1969

Apparatus and techniques for the plane wave analysis of acoustic filters

Victor Hugo Simon

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ABSTRACT

This thesis describes a method for determining the reflection and transmission characteristics of acoustic filters under static (i.e., no flow) conditions. These characteristics are obtained by the combination of linear plane wave acoustic theory with a sequential experimental procedure. The experimental data necessary for the calculation of the reflection characteristics were obtained from measurements made on the inlet side of the filter with a standing wave tube apparatus. In order to calculate the transmission characteristics of the filter, the standing wave measurements were augmented with measurements taken on both the inlet and outlet side of the filter. This was accomplished with a transmission tube apparatus. The calculations were simplified by terminating the filter anechoically.

A standing wave tube and transmission tube apparatus, designed and built by a previous investigator, were used to determine the reflection and transmission characteristics of seven reactive filter elements. These elements consisted of bends, coils, and Tees in 0.430-inch I.D. copper refrigeration tubing. The frequency range over which experimental measurements were made is from 500 to 2000 Hz. All calculations of the reflection and transmission characteristics were performed by digital computer. The results indicate that the transmission characteristics of these particular configurations are too high for effective sound attenuation. Difficulties encountered with the apparatus and experimental procedure are discussed and recommendations for their improvement are made.

The design, construction, and evaluation of a 2-inch I.D. standing wave
tube apparatus are described. It is believed that, with minor modifications, this apparatus is capable of analyzing the reflection characteristics of acoustic filters under operating conditions of steady flow. The reflection characteristics, obtained from measurements of a solid piston termination, indicate that the apparatus produces very accurate and consistent results over a frequency range from 50 to 3500 Hz. No steady flow measurements were made.

The mechanism of attenuation and its effect upon sound waves propagating through tubing are discussed. A method of measuring the effects produced by attenuation in 0.430-inch I.D. hardened copper refrigeration tubing is also described. Experimental measurements of the attenuation were made over a frequency range from 500 to 3000 Hz. The results of these measurements were used to calculate the attenuation constant. The experimental and theoretical values of the attenuation constant are in fairly close (+10%) agreement. The results of the experimental measurements also indicate that the attenuation is very sensitive to temperature and humidity variations. Recommendations for an improved experimental technique are given.

The failure of the linear plane wave acoustic theory, due to the excitation and propagation of higher order transverse modes of vibration, is considered. An experimental method of investigating the effects produced by these higher modes is described. The results obtained from this particular examination, however, are not conclusive. Suggestions for obtaining more conclusive data are mentioned.
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NOMENCLATURE

\( A_m \) = magnitude of incident sound pressure wave \( A_1 \)

\( A_1 \) = complex value of the incident sound pressure wave at filter discontinuity inlet

\( A_1' \) = complex value of the incident sound pressure wave at a distance \( x_1 \) from the filter or discontinuity inlet

\( A_2 \) = complex value of the transmitted sound pressure wave at filter or discontinuity outlet

\( A_{2t} \) = complex value of the total transmitted sound pressure wave at an expansion chamber outlet.

\( B_m \) = magnitude of the reflected sound pressure wave \( B_1 \)

\( B_1 \) = complex value of the reflected sound pressure wave at a filter or discontinuity inlet

\( B_{1t} \) = complex value of the total reflected sound pressure wave at an expansion chamber inlet

\( C_2 \) = complex value of the transmitted sound pressure wave at a distance \( x_2 \) from the filter or discontinuity outlet

\( c \) = free space value of the speed of sound, cm/sec

\( c' \) = speed of sound in a tube, cm/sec

\( c_p \) = specific heat at constant pressure, cal/gm - °C

\( c_v \) = specific heat at constant volume, cal/gm - °C

\( d \) = internal diameter of a tube

\( \text{db} \) = unit of sound pressure level (S.L.). \( S, L = 20 \log_{10} \frac{P}{P_{\text{ref}}} \) db
\( E_{A_1} \) = average energy in the incident sound wave \( A_1 \)

\( E_{A_2} \) = average energy in the transmitted sound wave \( A_2 \)

\( E_{B_1} \) = average energy in the reflected sound wave \( B_1 \)

\( e \) = base of the natural logarithms

\( f \) = frequency in Hz

\( f_c \) = critical frequency defining the limit for plane waves

\( j = [-1]^{1/2} \)

\( K \) = thermal conductivity, cal/sec - cm - °C

\( k = \frac{\omega}{c} \)

\( L \) = length of an expansion chamber

\( L_h \) = thickness of thermal boundary layer, cm

\( L_v \) = thickness of viscous boundary layer, cm

\( l \) = length of standing wave tube

\( N, n \) = an integer

\( P_{\text{max}} \) = maximum sound pressure found in a standing wave

\( P_{\text{min}} \) = minimum sound pressure found in a standing wave

\( P_0 \) = static pressure

\( P_1, P_2 \) = measured values of total sound pressure at locations \( x_1 \) and \( x_2 \) (respectively) on the inlet and outlet sides of a filter or discontinuity

\( P' = \) instantaneous pressure = \( P_0 + p \)

\( p \) = incremental or acoustic pressure

\( p(x) \) = maximum value of the sound pressure at a given \( x \) location in a standing wave that fluctuates sinusoidally with time
\[ p_{0x} = \text{sound pressure at } x = 0 \]
\[ p_x = \text{sound pressure at any } x \text{ location} \]
\[ p_{\text{ref}} = \text{reference pressure} = 0.0002 \times 10^{-6} \text{ bars} \]
\[ q = \text{volume velocity} = uS \]
\[ q_{1t} = \text{net volume velocity on inlet side of a plane discontinuity} \]
\[ q_{2t} = \text{net volume velocity on outlet side of a plane discontinuity} \]
\[ q', r', s' = \text{fluid particle displacement in the } x, y, z \text{ directions respectively} \]
\[ R = \text{complex reflection factor} = \text{ratio of the reflected wave to the incident wave} \]
\[ r, r_0 = \text{internal radius of a tube} \]
\[ S, S_1, S_2 = \text{cross sectional areas of tubing} \]
\[ T = \text{complex transmission factor} = \text{ratio of the transmitted wave to the incident wave} \]
\[ t = \text{time, sec} \]
\[ u, v, w = \text{fluid particle velocity in the } x, y, z \text{ directions respectively, i.e.,} \]
\[ u = \frac{\partial q'}{\partial t}, \quad v = \frac{\partial r'}{\partial t}, \quad w = \frac{\partial s'}{\partial t} \]
\[ x, y, z = \text{cartesian coordinates of a particle in the fluid} \]
\[ x_{\text{max}} = \text{location of maximum sound pressure in a standing wave relative to the location of a filter or discontinuity} \]
\[ x_{\text{max}} = x_{\text{max}} \text{ and } x_{\text{min}} \]
\[ x_{\text{min}} = \text{location of minimum sound pressure in a standing wave relative to the location of a filter or discontinuity} \]
\( x_{rx} \) = location of an arbitrary sound pressure relative to the minimum sound pressure location

\( z_0 \) = characteristic impedance of the medium

**Greek Symbols**

\( \alpha \) = attenuation constant, cm\(^{-1}\)

\( \gamma \) = ratio of specific heats \( \frac{\rho}{c_v} \)

\( \Theta_{R,\theta} \) = phase angle (radians) between incident and reflected sound pressure waves

\( \Theta_T \) = phase angle between incident and transmitted sound pressure waves

\( \lambda \) = wavelength of sound, cm

\( \lambda_c \) = critical wavelength defining the limit for plane waves, cm

\( \nu \) = kinematic viscosity, cm\(^2\)/sec

\( \rho \) = incremental mass density, gm/cm\(^3\)

\( \rho_0 \) = static mass density, gm/cm\(^3\)

\( \rho' \) = instantaneous mass density = \( \rho_0 + \rho \), gm/cm\(^3\)

\( \Phi = 2 \frac{\omega \rho'}{c'} x_{rx} \)

\( \omega \) = circular frequency = \( 2\pi f \), radians/sec
I. INTRODUCTION

A sound wave is defined as a mechanical wave propagating through an elastic medium with a frequency range such that the sensation of hearing is produced in the human ear and brain. The frequency range of sound waves is from 20 to 20,000 Hz and is denoted as the "audible" range. Mechanical waves with a frequency below 20 Hz are denoted as "infrasonic" and are usually produced by large source disturbances such as earthquakes. At the other end of the scale are the mechanical waves with a frequency exceeding 20,000 Hz. These waves are denoted "ultrasonic" and can be produced by a piezoelectric crystal in resonance with a high frequency electric field [20].

A sound wave may be thought of as either a pressure wave or a displacement wave; the pressure fluctuation being 90 degrees out of phase with the displacement of the medium. The speed of propagation of the sound wave is proportional to the elasticity of the medium with the propagation velocity increasing with increasing elasticity.

Noise is defined as unwanted sound. It can be a sound wave of a singular frequency, but is in general a sound wave possessing a very irregular waveform. Such a waveform can be expressed as the superposition of a large number of periodic waves with singular frequency contents.

Noise can not only be irritating and distracting to exposed personnel; but, with sufficient intensity, can be physically dangerous. During the past few years the subject of noise reduction or control has become increasingly

*Numbers in brackets refer to references listed in the Bibliography.
more important. This has been caused by the general public's demand for a quieter environment. Consequently, the noise level in aircraft, automobiles, and air conditioning systems (to mention just a few) has become an important design parameter.

Sound energy (noise) is transmitted from its source to a receiver via a number of paths. It may be transmitted to a receiver directly from the source generating the disturbance such as the sound broadcast from the compressor or tailpipe of a jet engine. The receiver may also detect the sound indirectly from structural vibrations produced by rotating or reciprocating machinery. Consider the case of a refrigeration or air conditioning system. Vibrations from the motor and compressor produce vibrations of the structure upon which the unit is mounted. These structural vibrations produce noise. The compressor also produces pressure oscillations in the refrigerant. If these oscillations contain frequencies at or near the resonant frequencies of the piping system, they excite the piping into vibration. The vibrations of the piping then produce pressure oscillations in the surrounding medium (air) which the receiver detects as noise.

If the noise cannot be eliminated at its source then one of the following procedures may be employed to reduce the noise level. Noise producing structural vibrations can be eliminated if the devices producing these vibrations are shock mounted or otherwise isolated. Two other techniques used for noise reduction are:

1) to dissipate or absorb all or part of the unwanted sound before it impinges on the receiver;
2) to reflect the sound away from the receiver before it reaches the receiver's location.

It is usually necessary to employ a combination of these two methods to achieve effective sound attenuation.

The general mathematical expressions describing acoustic wave behavior are three dimensional with the waves usually having complex excitations. When the interactions of such waves at boundaries and discontinuities in the sound field are considered, the problem becomes even more complicated. Such problems do not lend themselves to exact solutions by present methods. This difficulty is alleviated if the sound waves are assumed "plane". If an imaginary plane is passed through such a wave so that it is perpendicular to the direction of wave propagation, it is found that the pressure and particle velocity are constant over the imaginary surface. In many instances the use of plane wave theory is not unduly restrictive and provides a means of obtaining theoretical solutions to many problems encountered in acoustic analysis. Plane wave acoustic theory is applied to the problems encountered in this investigation. Its limitations are also presented.

The noise produced by pressure oscillations from a refrigeration compressor or jet engine, for example, can be reduced by acoustic filters which eliminate or attenuate the pressure oscillations. Ideally, acoustic filters are classified [4] as either "dissipative" or "reactive". A dissipative filter's performance is primarily determined by the presence of a flow-resistive sound absorbant material and usually has wideband noise reduction characteristics. A reactive filter on the other hand does not depend on such
absorbant material for sound attenuation but rather upon sound reflections produced by its geometrical configuration. Reactive filters usually possess narrowband attenuation characteristics. In practice acoustic filters are often a combination of these two classifications.

Historically, the methods used in acoustic filter design have been based on trial and error experimentation. This is not only costly and time consuming, but impractical as well since a filter so designed will rarely function with any degree of efficiency when placed in a system with slightly different operating conditions. Thus a systematic analytic method for the design of acoustic filters is needed. Several authors [3, 4] give requirements for filter performance but fail to include any set specifications pertaining to their design.

Gatley [17] has shown that a combination of plane wave acoustic theory and experimental data can be used to predict the performance of small filters in an operating refrigeration system. This is accomplished by knowing the characteristics of the filter's component elements. The term "small" is used to denote the fact that the filter's physical dimensions are small in comparison to the wavelengths of sound passing through it.

One of the objectives of this investigation was to obtain, by use of plane wave theory combined with experimental measurements, the acoustic characteristics of several small reactive filter elements. This data could be added to a "library" of filter element characteristics. Once such a library is established, the overall performance of filters comprised of such component elements could then be predicted by using plane wave theory. This
would enable the design of a filter, for a specific application, to be carried out in a fairly straightforward manner.

One of the limitations of the standing wave apparatus used in this investigation was that only the static (i.e. no flow) values of the acoustic characteristics could be obtained. Since most filters are required to cope with a steady flow of fluid as well as pressure perturbations, it can be seen that this method of analysis may be somewhat lacking. Thus, another objective of this investigation was to design, construct, and evaluate a standing wave tube apparatus that is (at a future date) capable of analyzing filters under operating conditions of steady flow.

The remaining phases of this investigation were concerned with the measurement of the attenuation of plane sound waves in copper tubing and the examination of a specific filter configuration in which the plane wave theory was no longer valid.

The four experimental phases of this investigation are presented in four separate sections. Each section presents the experimental procedure, apparatus, results, discussion of results, conclusions, and recommendations for that particular phase.
II. LITERATURE REVIEW

A. The Mechanism of Attenuation and Its Effect upon Sound Waves Propagating through Tubes

When a sound wave propagates through air a certain amount of the energy present in the sound wave is lost or dissipated. In free space, this energy loss is due primarily to three physical properties of the medium. The first two (and most pronounced) are viscosity and heat conduction in the medium. The third is involved with the exchange of translational, rotational, and vibrational energy between colliding molecules. This particular mechanism of energy dissipation is often termed "molecular absorption", "anomalous absorption", or "thermal relaxation" and can become a significant component of the overall attenuation in polyatomic gases. [2, 18, 28, 42]

A sound wave propagating through an acoustical waveguide, such as a circular tube, encounters much greater attenuation than when traveling through free space.* This is due to viscous and thermal boundary layers existing at the tube wall. The existence of plane waves, as assumed in this thesis, implies the use of such a waveguide. Rayleigh [47] has shown, for the case of a circular tube, that a sound wave will eventually become plane if the driving frequency is lower than the frequency corresponding to the first transverse mode of vibration. This will be discussed in greater detail later.

*The attenuation in free space due to viscosity and heat conduction is negligible when compared to the attenuation caused by the boundary layers in small diameter tubes at frequencies below 20,000 Hz. [2, 17]
The effects of attenuation upon a sound wave propagating through a circular tube are a decrease in the sound pressure and propagation velocity of the wave. Helmholtz (1863) was the first to investigate the attenuation of sound waves in tubes. He presented a formula for the decrease in the propagation velocity due to attenuation but considered only the effect of viscosity in his derivation. Kundt (1868) measured the propagation velocity of sound in tubes and found that the effect of attenuation was greater than that predicted by Helmholtz's formula. Kundt suggested that heat conduction might be a cause for the discrepancy between experimental and theoretical values. Prompted by the work of Helmholtz and Kundt, Kirchhoff (1868) presented the complete theory for both the decrease in propagation velocity and sound pressure [60] due to the viscous and thermal boundary layers at the tube wall.

The decrease in sound pressure along the tube due to attenuation is exponential and increases with increasing distance. It is of the form [42]

\[ p_x = p_{0x} e^{-\alpha x} \]

where

- \( p_{0x} \) = sound pressure at \( x = 0 \)
- \( p_x \) = sound pressure at any \( x \) location
- \( e \) = base of the natural logarithms
- \( \alpha \) = attenuation constant

Kirchhoff's formula [60] for the attenuation constant is

\[ \alpha = \frac{1}{c_r} \left( \frac{\omega}{2} \right)^{1/2} \left[ (\nu)^{1/2} + (\gamma - 1) \left( \frac{K}{\rho_0 c_p} \right)^{1/2} \right] \]
where

\[ c = \text{propagation velocity of sound in free space, cm/sec} \]

\[ r = \text{inside radius of the tube, cm} \]

\[ \omega = \text{circular frequency} = 2\pi f, \text{ radians/sec} \]

\[ \nu = \text{kinematic viscosity, cm}^2/\text{sec} \]

\[ \gamma = \text{ratio of specific heats} = 1.4 \text{ for air} \]

\[ K = \text{thermal conductivity, cal/sec - cm - }^\circ\text{C} \]

\[ \rho_0 = \text{static mass density, gm/cm}^3 \]

\[ c_p = \text{specific heat at constant pressure, cal/gm - }^\circ\text{C} \]

Note: \[ \frac{K}{\rho_0 c_p} = \text{thermal diffusivity, cm}^2/\text{sec} \]

The theory presented by Kirchhoff [60] is valid for the case of "wide" tubes. This is a term that is applied to tubes in which the attenuation is caused chiefly by the viscous and thermal boundary layers at the tube wall. A "narrow" tube has a radius that is much less than the thickness of the boundary layer. In this case, at low frequencies, it is assumed that only viscous forces are present and that isothermal conditions exist. The "very wide" tube differs from the "wide" tube in that the sound energy is assumed concentrated at the tube walls and is not constant over the tube cross section as is the case for the "wide" tube. These three cases are considered in much more detail by Weston [60]. The tubes dealt with in this thesis fall into the "wide" category.

The other assumptions made by Kirchhoff [60] in his derivation of the attenuation constant are:
1) only plane sound waves exist;
2) the medium is homogeneous and only the effects of viscosity and heat conduction are present;
3) the sound wave amplitude is assumed small enough such that no circulation or turbulence occurs;
4) the sound wave is harmonic in nature, i.e., sound pulses are not considered;
5) the tube is considered infinitely long so that the velocity profiles are stable and no end effects are encountered;
6) the inside surface of the tube is assumed smooth, impervious, rigid, and symmetrical;
7) there is no fluid motion at the tube wall;
8) the gas temperature at the tube wall is constant;
9) the sound wavelength is much greater than the molecular mean free path.*

For dry air at 68°F and 760 mm Hg, Kirchhoff's expression for the attenuation constant reduces to [1, 60]

\[ \alpha = 2.96 \times 10^{-5} \frac{f^{1/2}}{r} \text{ cm}^{-1} \]

where

\[ f = \text{frequency in Hz} \]
\[ r = \text{tube radius in cm} \]

*This assumption is violated if the frequency exceeds \(10^9\) Hz in air at normal pressures and temperatures [60].
The general form of Kirchhoff's equation for the attenuation constant may be expressed as

$$\alpha = \frac{1}{2r} \left[ \frac{\omega}{c} L_v + (\gamma - 1) \frac{\omega}{c} L_h \right]$$

where

$$L_v = \left[ \frac{2\nu}{\omega} \right]^{1/2} = \text{thickness of viscous boundary layer, cm}$$

$$L_h = \left[ \frac{2K}{\omega \rho \rho_0 c_v} \right]^{1/2} = \text{thickness of thermal boundary layer, cm}$$

All other notation is the same as that presented previously. This expression is similar to the expression for the attenuation constant presented by Morse and Ingard [42].

Conflicting views are present in the literature concerning the expression for the thermal boundary layer thickness $L_h$. Morse and Ingard [42] stated that

$$L_h = \left[ \frac{2K}{\omega \rho \rho_0 c_v} \right]^{1/2}$$

while Rayleigh [47], Fay [15], Weston [60], and Shields, Lee, and Wiley [53] claimed that $c_p$ (instead of $c_v$) should appear in the denominator of such a term.* Fay [15] stated that the thermal effects should be calculated at constant pressure rather than constant volume since the thermal influence of the tube walls is not great enough to cause appreciable changes in pressure.

After further investigation of the derivation of the expression presented by Morse and Ingard [42] it is believed that the discrepancy involving the

*The expressions for the attenuation constant presented by these authors must be rearranged into forms similar to that presented by Morse and Ingard for this to become clear.
specific heats arises from certain simplifications these authors made in reducing the generalized energy equation to fit the problem at hand. If numerical values for the physical properties are inserted in both expressions for the attenuation constant it is found that the Kirchhoff expression is smaller than the Morse-Ingard expression by approximately 7 percent.

Beranek [21] gave the value of the Kirchhoff attenuation constant as

$$\alpha = 2.77 \times 10^{-5} \frac{f^{1/2}}{r} \text{ cm}^{-1}$$

which he stated is about 15 percent low when compared with experimental measurements.

Fay [15] experimentally investigated the attenuation of sound waves propagating through a rigid "wide" tube of 0.749 inch diameter. The tube was packed in sand to prevent vibrations of the tube wall and to provide some form of thermal insulation. Dry air at a very low flow rate (1 ft$^3$/hr) was passed through the tube to eliminate any effects produced by variations in humidity. The range of frequencies investigated was up to 5000 Hz. Fay found that in this frequency range, Kirchhoff's equation involving the square root of the frequency was valid within a small percentage. He stated that a closer agreement between theoretical and experimental values could be obtained if a linear frequency term were added to Kirchhoff's equation. Fay's expression for the attenuation constant is

$$\alpha = 2.92 \times 10^{-5} f^{1/2} + 6.75 \times 10^{-8} f \text{ cm}^{-1}$$

for a 0.749 inch diameter tube. He concluded that the linear term in this expression was due to absorption in the air alone unless there was an
additional influence of the tube wall not predicted by Kirchhoff's analysis.

The molecular absorption (or anomalous sound absorption) in air was investigated by Knudsen [30, 15, 21] using reverberant chamber techniques. It was found that this absorption is proportional to the first power of the frequency. Fay stated that the additional attenuation (not predicted by Kirchhoff) found in the tube is much greater in magnitude than the attenuation due to molecular absorption found by Knudsen in the reverberant chamber. Fay concluded that until further measurements were made to determine whether or not the coefficient of the linear term is a function of the tube diameter, no definite correlation with Knudsen's data can be made.

Another conclusion drawn by Fay was that variations in humidity could cause large inconsistencies in the results obtained from attenuation measurements. This was observed in measurements made without the dried air purging.

The effects of water vapor (and also temperature) upon the sound absorption in air and other gas mixtures has been investigated by Knudsen [29, 30], Chrisler and Miller [11], Harlow and Kitching [21], and (most recently) Harris [22, 23]. In brief, these investigators stated that the anomalous sound absorption depends in a very characteristic way on the amount of water vapor present as an impurity. The relaxation frequencies, and hence the molecular absorption, of a gas are very sensitive to small amounts of certain impurity gases [21]. Knudsen, investigating the absorption of sound in oxygen [29], found that the relaxation frequencies are a linear function of the impurity gas concentration for impurity gases such as N₂, NH₃, H₂S,
and C$_2$H$_2$. This is not the case, however, when water vapor is present as an impurity. He found that with water vapor present, the relaxation frequencies cease to be a linear function of the impurity concentration and become instead a quadratic function of the water molecule concentration [21, 29].

A more rigorous consideration of the molecular absorption of sound in tubes was presented by Shields, Lee, and Wiley [53]. They stated that the relaxation phenomenon in gases is also a function of the ratio of the driving frequency to the static pressure of the gas, i.e., $f/\rho_0$.

Beranek [2] compared the results of Fay [15] with the results of other investigations of the attenuation of sound in tubes. He found that at higher frequencies, Fay's expression for the attenuation constant gave values that were 10 to 20 percent higher than measured values. After taking the results of numerous investigators into consideration, including Fay's, Beranek concluded that the attenuation constant is best described by Kirchhoff's equation increased by 15 percent.* Using his numerical evaluation of the Kirchhoff attenuation constant and increasing it by 15 percent gave the attenuation constant as

$$\alpha = 3.18 \times 10^{-5} \frac{f^{1/2}}{r} \text{ cm}^{-1}.$$  

This is the expression for the theoretical attenuation constant used in this thesis. The numerical values of the attenuation constant as functions of the frequencies and tube radii used in this investigation are presented in Appendix 1.

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*This is for the case of normal undried air in "wide" tubes with thin walls exposed to the surroundings. As will be mentioned in Section IV this corresponds closely to the conditions under which experimental measurements were made in this investigation.
These theoretical values of the attenuation constant are in close agreement with the attenuation constants obtained from experimental measurements made in this investigation (see Section IV).

Kirchhoff, using the same assumptions as presented in the attenuation constant derivation, gave the equation \([14, 53, 60]\) for the speed of sound in a "wide" tube of radius \(r\) as

\[
c' = c \left[ 1 - \frac{L_{vh}}{2r[\pi f]} \right]^{1/2} \text{ cm/sec}
\]

where

\[
L_{vh} = [\nu]^{1/2} + (\gamma - 1) \left[ \frac{K}{\rho_0 c_p} \right]^{1/2}
\]

\(c = \text{adiabatic speed of sound in free space} = [\gamma RT]^{1/2}\)

for an ideal gas = 1494 \([T]^{1/2}\) cm/sec for air \([52]\)

\(R = \text{ideal gas constant}\)

\(T = \text{static temperature in degrees Rankine}\)

\(\nu = \text{kinematic viscosity, cm}^2/\text{sec}\)

\(\gamma = \text{ratio of specific heats} = 1.4\) for air

\[
\frac{K}{\rho_0 c_p} = \text{thermal diffusivity, cm}^2/\text{sec}
\]

Using the numerical values of the physical properties as given by Chapman \([10]\), Kirchhoff's expression for the speed of sound in a tube of radius \(r\) is

\[
c' = 34,529 \left[ 1 - \frac{0.579}{2r[\pi f]} \right] \text{ cm/sec}
\]

or

\[
c' = 34,529 \left[ 1 - \frac{0.163}{r[\pi f]}^{1/2} \right] \text{ cm/sec}
\]
for dry air at 74°F and 760 mm Hg. This agrees with the value for Kirchhoff's equation given by Weston [60].

Kirchhoff's equation for the speed of sound correction due to tube effects was experimentally verified by Scott [51, 9] and El Hakeem [14] who found that the theoretical and experimental values are in close agreement.

The numerical values of the theoretical speed of sound in tubes obtained from the evaluation of Kirchhoff's equation are presented in Appendix 2 as functions of the frequencies and tube radii used in this investigation. Included in Appendix 2 are the values of the wavelength corresponding to these values of the speed of sound. They are obtained from the relationship

$$\lambda = \frac{c'}{f}$$

where

$\lambda$ = wavelength in cm

c' = speed of sound in tube (cm/sec)

f = frequency in Hz

As will be mentioned later (see Section V and Appendix 2) these theoretical values of the wavelength are in very good agreement with the values obtained from experimental measurements made in this investigation.

B. The Excitation and Propagation of Sound Waves Containing Higher Order Transverse Modes of Vibration

The existence of higher order transverse modes of vibration can be brought about by the inability of plane waves to satisfy all the boundary conditions imposed by a discontinuity or other specific configuration. * When

*Transverse modes may also be generated by a vibrating element or source.
such vibrations are present they produce effects that naturally cannot be predicted by plane wave theory.

In a tube, transverse modes are excited if the wavelength of a sound wave propagating through it is less than a certain critical value relative to the radius of the tube. This value establishes a critical frequency that cannot be exceeded if plane waves only are to exist in the tube.

Rayleigh [47] predicted that a wave propagating in a tube can be considered plane if its frequency is lower than the frequency corresponding to the first transverse mode of vibration. He also stated that even if a wave does contain transverse modes it will eventually become plane if the above relationship exists.* For the extreme case, he considered a point source disturbance situated in a long tube of radius $r$. He predicted that a wave produced by such a source will become plane if $\lambda > 3.1413r$. If this condition exists, the effect of transverse modes may be neglected. Rayleigh also stated that if the source were symmetrical over the tube cross section (i.e., a vibrating piston of radius $r$), the wave could be considered plane if the wavelength were approximately half the value of the point source case. In either case he did not specify how far the wave must propagate in the tube before it becomes entirely plane.

Hartig and Swanson [24] were among the first to experimentally investigate the effect of transverse modes in tubing. They stated that the first transverse mode is excited and propagated when $\lambda \approx 1.64r$ (or 0.82$d$). The frequency

---

*This concept was mentioned previously in this thesis when it was stated that the existence of plane waves implied the use of an acoustical waveguide such as a tube.
corresponding to this value of the wavelength is the critical frequency previously mentioned and defines the upper limit for the existence of plane waves in the tube. The derivation of this value appears in Section III.

Several investigations have experimentally established critical wavelengths (and hence frequencies) that define the limits for plane wave propagation in tubes. Beranek [5] stated that, under proper conditions of excitation, the first transverse mode can occur at $\lambda = 3.41r$. However, for axial excitation he concluded that this mode will not be excited and the limit for plane waves can be extended to the value $\lambda \approx 1.7r$.

Sabine [49] stated that the first transverse mode is expected to occur at $\lambda = 4r^*$ but that no noticeable effects were observed until the second transverse mode was reached at $\lambda = 2r$. At this point he found that wide and irregular discrepancies occurred between the experimental data and the data obtained from plane wave theory. Sabine concluded that the first mode does not propagate because of the sound source arrangement of the experimental apparatus.

Scott [51, 17] claimed that higher modes could be neglected for $\lambda \geq 3.413r$ as predicted by Rayleigh. He stated that good agreement with plane wave theory, however, may be obtained until a limit of $\lambda = 1.73r$ is reached.

Berendt and Schmidt [7] stated that the limit for plane waves in an 8-inch diameter tube filled with air is 900 Hz. This corresponds to $\lambda = 3.67r$.

Miles [35, 36, 38] presented a theoretical consideration of the effect of plane discontinuities in cylindrical tubes. He stated that the plane wave theory

*Sabine's terminology is somewhat confusing. The tube in which his experimental data was taken was square; yet he refers to tube diameter. No mention of hydraulic diameter is made.
is violated once the first transverse mode has been excited in the tube. Miles gave the point at which this occurs as \( \lambda = 1.64r \) corresponding to the value presented by Hartig and Swanson [24].

A plane discontinuity is defined as an abrupt change in cross sectional area at a given location. Miles [38] gave several examples of such discontinuities in a cylindrical tube. In one of his examples the tube diameter increased from \( a_1 \) to \( a_2 \) at an axial location* \( z = z_1 \). Plane waves were assumed to exist in the tube section with diameter \( a_1 \). Miles stated that the plane wave theory is not capable of satisfying all of the boundary conditions at the discontinuity. In order to satisfy all these boundary conditions, higher order transverse modes of vibration must exist at the discontinuity.** He concluded that if the plane wave in the section with diameter \( a_1 \) has a wavelength such that \( \lambda \geq 0.82a_2 \), then these higher modes do not propagate and are attenuated with increasing distance in the section of diameter \( a_2 \). In this case only the principal mode or plane wave exists at large distances past the discontinuity. The distance required for the attenuation of these higher modes is determined by the value of the diameter-to-wavelength ratio. If \( a_2 \) is small in comparison to the wavelength, these modes are attenuated quite rapidly. Conversely if \( a_2 \) is large, then the higher modes propagate along with the principal mode and the effects produced by them become pronounced [38]. For discontinuities in general,

---

*This is Miles' notation.

**The analysis of plane discontinuities is presented in greater detail in Section III.
the most noticeable effect produced by the propagation of transverse modes is phase distortion of the transmitted and reflected waves \([35, 36, 38]\).

Davis, Stokes, Moore, and Stevens \([13]\) in investigating the acoustic characteristics of a wide variety of cylindrical muffler configurations found that the results predicted by the plane wave theory are in close agreement with experimental data until the first transverse mode is excited within the mufflers. They stated that this occurs whenever \(\lambda > 0.82d\) (where \(d\) is the internal diameter of the muffler).

Miwa and Igarashi \([39]\) stated:

"In Part I \([26]\), the attenuation characteristics of various acoustic filters were calculated theoretically from the four terminal constants of each section and the measured results agreed fairly well when the system was considered to be one dimensional. However, when the radius of the cylinder was large compared with the wavelength, obtained results showed particular attenuation, which could not be expected from the one dimensional theory."

Gatley \([17]\) and Buckley \([9]\) have also encountered discrepancies in experimental results at high frequencies that exhibit a departure from the linear one dimensional plane wave theory. These discrepancies were mainly in phase angle measurements which indicated phase distortion as predicted by Miles \([35, 36, 38]\). At this point it can be seen that three dimensional effects produced by higher order transverse modes of vibration can become significant as the frequency and/or tube diameter increase.

C. The Concept of Acoustic Impedances

Although impedance is not specifically dealt with in this investigation, an understanding of its physical meaning is important since it is a basic con-
cept of acoustic analysis.

Three different acoustic impedances are encountered in acoustic analysis [2, 3, 27]. They are

1) The "acoustic impedance"
2) The "specific acoustic impedance"
3) The "radiation (or mechanical) impedance".

Each of these impedances has a specific area of application in acoustic analysis.

The "acoustic impedance" is defined as the complex quotient of the instantaneous sound pressure divided by the instantaneous volume velocity (particle velocity x area) at a given location. This is analogous to the electrical impedance found in AC circuit theory. If this analogy is used to describe the acoustic impedance, the sound pressure is analogous to a voltage and the volume velocity is analogous to an electrical current. The real component of the complex acoustic impedance is termed the "acoustic resistance" and is associated with the dissipation of energy. The imaginary component of the impedance is termed the "acoustic reactance" and is associated with the effective mass and elasticity of the medium. The effective mass of the medium is taken into account by the "acoustic iner-tance" which is analogous to electrical inductance. The elasticity of the medium is described by the "acoustic compliance" which is analogous to electrical capacitance. The unit of acoustic impedance is the "acoustic ohm".

It should be stated at this point that the electrical analogy is very useful in the realm of acoustic analysis. Many acoustic systems may be reduced
to equivalent electrical circuits. These circuits may then be analyzed by
the application of AC circuit theory or by analog computer techniques.

The acoustic impedance is useful in discussing the sound produced
by vibrating surfaces and the transmission of this sound through lumped
acoustic elements, tubes, and horns [27].

The "specific acoustic impedance" is defined as the complex quotient
of the instantaneous sound pressure divided by the instantaneous particle vel-
ocity at a given location. In other words, it may be thought of as the acoustic
impedance placed on a per unit area basis. For the particular case of a free
plane progressive wave propagating in one direction through an ideal gas,
the specific acoustic impedance of the medium is a real quantity with a mag-
nitude equal to the product of the static density times the speed of propagation
of the wave. The product of these two quantities is a characteristic property
of the medium and hence is denoted as the "characteristic impedance" of the
medium. Since the characteristic impedance is a real quantity, it is also
known as the "characteristic resistance" of the medium. If the sound pressure
is known, the characteristic impedance provides a means of calculating the
particle velocity (See Section III).

The specific acoustic impedance is very useful when the transmission
of sound waves from one medium to another is considered. The unit of speci-
fic acoustic impedance is the "rayl".

The "radiation impedance" is defined as the complex quotient of the
instantaneous force divided by the instantaneous particle velocity at a given
location. Radiation impedance is related to the specific acoustic impedance
by a factor of area and to the acoustic impedance by a factor of area squared. The radiation impedance is also known as the "mechanical impedance". It is used primarily in calculating the coupling between sound waves and a driving source or driven load [27]. The unit of radiation impedance is the "mechanical ohm".

The reader may obtain more specific details concerning the acoustic impedances and their components by referring to almost any basic acoustics text.

D. The Experimental Measurement of Acoustic Impedance

The measurement of acoustic impedance has received considerable attention since the turn of the century. One of the first experimental investigators (1913) in this area was Taylor [57] who examined the acoustic absorption characteristics of a number of porous materials. Taylor's apparatus consisted of a wooden standing wave tube 9 cm. square and 115 cm. long. A source of sound (organ pipe) was placed at one end of the tube and the material in question at the other. The sound field was explored by a 0.5 inch diameter movable probe tube that was attached to a Rayleigh disk detector. The absorption of the material was obtained from the measurement of the magnitude and location of the sound pressure nodes in the standing wave tube. Taylor assumed that the sound waves in the tube were plane.

One of the first investigators to develop a technique for measuring the impedance of an acoustic element directly was Stewart [56]. His apparatus essentially consisted of an acoustic "conduit" and two telephone receivers (acting as sound generators) activated by a common electronic oscillator.
One of the telephone receivers acted as a sound source in a long uniform tube while the other acted as a comparison instrument or standard. The element whose impedance was to be measured was inserted as a branch in the tube. The sound wave that was transmitted past the branch was then counteracted by a sound wave from the comparison instrument until a null was detected by the observer. The data obtained from this procedure was combined with other measurements made with and without the element present to yield the resistive and reactive components of the complex acoustic impedance. Several different elements were tested.

Stewart stated that sources of error are temperature variations within the apparatus and source frequency fluctuations. He said that these can be avoided if the apparatus is insulated and rapid measurements are made.

Wente and Bedell [59] improved Taylor's method of analysis and discussed various ways of obtaining the acoustic impedance and absorption coefficient of porous materials by measuring the standing waves in tubes. The three methods they recommended for determining the impedance of a sample are:

1) Measure the magnitudes of, and relative phase angle between, two sound pressures in the tube that are separated by an accurately known distance;

2) Determine the absolute value of the sound pressure at points along a tube of constant length;

3) Measure the pressure at the source in a variable length tube.

Wente and Bedell stated that the first method is sensitive to temperature
variations. Also, if the distance between the two sound pressures is not accurately known, errors in phase measurement will result. The second method is rather insensitive to temperature fluctuations but requires a long probe tube that they said has too much inherent attenuation for accurate measurements. The third method requires an infinite sound source impedance. The authors considered the third method as the best and presented data obtained by it. They also stated that errors may be encountered if stray electromotive forces enter the measuring system or if extraneous vibrations are present in the tube.

Another direct method for determining the impedance of acoustic elements was presented by Flanders [16].

"The apparatus to be described in this paper measures acoustic impedance directly in terms of a known acoustic impedance and three balance readings of an electrical potentiometer. The only assumptions involved in the method are that the elements of the apparatus be invariable during a measurement, and that the value of the comparison acoustic impedance be known accurately."

Flanders used a closed tube one-eighth of a wavelength long as an impedance standard. He stated that since the potentiometer readings are used as ratios none of the response characteristics of the sound source or electrical hardware affect the accuracy of the measurements. Flanders said that the data obtained from experimental measurements agreed with the theory fairly well. Among the elements he investigated were a closed tube, a conical horn, an exponential horn, and an "infinite" tube. The "infinite" tube was represented by 112 feet of coiled tubing.

Sivian [54] presented a method for measuring the acoustic impedance
of circular and rectangular orifices. His apparatus consisted of a resonator with a sound source located at the entrance. The orifice was positioned within the cavity between the sound source and the closed end. The sound field was examined by means of a probe placed near the orifice. Sivian computed the impedance of the orifice by using the ratio of the pressure at the orifice to the pressure that existed in the closed volume behind it. He stated that the acoustic reactance was found to be relatively independent of the particle velocity through the orifice.

Robinson [48] described an acoustic impedance bridge method for determining the impedance of acoustic elements. The impedance of the unknown element was determined by balancing it against a known acoustic impedance. The point at which the system balanced was determined by a differential microphone. Robinson obtained good agreement between experimental data and theoretical values for a piston inside a tube but not for a Helmholtz resonator. He attributed this to error in the determination of the orifice conductivity of the resonator. He stated that the discrepancies encountered in his investigation could have been caused by:

1) The effects produced at the junction of two tubes of different diameters;
2) The dependence of the orifice conductivity on the position of the orifice in the tube;
3) The interference produced by microphone probes;
4) The microphone probes being placed too near the orifice.

Robinson also constructed a similar apparatus that was of more compact form.

Morrical [41] determined the absorption of commercial acoustic
material by use of a modified tube method. Sound waves that propagated through an 8-inch pipe struck a sample of material placed at a 45° angle to the axis of the pipe and were reflected down a side tube. The side tube was equipped with a microphone and measured the magnitude of the reflected sound wave. The sample was then replaced by a thick glass plate and the procedure was repeated. Comparison of the two sound pressure readings obtained gave the absorption of the sample.

Hall [19] described an impedance measuring apparatus that consisted of a 270° groove machined in a flat plate. This plate was covered by a second rotating plate upon which a microphone was mounted. The groove was 2.81-cm. square and had a circumference of 24 inches which allowed a frequency range of 270 to 6000 Hz. A sound source was located at one end of the groove and the element under test at the other. The sound field was explored by rotating the plate housing the microphone. Hall evaluated the acoustic impedance of the unknown element in terms of the characteristic impedance of the groove and thus eliminated the need for impedance standards which are required by the bridge method.

Hall stated that the tube attenuation affects the location of the pressure minimum with respect to the element. The location of the pressure maximum, however, is not affected to any great degree. He also mentioned that the measured magnitude of the sound pressure is less than the true magnitude since the microphone integrates the pressure over a finite area. This does not affect the results, however, since pressure ratios appear in the calculation of impedance.

Beranek [5] presented a summary of nine methods for measuring acoustic impedance stating the advantages and disadvantages of each. He also listed a
set of conditions upon which accurate experimental analysis depends. The most important are:

1) The method should be absolute and not dependent on a reference or standard impedance;
2) Temperature variations within the apparatus must be held to a minimum;
3) The pressure detector should not affect the sound field;
4) The frequency must be accurately known;
5) Residual noise in the system must be held to a minimum;
6) The tube should be smooth and leak-free;
7) Pressure or particle velocity ratios (instead of absolute values) should be used wherever possible.

Beranek used two standing wave tubes; one of 3-inch diameter for frequencies between 100 and 3000 Hz and another of 1.25-inch diameter for frequencies above 3000 Hz. This was done to minimize the effects of higher order transverse modes of vibration. He also presented a technique for obtaining high source impedance. Beranek's experimental results were in close agreement with results predicted by plane wave theory. Using the aforementioned techniques, Beranek obtained data that was previously unobtainable.

Sabine [49] stated that rapid measurements should be made to minimize the effect of drift errors. He also stressed the importance of using well-defined pressure minimums for distance measurements instead of broad pressure maximums. Sabine suggested that all electronic signals should be filtered to eliminate harmonics.
Scott [51,17] presented a list of requirements for accurate results in the 100-5000 Hz range that is similar to the list presented by Beranek [5]. The requirements not previously mentioned by Beranek are:

1) The tube should be rigid;

2) The location of the microphone with respect to the termination should be known to within 0.1 millimeter;

3) Symmetric readings should be taken on both sides of a pressure minimum to accurately determine its location.

Scott claimed that he obtained very accurate results by use of such methods. He also noted that the pressure minima are shifted by tube attenuation.

In a later article (1947) Beranek [6] reported several improvements made on the previous apparatus. One of these improvements consisted of enclosing the standing wave tube in a water jacket to eliminate errors caused by temperature variations. Beranek also investigated the effect of a traveling microphone inside the tube. He found that the presence of a 1.6 cm. diameter microphone shifted the nodal points of the standing wave from 1/8 to 3/8 inches in the frequency range from 100 to 4000 Hz. When this microphone was replaced by a smaller microphone of 1 cm. diameter, the maximum shift was reduced to 1/10 inch. The 1 cm. microphone affected the magnitude of the standing wave by 0.1 to 0.4 db.*

*The decibel is a means of expressing the sound pressure level (S. L.) which is defined as

\[ S. \text{ L.} = 20 \log_{10} \frac{p}{p_{\text{ref}}} \text{ db} \]

where

\[ p = \text{acoustic or sound pressure} \]
\[ p_{\text{ref}} = \text{reference pressure} = 0.0002 \times 10^{-6} \text{ bars} \]
Cook [12] and Mawardi [34] described methods for determining the impedance of acoustic materials that do not require the formation of standing waves, movable microphones, movable samples, or movable sources. Their apparatus consisted of cavities or tubes whose physical dimensions were small compared with the wavelengths of interest. This effectively eliminated standing waves within the cavities. The sample to be examined was placed at one end of the tube or cavity and a very high impedance source at the other. Sound pressure measurements were made in the cavity with the sample in place. The sample was then replaced with a rigid termination (a steel plate) and the procedure repeated. Comparison of the two measurements thus obtained gave the acoustic impedance of the sample. One of the advantages of not using standing waves is that long tubes are not required for low frequency measurements. Mawardi [34] stated that the method is simple, rapid, and precise. He also considered the effects produced by finite source impedance and heat losses at the cavity walls. The physical dimensions of the cavities are determined by the frequency range desired.

The firm of Brue1 and Kjaer [8] manufacture a standing wave tube apparatus capable of measuring the absorption and impedance of acoustic materials. The apparatus has a frequency range of 95 to 6500 Hz. The sound field is explored by a probe, connected to a movable microphone, that passes through the center of the sound source. The microphone is mounted on a carriage that moves along a scale indicating the distance from the test sample to the probe entrance. The manufacturers state that one of the advantages of the apparatus is that small (10 cm. in diameter) samples may be used. They
claim that the measurements are quickly and easily performed and are "perfectly reproducible".

The measurement of the absorption and impedance of acoustic materials has been standardized (1958) by the American Society of Testing Materials [1].

"This method of test is limited to the use of apparatus consisting of a tube of uniform cross-section and fixed length, excited by a single tone of selected frequency, in which the standing wave pattern in front of a specimen upon which plane waves impinge at normal incidence is explored by means of a moving probe tube or microphone."

Listed in this report are specifications for the construction of a standing wave tube as well as procedures for measurement of the acoustic characteristics of the sample. The most pertinent of these are:

1) For measurements involving two consecutive pressure minima, *

the length of the tube, 1, in feet must be at least**

\[ l = \frac{1000}{f_{\text{min}}} \]

where \( f_{\text{min}} \) is lowest frequency at which measurements are to be made.

2) For measurements involving one pressure minimum, the length of the tube, 1, in feet must be at least

\[ l = \frac{330}{f_{\text{min}}} \]

*The report reveals that this method yields the best results.

**This length is necessary if two consecutive pressure minima are to be found in the tube.
3) A pressure minimum shall not be measured any closer to the source than one tube diameter.*

4) For a cylindrical tube, the inside diameter, d, in inches, shall not be greater than

\[ d = \frac{8000}{f_{\text{max}}} \]

where \( f_{\text{max}} \) is the highest frequency at which measurements are to be made.**

5) The interior cross sectional area of the tube shall be uniform to within 0.2 per cent over the entire length of the tube.

6) The tube walls shall be heavy enough so that no sound energy is lost by their vibration. For steel or brass tubes of 3 inch I.D. or less, 1/4 inch wall thickness is sufficient. For larger tubing, thicker walls and a sand jacket are recommended.

7) The microphone or probe tube as well as any supporting structure inside the tube shall not have a cross sectional area greater than 5 per cent of the cross sectional area of the tube.

8) The microphone probe tube shall not have a wall thickness less than 1/8 of its outside diameter.

9) The microphone output shall be filtered.

10) The sound source shall be mounted directly on the end of the tube or to a 45 or 90 degree elbow.

*This is to avoid near field distortion

**This is to minimize the effects of higher order transverse modes of vibration.
11) The electronic equipment shall be stable.

12) The measuring system shall be capable of determining the microphone location to within \( \pm 0.2 \) millimeter.*

13) The seal between the sample and the end of the tube shall be as airtight as possible.

14) The sample shall not be mounted so that it is distorted or warped.

A specific apparatus described in this report employs a long, movable, microphone probe that extends through the center of the source and traverses the axis of the tube. If such a microphone-probe configuration is used, a correction factor must be applied to obtain the true location of the pressure minima. This is due to the fact that the probe, and hence the microphone, respond to sound pressures a short distance away from the probe mouth instead of directly at the mouth. In other words, probe end effects must be considered.

Summarized in the A.S.T.M. report are the developments made to date in the design and operation of standing wave or impedance tubes. Berendt and Schmidt [7] incorporated the techniques embodied in this report to design a portable standing wave tube that was used for absorption measurements of acoustic material in the field. They reported that very good results have been obtained in the 400 to 900 Hz region.

The measurement of impedance by the standing wave method has been improved upon over the years but basically has not been changed since its conception. The other methods outlined above have merit but have not

---

*For a frequency range of 125 to 4000 Hz.
surpassed the standing wave method of analysis.

In general the apparatus and techniques previously mentioned have been involved with the measurement of the absorption and/or impedance of acoustic materials and closed cavity type elements. In these instances the acoustic energy has been absorbed and reflected by the sample or element and has not been transmitted past it. This is not the case, however, when flow-through acoustic filters or filter elements are considered, as will be shown in the next section.

E. The Analysis of Acoustic Filters and Filter Elements

One of the first investigators (1922) to consider the analysis of acoustic filters was Stewart [55]. He derived the impedance relationships for several filters that consisted of series and parallel combinations of like elements. This was accomplished by using a lumped parameter approach. From the formulas thus obtained, Stewart constructed three basic filters and measured their transmission characteristics by means of an impedance bridge. He stated that agreement between experimental and theoretical data was within 8 to 13 per cent.

The three filters constructed by Stewart consisted of:

1) A low frequency filter that attenuated sound waves below a certain frequency;
2) A high frequency filter that attenuated sound waves above a certain frequency,
3) A single band filter that attenuated sound waves with several frequencies.
The first two filters had rather sharp cut-off frequencies while the third had cut-off frequencies that were not well defined. Stewart was primarily interested in these cut-off frequencies and was not too concerned about the overall transmission characteristics of the filters. Stewart concluded that the attenuation of sound by such filters was not due to dissipation but to interference. He claimed that the high attenuation obtained by the combination of relatively few elements was remarkable.

Mason [33] developed equivalent electrical networks for acoustic filters using lumped parameter approximations borrowed from electrical transmission line theory. He claimed that it is possible to determine the physical dimensions of an acoustic filter having given attenuation and impedance characteristics from such networks. With the exception of two expansion chambers, the filters considered by Mason were all of the side branch configuration.

Miles [35, 36, 37, 38] presented a rigorous theoretical consideration of various plane discontinuities in a series of four publications. The first publication [38] dealt with the reflection of sound from a discontinuity formed by a change of cross sectional area in a circular tube. The expressions for the pressure distribution in the vicinity of the discontinuity were presented as well as the previously mentioned effects of higher order transverse modes of vibration. Expressions for the reflected and transmitted waves at the discontinuity plus the reflection and transmission coefficients of the discontinuity were also developed. Miles' second publication [35] presented the same theoretical data mentioned above for the same type of discontinuity using an electrical transmission line analogy. In his third publication [36] he applied this
method to windows and changes of cross sectional area in both circular and rectangular tubes. The calculation of resonance in cavity type resonators was also discussed. Miles' fourth publication [37] gave a numerical example of the diffraction of plane waves by a right angle joint in a rectangular duct. No experimental verification of the theoretical results presented in these four publications has been made by Miles.

Lippert [31, 32] has experimentally measured the sound reflected and transmitted by right angle bend configurations in rectangular tubing. A complete description of the theory, apparatus, technique, and results was presented in two consecutive publication.

Lippert's apparatus consisted of two 3-inch square brass tubes 6-feet long; joined at a right angle. A sound source was located at one end and an anechoic* termination at the other. The anechoic termination was used so that only the transmitted wave existed downstream from the bend. The sound field was examined in either tube by means of a long probe attached to a traveling microphone.

Lippert calculated the complex reflection and transmission factors** of the bend from magnitude and phase measurements of the reflected and transmitted waves. He then used these complex ratios to determine the impedance matrix of the bend. Lippert stated that the reflection and transmission factors are more useful acoustic characteristics for filter analysis than is the impedance.

*An anechoic termination is non-reflecting, i.e., it absorbs all of the incident acoustic energy.

**The concept of reflection and transmission factors is described in detail in Section III.
Lippert's experimental results indicated that the characteristics of the right angle bend were accurately predicted by Miles [37]. Lippert also presented data obtained from measurements of the sound reflected and transmitted by a window placed in the input plane of the bend.

Davis, Stokes, Moore, and Stevens [13] performed a theoretical and experimental investigation of 77 different muffler configurations using plane wave theory. Experimental measurements were made using a loudspeaker source, a standing wave tube, and an anechoic termination. The standing wave on the inlet side of the muffler was examined by a sliding microphone mounted in a standing wave tube. The sound wave transmitted by the muffler was measured by three stationary microphones located at various positions along the outlet tubing leading to the anechoic termination. Two different sizes of this apparatus were constructed: one of 3-inch diameter, and a second with 12-inch diameter. Only three of the mufflers tested had 12-inch diameter inlets and outlets.

"The types of mufflers on which the most extensive tests were made are the single expansion chamber, the multiple expansion chamber, the single resonator, and the multiple resonator. ... A few mufflers were constructed of combinations of the above types. In addition, side-branch tubes with one end closed were investigated."

The attenuation of these mufflers was measured over a frequency range of 50 to 750 Hz. The measured attenuation was in close agreement with the attenuation predicted by plane wave theory up to 700 Hz. At this point higher order transverse modes of vibration began to propagate. The authors also considered the effects of steady flow through the mufflers and performed engine tests of several selected configurations. A general muffler design procedure along with three sets of design
curves showing the attenuation characteristics of anechoically terminated mufflers was presented. In regard to muffler design the authors stated:

"Even with the assistance of the information presented in this report, it is likely that a certain amount of trial and error will be necessary in muffler design when the goal is a very highly efficient muffler in terms of attenuation per unit of weight or volume."

Igarashi and Toyama [26] have theoretically and experimentally investigated the attenuation characteristics of several acoustic filters terminated anechoically. The filters consisted of expansion chambers, resonators, and combinations of the two. The effect of sound absorbing material placed within the filters was also considered. The authors used four terminal network theory to calculate the attenuation characteristics of the filters. The sound field was considered one dimensional, i.e., plane waves were assumed to exist. Inlet and outlet sound pressure and particle velocity were used as variables. The experimental measurements were made with the filter terminated anechoically at both ends. Fitting connected a sound source and fixed microphone to the filter's inlet and outlet respectively. The anechoic terminations, sound source, and microphone were packed in sand to prevent any system vibrations. The data output was automatically recorded on magnetic tape. The experimentally determined attenuation characteristics were in good agreement with the theoretically predicted values except at higher frequencies. In this region, the one dimensional theory was no longer applicable due to the presence of higher order transverse modes of vibration. In the second part of this report, Miwa and Igarashi [39] considered the three dimensional effects in greater detail.

Three methods of analyzing the performance of a filter or muffler were given
by Beranek [4]. These are "insertion loss (IL)", "transmission loss (TL)", and "noise reduction (NR)".

Insertion loss is defined as the difference in sound pressure magnitude measured before and after the insertion of the filter. The filter is located between the sound source and the point where the sound pressure measurements are made. The insertion loss assumes that the sound attenuation which is true if the source impedance is infinite. However, if the source impedance is finite, the insertion of a filter into the system will affect the output of the source. In this case the attenuation is caused not only by the filter but by the source as well.

The transmission loss is defined as the ratio of the sound energy incident on the filter inlet to the energy transmitted by the filter at the filter outlet. One of the disadvantages of the method is that it is difficult to separate the incident sound energy from the sound energy reflected by the filter. This can be done by indirect means however. Measurement of the transmitted energy is accomplished by terminating the filter anechoically. Of the three methods presented here, the transmission loss method of analysis gives the most accurate description of a filter's performance.

Noise reduction is defined as the difference between sound pressure magnitudes measured at the inlet and outlet of the filter. This method does not account for the reflections produced by the filter. Hence, the suitability of this method for describing a filter's performance is questionable. Noise reduction is also known as "sound power level difference" and "end differences".

Gatley [17] presented general methods for the design and evaluation of small acoustic filters used in the suction and discharge lines of refrigeration systems.
The design method required that the reflection and transmission characteristics of the filter's component elements be known. Once these characteristics had been established, the overall characteristics of the filter were determined using plane wave theory. This was accomplished by combining the elements in a sequential manner. This procedure took the first two elements of the filter and determined the overall characteristics of the pair. This pair was then combined with the next filter element and the process repeated. This was carried out until all of the filter elements had been included. Gatley also gave a theoretical analysis of plane discontinuities and expansion chambers.

Gatley presented three different methods for experimentally determining reflection factors. One of these is the standing wave method which he recommended as best. For accurate phase measurements, Gatley used two standing wave tubes of different lengths. The shorter of the two was used for high frequencies where the sound pressure minima are relatively close together. Transmission factors were determined by combining the reflection factors obtained from standing wave measurements with transmission measurements obtained from a separate apparatus which measured the sound pressure on both sides of the discontinuity or filter.* Both the reflection and transmission factor measurements were made with the system terminated anechoically to simplify the calculations.

Experimental reflection and transmission factors were determined for plane discontinuities and expansion chambers over a frequency range from 300 to 5000 Hz. These agreed well with the results predicted by plane wave theory.

*With the exception of a third standing wave tube, Gatley's apparatus was designed for measurements in 0.5-inch diameter tubing. This corresponds to the tubing sizes found in small refrigeration systems.
at the lower frequencies but not at the higher frequencies where the effects of higher order transverse modes of vibration become pronounced. As mentioned previously, these effects are most apparent in the phase angle data. The effects of microphones and microphone probes placed in the sound field were discussed as well as the general problems encountered in the experimental measurements.

Gatley also experimentally determined the characteristics of two prototype production mufflers and predicted their performance in an operating refrigeration system using equations developed from the plane wave theory. An experimental refrigeration system, using air or R-22 as a circulating medium, was built in order to experimentally measure the performance of the mufflers. Gatley stated that agreement between predicted and measured values was very good. Problems involved in measurement of the mufflers' performances due to system operating pressures were mentioned.

Buckley [9] has constructed and evaluated two standing wave tubes and corresponding transmission apparatus for the plane wave analysis of mufflers. The tubes were of 2-inch and 0.5-inch diameter corresponding to the tubing sizes found in automotive exhaust systems and small refrigeration systems respectively. Buckley described in detail the theoretical and experimental methods of analysis which were similar to Gatley's [17]. Using the small standing wave tube, Buckley obtained the reflection and transmission factors of a variable length expansion chamber over a frequency range from 300 to 4000 Hz. He also derived the theoretical expressions for the reflection and transmission factors of the expansion chamber. The experimental and theoretical characteristics were in
good agreement until the effects of higher order transverse modes of vibration were encountered at the higher frequencies. As mentioned previously, these discrepancies were most apparent in the phase angle data. The 2-inch apparatus was used to evaluate the acoustic characteristics of a commercially available automobile muffler which did not lend itself to straightforward theoretical analysis. Buckley discussed this experimental analysis and compared the results so obtained to a noise spectrum of the same type of muffler measured under actual operating conditions on an automobile.

Buckley's small standing wave tube and transmission apparatus, with certain modifications was used in this investigation to determine the acoustic characteristics of several filter elements. The apparatus and its modifications are described in greater detail in Section V.

Many of the apparatus design specifications and filter analysis techniques presented in the literature review have been incorporated in this investigation.
III. THEORETICAL DETERMINATION OF ACOUSTIC FILTER CHARACTERISTICS

A. Derivation of the Wave Equation

The acoustic wave equation is a second order partial differential equation that expresses the mechanism of sound propagation in a medium. It is obtained by the combination of the Equation of Motion, the Continuity Equation, and the Gas Laws.

In this derivation the medium is a fluid subject to the following restrictions:

1) the fluid is ideal, i.e., it obeys the perfect gas laws and no dissipative forces such as viscosity are present;

2) the fluid is homogenous and isotropic.

Other assumptions made in the derivation are:

3) adiabatic and reversible compression and expansion of the fluid;

4) the acoustic pressure $p$ is much less than the static pressure $P_0$;

5) the incremental density $\rho$ is much less than the static density $\rho_0$;

6) the steady flow velocity of the medium is less than or equal to the particle velocity.

The assumption of an ideal fluid disregards the effect of shear forces produced by viscosity; however, the viscosity of air is small and hence the shear forces that exist are usually small in comparison to the pressure forces and can be neglected. If the shear forces become an important parameter, they may be accounted for by an attenuation factor.

For low pressures and moderate temperatures the behavior of air is predicted to a fair degree of accuracy by the perfect gas laws. This concept is presented in any basic text in Thermodynamics.
The assumption that the fluid is homogenous and isotropic simplifies
the calculations and is valid for air at room temperature and pressure. If the
medium were, for example, sea water with a variation in salinity with position,
then this assumption would be violated. This assumption also involves neglect-
ing the effect of gravitational forces.

From Thermodynamics it is known that rapid compression results in an
increase in temperature of the fluid and conversely rapid expansion results in
a decrease in temperature of the fluid. A sound wave traveling through a medium
produces rapid expansions and compressions of the medium about the ambient
pressure and hence fluctuations of the temperature about the ambient tempera-
ture. These fluctuations are in phase with the sound pressure and occur at the
same frequency as the sound wave. It has been shown [2, 3] that the speed of
propagation of a thermal diffusion wave in the audible frequency range is much
slower than the speed of propagation of a sound wave. Therefore, there is
Insufficient time for significant heat exchange and the alternating expansions
and compressions may be considered adiabatic. For small pressure fluctuations
about the ambient pressure, the expansions and compressions may also be con-
sidered reversible. Since the expansions and compressions are assumed adia-
Batic and reversible, they are isentropic.

The assumptions that the acoustic pressure and incremental density
are small in comparison to their corresponding static values constitutes an
upper limit for the sound pressure if the wave equation is to be valid. Beranek [3]
suggests a sound pressure limit of 110 db ($p_{\text{ref}} = 0.0002 \mu\text{bar}$) which is approx-
imately 0.006 per cent of the ambient pressure. This is considered a very
conservative limit. This restriction is also necessary for the isentropic relations to be valid since large rapid pressure fluctuations about the ambient pressure could not be considered reversible.

The effect of steady flow may be neglected if the steady flow velocity of the medium is less than or equal to the particle velocity resulting from the acoustic pressure perturbation [58].

In this derivation a particle will be considered as an infinitesimal mass of fluid which possesses constant values of pressure, density, and velocity throughout its volume.

The notation is as follows:

- \( x, y, z \) Cartesian coordinates of a particle in the fluid
- \( q', r', s' \) fluid particle displacement in the \( x, y, z \) directions respectively
- \( u, v, w \) fluid particle velocity in the \( x, y, z \) directions respectively, i.e.,
  \[ u = \frac{\partial q'}{\partial t}, \quad v = \frac{\partial r'}{\partial t}, \quad w = \frac{\partial s'}{\partial t} \]
- \( p' \) instantaneous pressure = \( p_0 + p \)
- \( p_0 \) static pressure
- \( p \) incremental or acoustic pressure
- \( \rho' \) instantaneous mass density = \( \rho_0 + \rho \)
- \( \rho_0 \) static mass density
- \( \rho \) incremental mass density
- \( \gamma \) ratio of specific heats \( \left( \frac{c_p}{c_v} \right) = 1.4 \) for air

Consider a fluid volume element (Fig. 1) of dimensions \( dx, dy, \) and \( dz \). The Momentum Equation, or the Equation of Motion, states that the summation of forces in a particular direction equals the time rate of change of momentum in
Fig. 1 Fluid Volume Element
that direction. In the x direction, the net force acting to accelerate the volume element is

\[ \left[ P' - (P' + \frac{\partial P'}{\partial x} \, dx) \right] \, dydz = -\frac{\partial P'}{\partial x} \, dx \, dydz. \]

The time rate of change in momentum in the x direction is

\[ \frac{\partial (\rho'u \, dx \, dydz)}{\partial t} \]

hence the summation of forces in the x direction yields

\[ -\frac{\partial P'}{\partial x} = \frac{\partial (\rho'u)}{\partial t}. \]

Now \( P' = P_0 + p \) and

\[ \frac{\partial P'}{\partial x} = \frac{\partial P}{\partial x} + \frac{\partial p}{\partial x} \]

therefore

\[ -\frac{\partial P'}{\partial x} = -\frac{\partial p}{\partial x} = \frac{\partial (\rho'u)}{\partial t}. \] (1a)

Likewise in the y and z directions we obtain

\[ -\frac{\partial p}{\partial y} = \frac{\partial (\rho'v)}{\partial t} \] (1b)

\[ -\frac{\partial p}{\partial z} = \frac{\partial (\rho'w)}{\partial t}. \] (1c)

Differentiating (1a) with respect to x, (1b) with respect to y, (1c) with respect to z, and adding yields

\[ - \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) p \overset{\partial}{\rightarrow} \frac{\partial}{\partial t} \left[ \frac{\partial (\rho'u)}{\partial x} + \frac{\partial (\rho'v)}{\partial y} + \frac{\partial (\rho'w)}{\partial z} \right]. \]

(2)

The Continuity Equation states that the net influx of mass into the volume element equals the rate of mass increase within the element. The net mass
influx into the element due to flow in the x direction is

\[ [p'u - (\rho' + \frac{\partial(\rho' u)}{\partial x}) \, dx] \, dydz = - \frac{\partial(\rho' u)}{\partial x} \, dx \, dy \, dz. \]  

(3a)

Similarly in the y and z directions:

\[ [p'v - (\rho' + \frac{\partial(\rho' v)}{\partial y}) \, dy] \, dz \, dx = - \frac{\partial(\rho' v)}{\partial y} \, dy \, dz \, dx. \]  

(3b)

\[ [p'w - (\rho' + \frac{\partial(\rho' w)}{\partial z}) \, dz] \, dx \, dy = - \frac{\partial(\rho' w)}{\partial z} \, dz \, dx \, dy. \]  

(3c)

The rate of mass increase within the element equals

\[ \frac{\partial}{\partial t} (\rho' \, dx \, dy \, dz). \]  

(3d)

Summing (3a), (3b), (3c), equating to (3d), and simplifying yields

\[ \frac{\partial(\rho' u)}{\partial x} + \frac{\partial(\rho' v)}{\partial y} + \frac{\partial(\rho' w)}{\partial z} = - \frac{\partial \rho'}{\partial t}. \]  

(4)

Comparison of (4) with (2) shows that the right hand side of (2) is equal to the partial derivative with respect to time of (4). Hence (2) reduces to

\[ \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) p = \frac{\partial^2 \rho'}{\partial t^2}. \]

or

\[ \nabla^2 p = \frac{\partial^2 \rho'}{\partial t^2}. \]  

(5)

where \( \nabla = \) the del operator in Cartesian coordinates.

Now

\( \rho' = \rho_0 + \rho \)

and

\[ \frac{\partial \rho'}{\partial t} = \frac{\partial \rho_0}{\partial t} + \frac{\partial \rho}{\partial t}. \]
Therefore

\[
\frac{c^2 \rho'}{\alpha t^2} = \frac{\partial^2 \rho}{\partial t^2}.
\]  

(6)

For an isentropic process, the Ideal Gas Law yields

\[
\frac{P'}{(\rho')^\gamma} = \text{constant}.
\]

Differentiation yields

\[
\frac{dP'}{P'} = \gamma \frac{d\rho'}{\rho'}.
\]  

(7)

Now \( P' = P_0 + p, \rho' = \rho_0 + \rho \), and since it is assumed that \( \rho << \rho_0 \) and \( p << P_0 \), we have

\[
dP' = p \quad d\rho' = \rho
\]

\[
P' = P_0 \quad \rho' = \rho_0.
\]

Therefore substituting the above in (7) and rearranging yields

\[
\rho = \frac{\rho_0 P}{P_0^\gamma}.
\]  

(8)

where \( \rho_0, P_0 \), and \( \gamma \) are constant.

Substituting (8) into (6) and substituting this result into (5) yields

\[
\nabla^2 p = \frac{\rho_0}{P_0^\gamma} \frac{\partial^2 p}{\partial t^2}.
\]  

(9)

Defining \( c^2 = \frac{\gamma P_0}{\rho_0} \) and substituting into (9) yields the three-dimensional wave equation in Cartesian coordinates

\[
\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}
\]  

(10)

where \( c \) is the propagation velocity of sound.
For plane waves the sound pressure is a function of one dimension and
time only. If we choose the independent dimension as $x$ and remove the $y$ and
$z$ dependency, (10) becomes the plane wave equation

\[
\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}.
\]  

(11)

One of the requirements for plane waves is that no transverse modes
of vibration are present. The concept of the existence of these higher modes
is presented by Rayleigh [47], Hartig and Swanson [24], and Miles [35, 36, 38].
They state that the vibrational modes of the sound wave are a function of the
size and shape of the boundaries containing the sound wave relative to its
wavelength.

Consider a tube of inner radius $r_0$ and extending to infinity in the $z$
direction. It is assumed that there is no attenuation or other form of energy
loss existing within the tube. For this case, the three dimensional wave
equation in cylindrical coordinates is

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}.
\]  

(12)

The transverse modes of vibration are defined as those oscillations that are
independent of $z$, hence, for the transverse dimensions, (12) becomes

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}.
\]  

(13)

The solutions of equations (12) and (13) are given by Hartig and Swanson [24].
The solution of (13) is the sum of terms of the type

\[
J_n(kr) \left[ A \cos n\theta + B \sin n\theta \right] \sin kct
\]  

(14)
where

\[ J_n(kr) = \text{Bessel's function of the first kind}; \]

\[ k = \frac{\omega}{c}; \]

A and B = arbitrary constants.

Here

\[ \omega = \text{circular frequency} = 2\pi f \]

\[ c = \text{propagation velocity of sound in the medium.} \]

At the wall of the tube, i.e., at \( r = r_0 \), the boundary condition is that the transverse velocity is zero. This implies

\[
\frac{\partial p}{\partial r} \bigg|_{r=r_0} = 0
\]

and therefore

\[ J'_n(kr_0) = 0 \tag{16} \]

Equation (16) establishes the condition for the existence of higher modes and defines the point at which the plane wave theory fails to completely describe the physical situation. This point is designated as the frequency \( f_c \). The solution of (12) is similar to (14) but contains in addition a \( z \) term of the form

\[ e^{j(\omega t + k\Psi z)} \tag{17} \]

where

\[ \Psi = \left[ 1 - \left( \frac{f}{f_c} \right)^2 \right] \frac{1}{2}. \tag{18} \]

Now suppose that a source producing a higher order transverse mode of vibration exists at the tube section \( z = 0 \). It is assumed that the frequency of this
source, \( f \), is constant and that steady state conditions exist throughout the tube. If \( f_c \) is less than \( f \) then the exponential term in (17) is imaginary and the higher mode propagates with undiminished magnitude as \( z \) approaches infinity. In this case the minus sign appearing in (17) is used. For \( f_c \) greater than \( f \), (18) is imaginary which upon substituting in (17) indicates that the higher mode is damped with increasing \( z \) and hence does not propagate. In this instance, the positive sign in (17) is used.

The first root of (16) is 3.832 which yields

\[
kr_0 = 3.832 \tag{19}
\]

Since \( k = \frac{\omega}{c} = \frac{2\pi f}{c} \), this can be rewritten as

\[
f = \frac{3.832 c}{2\pi r} \equiv f_c \tag{20}
\]

which upon substituting the relationships \( c = f \frac{\lambda_c}{c} \), \( d = 2r_0 \), and rearranging becomes

\[
\lambda_c = \frac{\pi d}{3.832} \approx 0.82 d. \tag{21}
\]

Therefore, (21) states that the first transverse mode of vibration will propagate when the driving frequency is such that the wavelength of sound in the tube is approximately equal to 0.82 times the diameter of the tube. This frequency defines the upper limit for the existence of plane waves.

B. Solution of the Plane Wave Equation

The general solution to the plane wave equation (11), according to Beranek [3], can be expressed as the sum of two terms

\[
p(x, t) = f(t - \frac{x}{c}) + g(t + \frac{x}{c}) \tag{22}
\]
where $f$ and $g$ are arbitrary functions. The only restriction placed on $f$ and $g$

is that they both have continuous first and second derivatives.

Equation (22) states that at any instant in time $t$ and at any point in space $x$ the sound pressure is composed of two pressure waves traveling in opposite directions. One of these waves, $f(t - \frac{x}{c})$, is traveling away from the source of disturbance while the other wave, $g(t + \frac{x}{c})$, is traveling towards the source of disturbance. This is a general solution and is tailored to fit the physical situation.

Consider an acoustic driver coupled to an infinitely long tube. For plane waves and neglecting the effect of attenuation, it is seen that a generated sound wave propagates with undiminished amplitude to infinity. In this case

$$p(x, t) = f(t - \frac{x}{c}) + 0. \quad (23)$$

There is no reflected wave (wave traveling towards the source). The same result is obtained in a finite tube if the tube is terminated anechoically.

Next consider a finite tube that has some form of discontinuity (such as a filter) somewhere along its length. A generated sound wave traveling away from the source and impinging upon the discontinuity experiences the following effects:

1) part of the incident wave is transmitted by the discontinuity;

2) part of the incident wave may be absorbed by the discontinuity;

3) part of the incident wave is reflected by the discontinuity.

In this case

$$p(x, t) = f(t - \frac{x}{c}) + g(t + \frac{x}{c}) \quad (22)$$
since part of the incident wave is reflected and is traveling towards the source.

The summation of two waves traveling in opposite directions, in the steady state, yields a "standing wave". At any given x location along the standing wave pattern, the maximum value of sound pressure is fixed, while the instantaneous value of sound pressure varies sinusoidally with time. If the x location is changed the only effect is that the maximum value of the sound pressure is changed. This is presented graphically in Fig. 2.

Equation (22) also points out the fact that a wave traveling away from or towards the source is propagated without change of shape regardless of its initial shape. This is assuming that there is no attenuation, scattering, dispersion, or any other form of physical interaction of the wave with the surroundings. This fact is illustrated [3] for a wave traveling away from the source by assuming a value of \( f(t_1) \) for the sound pressure at \( x = 0 \) and \( t = t_1 \). At \( t = t_1 + t_2 \), the wave has traveled a distance equal to \( x = t_2 c \). Substitution of the values of \( x \) and \( t \) at time \( t = t_1 + t_2 \) into (22) and noting that \( g(t + \frac{x}{c}) = 0 \) yields the value of the sound pressure as

\[
p(x, t) = f(t_1 + t_2 - \frac{t_2 c}{c}) = f(t_1). 
\]

Therefore the wave has propagated without changing its shape. The same argument can be used to show that the wave traveling towards the source also propagates without change of shape.

From the theory of Fourier series, a steady state wave can be represented by an infinite linear summation of sine wave terms, where each sine wave term corresponds to a particular frequency component of the wave.
Fig. 2 Sound Pressure Distribution in Standing Wave
Since the wave in this case is propagated without change of shape, in the steady state, only those solutions that are sinusodial and vary in time with the same frequency as the source need be considered.

Returning to (11), the sinusodial solution is obtained by the separation of variables technique and is expressed as

\[ p(x, t) = (H \sin \frac{\omega x}{c} + I \cos \frac{\omega x}{c}) (J \sin \omega t + K \cos \omega t) \]  

(25)

where the constants \( H \) and \( I \) are determined by the boundary conditions and the constants \( J \) and \( K \) by the initial conditions. In this case \( \omega \) equals the circular frequency or \( 2\pi f \).

Following the procedure of Gatley [17] and Buckley [9], the solution of (11) can also be expressed in exponential notation as

\[ p(x, t) = (A_1 e^{j\frac{\omega x}{c}} + B_1 e^{-j\frac{\omega x}{c}}) e^{j\omega t} \]  

(26)

or

\[ p(x, t) = (A_m e^{j\frac{\omega x}{c}} + B_m e^{j(\theta - \frac{\omega x}{c})}) e^{j\omega t} \]  

(27)

where (See Fig. 3)

\[ p(x, t) = \text{The complex value of the sound pressure at any location } x \text{ and time } t. \]

\[ A_1 = \text{The complex value of a sound pressure wave traveling in the negative } x \text{ direction and having magnitude } A_m = |A_1| \text{ and phase angle of zero degrees.} \]

\[ B_1 = \text{The complex value of a sound pressure wave traveling in the posi-} \]

* The validity of this solution can be proved by direct substitution into (11).
Fig. 3 Sound Pressure Wave Relationship
tive x direction having magnitude $B_m = |B_1|$ and a phase angle of $\theta$ with respect to $A_1$.

In this solution and in all further analytic considerations, the positive x direction is away from the discontinuity or filter and is towards the source. Therefore, recalling (22) it is seen that

$$f(t - \frac{x}{c}) = B_m e^{\frac{j(\theta - \omega x)}{c}} e^{j\omega t} \quad (28)$$

and

$$g(t + \frac{x}{c}) = A_m e^{\frac{j\omega x}{c}} e^{j\omega t} \quad (29)$$

C. Derivation of Reflection Characteristics

Consider a finite tube with an acoustic driver at one end and an arbitrary filter at the other. As mentioned previously, a sound wave impinging on the filter is transmitted, reflected, and absorbed; the degree of transmission, reflection, and absorption depends upon the type of filter. The wave incident on the filter is denoted by $A_1$, the transmitted wave as $A_2$, and the reflected wave as $B_1$.

When the filter is terminated anechoically there is only the transmitted wave $A_2$ propagating downstream of the filter and $B_2$ is zero. Therefore, a standing wave, composed of $A_1$ and $B_1$, exits on the source side of the filter while a single wave, $A_2$, propagates on the other side of the filter to the anechoic termination. This situation is depicted in Fig. 4.

We may now define a reflection factor $R$ which expresses the complex ratio of the reflected wave to the incident wave as

$$R = \frac{B_1}{A_1} = \frac{B_m e^{j\theta}}{A_m} \quad (30)$$
Fig. 4 Sound Pressure Wave Relationship for an Arbitrary Filter Terminated Anechoically
where $e^{j\theta}$ expresses the phase relationship between the reflected and incident waves.

Likewise we may define a transmission factor $T$ as the complex ratio of the transmitted wave to the incident wave such that

$$T = \frac{A_2}{A_1}$$

(31)

where $A_2$ is complex and has a definite phase relationship with $A_1$. It can be seen that terminating the filter anechoically simplifies the determination of the transmission factor, i.e., only $A_2$ must be considered.

Up to this point in the derivation, the effect of attenuation has been neglected. As stated previously, the attenuation effect may be expressed as an exponential relation involving the change in magnitude of a sound wave with distance. The propagation velocity of the sound wave is also affected. Modifying (27) to account for the effect of attenuation yields the expression for the sound pressure

$$p(x, t) = (A_m e^{ax} e^{j\frac{\omega}{c}x} + B_m e^{-ax} e^{j(\theta - \frac{\omega}{c}x)}) e^{j\omega t}.$$  

(32)

Removing $e^{\frac{j\omega x}{c}}$ from the parenthesis and rewriting yields

$$p(x, t) = [A_m e^{ax} + B_m e^{-ax} e^{j(\theta - \frac{2\omega}{c}x)}] e^{j\frac{\omega}{c}x} e^{j\omega t}.$$  

(33)

Since $p(x, t)$ is a complex quantity, the magnitude is obtained by taking the square root of the sum of the squares of its components. The magnitude of $p(x, t)$ at any given $x$ location represents a maximum value of sound pressure at that point which fluctuates sinusoidally with time. If we keep this in mind
and perform the above operations we obtain the magnitude of the sound pressure as a function of $x$ only. Thus the squared scalar magnitude of the sound pressure is

$$p(x) = p(x, t) = \frac{2}{A} e^{2\alpha x} + \frac{2}{B} e^{-2\alpha x} + 2A B \cos \left( \theta - \frac{2\omega}{c}x \right).$$  \hspace{1cm} (34)

The locations of the maximum and minimum pressures are found by taking the partial derivative of (34) with respect to $x$ and equating the result to zero. Thus,

$$\frac{\partial p(x)}{\partial x} = 0 = 2\alpha \left( \frac{2}{A} e^{2\alpha x} - \frac{2}{B} e^{-2\alpha x} \right) + \frac{\omega}{c} A B \cos \left( \theta - \frac{2\omega}{c}x \right).$$  \hspace{1cm} (35)

Assuming that the value of attenuation is small, then the second term is large compared to the first and we have

$$\sin \left( \theta - \frac{2\omega}{c}x \right) = 0$$  \hspace{1cm} (36)

which implies

$$\theta - \frac{2\omega}{c}x_{\max \min} = \pm n\pi , \hspace{0.5cm} n = 1, 2, 3, \ldots$$

or

$$\theta = \frac{2\omega}{c}x_{\max \min} \pm n\pi , \hspace{0.5cm} n = 1, 2, 3, \ldots$$  \hspace{1cm} (37)

If the tube attenuation is not negligible then $\theta$ may be found from

$$\sin \left( \theta - \frac{2\omega}{c}x \right) = \frac{\alpha \left( B e^{-2\alpha x} - A e^{2\alpha x} \right)}{\omega A \frac{1}{m} B \frac{1}{m}}$$  \hspace{1cm} (38)

which upon substituting the magnitude of the reflection factor becomes

$$\sin \left( \theta - \frac{2\omega}{c}x \right) = \frac{\alpha \left( |R| e^{-2\alpha x} - \frac{1}{|R|} e^{2\alpha x} \right)}{\omega}.$$  \hspace{1cm} (39)
It can be seen that \( \theta \) will be most affected by low values of \( |R| \) and high values of \( \alpha \) and \( x \). Gatley [17] has shown that for \( |R| = 0.3 \), and \( x = 120 \text{ cm} \), the error caused by use of (37) instead of (39) is on the order of 2 degrees. The error will be reduced for larger values of \( |R| \) and smaller values of \( \alpha \) and \( x \). It should be noted that the numerical example just presented is an extreme case and that the simplified equation (37) is valid in almost all instances.

Equation (36) also implies that

\[
\cos \left( \theta - 2 \frac{\omega x}{c' \text{max}} \right) = \pm 1. \tag{40}
\]

Substitution of (40) into (34) yields

\[
p(x_{\text{max}}) = p_{\text{max}} = \frac{A_m e^{2\alpha x} + B_m e^{-2\alpha x}}{2} \tag{41}
\]

or

\[
p(x_{\text{max}}) = p_{\text{max}} = A_m e^{\alpha x_{\text{max}}} + B_m e^{-\alpha x_{\text{max}}} \tag{42}
\]

(for n even)

and

\[
p(x_{\text{min}}) = p_{\text{min}} = A_m e^{\alpha x_{\text{min}}} - B_m e^{-\alpha x_{\text{min}}} \tag{43}
\]

(for n odd)

Multiplying (42) by \( e^{\alpha x_{\text{min}}} \), and (43) by \( e^{-\alpha x_{\text{max}}} \), subtracting the resulting equations, and solving for the reflected wave magnitude, \( B_m \), yields

\[
B_m = \frac{p_{\text{max}} e^{\alpha x_{\text{min}}} - p_{\text{min}} e^{-\alpha x_{\text{max}}}}{e^{\alpha x_{\text{max}}} - e^{-\alpha x_{\text{min}}}} \tag{44}
\]
Multiplying (42) by \( e^{-\alpha x_{\text{min}}} \), (43) by \( e^{-\alpha x_{\text{max}}} \), adding the resulting equations, and solving for the incident wave magnitude, \( A_m \), yields

\[
A_m = \frac{p_{\text{max}} e^{-\alpha x_{\text{min}}} + p_{\text{min}} e^{-\alpha x_{\text{max}}}}{e^{-\alpha x_{\text{max}} e^{-\alpha x_{\text{min}}} + e^{-\alpha x_{\text{min}}} e^{-\alpha x_{\text{max}}}}}.
\]

The absolute value or magnitude of the reflection factor is

\[
|R| = \frac{B_m}{A_m}
\]

which becomes upon substitution of (44) and (45)

\[
|R| = \frac{p_{\text{max}} e^{-\alpha x_{\text{min}}} - p_{\text{min}} e^{-\alpha x_{\text{max}}}}{p_{\text{max}} e^{-\alpha x_{\text{min}}} + p_{\text{min}} e^{-\alpha x_{\text{max}}}}.
\]

The reflection angle, or the phase angle between the incident and reflected wave, is given by (37), namely

\[
\theta = 2\frac{\omega c}{e^\alpha x_{\text{max}} - e^\alpha x_{\text{min}}} \pm n\pi, \quad n = 1, 2, 3, \ldots
\]

Inspection of equations (37) and (47) shows that the location and measurement of the maximum and minimum pressure is required for the evaluation of the complex value of the reflection factor. As will be mentioned later, this actually involves the location of two adjacent minimum pressures and the measurement of the magnitude of the maximum pressure which is located midway between. Often at low frequencies it is not possible to measure two adjacent minimum pressures. Therefore, an alternate method involving the location and measurement of one pressure minimum and one arbitrary pressure must
be considered. Recall equations (34) and (40),

\[
p(x)^2 = A_m e^{2\alpha x} + B_m e^{-2\alpha x} + 2A_m B_m \cos \left( \theta - 2\frac{\omega}{c'} x \right)
\]  

(34)

\[
\cos \left( \theta - 2\frac{\omega}{c'} \frac{x}{\min} \right) = \pm 1.
\]  

(40)

Expressing \( x \) as a relative location yields

\[
x = x_{\min} + x_{rx}
\]

(48)

where

\( x = \) arbitrary pressure location relative to the filter;

\( x_{\min} = \) minimum pressure location relative to the filter;

\( x_{rx} = \) arbitrary pressure location relative to the minimum pressure location.

The last term of (34) becomes upon substitution of (48)

\[
2A_m B_m \cos \left( \theta - 2\frac{\omega}{c'} x_{\min} - 2\frac{\omega}{c'} x_{rx} \right)
\]

(49)

At a pressure minimum, (40) assumes a value of \(-1\) or

\[
\cos \left( \theta - 2\frac{\omega}{c'} \frac{x}{\min} \right) = -1
\]

(40)

hence

\[
\theta - 2\frac{\omega}{c'} \frac{x}{\min} = \pi.
\]

(50)

Substitution of (50) into (49) yields

\[
2A_m B_m \cos \left( \pi - 2\frac{\omega}{c'} x_{rx} \right).
\]

(51)
Defining
\[ \phi = 2 \omega_{x} e^{x_{x}} \]  
(52)
and substituting into (51) gives (34) the form
\[ p^{2}(x) = A_{m} e^{2\alpha x} + B_{m} e^{-2\alpha x} + 2A_{m} B_{m} \cos \phi. \]  
(53)
Recall equation (43)
\[ p_{min} = A_{m} e^{\alpha x_{min}} - B_{m} e^{-\alpha x_{min}} \]  
(43)
which upon rearranging yields
\[ B_{m} = A_{m} e^{\alpha x_{min}} - P_{min} e^{-\alpha x_{min}}. \]  
(54)
Substituting (54) into (53) gives the value of the arbitrary pressure, \( p(x) \), as
\[ p(x)^{2} = A_{m}^{2} e^{2\alpha x_{min}} + 2e^{2\alpha x_{min}} - x_{min} + 2e^{2\alpha x_{min}} \cos \phi + \]  
\[ A_{m}^{2} \left[ -2P_{min} e^{\alpha x_{min}} \cos \phi + e^{\alpha x_{min}} \right] + \]  
\[ [P_{min}^{2} e^{\alpha x_{min}} - x_{min}] \]  
(55)
which is a quadratic equation in \( A_{m} \). Applying the quadratic formula yields
\[ A_{m} = \frac{-b + \sqrt{b^{2} - 4ac}}{2a} \]  
(56)
where
\[ \overline{a} = [e^{2\alpha x_{min}} + e^{\alpha x_{min}} - x_{min} + 2e^{\alpha x_{min}} \cos \phi] \]  
(56a)
\[ b = \left[ -2p_{\text{min}} \right] \left[ e^{\alpha x_{\text{min}}} \cos \phi + e^{\alpha(3x_{\text{min}} - 2x)} \right] \]

\[ c = p^2_{\text{min}} \left[ e^{2\alpha(x_{\text{min}} - x)} - p^2(x) \right] \]  

Since \( A_m \) is the magnitude of the incident sound wave, it cannot be negative and hence only the positive root of (56) is applicable. If the value of attenuation is small, the exponential terms approach one and (56) reduces to

\[ A_m = \frac{1}{2} \left[ p_{\text{min}} + \left( p^2_{\text{min}} - 2 \left( \frac{p^2_{\text{min}}}{1 - \cos \phi} \right) \right)^{1/2} \right] \]

Substituting the expression for \( B_m \) from (54) into the definition of the reflection factor magnitude, \( |R| = \frac{B_m}{A_m} \), yields

\[ |R| = \frac{A_m e^{\alpha x_{\text{min}}} - p_{\text{min}} e^{\alpha x_{\text{min}}}}{A_m} \]

where \( A_m \) is determined from (52) and (56). The phase angle associated with \( R \) is, as before, found from (37).

D. Derivation of Transmission Characteristics

One of the most important characteristics of an acoustic filter is the transmission factor. The reflection factor is necessary for the determination of the transmission factor but by itself can yield only a limited evaluation of a filter's overall performance.

The transmission factor is determined by combining the complex value of the reflection factor with the values of, and relative phase angle be-
tween, the sound pressures obtained at known locations on either side of the filter.*

One of the most convenient methods for operating with complex numbers is to express all complex quantities as phasors. Using this procedure, the transmission factor can be calculated graphically as shown in Fig. 5. Phasor notation also lends itself very well to digital computer techniques.

The graphical calculation of the transmission factor begins by assuming that the value of the incident wave at the filter inlet, denoted by \( A_1 \), has a value of one at an angle of zero degrees. The reflected wave at the filter inlet, \( B_1 \), is then obtained from the complex product of the reflection factor and \( A_1 \). At location \( x_1 \) the incident wave \( A'_1 \) has a positive phase angle of \( \frac{\omega}{c} x_1 \) and due to attenuation has a magnitude greater than \( A_1 \) of \( |A'_1| = |A_1| e^{\alpha x_1} \).

Similarly, the reflected wave at \( x_1 \), \( B'_1 \), has a negative phase angle \( \frac{\omega}{c} x_1 \) and has a magnitude less than \( B_1 \) of \( |B'_1| = |B_1| e^{-\alpha x_1} \). Rotating \( A'_1 \) and \( B'_1 \) through the proper phase angle, correcting their magnitudes for the effect of attenuation, and adding them vectorially yields the total sound pressure vector, \( C_1 \), at location \( x_1 \). The measured values of total sound pressure at locations \( x_1 \) and \( x_2 \) (which is downstream of the filter outlet) are \( P_1 \) and \( P_2 \) respectively; hence multiplying \( C_1 \) by the ratio \( \frac{P_2}{P_1} \) and rotating the result through the relative phase angle between \( P_2 \) and \( P_1 \) yields the total sound pressure vector, \( C_2 \), of the transmitted wave at location \( x_2 \). Since the filter is terminated anechoically the total sound pressure at \( x_2 \) is due solely to the transmitted wave. To find \( A_2 \), the value of the transmitted wave at the filter

* This method is also described by Gatley [17] and Buckley [9].
A_1 = Incident Wave at Filter Inlet

B_1 = Reflected Wave at Filter Inlet

A'_1 = Incident Wave at x_1

B'_1 = Reflected Wave at x_1

C_1 = Total Sound Pressure at x_1

C_2 = Total Sound Pressure at x_2

A_2 = Transmitted Wave at Filter Outlet

T = \frac{A_2}{A_1} angle \Theta_T

Fig. 5 Graphical Determination of the Transmission Factor for an Arbitrary Filter
outlet, C₂ is rotated through the positive phase angle $\frac{\omega}{c} x_2$, and corrected for attenuation by increasing its magnitude by $e^{\alpha x_2}$, i.e., $|A_2| = |C_2| e^{\alpha x_2}$.

Since A₂ and A₁ are now known, the complex value of the transmission factor, T, can be determined by the use of equation (31)

$$T = \frac{A_2}{A_1}.$$ (31)

Thus the reflection and transmission characteristics of an arbitrary acoustic filter have now been established.

E. Theoretical Characteristics of a Plane Discontinuity

Up to this point in the discussion the acoustic filter has always been assumed to have an arbitrary configuration. The theoretical characteristics of some specific configurations will now be considered, namely, a plane discontinuity and an expansion chamber.

A plane discontinuity may be thought of as an abrupt change in cross-sectional area at a given x location. The inlets and outlets of expansion chambers and mufflers are good examples of plane discontinuities. An illustration of a plane discontinuity and the notation involved is illustrated in Fig. 6. The incident, reflected, and transmitted waves at the discontinuity are denoted by A₁, B₁, and A₂ respectively. In this case the cross-sectional area change is from $S_1$ to $S_2$ where $S_2$ is greater than $S_1$.*

Once a steady-state condition has been reached, the average energy flow in the incident wave equals the average energy flow in the reflected wave plus the average energy flow in the transmitted wave plus the average energy dissipated.

*The following procedure is applicable regardless of the relative magnitudes of $S_2$ and $S_1$. 
Fig. 6 Configuration and Sound Pressure Wave Relationship for a Plane Discontinuity Terminated Anechoically
At a plane discontinuity the energy dissipated is theoretically zero so that

\[ E_{A_1} = E_{B_1} + E_{A_2}. \]  

(59)

It is also assumed that the discontinuity is terminated anechoically so that only the transmitted wave \( A_2 \) exists beyond the discontinuity.

We now define the volume velocity, \( q \), as

\[ q = uS \]  

(60)

where \( u \) is the particle velocity and \( S \) is the cross-sectional area. The particle velocity for a free plane progressive wave is

\[ u(x, t) = \frac{p(x, t)}{z_0} \]  

(61)

where \( z_0 \), the characteristic impedance, is given by

\[ z_0 = \rho_0 c \]  

(62)

\[ \rho_0 = \text{static density of the medium} \]

\[ c = \text{speed of sound in the medium}. \]

Two physical requirements for plane waves [17] at a plane discontinuity are:

1) \( p_1 = p_2 \);

2) the net volume velocity \( q_{1t} \) equals the net volume velocity \( q_{2t} \).

Requirement (1) states that the sound pressures over both cross sections are equal. This implies that

\[ A_2 = A_1 + B_1. \]  

(63)

It is also true that at a plane discontinuity there is no phase shift between the
incident, reflected, and transmitted waves. This fact allows us to write

\[ |A_2| = |A_1| + |B_1|. \]  

(64)

Requirement (2) implies that the particle velocity over the cross-sectional area \( S_2 \) is constant. This condition definitely does not exist and therefore requires that higher order transverse modes of vibration are present at the discontinuity [35, 38]. When the higher modes of vibration are neglected, the discrepancy between theoretical and experimental results becomes significant as the frequency is increased [17].

Using (60), (61), (62), and equating volume velocities yields

\[ \left( \frac{|A_1|}{\rho_0 c} - \frac{|B_1|}{\rho_0 c} \right) S_1 = \frac{|A_2|}{\rho_0 c} S_2 \]  

(65)

which reduces to

\[ |A_1| S_1 - |B_1| S_1 = |A_2| S_2. \]  

(66)

The complex reflection factor, \( R \), and the complex transmission factor, \( T \), reduce to scalar quantities since there is no phase shift at a plane discontinuity. Hence

\[ R = \frac{|B_1|}{|A_1|}. \]  

(67)

and

\[ T = \frac{|A_2|}{|A_1|}. \]  

(68)

Substituting the expression for \( |A_2| \) from (64) in (66), dividing the result by \( |A_1| \), and applying (67) yields the reflection factor as

\[ R = \frac{S_1 - S_2}{S_2 + S_1}. \]  

(69)
Substituting the expression for $|B_1|$ from (64) in (66), dividing the result by $|A_1|$, and applying (68) yields the transmission factor as

$$T = \frac{2S_1}{S_2 + S_1}.$$  \hspace{1cm} (70)

If the same analysis is applied to a plane discontinuity having a cross-sectional area decrease from $S_2$ to $S_1$, instead of an increase from $S_1$ to $S_2$, the reflection and transmission factors are found to be

$$R = \frac{S_2 - S_1}{S_2 + S_1},$$  \hspace{1cm} (71)

$$T = \frac{2S_2}{S_2 + S_1}.$$  \hspace{1cm} (72)

Inspection of (69), (70), (71), and (72) shows that, for plane waves at a plane discontinuity, the reflection and transmission factors are independent of frequency and are a function of the cross-sectional area ratios only.

F. Theoretical Characteristics of an Expansion Chamber

An expansion chamber consists of two plane discontinuities with a connecting tube in between. In this consideration, the expansion chamber consists of a plane discontinuity having a cross-sectional area increase of $S_1$ to $S_2$, a connecting tube of constant cross-sectional area $S_2$ and length $L$, and a plane discontinuity having a cross-sectional area decrease from $S_2$ to $S_1$. The expansion chamber, corresponding notation, and previously developed reflection and transmission characteristics for the plane discontinuities are presented in Fig. 7.
$S_1 =$ Internal Cross Sectional Area of Inlet and Outlet

$S_2 =$ Internal Cross Sectional Area of Chamber

$$ R_1 = \frac{S_1 - S_2}{S_2 + S_1} = R_4 \quad R_2 = \frac{S_2 - S_1}{S_2 + S_1} = R_3 $$

$$ T_1 = \frac{2S_1}{S_2 + S_1} = T_4 \quad T_2 = \frac{2S_2}{S_2 + S_1} = T_3 $$

Fig. 7 Analysis of Simple Expansion Chamber
In all previous analyses it is assumed that anechoic conditions exist on the outlet side of the discontinuity, i.e., the discontinuity is terminated anechoically. If a second discontinuity is located after the first, as in the case of an expansion chamber, then the analysis becomes somewhat complicated. This is due to the fact that the second discontinuity affects the transmitted and reflected waves from the first discontinuity.

In the following analysis of an expansion chamber* it is assumed that anechoic conditions exist on the outlet side of the chamber and that there is no attenuation or absorption within the chamber itself.

Consider a sound wave, $A_1$, impinging on the inlet of the expansion chamber. A certain amount of $A_1$ is reflected by the first discontinuity and the rest is transmitted. The transmitted wave upon striking the second discontinuity undergoes reflection and transmission with the result that part of the wave leaves the chamber and the rest of the wave is reflected back to the first discontinuity where reflection and transmission occur again. Since it is assumed that there is no attenuation or absorption in the chamber, an infinite number of reflections and transmissions occur. Fig. 8 gives the values of several of the reflected and transmitted waves in terms of the previously developed reflection and transmission characteristics for plane discontinuities.

The total reflected wave, $B_{1t}$, of the expansion chamber is composed of that part of the incident wave, $A_1$, which is reflected by the first discontinuity

*The following derivation is due to Gatley [17] and Buckley [9].
Fig. 8 Internal Reflections in an Expansion Chamber
Terminated Anechoically
plus an infinite number of internally reflected waves which are transmitted by the first discontinuity. The total transmitted wave, $A_{2t}$, is composed of that part of the incident wave that is transmitted by both discontinuities plus an infinite number of internally reflected waves which are transmitted by the second discontinuity. Referring to Fig. 8 the total reflected wave is

$$B_{1t} = A_1 R_1 + A_1 T_1 R_3 T_2 e^{-j\frac{2\omega L}{c}} + A_1 T_1 R_3 T_2 R_2 R_3 e^{-j\frac{4\omega L}{c}}$$

$$+ A_1 T_1 R_3 T_2 R_2 R_3 R_3 e^{-j\frac{6\omega L}{c}} + \ldots$$  \hspace{1cm} (73)

Inspection of (73) indicates that all terms after the first have a common factor of $A_1 T_1 R_3 T_2$. Therefore, (73) may be expressed as

$$B_{1t} = A_1 R_1 + A_1 T_1 R_3 T_2 [1 + R_2 R_3 e^{-j\frac{2\omega L}{c}} + R_2^2 R_3^2 e^{-j\frac{4\omega L}{c}} + \ldots]$$  \hspace{1cm} (74)

If the substitution $Z = R_2 R_3 e^{-j\frac{2\omega L}{c}}$ is made, the result is

$$B_{1t} = A_1 R_1 + A_1 T_1 R_3 T_2 [1 + Z + Z^2 + \ldots]$$  \hspace{1cm} (75)

where the infinite geometric series $1 + Z + Z^2 + Z^3 + \ldots$ converges to

$$\frac{1}{1 - Z} \text{ if } |Z| < 1.$$  

From Fig. 7 and (71)

$$R_2 = R_3 = \frac{S_2 - S_1}{S_2 + S_1}$$  \hspace{1cm} (76)

which is less than one for all $S_2 > S_1$. Therefore, $R_2 R_3 e^{-j\frac{2\omega L}{c}} = Z$ is also less than one. Hence the total reflected wave is given by
The complex value of the reflection factor for the expansion chamber is

\[ R = \frac{B_{1t}}{A_1} \]. Therefore

\[ R = R_1 + \frac{-j\frac{\omega}{c}L}{1 - R_2R_3 e^{-j\frac{2\omega}{c}L}} \]. (78)

Again referring to Fig. 8, the total transmitted wave is

\[ A_{2t} = A_1 T_1 T_3 e^{-j\frac{\omega}{c}L} + A_1 T_1 R_3 R_2 T_3 e^{-j\frac{3\omega}{c}L} + A_1 T_1 R_3 R_2 R_3 R_2 T_3 e^{-j\frac{5\omega}{c}L} + \ldots \] (79)

which has a common term \( A_1 T_1 T_3 e^{-j\frac{\omega}{c}L} \). Therefore, (79) may be expressed as

\[ A_{2t} = A_1 T_1 T_3 e^{-j\frac{\omega}{c}L} \left[ 1 + R_2 R_3 e^{-j\frac{2\omega}{c}L} + R_2 R_3 e^{-j\frac{4\omega}{c}L} + \ldots \right] \]. (80)

It is seen that the series term is the same as before so that

\[ A_{2t} = \frac{A_1 T_1 T_3 e^{-j\frac{\omega}{c}L}}{1 - R_2R_3 e^{-j\frac{2\omega}{c}L}} \]. (81)

The complex value of the transmission factor for the expansion chamber is

\[ T = \frac{A_{2t}}{A_1} \]. Therefore
Inspection of (82) indicates that the transmission characteristics of an expansion chamber are a function of frequency, cross-sectional area ratio, and chamber length. Also from (82) it can be seen that maximum transmission, or resonance, occurs at

\[ 2 \frac{\omega}{c} L = n \pi, \quad n = 2, 4, 6, \ldots \]  

and that minimum transmission occurs at

\[ 2 \frac{\omega}{c} L = n \pi, \quad n = 1, 3, 5, \ldots \]  

Buckley [9] states that the magnitudes of the reflection factor and transmission factor recur for \( 2 \frac{\omega}{c} L \) greater than 2\( \pi \) while the values of the reflection phase angle and transmission phase angle recur for \( 2 \frac{\omega}{c} L \) greater than 2\( \pi \) and 4\( \pi \) respectively.

As was stated in the analysis of plane discontinuities, the subscripted reflection and transmission factors appearing in (78), (82), and Fig 7 have zero phase angles, in this case, and are regarded as scalars. They may be obtained from equations (69), (70), (71), and (72) or from experimental measurements. It should be noted that if these characteristics are obtained from experimental measurements their phase angles may not necessarily be zero. Equations (78) and (82), however, are valid for scalar or complex values of the subscripted reflection and transmission factors.
An important observation to make is that the reflection and transmission characteristics for an expansion chamber can be expressed in terms of the characteristics of its elements.

A computer program for evaluating the theoretical reflection and transmission characteristics of anechoically terminated expansion chambers is presented in Appendix 5.
IV. EXPERIMENTAL DETERMINATION OF THE ATTENUATION CONSTANT

A. Introduction

The literature review revealed several different theoretical and experimental values for the attenuation constant. These values were obtained by a variety of methods and under various conditions which often were not elaborated upon in the literature. Hence, it was decided to experimentally measure the values of the attenuation constant under a set of conditions that closely approximated the conditions that would exist when actual experimental measurements were made. A secondary objective of these measurements was to investigate the effects of different microphone probe configurations and sound intensity levels upon the attenuation constants.

In this investigation the attenuation constant was measured in a 0.430-inch I.D. hardened copper refrigeration tubing. This diameter corresponded to the diameter of the standing wave tube and transmission tube apparatus used for the experimental analysis of acoustic filter elements (See Section V). The measurements were made at various sound intensity levels over a frequency range from 500 to 3000 Hz.

B. Experimental Apparatus

Recall the expression for the attenuation of a plane sound wave propagating through a tube, namely:

\[ p_x = p_{0x} e^{-\alpha x} \]

where

- \( p_{0x} \) = sound pressure at \( x = 0 \)
- \( p_x \) = sound pressure at any \( x \) location
\[ e = \text{base of the natural logarithms} \]
\[ \alpha = \text{attenuation constant} \]

Solving for \( \alpha \) yields

\[ \alpha = \frac{1}{x} \ln \left( \frac{p_{0x}}{p_x} \right) \]

Hence, the attenuation constant may be obtained from the measurement of the plane sound wave magnitude at two locations in a tube separated by a known distance.

Two methods of measuring the attenuation constant in small refrigeration tubing were examined. Both methods required relatively long lengths of tubing. The long lengths were required to produce measurable attenuation and to permit the wave generated by the sound source (an acoustic driver) to become plane before it reached the points at which measurements were made. The lengths of the tubing were determined primarily by the frequency range over which the attenuation constant was to be measured. For a frequency range from 500 to 5000 Hz the overall length was approximately 30 feet.

The first method examined employed two microphone stations separated by a long length of copper tubing. One end of the tubing was attached to an acoustic driver and the other end was left open to the surroundings. The microphone stations, driver, and open end were each separated by a distance of approximately 10 feet. A "tone burst generator" was used to gate a preselected number of cycles of output from the frequency generator to the amplifier and driver.* The tone burst was of sufficient duration so that the driver reached a steady state operation.

*The model and serial numbers of all equipment used in this investigation are listed in Appendix 7.
The burst was terminated, however, before the sound wave reflected from the open end reached either microphone station. In this manner, the incident wave magnitude could be measured at each microphone station. Only one microphone was used when measurements were made. This was accomplished by measuring the sound pressure at one microphone station at a time. The microphone station that was not in use was sealed by a dummy microphone plug. The microphone and plug were then interchanged and the second sound pressure measurement made. The reason for this procedure was to eliminate the need for two "matched" microphones which are not only difficult to obtain but expensive as well. The sound pressure at each microphone station was measured by means of an oscilloscope.* The output from the microphone was passed through a sound analyzer with a filter before it was fed to the oscilloscope. This was done to eliminate any harmonics or other extraneous frequencies from the microphone output, since the sound wave generated by the driver was not always of singular frequency content. Since the sound pressures at two locations, and the distance between these locations were known, the attenuation constant could be calculated.

Although this method was potentially attractive on paper, it was not so in practice. The measurements were time consuming since the microphone and plug had to be interchanged for each run. Another undesirable feature of this method was that two microphone stations could introduce another experimental variable if they were not identical in all respects. For these reasons, this method was rejected in favor of the method described below.

*The storage feature of the oscilloscope was particularly helpful here since it allowed the oscilloscope trace to be "frozen".
The method adopted for the attenuation measurements* used one microphone station mounted on a long length of tubing that was terminated with a steel piston. This method was similar to the previously described method in the respect that the same electronic equipment and apparatus layout were used. The tone burst duration was adjusted so that the incident wave was terminated before the wave reflected from the steel piston reached the microphone station. Therefore, by the use of a storage oscilloscope, the incident and reflected wave magnitudes could be individually measured at the microphone station. The microphone output was passed through the sound analyzer and into the storage oscilloscope in the same manner described previously. A photograph of a typical oscilloscope trace is presented in Fig. 9. It was assumed that the steel piston had a reflection factor of 1.0 at an angle of zero degrees. This meant that the incident wave was reflected from the piston face without energy loss or change of phase. Since the distance from the microphone station to the steel piston was known, measurement of the incident and reflected wave magnitudes provided all of the data necessary for the calculation of the attenuation constant. It should be noted that the effective length over which the attenuation was measured was twice the length of the copper tubing between the microphone station and the piston.

The experimental apparatus incorporating two microphone stations was modified to measure the attenuation constant utilizing only one microphone station. This was accomplished by sealing the end of the tubing with a steel piston fitted with an "O" ring and sealing the microphone station nearest the driver with

*A method somewhat similar to this was proposed as a means of acoustic filter analysis by Gatley [17].
Incident Wave

Reflected Wave

Frequency  500 Hz
Solid Steel Piston Termination
(Photograph Courtesy of W.S. Gatley)

Fig. 9 Photograph of Storage Oscilloscope Trace Illustrating the Effects Produced by Attenuation in Tubing
a dummy microphone plug. It was determined that any interference caused by the plugged microphone station was negligible. A schematic of the apparatus is shown in Fig. 10.

The microphone stations used for the measurement of the attenuation constant* were originally built by Buckley [9]. They consisted of machined brass cavities, equipped with probes, that were soldered to the walls of 0.430-inch brass tubes. These assemblies were joined to the system by soldered refrigeration couplings. The microphone cavity acted as an adaptor between the microphone and the probe which extended through the wall of the tubing. The microphone that was used was of the ceramic crystal dynamic type and was lightly clamped** in the cavity by a set screw that was threaded through the side of the cavity. A slip fit existed between the microphone and the cavity. A pressure relief channel was machined into the side of the cavity to prevent any damage to the microphone should it be withdrawn from the cavity too rapidly. The bottom of the cavity was fitted with a felt washer approximately 1/32 inch thick which minimized the transmission of any structural vibrations from the microphone cavity to the microphone. A dimensioned drawing of a microphone station without the felt washer is presented in Fig. 11.

The microphone stations originally featured the probes as integral parts

*These microphone stations were also used for transmission measurements (See Section V).

**If the microphone was clamped too tightly there was a possibility that the casing of the microphone would be distorted. This would in turn affect the response characteristics of the microphone.
Fig. 10 Schematic of Attenuation Measurement Apparatus

1. Hardened Copper Refrigeration Tubing 0.430" I.D.
2. Brass Tubing 0.430" I.D.
3. Soldered Coupling
4. Microphone
5. Microphone Cavity with Probe
6. Microphone Cavity Plug
7. Steel Piston with "O" Ring Seal
8. 40 Watt Acoustic Driver
Fig. 11 Microphone Station Detail

Pressure Relief Channel

Probe (Press Fit)

Slip Fit for Microphone

Allen Head Setscrew

Microphone Cavity (Brass)

Soldered

Brass Tubing (Approximately 18" in Length for Attenuation Measurements Shortened to 8.892" for Transmission Measurements)
of the microphone cavities. Since different probe configurations were to be examined, the original probes were removed, leaving holes in the bottoms of the cavities and in the walls of the brass tubes. These holes were enlarged to allow the new probes to be press fitted into place. The dimensions of the probes used for the attenuation measurements will be specified later.

Since several different probe configurations were to be examined, a convenient means of removing the probe from the microphone cavity was required. This was accomplished by silver soldering one end of the probe to a brass washer; this end of the probe was flush with the top face of the washer. When the probe was in position, the washer rested on the bottom of the cavity. * The bottom of the microphone cavity used for attenuation measurements was not completely flat, as shown in Fig. 11, but had a slight conical depression** around the probe which extended almost to the cavity walls. The probe was removed by grasping the edge of the washer which projected into the cavity, with a pair of needle-nose pliers. When the felt washer was placed over the probe washer, the microphone rested upon a flat surface. The probe washer also acted as a stop which allowed the probe to be inserted an accurately known distance into the sound field.

C. **Experimental Procedure**

The frequencies at which attenuation measurements were made were

---

*This detail is omitted from Fig. 11.

**Before the microphone stations were used for transmission measurements the bottoms of the microphone cavities were machined flat and the probes (without washers) permanently press fitted into place as shown in Fig. 11 (See Section V for further detail).
500, 1000, 1500, 2000, 2500, and 3000 Hz. The sound intensity level at each of these frequencies was determined by the frequency generator output voltage which was set at values between 0.10 and 1.30 volts for each run. Four different probe configurations were investigated and will be described later.

The procedure involved in measuring the attenuation constant was relatively straightforward. The electronic equipment was allowed to stabilize by allowing a warm-up period of at least an hour. During this time the probe and microphone were inserted into the microphone cavity and all connections and seals checked for tightness. The frequency generator was then set for the desired output level and frequency. The output level was read from a voltmeter on the frequency generator and the output frequency determined by an electronic digital frequency counter.* The sound analyzer was then set to "all pass" (no filter) and the tone burst duration determined by observing the incident and reflected waves on the storage oscilloscope. The duration was adjusted so that the incident and reflected waves contained the maximum possible number of cycles and yet were still distinguishable. The number of cycles increased as the frequency increased. The sound analyzer was then placed in the 1/3 or 1/10-octave filter mode and the incident and reflected wave magnitude read from the oscilloscope trace. The 1/3-octave filter was used for frequencies below 2500 Hz while the 1/10-octave filter was used for measurements at 2500 and 3000 Hz. The 1/10-octave filter could not be used below 2500 Hz since its transient response time was too slow. The frequency generator

*The frequency counter was required due to the fact that the frequency generator scale did not give an accurate indication of the output frequency.
output level was then set at a new value and the measurement of the wave magnitudes were repeated. No further adjustments were required until the frequency was changed. At this time the tone burst duration and sound analyzer filter were reset.

As mentioned previously, four different probe configurations were examined. The probes were fabricated from 1/16-inch and 1/8-inch diameter* hardened brass tubing. The four configurations were:

1) 1/16-inch diameter probe, inserted in the sound field 1/16 inches;
2) 1/16-inch diameter probe, inserted in the sound field 1/3 of the copper tube diameter or 0.143 inches;
3) 1/8-inch diameter probe, inserted in the sound field 1/16 inches;
4) same probe as #3 only in this case the probe and microphone cavity were loosely packed with steel wool.

A separate microphone station was used for each of the two probe diameters. In this manner, each microphone station had to be fitted for only one probe diameter instead of two. The previously mentioned disadvantage of two microphone stations was not as pronounced in this case since all sound pressure measurements were made at one microphone station.

In addition to the four probe configurations, two sets of attenuation measurements were made with the probes removed, i.e., with only holes in the microphone cavity bottom and tube wall. The holes were a few thousandths

*The 1/16-inch diameter tubing had an outside diameter of 0.0635 inches and an internal diameter of 0.030 inches. The 1/8-inch diameter tubing had an outside diameter of 0.125 inches and an internal diameter of 0.095 inches.
of an inch larger than the outside diameters of the probes.

The results of the attenuation measurements are presented in Table I. Also appearing in Table I are the theoretical values of the attenuation constant obtained from the modified Kirchhoff formula namely:

\[ \alpha = 3.18 \times 10^{-5} \frac{f^{1/2}}{r} \text{ cm}^{-1} \]

This formula was previously discussed in Section II

D. Discussion of Results

The results obtained from the attenuation measurements were difficult to analyze. The primary reason for this was that the measurements were affected by variations in temperature and humidity. This was noticed when the results obtained from a given probe configuration on one day could not be duplicated on another.* This can be seen in Table I. The data were repeatable, however, if all measurements were made on the same day which suggested that the method of measuring the attenuation constant was reliable. Unfortunately, the laboratory in which the attenuation measurements were made was not air conditioned and hence the temperature and humidity conditions were impossible to control. For this reason, as many measurements as possible were made on the same day. The values of the attenuation constant obtained from measurements taken on the same day are denoted by matching asterisks in Table I.

Despite the fact that the attenuation measurements were influenced by temperature and humidity variations, the experimental values of the attenuation

*This difficulty was also experienced by Fay [15].
TABLE I

Measured Values of the Attenuation Constant in 0.430-inch I.D. Copper Tubing for Various Probe Configurations and Intensity Levels

(Note: All Values of $\alpha$ are $10^3$ cm, Matching Starred Values Indicate Data Taken on Same Day)

<table>
<thead>
<tr>
<th>Frequency Generator Output (Volts)</th>
<th>$\alpha$** Probe</th>
<th>$\alpha$** Probe</th>
<th>$\alpha$* Probe</th>
<th>$\alpha$* Probe</th>
<th>$\alpha$ Probe</th>
<th>$\alpha$* No Probe</th>
<th>$\alpha$* No Probe</th>
<th>Theoretical Value of Attenuation Constant</th>
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</thead>
<tbody>
<tr>
<td>Frequency = 500 Hz</td>
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<tr>
<td>0.50</td>
<td>2.93</td>
<td>3.19</td>
<td>3.11</td>
<td>3.42</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>0.75</td>
<td>3.05</td>
<td>3.19</td>
<td>3.11</td>
<td>3.42</td>
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<tr>
<td>1.00</td>
<td>3.05</td>
<td>3.19</td>
<td>3.05</td>
<td>3.42</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>1.30</td>
<td>3.05</td>
<td>3.19</td>
<td>3.05</td>
<td>3.42</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
constant obtained from probe measurements were all within approximately ± 10% of the theoretical value. The experimental values of the attenuation constant obtained from measurements made with the 1/16-inch diameter probes were greater than the theoretical values at frequencies below 1500 Hz. At 1500 Hz and above, the experimental values were less than the theoretical.

The experimental values of the attenuation constant obtained from measurements made with the 1/8-inch diameter probe were very close to the theoretical values except at 1500 Hz where the deviation approached 10%. This suggested that 1500 Hz could have been a resonant frequency of the microphone cavity or probe. For this reason, the cavity and probe were loosely packed with steel wool and the measurements were repeated. It was assumed that the steel wool would change the response characteristics of the cavity and probe but as can be seen in Table I the results of this test did not deviate appreciably from the previous data.

It appeared that the length of the probes did not affect the measurement to a great degree; however, the data obtained without the probes exhibit rather large deviations from both the theoretical values and the experimental values obtained from probe measurements. This was possibly due to the influence of the boundary layer at the tube wall.

The variations of the experimental values of the attenuation constant with intensity could have been caused by turbulence occurring in the boundary layer or vibration of the tube wall. In several instances the attenuation was observed to increase as the intensity increased. This implied that the energy
losses were increasing with intensity. Both turbulence in the boundary layer and vibration of the tube walls represent energy loss mechanisms that could be functions of the sound intensity.

One of the disadvantages of the method used to measure the attenuation constant was that it was difficult to read the oscilloscope accurately. The error encountered by using the oscilloscope, however, was determined to be very small.

At frequencies above 3000 Hz, accurate measurements became very difficult to make. This was due to the fact that sharp well-defined waveforms could not be obtained on the oscilloscope. It is believed that this problem could be solved by using narrower band width filters in the microphone output circuit.

E. Conclusions

It was concluded that more information was required before any definite correlation of the data could be made. This would require a detailed investigation performed under controlled conditions of humidity and temperature.

It was apparent that the method of measuring the attenuation constant was valid and was useful over a frequency range from 500 to 3000 Hz.

By examining the data obtained from the attenuation measurements, it can be seen that the experimental values of the attenuation constant tend to group around the theoretical values of the attenuation constant. In view of this and the number of experimental variables involved, it was decided that the theoretical value of the attenuation constant, obtained from the modified Kirchhoff formula, would be used in all further experimental measurements.
F. Recommendations

The method used for the attenuation measurements in this investigation was practical but it is believed that more information could be obtained from a method that did not incorporate a tone burst. This could be accomplished by using an apparatus similar to the previously described apparatus that incorporated two microphone stations. An anechoic termination would be used so that only a sound wave traveling in one direction would exist in the tube. The acoustic driver would operate continuously. The microphone stations would be matched as closely as possible especially with regard to the microphone cavities and the probes. The sound pressure measurements would be made with one microphone at one microphone station at a time. The microphone output could be filtered regardless of the frequency since the filter response time would no longer be important. The microphone output could also be measured with an accurate voltmeter instead of an oscilloscope. This method would also allow the sound intensity inside the tube to be read directly. This would be a valuable parameter to have at hand when the analysis of results is undertaken. The output of the microphone could also be quite easily put on magnetic tape. The greatest drawback of this proposed method is that a completely anechoic termination is required for accurate results. Such a termination is fairly difficult to achieve.

For best results, all measurements of the attenuation constant should be made under controlled conditions of temperature and humidity. This could be accomplished by placing the apparatus in an air conditioned environment.* The temperature and humidity should also be recorded at the time of each data run.

*All other measurements made in this investigation were done under such conditions.
V. EXPERIMENTAL DETERMINATION OF THE ACOUSTIC CHARACTERISTICS OF SMALL REACTIVE FILTER ELEMENTS

A. Introduction

As stated previously in the introduction of this thesis, one of the objectives of this investigation was to determine the acoustic characteristics of several small reactive filter elements. The reflection and transmission characteristics of seven filter elements were measured at frequencies of 500, 1000, 1500, and 2000 Hz.* The filter elements consisted of bends, coils, and Tees. These elements were fabricated from 0.430 inch I.D. copper refrigeration tubing and refrigeration fittings. The filter elements will be described in greater detail later.

B. Experimental Determination of Reflection Factors: Apparatus and Procedure

The reflection factor measurements were made with a 0.430-inch I.D. standing wave tube constructed by Buckley [9]. All measurements were made with the filter elements terminated anechoically. For the frequencies at which these measurements were taken the expression for the reflection factor magnitude involving a sound pressure maximum and sound pressure minimum** applied, namely:

\[ |R| = \frac{p_{\text{max}} e^{\alpha x_{\text{min}}} - p_{\text{min}} e^{\alpha x_{\text{max}}}}{p_{\text{max}} e^{\alpha x_{\text{min}}} + p_{\text{min}} e^{\alpha x_{\text{max}}}} \]  

(47)

*The greatest part of most noise spectra is below 5000 Hz [9, 17].

**Obtaining the reflection factor from the measurement of one pressure minimum and another arbitrary pressure is discussed in Section VII.
where

\[ P_{\text{max}} = \text{maximum sound pressure} \]
\[ P_{\text{min}} = \text{minimum sound pressure} \]
\[ x_{\text{max}} = \text{location of maximum sound pressure relative to the filter element} \]
\[ x_{\text{min}} = \text{location of minimum sound pressure relative to the filter element} \]
\[ \alpha = \text{attenuation constant} \]
\[ e = \text{base of the natural logarithms} \]

In practice, the determination of the reflection factor magnitude involved the measurement of the magnitudes of a sound pressure maximum and a sound pressure minimum and the measurement of the locations of the two adjacent sound pressure minima on either side of the sound pressure maximum. As can be seen in Fig. 2, the sound pressure maxima in a standing wave are rather broad while the sound pressure minima are sharp and well-defined. Because of this fact, the location of a sound pressure minimum can be measured with greater accuracy than a sound pressure maximum. Therefore, the location of the sound pressure maximum was determined from the relationship

\[ x_{\text{max}} = \frac{x_{2 \text{ min}} - x_{1 \text{ min}}}{2} + x_{1 \text{ min}} \]

where \( x_{1 \text{ min}} \) and \( x_{2 \text{ min}} \) are the locations of two adjacent pressure minima.*

The location of a sound pressure minimum was determined by measuring the locations of equal sound pressures on either side of the minimum sound pressure.

*A similar procedure is described by Gatley [17] and Buckley [9].
sure and taking the mean of the results. This procedure was suggested by several investigators \([9,17,51]\) and was done in the interest of greater accuracy. In all instances, the sound pressure minimum nearest the termination was taken as the starting point or \(x_{\text{1 min}}\). The reason for this will be explained shortly.

The calculation of the reflection factor phase angle requires that the distance from the termination to a sound pressure minimum or maximum is known accurately. As mentioned previously, the location of a sound pressure minimum can be determined with greater accuracy than a sound pressure maximum. Therefore, using a sound pressure minimum, the expression for the phase angle is

\[
\theta = 2 \frac{\omega}{c'} \frac{x_{\text{1 min}}}{m} \pm n\pi, \quad n = 1, 3, 5, \ldots \quad (37)
\]

Since \(\omega = 2\pi f\) and \(c' = f\lambda\), this can be expressed as

\[
\theta = \frac{4\pi}{\lambda} \frac{x_{\text{1 min}}}{m} \pm n\pi, \quad n = 1, 3, 5, \ldots
\]

The distance between two adjacent sound pressure minima is one half of a wavelength so that

\[
\theta = \frac{2\pi}{\lambda} \frac{x_{\text{1 min}}}{x_{\text{2 min}} - x_{\text{1 min}}} \pm n\pi, \quad n = 1, 3, 5, \ldots
\]

For accurate phase angle calculations, the measurement of \(\lambda/2\) must be made to within approximately \(1/3\%\) \([9, 17]\). Due to the configuration of the standing wave tube it was possible to come only within 33 inches of the termination. At higher frequencies several wavelengths existed in this distance. If the first sound pressure minimum nearest the filter was used in the phase angle.
calculations, a small error in measurement of the wavelength would result in a small error in the phase angle. However, if another sound pressure minimum farther away from the termination was used, an additional error would be accumulated for each integral wavelength that existed between the termination and the measured sound pressure minimum. This was the reason for measuring the sound pressure minima that were as close as possible to the termination. The error in phase angle calculations decreased in magnitude as the frequency decreased due to the fact that the wavelength increased. With a distance of 33 inches between the measured sound pressure minimum and the termination, a 1/3% error in the measurement of \( \lambda/2 \) at 4000 Hz resulted in an error of 20 degrees in the phase angle. At 500 Hz, however, the same measurement error produced only a 3 degree error in the phase angle [9].

The standing wave tube used for the reflection factor measurements consisted of a 90-inch length of 0.430-inch I.D. stainless steel tubing coupled at one end to an acoustic driver and mounted along a length of 6 inch steel channel. A 24-inch slot 1/16-inch wide was machined into the wall of the tube so that a microphone probe could be inserted through the tube wall. * The slot was centered along the length of the tube and was sealed at all times by a concentric 0.500-inch I.D. brass tube. The two tubes were lapped together for a sliding fit. A thin film of oil between the two tubes provided lubrication and

*Most standing wave tubes described in the literature featured a probe that was inserted axially into the tube. Due to the small diameter of this standing wave tube, this approach was not feasible.
acted as a sealing mechanism between them. The slotted stainless steel tube was clamped firmly in position by a setscrew bracket near the termination end and by a setscrew threaded through the driver coupling at the other. Therefore, the slotted tube was fixed while the brass tube was free to slide along the length of the stainless steel tube. The sliding brass tube passed through two guides which were placed just past the ends of the slot and served to stabilize the tube assembly. A microphone cavity with a felt washer, identical to the one depicted in Fig. 11, was soldered to the brass tube and was fitted with a probe* which extended through the walls of both tubes and into the sound field 1/16-inch. The sound field was explored by sliding the brass tube, upon which the microphone assembly was mounted, along the length of the slot.** This was accomplished by means of a powered traversing system which was attached to the sliding brass tube. The traversing system consisted of a leadscrew which was mounted parallel to the standing wave tube and was driven by a reversible, variable speed, electric drill motor. A short length of brass tubing was soldered perpendicularly to the sliding brass tube near the microphone cavity. This length of tubing slipped into a short length of concentric brass tubing that was soldered to a split nut threaded onto the leadscrew. This arrangement translated the rotational motion of the leadscrew into longitudinal mo-

*The probe was machined from a surgical hypodermic needle and had an internal diameter of 0.050 inches. The outside diameter was 0.056 inches and the overall length was approximately 0.3 inch. This probe configuration was not used by Buckley [9]. Buckley used a probe but it was missing when this investigation was begun. He does not give the exact dimensions of the probe used in his investigation.

**Gatley [17] measured the effects of probes and slots on the sound field and found that the interference produced by them was negligible if the slot and probe were small in comparison to the internal diameter of the tube.
tion* of the sliding brass tube. Limit switches were placed at both ends of the slot to prevent the microphone from being driven past the slot. A handwheel was attached to the leadscrew to enable fine adjustments to be made. The drill motor was equipped with a "dog clutch" which permitted the motor to be disengaged from the leadscrew when fine adjustments were made. The traversing system was actually more of a convenience than a necessity since the sliding friction between the two tubes was not very great. However, this system did keep the brass tube properly oriented with respect to the slot which was essential.

The modifications of Buckley's [9] standing wave tube were primarily in the measuring system. The original system, which will not be described here, was replaced by a system that made the measurement of sound pressure locations more rapid and, it is believed, more accurate. This system consisted of a pointer, mounted on the sliding brass tube, that traversed a fixed 2 meter scale (meter stick) attached by brackets to the steel channel base. The meter stick was positioned so that the zero end accurately coincided with the termination end of the standing wave tube. The pointer was attached to the brass tube by means of a modified hose clamp and was positioned an accurately known distance (2.71 cm) from the center of the microphone probe. This distance had to be added to all measurements before any calculations were made. The distance from the end of the standing wave tube to the termination was also dealt with in a like manner. The only other modification of the original standing

*The maximum speed at which the sliding brass tube could be driven was 2 inches per second [9].
wave tube apparatus was that the clamp near the termination end of the standing wave tube was replaced by a clamp that prevented the slotted tube from twisting as well as moving longitudinally. This was done to prevent the probe from being sheared off when a termination was attached to the standing wave tube. The terminations were joined to the standing wave tube by slip fit couplings with "O" ring seals. It was often necessary to twist these back and forth to get them onto the tubing due to the tight fit of the "O" rings. This in turn would have twisted the slotted tube if this modification had not been made. A simplified schematic of the standing wave tube apparatus with the associated electronics is depicted in Fig. 12. A photograph of the standing wave tube apparatus is shown in Fig. 13.

The sound pressure magnitude was read from a sensitive vacuum tube voltmeter. The microphone output was passed through a sound analyzer with a 1/10-octave filter before it was measured by the voltmeter. A dual beam oscilloscope was used to examine the microphone output before and after the 1/10-octave sound analyzer filter. In this manner, it was possible to observe any higher harmonics or distortion occurring in the microphone output and voltmeter input. If higher harmonics or distortion were present in either of these, the data obtained under such circumstances was noted accordingly. All measurements were made as rapidly as possible to minimize the effects of drift errors.

The standing wave tube apparatus was evaluated by measuring the reflection characteristics of a solid steel piston. As mentioned previously, such a termination was assumed to have a reflection factor of one with a phase
1. 40 Watt Acoustic Driver
2. Fixed Stainless Steel Tube 0.430" I.D.
3. Movable Brass Tube 0.500" I.D.
4. Variable Speed Reversible Drill Motor
5. Dog Clutch
6. Bearing
7. Limit Switch
8. Leadscrew
9. Hand Wheel Fine Adjust
10. Microphone Cavity and Probe
11. 1/16" x 24" Slot
12. Meter Stick
13. Pointer
14. Acoustic Filter
15. Anechoic Termination

Fig. 12 Schematic of 0.430-Inch I.D. Standing Wave Tube Apparatus
Fig. 13 Photograph of 0.430-Inch I.D. Standing Wave Tube Apparatus
angle of zero degrees. The piston was placed as close as possible to the end of the standing wave tube which corresponded to the location of a filter element. The results of the evaluation measurements are presented in Table II. All of the reflection factor calculations were performed by digital computer. The program used for these calculations is given in Appendix 3. The values of the wavelengths measured in this evaluation are compared with calculated values and are presented in Appendix 2.

As mentioned in Section IV, the values of the attenuation constant were obtained from the modified Kirchhoff formula. These values are presented in Appendix 1. Table II also gives the values of the reflection factor magnitudes obtained from calculations that neglected the effect of attenuation. As can be seen in Table II, the effect of attenuation is quite apparent.

The frequency generator output level was increased as the frequency increased due to the fact that the driver output decreased with increasing frequency. This procedure was incorporated in all measurements made in this investigation. This drop in output is also shown in Table II.

It was found that the gain setting of the sound analyzer affected the results obtained from the sound pressure measurements. This in turn affected the calculated magnitudes of the reflection factors as can be seen in Table II. When a low gain setting was used, the minimum sound pressure produced a voltmeter reading that was very near the scale* zero which was difficult to read accurately. Furthermore, most meters do not give accurate readings

*Two voltmeter scales were used; one for measurement of sound pressure maxima and another more sensitive scale for sound pressure minima.
### TABLE II

Evaluation of 0.430-Inch I.D. Standing Wave Tube
(Solid Piston Termination)

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Frequency Generator Output (Volts)</th>
<th>Maximum Sound Pressure Measured (db)</th>
<th>Minimum Sound Pressure Measured (db)</th>
<th></th>
<th></th>
<th>Phase Angle (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.045</td>
<td>132</td>
<td>112</td>
<td>0.789</td>
<td>0.985</td>
<td>6.14</td>
</tr>
<tr>
<td>500</td>
<td>0.045</td>
<td>132</td>
<td>112</td>
<td>0.799</td>
<td>0.997</td>
<td>3.81</td>
</tr>
<tr>
<td>1000*</td>
<td>0.045</td>
<td>124</td>
<td>110</td>
<td>0.667</td>
<td>0.942</td>
<td>357.74</td>
</tr>
<tr>
<td>1000</td>
<td>0.045</td>
<td>124</td>
<td>110</td>
<td>0.743</td>
<td>1.049</td>
<td>357.85</td>
</tr>
<tr>
<td>1500</td>
<td>0.25</td>
<td>128</td>
<td>113</td>
<td>0.686</td>
<td>1.009</td>
<td>3.77</td>
</tr>
<tr>
<td>1500</td>
<td>0.25</td>
<td>128</td>
<td>113</td>
<td>0.680</td>
<td>1.000</td>
<td>0.31</td>
</tr>
<tr>
<td>2000</td>
<td>0.25</td>
<td>125</td>
<td>113</td>
<td>0.581</td>
<td>0.927</td>
<td>355.71</td>
</tr>
<tr>
<td>2000*</td>
<td>0.25</td>
<td>125</td>
<td>113</td>
<td>0.567</td>
<td>0.905</td>
<td>8.40</td>
</tr>
<tr>
<td>2500</td>
<td>0.50</td>
<td>122</td>
<td>110</td>
<td>0.666</td>
<td>1.098</td>
<td>2.75</td>
</tr>
<tr>
<td>2500</td>
<td>0.50</td>
<td>122</td>
<td>110</td>
<td>0.649</td>
<td>1.070</td>
<td>28.30</td>
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<tr>
<td>3000</td>
<td>0.50</td>
<td>118</td>
<td>108</td>
<td>0.602</td>
<td>1.060</td>
<td>14.80</td>
</tr>
<tr>
<td>3000*</td>
<td>0.50</td>
<td>118</td>
<td>108</td>
<td>0.595</td>
<td>1.047</td>
<td>8.64</td>
</tr>
<tr>
<td>3500</td>
<td>0.75</td>
<td>111</td>
<td>100</td>
<td>0.599</td>
<td>1.080</td>
<td>32.02</td>
</tr>
<tr>
<td>4000</td>
<td>0.75</td>
<td>108</td>
<td>98</td>
<td>0.590</td>
<td>1.128</td>
<td>346.61</td>
</tr>
<tr>
<td>4500</td>
<td>0.75</td>
<td>91</td>
<td>80</td>
<td>0.564</td>
<td>1.102</td>
<td>302.43</td>
</tr>
<tr>
<td>5000</td>
<td>0.75</td>
<td>86</td>
<td>77</td>
<td>0.576</td>
<td>1.185</td>
<td>140.69</td>
</tr>
</tbody>
</table>

*These values were obtained with too low a gain setting of the sound analyzer. See text for detail.
at the extremes of their scales. Therefore, all further measurements were made with the gain set as high as possible. The gain could only be increased to a certain point, however, since too high a setting would produce a full or off scale reading at a sound pressure maximum. The gain settings did not remain constant for all frequencies due to the aforementioned decrease in driver output.

Inspection of the data presented in Table II indicated that the phase angle measurements began to deteriorate at frequencies above 2000 Hz. It is believed that this was caused primarily by the fact that the piston was located 33 inches from the end of the slot. At 2000 Hz, over six wavelengths existed in this distance. Thus, as far as phase angle calculations were concerned, any error made in measuring the value of the wavelength was compounded by at least a factor of six. It is believed that the measuring system was simply not accurate enough to cope with this situation even though it could be read to $\pm 0.2$ millimeter. Phase distortion caused by higher order transverse modes of vibration was ruled out since, according to the formula given by Hartig and Swanson [24] (see Section III), this should not have occurred until a frequency of approximately 37 KHz was reached.

Above 3500 Hz the deviation between the theoretical and experimental values of the reflection factor magnitude also became fairly large. The fact that the experimental values were greater than one suggested that the values of the attenuation constant were possibly too high at these frequencies.

The standing wave measurements could have been affected by leakage occurring between the concentric tubes [9]. A completely airtight seal did
not exist between the two tubes due to the fact that several thousandths of an inch clearance was necessary for the sliding fit. It was assumed that this clearance was not constant over the tube length. Therefore the magnitude of the error due to leakage would not have been constant either but would have varied as the brass tube was moved along the slotted stainless steel tube. It was believed, however, that the film of oil between the tubes tended to minimize such effects. The oil film did have one disadvantage. It was found that if large amounts of oil were used, the oil had a tendency to accumulate in the probe. This in turn produced very erratic sound pressure readings.* This difficulty was avoided by carefully inspecting and cleaning (if necessary) the probe before each data run. The quantity of oil between the two tubes was also reduced which helped alleviate the problem.

Below 450 Hz, two adjacent sound pressure minimums could not be found in the slot. Buckley [9] reported that measurements below this frequency often required that extensions be added to the standing wave tube to even get one sound pressure minimum in the slot. This was a trial and error procedure with the lengths of the extensions changing for each filter or element tested. It was therefore decided not to make measurements below 500 Hz with this apparatus. Due to the difficulty encountered with the phase angle measurements above 2000 Hz, this frequency was set as the upper limit for the measurements. Therefore, all measurements were made in the 500 to 2000 Hz region.

*Erratic sound pressure readings were also obtained if the probe was removed from the microphone cavity. This was similar to the effects observed when attenuation measurements were made without a probe (see Section IV).
All measurements were made in an air conditioned laboratory where the temperature never varied more than 2°F from 74°F. The variation in the humidity was not known.

In order to determine the reflection and transmission characteristics of a filter or filter element, an anechoic termination was required. This was accomplished by placing 4 feet of packed 00 stainless steel wool in a 7 foot section of 0.430-inch I.D. copper tubing. The remaining 3 foot air space was sealed off by a steel piston fitted with an "O" ring. The fraction of the sound energy that was not absorbed by the steel wool and entered the air space was reflected back into the steel wool by the piston. The evaluation of the anechoic termination is presented in Table III. As can be seen in this table, the termination was not completely anechoic. The termination was acceptable, however, due to the fact that the reflections produced by it were of small magnitude.

C. Experimental Determination of Transmission Factors: Apparatus and Procedure

As will be recalled from Section III, the transmission factor was determined by combining the data obtained from reflection factor measurements with the data obtained from transmission measurements. The transmission measurements consisted of determining the magnitude and relative phase angle between the sound pressures that existed at a known distance on either side of the filter. Since the calculations for determining the transmission factor from the transmission measurements have been presented in detail in Section III, they will not be repeated here.

The transmission measurements were performed after all of the
### TABLE III

Evaluation of 0.430-Inch I.D. Anechoic Termination

| Frequency (Hz) | Frequency Generator Output (Volts) | |R| |
|----------------|-----------------------------------|---|---|
| 500            | 0.045                             | 0.063 |
| 1000           | 0.045                             | 0.069 |
| 1500           | 0.25                              | 0.029 |
| 2000           | 0.25                              | 0.033 |
reflection factor data had been obtained. The standing wave tube was uncoupled from the acoustic driver and removed from the steel channel base. A 4-foot (approximately) length of 0.430-inch I.D. hardened copper refrigeration tubing was then coupled to the driver. This was in turn joined by a slip fit coupling to the transmission tube apparatus. This apparatus consisted of a microphone station coupled to either side of the filter or filter element. The apparatus was terminated with the previously described anechoic termination. As mentioned previously in Section IV, one microphone was used for all measurements. The microphone station not in use was sealed by a dummy microphone plug. The microphone stations were the same ones that were used for the attenuation measurements (see Fig. 11) but with the following changes:

1) the length of the brass tubing was shortened to 8.892 inches;

2) the conical bottom of the microphone cavity was machined flat;

3) the probe was permanently press-fitted into place.

The brass tubing was shortened due to the fact that the soldered couplings used in the attenuation measurements had left a sufficient build-up of solder on the tube such that the slip fit couplings were difficult to install and remove. The tubing was shortened very carefully so that the microphone probe was exactly in the center of the length of tubing. This permitted either end of the microphone station to be coupled to the filter. The distance from the microphone probe to the end of the brass tube was purely arbitrary. This was also true of the distance from the probe to the filter itself. The only restriction placed on the distance from probe to filter is that it have a value of three tube
diameters* to avoid the possibility of "near-field" distortion. The probes were machined from a surgical hypodermic needle and had internal diameters of 0.050 inches and external diameters of 0.056 inches. They were inserted in the sound field 1/16 inch. Since larger probes had been used in the attenuation measurements, the holes in the microphone cavities and tube walls were filled with silver solder and redrilled to accept the new probes. At this stage of the operation it was decided to machine the bottoms of the microphone cavities flat as shown in Fig. 11. The felt washer in the bottom of the cavity described in Section IV was retained. A schematic of the transmission measurement apparatus with the associated electronics is presented in Fig. 14. A photograph of the microphone stations mounted on an expansion chamber (see Section VI) is shown in Fig. 15.

The sound pressure magnitude was measured by the same electronics used in the reflection factor measurements. As was done in the reflection factor measurements, the sound analyzer 1/10-octave filter was used, with the dual beam oscilloscope monitoring the microphone output and voltmeter input.

The phase angle between the two sound pressures was determined by use of a precision electronic phase meter.** One input of the phase meter was connected to the microphone output circuit (see Fig. 14) in parallel with the sound analyzer. The other input of the device was connected to the fre-

*Buckley [9] gives a value of three tube diameters while the ASTM Report [1] states that one tube diameter is sufficient.

**A photograph of all of the electronic equipment used in all phases of this investigation is shown in Fig. 16.
1. 40 Watt Acoustic Driver
2. Hardened Copper Refrigeration Tubing 0.430" I.D.
3. Slip Fit Connector with "O" Ring Seals
4. Brass Tubing 0.430" I.D.
5. Microphone Cavity with Probe
6. Microphone Cavity Plug
7. Acoustic Filter
8. Microphone
9. Anechoic Termination

Fig. 14 Schematic of Transmission Measurement Apparatus
Fig. 15 Photograph of Transmission Measurement Microphone Stations Mounted on an Expansion Chamber
Fig. 16 Photograph of Electronic Equipment Used for Acoustic Measurements
quency generator output in parallel with the amplifier. The frequency generator output served as a reference. The sound pressure was first measured on the driver side or input of the filter. At this time the phase angle between this sound pressure and the reference was recorded. The microphone and plug were then interchanged and the process was repeated. The phase angle that existed between the two sound pressures was obtained by subtracting the phase angle measured at the input side of the filter from the phase angle measured at the output side of the filter.

The data obtained from the transmission measurements along with the data obtained from the reflection factor measurements were fed into a digital computer which calculated the values of the transmission factor. The program for the calculation of the transmission factors is given in Appendix 4.

D. Description of Filter Elements Examined with Results of Experimental Measurements

As was mentioned in the introduction of this section, the filter elements examined in this phase of the investigation consisted of bends, coils, and Tees. Such configurations are commonly found in refrigeration and other flow handling systems. The acoustic characteristics of these configurations do not generally lend themselves to straightforward analysis. Therefore, the reflection and transmission characteristics of these configurations were experimentally determined by the techniques described in the previous sections of this report.

The filter elements were all fabricated from 0.430-inch I.D. copper refrigeration tubing and sweat-soldered refrigeration fittings. They consisted of:
1) bends of 45 and 90 degrees;  
2) 3 inch diameter coils of one and two turns;  
3) 6 inch diameter coils of one and two turns;  
4) a tunable sidearm Tee.*  

A photograph of these filter elements along with an expansion chamber (see Section VI) is shown in Fig. 17. Detail of the tunable sidearm Tee is presented in Fig. 18.

The Tee was tuned for minimum transmission at 1000 Hz. This was accomplished by utilizing the formula presented in Fig. 18. The value of the wavelength used in this calculation was obtained from the previously mentioned reflection factor measurements of a solid piston.

The reflection and transmission factors of the filter elements are given in tabular form by Table IV and are displayed graphically in Figs. 19 through 26.

E. Discussion of Results

The discussion of the data presented in Table IV and Figs. 19 through 26 is somewhat hampered by the fact that no theoretical or previously measured values are available for comparison.

As can be seen in Table IV, the highest attenuation was obtained from the tunable sidearm Tee at 1000 Hz. However, this configuration had very narrow band attenuation characteristics. All of the other configurations had rather low attenuation characteristics that did not vary appreciably with

*This is actually a filter, not a filter element.
Fig. 17 Photograph of Filter Elements and Expansion Chamber
Note:
Minimum sound transmission occurs [61] at outlet whenever

\[ 2D_1 = (2N - 1) \frac{\lambda}{2} \]

where \( N \) is any integer. If piston is removed and outlet is sealed, the sound transmitted is a minimum whenever

\[ 2D_2 = (2N - 1) \frac{\lambda}{2} \]

Fig. 18 Detail of Tunable Sidearm Tee
<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Reflection Factor</th>
<th>Transmission Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude</td>
<td>Phase Angle (Degrees)</td>
</tr>
<tr>
<td>Tunable Sidearm Tee</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>0.521</td>
<td>254.18</td>
</tr>
<tr>
<td>1000</td>
<td>0.767</td>
<td>186.30</td>
</tr>
<tr>
<td>1500</td>
<td>0.464</td>
<td>138.52</td>
</tr>
<tr>
<td>2000</td>
<td>0.164</td>
<td>170.62</td>
</tr>
<tr>
<td>90 Degree Bend</td>
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<td></td>
</tr>
<tr>
<td>500</td>
<td>0.051</td>
<td>176.69</td>
</tr>
<tr>
<td>1000</td>
<td>0.154</td>
<td>227.46</td>
</tr>
<tr>
<td>1500</td>
<td>0.060</td>
<td>272.74</td>
</tr>
<tr>
<td>2000</td>
<td>0.182</td>
<td>307.07</td>
</tr>
<tr>
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<tr>
<td>500</td>
<td>0.070</td>
<td>165.40</td>
</tr>
<tr>
<td>1000</td>
<td>0.121</td>
<td>335.35</td>
</tr>
<tr>
<td>1500</td>
<td>0.055</td>
<td>149.81</td>
</tr>
<tr>
<td>2000</td>
<td>0.091</td>
<td>31.98</td>
</tr>
<tr>
<td>3-Inch Diameter Coil: One Turn</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>0.223</td>
<td>291.73</td>
</tr>
<tr>
<td>1000</td>
<td>0.139</td>
<td>263.34</td>
</tr>
<tr>
<td>1500</td>
<td>0.039</td>
<td>118.29</td>
</tr>
<tr>
<td>2000</td>
<td>0.125</td>
<td>134.17</td>
</tr>
<tr>
<td>3-Inch Diameter Coil: Two Turns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>0.167</td>
<td>328.55</td>
</tr>
<tr>
<td>1000</td>
<td>0.162</td>
<td>345.10</td>
</tr>
<tr>
<td>1500</td>
<td>0.087</td>
<td>311.71</td>
</tr>
<tr>
<td>2000</td>
<td>0.106</td>
<td>178.10</td>
</tr>
<tr>
<td>6-Inch Diameter Coil: One Turn</td>
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<td></td>
</tr>
<tr>
<td>500</td>
<td>0.161</td>
<td>327.63</td>
</tr>
<tr>
<td>1000</td>
<td>0.000</td>
<td>0.00</td>
</tr>
<tr>
<td>1500</td>
<td>0.091</td>
<td>241.10</td>
</tr>
<tr>
<td>2000</td>
<td>0.000</td>
<td>0.00</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>REFLECTION FACTOR</td>
<td>TRANSMISSION FACTOR</td>
</tr>
<tr>
<td>---------------</td>
<td>-------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td></td>
<td>Magnitude</td>
<td>Phase Angle (Degrees)</td>
</tr>
<tr>
<td>6-Inch Diameter Coil: Two Turns</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.104</td>
<td>81.80</td>
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<tr>
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<td>0.000</td>
<td>0.00</td>
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<tr>
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<td>0.075</td>
<td>218.21</td>
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<tr>
<td>2000</td>
<td>0.046</td>
<td>109.64</td>
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</table>
Fig. 19 Reflection Characteristics of 0.430-Inch I.D. Tunable Sidearm Tee Tuned for Minimum Transmission at 1000 Hz
Fig. 20 Transmission Characteristics of 0.430-Inch I.D. Tunable Sidearm Tee Tuned for Minimum Transmission at 1000 Hz
Fig. 21 Reflection Characteristics of Bends in 0.430-Inch I.D. Hardened Copper Refrigeration Tubing  ○ 90 Degree Bend  △ 45 Degree Bend
Fig. 22 Transmission Characteristics of Bends in 0.430-Inch I.D. Hardened Copper Refrigeration Tubing  

- 90 Degree Bend  
- Δ 45 Degree Bend
Fig. 23 Reflection Characteristics of 3-Inch Diameter Coils in 0.430-Inch I.D. Soft Copper Tubing. ○ One Turn Coil  △ Two Turn Coil
Fig. 24 Transmission Characteristics of 3-Inch Diameter Coils in 0.430-Inch I.D. Soft Copper Tubing

- One Turn Coil
- Two Turn Coil
Fig. 25 Reflection Characteristics of 6-Inch Diameter Coils in 0.430-Inch I.D. Soft Copper Tubing  ○ One Turn Coil  △ Two Turn Coil
Fig. 26 Transmission Characteristics of 6-Inch Diameter Coils in 0.430-Inch I.D. Soft Copper Tubing  ○ One Turn Coil  △ Two Turn Coil
frequency. In most instances, the attenuation characteristics of the coils differed very little from those of the bends.

As mentioned previously, the anechoic termination was not completely reflection free. Reflection factor measurements of the 6-inch diameter coils, however, produced three values of the reflection factor that had a magnitude of zero. It is believed that the sound energy reflected from the anechoic termination was reflected back into the termination by the coil and hence did not reach the standing wave tube. If this was the case, a standing wave would have been established between the filter element and the anechoic termination. The magnitude of the transmission factor obtained under such conditions would be smaller than the anechoic value if the outlet sound pressure was measured near a sound pressure minimum and greater than the anechoic value if it was measured near a sound pressure maximum. The effect on the phase angle is not known. Thus the fact that the termination was not completely anechoic could explain why the transmission factors did not have magnitudes of one at the frequencies where the reflection factors had magnitudes of zero. The same reasoning applies to the three values of the transmission factor that appeared elsewhere in the results, with magnitudes greater than one. It is believed, however, that the magnitudes of the reflections produced by the anechoic termination were not great enough to account for all of this deviation.

All sound pressure and distance measurements were each measured twice before they were recorded. In practically all instances, no deviation between the two measurements was found. If a deviation did occur, the average of the two measurements was recorded. As mentioned previously, all measure-
ments were made in an air conditioned laboratory to minimize the effects of temperature and humidity variations.

The standing wave measurements were found to be very tedious and time consuming. This was due primarily to the experimental method itself. However, additional difficulty was encountered with the traversing system and the vacuum tube voltmeter. The traversing system proved to be awkward for fine adjustments. Two different voltmeter scales had to be used for the sound pressure measurements; one for sound pressure maxima (0.0 to 1.0) and another more sensitive scale (0.0 to 0.1) for sound pressure minima. The meter had to be zeroed on each scale which was time consuming. This could also have been another source of error. The fact that the sound analyzer gain had to be adjusted at each frequency also added to the time involved in making measurements. The transmission measurements presented no difficulties other than the sound analyzer and voltmeter adjustments. No evidence of higher harmonics or distortion was ever found in the voltmeter input. This was true for both the reflection and transmission factor measurements.

F. Conclusions

It was concluded that the effect of tube attenuation should be considered in all calculations of the reflection and transmission factor magnitude in the frequency range from 500 to 5000 Hz. The effect of attenuation upon phase angle calculations was not as pronounced and hence was neglected. These conclusions have been supported by the work of other investigators [9,17].

The two most prominent drawbacks of the standing wave tube apparatus used in this phase of the investigation were:
1) measurements could not be made close enough to the termination for accurate phase angle results at higher frequencies;

2) the possibility of leakage between the two concentric tubes presented an unpredictable source of error.

The transmission apparatus was considered acceptable in all respects.

With the exception of the tunable sidearm Tee, the transmission characteristics of the filter element configurations examined were too high for any significant sound attenuation. The attenuation characteristics of the tunable sidearm Tee were such that it was effective at only the frequency for which it was tuned. It was noted, however, that the attenuation at this frequency was appreciable.

G. Recommendations

It is believed that several improvements of the apparatus used for the previously described measurements should be made. An anechoic termination that produced smaller reflections would definitely contribute to better results. This would be true for both the reflection and transmission factor measurements. Anechoic terminations, with various configurations, have been constructed and evaluated by several investigators [9,13,17,31]. A termination similar to one of these might produce smaller reflections.

A better quality voltmeter would also be a valuable improvement. If the voltmeter did not have to be zeroed for each scale, considerable time could be saved in making measurements.

The standing wave tube could be improved upon by removing the powered traversing system. This would allow the microphone to be moved along the slot
by hand. This would make the standing wave measurements somewhat less tedious. A guide mechanism would have to be installed to keep the brass tube aligned with the slot if the traversing system were removed. It is believed, however, that much better results could be obtained if a 0.430-inch I.D. standing wave tube was built similar to the 2-inch I.D. tube described in Section VII.
VI. INVESTIGATION OF HIGHER ORDER TRANSVERSE MODES OF VIBRATION

A. Introduction

In this phase of the investigation the effects of higher order transverse modes of vibration upon experimental measurements were examined. The objective of this examination was to determine if correction factors could be obtained that would permit the use of one dimensional plane wave theory in a region where three dimensional effects could not be neglected. In other words, such correction factors would expand the capabilities of the one dimensional theory.

The examination consisted of reflection and transmission measurements of an expansion chamber within which the plane wave theory was not valid. The previously described 0.430-inch I.D. apparatus was used for these measurements. Since the measurements were made in the frequency range from 500 to 2000 Hz, the plane wave theory was applicable to the standing wave and transmission tube apparatus. Thus, the three dimensional effects were isolated in the expansion chamber.

B. Experimental Apparatus, Procedure, and Discussion of Results

The experimental apparatus consisted of the standing wave and transmission tube equipment plus an expansion chamber. The expansion chamber was fabricated from a length of 10-inch steel pipe that was 12.46 inches long and had an internal diameter of 9.96 inches. The ends of the pipe were capped by aluminum plates which were fastened to the pipe by machine screws. The
inlet and outlet of the expansion chamber consisted of short lengths of 0.430-inch I.D. brass tubing that were attached to the aluminum end plates by machined aluminum flanges. All joints were sealed with gasket cement. The expansion chamber is shown in Figs. 15 and 17.

Due to the large diameter of the expansion chamber, the three dimensional effects could be produced at relatively low frequencies. As mentioned previously in Section III, the upper limit for plane waves in tubing is given by the expression

$$\lambda_c \equiv 0.82d.$$  \hspace{1cm} (21)

Using the relationship $$f_c = \frac{c}{\lambda c}$$, the value $$d = 9.96$$ in. = 25.30 cm, and the free space value* of the speed of sound at 74°F ($$c = 34,529$$ cm/sec) gives the critical frequency for the expansion chamber as

$$f_c = 1665$$ Hz.

It was assumed that above this frequency the plane wave theory was no longer valid inside the expansion chamber.

As can be recalled from Section III, the plane wave theory was used to derive the theoretical expressions for the reflection and transmission characteristics of an expansion chamber. Other investigators [9, 13, 17] have reported that the theoretical values obtained from such expressions were in very good agreement with experimentally measured values until the point was reached where higher order transverse modes of vibration occurred within the chamber. Thus it was assumed that the measured reflection and trans-

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*For the tube diameter and frequency range considered, the effect of tube attenuation was negligible.
mission factors of the 10 inch expansion chamber would deviate drastically from the theoretical values at frequencies above 1665 Hz but would be in close agreement with the theoretical values at frequencies below 1665 Hz. The frequencies at which measurements were made were 500, 750, 1000, 1250, 1500, 1600, 1700, 1800, 1900, and 2000 Hz. The reflection factor measurements were made and presented no difficulties. However, when the transmission measurements were made it was found that the transmission at these frequencies was zero. The theoretical reflection and transmission factors of the expansion chamber were obtained with the aid of the computer program presented in Appendix 5. These characteristics were evaluated over a frequency range from 0 to 5000 Hz at 50 Hz intervals. The largest theoretical transmission factor occurred at 550 Hz and had a value of 0.153 at 93.8 degrees. The next largest value was 0.077 at -85.6 degrees which occurred at 1100 Hz. The theoretical values of the transmission factor that were obtained at the frequencies where the measurements had been made never had a value greater than 0.007. It was obvious that this particular expansion chamber possessed very narrow band transmission characteristics. This was attributed to the severe change in cross sectional area. Because of the poor transmission characteristics of the expansion chamber in the frequency range from 500 to 2000 Hz, all further reflection and transmission factor measurements were abandoned.

As was mentioned previously, the transmission was zero at the frequencies at which the reflection factor measurements had been made. However, it was possible to obtain measurable sound pressures at the filter outlet at seven frequencies between 500 and 2000 Hz. These frequencies were
found by setting the sound analyzer on "all pass" (no filter) and slowly increasing the frequency from 500 to 2000 Hz. The frequency generator output level was also increased as the frequency was increased. The seven frequencies located in this manner were very well defined in the sense that if the frequency was varied even a few cycles either way the transmission dropped to zero. The use of the 1/10-octave filter at these frequencies did not affect the sound pressure readings. It was therefore assumed that these frequencies did not contain any higher harmonics.

The seven frequencies at which measurable transmission occurred were 547, 1095, 1653, 1663, 1700, 1751, and 1995 Hz. Four of these frequencies produced sound pressure readings that were an order of magnitude greater than the other three. These were 547, 1095, 1663, and 1751 Hz. This suggested that these frequencies were resonant frequencies of the expansion chamber. According to the plane wave theory (see Section III), resonance occurs in an expansion chamber whenever

\[ \frac{2 \omega_c}{L} = n\pi \quad , \quad n = 2, 4, 6, \ldots \]  

This expression gave three values of the resonant frequency in the frequency range from 500 to 2000 Hz. These were 546, 1092, and 1637 Hz. Thus, it can be seen that the first two resonant frequencies were very accurately predicted by the plane wave theory. However, above 1600 Hz the plane wave theory failed in the respect that the third predicted resonant frequency deviated from the experimental value by nearly 30 Hz. The plane wave theory also predicted that at 1750 Hz the transmission factor would be 0.006 at 90.3 degrees.
Due to the magnitude of the sound pressure observed at this frequency, this could not have been the case. The remaining frequencies of 1653, 1700, and 1995 Hz also deviated from the theoretical values in much the same manner.

C. Conclusions and Recommendations

It was concluded that the plane wave theory failed to describe the characteristics of the expansion chamber above 1600 Hz. Whether or not this was caused by the presence of higher order transverse modes of vibration could not be definitely ascertained from the available experimental data. Because of insufficient experimental data, the exact point at which the plane wave theory failed was also unknown.

It is believed that more conclusive data could be obtained by using an expansion chamber that did not have such narrow band transmission characteristics. In order to accomplish this, the expansion chamber would have to have a smaller diameter and a less drastic cross sectional area change. This would raise the critical frequency of the expansion chamber which would require measurements at higher frequencies. Due to the previously mentioned high frequency limitations, such measurements could not be made with the 0.430-inch I.D. standing wave tube apparatus.
VII. THE DESIGN, DEVELOPMENT, AND EVALUATION OF A 2-INCH I.D. STANDING WAVE TUBE APPARATUS

A. Introduction

As was stated in the introduction of this thesis, one of the disadvantages of the standing wave tube apparatus used in this investigation was that only the static (no flow) values of the acoustic characteristics could be obtained. Since most filters operate under steady flow conditions, this method of analysis could be somewhat lacking. The objective of this phase of the investigation was to design, construct, and evaluate a standing wave tube apparatus that, at a future date, would be capable of analyzing the reflection characteristics of filters under operating conditions of steady flow. The standing wave tube was to be fabricated from 2-inch I.D. tubing corresponding to the tubing sizes frequently found in automotive exhaust systems. This apparatus was also to be designed so that the previously described difficulties encountered with the 0.430-inch I.D. standing wave tube were eliminated. The frequency range over which the tube was to be used was from 50 to 5000 Hz.

B. Description of Experimental Apparatus

In addition to the 0.430-inch I.D. standing wave tube, Buckley [9] also constructed a 2-inch O.D. standing wave tube. This apparatus was identical to the smaller tube and possessed the same disadvantages of a potential leakage between the two concentric tubes and high frequency measurement limitations. These drawbacks were attributable to the basic design itself, i.e., two concentric tubes. It was therefore decided not to incorporate two concentric tubes in the design of the 2-inch I.D. tube.
The 2-inch I.D. standing wave tube consisted of a 6 foot length of 2 1/2-inch O.D. aluminum tubing coupled at one end to an acoustic driver* and mounted along an 8 foot length of 8 inch steel channel. The sound field was explored by a microphone probe inserted through the wall of the tube. This was accomplished in the following manner. The top of the tube was milled flat for a distance of 60 inches. The flat area was centered along the length of the tube and was brought to within 0.045 inches of the internal diameter of the tube at the point of tangency. A 52-inch slot 1/8-inch wide was then machined down the center of the flat area for insertion of the microphone probe. The slot was sealed by a 1.0-inch wide continuous spring steel band. The probe and microphone were mounted in an aluminum block that was attached to the steel band by two dowel pins, four small machine screws, and gasket cement. The relationship between the tube, steel band, and microphone block assembly is shown in Fig. 27. Detail of the aluminum tube is presented in Fig. 28.

Sound pressure measurements were made by sliding the microphone block assembly and steel strip along the length of the slot. Brass guide rails that extended the length of the flat area were positioned on either side of the microphone block assembly to keep the probe and the steel band aligned with the slot. The guide rails were held in place by 8 cradle supports spaced along the length of the tube. These supports also mounted the tube to the steel channel base and kept the whole apparatus in alignment. The guide rail and cradle detail is shown in the cross sectional view of the standing wave tube apparatus

*The driver used in previous phases of this investigation was replaced with a better quality J.B. Lansing unit. See Appendix 7 for details.
Microphone Block Assembly

Dowel Pin

Microphone Probe

2" I.D. Aluminum Tube

0.020" x 1.0" Spring Steel Band

*Note: All Support Structure Removed for Clarity

NOT TO SCALE

Fig. 27 Exploded View of 2-Inch I.D., Standing Wave Tube Apparatus
Fig. 28 Detail of 2-Inch I.D. Aluminum Tube
presented in Fig. 29. Additional detail is presented in Fig. 30.

The steel band was pressed firmly against the flat area of the tube by two lengths of surgical tubing which were placed along the inside faces of the guide rails and pressurized by compressed air. The expansion of the tubes, in directions other than towards the steel band, was prevented by lengths of brass angle which were fastened to the guide rails as shown in Figs. 29 and 30. The surfaces of the steel band, aluminum tube, and surgical tubing were all coated with a heavy silicon grease. By these procedures an airtight seal was obtained between the steel band and the aluminum tube. The silicon grease also reduced the effort required to move the microphone block and steel band when the surgical tubing was under pressure. The two lengths of surgical tubing were coupled together at one end by a section of copper tubing so that the pressure exerted on the steel band by each tube was the same. Thus, the two lengths of surgical tubing were essentially formed into a loop. One end of the loop was sealed by a plugged copper fitting while the other end was connected by a pressure regulator to a cylinder of compressed air. To seal the steel band and aluminum tube as tightly as possible, a pressure of approximately 14 psig was maintained in the surgical tubing. This was the highest pressure that could be used without binding the microphone block and steel band to the point where movement was difficult. At pressures above 20 psig, the microphone block and steel band could not be moved.

The seal between the steel band and aluminum tube at each end of the slot was maintained by steel rollers that kept the steel band in firm contact with the flat area of the tube. This was the reason for not extending the slot
Fig. 29 Cross Section of 2-Inch I.D. Standing Wave Tube Apparatus
Fig. 30 Photograph of 2-Inch I.D. Standing Wave Tube Apparatus with Brass Guide Rail and Brass Angle Removed
the full length of the flat area. After passing under the roller, the steel band moved away from the aluminum tube and passed over a pulley mounted above the aluminum tube beyond the end of the slot. This was done at both ends of the slot. The ends of the steel band were then bolted together to form a continuous band. This detail can be seen in Figs. 31 and 32.

The locations of the sound pressures were measured by a system that incorporated a cursor mounted on the movable microphone block and a fixed 2 meter scale (meter stick) attached along the top of the cradles. The hairline of the cursor was accurately aligned with the center of the microphone probe and the zero end of the meter stick was accurately aligned with the filter end of the aluminum tube. The configuration of the apparatus allowed sound pressure measurements to be made to within approximately 12 inches of the termination. Detail of the measuring system can be seen in Figs. 29, 30, 32, and 33.

With the exception of one modification, the sound pressure magnitude was measured with the same electronics used in the previously described standing wave measurements (see Section V). The modification consisted of the insertion of a preamplifier into the microphone output circuit to provide a better impedance match between the microphone and microphone cable. Since a longer cable (25 feet vs 12 feet) was used with the standing wave tube apparatus, a good impedance match was required to reduce the effect of signal attenuation by the cable. With this particular cable length, it was not necessary to utilize the gain feature of the preamplifier. The microphone and preamplifier composed an integral assembly which was inserted into the cavity.
Fig. 31 Side View Photograph of 2-Inch I.D. Standing Wave Tube Apparatus
Fig. 32 End View Photograph of 2-Inch I.D. Standing Wave Tube Apparatus
Fig. 33 Top View Photograph of 2-Inch I.D. Standing Wave Tube Apparatus
machined in the microphone block assembly. The microphone preamplifier assembly was lightly clamped in the microphone cavity by a setscrew threaded through the wall of the block (see Fig. 29). A felt washer was placed in the bottom of the cavity to minimize the transmission of any structural vibrations to the microphone. The microphone cavity was not equipped with a pressure relief channel since it was not known how the microphone cavity would be sealed when steady flow was introduced into the tube. Therefore, when the microphone assembly was removed, extreme care had to be used to prevent damage to the microphone.

The microphone probe* was machined from a surgical hypodermic needle and extended into the sound field 1/16 inch. An error in one of the machining operations prevented a press fit of the microphone probe into the microphone block. Although a tight slip fit existed, it was possible that the probe could have fallen out of the block into the aluminum tube. This was avoided by attaching one end of the probe to an aluminum washer with epoxy cement (see Fig. 29). The washer was the same diameter as the microphone cavity. Any leaks between the probe and the microphone block were prevented by the gasket cement that was used where the microphone block assembly was joined to the steel band.

Due to the number of, and relationships between, the component elements of this apparatus, a definite assembly and disassembly sequence existed. These sequences are presented in Appendix 6 for the benefit of the investigators using

*The probe had an internal diameter of 0.054 inches and an external diameter of 0.075 inches.
this apparatus in further research.

C. Evaluation Procedure and Discussion of Results

The 2-inch I.D. standing wave tube was evaluated, in the same manner as the 0.430-inch I.D. apparatus, by measuring the reflection characteristics of a solid piston. The piston was positioned near the end of the standing wave tube which corresponded to the location of a filter. Measurements were made over a frequency range from 50 to 5000 Hz.

Due to the long slot length of the 2-inch I.D. apparatus, two sound pressure minima could be found in the slot at frequencies as low as 200 Hz. At this and higher frequencies, the experimental procedures and calculations used for the determination of the reflection factor were identical to those presented in Section V.

At frequencies below 200 Hz, only one sound pressure minimum could be found in the slot. * As will be recalled from Section III, the reflection factor can be obtained from the measurement of the magnitudes and locations of one sound pressure minimum and another arbitrary sound pressure. Therefore, at frequencies below 200 Hz the reflection factor magnitude was determined from the relationship

\[
|R| = \frac{A_m e^{2\alpha x \text{min}} - P_m e^{\alpha x \text{min}}}{A_m}
\]

where

\[
A_m = \frac{1}{2} \left[ P_{\text{min}} + \left( P_{\text{min}}^2 - 2 \frac{P_{\text{min}}^2 - P^2(x)}{1 - \cos \theta} \right)^{1/2} \right]
\]

\[
(58)
\]

\[
(57)
\]

*At 50 Hz a 3-foot extension had to be added to the standing wave tube in order to obtain a sound pressure minimum in the slot.
and

$$\phi = 2\frac{\omega}{c'} x_{rx}$$  \hspace{1cm} (52)

here

- $p_{min} = \text{minimum sound pressure}$
- $p(x) = \text{arbitrary reference sound pressure}$
- $x_{min} = \text{location of minimum sound pressure relative to the termination}$
- $x_{rx} = \text{location of arbitrary reference sound pressure relative to the minimum sound pressure}$
- $c' = \text{speed of sound in the tube}$
- $\omega = 2\pi f$
- $\alpha = \text{attenuation constant}$
- $e = \text{base of the natural logarithms}$

Since the distance between two sound pressure minima could not be measured, the expression for the reflection factor phase angle was

$$\theta = 2\frac{\omega}{c'} x_{min} + n\pi$$  \hspace{1cm} n = 1, 3, 5, \ldots  \hspace{1cm} (37)

It can be seen from these expressions that the value of the speed of sound in the tube (or its equivalent, $f\lambda$) must be known in order to calculate the reflection factor. The wavelength data obtained from the reflection factor measurements at frequencies above 200 Hz indicated that the calculated values of the wavelength in the tube were in close agreement with measured values*

(see Appendix 2). Since the calculated values of the wavelength were obtained

*Other investigators [9, 17] have suggested "scaling" the wavelengths measured at higher frequencies down to the frequency of interest. However, this procedure assumes that the speed of sound in the standing wave tube is independent of frequency. This is generally not the case especially in small diameter tubing.
from the calculated values of the speed of sound in the tube, it was concluded that the calculated values of the speed of sound were accurate. Therefore, the calculated values of the speed of sound in the tube were used in all calculations of the reflection factor at frequencies below 200 Hz.

The arbitrary sound pressure that was measured was the largest sound pressure found in the slot. The minimum sound pressure was measured in the same manner described in Section V. All sound pressure and distance measurements were fed into a digital computer which calculated the reflection factors. The program is presented in Appendix 3.

The results of the reflection factor measurements are listed in Table V. As can be seen in this table, the experimental values of the reflection factor were in excellent agreement with the theoretical values up to a frequency of 4000 Hz. At this point and beyond, the deviation between the theoretical and experimental values was considerable. The design of the 2-inch I.D. standing wave tube enabled measurements to be made within approximately 12 inches of the termination. At 4000 Hz, approximately three wavelengths existed in this distance. Thus, any error made in the measurement of the wavelength was compounded by at least a factor of three. In order to determine whether or not the measuring system was at fault, the phase angle measurements at and above 4000 Hz were repeated with the piston placed at the end of the slot. In this manner, the first sound pressure minimum could be measured. The results of these measurements produced phase angles that still had values of approximately 30 degrees. If phase distortion was not being produced by higher order transverse modes of vibration, this indicated that the \( \pm 0.2 \) millimeter accuracy
### TABLE V

Evaluation of 2-Inch I.D. Standing Wave Tube (Solid Piston Termination)

| Frequency (Hz) | Frequency Generator Output (Volts) | Maximum Sound Pressure Measured (db) | Minimum Sound Pressure Measured (db) | $|R|$ for $\alpha = 0$ | $|R|$ for $\alpha \neq 0$ | Phase Angle (Degrees) |
|----------------|------------------------------------|-------------------------------------|-------------------------------------|-----------------|-----------------|-------------------|
| 50*            | 0.25                               | 110                                 | 90                                  | 0.979           | 1.009           | -1.12             |
| 75*            | 0.25                               | 140                                 | 102                                 | 0.983           | 1.008           | -1.22             |
| 100*           | 0.25                               | 140                                 | 102                                 | 0.982           | 1.004           | -0.86             |
| 125*           | 0.25                               | 137                                 | 100                                 | 0.979           | 0.998           | -1.23             |
| 150*           | 0.25                               | 140                                 | 100                                 | 0.986           | 1.003           | 0.45              |
| 175*           | 0.15                               | 140                                 | 105                                 | 0.989           | 1.005           | -0.14             |
| 200            | 0.45                               | 135                                 | 91                                  | 0.991           | 1.006           | 0.23              |
| 300            | 0.45                               | 138                                 | 92                                  | 0.994           | 1.007           | -0.03             |
| 400            | 0.45                               | 139                                 | 93                                  | 0.995           | 1.006           | -1.35             |
| 500            | 0.45                               | 134                                 | 100                                 | 0.962           | 0.991           | 358.80            |
| 600            | 0.45                               | 130                                 | 92                                  | 0.978           | 1.005           | 356.60            |
| 700            | 0.45                               | 128                                 | 89                                  | 0.981           | 1.006           | 359.12            |
| 800            | 0.45                               | 126                                 | 89                                  | 0.977           | 1.000           | 358.41            |
| 900            | 0.45                               | 126                                 | 88                                  | 0.984           | 1.000           | 1.04              |
| 1000           | 0.45                               | 125                                 | 88                                  | 0.974           | 0.994           | 0.05              |
| 1500           | 0.25                               | 126                                 | 91                                  | 0.970           | 0.998           | 0.31              |
| 2000           | 0.50                               | 120                                 | 83                                  | 0.969           | 0.992           | 357.29            |
| 2500           | 0.50                               | 127                                 | 89                                  | 0.976           | 1.006           | 1.43              |
| 3000           | 0.50                               | 116                                 | 77                                  | 0.977           | 1.012           | 354.53            |
| 3500           | 0.50                               | 113                                 | 83                                  | 0.928           | 0.959           | 359.82            |
| 4000           | 0.50                               | 114                                 | 86                                  | 0.912           | 0.947           | 120.66            |
| 4500           | 0.50                               | 111                                 | 86                                  | 0.892           | 0.925           | 343.47            |
| 5000           | 0.50                               | 114                                 | 90                                  | 0.890           | 0.926           | 36.64             |

*Reflection factor measurements made using only one sound pressure minimum.
of the measuring system was not capable of producing acceptable results at these higher frequencies. According to the theory of Hartig and Swanson [24], phase distortion due to higher order transverse modes of vibration would not have occurred until a frequency of approximately 8000 Hz was reached. However according to the A.S.T.M. report [1] (see Section II) the recommended frequency limit for a standing wave tube of this diameter was 4000 Hz.

It was also observed that the effect of attenuation was not nearly as pronounced in the 2 inch I.D. tube as it was in the 0.430 inch I.D. tube. This can be seen in the reflection factor magnitudes presented in Table V and in the speed of sound values presented in Appendix 2. Because of this fact, the calculation of the reflection factor below 200 Hz was rather insensitive to the value of the speed of sound used. When the free space values of the speed of sound were used in these calculations, the magnitudes of the reflection factors were unaffected while the phase angles only exhibited a maximum change of approximately 2 degrees. The decrease in the effect of attenuation was to be expected due to the increased diameter of the tube.

As can be seen in Table V, the output of the driver dropped considerably at the extremes of the frequency range. This required adjustment of the frequency generator output level as the frequency was varied. The sound analyzer gain was also adjusted for the reasons previously mentioned in Section V. It can also be seen in Table V that very high sound pressure levels existed within the tube, especially at the lower frequencies. These high levels evidently did not affect the results obtained from the reflection factor measurements of a solid piston.
In general, the measurements obtained in the 50 to 3500 Hz range were very consistent and accurate. The only difficulty encountered in the measurements was with the voltmeter. This problem was described previously in Section V. Since the same voltmeter was used for these measurements, this problem was to be expected. It was also found that moving the microphone block by hand reduced the tedium of the measurements.

A static pressure test of the steel strip and aluminum tube seal was not performed but it was observed that if measurements were made without air pressure in the surgical tubing, erratic results were obtained. These erratic results were apparently caused by leakage occurring between the steel band and the tube and by sound radiation from the steel band.

D. Conclusions

It was concluded that the 2-inch I.D. standing wave tube was capable of accurate reflection factor measurements over a frequency range from 50 to 3500 Hz. This was due to two basic features of the apparatus

1) sound pressures could be measured to within approximately 12 inches of the termination;

2) the system was airtight.

Whether or not the measurements at and above 4000 Hz were influenced by higher order transverse modes of vibration was not determined. In order to do this, measurements would have to be made with a more accurate measuring system.

It is believed that this apparatus, with a minimum number of modifications, will be capable of handling steady flow.
E. Recommendations

It is believed that the measuring system could be modified to give more accurate results at higher frequencies. This could be accomplished by the addition of a micrometer type fine adjustment mechanism to the existing system.

It is also recommended that all further measurements be made at lower values of the sound pressure level. One reason for this is that at very high sound pressure levels the validity of the assumptions made in deriving the plane wave equation (see Section III) becomes somewhat questionable. Also, unlike a solid piston, certain filters or filter elements may become nonlinear at high sound pressure levels.

The previously mentioned recommendations that a better quality voltmeter be used also applies here.
VIII. BIBLIOGRAPHY


8. BRUEL and KJAER, (1959) Instructions and Applications for Standing Wave Apparatus Type 4002, Brueel & Kjaer, Copenhagen, Denmark. 16 p.


IX. APPENDIX
Appendix 1

Calculated Attenuation Constants

The values of the attenuation constants used in this investigation are presented here for convenience. They are obtained from the modified Kirchhoff equation presented previously in this report

\[ \alpha = 3.18 \times 10^{-5} \left( \frac{f}{r} \right)^{1/2} \]

where

- \( f \) = frequency in Hz
- \( r \) = tube radius in centimeters

A desk calculator and mathematical tables were used in the evaluation of the above expression. The values for both the 0.430- and 2-inch I.D. tubes are as follows:

<table>
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<tr>
<th>Frequency (Hz)</th>
<th>0.430-Inch I.D. Tube (x 10^4 cm)</th>
<th>2-Inch I.D. Tube (x 10^4 cm)</th>
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<td>(x 10^4 cm)</td>
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Appendix 2

Speed and Wavelength of Sound

in 0.430-Inch and 2-Inch I.D. Tubes

When only one pressure minimum can be detected in the standing wave tube, the value of the speed of sound in the tube must be known in order to calculate the reflection and transmission characteristics of the filter (see Appendices 3 and 4). This occurs at frequencies below 450 Hz in the 0.430-inch standing wave tube and below 200 Hz in the 2-inch standing wave tube.

The values of the speed of sound are obtained from Kirchhoff's equation for the speed of sound in a tube presented previously in this report, namely,

\[ c' = c\left[1 - \frac{0.579}{2r[\pi f]^{1/2}}\right] \text{ cm/sec} \]

where

- \( c = \text{speed of sound in free space} \)
- \( r = \text{radius of tube in centimeters} \)
- \( f = \text{frequency in Hz} \)

All physical properties were evaluated for dry air at 74°F and 760 mm Hg and were obtained from Chapman [10]. For these conditions the adiabatic speed of sound in free space, \( c \), has a value of 34,529 cm/sec.

The wavelength corresponding to the value of the speed of sound at a given frequency and tube radius is obtained from the relationship

\[ \lambda_k = \frac{c'}{f} \]

The wavelength in free space is obtained from the same relationship; in this case \( c' = c \) and \( \lambda_k = \lambda \).

The equations for the speed and wavelength of sound were evaluated
by use of an IBM 360/50 Digital Computer.

The data tables compare the deviation of the calculated wavelengths in the tubes with measured values of the wavelength (where available) obtained from reflection factor measurements of solid pistons. The measured wavelength is denoted as $\lambda_m$. As can be seen in the tables, the calculated and measured values are in very good agreement. Notice also that as the frequency and tube diameter increase, the effects of attenuation upon the speed and wavelength of sound in the tube diminish.

The data is presented on the following pages.
<table>
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<tr>
<th>Frequency (Hz)</th>
<th>Calculated* Speed of Sound in 0.430-Inch I.D. Tube c' (cm/sec)</th>
<th>Calculated* Speed of Sound in 2-Inch I.D. Tube c' (cm/sec)</th>
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*Calculated value of the speed of sound in free space is 34,529 cm/sec.
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<th>Calculated Wavelength in 0.430-Inch I.D. tube $\lambda_k$ (cm)</th>
<th>Calculated Wavelength in Free Space $\lambda$ (cm)</th>
<th>Measured Wavelength in 0.430-Inch I.D. tube $\lambda_m$ (cm)</th>
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<td>+0.22</td>
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<td>43.16</td>
<td>11.51</td>
<td>-0.17</td>
<td>-0.09</td>
</tr>
<tr>
<td>1500</td>
<td>38.28</td>
<td>38.36</td>
<td>9.84</td>
<td>-0.81</td>
<td>-0.81</td>
</tr>
<tr>
<td>2000</td>
<td>34.46</td>
<td>34.53</td>
<td>8.63</td>
<td>-0.63</td>
<td>-0.63</td>
</tr>
<tr>
<td>3000</td>
<td>26.87</td>
<td>23.99</td>
<td>7.67</td>
<td>+0.35</td>
<td>+0.35</td>
</tr>
<tr>
<td>3500</td>
<td>22.93</td>
<td>22.94</td>
<td>6.91</td>
<td>+0.01</td>
<td>+0.01</td>
</tr>
<tr>
<td>4000</td>
<td>19.67</td>
<td>19.68</td>
<td>6.31</td>
<td>+0.03</td>
<td>+0.03</td>
</tr>
<tr>
<td>4500</td>
<td>17.24</td>
<td>17.26</td>
<td>5.96</td>
<td>+0.14</td>
<td>+0.14</td>
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<td>15.13</td>
<td>5.59</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
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<td>13.34</td>
<td>5.14</td>
<td>-0.05</td>
<td>-0.05</td>
</tr>
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<td>11.71</td>
<td>11.72</td>
<td>4.71</td>
<td>-0.06</td>
<td>-0.06</td>
</tr>
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<td>10.27</td>
<td>4.32</td>
<td>-0.02</td>
<td>-0.02</td>
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<td>9.00</td>
<td>9.01</td>
<td>3.95</td>
<td>-0.02</td>
<td>-0.02</td>
</tr>
<tr>
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<td>7.90</td>
<td>7.91</td>
<td>3.61</td>
<td>-0.03</td>
<td>-0.03</td>
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</table>
Appendix 3

Computer Programs for the Calculation
of Experimental Reflection Factors

The data obtained from the standing wave measurements is best analyzed by digital computer techniques. This insures accuracy and minimizes the effort involved in determining the characteristics of a given filter. In the following programs, WATFOR program language is used in an IBM 360/50 Digital Computer.

As described previously in this report, there are two techniques available for the calculation of the reflection factor. The first involves the measurement and location of two sound pressure minima while the second involves the measurement and location of one sound pressure minimum and another arbitrary sound pressure. As mentioned before, the second technique is required due to the fact that at low frequencies two sound pressure minima do not always exist in the distance traversed by the microphone. In the 2-inch standing wave tube two sound pressure minima cannot be detected below 200 Hz. This frequency increases to 450 Hz for the 0.430-inch standing wave tube. Therefore, the use of two separate programs is required. One program is used when two sound pressure minima can be located and the other is used when only one sound pressure minimum can be found in the distance traversed by the microphone.

The computer program used for the calculation of experimental reflection factors using two sound pressure minima makes use of equations (37) and (47) presented previously in this report. The notation is as follows:

\[ \text{ATT} = \text{attenuation constant} \]
DELTA = correction factor for the discrepancy between the measured
and actual distance from the discontinuity to the sound pressure minima

DELTA = distance between first and second sound pressure minimums
DELTA = DELTAX
F = frequency in Hz
M = number of input data cards
PHASE = phase angle associated with reflection factor
PMAX = maximum sound pressure
PMIN = minimum sound pressure
R1 = calculated magnitude of the reflection factor neglecting attenuation
R2 = calculated magnitude of the reflection factor considering the effect of attenuation
RFATT = R2
RFO = R1
X1, X2 = measured location (distance from discontinuity) of equal sound pressure magnitudes on either side of the first sound pressure minimum
X3, X4 = measured location (distance from discontinuity) of equal sound pressure magnitudes on either side of the second sound pressure minimum
X1MIN = actual distance from the discontinuity to the first sound pressure minimum
\[ X_{2\text{MIN}} = \text{actual distance from the discontinuity to the second sound pressure minimum} \]

\[ X_{\text{MAX}} = \text{actual distance from the discontinuity to the sound pressure maximum} \]

The computer program for calculating experimental reflection factors using two sound pressure minimums along with a sample of output is presented on the next page. It should be noted that this program permits the raw data to be fed directly into the computer with no preliminary hand calculations being required. Note also that anechoic conditions are assumed to exist on the outlet side (or downstream) of the filter.
C PROGRAM FOR THE CALCULATION OF EXPERIMENTAL REFLECTION FACTORS USING TWO SOUND PRESSURE MINIMUMS
C FREQUENCY IN CYCLES PER SECOND
C SOUND PRESSURE IN VOLTS
C X IN CENTIMETERS
C PHASE ANGLE IN DEGREES
C ATTENUATION FACTOR PER CENTIMETER
C OUTPUT IS IN THE FOLLOWING ORDER
C
C 2.0 INCH S.W.T. DATA OF SEPT 19, 1968 RIGID PISTON TERMINATION
1 WRITE(3,200)
2 DO 10 J=1,M
3 READ(1,100)F, PMAX, PMIN, X1, X2, X3, X4, ATT, DELTA
4 X1MIN=((X2+X1)/2.0)+DELTA
5 X2MIN=((X4+X3)/2.0)+DELTA
6 DELTB=X2MIN-X1MIN
7 XMAX=(DELTB/2.0)+X1MIN
8 R1=(PMAX-PMIN)/(PMAX+PMIN)
9 A=ATT*X1MIN
10 B=ATT*XMAX
11 C=EXP(A)
12 D=EXP(B)
13 R2=((PMAX*C)-(PMIN*D))/((PMAX/C)+(PMIN/D))
14 Z=X1MIN/DELTB
15 DO 2 N=1,99,2
16 PI=3.1416
17 PH=(2.0*PI*Z)-N*PI
18 PHASE=PH*(180.0/PI)
19 IF(ABS(PHASE)-360.0)1,1,2
20 2 CONTINUE
21 1 WRITE(3,201)F, PMAX, PMIN, X1 MIN, X2MIN, DELTB, XMAX, R1, ATT, R2, PHASE
22 10 CONTINUE
23     CALL EXIT
24 100 FORMAT(9F8.0)
25 200 FORMAT(3X,'FREQUENCY',5X,'PMAX',5X,'PMIN',5X,'X1MIN',6X,'X2MIN',6X
5,'DELTAX',5X,'XMAX',7X,'RFO',7X,'ATTENUATION',4X,'RFATT',5X,'PHASE
6 ANGLE')
7F5.3,5X,E10.3,5X,F5.3,5X,F7.2)
27     END

/DATA
FREQUENCY  PMAX  PMIN  X1MIN  X2MIN  DELTAX  XMAX  RFO  ATTENUATION  RFATT  PHASE ANGLE
200.0    0.581  0.001  43.13   129.28  86.15   86.20  0.991  0.177E-03  1.006  0.23
300.0    0.340  0.001  28.68   86.04  57.36   57.35  0.994  0.217E-03  1.007  -0.03
400.0    0.388  0.004  21.47   64.74  43.26   43.10  0.995  0.250E-03  1.006  -1.35
500.0    0.209  0.004  51.68   86.21  34.53   68.94  0.962  0.280E-03  0.991  358.80
   ...  ...  ...  ...  ...  ...  ...  ...  ...  ...  ...
3500.0   0.534  0.020  22.21   27.14  4.93    24.67  0.928  0.741E-03  0.959  359.82
4000.0   0.605  0.028  23.89   27.99  4.10    25.94  0.912  0.792E-03  0.947  120.66
4500.0   0.440  0.025  21.08   24.94  3.86    23.01  0.892  0.840E-03  0.925  343.47
5000.0   0.635  0.037  22.54   25.96  3.41    24.25  0.890  0.885E-03  0.926  36.64

OBJECT CODE= 1792 BYTES, ARRAY AREA= 0 BYTES, UNUSED= 48208 BYTES

HASP JOB STATISTICS-- 58 CARDS READ-- 60 LINES PRINTED-- 0 CARDS PUNCHED-- 1.68 SEC. CPU TIME
The computer program for the calculation of experimental reflection factors using one sound pressure minimum and another arbitrary sound pressure requires that all distance measurements appearing in the calculations be corrected for any discrepancies between measured and actual distances before they are fed into the computer. This program utilizes the previously presented equations (37), (52), (57), and (58) in calculating the reflection factors. The speed of sound is obtained from Appendix 2. The notation is as follows:

- \( AO \) = calculated magnitude of the incident sound pressure wave
- \( ATT \) = attenuation constant
- \( C \) = speed of sound in the standing wave tube
- \( F \) = frequency in Hz
- \( M \) = number of input data cards
- \( PHASE \) = phase angle associated with the reflection factor
- \( PMIN \) = minimum pressure
- \( PREF \) = arbitrary pressure
- \( RA \) = calculated magnitude of the reflection factor considering the effect of attenuation
- \( RFATT \) = RA
- \( RFO \) = calculated magnitude of the reflection factor neglecting attenuation
- \( RO \) = RFO
- \( XMIN \) = distance of sound pressure minimum from discontinuity
- \( XREF \) = arbitrary pressure location relative to the minimum pressure location

This program for the calculation of low frequency experimental reflection factors along with a sample of data output is presented on the next page.
PROGRAM TO CALCULATE LOW FREQUENCY EXPERIMENTAL REFLECTION FACTORS

DATA IS IN THE FOLLOWING ORDER

LARGE STANDING WAVE TUBE  SEPT 19, 1968

PI=3.1416
WRITE(3, 200)
DO 10 J=1, M
READ(1,100)F, ATT, C, PREF, PMIN, XREF, XMIN
B=(4.0*PI*F)/C
Z=B*XMIN
DO 2 N=-1,99,2
PH=Z-N*PI
PHASE=PH*(180.0/PI)
IF(ABS(PHASE)>360.0)1,1,2
2 CONTINUE
1 ETA=B*ABS(XMIN-XREF)
DEM=1.0-COS(ETA)
AO=(PMIN+SQRT(PMIN**2-(2.0*(PMIN**2-PREF**2)/DEM)))/2.0
RO=(AO-PMIN)/AO
RA=((AO*EXP(2.0*ATT*XMIN))-(PMIN*EXP(ATT*XMIN)))/AO
WRITE(3,201)F, PREF, XREF, PMIN, XMIN, RO, RA, PHASE
10 CONTINUE
CALL EXIT
WRITE(3,200)
FORMAT(7F10.0)
200 FORMAT(3X,'FREQUENCY', 2X, 'PREF', 8X, 'XREF', 8X, 'PMIN', 8X, 'XMIN', 8X,'RFATT', 9X,'RFATT', 6X,'PHASE')
201 FORMAT(3X, F6.1, 5X, F7.3, 5X, F7.2, 5X, F7.3, 5X, F7.2, 5X, F7.3, 5X, F7.3, 5X, 2F8.2)
END
<table>
<thead>
<tr>
<th>FREQUENCY</th>
<th>PREF</th>
<th>XREF</th>
<th>PMIN</th>
<th>XMIN</th>
<th>RFO</th>
<th>RFATT</th>
<th>PHASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.0</td>
<td>0.090</td>
<td>115.54</td>
<td>0.002</td>
<td>170.01</td>
<td>0.979</td>
<td>1.009</td>
<td>-1.20</td>
</tr>
<tr>
<td>75.0</td>
<td>0.400</td>
<td>39.73</td>
<td>0.004</td>
<td>113.47</td>
<td>0.983</td>
<td>1.008</td>
<td>-1.28</td>
</tr>
<tr>
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<td>42.33</td>
<td>0.005</td>
<td>85.36</td>
<td>0.982</td>
<td>1.004</td>
<td>-0.91</td>
</tr>
<tr>
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<td>0.300</td>
<td>26.72</td>
<td>0.004</td>
<td>68.19</td>
<td>0.979</td>
<td>0.998</td>
<td>-1.28</td>
</tr>
<tr>
<td>150.0</td>
<td>0.400</td>
<td>27.61</td>
<td>0.004</td>
<td>57.39</td>
<td>0.986</td>
<td>1.003</td>
<td>0.40</td>
</tr>
<tr>
<td>175.0</td>
<td>0.400</td>
<td>36.79</td>
<td>0.006</td>
<td>49.05</td>
<td>0.989</td>
<td>1.005</td>
<td>-0.18</td>
</tr>
</tbody>
</table>

OBJECT CODE= 1624 BYTES, ARRAY AREA= 0 BYTES, UNUSED= 48376 BYTES

HASP JOB STATISTICS-- 38 CARDS READ-- 48 LINES PRINTED-- 0 CARDS PUNCHED-- 1.38 SEC. CPU TIME
Appendix 4

Computer Program for the Calculation of Experimental Transmission Factors

The graphical calculation of the experimental transmission factor, presented previously in this report, is adapted to digital computer solution by the use of complex numbers and phasor notation. *

The computer program used for calculating the experimental transmission factors uses data obtained from the experimental reflection factor programs in addition to the data obtained from transmission measurements. This additional data consists of the reflection factor magnitude, reflection factor phase angle, and the distance between the first measured sound pressure minimum and the second sound pressure minimum, \( \frac{\lambda}{2} \). The output from the program that uses two sound pressure minima to calculate the reflection characteristics contains all of this required data. The output from the program that is used when two sound pressure minima cannot be found, however, supplies only the value of the reflection factor magnitude and reflection factor phase angle. In this case the distance between the sound pressure minima must be calculated from the relation

\[
\frac{\lambda}{2} = \frac{c'}{2f}
\]

where

\( c' \) = speed of sound in the standing wave tube obtained from Appendix 2

\( f \) = frequency

The notation used in the program for the calculation of the experimental transmission factors is as follows:

* A similar program is given by Buckley [9].
A = phase angle between BO and B1 = -Z

ANGLE = calculated reflection factor phase angle expressed in radians

AO = incident sound pressure wave at discontinuity inlet with assumed value of one and phase angle zero

A1 = calculated incident sound pressure wave at x₁

A2 = calculated transmitted wave at outlet of discontinuity

ATT = attenuation constant

BO = calculated reflected sound pressure wave at discontinuity inlet

B1 = calculated reflected sound pressure wave at x₁

C1 = calculated total sound pressure at x₁

C2 = calculated total sound pressure at x₂

CX = measured relative phase angle between P₁ and P₂ expressed in radians

DELTA = distance between first and second sound pressure minima or 0.5 times the wavelength

F = frequency in Hz

M = number of input data cards

P = calculated transmission phase angle expressed in degrees

P₁ = measured value of the sound pressure at the microphone station located at x₁ on the inlet side of the discontinuity

P₂ = measured value of the sound pressure at the microphone station located at x₂ on the outlet side of the discontinuity

R = magnitude of calculated reflection factor considering the effects of attenuation
RANG = calculated reflection factor phase angle expressed in degrees

RO = complex value of the calculated reflection factor

RP = pressure ratio \( \frac{P_2}{P_1} \)

TANG = measured relative phase angle between P1 and P2 expressed in degrees

VAL = calculated magnitude of the transmission factor

W1 = distance from the discontinuity inlet to the microphone station located at \( x_1 \) on the inlet side of the discontinuity

W2 = distance from the discontinuity outlet to the microphone station located at \( x_2 \) on the outlet side of the discontinuity

Z = phase angle between AO and A1 due to path distance W1

Z2 = phase angle between C2 and A2 due to path distance W2

The program for the calculation of experimental transmission factors along with a sample of data output is presented on the following page.

Note: Anechoic conditions are assumed to exist beyond location \( x_2 \).
C PROGRAM FOR THE CALCULATION OF EXPERIMENTAL TRANSMISSION FACTORS
C DATA IS IN FOLLOWING ORDER
C TEE TUNED FOR 1000 HZ
C 90 DEGREE BEND
C 45 DEGREE BEND
C 3" DIA 1 TURN COIL
C 3" DIA 2 TURN COIL
C 6" DIA 1 TURN COIL
C 6" DIA 2 TURN COIL
1 COMPLEX RO, BO, Y, A1, V, B1, C1, Z1, Y1, A2, CMPLX, CEXP
2 WRITE(3, 200)
3 PI=3.1416
4 DO 10 J=1, M
5 READ(1, 100)F, ATT, R, RANG, DELTA, W1, W2, P1, P2, TANG
6 AO=1.0
7 ANGLE=RANG/57.3
8 F1=R*COS(ANGLE)
9 F2=R*SIN(ANGLE)
10 RO=CMPLX(F1, F2)
11 BO=RO*AO
12 Z=(PI*W1)/DELTA
13 Y=CMPLX(0.0, Z)
14 D1=ATT*W1
15 D1P=EXP(D1)
16 A1=D1P*AO*CEXP(Y)
17 A=-1.0*Z
18 V=CMPLX(0.0, A)
19 D2=-D1
20 D2P=EXP(D2)
21 B1=D2P*BO*CEXP(V)
22  \[ C_1 = A_1 + B_1 \]
23  \[ R_P = P_2 / P_1 \]
24  \[ C_X = \text{TANG} / 57.3 \]
25  \[ Z_1 = \text{CMPLX}(0, 0, C_X) \]
26  \[ C_2 = C_1 * R_P * \text{CEXP}(Z_1) \]
27  \[ D_3 = A T T * W_2 \]
28  \[ D_3 P = \text{EXP}(D_3) \]
29  \[ Z_2 = (\pi * W_2) / \text{DELT}\_A \]
30  \[ Y_1 = \text{CMPLX}(0, 0, Z_2) \]
31  \[ A_2 = D_3 P * C_2 * \text{CEXP}(Y_1) \]
32  \[ G = \text{REAL}(A_2) \]
33  \[ H = \text{AIMAG}(A_2) \]
34  \[ \text{VAL} = \text{SQRT}(G^2 + H^2) \]
35  \[ \text{IF}(G) 12, 13, 12 \]
36  \[ \text{IF}(H) 14, 14, 15 \]
37  \[ P = -90, 0 \]
38  \[ \text{GO TO} 20 \]
39  \[ P = 90, 0 \]
40  \[ \text{GO TO} 20 \]
41  \[ P = (\text{ATAN} 2(\text{ABS}(H), \text{ABS}(G))) * 57.3 \]
42  \[ \text{IF}(G) 91, 17, 17 \]
43  \[ \text{IF}(H) 18, 18, 19 \]
44  \[ \text{IF}(H) 21, 21, 20 \]
45  \[ P = P - 180, 0 \]
46  \[ \text{GO TO} 20 \]
47  \[ P = 180, 0 - P \]
48  \[ \text{GO TO} 20 \]
49  \[ P = -1, 0 * P \]
50  \[ \text{WRITE}(3, 201) F, R, \text{RANG}, \text{VAL}, P \]
51  \[ \text{CONTINUE} \]
52  \[ \text{CALL EXIT} \]
G3 100 FORMAT(10F7.0)
54 200 FORMAT(3X, 'FREQUENCY', 3X, 'REFLECTION FACTOR', 3X, 'REFLECTION ANGLE'
1, 3X, 'TRANSMISSION FACTOR', 3X, 'TRANSMISSION ANGLE')
55 201 FORMAT(5X, F6.1, 8X, F7.3, 13X, F8.2, 14X, F7.3, 13X, F8.2)
56 END

/DATA

<table>
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<th>FREQUENCY</th>
<th>REFLECTION FACTOR</th>
<th>REFLECTION ANGLE</th>
<th>TRANSMISSION FACTOR</th>
<th>TRANSMISSION ANGLE</th>
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<td>0.767</td>
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</tr>
<tr>
<td>1500.0</td>
<td>0.464</td>
<td>138.52</td>
<td>0.723</td>
<td>26.69</td>
</tr>
<tr>
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<td>0.164</td>
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<td>11.15</td>
</tr>
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</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500.0</td>
<td>0.104</td>
<td>81.80</td>
<td>0.689</td>
<td>171.31</td>
</tr>
<tr>
<td>1000.0</td>
<td>0.000</td>
<td>0.00</td>
<td>0.784</td>
<td>-34.11</td>
</tr>
<tr>
<td>1500.0</td>
<td>0.075</td>
<td>218.21</td>
<td>0.780</td>
<td>155.01</td>
</tr>
<tr>
<td>2000.0</td>
<td>0.046</td>
<td>109.64</td>
<td>0.689</td>
<td>106.91</td>
</tr>
</tbody>
</table>

OBJECT CODE= 2944 BYTES, ARRAY AREA= 0 BYTES, UNUSED= 47056 BYTES

HASP JOB STATISTICS-- 98 CARDS READ-- 100 LINES PRINTED-- 0 CARDS PUNCHED-- 3.42 SEC. CPU TIME
Appendix 5

Computer Program for the Calculation of the Theoretical Characteristics of an Expansion Chamber

The theoretical plane wave reflection and transmission factors of an expansion chamber, with the configuration described previously in this report, are calculated by the following program. This program utilizes equations (69), (70), (71), (72), (78), and (82) to compute these characteristics for given values of chamber diameter and length, inlet-outlet diameter, and frequency. As mentioned in the derivation of (78) and (82), anechoic conditions are assumed to exist on the outlet side of the expansion chamber. The notation is as follows:

- \( C \) = speed of sound (Note: It is assumed that there is no attenuation inside the chamber; therefore, the speed of sound in this case is the same as in free space.)
- \( D_1 \) = internal diameter of inlet and outlet
- \( D_2 \) = internal diameter of chamber
- \( F \) = frequency in Hz
- \( M \) = number of input data cards
- \( PR \) = calculated expansion chamber reflection factor phase angle expressed in degrees
- \( PT \) = calculated expansion chamber transmission factor phase angle expressed in degrees
- \( R_1, R_2 \) = theoretical reflection factors at inlet discontinuity
- \( R_3 \) = theoretical reflection factor at outlet discontinuity
- \( R_{1C}, R_{2C}, R_{3C} \) = theoretical reflection factors \( R_1, R_2, \) and \( R_3 \) expressed
as complex numbers

RTC = calculated complex value of expansion chamber reflection factor

S₁ = internal cross sectional area of inlet and outlet

S₂ = internal cross sectional area of chamber

T₁, T₂ = theoretical transmission factors at inlet discontinuity

T₃ = theoretical transmission factor at outlet discontinuity

T₁C, T₂C, T₃C = theoretical transmission factors T₁, T₂, and T₃ expressed as complex numbers

TTC = calculated complex value of expansion chamber transmission factor

VALR = calculated magnitude of expansion chamber reflection factor

VALT = calculated magnitude of expansion chamber transmission factor

W = chamber length

The program for the calculation of the theoretical characteristics of an expansion chamber along with a sample of output is presented on the next page.
C PROGRAM TO SOLVE FOR THE THEORETICAL CHARACTERISTICS OF AN
C EXPANSION CHAMBER
C W=CHAMBER LENGTH
C D1=INLET DIAMETER
C D2=CHAMBER DIAMETER
C DATA IS IN THE FOLLOWING ORDER 10" EXPANSION CHAMBER WITH 2" INLET
1 COMPLEX E1, E2, R1C, R2C, R3C, T1C, T2C, T3C, DEMC1, DEMC2, RTC, TTC, CMPLX, CE

1XP, E3, E4
2 PI=3.1416
3 W=12.455
4 C=13596.
5 D1=2.00
6 D2=9.961
7 S1=PI*(D1**2)/4.0
8 S2=PI*(D2**2)/4.0
9 DEM=S2+S1
10 R1=(S1-S2)/DEM
11 R2=(S2-S1)/DEM
12 R3=R2
13 T1=(2.0*S1)/DEM
14 T2=(2.0*S2)/DEM
15 T3=T2
16 WRITE(3,200)
17 DO 10 J=1,M
18 READ(1,100)F
19 A1=(2.0*PI*W*F)/C
20 A2=A1*2.0
21 E1=CMPLX(0.0,A1)
22 E2=CMPLX(0.0,A2)
23 E3=-1.0*E1
24 \[ E_4 = -1.0 \times E_2 \]
25 \[ R_1 C = \text{CmplX}(R_1, 0.0) \]
26 \[ R_2 C = \text{CmplX}(R_2, 0.0) \]
27 \[ R_3 C = \text{CmplX}(R_3, 0.0) \]
28 \[ T_1 C = \text{CmplX}(T_1, 0.0) \]
29 \[ T_2 C = \text{CmplX}(T_2, 0.0) \]
30 \[ T_3 C = \text{CmplX}(T_3, 0.0) \]
31 \[ \text{DEMC1} = \text{CmplX}(1.0, 0.0) \]
32 \[ \text{DEMC2} = \text{DEMC1} \cdot (R_2 C \times R_3 C \times \text{CEXP}(E_4)) \]
33 \[ \text{RTC} = R_1 C \cdot (R_3 C \times T_1 C \times T_2 C \times \text{CEXP}(E_4) \div \text{DEMC2}) \]
34 \[ \text{TTC} = (T_1 C \times T_3 C \times \text{CEXP}(E_3) \div \text{DEMC2}) \]
35 \[ G = \text{REAL}(\text{RTC}) \]
36 \[ H = \text{AIMAG}(\text{RTC}) \]
37 \[ \text{VALR} = \sqrt{G^2 + H^2} \]
38 \[ \text{IF}(G)12,13,12 \]
39 \[ \text{IF}(H)14,14,15 \]
40 \[ 13 \text{ IF}(H)14,14,15 \]
41 \[ \text{PR} = -90.0 \]
42 \[ \text{GO TO 20} \]
43 \[ 15 \text{ PR} = 90.0 \]
44 \[ \text{GO TO 20} \]
45 \[ 12 \text{ PR} = (\text{ATAN2}(\text{ABS}(H), \text{ABS}(G))) \times 57.3 \]
46 \[ \text{IF}(G)91,17,17 \]
47 \[ 91 \text{ IF}(H)18,18,19 \]
48 \[ 17 \text{ IF}(H)21,21,20 \]
49 \[ 18 \text{ PR} = \text{PR} - 180.0 \]
50 \[ \text{GO TO 20} \]
51 \[ 19 \text{ PR} = 180.0 - \text{PR} \]
52 \[ \text{GO TO 20} \]
53 \[ 21 \text{ PR} = -1.0 \times \text{PR} \]
54 \[ \text{GO TO 20} \]
55 \[ 20 \text{ T} = \text{REAL}(\text{TTC}) \]
56 \[ \text{U} = \text{AIMAG}(\text{TTC}) \]
VALT = SQRT(T**2 + U**2)

IF(T) 22, 23, 22
23 IF(U) 24, 24, 25
24 PT = -90.0
25 GO TO 30
26 PT = 90.0
27 GO TO 30
28 PT = (ATAN2(ABS(U), ABS(T))) * 57.3
29 IF(T) 92, 27, 27
30 IF(U) 28, 28, 29
31 PT = PT - 180.0
32 GO TO 30
33 PT = 180.0 - PT
34 GO TO 30
35 PT = -1.0 * PT
36 WRITE (3, 201) F, VALR, PR, VALT, PT
37 10 CONTINUE
38 CALL EXIT
39 100 FORMAT (F10.0)
40 200 FORMAT (3X, 'FREQUENCY', 2X, 'REFLECTION FACTOR', 2X, 'REFLECTION ANGLE'
41 2, 2X, 'TRANSMISSION FACTOR', 2X, 'TRANSMISSION ANGLE')
42 201 FORMAT (3X, F6.1, 10X, F7.3, 11X, F7.2, 15X, F7.3, 12X, F7.2)
43 END

/Data
FREQUENCY REFLECTION FACTOR REFLECTION ANGLE TRANSMISSION FACTOR TRANSMISSION ANGLE
0.0 0.000 0.00 1.000 0.00
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100.0 0.989 -172.93 0.147 -82.93
150.0 0.994 -176.06 0.106 -86.07
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**OBJECT CODE= 3640 BYTES, ARRAY AREA= 0 BYTES, UNUSED= 46360 BYTES**

**HASP JOB STATISTICS-- 169 CARDS READ-- 179 LINES PRINTED-- 0 CARDS PUNCHED-- 13.35 SEC. CPU TIME**
Appendix 6

Assembly and Disassembly Procedure for the 2-Inch I.D. Standing Wave Tube

Disassembly Procedure:

1. Slide the microphone block towards the driver end of the tube until the end of the slot is reached.
2. Remove the setscrew clamping the microphone in the microphone block and slowly withdraw the microphone.
3. Remove all external connections to the driver and the pressurization system. Also remove any termination attached to the tube.
4. Remove the two machine screws attaching the cursor scale and bracket to the microphone block and remove the scale and bracket.
5. Remove the eight flat head screws attaching the meter stick scale to the cradle supports and remove the meter stick.
6. Loosen the nuts holding the rollers in contact with the metal band until they are free to move in a vertical direction.
7. Remove the two round head screws and nuts joining the ends of the metal band. (Note: It may be necessary to loosen the nuts on one of the large pulleys to achieve sufficient slack in the band.)
8. Remove the four allen head cap screws attaching the pulley-roller assembly to the base and remove the assembly. Repeat for the other assembly.
9. Remove the flare coupling attaching the pressurization tubing to the air inlet tee. Do not remove the hose clamp holding the surgical tubing to the copper tubing.
10. Remove the machine screw attaching the pressurization tubing termination to the side of the steel channel base.

11. Remove the hose clamps at the pressurization tubing splice and remove the copper tubing joining the two ends of the surgical tubing.

12. Remove the ten flat head screws attaching the brass angle to the guide rail. It will be necessary to move the microphone block to gain access to all of the screws. After the screws have been removed, move the microphone block back to its former position. Repeat for the other angle.

13. Carefully slide the brass angles towards the termination end of the standing wave tube until they clear the microphone block. The angles may now be removed. (Note: The angles and guide rails are matched and are not to be interchanged.)

14. Remove the pressurization tubing by withdrawing it from the driver end of the standing wave tube.

15. Remove the thirty two hex head bolts attaching the brass guide rails to the cradle supports.

16. Carefully slide the guide rails towards the termination end of the standing wave tube until they clear the microphone block. The rails may now be removed. (Note: The rails must be returned to their original location and are not to be interchanged.)

17. The microphone block and attached metal band may now be removed.

18. Remove the driver assembly by removing the two hex head bolts (attaching the driver and flanged coupling to the supports mounted on the steel channel base) and carefully twisting and pulling the assembly off of the standing
Separate the driver from the flanged coupling by removing the four allen head cap screws. It may be necessary to gently pry the two apart since the driver gasket has a tendency to stick.

Remove the allen head positioning screw located on the cradle support nearest the termination end of the standing wave tube.

Remove the two hex head bolts attaching the cradle supports (nearest the ends) to the steel channel base. The entire standing wave tube may now be removed from the base.

Carefully mark each cradle support with respect to its position on the standing wave tube and slide off of the tube. (Note: The cradles must be returned to their initial position and may not be interchanged or reversed.)

Assembly Procedures:

The assembly procedure is the reversal of steps 1 through 22 but with the following additions.

1. When replacing bolts and screws do not tighten any one bolt or screw until all others in that particular assembly have been installed. There is no specific torquing sequence but it should take several incremental tightening operations before the final torque is attained. Torque all bolts and screws securely but use caution not to shear them or strip the threads. A small wrench or socket and small screwdriver are recommended.

2. In steps 17, 16, 14, and 13, apply lubricants to the surfaces in sliding contact. Silicon grease is recommended.
Appendix 7

List of Equipment


5. General Radio Co. Tone Burst Generator, Type 1396, Serial No. 983.


12. AD YU Electronics, Inc. Precision Phase Meter, Type 406L, Serial No. 3399L.

13. Tektronix Storage Oscilloscope, Type 564, Serial No. 007764.

14. Tektronix Dual Beam Oscilloscope, Type 556, Serial No. 002267.

15. Dynakit Mark IV 40 watt Amplifier, Serial No. 8741024.


X. VITA

Victor Hugo Simon III was born on February 8, 1946, in Springfield, Missouri. He attended Saint Agnes Parochial Grade School until June 1959. In September 1959 he enrolled at Parkview High School, also in Springfield. During the summer of 1962 he was a participant in the National Science Foundation Summer Program for Secondary School Students held at the University of Missouri - Rolla. He was graduated from Parkview High School in June 1963.

In September 1963 Mr. Simon enrolled at the University of Missouri - Rolla in the Department of Physics. During the summers of 1965 and 1966 he was a participant in a National Science Foundation Undergraduate Research Program held there and worked as a research assistant while involved with the program. In January 1967 he began working towards the Master of Science Degree in Mechanical Engineering. He received the Bachelor of Science Degree in Physics in June 1967. From September 1967 to July 1968 he held a research assistship sponsored by a National Science Foundation Research Initiation Grant under the supervision of Dr. W.S. Catley of the Department of Mechanical and Aerospace Engineering.

Mr. Simon is a member of Sigma Pi Sigma, Tau Beta Pi, and Kappa Sigma Social Fraternity. He is a United States citizen.