Further investigation of the traveling-wave transistor

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FURTHER INVESTIGATION OF
THE TRAVELING-WAVE TRANSISTOR

by 440
ROBERT JAMES FEUGATE JR., 1946

A
THESIS

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ABSTRACT

A brief introduction to the concept of a traveling-wave field-effect transistor is presented. Analytic solutions for certain lossless and lossy special cases are derived and a numerical method for solution of the general case developed. A possible computer program for implementing this numerically is given and utilized to compare the traveling-wave transistor to a conventional field-effect transistor amplifier.
PREFACE

The author takes this opportunity to express his appreciation to the faculty of the UMR Electrical Engineering Department for the invaluable assistance rendered him in the research summarized in this thesis. In particular, recognition should go to Dr. E.C. Bertnolli for his suggestion to use the superposition method in solving the distortionless cases presented, and to Dr. N.G. Dillman for his continuing advice and encouragement.

The notation used in this thesis does, in general, conform to standard usage. Capital letters, V and I, refer to transform voltage and current, with subscripts 2 and 1 denoting variables on the output and input lines, respectively. As is standard, lower case r,l,g, and c represent the per meter transmission line parameters, while $G_m$ is the transconductance per unit length coupling the two lines. Other notation is defined either in the body of the paper or on the associated figures.
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I. INTRODUCTION

In 1965, George W. McIver proposed, in the Proceedings of the IEEE\textsuperscript{1}, a device he termed the "traveling-wave transistor" and derived certain equations describing the device's operation. Dr. Norman Dillman, of the UMR faculty, became interested in the possibility of constructing a practical model of the transistor and instituted research aimed at completing the analysis Mr. McIver had begun. The result is the theoretical analysis contained in this paper.

By way of introduction, compare the geometry of microstrip transmission (see Fig. 1-a) with a cross-section of the traveling-wave transistor. The transistor itself is simply an insulated gate field effect transistor, with the transverse dimension (channel "width") much greater than usual. It is immediately apparent that the microstrip and the gate region of the MOSFET are essentially the same configuration: a long metal strip (the gate metallization) laid over a ground plane (the channel), and a separating dielectric (the silicon dioxide). Similarly, the drain region comprises two parallel conductors with intervening dielectric, although the resemblance to microstrip is more remote. These transmission line-like gate and drain geometries suggest application of distributed parameter analysis to this elongated MOSFET; doing that results in the equivalent circuit of Fig. 2 (note that transistor input and output capacitances have
been absorbed into the transmission lines).

Here again, the drawing looks familiar; it is identical to the equivalent circuit of a distributed amplifier\(^2\). Presumably, performance like that of distributed amplifiers can be expected. That is, the outputs of all the differential elements will add, and the cutoff frequency of the traveling-wave transistor will become much higher than a lumped analysis would indicate.

It seems now that a new high-frequency amplifier has been discovered, one that has transmission lines for input and output terminations and may possibly (pending more investigation) be capable of wide-band performance. Before the analysis is begun, certain necessary assumptions will be made:

1. It will be assumed that, in normal operation, both the input and output ports are terminated in the lines' characteristic impedances
   \[ R_{01} = \sqrt{\frac{1}{L_1 C_1}}, \quad R_{02} = \sqrt{\frac{1}{L_2 C_2}} \], see Fig. 3.

2. The phase velocities of the gate and drain transmission lines will be assumed equal
   \[ v_{ph} = \frac{1}{\sqrt{1/c}} \). This is, of course, also a requirement for conventional distributed amplifiers. If it were not so, components from each of the differential current sources would arrive at the output end of the drain line at different times, causing phase distortion.
3. It will be assumed that the transistor has some finite gate-to-drain transconductance per unit length, $G_m$.

Necessary background having been supplied, it is now possible to begin deriving equations describing the transistor. First, the formula McIver originally developed will be re-derived, using a different, much simpler approach. Then, several important special cases involving lossy, but distortionless, gate and drain lines will be examined. After that, the requirements of distortionless lines and of unilateral coupling (coupling only through device transconductance) will be lifted. Finally, the problems involved in translating the theory into a functioning device will be discussed.
II. REVIEW OF LITERATURE

Since the literature dealing with insulated-gate field-effect transistors and that concerning microstrip transmission lines have both been extensively catalogued, there is no need to discuss them here. On the other hand, only a few references have appeared which deal with the transistor discussed in this paper. Each reference appeared as a letter in the Proceedings of the IEEE; they are listed below chronologically, with brief comments following the reference.


--- (1965) A Traveling Wave Transistor.

These are the two papers in which McIver originally proposes the device and initiates solution of the problem. The lossless case (see below for terminology) is solved and special-conditions lossy solutions are presented, along with an approximate result for the problem in which coupling exists both in the forward direction (through transconductance) and in the reverse direction (by feedback capacitance). Because he imposes requirements of special line terminations, usefulness of McIver's results is limited. The IEEE paper contains a serious error; the assumption (untrue) is made
that waves travel in only one direction on the output transmission line.


Mr. Kopp, using the coupled-mode technique originally applied to traveling-wave tubes, carries out the solution for lossless lines in a manner different from that of McIver.


Mr. Jutzi approaches the problem of coupled transmission lines in general, concluding that such lines exhibit simultaneously two different propagation constants. These, in turn, give rise to two reflection coefficients at each termination. Assuming special conditions on these reflection coefficients, Jutzi develops a rather complicated gain expression, somewhat different in form from McIver's. Jutzi's results, while more general, are less readily applied to the problem at hand.


The authors, using less analytic methods than the previous discussions, carry out a treatment similar to Jutzi's finding that, under certain conditions, waves grow exponentially with time. Experimental results, gained using con-
ventional distributed amplifiers, are presented which seem to support that conclusion. Neither expressions for terminal voltages or currents nor many details about the experimental portion are presented.


Using methods he presented in a previous paper, Lindquist presents another possible solution method for the lossless case; his method is, however, more general, applicable to any unilateral case. Lindquist's technique is powerful, can be used to find the entire h-parameter set, and appears most useful of the solutions yet presented. He also notes the error mentioned above in McIver's first paper.
III. SOLUTIONS TO THE TRAVELING WAVE TRANSISTOR PROBLEM

A. **Lossless Transmission Lines**

If the transistor and its transmission lines are assumed ideal, losses due to resistive components do not exist, and the equivalent circuit remains as in Fig. 2, with terminations as shown to prevent reflected waves. This is the case studied in McIver's 1965 article, although he omitted the terminating resistance at \( x=0 \) on the drain line, stating that it was not necessary, since no current waves would travel to the left on the drain transmission line. This is, as noted by Lindquist\(^3\), an error; however, McIver's TRW paper does not make this mistake.

Now, it is possible to proceed from the equivalent circuit by writing the telegrapher's equations, modified to include the effect of \( G_m \), and solve for the various voltages and currents as a boundary value problem, but this is a rather involved process. The same end can be achieved more simply by an application of the superposition theorem.

To begin with, consider the characteristic impedances of each transmission line. Since the gate line is terminated on the right in its characteristic impedance, it will appear infinitely long and have an input impedance of \( R_{01} = \sqrt{1/\gamma_1} \). On the output line, the differential current generators, while they are controlled sources, have outputs which are
independent both of one another and of conditions along the

drain transmission line. Therefore, they do not alter the

characteristic impedance, and this line, too, is "flat",

having voltage standing wave ratio (VSWR) of unity and

characteristic impedance of $\sqrt{\frac{1}{c_2}}$.

Bearing these facts in mind, consider a single current
generator (see Fig. 4). It will see a load of two parallel

resistances of $R_{02}$; one resistor represents the line to the

left of the generator, the other the portion to the right

of the generator. Then, just as in the conventional dis-

tributed amplifier, the generator's output current will

divide equally, half becoming a current wave traveling along

the line to the right, half traveling to the left. Let each

half of the generator's output be labeled $I_2(x)$. In accor-

dance with standard transmission line theory,

$$v_1(x) = v_1(0)e^{-j\beta x} = \left(\frac{E_{in}}{2}\right)e^{-j\beta x},$$

where $\beta = \frac{\omega}{v_{ph}} = \frac{\omega}{\sqrt{1c_1}} = \frac{\omega}{\sqrt{1c_2}}$

(the last equality comes from the assumption of equal phase

velocities). Then $I_2(x_k) = -\left(\frac{E_{in}}{4}\right)G_m e^{-j\beta x_k}$

and $I_2(D) = \int_0^D \left(-\left(\frac{E_{in}}{4}\right)G_m e^{-j\beta D} \right) dx$.

The total output current is

$$I_2(D) = \sum_{k} I_2(D) = \sum_{k} \left(-\left(\frac{E_{in}}{4}\right)G_m e^{-j\beta D} \right);$$

taking the limit as $\Delta x \to 0$,

$$I_2(D) = \left(-\left(\frac{E_{in}}{4}\right)G_m e^{-j\beta D} \right)$$

(1).

In exactly the same manner, the current $I_2(0)$ can be

calculated as

$$I_2(0) = j(G_m E_{in}/8\beta) \left(1 - e^{-j2\beta D} \right)$$

(2).
At first glance, it appears that the two outputs could easily be coupled to produce added gain. This is, unfortunately, not the case, for examination of the expression for $I_2(0)$ reveals that this current does not have linear phase shift; hence, it is not free of distortion. Moreover, for wavelengths such that $\lambda_{\text{line}} = 2D/n$, $n=1,2,...$ the current at $x=0$ is zero; obviously, considerable amplitude distortion accompanies the previously mentioned phase distortion. These two characteristics make the left-end output unsuitable for wide-band amplification, exactly opposite the desired result. Therefore the drain current at zero will be ignored; again, this parallels the lumped distributed amplifier case.

Now, transducer gain is easily calculated as

$$G_T = \left( \frac{G_m D^2 R_{01} R_{02}}{4} \right)$$ (3).

It would seem, then, that the traveling-wave transistor is capable of arbitrarily large gain, increasing with the square of channel width. Furthermore, the only frequency-dependent term is $G_m$, seeming to promise very wide band response.

It is to be suspected, though, that this is not the whole story; the derivation thus far has neglected all losses and any stray coupling between drain and gate. Both these factors will now be considered, first through some important special instances, then by consideration of the general case.
B. Distortionless Lines

The first set of special cases to be discussed assumes that the input and output lines, while they may be lossy, remain distortionless; that is, their parameters obey the condition $rc=1g$. Now, the lines themselves introduce neither phase nor amplitude distortion, but signals traveling along the lines attenuate as they propagate, in this manner:

$$v(x) = v(0)e^{-\gamma x} = v(0)e^{-(\alpha+j\beta)x},$$

where $\gamma = \alpha+j\beta = \sqrt{rg} + j\omega \sqrt{1/c}$. With distortionless lines, the characteristic impedance remains equal to $\sqrt{1/c}$.

The reasons for examining the traveling-wave transistor with distortionless lines, which is a very special case, hard to obtain in practice, are two-fold. First, this yields solutions which are both easy to find and relatively simple in form, whereas non-distortionless lines produce exceedingly complicated expressions. Secondly, distortionless lines give a measure of the optimum performance to be expected.

C. Lossy Drain and Lossless Gate Lines

The first lossy case to be considered allows the drain line to become lossy, while the gate is still lossless. The opposite, with a lossy gate and a lossless drain, has results of exactly the same form, and, therefore, will not be considered separately. Now, the current at $x=D$ from a differential
generator has the form,
\[ I_2(D) = \hat{I}_2(x) e^{-j\beta(D-x)} e^{-\alpha(D-x)} \]

Again, it should be noted that \( \hat{I}_2(x) \) is one-half the differential generators output current. Carrying out the solution exactly as before,
\[ I_2(D) = \int_0^D -\frac{1}{4G_m E_m} e^{-j\beta x} e^{-j\beta(D-x)} e^{-\alpha(D-x)} dx \]
\[ = -\frac{1}{4G_m E_m} \frac{1}{(1-e^{-\alpha D}/\alpha)} e^{-j\beta D} \] (4).

Again finding the transducer gain,
\[ G_T = \frac{G_m^2 R_{01} R_{02}}{4} \left( \frac{1-2e^{-\alpha D}+e^{-2\alpha D}}{\alpha^2} \right) \] (5).

Note that the gain is now no longer unbounded, but converges to a maximum value of \( (G_m^2 R_{01} R_{02})/4\alpha^2 \) as the channel becomes very wide. \( G_T \) is more than 90% of this final value for a width of \( 3/\alpha \).

D. Both Lines Lossy, With Equal Attenuation Constants

Next the propagation constants of the two lines are set equal; that is, \( \sqrt{1/c_1} = \sqrt{1/c_2} = \beta/\omega \) and \( r_{1g_1} = r_{2g_2} = \alpha^2 \), although \( R_{01} \) doesn't necessarily equal \( R_{02} \). Under these conditions,
\[ V_1(x) = (E_{in}/2) e^{-\alpha x} e^{-j\beta x}, \text{ and} \]
\[ \hat{I}_2(D) = \hat{I}_2(x) e^{-\alpha(D-x)} e^{-j\beta(D-x)} \]
\[ = -(E_{in}/4) G_m e^{-\alpha D} e^{-j\beta D} dx. \]

Performing the integration gives
\[ I_2(D) = -(E_{in}/4) G_m e^{-\alpha D} e^{-j\beta D} \]
and \[ G_T = (G_m^2/4)D^2 e^{-2\alpha D}R_{01}R_{02} \quad (6). \]

Inspection of equation (6) indicates that the gain has a maximum and that increasing \( D \) beyond an optimum value causes a decrease in gain, rather than the continuous increase observed previously. Transducer gain is then maximized by differentiating \( G_T \) and setting the derivative to zero:

\[ \frac{dG_T}{dD} = (G_m^2 R_{01} R_{02}/4)(2D e^{-2\alpha D} - 2\alpha D^2 e^{-2\alpha D}) = 0. \]

Solving yields \( D_{\text{opt}} = 1/\alpha \), and \( G_{T,\text{max}} = (G_m^2 R_{01} R_{02})/(4e^{2\alpha^2}) \).

E. Both Lines Lossy, Unequal Attenuation Constants

Finally, consider distortionless gate and drain lines in which the attenuation constants are unequal. Still, however, the lines are required to have identical phase constants, creating the following relationships:

\[ \alpha_1 = \sqrt{r_1 g_1}, \quad \alpha_2 = \sqrt{r_2 g_2}, \quad \text{and} \quad \beta = \omega \sqrt{l_1 c_1} = \omega \sqrt{l_2 c_2}. \]

In the same manner as before, solutions are found as

\[ I_2(D) = -(E_i n/4)G_m e^{-j\beta D}(e^{-\alpha_2 D} - e^{-\alpha_1 D})/(\alpha_2 - \alpha_1) \quad (7) \]

and

\[ G_T = \frac{G_m^2}{4} \left( \frac{e^{-2\alpha_1 D} - e^{-(\alpha_1 + \alpha_2) D} + e^{-2\alpha_2 D}}{(\alpha_2 - \alpha_1)^2} \right) R_{01} R_{02} \quad (8). \]

In this instance, \( G_T \) varies through the sum of three exponentials, making operation on \( G_T \) to maximize transducer gain difficult. The same end is accomplished by maximizing \( I_2(D) \), giving the result \( D_{\text{opt}} = (\ln \alpha_2 - \ln \alpha_1)/(\alpha_2 - \alpha_1) \).
At this point certain trends are beginning to become apparent. For one thing, when losses are considered, it is found that the transistor has an optimum channel width which produces maximum gain. Plotting $G_T$ versus line length, $D$, for various attenuation constants shows that this optimum length is shortest (and maximum gain smallest) for $a_2 = a_1$. It still appears that high frequency cutoff is due to deterioration of transconductance alone; this is true, so far, but it is the result of assuming unilateral coupling and distortionless transmission lines. When these restrictions are relaxed, gain variations due to the lines themselves will appear.

F. A General Solution

A truly general analytic solution, describing the voltages and currents as a function of $x$, the distance along the line, is no small undertaking: straightforward solutions of the boundary-value problems are impractical, due to too few boundary conditions, and approaches using linear algebra are extremely tedious, perhaps simply too complicated for hand computation. Mr. Vernon Stanley, a doctoral candidate at UMR, is attacking the problem with the latter technique, but as yet has arrived at no final answer. It is possible, however, to solve the problem for the terminal voltages and currents through numerical means. The major objection to such an approach is that it requires a foreknowledge of the
line parameters, $r, l, c,$ and $g,$ along with device transconductance per meter and feedback capacitance per meter, in addition to the values of the terminating impedances. This is not the disadvantage it first appears to be, however, for the equations resulting from a generalized solution will likely be quite complex, judging from those for non-distortionless conventional lines. Discerning trends and dependencies would probably require numerical substitutions, even if general analytic solutions were available. The major drawback, then, to a numerical approach is that machine computation is mandatory.

To begin the generalized treatment, consider a section of the line which is $\Delta x$ long (see Fig. 5). This represents the completely general case, except that inductive coupling is not considered. It would be entirely possible to handle mutual inductances, but in this problem transformer action will be considered negligible. Now, Kirchhoff's laws are utilized to develop a system of differential equations. One such equation will be derived, by way of illustration.

Writing a voltage law equation, ($s$ is the Laplace transform variable)

$$-V_2(x) + I_2(x)(r_2 + s l_2) \Delta x + V_2(x + \Delta x) = 0,$$

and

$$V_2(x + \Delta x) - V_2(x) \quad /\Delta x = -I_2(x)(r_2 + s l_2).$$

Taking the limit as $\Delta x \to 0$ gives

$$dV_2/dx = -I_2(x)(r_2 + s l_2) \quad (9-a)$$
Further application of KCL and KVL yield

\[ \frac{dV_1}{dx} = -I_1(x)(r_1+s_1) \]  \hspace{1cm} (9-b)

\[ \frac{dI_1}{dx} = V_1(-g_1-g_c-s(c_c+c_1)) + V_2(g_c+s_c) \]  \hspace{1cm} (9-c)

and \[ \frac{dI_2}{dx} = V_1(-g_m+g_c+s_c) + V_2(-g_c-g_2-s(c_c+c_2)) \]  \hspace{1cm} (9-d)

It is also possible to write the equations above in matrix form as

\[
\begin{bmatrix}
V_2(x) \\
V_1(x) \\
I_2(x) \\
I_1(x)
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & -r_2-s_2 & 0 \\
0 & 0 & 0 & -r_1-s_1 \\
-g_c-g_2-s(c_c+c_1) & -g_m+g_c+s_c & 0 & 0 \\
g_c+s_c & -g_1-g_c-s(c_c+c_1) & 0 & 0
\end{bmatrix}
\begin{bmatrix}
V_2(x) \\
V_1(x) \\
I_2(x) \\
I_1(x)
\end{bmatrix}
\]  \hspace{1cm} (10).

Let the 4x4 matrix be denoted as SDEQ (to conform to the notation of the appendix): then, from linear algebra

\[
\begin{bmatrix}
V_2(x) \\
V_1(x) \\
I_2(x) \\
I_1(x)
\end{bmatrix}
= e^{(SDEQ)x}
\begin{bmatrix}
V_2(0) \\
V_1(0) \\
I_2(0) \\
I_1(0)
\end{bmatrix}
= SXFR
\begin{bmatrix}
V_2(0) \\
V_1(0) \\
I_2(0) \\
I_1(0)
\end{bmatrix}
\]  \hspace{1cm} (11).

where \( SXFR = e^{(SDEQ)x} \). Although the solution so far is in terms of \( s \), it is necessary to change to phasors for the steady-state calculations which follow. Therefore, let \( s = j\omega \).
Since the quantities of interest are the terminal voltages and currents, let \( x = D \). Now, because all the voltages and currents are still unknown, the situation is essentially that of 8 unknowns, and only four equations. Things are not hopeless, however, for it has been assumed that the lines' terminations are already known. By reference to Fig. 5, it is apparent that

\[
I_1(0) = (E_{in} - V_1(0))/R_{01}, \quad I_2(0) = -V_2(0)/R_{02},
\]

\[
I_1(D) = V_1(D)/R_{01}, \quad \text{and} \quad I_2(D) = V_2(D)/R_{02}.
\]

Note that, while the terminations were assumed to be real, this is not a necessary condition; resistive terminations are simply the most likely operating conditions. Substituting these relationships into equation (11) and applying matrix algebra produces the result:

\[
\begin{bmatrix}
\frac{-E_{in}s_{14}}{R_{01}} \\
\frac{-E_{in}s_{24}}{R_{01}} \\
\frac{-E_{in}s_{34}}{R_{01}} \\
\frac{-E_{in}s_{44}}{R_{01}}
\end{bmatrix} =
\begin{bmatrix}
s_{12} & -s_{14} & s_{11} & -s_{13} & 0 & -1 \\
s_{22} & -s_{24} & s_{21} & -s_{23} & 0 & -1 \\
s_{32} & -s_{34} & s_{31} & -s_{33} & 0 & -\frac{1}{R_{02}} \\
s_{42} & -s_{44} & s_{41} & -s_{43} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
V_1(0) \\
V_2(0) \\
V_1(D) \\
V_2(D)
\end{bmatrix}
\]

(12)

where \( s_{mn} \) is the result in the \( m \)th row and \( n \)th column of the matrix \( S_{XFR} = e^{(SDEQ)x} \). In the notation of the appendix, equation (12) is written \( \text{SOLN} = \text{SDELTA} \cdot \begin{bmatrix} V_1(0) \\ V_2(0) \\ V_1(D) \\ V_2(D) \end{bmatrix} \). Then, it is
obvious that

\[
\begin{bmatrix}
V_1(0) \\
V_2(0) \\
V_1(D) \\
V_2(D)
\end{bmatrix} = (SDELTA)^{-1}(SOLN).
\]

Now, since the output power is given by \((V_2(D))^2/R_{02}\), and the power supplied by a matched generator is \(|E_{in}|^2/4R_{01}\), the transducer gain is easily calculated as \(4(V_2(D))^2R_{01}/R_{02}\) for \(E_{in}=1.0/0^\circ\); input impedance is \(Z_{in}=V_1(0)/I_1(0)\), the current calculated by the formula given above. This same approach can be utilized to find output impedance, by shifting the generator to the output terminals; in that case, \(I_1(0)=-V_1(0)/R_{01}, I_2(D)=[V_2(D)-E_{in}]/R_{02}\) and the other currents are unchanged. The same matrix equations as before hold, except now, the SOLN matrix is given by

\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & -E_{in} \\
R_{02} & 0 & 0
\end{bmatrix}
\]

Once \(V_2(D)\) is determined, \(I_2(D)\) can be found and output impedance calculated by \(Z_{out}=-V_2(D)/I_2(D)\).

So far, the necessary operations have been indicated, but not performed. The required computations, particularly evaluating \(e^{(SDEQ)x}\) and finding \((SDELTA)^{-1}\), are rather complicated operations, almost impossibly cumbersome for hand calculations. If, on the other hand, numerical values for all the per meter parameters are known, machines can be utilized to determine the terminal voltages. A large portion of this research has been directed toward developing a computer program to perform these calculations.
G. Computer Program to Complete the Solution

To carry out the matrix calculations needed to determine the operating voltages, a Fortran IV (G level) program was written. The program itself appears as an appendix, along with explanation of input and output data formats, while a simplified flow chart is given in Fig. 6.

Initially the various preparatory commands are given the computer: necessary memory space is reserved for arrays, the variables labeled L1 and L2, representing the input and output line inductance per unit meter, respectively, are declared to be floating point numbers (so that the familiar inductance label can be preserved), and all variables and matrices starting with "S" are established as being complex. Next, the values for device parameters, terminal impedances, length, and operating frequency are read into the machine. At this point, actual calculations begin; the elements of the matrix called SDEQ are calculated and all other matrices set to initial values. A counter, K, is set to unity. In the loop formed by the next steps, e^{(SDEQ)D=SXFR} is evaluated. First, STERM=(SDEQ)^K/K! is calculated and added to the matrix SXFR; then STERM is checked, to determine if its elements are smaller than some arbitrary limit. If the inequality is true, the program passes out of the loop; if STERM is too large, K is incremented and a new STERM found and added into the series. The
result is a truncated series of the form
\[
\frac{1}{1!} + \frac{[SDEQ]D}{2!} + \cdots \frac{[SDEQ]^nD^n}{n!} = e^{(SDEQ)D} = SXFR^4
\]

This is not a particularly efficient method for finding $e^A$, where $A$ is a matrix, but other methods require finding the roots of polynomials with complex coefficients, a subroutine not available at the UMR computer center at the time when the program was written.

When SXFR is calculated to the desired precision, the matrices SOLN and SDELTA are formulated as defined above. Then, using the Scientific Subroutines Package library routine CINVRT, SDELTA$^{-1}$ is calculated. Straight-forward matrix and scalar computations give the terminal voltages, transducer gain, and output impedance, all of which, along with frequency of operation and other pertinent information, are printed out. Then, for a frequency response computation, gain is checked and, if the device is still below its cutoff frequency, operating frequency is incremented and the process begun again. For calculations other than frequency response (for instance, variation of gain as channel width is changed), the cards controlling the outermost loop can easily be changed.
IV APPLYING THE SOLUTION TO A TRANSISTOR

With a general solution method available, it is time to develop some of the per meter parameters of a possible traveling-wave transistor and, using the procedure just discussed, determine if, after all, the traveling-wave configuration possesses any clear-cut advantage over conventional field-effect transistors. It is not intended at this time to develop an accurate model for the transistor. Consequently, several gross simplifying assumptions will be made. These simplifications will, however, tend to lower time constants and otherwise give an optimistic performance picture. The results will then set limits on the best performance to be obtained.

It will be assumed that the MOSFET under consideration is an enhancement-mode device with the geometry given in Fig. 7. The gate metallization is very thin, a few hundred angstroms. This is contrary to the conventional practice, which has metal thicknesses of several times the oxide thickness, and would severely limit the input power of any actual device. Despite the extreme thinness, losses in the gate metal (and, for that matter, all other losses) will be ignored. This is, of course, a considerable oversimplification, but it will be retained as a first approximation.
Another assumption will be to neglect the effect of surface states on threshold voltage. This is not a particularly serious limitation, since the surface states can be accounted for simply by increasing $V_{th}$. With this approximation, the pinch-off voltage is given by

$$V_{po} = V_g - K \sqrt{V_g}$$

where $K = h \sqrt{2\varepsilon q N_A / \varepsilon_d}$

$V_g$ = gate bias voltage = 10v.

$h$ = oxide thickness = 3000Å

$W$ = channel length ("line width") = 1 mil.

$N_A$ = substrate doping level = $10^{15}$/cm$^3$

$\varepsilon_d$ = oxide permittivity = $3.8\varepsilon_0$

$\varepsilon$ = substrate permittivity = $12\varepsilon_0$.

For the assumed geometry and bias, $K = 1.642$ and $V_{po} = 4.8v$.

Proceeding to calculate device properties,

$G_m =$ transconductance per unit length

$$= g_m / D = \mu_n \varepsilon_d V_{po} / Wh (V_D - V_{po})$$

$$= 0.529 \text{ mhos per meter.}$$

The input capacitance is given by $C/D = \varepsilon_d W \gamma / h = \varepsilon_d W / h_{eff}$, the equation for a pair of parallel plates of separation $h_{eff}$ $W$ units in width and one unit long. It will be

*As is pointed out in the text edited by J.T. Wallmark and H. Johnson (Field Effect Transistors, Prentice-Hall Englewood Cliffs, N.J., 1966), the mobility of carriers in the channel of a MOSFET is quite different from the bulk value. Since their measured data seems to indicate that electron mobility in an n-channel device can be around a sixth of the bulk mobility, $\mu_n$ was assumed to be 250 cm/c-sec. in these calculations.
assumed the \( h_{\text{eff}} = h/\alpha \) represents the effective height of the gate metal above an apparent ground plane when the transmission line aspects of the problem are discussed shortly. The factor \( \gamma \) accounts for the fact MOS capacitance decreases with increasing bias. Gamma will be assumed to remain at its pinch-off value for \( V_D > V_{po} \). Substituting numbers,

\[
\gamma = 1 - \frac{5V_D^2 + 4KV_D^{3/2}}{60[V_g - \frac{1}{2}V_D - 2/3(K V_D)^2]} = 0.886,
\]

\[
h_{\text{eff}} = 3390 \Omega, \text{ and } c_1 = \frac{c_{in}}{D} = 2520 \text{ pf./m.}
\]

Now, it is necessary to calculate the impedance of the stripline formed by the gate metallization and the channel. Using the equation \(^7\)

\[
R_0 = \frac{377h_{\text{eff}}}{W\varepsilon_{dr} [1 + 1.735(\varepsilon_{dr})^{-0.0724}(W/h_{\text{eff}})^{-0.836}]} \]

it is determined that \( R_{01} = 2.48 \text{ ohms.} \) Furthermore, since

\[
R_{01} = \sqrt{\frac{1}{l_1/c_1}}, \quad l_1 = R_{01}^2 c_1 = 15.5 \text{ nh./m.}
\]

The final parameter needed is the feedback capacitance, \( c_c \); assuming a gate drain overlap of 2 microns (a typical number), \( c_c = \varepsilon_d w_{\text{overlap}}/h = 224 \text{ pf./m.} \). To properly model the transistor similar calculations must be carried out for \( l_2 \) and \( c_2 \) of the drain transmission line. This is not as easily done, however, since the drain fits no standard geometry. Since a sophisticated model is not the primary
purpose here, the obvious assumption will be made that the
two lines are identical.

Now, all data, except the length of the lines, neces-
sary to carry out frequency response calculations have been
assembled. Before proceeding to the calculations them-
selves, it is necessary to digress and consider some impor-
tant points. Neglect, for the moment, feedback capacitance;
then, transducer gain is given by
\[ G_T = \left( \frac{G_m}{2} D^2 R_{01} R_{02} \right)/4. \]
Substituting the values found above, and solving for D, it
appears that, for unity gain, the device must be 1.82 meters
long! This dramatically emphasizes the trade-off that may
preclude development of a truly practical traveling-wave
transistor. As line width decreases, \( G_m, R_{01}, \) and \( R_{02} \) all
increase, resulting in an extremely rapid increase in \( G_T. \)
Unfortunately, very small geometries are very difficult to
produce, and before high gain can be expected, the device
becomes too small for present techniques. On the other
hand, an extremely long device, even if formed in some sort
of spiral or meander pattern, would require large substrate
areas, producing low yields and requiring bulky packages.

A. Comparing a Traveling Wave Device to a Conventional
Transistor

The familiar MOSFET, then, possesses at least one
advantage over the distributed parameter device - much
larger gain. Even when working at the same impedance levels
(see Fig. 8), the gain of the conventional amplifier is
given by $G_T = g_m^2 R_0 R_{02}$ (since there is no need to match the output to the line it drives). It seems that a conventional device half as long as the traveling-wave transistor provides the same gain. This suggests a basis for comparing a traveling-wave amplifier with the more usual one. Given two transistors of identical cross-section; connect one as a traveling wave transistor, the other as a conventional MOSFET (perhaps with many parallel connections to the gate to minimize distributed effects). The lengths are adjusted so that the conventional device in the circuit of Fig. 8 provides the same gain as does the traveling wave FET.

Which device has the better frequency response? If it is the traveling-wave transistor, then perhaps pushing dimensions down to the state of the art will produce useful wideband amplification. If the conventional device exhibits the better response, then the usefulness of the traveling wave configuration seems limited.

This is exactly what was done. The ordinary FET was represented by the simplified equivalent circuit of Fig. 9 and an ECAP analysis performed; the distributed parameter amplifier was analyzed by the program previously discussed. So that the traveling-wave transistor could provide useful gain, the rather unlikely length of two meters was assumed, making the conventional MOSFET a meter long. The results are plotted in Fig. 10.
It is apparent from the curves that, while the performance of both circuits starts to deteriorate at approximately the same frequency, the traveling-wave transistor's gain falls off somewhat more slowly. The new configuration doesn't exhibit the dramatic improvement hoped for, but it does have a significantly wider passband, and, if fabricated to dimensions offering both useful gain and reasonable length, could have potential in wideband circuits. This comparison is by no means a final judgment on the merits of the traveling wave transistor, for no attempt at optimization has been made, nor has a complete model been developed. The results, while not spectacular, are encouraging; further investigation may disclose more favorable geometries with correspondingly greater passbands.
V. SUMMARY AND CONCLUSIONS

Initial treatments of the traveling-wave transistor problem seemed to disclose a new device, of extremely wide frequency response, which has a power gain limited only by the length of the device. Closer examination, including losses, reveals, unfortunately, that this is not true in fact. Once losses are introduced, gain no longer simply increases with increasing length but rather reaches a maximum at some optimum length, decreasing again as the transistor is made still longer. Expressions for power gain and optimum length have been developed for several special cases involving distortionless transmission lines.

The next refinement, accounting for possible feedback capacitance from drain line to gate line, complicates the solution seriously. While, as yet, no completely analytic result for this new problem has appeared, it is possible to solve the problem by numerical means. The steps to accomplish such a solution were presented and a Fortran program written to accomplish the calculations.

Then, the first steps were taken toward evaluating all the per meter transmission line parameters for the transistor. In the process of these computations, the need for utilizing theory based both upon transmission line aspects and upon semiconductor device concepts was illustrated.
Finally, the parameters found were substituted into the computer program previously developed to carry out a frequency response analysis. This analysis showed that the configuration utilizing distributed effects does indeed have a wider response band than a conventional amplifier and at the same time demonstrated that the presence of feedback capacitance in the traveling wave transistor results in a high-frequency cutoff.

Research possibilities dealing with the new transistor are not in any sense exhausted. First of all, the circuit aspects of the problem remain. An analytic solution has yet to be carried out. Allied with this is the question of stability; with a feedback path existing, what conditions will cause the transistor to oscillate? Second, how can performance be improved? A numerical solution now being available a more detailed investigation than previously possible can be done. The computer program presented is easily modified to calculate any performance figure dependent upon terminal voltages and currents; it could be utilized as part of a routine to determine the most satisfactory device dimensions.

Next, of course, is the actual fabrication of such a distributed parameter amplifier and its experimental evaluation. With this would come improved modeling, along with proof or denial of operating characteristics superior to conventional MOSFETs. A final suggestion, while no less
important, is in the realm of computer science, rather than electrical engineering. The series used in evaluating $e^A$, where $A$ is a matrix, is absolutely and uniformly convergent but converges rather slowly. At high frequencies, where the elements of the matrix become relatively large, the individual terms of the series become too large for the computer to handle before the series converges adequately. Obviously, this tends to limit the possible range at high frequency calculations. Evaluating the exponential by use of Sylvester's theorem is not possible, for then it is necessary to find the eigenvalues of a complex matrix, a capability not presently available at UMR. Some way of circumventing this obstacle is needed before the solution described herein can obtain its maximum usefulness.
Fig. 1. Comparison of microstrip and traveling-wave transistor geometries.
Fig. 2 Distributed Parameter Equivalent Circuit for the Traveling-Wave Transistor
Fig. 3. Traveling wave transistor operating configuration.

Fig. 4. A single differential generator.
Fig. 5 - Equivalent Circuit for the Generalized Case
Fig. 6. Simplified flow chart to accomplish numerical solution for the general case.
Substrate

Gate

Source

$N_A = 10^{15}$ cm$^{-3}$

W = 1 mil

Fig. 7. Cross section of the geometry to be used.

Fig. 8. A conventional amplifier circuit; it must match the input transmission line but need not match the output line (biasing circuitry is omitted).

Fig. 9. Simplified MOSFET equivalent circuit.
Fig. 10  Response Curves for MOSFET and Traveling-Wave Transistor
SAMPLE COMPUTER PROGRAM

THE PROGRAM BELOW CALCULATES TERMINAL VOLTAGES, INPUT
IMPEDEACE, AND TRANSDUCER GAIN AS A FUNCTION OF
FREQUENCY. BEGINNING WITH SOME SELECTED INITIAL
FREQUENCY, SAY 1000 HERTZ, THE CALCULATIONS ARE
DONE AT 1000; 2000; 3000; ... 9000; 10,000; 20,000 AND
SO ON UNTIL GAIN IS DOWN TO -30DB.

THE LINE PARAMETERS AND DEVICE LENGTH ARE READ
IN BY A 5F13.3 FORMAT IN THE ORDER INDICATED IN
THE PROGRAM'S READ STATEMENT. THE FIRST INPUT DATA
IS THE INITIAL FREQUENCY, AND TAKES THE FIRST TWO
DATA SLOTS. THE SLOT ENDING IN COLUMN 13 ALWAYS
MUST BE 0.0, AND THE ONE ENDING IN COLUMN 26 ALWAYS
CONTAINS THE INITIAL FREQUENCY.

IMPLICIT COMPLEX($)
REAL*4 LL1,LL2
ODIMENSION SDELA(4,4),SOLN(4),SB1(4),SC(4),
1IP(4),IQ(4)
ODIMENSION SDEQ(4,4),SXFR(4,4),STERM(4,4),
1SMULT(4,4)
1READ (1,100)SF,LL1,LL2,GM,C1,C2,CC,G1,G2,GC,R1,
R2,RO1,RO2,0
100 FORMAT (5F13.3)
1WRITE (3,101)SF,LL1,LL2,GM,C1,C2,CC,G1,G2,GC,R1,
R2,RO1,RO2,0
101 FORMAT (4E18.8)
SI=SF
10 S=6.2832*SF
DO 20 I=1.4
DO 20 J=1.4
SDELA(I,J)=(0.0,0.0)
SXFR(I,J)=(0.0,0.0)
STERM(I,J)=(0.0,0.0)
20 SDEQ(I,J)=(0.0,0.0)
SDEQ(1,3)=-R2-S*L2
SDEQ(2,4)=-R1-S*L1
SDEQ(3,1)=-G2-GC-S*(C2+CC)
SDEQ(3,2)=-GM+(S*CC)+GC
SDEQ(4,1)=-S*(CC)+GC
SDEQ(4,2)=-G1-GC-S*(C1+CC)
DO 40 I=1.4
SXFR(I,1)=(1.0,0.0)
40 STERM(I,1)=(1.0,0.0)
K=1
50 DO 60 I=1.4
DO 60 J=1.4
SUM=(0.0,0.0)
59 SUM=SUM+STERM(I,N)*SDEQ(N,J)
60 SMULT(I,J)=(SUM*0)/K
DO 70 I=1,4
DO 70 J=1,4
STERM(I,J)=SMULT(I,J)
70 SXFR(I,J)=SXFR(I,J)+STERM(I,J)
K=K+1
IF (K.EQ.40) GO TO 80
DO 80 I=1,4
DO 80 J=1,4
IF (CABS(STERM(I,J))-0.02) 80,80,50
80 CONTINUE
EIN=1.00
DO 90 I=1,4
SOLN(I)=(-EIN/RO1)*SXFR(I,4)
SDelta(I,1)=SXFR(I,2)-(SXFR(I,4)/RO1)
90 SDelta(I,2)=SXFR(I,1)-(SXFR(I,3)/RO2)
SDelta(I,3)=(0.0,0.0)
SDelta(I,4)=(-1.0,0.0)
SDelta(2,3)=(-1.0,0.0)
SDelta(2,4)=(0.0,0.0)
SDelta(3,3)=(0.0,0.0)
SDelta(3,4)=(-1/RO2)
SDelta(4,3)=(0.0,0.0)
EPS=0.00001
ND=4
CALL CINVRT(SDELTA,ND,N,EPS,SD,SB,SC,IP,IQ,KEY)
IF (KEY.EQ.2) GO TO 110
GO TO 120
110 WRITE (3,500)
500 FORMAT('0 THE MATRIX SDELTA IS SINGULAR, 1 NO SOLUTION IS POSSIBLE')
GO TO 190
120 DO 180 I=1,4
SVOLT=(0.0,0.0)
130 DO 130 J=1,4
SVOLT=SVOLT+SDelta(I,J)*SOLN(J)
130 X=CABS(SVOLT)
IF (REAL(SVOLT)) 132,131,136
131 IF (AIMAG(SVOLT)) 134,134,135
134 Y=-90.0
GO TO 133
135 Y=90.0
GO TO 133
132 Y=(57.3*ATAN(AIMAG(SVOLT)/REAL(SVOLT)))+180.0
GO TO 133
136 Y=57.3*ATAN(AIMAG(SVOLT)/REAL(SVOLT))
133 CONTINUE
GO TO (140,150,160,170),I
140 WRITE (3,600)RO1,RO2,X,Y
600 FORMAT('1 THE TERMINATING RESISTORS OF',F5.2,' AND',F5.2,
2 OHMS, RESPECTIVELY. THE TERMINAL VOLTAGES ARE',
3/F,35X,'$V(0)=',F9.4,' VOLTS, AT AN ANGLE OF',
4/F7.1,' DEGREES.')
SZIN=(SVOLT*RO1)/(1.0-SVOLT)
GO TO 180
50 WRITE (3,700) X, Y
7000 FORMAT (/35X, 'V2(0)=', F9.4, ' VOLTS, AT AN ANGLE
1 OF', F7.1, ' DEGREES.')
GO TO 180
160 WRITE (3,800) X, Y
3000 FORMAT (/35X, 'V1(0)=', F9.4, ' VOLTS, AT AN ANGLE
1 OF', F7.1, ' DEGREES.')
GO TO 180
170 WRITE (3,900) X, Y
9000 FORMAT (/35X, 'V2(0)=', F9.4, ' VOLTS, AT AN ANGLE
1 OF', F7.1, ' DEGREES.')
FREQ = AIMAG(SF)
GT = (4*(X**2)*R01)/R02
GTDB = 10.0*ALOG10(GT)
WRITE (3,1000) GTDB, FREQ
0000 FORMAT ('0',20X, 'THE DEVICE'S TRANSDUCER
1 GAIN IS', F5.2, ' DECIBELS AT ', E10.3, ' HZ.')
WRITE (3,1200) ZIN
2000 FORMAT ('0',20X, 'THE DEVICE'S INPUT IMPEDANCE IS ',
12F9.4, ' OHMS')
IF (GTDB+3.0) 210,210,180
180 CONTINUE
190 IF (AIMAG(SF)-9.5*AIMAG(SI)) 220,230,230
220 SF = SF + SI
GO TO 10
230 SI = SF
SF = SF + SI
GO TO 10
210 CONTINUE
STOP
ENDE.
Sample Set of Input Data Cards
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The Traveling-Wave Transistor


Field-Effect Transistors


**strip Transmission Lines**


VITA

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On January 27, 1968, he became married to the former Julia E. Taylor, and presently resides in Rolla, Missouri. He is a member of Triangle Fraternity, Eta Kappa Pi, Tau Beta Pi, Phi Eta Sigma, Phi Kappa Phi, and Blue Key Fraternity.