Digital simulation of a Costas loop demodulator in Gaussian noise and CW interference

Ajay Mugatlal Mehta

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DIGITAL SIMULATION OF A COSTAS LOOP DEMODULATOR IN GAUSSIAN NOISE AND CW INTERFERENCE

By

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THESIS

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ABSTRACT

A digital computer model of a Costas loop has been employed to study the effects of Gaussian noise and continuous-wave interference on the detection of biphase modulated signals. The dependence of bit-error probability, mean-square error between input and output, phase-error variance and threshold on signal-to-Gaussian-noise power ratio, interference-to-Gaussian-noise power ratio and frequency offset of the interference have been investigated and compared with previously obtained experimental results. The model has been verified for its accuracy in the case of no noise and when Gaussian noise is present at its input.
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I. INTRODUCTION

A numerical technique for conducting statistical sampling experiments on a model of a system to obtain a probabilistic approximation to the behavior of the system is called Monte Carlo simulation. The model of the system must be describable in a logical and/or mathematical manner. In general, the simulation is carried on a digital computer, not because of any basic relationship, but because of the very large amount of calculations required.

This thesis is concerned with the development of a digital computer model of a Costas loop demodulator and simulating the effects of Gaussian noise and continuous-wave (cw) interference on the demodulation of phase shift keyed (PSK) signals. Interference is commonly encountered in many applications of Costas loops, e.g., in telemetry systems where the interference is due to extraneous sources. The simulation technique consists of integrating the loop equations by using the Runge-Kutta method of order IV [1].

Costas loops are similar to phase-lock loops (PLLs) in that both can be used to demodulate signals which are digitally phase modulated. The difference, however, is that a PLL requires carrier component to lock onto, but a
Costas loop does not. A more complete description of the operation of a Costas loop will be given later. Because of the intractable mathematical nature of the problem, computer modeling appears to be one of few ways available for obtaining useful results.

Digital computer simulation of PLL's in Gaussian noise has been carried out before [2,3]. Simulation of PLL operation in multipath backgrounds has also been done previously [4], and theoretical analyses of Costas loops operating in Gaussian noise are also obtainable in the literature [5]. However, few results are available on the effects of interference on such devices.

The performance of the Costas loop considered in this thesis is characterized in terms of (1) bit-error probability, (2) normalized mean-square error between input and output, (3) threshold, and (4) phase-error variance. Results are obtained for different values of (i) signal-to-Gaussian-noise power ratio, (ii) interference-to-Gaussian-noise power ratio, and (iii) frequency offset of interference. Previously obtained experimental results [6] will be used to compare with and verify the accuracy of the simulation.
II. DERIVATION OF THE COMPUTER MODEL

A. Equations for Costas Loop Operation in Noise and Interference

In this chapter the equations describing the operation of the Costas loop are derived. These equations will then be used to develop the computer model. Figure 1 shows a block diagram representation of a Costas loop demodulator. The equations which describe the operation of the loop with signal, Gaussian noise, and cw interference at its input will now be obtained. The digital computer model consists of a numerical solution of these equations, as will be described later.

Let \( y(t) \) be the input to the Costas loop. It consists of the suppressed carrier signal, \( s(t) \), plus Gaussian noise, \( n_g(t) \), plus cw interference, \( n_i(t) \). Thus, \( y(t) \) can be written as

\[
y(t) = s(t) + n_g(t) + n_i(t) \quad (2-1)
\]

The signal \( s(t) \) is represented as

\[
s(t) = \sqrt{2} A m(t) \sin(\omega_c t + \theta), \quad (2-2)
\]

where
Figure 1. Block Diagram of a Costas Loop Demodulator

NOTE: LPF = LOWPASS FILTER
A = constant amplitude factor,
\[ m(t) = \text{modulation or signal envelope}, \]
\[ \omega_0 = \text{frequency of the carrier in radians/second}, \]
and
\[ \theta = \text{random phase angle of the carrier}, \]

Assuming that the noise \( n_g(t) \) is narrow-band, zero-mean, and Gaussian, it can be represented as [7]

\[
n_g(t) = \sqrt{2} n_1(t) \cos(\omega_0 t + \theta) + \sqrt{2} n_2(t) \sin(\omega_0 t + \theta) \quad (2-3)
\]

where \( n_1(t) \) and \( n_2(t) \) are statistically independent sample functions of a jointly stationary Gaussian process. The power spectral densities of \( n_1(t) \) and \( n_2(t) \) are \( N_0 \) watts/Hz, single-sided, over the passband 0 to \( W/2 \) Hz and zero elsewhere, if that of \( n_g(t) \) is \( N_0 \) watts/Hz, single-sided, over the passband \( f_0 - W/2 \) to \( f_0 + W/2 \) Hz (\( f_0 = \omega_0 / 2\pi \)) and zero elsewhere. Thus, 

\[
n^2_g(t) = 2n^2_1(t) = 2n^2_2(t) = N_0 W \quad (2-4)
\]

The cw interference can be represented as

\[
n_i(t) = \sqrt{2} b_o \sin[(\omega_0 + \Delta \omega) t + \theta + \delta] \\
= \sqrt{2} b_o \sin(\Delta \omega t + \delta) \cos(\omega_0 t + \theta) \\
+ \sqrt{2} b_o \cos(\Delta \omega t + \delta) \sin(\omega_0 t + \theta), \quad (2-5)
\]
where the randomly varying phase difference between the signal and the interference is represented as $\delta$.

Equation (2-5) can be written in the same form as Eq. (2-3), so that

\[ n_i(t) = \sqrt{2} n_{ic} \cos(\omega_o t + \theta) + \sqrt{2} n_{is} \sin(\omega_o t + \theta) \]  

(2-6)

where

\[ n_{ic} = b_0 \sin(\Delta \omega t + \delta) \]  

(2-7a)

and

\[ n_{is} = b_0 \cos(\Delta \omega t + \delta) \]  

(2-7b)

The lowpass filters LPF 1 and LPF 2 are assumed to have the following effects on the signal, noise, and interference components at points (1) and (2) in Figure 1:

1. The low frequency components of signal are passed without distortion;
2. The low frequency components of the noise are passed without distortion;
3. All double frequency components are completely rejected;
4. The only effect on the cw interference is attenuation and phase shift of its low frequency components in accordance with the filter transfer function, which is represented as
\[ H(\omega) = B(\omega) \exp[j\theta(\omega)], \tag{2-8} \]

where

\[ B(\omega) = \text{amplitude response function of the lowpass filters} \]

and

\[ \theta(\omega) = \text{phase shift function of the lowpass filters}. \]

Let the output of the VCO (point (7) in Figure 1) be

\[ V_7(t) = \sqrt{2} \cos(\omega_0 t + \hat{\theta}) \tag{2-9} \]

where \( \omega_0/2\pi \) is the free-running frequency of the VCO in Hz, and \( \hat{\theta} \) is the Costas loop's estimate of the unknown phase \( \theta \). The output \( V_3(t) \) of LPF 1 is the same as the low frequency components of the output \( V_1(t) \) of the upper phase detector except that the low frequency components of interference will be attenuated and phase shifted according to the transfer function \( H(\omega) \) as defined by Eq. (2-8). Thus,

\[
V_3(t) = A m(t) \cos \phi - n_1(t) \sin \phi + n_2(t) \cos \phi \\
- b_0 B(\Delta \omega) \sin[\Delta \omega t + \delta + \theta(\Delta \omega)] \sin \phi \\
+ b_0 B(\Delta \omega) \cos[\Delta \omega t + \delta + \theta(\Delta \omega)] \cos \phi \tag{2-10}
\]

where

\[ \phi = \theta - \hat{\theta} \tag{2-11} \]

is the phase-error.
Similarly, the output of LPF 2 can be written as

\[ V_4(t) = A_m(t)\sin \theta + n_1(t)\cos \theta + n_2(t)\sin \theta \]
\[ + b_0 B(\Delta \omega) \sin[\Delta \omega t + \delta + \theta(\Delta \omega)]\cos \theta \]
\[ + b_0 B(\Delta \omega) \cos[\Delta \omega t + \delta + \theta(\Delta \omega)]\sin \theta \]  

(2-12)

The differential equation describing the loop is

\[ \frac{d\hat{\theta}(t)}{dt} = K_v F(p) V_5(t) \]  

(2-13)

where

\[ p = \frac{d}{dt} \]  is the differential operator,

\[ K_v \]  is the multiplying constant for the VCO,

\[ F(p) \]  is the transfer function of the loop filter LPF 3,

and

\[ V_5(t) \]  is the voltage at point (5) in Figure 1.

From Figure 1,

\[ V_5(t) = K_m V_3(t) V_4(t), \]  

(2-14)

where \( K_m \)  is the multiplier constant.

In practice, the characteristics of a second order loop are conveniently specified in terms of its damping factor \( \zeta \)  and the natural frequency of oscillation \( \omega_n \), or the equivalent noise bandwidth, \( W_L \), of the linearized loop.
These parameters can be defined in terms of various loop and signal constants. From Eqs. (2-10) and (2-14), the differential equation for the linearized loop with only signal present at the input is

\[ \frac{d\hat{\theta}}{dt} = K_v K_m A^2 F(p) (\theta - \hat{\theta}) \]  

where the approximation \( \sin 2\theta \approx 2\theta \) has been used. The closed loop transfer function \( H_o(s) \) can be defined as the ratio of the Laplace transform of \( \theta(t) \) to that of \( \hat{\theta}(t) \), and from Eq. (2-15), is

\[ H_o(s) = \frac{Y(s)}{1+Y(s)}, \]  

where

\[ Y(s) = K_v K_m A^2 F(s)/s \]  

is referred to as the open loop transfer function.

Equation (2-17) is written under the assumption that \( m(t)=\pm 1 \), i.e., \( m(t) \) is a binary digital sequence.

Assuming \( F(s) \) to be the filter transfer function of a perfect second-order active loop, i.e.,

\[ F(s) = 1 + \frac{a}{s}, \]  

it can be easily shown that

\[ H_o(s) = \frac{2\zeta \omega_n s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]  

(2-19)
where

\[ a = \frac{\omega_n}{2\zeta} \]  \hspace{1cm} (2-20)

and

\[ 2\zeta \omega_n = A^2 K_v K_m \]  \hspace{1cm} (2-21)

A commonly used value for the damping factor in PLL work is \( \zeta = 0.707 \) [8]. This value for \( \zeta \) will be used here. The natural frequency \( \omega_n \) is conveniently specified in terms of \( \zeta \) and the equivalent noise bandwidth \( W_L \) of the loop, by

\[ \omega_n = \frac{4\zeta W_L}{1 + 4\zeta^2} \]  \hspace{1cm} (2-22)

The parameter \( W_L \) is defined by

\[ W_L = 2B_L = \int_{-\infty}^{\infty} \left| \frac{Y(f)}{1+Y(f)} \right|^2 df \]  \hspace{1cm} (2-23)

where \( Y(f) \) is given by Eq. (2-17) with \( s = j2\pi f \).

Even for the nonlinear case, it is usually most convenient to describe the second-order loop in terms of \( \zeta \) and \( \omega_n \) (or \( W_L \)) which are parameters that have been defined for the linearized loop. When Eq. (2-13) is expressed in terms of \( \zeta \) and \( \omega_n \), for a loop filter with transfer function given by Eq. (2-18), it becomes

\[ \frac{d^2 \hat{\theta}}{dt^2} = 2\zeta \omega_n \frac{d}{dt} \left( \frac{V_5}{K_m} \right) + \omega_n^2 \left( \frac{V_5}{K_m} \right) \]  \hspace{1cm} (2-24)
This last equation along with Equations (2-10), (2-12), (2-14) and (2-22) completely describes the loop action in terms of the desired parameters $W_L$ and $\zeta$ in response to the signal, noise and interference.

For the purpose of numerical integration, Eq. (2-24) can be represented as two simultaneous differential equations of first order [1]. If $\theta = y$, and $z$ is introduced as a dummy variable, Eq. (2-24) is equivalent to

$$\frac{dy}{dt} = z + 2\zeta \omega_n \frac{V_5}{K_m^2} \quad (2-25)$$

and

$$\frac{dz}{dt} = \omega_n \frac{V_5}{K_m^2} \quad (2-26)$$

These are the equations which are numerically integrated in the digital computer model of the loop. This model will be described in the next section.

B. Implementation of the Loop Equations as a Computer Model

The equations derived in the previous section that describe the Costas loop operation are first represented as a flow diagram to facilitate the development of the computer program for the model. Two such flow diagrams are shown in Appendices A and C. The first one is for the noiseless case and is a much simplified version of
the second one, which includes Gaussian noise and cw interference. The flow diagrams are presented in the logical form and show the major steps implemented in the computer program (Appendices B and D) written for the model.

From the flow diagram for the model with interference it will be noted that a limiter is included in the in-phase channel of the loop. This was done mainly to simulate the actual loop used to obtain the experimental results which included a limiter. It stabilizes the loop response at high noise and interference levels.

The results presented in the next section were obtained only for a few specific cases, because (i) only a limited number of experimental results [6] were available for comparison and verification of the model and (ii) availability of computer time was limited. However, the flow diagram presented in Appendix C is quite general and is flexible enough to allow a selection of loop characteristics such as amplitude response function $B(\omega)$ of the in-phase and quadrature-phase channel filters (LPF1 and LPF2) to the low frequency components of interference, the loop bandwidth $W_L$, and the damping factor $\zeta$. Two types of modulating signals can be selected, namely, a random binary sequence or periodic binary sequences such as a pseudo-random maximal-length
sequence [9]. In practice, purely random sequences with known properties are difficult to generate and use. Therefore, periodic pseudo-random sequences of finite length are often employed to simulate digital data.

A normal random number generator (available in the Scientific Subroutine Package (SSP) memory of the computer) was employed to generate noise samples for the two Gaussian processes \( n_1(t) \) and \( n_2(t) \) (refer to Figure 7). The standard deviation for these Gaussian samples was calculated as follows.

Assume a sample is taken every

\[
T_I = \frac{T_b}{SPB} \text{ seconds,} \tag{2-27}
\]

where

\( T_b = \text{period of a data bit,} \)

and

\( SPB = \text{number of samples per bit} \)

Also, let the variance of each noise sample be \( \sigma^2 \). We want \( \sigma^2 \) in terms of the signal-to-Gaussian-noise ratio (SNR) and \( SPB \). Let the noise bandwidth be \( B \text{ Hz} \) and the noise power spectral density be \( N_0 \text{ watts/Hz} \), as shown in Figure 2.
Thus,

$$\sigma^2 = N_0 B$$

(2-28)

and, for the samples to be independent, we must sample at the Nyquist rate \([10]\), or

$$T_I = \frac{1}{2B}$$

(2-29)

Substituting for \(B\) in Equation (2-28) from Equation (2-29) we obtain

$$\sigma^2 = \frac{N_0}{2T_I}$$

(2-30)

The signal-to-Gaussian noise ratio is defined as

$$\text{SNR} = \frac{A^2 T_b}{N_0}$$

(2-31)

where the noise power is measured in a bit-rate bandwidth \(f_b = 1/T_b\). From Equations (2-27), (2-30) and (2-31) we obtain

$$\sigma^2 = \frac{\text{SPB}}{2 \times \text{SNR}}$$

(2-32a)
where \( A = 1 \) for convenience (i.e., all power levels are normalized to the power in the signal). Thus, the standard deviation \( \sigma \) of each noise sample is

\[
\sigma = \left( \frac{\text{SPB}}{2 \times \text{SNR}} \right)^{1/2}.
\] (2-32b)

The filters LPF \( 1 \) and LPF \( 2 \) were assumed to be a cascade of two one-pole lowpass stages, having an amplitude response function

\[
B(f) = \frac{1}{1 + (f/\omega)^2}
\] (2-33)

where \( \omega \) is the cut-off frequency of the filter.

From Equation (2-5), the interference power is \( b_0^2 \). However, the signal power has been assumed to be unity (for convenience), so the amplitude \( b_0 \) of the cw interference can be calculated for the model as

\[
b_0^2 = \text{Interference Power} = \frac{\text{Interference Power}}{\text{Signal Power}} = \frac{\text{INR}}{\text{SNR}}
\] (2-34a)

or

\[
b_0 = \left( \frac{\text{INR}}{\text{SNR}} \right)^{1/2}.
\] (2-34b)
The relations derived in this section were employed to calculate different parameters for the computer model from the given values of the standard parameters $\zeta$, $W_L$, INR, SNR, $\Delta f$, $f_b$, etc. for the Costas loop.
III. VERIFICATION OF THE MODEL AND RESULTS

Once the computer model was obtained, its validity and accuracy were tested by comparing simulation results with known theoretical results for loop operation under various conditions but without cw interference. The effects of cw interference on Costas loop operation were then studied for a few specific cases by using the model.

A. Verification of Model

The major steps taken in verifying the accuracy of the digital computer model of a Costas loop operating in interference were the following:

1. Selection of a proper method for solving the differential equation of the Costas loop

Before adopting the Runge-Kutta method of order IV for the differential equation of the loop, the RKGS and HPCG subroutines available in the SSP memory of the computer were first used. Neither of these methods were suitable, as in trying to satisfy a specified error, they have the inherent tendency to reduce the step size. This in turn, made it impossible to calculate the standard deviation of the Gaussian-noise samples used to simulate
n_1(t) and n_2(t) from the known signal-to-Gaussian-noise ratio, because of the resulting unknown number of samples used per bit. The Runge-Kutta method of order IV for solving two simultaneous differential equations of first order [1] with a fixed size integration (i.e. sampling) interval, h, was then used and found acceptable.

2. Testing the model for transient response under noiseless conditions

Testing of the model for transient response was accomplished for two inputs, namely, signal with initial frequency offset from the VCO frequency, and an input signal with increasing frequency offset.

Case 1: Constant Frequency Offset

For the constant frequency offset case, the input signal is

\[ s(t) = \sqrt{2} A m(t) \cos(\omega_0 t + \theta) \]  

(3-1)

where

\[ \theta = \Omega t u(t) \]  

(3-2)

That is, the initial frequency offset of the signal is \( \Omega \) radians/second from the initial frequency of the VCO.

With small phase error and for no noise and interference, Eqs.(2-10), (2-12) and (2-14) simplify to

\[ \frac{V_5(t)}{K_m} = \frac{1}{2} \sin 2\theta = \varnothing , \]  

(3-3)
when $A = 1$ (assumed for convenience).

Substituting into Eq. (2-24) and solving for the phase error $\varphi$ we get, for $\zeta = 0.707$,

$$\varphi(t) = \sqrt{2} \frac{\omega}{\omega_n} \exp\left(-\frac{\omega_n t}{\sqrt{2}}\right) \sin\left(\frac{\omega_n t}{\sqrt{2}}\right) u(t). \quad (3-4)$$

Linear operation of the loop imposes the condition that $|\varphi_{\text{max}}| \ll 1$ radian. It can be shown that in terms of $\Omega$ this requires

$$\Omega \ll 3.2W_L \quad (3-5)$$

for the damping factor used.

A plot of phase error, $\varphi$, versus time, as obtained from Eq. (3-4), is shown in Figure 3. The phase error obtained from the computer model is also shown. For $\Omega=10$ and $W_L=100$, the condition of linearity is satisfied and the two plots coincide exactly. This indicates that the basic computer model is accurate for noiseless inputs. Also shown in Figure 3 is the response of the model for $\Omega=200$, which shows the deviation from linearity when a value of $\Omega$ is used that does not satisfy the inequality in Eq. (3-5).

Case 2. Linearly increasing frequency

The phase of the carrier for a linearly increasing frequency is
Figure 3. Transient Response of the Model - Noiseless Case 1.
\[ \theta = \frac{1}{2}Dt^2 u(t). \] (3-6)

The solution of the linearized differential equation of the loop for this input phase is

\[ \phi(t) = \frac{D}{\omega_n^2} [1 - \sqrt{2} \exp(-\frac{\omega_n t}{\sqrt{2}})\cos(\frac{\omega_n t}{\sqrt{2}} - \frac{\pi}{4})] \] (3-7)

and the condition of linearity, \(|\phi_{\max}| << 1\), leads to

\[ D << \frac{8}{9} W_L^2 \] (3-8)

for \( \zeta = 0.707 \).

A comparison of \( \phi(t) \) calculated from the linearized differential equation with \( \phi(t) \) obtained from the computer model is given in Fig. 4 for two values of \( D \). For \( D=10 \), the condition for linearity, Eq. (3-8), is satisfied, while for \( D=10000 \), the model is operating in the non-linear region. Again, as in the previous case, the plot obtained from the linearized solution coincides exactly with the response of the model as long as condition for linearity is satisfied. This further verifies the accuracy of the model under noiseless conditions.

3. Selection of proper seeds for the random Gaussian number generators

Seed for a random number generator is a number to be specified and used as a starting point in generating the random number sequence. Proper selection of seeds
Case 2. $\theta = \frac{1}{2}Dt^2$

$W_L = 200$

Bits = 200

Figure 4. Transient Response of the Model-Noiseless Case 2.
for the random number generators had a significant influence on the number of replications of the simulation model required to reach steady state conditions. Good seeds were selected by trial and error. The sample mean and standard deviation of the output of the Gaussian random number generator used to generate the noise samples were plotted against the number of iterations. The two seeds selected for generating $n_1(t)$ and $n_2(t)$ were the ones that generated processes for which the mean and the standard deviation settled down to their nominal values in minimum number of iterations. Example of the effect of seed selection is given in Fig. 5.

4. Selecting the number of initial bits which could be considered as the transient response of the model

The initial bits that constitute the transient response of the model are omitted from the calculation of bit-error probability, mean-square error, cycle-skips, etc. to allow the transient response of the model to die out. The number of bits omitted was selected in conjunction with step 3 above, and for the parameters and bit-rates chosen was about 200 bits, as can be seen from Fig. 5.
Figure 5: Example of the Effect of Seed Selection

- example of good seed (123456789)
- example of bad seed (987654321)
5. Testing the model when Gaussian noise is present

The model was run for signal-to-noise power ratios (noise referred to a bit-rate bandwidth) of 0, 1, 2, 3 and 4 decibels (SNRDB=0, 1, 2, 3 and 4). The bit-error probability \( P_E \) obtained from the model was compared with the theoretical values obtained by Didday and Lindsey [5] for a first-order Costas loop (i.e., where LPF3, shown in Figure 1 is absent). This comparison is shown in Table 1.

The phase-error variance is one more yardstick to test the model. The variance \( \sigma^2 \) of the phase-error was obtained from the model and compared with the theoretical results calculated from a linearized analysis of the loop. It has been shown [5] that for high signal-to-noise power ratios the following approximate relation holds:

\[
\sigma^2 = \frac{W_L}{W} \left( \frac{1}{\text{SNRBR}} + \frac{1}{2(\text{SNRBR})^2} \right)
\]  

(3-9)

where \( \text{SNRBR} \) is the signal-to-Gaussian-noise power ratio in a bit-rate bandwidth. A comparison between \( \sigma^2 \) obtained from the computer model and the corresponding values for a linearized model given by Eq. (3-9) is presented in Table 2. This was done for SNRDB=7, 8, and 9 dB.
Table 1: Bit-Error Probability vs SNRDB*  

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<tr>
<th>SNRDB</th>
<th>PE (_{\text{Theoretical}})</th>
<th>PE (_{\text{Model + (with limiter)}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.078</td>
<td>0.099</td>
</tr>
<tr>
<td>1</td>
<td>0.055</td>
<td>0.076</td>
</tr>
<tr>
<td>2</td>
<td>0.035</td>
<td>0.041</td>
</tr>
<tr>
<td>3</td>
<td>0.022</td>
<td>0.024</td>
</tr>
<tr>
<td>4</td>
<td>0.010</td>
<td>0.012</td>
</tr>
</tbody>
</table>

* Signal-to-Gaussian-noise power ratio measured in a bit-rate bandwidth and expressed in decibels.

+ A step-limiter was introduced in the in-phase channel of the Costas loop.

Table 2: Phase-Error Variance vs SNRDB  

<table>
<thead>
<tr>
<th>SNRDB</th>
<th>(\sigma^2_\phi) (_{\text{Theoretical}})</th>
<th>(\sigma^2_\phi) (_{\text{Model with limiter}})</th>
<th>(\sigma^2_\phi) (_{\text{Model without limiter}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.00219</td>
<td>0.00295</td>
<td>0.00236</td>
</tr>
<tr>
<td>8</td>
<td>0.00171</td>
<td>0.00161</td>
<td>0.00167</td>
</tr>
<tr>
<td>9</td>
<td>0.00134</td>
<td>0.00118</td>
<td>0.00120</td>
</tr>
</tbody>
</table>
The mean of the phase-error is expected to be zero because of a zero mean assumed for the Gaussian noise. The values obtained from the computer model were very small, but not quite equal to zero. The reason for this is discussed in the next chapter.

Threshold can be defined as the signal-to-noise ratio at which a loop looses lock which is more or less arbitrarily defined.

An arbitrary method of determining the occurrence of loss of lock in the computer model is used. Each time the absolute value of the phase-error $\phi$ exceeds the threshold value, $\pi/4$ radian, a cycle-skip results and is noted. A count is also kept of the number of cycle-skips in every ten consecutive input bits, and a loss of lock is said to occur when this count exceeds the value five. The value of the SNR for which this occurs is defined as the threshold value for the model. Also note that after a cycle-skip has been registered, the conditions in the loop (model) are initialized to those at time $t=0$ second.

It has been suggested [5] that threshold occurs in a Costas loop at $\sigma^2 \approx 1/8$. With $W/W_L=100$ in Eq. (3-9), the value used in the model, a threshold of -9.2 dB results (measured in a bit-rate bandwidth).
<table>
<thead>
<tr>
<th>SPB*</th>
<th>Model (without limiter)</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-10dB⁺</td>
<td>-9.2dB</td>
<td>-7dB</td>
</tr>
<tr>
<td>20</td>
<td>-9dB</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* SPB - Noise samples taken per bit

⁺ Signal-to-Gaussian-noise power ratio (measured in a bit-rate bandwidth) expressed in decibels.

Table 3: Threshold Results for Costas Loop
Threshold obtained from the model is compared with this value and the previously obtained experimental value, in Table 3.

Results were also obtained for phase-error variance for the case when a limiter was inserted in the in-phase channel of the loop. These results are included in Table 2 and conclusions drawn in the next chapter. The limiting process has an interesting effect on the threshold. This is discussed in the next chapter.

The effect on the threshold of varying the number of samples per input bit was also investigated. The results are included in Table 3 and the comments follow in the next chapter.

B. Results for interference backgrounds

The final step was to obtain results for CW interference once the model was tested and verified (steps 1 through 5).

Bit-error probability and normalized mean-square error between input and output were computed for different signal-to-Gaussian-noise power ratios, interference-to-Gaussian noise power ratios and frequency offsets of the interference. These are compared with previously obtained experimental results in Tables 4 and 5.
<table>
<thead>
<tr>
<th>SNRDB</th>
<th>INRDB</th>
<th>$\Delta f=10\text{KHz}$</th>
<th>$\Delta f=5\text{KHz}$</th>
<th>$\Delta f=0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Experimental MSE</td>
<td>Model MSE</td>
<td>Experimental MSE</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.140</td>
<td>0.166</td>
<td>0.160</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.140</td>
<td>0.167</td>
<td>0.190</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.155</td>
<td>0.170</td>
<td>0.230</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0.120</td>
<td>0.136</td>
<td>0.130</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.120</td>
<td>0.136</td>
<td>0.150</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.125</td>
<td>0.137</td>
<td>0.170</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0.100</td>
<td>0.108</td>
<td>0.105</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.100</td>
<td>0.108</td>
<td>0.130</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.105</td>
<td>0.109</td>
<td>0.150</td>
</tr>
</tbody>
</table>

Notes: INRDB = cw interference-to-Gaussian-noise power ratio (in decibels)
$\Delta f$ = frequency-offset of interference from the carrier frequency $f_c$ Hz
MSE = normalized mean-square error between input and output

Table 4: Summary of Results for MSE in Presence of cw Interference
<table>
<thead>
<tr>
<th>SNRDB</th>
<th>INRDB</th>
<th>$\Delta f=10\text{kHz}$</th>
<th>$\Delta f=5\text{kHz}$</th>
<th>$\Delta f=0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Experimental $P_E$</td>
<td>Model $P_E$</td>
<td>Experimental $P_E$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.002</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>0.002</td>
<td>0.001</td>
<td>0.0025</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.002</td>
<td>0.001</td>
<td>0.003</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0.0005</td>
<td>0.001</td>
<td>0.0006</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0005</td>
<td>0.0</td>
<td>0.0009</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.0005</td>
<td>0.0</td>
<td>0.001</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0.001</td>
<td>0.0</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.001</td>
<td>0.0</td>
<td>0.0003</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.0001</td>
<td>0.0</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

Table 5: Summary of Results for $P_E$ in Presence of cw Interference
IV. DISCUSSION OF RESULTS AND CONCLUSIONS

From the results presented in the previous chapter, the following comments can be made and conclusions drawn:

A. For the ideal noiseless case, the transient response (Figs. 3 and 4) of the model coincided closely with the analytically calculated response of the linearized \((\Sin2\varphi=2\varphi)\) loop, when the conditions of linearity were satisfied. This verified the accuracy of the computer model for noiseless conditions.

B. Selection of seeds for the random number generators (subroutine GAUSS in the computer program shown in Appendix D) had a significant influence on the length of the initial transient response of the model and on the values for \(P_E\) and \(\sigma^2\) calculated from the model. A bad seed had the effect of increasing the \(P_E\) and \(\sigma^2\) beyond the values expected theoretically, while a good seed tended to produce more acceptable values for these quantities.

C. For the Gaussian noise case, the \(P_E\) calculated from the model was always higher than the theoretically calculated values (Table 1). This can be attributed
to (i) the theoretical values used for comparison are for a first-order Costas loop (i.e. when LPF 3 shown in Figure 1 is absent) while the model is for the second-order Costas loop, and (ii) for the number of noise samples used in the model, the noise may not have been perfectly Gaussian in character. Also, the two noise processes \( n_1(t) \) and \( n_2(t) \) were not truly independent statistically, as they should be.

D. For high signal-to-Gaussian-noise ratios (SNRDB=7,8, and 9), the introduction of a limiter in the in-phase channel of the loop (model) did not have any noticeable effect on the values of \( \sigma_\phi^2 \) (Table 2). With or without a limiter, the values of \( \sigma_\phi^2 \) calculated from the model, compared favorably with the theoretical values calculated from Eq. (3-9) which holds for high signal-to-noise ratios.

E. Without a limiter in the in-phase channel, the value obtained for the threshold was close to the theoretically predicted value in the presence of Gaussian noise alone. Increasing the number of noise samples per bit had the interesting effect of raising the threshold. However, the most
interesting result found was that the limiter in the in-phase channel of the loop had the effect of stabilizing the loop near threshold and as a result, the loop did not go out of lock even at \(-11\text{dB (SNRDB)}\) and had only 4 cycle-skips in 800 bits.

F. The results obtained for the cw interference were compared only with previously obtained experimental results. No theoretical results are available in the literature for the performance of Costas loops operating in cw interference. From the majority of the results obtained for the MSE (Table 4) it can be concluded that the effects of interference as obtained with the model were much less severe than shown by the corresponding experimental results. Possible explanations for this are: (i) measurement error in obtaining the experimental results; (ii) inadequate representation of the cw interference in the model. However, no such conclusion could be made from the results for the \(P_E\). For the very low values of \(P_E\) corresponding to high SNRDB's, the model did not employ a sufficient number of bits for good accuracy. Only 1000 bits were used due to limited availability of computer time.
REFERENCES


VITA

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He has been enrolled as a Master's candidate in the Electrical Engineering department of the University of Missouri - Rolla since September, 1968.

Mr. Mehta is a member of IEEE and Eta Kappa Nu.
APPENDIX A

FLOW DIAGRAM - NOISELESS CASE
Define Eqs. (2-25) and (2-26) to be solved for y and z

Input: ζ, W_L, f_b, SPB, Bits

Calculate \( \omega_n, T_b' \)

\( H \)

\( IJ = SPB \times \text{Bits} \)

\( D = 10 \)
\( m(t) = 1 \)

DO 3
I=1, IJ

\( \theta = \frac{1}{2} Dt^2 \)

\( \phi(t) = \theta - y \)

Solve for y and z using Runge-Kutta method of order IV with integration interval H

Calculate \( \phi(t) \) for linearized loop from Eq. (3-7)

Figure 6: Flow Diagram for Computer Model - Noiseless Case 2
APPENDIX B

COMPUTER PROGRAM - NOISELESS CASE
COMPUTER PROGRAM FOR DIGITAL SIMULATION OF COSTAS LOOP OPERATION

CASE OF LINEARLY INCREASING FREQUENCY AT THE INPUT

PI STORES THE VALUE OF PHI AT THE END OF EACH BIT, TPHI STORES THE THEORETICALLY CALCULATED VALUE OF PHI AT THE END OF EACH BIT

DEFINE THE TWO SIMULTANEOUS DIFFERENTIAL EQUATIONS OF THE COSTAS LOOP T - TIME IN SECONDS, Y = 0, Z - DUMMY VARIABLE INTRODUCED

F1(T,Y,Z) = Y + 2. * Z / PI * WN * V5
G1(T,Y,Z) = WN * V5

ZETA - DAMPING FACTOR, WL - EQUIVALENT NOISE BANDWIDTH
WN - NATURAL FREQUENCY, WI - INPUT SIGNAL BANDWIDTH
FB - BIT-RATE BANDWIDTH, TB - PERIOD OF AN INPUT BIT
SPR - GAUSSIAN-NOISE SAMPLES TAKEN PER BIT, BITS - TOTAL NUMBER OF BITS CONSIDERED, H - INTEGRATION AND SAMPLING INTERVAL, SAMPLE - TOTAL NUMBER OF SAMPLES
E - MODULATING DIGITAL SIGNAL

ZETA = 0.707
PI = 3.1416
WL = 200.
WN = 4. * PI / (1. + 4. * ZETA ** 2)
WI = 10000.
FB = WI / 2.
TB = 1. / FB
SPR = 20.
NSPR = SPR
BIT = 200.
NMAX = BIT
H = TH / SPR
SAMPLE = SPR * BIT
I1 = SAMPLE
CALL PENPOS(12HMEHTA AJAY M, 12, 1)
D - RATE OF INCREASE IN THE INPUT FREQUENCY (HZ/SEC)

D=10.

DO LOOP FOR TWO VALUES OF D (D = 10, CONDITION OF LINEARITY SATISFIED ; AND D = 10000, NONLINEAR CASE)

DO 55 K=1,2

T=0.
Y=0.
Z=0.
F=1.

TPHI(1)=0.
P1(1)=0.

MAIN DO LOOP PERFORMING ITERATIONS

DO 3 I=1,10

THETA=D*T*T/2.
PHI=THETA-Y

V3=F*COS(PHI)
V4=F*SIN(PHI)
V5=V3*V4

EMPLOYING RUNGE-KUTTA METHOD OF ORDER IV TO SOLVE THE LOOP EQUATIONS

A1=H*FL(T,Y,Z)
B1=H*GL(T,Y,Z)
A2=H*FL(T+H/2.,Y+A1/2.,Z+B1/2.)
B2=H*GL(T+H/2.,Y+A1/2.,Z+B1/2.)
A3=H*FL(T+H/2.,Y+A2/2.,Z+B2/2.)
B3=H*GL(T+H/2.,Y+A2/2.,Z+B2/2.)
A4=H*FL(T+H,Y+A3,Z+B3)
B4=H*GL(T+H,Y+A3,Z+B3)

T=T+H
C CHECK FOR THE END OF A BIT
IF(MOD(T,NSPB).EQ.13,4,3)

C CONTINUE
J=J/SPR+1.
PI(J)=PHI
TPHI(J)=D*(1.-1.414*EXP(-WN*T*0.707)*COS(WN*T*0.707-PI/4.))/WN/WN

C GENERATING A RANDOM DIGITAL MODULATING SIGNAL
C
IF(RAND(0) LT 0.5)F=-1.
IF(RAND(0) GE 0.5)F=1.

C CONTINUE
PHIMIN=0.
PHIMAX=D*(1.+EXP(-PI))/WN/WN
WRITE(3,100)PHIMIN,PHIMAX
100 FORMAT(1H,'PHIMIN =',F1.4,10X,'PHIMAX =',F7.4)
YMIN=1.6*PHIMIN
YMAX=1.6*PHIMAX
CALL NEWPLT(2.0,2.0,10.0)
CALL ORIGIN(0.0,0.0)
CALL TSCLAE(0.0,BITS*TR,8.0)
CALL YSCALE(YMIN,YMAX,5.0)
CALL TPLT(P1,NMAX,7,10*(K-1)+3)
D=10000.
55 CONTINUE

CALL NEWPLT(2.0,2.0,10.0)
CALL ORIGIN(0.0,0.0)
CALL TSCLAE(0.0,BITS*TR,8.0)
CALL YSCALE(YMIN,YMAX,5.0)
CALL TPLT(TPHI,NMAX,1.+1)
CALL TAXIS(SPR*TR)
CALL YAXIS(PHIMAX/5.)
CALL ENDPLT
CALL LSTPLT
CALL EXIT
STOP
END
APPENDIX C

FLOW DIAGRAM - GAUSSIAN NOISE AND CW INTERFERENCE PRESENT

Nomenclature for notations used in Appendix C (but not defined earlier)

Bits - Total number of bits considered

OMIT - Number of initial bits omitted as transient response of the computer model

IJ - Total number of noise samples considered

NI - Number of initial samples omitted as transient response

NUSKIP - Count of the number of cycle-skips per every 10 consecutive bits

NSKIP - Number of cycle-skips
Define Eqs. (2-25) and (2-26) to be solved for y and z.

Define the low-frequency amplitude response function of the filters LPF1 and LPF2.

Input: \( WL, \) \( f_{h}, SPB, \) \( Bits, \) \( SNRDB, \) \( INRDB, \) \( OMIT \)

Calculate \( \omega, T_{b}, H, \) \( SNR, INR, b, \) \( I/J = SPB*Bits, \) \( NI = SPB*OMIT \)

\( f_{h} = 10000 \)

Do 7 \( I = 1, 3 \)

\( \Delta f = 10000 \)

\( \omega, n, n_{1}, n_{2} = 0 \)

INITIALIZE m(t) = -1

INITIALIZE ALL SUMMATIONS

\( \text{NSKIP} = \text{NSKIP} + 1 \)

Write "the loop is out of lock"

\( \text{STOP} \)

\( \theta = \theta - y \)

Do 3 \( j = 1, 11 \)

\( \text{RESET} y, z, n, n_{1}, n_{2} \)

\( \text{NSKIP} = \text{NSKIP} + 1 \)

\( \text{YES} \)

\( \text{NO} \)

\( \text{YES} \)

Figure 7: Flow Diagram for Computer Model - Gaussian Noise Plus CW Interference Present
Figure 7 (contd.): Flow Diagram for Computer Model - Gaussian Noise Plus cw Interference Present
Select \( m(t) \) for the next bit from the stored maximal length pseudo-random sequence.

Generate random phase \( \delta(0, 2\pi) \) for the next bit.

\[
\begin{align*}
\text{3} & \\
\text{CONTINUE} & \\
\end{align*}
\]

\[
P_E = \frac{\sum \text{Error}}{\text{Bits-Omit}} \\
\text{MSE} = \frac{\sum (m(t) - v_3)^2}{4\sum m^2(t)}
\]

\[
\bar{\phi}_{\text{mean}} = \frac{\sum \phi}{(I-NI)} \\
\sigma^2 = \frac{\sum \phi^2}{(I-NI)} - \bar{\phi}_{\text{mean}}^2
\]

\[\Delta f = \Delta f - 5000\]

\[
\begin{align*}
\text{7} & \\
\text{CONTINUE} & \\
\end{align*}
\]

END

Figure 7 (contd.): Flow Diagram for Computer Model - Gaussian Noise Plus cw Interference
APPENDIX D

COMPUTER PROGRAM - GAUSSIAN NOISE
AND CW INTERFERENCE PRESENT
FORTRAN IV LEVEL 19

MAIN

DATE = 70132

22/4/4/00

COMPUTER PROGRAM FOR DIGITAL SIMULATION OF COSTAS LOOP OPERATION

CASE OF SIGNAL CORRUPTED BY GAUSSIAN NOISE AND CW INTERFERENCE

REAL*4 INR, INRG, NI, N2
DIMENSION F2(31)
DIMENSION IJKLM(3), JKLW(N3)

DEFINE THE TWO SIMULTANEOUS DIFFERENTIAL EQUATIONS OF FIRST-ORDER
DESCRIBING THE COSTAS LOOP
T - TIME IN SECONDS, Y - \theta, Z - DUMMY VARIABLE INTRODUCED

F1(T, Y, Z) = Z + 2 * \text{REAL}(X) * \text{IF} \text{INRG} = 0
G1(T, Y, Z) = \text{REAL}(X) * \text{IF} \text{INRG} = 1

DEFINE AMPLITUDE RESPONSE FUNCTION OF THE FILTERS LPF 1 AND LPF 2
FOR THE LOW-FREQUENCY COMPONENTS OF THE CW INTERFERENCE

PBI = \text{DELTAF} / \text{N}, \text{ALPHA} = 1 / (1 + (\text{DELTAF} / \text{ALPHA})^4)
N = 2
ALPHA = 5.3E3

CONSTANTS AND PARAMETERS DEFINED
PI = 3.1416
THRESH = PI/4.

ZETA - DAMPING FACTOR, WL - EQUIVALENT NOISE BANDWIDTH
WN - NATURAL FREQUENCY, WI - INPUT SIGNAL BANDWIDTH
FR - BIT-RATE BANDWIDTH, TR - PERIOD OF AN INPUT BIT
SPB - GAUSSIAN-NOISE SAMPLES TAKEN PER BIT, BITS -
TOTAL NUMBER OF BITS CONSIDERED, H - INTEGRATION AND
SAMPLING INTERVAL, SAMPLE - TOTAL NUMBER OF SAMPLES
F - MODULATING DIGITAL SIGNAL, CMIT - NUMBER OF INITIAL
RITS OMITTED AS TRANSIENT RESPONSE OF THE MODEL

ZETA = 0.707
WL = 100.
WN = 4.9 * ZETA * WL / (1 + 4.9 * ZETA * 2)
WI = 10000.
FR = WI / 2.
TH = 1.7F
SPB = 20.
NSPB=SPB
OMIT=200.
NI=OMIT*SPB
H=13/SPB
BITS=700.
SAMPLE=SPB*BITS
I1=SAMPLE

DEFINE THE MAXIMAL-LENGTH PSEUDO-RANDOM DIGITAL MODULATING SEQUENCE OF +1'S AND -1'S
READ(1,201)(F2(1L), L=1,31)

INPUT: SIGNAL-TO-GAUSSIAN-NOISE POWER RATIO IN DECIBELS
READ(1,1041)SNRDB
SNR=10.**(SNRDB/10.)

CALCULATE THE STANDARD DEVIATION OF RANDOM GAUSSIAN SAMPLING PROCESSES \(N_1\) AND \(N_2\)
STODEV=SQRT(SPBT/SNR/2.)

CALCULATE ESTIMATED VALUE OF VARIANCE(\(PHI\)), HOLDS GOOD AT HIGH SNR'S
VARPHI=WL/WI**[(1./SNR+0.5/SNR**2)]

INPUT: INTERFERENCE-TO-GAUSSIAN-NOISE POWER RATIO IN DECIBELS
READ(1,1041)INRDB
INP=10.**(INRDB/10.)

CALCULATE THE AMPLITUDE OF CW INTERFERENCE
A=SQR(INR/SNR)

DO LOOP FOR DIFFERENT VALUES OF FREQUENCY-OFFSET OF INTERFERENCE
DELTAF=10000.
DO 7 II=1,3
WRITE(3,1011)SNRDB,STODEV,INRDB,N,ALPHA,DELTAF
J=KLMN(II)=123456789
IJKL(M)=753214587
C=BR(DELTAF,N,ALPHA)
DELTAW=2.*C*DELTAF

INITIALIZE THE CONDITION OF THE LOOP
I=0.
Y=0.
Z=0.
F=-1.
N1=0.
N2=0.
BIGPHI1=0.
BIGPHI2=0.
BIGERR=0.
BIGO=0.
BIGF2=0.
BIGF2Z=0.
DELTA=0.
THETA=0.
NUSKIP=0.
NSKIP=0.

C MAIN LOOP PERFORMING ITERATIONS
DO 3 I=1,1

C PHI = PHASE ERROR
PHI=THETA-Y

C CHECK FOR A CYCLE-SKIP
IF(ABS(PHI).GE.THRESH)GO TO 11

C GO TO 12
11 CONTINUE

C IF NUMBER OF CYCLE-SKIPS PER EVERY 10 CONSECUTIVE BITS IS GREATER
C THAN 5, DECLARE THE LOOP AS BEING OUT OF LOCK
C IF(NUSKIP.GE.5)GO TO 66

C CONTINUE

20 CONTINUE

C IF NUMBER OF CYCLE-SKIPS PER EVERY 10 CONSECUTIVE BITS IS GREATER
C THAN 5, DECLARE THE LOOP AS BEING OUT OF LOCK
C IF(NUSKIP.GE.5)GO TO 66

C IN CASE OF CYCLE-SKIP, RESET THE CONDITION OF THE LOOP AS AT TIME
C T=0, THIS IS EQUIVALENT TO MANUALLY resetting IN AN EXPERIMENT

C GO TO 12
10072 WRITE(3,103)
10073 STOP
10074 12 CONTINUE
10075 X=DELTAW*T+DELTA
10076 V3=(F+N2+B*C*COS(X))*COS(PHI)-(N1+B*C*SIN(X))*SIN(PHI)
10077 V4=(F+N2+B*C*COS(X))*SIN(PHI)+(N1+B*C*SIN(X))*COS(PHI)
10078 CASE OF OUTPUT THROUGH A LIMITER IN THE IN-PHASE CHANNEL OF THE LOOP
10079 IF(V3.GE.0.)*V3=1.
10080 IF(V3.LE.0.)*V3=-1.
10081 V5=V3*V4
10082 IF(I.LE.N1)GO TO 21
10083 R1=PHI=R1+PHI+PHI
10084 R1=V3
10085 FQ=F-0.
10086 BIGO=BIGO+Q
10087 BIGFO2=BIGFO2+F0*FQ
10088 BIGF2=BIGF2+(F-(F)**2)
10089 21 CONTINUE
10090 RUNGE-KUTTA METHOD OF ORDER IV USED FOR NUMERICAL INTEGRATION
10091 A1=H*F1(T,Y,Z)
10092 B1=H*G1(T,Y,Z)
10093 A2=H*F1(T+H/2.,Y+Y1/2.,Z+B1/2.)
10094 B2=H*G1(T+H/2.,Y+Y1/2.,Z+B1/2.)
10095 A3=H*F1(T+H/2.,Y+Y2/2.,Z+R2/2.)
10096 B3=H*G1(T+H/2.,Y+Y2/2.,Z+R2/2.)
10097 A4=H*F1(T+H,Y+A3/2.,Z+B3)
10098 B4=H*G1(T+H,Y+A3/2.,Z+B3)
10101 T=T+H
10102 GENERATE SAMPLES FOR THE TWO RANDOM GAUSSIAN PROCESSES N1 AND N2
10103 CALL GAUSS(IJKLM(11),STDEV,O.,N1).
10104 CALL GAUSS(IJKLM(11),STDEV,O.,N2).
10105 CHECK FOR THE END OF A BIT
10106 IF(MOD(I,NSPB))13,4,3
10107 4 CONTINUE
J=FLOAT(J)/SPR
IF(J.LE.200)GO TO 22
IF(MOD(J,10).EQ.0)NSKIP=0
IF(BIGO.LT.0.)FOUT=-1.
IF(BIGO.GE.0.)FOUT=1.

COMPUTING THE NUMBER OF ERRORS
IF(F.NE.FOUT)BIGERR=BIGERR+1.
BIGO=0.
22 CONTINUE
JJ=MOD(J+1,31)
IF(JJ.EQ.0)JJ=31

GENERATING A PSEUDO-RANDOM FINITE-LENGTH DIGITAL MODULATING SIGNAL
F=FZ(JJ)

GENERATING A RANDOM PHASE, DELTA, FOR THE CW INTERFERENCE
72=MAG(J)
DELTA=2.*PI*72

CONTINUE
PE - BIT-ERROR PROBABILITY, SOMAN - MEAN-SQUARE ERROR BETWEEN INPUT AND OUTPUT, PHIM - MEAN OF PHI
PHIVAR - VARIANCE OF PHI
BIGERR=BITZ(BitS-OMIT)
SOMAN=BITGZ(BITG2)
PHIM=BITPHI(BitS-OMIT)
PHIVAR=BITPH2(BitS-OMIT)-PHIM\^2
WRITE(3,100)PF,PE,SOMAN,PHIM,PHIVAR,NSKIP
WRITE(3,101)PE,SOMAN,PHIM,PHIVAR,NSKIP
DELTA=DELTA-5000.
7 CONTINUE
100 FORMAT(/1X,'PE=',F5.3,10X,'MEAN SQ. ERR.=',F9.4,10X,'MEAN PHI=',F8.
/5,10X,'VARI. PHI.=',F12.5,10X,'CYCLES SKIPPED=',[3]
101 FORMAT('/1X','SNRDB = ',F5.1,10X,'STD. DEVIATION = ',F7.4,10X,'INRDB
= ',F5.1,10X,'N = ',F5.1,10X,'ALPHA = ',F16.7,10X,'DELTAF = ',F16.7)
103 FORMAT(/10X,'THE LOOP IS OUT OF LOCK/')
104 FORMAT(F7.2)
201 FORMAT(F4.1)
STOP
END