficial velocity, \( v \), as follows:

\[
va = \frac{vLa}{\mathcal{g}L} \tag{25}
\]

Correcting Poiseuille's law for actual flow conditions gives:

\[
va = \frac{m^2}{\mathcal{g}} \frac{\Delta p}{\mu L_a} \tag{26}
\]

Hence,

\[
\frac{v}{\mathcal{g}} \frac{La}{L} = \frac{m^2}{k_0} \frac{\Delta p}{L_a} \]

or,

\[
v = \frac{\mathcal{g} m^2}{k_0} \frac{\Delta p}{\mu L} \left( \frac{L}{L_a} \right)^2 \tag{27}
\]

For porous media, the mean hydraulic radius, \( m \), is equal to the ratio of porosity to surface area per unit bulk volume. This relation is expressed as follows:

\[
m = \frac{\mathcal{g}}{S} \tag{28}
\]

Since \( S_p \) was defined as the surface area per unit pore volume, it can be written as:

\[
S_p = \frac{S}{\mathcal{g}} = \frac{1}{m} \tag{29}
\]
Darcy's law for laminar flow through porous media can be written as follows:

\[ v = \frac{K}{\mu} \frac{\Delta p}{L} \]  

(30)

where

- \( v \) = flow velocity
- \( K \) = permeability
- \( \mu \) = viscosity
- \( L \) = length
- \( \Delta p \) = pressure drop

Combining Equation (30) and (27) yields

\[ \frac{\varnothing m^2}{k_o} \frac{\Delta p}{\mu L} \left( \frac{L}{La} \right)^2 = \frac{K\Delta p}{\mu L} \]

or

\[ K = \frac{\varnothing m^2}{k_o} \left( \frac{L}{La} \right)^2 \]  

(31)

This equation is known as the Kozeny–Carman equation.

Substituting the value of \( m \) into Equation (31) yields,

\[ K = \frac{\varnothing}{k_o} S \frac{\Delta p}{p^2} \left( \frac{L}{La} \right)^2 \]  

(32)

Define \( \left( \frac{La}{L} \right)^2 \) as tortuosity of the porous medium (T) and further define

\[ k_z = k_o \cdot T \]  

(33)
This result, $k_z$, is called the Kozeny constant. Carman was responsible for writing the Kozeny constant. He concluded that the value of $k_z$ is 5.0 in all cases. However, further investigation has suggested a possible variation in $k_z$ from 3.0 (or less) to 6.0.

Regrouping terms in Equation (32) yields the following expression containing the Kozeny constant, $k_z$:

$$k = \frac{g}{k_z \cdot Sp^2} \quad (34)$$

In the following paragraphs theoretical developments are presented with three different porous models characterized by the tortuosity of the porous media which correlate porosity, permeability and relative permeability to the wetting phase of a water-wet rock with other measurable quantities of porous media.

A. Porous Model 1: $T = F^2 \varphi^2 (1 - Swc)^2$

Wyllie and Rose have suggested that the measured values of formation resistivity factor allow direct evaluation of tortuosity for a porous medium. As defined by Archie, the true formation factor is:

$$F = \frac{R_o}{R_w} \quad (35)$$

where
\[ F = \text{true formation factor} \]

\[ R_0 = \text{resistivity of the rock when 100 per cent saturated with an electrolyte (usually brine)} \]

\[ R_w = \text{resistivity of the electrolyte} \]

Resistivity is given by:

\[ R = \frac{rA}{L} \quad (36) \]

where

- \( R \) = resistivity
- \( A \) = cross sectional area of the conductor
- \( r \) = resistance across the conductor
- \( L \) = length of the conductor

Consider a cube of length, \( L \), and cross sectional area \( A \). Resistance across this unit cube of electrolyte having resistivity, \( R_w \), is:

\[ r_1 = \frac{R_w L}{A} \quad (37) \]

To illustrate the resistivity concept, reference is made to Figure 3 which represents a cube of porous rock which is 100 per cent saturated with an electrolyte of resistivity, \( R_w \). Considering solids to be non-conductive, the electrical flow must then be through the electrolyte filled pores. The cross-sectional area available for conduction is \( \mathcal{A} \). The average path length that an ion must traverse is the tortuous path, \( L_a \).
Figure 3: Straight Capillary Tube Model of Porous Media (presented by Amyx, Bass and Whiting to illustrate the model proposed by Wyllie et al.9,17).
Pirson suggested that in a completely brine-saturated rock the electric current will not flow effectively through the full volume of brine. This is analogous to fluid flow in porous media at 100 per cent saturation where all the fluid is not movable. The immovable brine is the irreducible wetting phase saturation, Swc. If the passage of fluid and electric current is considered to be equivalent then the irreducible brine saturation, which occupies the cross capillaries through which there is no pressure differential or potential drop, behaves like nonconductive mineral framework.

Therefore, the cross sectional area available for conduction in the porous cube under consideration is $\phi A(1 - Swc)$. The resistance, $r_2$, of such a cube can be written as:

$$r_2 = \frac{R \omega A}{\phi A(1 - Swc)}$$  \hspace{1cm} (38)

By definition, $R_0 = \frac{r_2 A}{L}$  \hspace{1cm} (39)

Hence, $R_0 = \frac{R \omega L A}{L\phi A(1 - Swc)}$  \hspace{1cm} (40)

Therefore, $F = \frac{R_0}{R_w}$

$$= \left(\frac{L a}{L}\right) \frac{1}{\phi (1 - Swc)}$$

$$= \frac{\sqrt{T}}{\phi (1 - Swc)}$$  \hspace{1cm} (41)
where $T$ is the tortuosity, which is being characterized here by the following relationship:

$$T = F^2 \varphi^2 (1-Swc)^2 \quad (42)$$

This definition leads to the following equation characterizing $k_z$, the Kozeny constant:

$$k_z = k_o * F^2 \varphi^2 (1-Swc)^2 \quad (43)$$

(1) Calculation of Absolute Permeability

Equation (34) can now be rearranged as:

$$K = \frac{\varphi}{k_o F^2 \varphi^2 (1-Swc)^2 S^2 p^2} \quad (44)$$

$$= \frac{1}{k_o F^2 \varphi (1-Swc)^2 S^2 p^2}$$

In a circular capillary tube, the capillary rise of a liquid which has a contact angle of zero with the material of the tube, is given by:

$$P_c = \frac{2\nu}{r} = \rho gh \quad (45)$$

or,

$$\frac{\nu}{\rho gh} = \frac{r}{2} = \frac{d}{4} = m \quad (46)$$
Again, for a porous medium,

\[ m = \frac{\varrho}{S} = \frac{1}{Sp} \]  \hspace{1cm} (47)

Hence,

\[ \frac{\varrho}{S} = \frac{v}{\rho gh} = \frac{1}{Sp} \]  \hspace{1cm} (48)

For a porous medium at 100 per cent saturation with a wetting phase,

\[ P_c = P_d \]  \hspace{1cm} (49)

Hence,

\[ P_d = \rho gh \]

Thus, from Equation (46)

\[ \frac{P_d}{v} = Sp \]  \hspace{1cm} (50)

Therefore, referring back to Equation (44), the following relationship can be written:

\[ K = \frac{v^2}{k_o F^2 \varrho (1-Swc)^2 P_d^2} \]  \hspace{1cm} (51)

where \( K \) is the absolute permeability of the porous medium.
(2) Calculation of Porosity

Leverett's J-function relationship is again written as:

\[ J(S_w) = \left( \frac{P_c}{v} \right) \left( \frac{K}{\varrho} \right)^{0.5} \]

Rose and Bruce\(^5\) wrote the equation as

\[
\frac{[J(S_w)]}{S_w+1} = \left( \frac{P_d}{v} \right) \left( \frac{K}{\varrho} \right)^{0.5}
\]  \hspace{1cm} (52)

in order to yield the displacement pressure. Thus,

\[
P_d = \frac{[J(S_w)]}{S_w+1} \frac{v}{\sqrt{\frac{K}{\varrho}}}
\]  \hspace{1cm} (53)

rewriting Equation (51) and rearranging yields:

\[
K = \frac{\nu^2 \cdot K}{\varrho \cdot k_o \cdot F^2 \cdot \varrho \cdot (1 - Swc)^2 \cdot \nu^2 \cdot [J(S_w)]^2 \cdot Sw+1}
\]

or,

\[
\varrho = \frac{1}{\sqrt{K_o \cdot F \cdot (1 - Swc) \cdot [J(S_w)]}} \cdot Sw+1
\]  \hspace{1cm} (54)

Applying the Rose and Bruce correlation to a uniform porous medium yields:
Substituting this result into Equation (54) yields an equation for porosity as follows:

\[ \varphi = \frac{\sqrt{K_z}}{\sqrt{K_0} F (1 - Sw_c)} \]

(3) Calculation of Relative Permeability for Wetting Phase

When a partial wetting phase saturation exists in the rock, the effective tortuosity, \( T_e \), concept is still useful since the non-wetting phase is nonconductive to electricity.

Hence,

\[ T_e = F_e^2 \varphi^2 (Sw_i - Sw_c)^2 \]

where \( F_e \) is the effective formation resistivity factor at the saturation, \( Sw_i \).

From the Kozeny-Carman Equation (34), the effective permeability, \( K_e \), for the wetting phase at any partial saturation can be written as:

\[ K_e = \frac{\varphi_e}{k_{ze} \cdot Sp^2} \]

where \( \varphi_e \) is the effective porosity at saturation, \( Sw_i \), and can be expressed as
\[ \phi_e = \phi * Swi, \]  

(58)

and \( k_{ze} \) is the effective Kozeny constant at the same saturation. The Kozeny constant, then, is written

\[ k_{ze} = k_o * Fe^2 \phi^2 (Swi - Swc)^2 \]  

(59)

Based upon Equation (48),

\[ Sp = \frac{P_c}{\nu} \]  

(60)

where \( P_c \) is the capillary pressure at saturation, \( Swi \).

Hence, the effective permeability for the wetting phase at any partial saturation, \( Swi \), can be derived from Equation (57) as

\[ K_e = \frac{Swi \cdot \nu^2}{k_o Fe^2 \phi^2 (Swi - Swc)^2 P_c^2} \]  

(61)

The wetting phase relative permeability is thus expressed as

\[ K_{rw} = \frac{k_e}{K} = \left( \frac{Swi \cdot \nu^2}{k_o Fe^2 \phi^2 (Swi - Swc)^2 P_c^2} \right) \left( \frac{\nu^2}{k_o P^2 \phi^2 (1 - Swc)^2 P_d^2} \right) \]

\[ = Swi \left( \frac{Fe}{P} \right)^2 \left( \frac{P_d}{P_c} \right)^2 \frac{(1 - Swc)^2}{(Swi - Swc)^2} \]  

(62)

The ratio \( \left( \frac{P}{Fe} \right) \) can be expressed as follows:
\( \frac{F}{F_e} = \frac{(R_o/R_w)}{(R_t/R_w)} = \frac{R_o}{R_t} = \frac{1}{I} \)  \( (63) \)

\( R_t \) is the true resistivity at partial saturation, \( Swi \) and \( I = R_t/R_o \), the resistivity index of the formation.

Hence, introducing the results of Equation (63) in Equation (62) yields

\[ K_{rw} = Swi \cdot \left( \frac{1}{I^2} \right) \cdot \left( \frac{P_d}{P_c} \right)^2 \left( \frac{1-Swc}{Swi-Swc} \right)^2 \]  \( (64) \)

In the above equation, it should be noted that as \( Swi \) approaches \( Swc \) the capillary pressure term, \( P_c \), tends toward infinity. However, the rate of change of \( P_c \) near \( Swc \) is large, and it is assumed that the limit of the function goes to zero, as follows:

\[ \lim_{Swi \to Swc} \frac{P_d}{P_c} \left( \frac{1-Swc}{Swi-Swc} \right) = 0 \]  \( (65) \)

B. Porous Model 2: \( T = F \phi (1-Swc) \)

Cornell and Katz\(^{19} \) have presented a slightly different model which is illustrated in Figure 4. In the simplest form of this model the pores can be considered uniform in cross section but oriented so that they have an effective length, \( La \), which is greater than \( L \). The cross sectional area available for flow is considered constant at each plane in the model. The effective cross-sectional area
Figure 4: Inclined Capillary-Tube Model of Porous Media (after Cornell and Katz)
$A_a$ is the area normal to the direction of flow in the pore. This parameter is developed as follows:

$$A_a = A_1$$  (66)

$$A_1' = A_1 \left( \frac{L}{La} \right)$$  (67)

$$A_1 = \theta A$$  (68)

$$A_a = \theta A \left( \frac{L}{La} \right)$$  (69)

By logic similar to that developed for model 1, the resistance of a porous cube is given by:

$$r_2 = \frac{R_w L_a}{A_a (1-Swc)}$$  (70)

$R_o$ can be developed as

$$R_o = \frac{r_2 A}{L} = R_w \left( \frac{La}{L} \right)^2 \frac{1}{\theta (1-Swc)}$$  (71)

and the formation resistivity factor, $F$, can be expressed as

$$F = \frac{R_o}{R_w} = \left( \frac{La}{L} \right)^2 \frac{1}{\theta (1-Swc)}$$

$$= \frac{T}{\theta (1-Swc)}$$  (72)

and

$$T = F \theta (1-Swc)$$  (73)
Therefore, the Kozeny constant is

\[ k_z = \frac{k_o \varnothing}{(1-Sw)P_d^2} \]  

(74)

(1) Calculation of Absolute Permeability

Equation (34) now can be written as

\[ K = \frac{\varnothing}{k_o F (1-Sw) P_d^2} \]

\[ = \frac{1}{k_o F (1-Sw) P_d^2} \]  

(75)

By an approach similar to that used to develop Model 1, a relationship can be derived for absolute permeability as

\[ K = \frac{\nu^2}{k_o F (1-Sw) P_d^2} \]  

(76)

(2) Calculation of Porosity

Similarly an expression can be derived for porosity by combining Equations (53) and (76) to yield:

\[ \varnothing = \frac{1}{k_o F (1-Sw)(J(Sw))^2} \]

\[ \frac{1}{Sw+1} \]  

(77)
Introducing Rose and Bruce's correlation, Equation (9) into Equation (77) the porosity of a porous media can be expressed as

\[ \phi = \frac{k_z}{k_o F(1-S_{wc})} \]  

(78)

(3) Calculation of Relative Permeability for Wetting Phase

With an approach similar to that used in developing Model 1, the effective tortuosity to a partial wetting phase can be expressed as

\[ T_e = \frac{F e}{\phi} (S_{wi}-S_{wc}) \]  

(79)

and the corresponding Kozeny constant becomes,

\[ k_{ze} = k_o F e \phi (S_{wi}-S_{wc}) \]  

(80)

The effective porosity is written as

\[ \phi_e = \phi S_{wi} \]  

(81)

Thus, the effective permeability for the wetting phase at the saturation, \( S_{wi} \), is expressed as

\[ K_e = \frac{S_{wi} v^2}{k_o F e (S_{wi}-S_{wc}) P_c} \]  

(82)

Then, relative permeability to the wetting phase, \( K_{rw} \), can be written as:
To make this equation valid at $Swi=Swc$ we make the same assumption as in Equation (65). That is:

$$\lim_{Swi \to Swc} \left[ \frac{P_d}{P_c} \frac{1-Swc}{Swi-Swc} \right] = 0 \quad (65)$$

C. Porous Model 3: $T = [F\varnothing(1-Swc)]^{1.33}$

Winsauer, et al.\textsuperscript{20} devised a method of determining tortuosity by transit time of ions flowing through the rock under a potential difference. The data obtained were correlated with the product $F\varnothing$ and are given as follows:

$$\left(\frac{L_a}{L}\right)^{1.67} = F\varnothing$$

or

$$T = \left(\frac{L_a}{L}\right)^2 = (F\varnothing)^{-\frac{2}{1.67}} \quad (84)$$

Introducing the correction to the tortuosity factor to account for irreducible wetting phase saturation, Swc,
yields a relationship for tortuosity in a porous medium 100 per cent saturated with the wetting phase. A slight variation of this relationship was presented by Amyx, et al.\textsuperscript{21}, and is shown as follows:

\[ T = [F(1-Swc)\phi]^{1.33} \] \hspace{1cm} (85)

(1) **Calculation of Absolute Permeability**

Using the above function for tortuosity factor a relationship for absolute permeability is derived as follows:

\[ K = \frac{v^2}{k_o F^{1.33} \phi^{0.33} (1-Swc)^{1.33} \rho_d^2} \] \hspace{1cm} (86)

(2) **Calculation of Porosity**

The porosity of a porous medium can be derived as

\[ \phi = \left[ \frac{1}{k_o F^{1.33} (1-Swc)^{1.33} [J(Sw)]^2} \right]^{1/1.33} \] \hspace{1cm} (87)

Introducing Rose and Bruce's correlation, Equation (9), into Equation (87) yields an equation for porosity as follows:

\[ \phi = \left[ \frac{k_z}{k_o F^{1.33} (1-Swc)^{1.33}} \right]^{0.75188} \] \hspace{1cm} (88)
(3) **Calculation for Relative Permeability for Wetting Phase**

At a partial saturation of the wetting phase, Swi, the effective tortuosity of the porous medium can be written as:

\[ T_e = \frac{1}{2} \left( \frac{\gamma}{\gamma_w} \right)^{1.33} (S_{wi} - S_{wc})^{1.33} \]  \hspace{1cm} (89)

Hence, the equation for effective permeability for wetting phase at any partial saturation, Swi, can be stated as:

\[ K_e = \frac{S_{wi}}{k_0} \left( \frac{\gamma}{\gamma_w} \right)^{1.33} (S_{wi} - S_{wc})^{1.33} \]  \hspace{1cm} (90)

Therefore, relative permeability for the wetting phase at any desired saturation, Swi, is derived as

\[ K_{rw} = S_{wi} \frac{1}{(T_e)^{1.33}} \left( \frac{P_d}{P_c} \right)^2 \frac{1 - S_{wc}}{S_{wi} - S_{wc}}^{1.33} \]

\[ = S_{wi} \left( \frac{1}{T_e} \right)^{1.33} \left( \frac{P_d}{P_c} \right)^2 \frac{1 - S_{wc}}{S_{wi} - S_{wc}}^{1.33} \]  \hspace{1cm} (91)

The behavior of the equation at the limit, Swi=Swc, is again found to be

\[ \lim_{S_{wi} \to S_{wc}} \left[ \left( \frac{P_d}{P_c} \right)^2 \frac{1 - S_{wc}}{S_{wi} - S_{wc}}^{1.33} \right] = 0 \]  \hspace{1cm} (92)
IV. RESULTS AND DISCUSSION

Data available from literature have been taken to check the validity of the theoretical equations developed in Chapter III.

Introducing conversion factors and generalizing, Equations (51), (76) and (86) for predicting absolute permeability reduce to

\[ K = 0.021331618 \frac{\nu^2 k_0 \phi (1-Swc)^2 P_d^2}{k_0 \phi (1-Swc) P_d^2} \]  \hspace{1cm} (93)

\[ K = 0.021331618 \frac{\nu^2 k_0 \phi (1-Swc)^2 P_d^2}{k_0 \phi (1-Swc) P_d^2} \]  \hspace{1cm} (94)

and

\[ K = 0.021331618 \frac{\nu^2 k_0 \phi^{1.33} 0.33 (1-Swc)^{1.33} P_d^2}{k_0 \phi^{1.33} 0.33 (1-Swc)^{1.33} P_d^2} \]  \hspace{1cm} (95)

respectively, where

\[ \nu = \text{surface tension, dynes/cm}^2 \]

\[ P_d = \text{displacement pressure, psi} \]

\[ \phi = \text{fractional porosity} \]

\[ K = \text{permeability, darcy} \]
Data used for testing Equations (93), (94), and (95) were obtained from Wyllie and Rose\textsuperscript{17}. These data and computed data using Equations (93), (94) and (95) are presented in Tables I, II and III, respectively. Permeability is also presented in these tables as a function of the shape factor, $k_0$, which was varied from 2.0 to 3.0. As the values of irreducible water saturation, $S_{wc}$, were not available in Wyllie and Rose's paper, these were assumed to be 20 per cent in all cases.

Through comparison of the measured and the computed permeabilities, it may be noted that results from Equations (93) using a value of $k_0 = 2.7$ yields values closer to the measured permeabilities than do Equations (94) and (95). The good quantitative agreement for most of the data presented in Table I suggest that the correlation by Equation (93) seems most useful.

Equations (55), (78) and (88) represent correlations for porosity. In these equations, no capillary pressure data are necessary for computation. Data used for testing Equations (55), (78) and (88) were obtained from Wyllie and Rose\textsuperscript{17}. These data and predicted porosity are presented in Tables IV, V and VI. As it is known that the Kozeny constant, $k_z$, and shape factor $k_0$, are not constant for all reservoir rocks, they were varied...
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<th>$F$</th>
<th>Swc (assumed frac)</th>
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TABLE II

ABOLUTE PERMEABILITY CALCULATED BY EQUATION(94)
USING DATA FROM WYLLIE AND ROSE 17

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### TABLE III

**Absolute Permeability Calculated by Equation (95)**

*Using Data from Wyllie and Rose*¹⁷

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<tr>
<td>Pyrex</td>
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<tr>
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<td>23.2</td>
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<td>60.8</td>
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</table>
TABLE IV
POROSITY CALCULATED BY EQUATION (55)* USING DATA FROM WYLLIE AND ROSE\textsuperscript{17}

<table>
<thead>
<tr>
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<td>23.2</td>
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<td>18.94</td>
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</table>

*Equation (55) is \[ \varphi = \left[ \frac{K_z}{\sqrt{K_o F (1-Swc)}} \right] \]

where \( K_z = \) Kozeny constant  
\( K_o = \) Shape factor  
\( F = \) True formation resistivity factor  
\( Swc = \) Irreducible Connate water saturation
TABLE V

POROSITY CALCULATED BY EQUATION (78)* USING DATA FROM WYLLIE AND ROSE17

<table>
<thead>
<tr>
<th>Core Description</th>
<th>F</th>
<th>(assumed) Porosity (frac)</th>
<th>Measured Porosity (pct)</th>
<th>Calculated Porosity (pct) Kz=4.0 K0=2.1</th>
<th>Calculated Porosity (pct) Kz=4.5 K0=2.1</th>
<th>Calculated Porosity (pct) Kz=6.0 K0=2.5</th>
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<tr>
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<td>26.55</td>
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<td>11.9</td>
<td>.2</td>
<td>26.5</td>
<td>20.00</td>
<td>22.50</td>
<td>25.21</td>
</tr>
<tr>
<td>&quot;</td>
<td>13.0</td>
<td>.2</td>
<td>26.2</td>
<td>18.31</td>
<td>20.60</td>
<td>23.07</td>
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<td>.2</td>
<td>19.8</td>
<td>19.20</td>
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<td>24.19</td>
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<td>23.6</td>
<td>20.703</td>
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<td>.2</td>
<td>32.5</td>
<td>37.20</td>
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<td>46.87</td>
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<td>23.2</td>
<td>21.84</td>
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<td>27.52</td>
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</table>

*Equation (78) is \( \phi = \frac{K_z}{K_o F (1-Swc)} \)

where \( K_z = \) Kozeny constant \( F = \) True formation resistivity factor \( K_o = \) Shape factor \( Swc = \) Irreducible connate water saturation
### TABLE VI

**POROSITY CALCULATED BY EQUATION (88)\* USING DATA FROM WYLIE AND ROSE\(^{17}\)**

<table>
<thead>
<tr>
<th>Core Description</th>
<th>F</th>
<th>Swc (assumed) (frac.)</th>
<th>Measured Porosity (pct.)</th>
<th>Calculated Porosity (pct) (K_z=4.5) (K_o=2.1)</th>
<th>Calculated Porosity (pct) (K_z=5.0) (K_o=2)</th>
<th>Calculated Porosity (pct) (K_z=6.0) (K_o=2.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alundum</td>
<td>11.3</td>
<td>.2</td>
<td>25.39</td>
<td>19.62</td>
<td>22.03</td>
<td>24.36</td>
</tr>
<tr>
<td></td>
<td>11.9</td>
<td>.2</td>
<td>26.5</td>
<td>18.63</td>
<td>20.92</td>
<td>23.13</td>
</tr>
<tr>
<td></td>
<td>13.0</td>
<td>.2</td>
<td>26.2</td>
<td>17.05</td>
<td>19.15</td>
<td>21.17</td>
</tr>
<tr>
<td>Nichols Buff</td>
<td>12.4</td>
<td>.2</td>
<td>19.8</td>
<td>17.88</td>
<td>20.08</td>
<td>22.19</td>
</tr>
<tr>
<td>Alundum</td>
<td>11.5</td>
<td>.2</td>
<td>23.6</td>
<td>19.28</td>
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<td>23.93</td>
</tr>
<tr>
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<td>.2</td>
<td>32.5</td>
<td>34.64</td>
<td>38.89</td>
<td>43.00</td>
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<tr>
<td>Alundum</td>
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<td>.2</td>
<td>23.2</td>
<td>20.34</td>
<td>22.84</td>
<td>25.25</td>
</tr>
</tbody>
</table>

\*Equation (88) is

\[
\phi = \frac{K_z}{K_o F^{1.33} (1-Swc)^{1.33}}^{.75188}
\]

where

- \(K_z\) = Kozeny constant
- \(K_o\) = Shape factor
- \(F\) = True formation resistivity factor
- Swc = Irreducible connate water saturation
in order to obtain better results. $k_z$ was varied within the range of 3.0 to 6.0 and $k_0$ was varied within the range of 2.0 to 3.0. As the values of irreducible water saturations, $Swc$, were not available, they were assumed to be 20 per cent in all cases. The better results obtained from varying $k_z$ and $k_0$ are presented in the respective tables.

Comparing the measured data and the computed data, it can be concluded that prediction by Equation (88), with $k_z = 6.$ and $k_0 = 2.1$, gives better quantitative agreement with measured porosities. However, Equation (78) also proved accurate.

Equations (54), (77) and (87) for predictions of porosity were used to compute porosity of gas cap rocks. Original data were obtained from Thomas. Irreducible water saturation values and J-function values at unity water saturations were obtained from the capillary pressure curves presented in that work. These data and the predicted porosities are presented in Table VII.

It should be noted that the predicted porosities are considerably higher than the measured porosities. This is indicative that the concept of simple porous models used to characterize the tortuosity of porous media, fails for very tight rocks.
TABLE VII

POROSITY CALCULATED BY EQUATIONS (54), (77) AND (87) USING DATA FROM THOMAS\textsuperscript{22}

(Cap Rocks)

<table>
<thead>
<tr>
<th>Core Description</th>
<th>$[J(Sw)]_{Sw+1}$</th>
<th>$F$</th>
<th>Swc (frac)</th>
<th>Permeability to water (md)</th>
<th>Measured Porosity (pct)</th>
<th>Porosity Calculated from Eqn. (54) (pct)</th>
<th>Eqn. (77) (pct)</th>
<th>Eqn. (87) (pct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paradox Limestone</td>
<td>.0519</td>
<td>77</td>
<td>.2</td>
<td>.0742</td>
<td>10.</td>
<td>20.85</td>
<td>267.856</td>
<td>92.54</td>
</tr>
<tr>
<td>Dean Sandstone</td>
<td>.03869</td>
<td>67.6</td>
<td>.22</td>
<td>.001</td>
<td>9.48</td>
<td>32.67</td>
<td>563.004</td>
<td>75.45</td>
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<td>Paradox Limestone</td>
<td>.01811</td>
<td>317</td>
<td>.015</td>
<td>.00026</td>
<td>3.26</td>
<td>11.78</td>
<td>433.82</td>
<td>137.09</td>
</tr>
<tr>
<td>Eau Claire Sandstone</td>
<td>.04930</td>
<td>59.3</td>
<td>.015</td>
<td>.178</td>
<td>15.9</td>
<td>23.15</td>
<td>313.06</td>
<td>72.48</td>
</tr>
<tr>
<td>Sprayberry Sandstone</td>
<td>.05518</td>
<td>60.5</td>
<td>.23</td>
<td>.00152</td>
<td>11.9</td>
<td>25.93</td>
<td>373.31</td>
<td>85.97</td>
</tr>
</tbody>
</table>
For the tight media few empirical relations for tortuosity factor and porosity of the rock were attempted due to apparent weakness of the theory. However, one of the better relationships proved to be

\[ T = F^3 \varphi^3 (1 - Swc)^3, \tag{96} \]

so that

\[ \varphi = \left[ \frac{1}{k_0 F^3 (1 - Swc)^3 [J(Sw)]^2} \right]^{1/333} \tag{97} \]

Equation (97) yields reasonable agreement with most of the measured data. The basic data and the computed porosities are presented in Table VIII.

Equations (64), (83) and (91) represent correlations for relative permeability to the wetting phase (water). Data used to test these three equations were obtained from Leverett\textsuperscript{1,23} and from Fatt and Dykstra\textsuperscript{13}. The values of resistivity index, I, at a particular saturation, Swi, were taken as cited by various authors\textsuperscript{9,24}. These are values which they used for similar types of relative permeability calculations. These data and the computed relative permeabilities to water are shown in Tables IX, X and XI. It may be noted that Equation (64) predicts better results for unconsolidated sands (Table IX) than do Equations (83) and (91). In case
## TABLE VIII
POROSITY CALCULATED BY EMPIRICAL EQUATION (97) USING DATA FROM THOMAS

(Cap Rocks)

<table>
<thead>
<tr>
<th>Core Description</th>
<th>$[J(Sw)]/Sw+1$</th>
<th>$F$</th>
<th>$Swc$ (fac.)</th>
<th>Permeability to water (md)</th>
<th>Measured Porosity (percent)</th>
<th>Porosity Calculated From Empirical Equation (97) $k_o = 2.25$</th>
</tr>
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<tbody>
<tr>
<td>Paradox Limestone</td>
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<td>77</td>
<td>.20</td>
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<td>67.6</td>
<td>.22</td>
<td>.001</td>
<td>9.48</td>
<td>12.67</td>
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<tr>
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<td>317</td>
<td>.015</td>
<td>.00026</td>
<td>3.26</td>
<td>3.55</td>
</tr>
<tr>
<td>Eau Claire Sand Stone</td>
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<td>.015</td>
<td>.178</td>
<td>15.9</td>
<td>9.74</td>
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<tr>
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<td>60.5</td>
<td>.25</td>
<td>.00152</td>
<td>11.9</td>
<td>11.32</td>
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</table>
TABLE IX

WETTING PHASE RELATIVE PERMEABILITY
CALCULATED BY EQUATIONS (64), (83)
AND (91) USING DATA FROM LEVERETT1,23

UNCONSOLIDATED SAND
(Absolute permeability: 3.8 D)

<table>
<thead>
<tr>
<th>Swi</th>
<th>$\frac{1}{I}$</th>
<th>$\frac{P_d}{P_c}$</th>
<th>Swc (frac.)</th>
<th>Swi - Swc (frac.)</th>
<th>$K_{rw}$ (observed) (frac.)</th>
<th>$K_{rw}$ calculated from (frac.)</th>
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<td></td>
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<td>-</td>
<td>.2</td>
<td>0.0</td>
<td>-</td>
<td>0.0</td>
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<tr>
<td>.3</td>
<td>0.08</td>
<td>.745</td>
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<td>0.1</td>
<td>.03</td>
<td>.068</td>
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<td>.16</td>
<td>.80</td>
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<td>0.2</td>
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<td>.845</td>
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<td>.185</td>
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<td>.2974</td>
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<td>.4260</td>
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<td>.68</td>
<td>.94</td>
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<td>.581</td>
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<td>.98</td>
<td>.2</td>
<td>0.8</td>
<td>.76</td>
<td>.7966</td>
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</table>
TABLE X

WETTING PHASE RELATIVE PERMEABILITY CALCULATED BY EQUATIONS (64), (83) AND (91) USING DATA FROM FATT AND DYKSTRA - CONSOLIDATED SAND (ABSOLUTE PERMEABILITY: 125 md)

<table>
<thead>
<tr>
<th>Swi</th>
<th>( \frac{l}{I} )</th>
<th>( \frac{P_d}{P_c} ) (frac)</th>
<th>Swc (frac)</th>
<th>( K_{rw} ) (observed frac)</th>
<th>( K_{rw} ) Calculated From (frac.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Eqn. (64)</td>
<td>Eqn. (83)</td>
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<td>.06</td>
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<td>.4</td>
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<td>.84</td>
<td>.666</td>
<td>.4</td>
<td>.76</td>
<td>.4351</td>
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</table>
TABLE XI

WETTING PHASE RELATIVE PERMEABILITY CALCULATED FROM EQUATIONS (64), (83) AND (91) USING DATA FROM FATT AND DYKSTRA — CONSOLIDATED SAND

(Absolute permeability: 47 md)

<table>
<thead>
<tr>
<th>Swi</th>
<th>( \frac{1}{I} )</th>
<th>( \frac{P_d}{P_c} )</th>
<th>Swc (frac)</th>
<th>( K_{rw} ) (frac) (observed)</th>
<th>( K_{rw} ) (frac) Calculated From</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Eqn. (64)</td>
<td>Eqn. (83)</td>
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<tr>
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<td>0.0</td>
<td>0.0</td>
<td>.32</td>
<td>0</td>
<td>0.0</td>
</tr>
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<td>.40</td>
<td>.16</td>
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<td>.32</td>
<td>.006</td>
<td>.0028</td>
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<td>.27</td>
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<td>.02</td>
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<tr>
<td>.60</td>
<td>.40</td>
<td>.1913</td>
<td>.32</td>
<td>.04</td>
<td>.0207</td>
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<td>.70</td>
<td>.53</td>
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<td>.075</td>
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<td>.68</td>
<td>.3666</td>
<td>.32</td>
<td>.18</td>
<td>.0997</td>
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<tr>
<td>.90</td>
<td>.84</td>
<td>.55</td>
<td>.32</td>
<td>.50</td>
<td>.2641</td>
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</tbody>
</table>
of consolidated sands, all three equations predict approximately similar results.

The data predicted by Equation (64) were plotted in Figures 5, 6, and 7 and were compared with the observed relative permeability curves obtained by Leverett$^{1,23}$ and Fatt and Dykstra$^{13}$. In Figures 6 and 7 the computed relative permeability curves from Equation (64) were also compared with the curves obtained by Fatt et al$^{13}$, from Equation (20).

It is noted that the computed relative permeability curves are not in perfect agreement with observed values. However, the computational techniques are better in general than earlier methods.
Figure 5: Comparison of Measured and Calculated Relative Permeability for Unconsolidated Sand
Figure 6: Comparison of Measured and Calculated Relative-Water Permeability Curves and Capillary Pressure Curves in Water-Oil System.

Sand: Basal Tuscaloosa (Miss.), Permeability: 125 md., Porosity: 15 per cent. Data from Figure 3 of Fatt and Dykstra.
Figure 7: Comparison of Measured and Calculated Relative-Water Permeability Curves and Capillary Pressure Curve in Water-Oil System. Sand: Vaqueros, Northern San Joaquin Valley (Calif.). Permeability: 47 md. Porosity: 12 per cent. Data from Figure 11 of Fatt and Dykstra.
V. CONCLUSIONS

The following conclusions are based on the results obtained in this study:

1. An equation (Equation (93)) has been derived for predicting the absolute permeability of a porous medium. The principal assumption is that the porous medium is composed of a bundle of capillary conduits. The best equation to define the tortuosity of a porous medium was found to be:

\[ T = F^2 \gamma^2 (1-Swc)^2 \]  

(42)

This is the definition of tortuosity used in Equation (93) to predict permeability.

Equation (93) correlates displacement pressure, surface tension, connate water saturation, porosity and true formation resistivity factor with absolute permeability. Experimental data have been presented to show that the equation provides a fairly reliable method of calculating permeability from the capillary pressure curve and electrical properties of a rock, and that the method shows improvement over other techniques previously available.
2. A suitable equation (Equation (88)) has been derived to predict the porosity of a porous medium. It is found that this equation predicts better results when the Kozeny constant, $k_z = 6.0$ and the shape factor, $k_0 = 2.1$. A comparison of the observed porosities and those calculated by Equation (88) indicate that equation is reliable.

3. One empirical correlation (Equation (97)) for characterizing the tortuosity of very tight rocks, such as cap rocks, has been found. Calculated porosities using this definition of tortuosity factor was found to be in good agreement with observed porosities.

4. An equation (Equation (64)) was derived for predicting relative permeabilities to water in porous medium. This formula was derived on the assumption that there are no conductive materials except the capillary network of water filaments distributed in a porous medium and that the rock is completely water-wet. Variations between the calculated and observed values of relative permeabilities are believed to be due to limitations of these assumptions.

The results predicted by this correlation were found to be sensitive to the irreducible water saturation, Swc. It was found that the predicted results were more reliable for lower values of Swc.
The equation can only be used to predict the relative permeability to water (wetting phase) and it is probable that drainage capillary pressure curves yield better results than imbibition curves.

In spite of weakness in the technique as described above, comparison of calculated and observed relative permeabilities indicate that the technique is a reliable tool for predicting wetting-phase (water) relative permeabilities.
VI. APPENDICES
### APPENDIX A

#### Nomenclature

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Dimension</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>(F/L²)</td>
<td>Atm</td>
</tr>
<tr>
<td>P&lt;sub&gt;c&lt;/sub&gt;</td>
<td>(F/L²)</td>
<td>dynes/cm²</td>
</tr>
<tr>
<td>P&lt;sub&gt;d&lt;/sub&gt;</td>
<td>(F/L²)</td>
<td>dynes/cm²</td>
</tr>
<tr>
<td>q</td>
<td>(L&lt;sup&gt;3&lt;/sup&gt;/T)</td>
<td>cc/sec.</td>
</tr>
<tr>
<td>Q</td>
<td>(L&lt;sup&gt;3&lt;/sup&gt;)</td>
<td>cc</td>
</tr>
<tr>
<td>r</td>
<td>(L)</td>
<td>cm</td>
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</tr>
<tr>
<td>R</td>
<td></td>
<td>Ohm</td>
</tr>
<tr>
<td>R&lt;sub&gt;o&lt;/sub&gt;</td>
<td></td>
<td>Ohm-cm</td>
</tr>
<tr>
<td>R&lt;sub&gt;1&lt;/sub&gt; &amp; R&lt;sub&gt;2&lt;/sub&gt;</td>
<td>(L)</td>
<td>cm</td>
</tr>
<tr>
<td>Swi</td>
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<td>fraction</td>
</tr>
<tr>
<td>Swc</td>
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<td>fraction</td>
</tr>
<tr>
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<td>L&lt;sub&gt;T&lt;/sub&gt;</td>
<td>cm/sec</td>
</tr>
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<td>cm²</td>
</tr>
<tr>
<td>m</td>
<td>L</td>
<td>cm</td>
</tr>
<tr>
<td>S&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>L&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>cm&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
</tbody>
</table>
### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Dimension</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_p$</td>
<td>Surface area per unit pore volume of pore space</td>
<td>$\frac{1}{L}$</td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>$k_0$</td>
<td>Shape factor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_z$</td>
<td>Kozeny constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J(Sw)$</td>
<td>Leverett's J-function</td>
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</tr>
</tbody>
</table>

### Greek Letters

<table>
<thead>
<tr>
<th>Letter</th>
<th>Description</th>
<th>Dimension</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>Contact angle between wetting fluid and solid surface</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>Fluid viscosity</td>
<td>(M/LT)</td>
<td>Cp</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Porosity</td>
<td></td>
<td>fraction</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Surface tension between two immiscible fluids</td>
<td>(F/L)</td>
<td>dynes/cm</td>
</tr>
<tr>
<td>$T$</td>
<td>Tortuosity of a porous medium</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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APPENDIX B
Conversion Factors

1. **Leverett J-function**

   Leverett J-function is given as
   \[ J(S_w) = \frac{p_c}{\nu} \sqrt{\frac{K}{\phi}} \]

   where
   - \( p_c = \text{dynes/cm}^2 \)
   - \( \nu = \text{dynes/cm} \)
   - \( K = \text{cm}^2 \)
   - \( \phi = \text{fraction} \)

   To express this equation in practical units, that is:
   - \( p_c = \text{psi} \)
   - \( \nu = \text{dynes/cm} \)
   - \( K = \text{md} \)
   - \( \phi = \text{per cent} \)

   first we have to change the unit of permeability from cm\(^2\) to md.

   Unit of permeability is cm\(^2\), when units of the following universal equation
   \[ K = \left( \frac{Q}{A} \right) \mu \left( \frac{L}{\Delta P} \right) \]
are

\[ Q = \text{cc/sec} \]
\[ A = \text{cm}^2 \]
\[ \mu = \text{poise} \]
\[ L = \text{cm} \]
\[ \Delta P = \text{dynes/cm}^2 \]

Hence, converting to original C.G.S. units, we get

\[ [K(\text{md}) \times 10^{-3}] = \frac{Q}{A} \frac{(\text{cm/sec}) \mu (p \times 10^{-2}) \frac{L}{\Delta P} \frac{\text{cm}}{\text{dynes/cm}^2 \times 1}}{1.013 \times 10^6} \]

or,

\[ K_{md} = K(\text{cm}^2) [(10^2) \times (10^3) \times (1.013) \times 10^6)] \]

or,

\[ K(\text{cm}^2) = \frac{K_{md}}{1.013 \times 10^{11}} \]

Hence, \( J(\text{Sw}) \) in practical units can be expressed as

\[
J(\text{Sw}) = \frac{P_c (\text{Psi}) \times 1.013 \times 10^6}{(14.7)^\frac{1}{\nu}} \frac{\sqrt{\frac{K_{(\text{md})}}{1.013 \times 10^{11}}}}{\sqrt{\frac{g (\text{P.C}) \times 10^{-2}}{\nu}}} \]

\[ = 2.17 \frac{P_c}{\nu} \sqrt{\frac{K}{g}} \]
2. **Absolute Permeability Equations**

Let us take the case of Equation (51) for absolute permeability calculations:

\[
K = \frac{v^2}{k_0 \times F^2 \times \theta \times P_d^2 (1 - Swc)^2}
\]

where

- \( v = \text{dynes/cm} \)
- \( \theta = \text{fraction} \)
- \( P_d = \text{dynes/cm}^2 \)
- \( Swc = \text{fraction} \)
- \( k_0 = \text{Shape factor} \)
- \( K = \text{cm}^2 \)

Converting this equation in practical units, that is:

\[
K = \text{darcy}
\]

\( v = \text{dynes/cm} \)

\( \theta = \text{fraction} \)

\( P_d = \text{psi} \)

\( Swc = \text{fraction} \)

we will get

\[
\frac{K(d)}{1.013 \times 10^8} = \frac{v^2}{k_0 \times F^2 \times \theta \times P_d^2 \left(\frac{\text{psi}}{14.7 \times 1.013 \times 10^6}\right)^2 (1 - Swc)^2}
\]

or,

\[
K = 0.021331688 \frac{v^2}{k_0 \times F^2 \times \theta \times P_d^2 (1 - Swc)^2}
\]
VII. BIBLIOGRAPHY


VIII. VITA

Prasanta Kumar Guharoy was born on March 10, 1943 in Patna, India. He is a graduate of Indian School of Mines in Dhanbad, India, and received a Bachelor of Science Degree, with honors, in Petroleum Engineering.

He is presently a member of Pi Epsilon Tau, National Honor Society of Petroleum Engineers, and Junior Member of Society of Petroleum Engineers of AIME.

He has previously been employed by Oil and Natural Gas Commission, India, January 1967 to January 1969; Oil and Natural Gas Commission, India, Summer, 1964; Assam Oil Company, Digboi, India, Summer 1965, and Oil India Limited, Nahorkatya, India, Summer 1965.

He has been enrolled in the graduate school of University of Missouri-Rolla since January 1969.