An analysis of pressure build-up theory

Hansraj J. Patel

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AN ANALYSIS OF PRESSURE BUILD-UP THEORY

BY

HANSRAJ J. PATEL

A

THESIS

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Approved by

J. P. Gover (advisor)

Charles E. Ortthe

Ray L. Morgan
ABSTRACT

Various theories regarding the pressure build-up characteristics for producing wells are analyzed. The merits and limitations of each theory are discussed. The error involved in plotting the pressure build-up data for the case of well-bore pressure versus shut-in time is pointed out. Equations have been presented for the various quantitative determinations of reservoir properties which can be obtained from typical pressure build-up data.

Utilizing all the previous methods presented in the literature, a recommended procedure for field application is proposed. One oil well and three gas wells have been analyzed using the recommended procedure with the attendant calculations given in detail. Two gas wells have been analyzed using both the cumulative production since completion of the well and cumulative production since the last build-up test of the well. The small error introduced by using the more accessible cumulative production since the last test is noted for each of the gas wells investigated.
ACKNOWLEDGMENTS

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INTRODUCTION

When a producing well is shut-in, its bottom-hole pressure will increase. If the shut-in time is sufficiently long, the bottom-hole pressure will approach that pressure defined as the static bottom-hole pressure. The rate of the pressure build-up is a function of certain properties of the reservoir rock, the fluids therein and the previous production history.

Pressure build-up characteristics of wells following shut-in have been used for many years by petroleum engineers to obtain static bottom-hole pressure by extrapolation of build-up data to avoid the long time requirements for such wells to reach static pressure. Many authors (1-8 incl.) have pointed out various additional uses of the pressure build-up characteristics, the most important of which are:

1.) Formation damage limit \(^{(1)}\)
2.) Early quantitative determination of reserves \(^{(2)}\)
3.) Skin effect \(^{(3, 4)}\)
4.) Damage factor \(^{(5, 6)}\)
5.) Gas well stabilization factor \(^{(7)}\)
6.) Evaluation of acid treatments \(^{(8)}\)

As early as 1937, Muskat \(^{(9)}\) suggested and derived a relationship between the slope of the straight line portion of the typical build-up curve and the permeability of the formation in the immediate vicinity of the well-bore. The author pointed out that the typical bottom-hole pressure increase with time yielded a stretched out "S" or sigmoid type curve.
 Basically, an analysis of pressure build-up data is dependent upon obtaining exact solutions of the fundamental differential equation of continuity. (10)

\[
\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} = \frac{f \mu c}{k} \frac{\partial p}{\partial t}
\]

Exact solutions of the above fundamental differential equation of continuity have been obtained for various boundary conditions, assuming the well as a point source or a radial source. There are several other assumptions inherent in the solution which will be discussed later.

The purpose of this study is three fold:

1.) To bring together the several methods suggested for pressure build-up analysis;

2.) To point out similarities and differences between these methods and

3.) To derive a procedure for pressure build-up analysis utilizing the best suggestions from all previous methods presented in the literature.
LITERATURE REVIEW

Pressure build-up curves were analysed as early as 1937 by Muskat who suggested a relationship between the slope of the straight line portion of the typical build-up curve and the permeability of the formation in the immediate vicinity of the well bore.

Since that time, several methods (11, 12, 13) have been proposed for analysis of pressure build-up data based on solutions of the unsteady-state equation of continuity for Darcy flow in radial system:

For gas:
\[
\frac{\partial^2 P}{\partial r^2} + \frac{1}{r} \frac{\partial P}{\partial r} = \frac{f \mu \partial P}{k \partial t}
\]
(1)

For slightly compressible liquid:
\[
\frac{\partial^2 P}{\partial r^2} + \frac{1}{r} \frac{\partial P}{\partial r} = \frac{f \mu c}{k c} \frac{\partial P}{\partial t}
\]
(2)

The symbols used are in compliance with the standards set forth by the American Petroleum Institute in 1956. Explanations are given in the nomenclature.

Several assumptions are inherent in the solution of these equations. The assumptions, which are valid for both equations, are:

1.) An undersaturated, single phase fluid is flowing in the reservoir.

2.) The viscosity, compressibility and density of the flowing fluid are assumed constant under all reservoir conditions.

3.) The production rate is a stabilized rate prior to shut-in.

4.) The well is shut-in at the sand-face, so that no fluids are produced into the well-bore after shut-in.
5.) The sand comprising the reservoir is uniform in its properties, and

6.) the reservoir is a horizontal, circular cylinder.

The first step usually taken in the development of the methods for analysis of reservoir data is to describe the reservoir as a hypothetical model. (The properties of the hypothetical model should be as close as possible to those of the reservoir). Mathematical equations relating various reservoir quantities have been derived for this hypothetical model. Various necessary conditions accompany the rigorous derivation of these equations to satisfy the assumed limitations of applicability. In most cases, applications of the methods are meaningful only when the conditions assumed in the derivation closely approximate those met in the actual reservoir.

The methods proposed in the literature for pressure build-up analysis can be classified according to the reservoir boundary and initial conditions imposed (assumed) in the solution of the equation of continuity. These conditions assumed for the solution can be grouped into two main groups:

Group I.

A small inner boundary (the well bore radius) over which the steady-state flow of compressible fluid is constant and a large but finite outer reservoir (the drainage radius) at which:

a.) The pressure remains constant after shut-in, or

b.) there is no influx of fluid across the boundary after shut-in.
Group II.

a.) A small inner boundary (the well bore radius) over which the steady-state flow of compressible fluid is constant, and an infinite outer reservoir boundary at which the pressure remains constant after shut-in.

b.) The inner boundary is vanishingly small.

Miller, Dyes and Hutchinson Method:

Miller, Dyes and Hutchinson (13) have presented a method for pressure build-up analysis to determine static reservoir pressure and permeabilities. Their approach investigates a system with:

1.) A compressible fluid flowing radially.

2.) The pressure at the finite outer boundary remains constant after shut-in and there is no influx of fluid across this boundary.

Their equations define the build-up characteristics for the shut-in well as a function of:

1.) Shut-in time ($\Delta t$).

2.) Effective permeability ($k_e$) and porosity ($f$) of the drainage area.

3.) Viscosity ($\mu$) and compressibility ($c$) of the flowing fluid.

4.) Production rate prior to shut-in ($g$) and

5.) Drainage radius of the well ($r_e$).

The permeability obtained from their investigation is the effective permeability of the formation, which is an average value that assumes radial flow but excludes the damaged, or improved zone in the immediate well-bore vicinity. The effective permeability is a measure of the inherent capacity of the undamaged formation to transport fluid.
average permeability calculated from productivity index includes the effect of damaged or improved zone. Comparison of average and effective permeability indicates the extent to which the well-bore has been damaged or improved.

They gave the following equations:

Stabilized time for steady-state conditions

\[ t_s = \frac{50 \, f \mu_e r_e^2}{k} \]  \hspace{1cm} (3)

\[ t_s = \frac{50 \, f \, c_t \mu_o r_e^2}{k \left( \frac{\gamma_o + \gamma_g R}{\beta_o} \right)} \]  \hspace{1cm} (4)

\[ t_{si} = \frac{190 \, f \, c \mu_o r_e^2}{k} \]  \hspace{1cm} (5)

\[ t_{si} = \frac{190 \, f \, c_t \mu_o r_e^2}{k \left( \frac{\gamma_o + \gamma_g R}{\beta_o} \right)} \]  \hspace{1cm} (6)

Equations (3) and (4) give the time (in days) required to approximate a steady-state condition to obtain a stabilized production rate prior to shut-in. Equations (5) and (6) give the prior estimation of shut-in time in hours required to obtain satisfactory data. Equations (3) and (5) are for single-phase flow, while (4) and (6) are for two-phase flow.

Horner Method:

Horner\(^{(10)}\) has proposed a method utilizing the conditions described under Group II.

Equation (2) was applied as the basic equation relating the pressure
relationship for the hypothetical model. By making use of the point source solution, an equation which describes at time $t$ the well-bore shut-in pressure was presented:

$$p_w = p_o + \frac{q \mu}{4 \pi kh} Ei \left( -\frac{r_w^2 f \mu c}{4k t} \right)$$  \hspace{1cm} (7)

Where:

- $p_w$ = Well-bore pressure at time $t$
- $p_o$ = Static reservoir pressure
- $Ei (-x) = - \int_x^{\infty} \frac{e^{-u}}{u} du$.

The error introduced by Horner in the above equation as the basic solution for the case of an infinite reservoir is in considering the well radius as being infinitely small.

For constant production rate, well-bore shut-in pressure at any time $\Delta t$ after shut-in in a single well in an infinite reservoir is:

$$p_w = p_o + \frac{q \mu}{4 \pi kh} \left\{ Ei \left( -\frac{r_w^2 f \mu c}{4k (t_o + \Delta t)} \right) - Ei \left( -\frac{r_w^2 f \mu c}{4k \Delta t} \right) \right\}$$  \hspace{1cm} (8)

By approximating the $Ei (-x)$ function into logarithmic equality, Horner gave as a final equation:

$$p_w = p_o - \frac{q \mu}{4 \pi kh} \ln \left( \frac{t_o + \Delta t}{\Delta t} \right).$$  \hspace{1cm} (9)

Horner showed that the error introduced by this approximation is very small. As soon as $\Delta t > 25 \frac{r_w^2 f \mu c}{k}$, the error in $p_w$ will decrease to 0.25 percent. This condition will be satisfied within a matter of seconds after the closing in of the well.

The above equations have been derived for a well which produces at
a constant rate, \( q \), from time zero to time \( t_0 \), and is then shut-in. However, such conditions hardly exist in reality. Some correction must then be applied to take into account the varying rates of production during the producing history of the well. However, the final results are laborious in application and are more precise than is required in most applications.

It was further pointed out that the outer boundary conditions assumed in this procedure are most nearly fulfilled by new wells in a new reservoir. An incorrect estimate of static reservoir pressure may be obtained when these conditions are not met. All of the previous equations can only be expected to be applicable in the case of a well which has not yet produced sufficient volumes such that the over-all static reservoir pressure has not decreased, i.e. a new well in which the effects of the reservoir boundary have not yet become apparent.

A modification of this method of build-up curve analysis for application to a reservoir with a finite reservoir boundary, which is essentially a material balance relationship used to approximate the change in boundary conditions, was also suggested.

**Thomas Method**

Thomas\(^\text{6}\) extended Horner's method of pressure build-up analysis by considering, in addition, a "damage factor". The damage factor is a measure of the reduced productivity resulting from a low permeability zone surrounding the well-bore, and is based on comparisons of the effective permeability from the build-up curve and permeability calculated using the productivity index. Thomas gave an equation for damage factor:

\[
D = 1 - \frac{2m \ln \left( \frac{r_e}{r_w} \right)}{P_0 - P_f}.
\]
Where: \( m \) is a slope of the pressure build-up curve.

Thomas pointed out that calculated effective permeability \((k_e)\) and damage factor \((D)\) are more reliable than the static pressure determination from pressure build-up data. It was further pointed out that the wells completed in reservoir which has produced a small fraction of their fluids will give better approximation when equation (10) is used.

This extension of Thomas’s on Horner’s procedures presents:

1.) Graphical estimations of static reservoir pressure \((p_0)\).
2.) Determination of the productive capacity of the productive horizon away from the well-bore \((kh)_e\), and
3.) The degree to which the formation adjacent to the well-bore has been damaged by completion or other causes \((D)\).

Van Everdingen’s Investigations:

Van Everdingen(4) has used the basic set of boundary conditions set forth in group II. Van Everdingen presented the following equations which describe the pressure differentials existant under compressible fluid flow conditions:

A. Steady-state flow:

\[
p_0 - p_f = \frac{q}{2\pi kh} \left( \ln \left( \frac{k_t \omega}{f_{pcr_w}} \right) - \frac{1}{2} \right).
\]  

\((11)\)

B. Unsteady-state flow:

1.) Considering the well as a unit circle source:

\[
p_0 - p_w = \frac{q}{4\pi kh} \left[ \ln \left( \frac{k_t \omega}{f_{pcr_w}} \right) + 0.809 \right].
\]  

\((12)\)

2.) Considering the well as a point source:

\[
p_0 - p_w = \frac{q}{4\pi kh} \ Ei \left( \frac{-f \ p_c r_w^2}{4 \ k t} \right).
\]  

\((13)\)
Equation (13) approaches equation (12) closely when the value of 
\[ \frac{f \mu c r_w^2}{4 k t} \] is less than 0.01. This condition will be satisfied within seconds after the closing in of the well.

Skin effect, \( S \), expresses the restriction to flow in the vicinity of the well-bore. A positive value of \( S \) indicates a restriction (skin) does exist, while a negative value indicates that the skin does not exist.

When this skin effect is considered, equation (12) becomes:

\[ P_0 - P_w = \frac{q_0 \mu c S_0}{4 \pi k_0 h} \left[ \ln \frac{r_o}{f} + \ln \frac{r_w}{c} + \frac{0.809 + 25}{t} \right] \]  
(14)

Or, if the skin effect is desired:

\[ S = \frac{1.151 (p_1 hr - p_f^2)}{m} - 1.151 \log \left[ \frac{q_0 T_z}{1.033 m h f r_w^2} \right] \]  
(15-A)

Or

\[ S = \frac{1.151 (p_1 hr - p_f)}{m} - 1.151 \log \left[ \frac{q_0 \rho c^2}{10.4 m h f c r_w^2} \right] \]  
(15-B)

There are two advantages of the skin-effect method in describing the well's conditions. They are:

1.) An estimate of fully build-up pressure is not required and
2.) the drainage radius, \( r_e \), need not be determined. However \( f, \mu, c, h \) must be known, although their effect is dampened by their being grouped in a logarithmic term.

The pressure drop in the "skin" or damaged zone near the well-bore is given by:

\[ \Delta P_{\text{skin}} = \Delta P(1 \text{ cycle}) \times 0.875. \]  
(16)

A measure of the efficiency of completion, acid treatment or clean-
ing job can be obtained by comparing the actual productivity index \((j)_{\text{actual}}\) and the ideal productivity index \((j)_{\text{ideal}}\). The ratio of these two quantities is:

\[
\text{Flow Efficiency} = \frac{\frac{j_{\text{actual}}}{j_{\text{ideal}}}} = \frac{P_0 - P_f - \Delta P_{\text{skin}}}{P_0 - P_f}.
\] (17)

In addition, the effect of storage capacity of casing and tubing on the pressure build-up curve was reviewed.

This method permits estimation of:

1.) The effective permeability of the formation \((k_e)\).

2.) Static reservoir pressure \((P_0)\), and

3.) the value representing the magnitude of the skin effect \((S)\).

**Hurst Analysis**

Hurst\(^{(3)}\) presented a method of build-up curve analysis similar to that presented by Van Everdingen. Hurst has explained the discrepancy between the theoretical constant-rate problem and actual constant-rate problem, by assuming that the rate of fluid entry into the well-bore from the formation is constant, and remains constant, thereafter.

Although this is the condition observed at the separator and stock tank from the time the fluid reaches the surface, it is not the condition that exists in the formation next to the well-bore. Further consideration reveals that the mechanical condition of a well is such, with the tubing suspended within the casing, that as soon as the annulus value is opened, the initial production is due to the unloading of the annulus to displace the fluid in the tubing. This unloading is very rapid in the beginning, and is quickly retarded by the stabilization of the bottom-hole pressure. Concurrent with this effect, fluid entry into the well from the
formation is initially zero, but rapidly increases to reach the constant-rate when such stabilization is established. The composite effect of the two reactions is the constant-rate of production observed at the surface. Thus, fluid influx into a well-bore is a variable-rate problem, which causes the initial pressure decline observed.

Hurst’s analysis gives an estimation of:

1.) The effective permeability of the formation \( k_e \)
2.) Static reservoir pressure \( p_o \), and
3.) damage around the well-bore \( D \).

Arps Analysis

Arps\(^{(5)}\) published an analysis in which graphical methods are used almost exclusively. Equations presented by Van Everdingen were applied graphically. Equation (17) was used to determine the completion factor. The completion factor of Arps corresponds to the flow efficiency of Van Everdingen, and also is related to the damage factor which is defined by Thomas as

\[
c_r = (1 - D)
\]

(18)

where \( c_r \) is condition ratio.

Two different approaches to the determination of condition ratio were suggested: (1) For finite depletion-type or solution gas drive-type reservoirs, and (2) For an infinite under-saturated reservoir. Variations were suggested to fit both the condition of no flow across a finite reservoir boundary and no flow across an infinite reservoir boundary. Arps analysis estimates all the same three quantities as estimated by Hurst’s method.

The Tracy Technique:

Tracy\(^{(14)}\) analysed pressure build-up curves in a manner similar to that of Miller, Dyes and Hutchinson. By comparing pressure build-up and
productivity index estimates of formation permeability, a measure of well damage is obtained. This measure is termed the "condition ratio".

The condition ratio is:

\[ cr = \frac{k h \text{ (from P1 data)}}{k h \text{ (from build-up data)}} \]  

(Complete equations for condition ratio are derived in appendix A and B).

If the value of the condition ratio is different from 1.0, the permeability next to the well-bore is different from that in the interwell area. If the condition ratio is greater than 1.0, the permeability near the well-bore is greater than the interwell permeability which implies an absence of permeability damages around the well-bore. A deep penetrating fracture treatment may substantially increase the production rate of the well. If the condition ratio is less than 1.0, the permeability near the well-bore is less than the interwell permeability which means that the well has a damaged zone adjacent to the well-bore. Any well with a condition ratio of less than 0.80 is considered a prospect for a stimulation treatment to overcome the apparent well-bore damage.

The author analysed the effect of after-production on pressure build-up curve. Production of reservoir fluids into the well-bore after shut-in would delay observed pressure build-up trends for appreciable periods of shut-in times. When this occurs it is often difficult to interpret the build-up curve. If observed pressures were corrected to obtain pressures that would have been observed in the absence of production after shut-in, it would make possible more positive identification of the straight line portion of the curve as well as the shut-in time required for a successful analysis.
McMohan Analysis:

McMohan (7) developed a method for determining gas well "stabilization factors" from build-up tests. This stabilization factor is defined as the ratio of a well's performance under pseudo-steady state conditions to the well's performance at the end of a given length of flowing time. It was further pointed out that the build-up tests are interchangeable with flow tests on the same wells. The stabilization factor of a well depends upon the well's own history as well as the status of surrounding wells.

McMohan analysed 319 pressure build-up test from 229 wells and 41 flow tests from 41 wells in the Hugoton Field of Kansas, Oklahoma, and Texas. It was further concluded from this analysis that in highly permeable reservoirs, the stabilization factor is approximately unity, but in tight, lower permeability reservoirs it can decrease to 0.25 or even lower values.
DISCUSSION

Methods of build-up curve analysis assuming an incompressible reservoir fluid are of limited applicability, and have only theoretical importance. All methods depend on a single mathematical foundation which is implied by the conditions set forth by equation (2).

When well-bore pressure is plotted against the logarithm of shut-in time, a typical inflection-type curve results. The pressure performance curve can be typically shown to be composed of three regions, as shown in Figure I.

Region I:

Much of the initial portion of region I may not be observed in the field build-up curve, since the abrupt rise in pressure noted is often completed in very short time after shut-in. The shape of the early portion varies with two factors, which are as follows:

1.) Fill-up or after-flow into the casing and tubing during pressure build-up operation and,

2.) a zone next to the well-bore with permeability different from that in the interwell area.

The earliest portion of initial build-up curve is of little use in establishing well-bore conditions. It was shown by Tracy, that the effect of after-flow cannot be considered negligible until 60 minutes build-up time has elapsed.

Region II:

The second region of the typical build-up curve is the
Figure I

Typical Ideal Pressure Build-up Curve

- Region I: After flow region
- Region II: Interwell region
- Region III: Outer boundary effects

Time after shut-in Hours

Wellbore pressure (psi)

Logarithmic scale
straight-line portion of interest. It follows immediately after the initial abrupt well-bore pressure increase due to fill-up.

This region can be used to determine an average value of the permeability throughout the interwell area as shown by a rearrangement of the slope relationship as follows:

\[ k_e = \frac{q_o \mu_o \beta_o}{h m} \]  

(20)

The straight-line portion of a pressure build-up curve is not interpreted in the same manner by all the authors:

Region III:

Ideally, the third region of the build-up curve displays a gradual levelling off in well-bore pressure as static reservoir pressure is approached. This region is seldom measured in field tests, due to the excessive shut-in times required. The most accurate measurement of static reservoir pressure is obtained from this final region of the build-up curve. This region represents the behavior of the well-bore pressure which is being effected by the boundary of the system from which the fluid is producing. Since between any two wells there is an interference point, all wells within a field must be considered bounded. If interference between wells is observed, it means that the static reservoir pressure is being approached during the build-up test.

Consider a single well in an infinite reservoir which is completed and first brought into production at time zero, and which subsequently produces at a constant rate, \( q \), until time \( t_o \) at which it is shut-in.
Then, ignoring the effect of the after-production, the well pressure \( p_w \) at time \( (t_o + \Delta t) \), where \( \Delta t \) represents the time of shut-in, may be obtained by superimposing two solutions of the equation:

\[
p_w = p_o + \frac{q \mu}{4 \pi k h} \text{Ei} \left( \frac{-r^2}{r k t} \right).
\] (21)

By substituting \( r_w \) for \( r \) and considering only the time since shut-in, equation (21) becomes

\[
p_w = p_o + \frac{q \mu}{4 \pi k h} \left[ \text{Ei} \left( \frac{-r_w^2}{4k (t_o + \Delta t)} \right) - \text{Ei} \left( \frac{-r_w^2}{4k \Delta t} \right) \right].
\] (22)

For small values of its argument, the Ei function may be approximated by a logarithmic function. If this approximation is made in equation (22), the following basic build-up equation for a single well in an infinite reservoir is obtained.

\[
p_w = p_o + \frac{q \mu}{4 \pi k h} \ln \left( \frac{t_o + \Delta t}{\Delta t} \right)
\] (23)

Thus, in the case of a well which has produced uniformly since completion at a rate \( q \) from an infinite reservoir, it may be expected that the bottom-hole pressure would build-up in accordance with equation (23). If \( p_w \) is plotted versus \( \ln \left( \frac{t_o + \Delta t}{\Delta t} \right) \), the points would be expected to fall on a straight line for the ideal case.

To argument this ideal case, two general methods of plotting build-up data are applied: (1) well-bore pressure \( (p_w) \) is plotted versus log of shut-in time \( (\Delta t) \) and, (2) well-bore pressure \( (p_w) \) is plotted versus log \( \left( \frac{t_o + \Delta t}{\Delta t} \right) \), or versus log \( \left( \frac{\Delta t}{t_o + \Delta t} \right) \).

In this investigation, a plot of \( (p_w) \) versus log \( \left( \frac{\Delta t}{t_o + \Delta t} \right) \) has been selected due to the ease of extrapolating the value of \( \left( \frac{\Delta t}{t_o + \Delta t} \right) \) to one for the purpose of finding the static reservoir pressure.
after very long times. If the other parameters were selected, then the value of time parameter must be extrapolated to a very long time (infinite shut-in time) for the purpose of finding the static reservoir pressure.

The slope of the linear portion of these plots is used for the determination of the interwell permeability. Methods expressing skin damage or well-bore condition also utilize the slope of this region of the build-up curve.

Some authors have used the time ratio plot in an infinite reservoir analysis while others have applied a $p_w$ vs. $\Delta t$ approach, assuming that $\Delta t$ is small compared to time $t_0$. Thomas noted that a $p_w$ vs. $\Delta t$ plot may be applied if the shut-in time is a small fraction of flowing time, when it is acknowledged that as $\Delta t$ becomes an appreciable fraction of $t_0$, the error increases and extrapolation to infinite shut-in time becomes impossible.

The time intervals representing infinite behavior may be of a long duration as in the case of large drainage areas or when one or more of the factors comprising the diffusivity constant, $\frac{k}{f \mu c}$, impose large transient times. A tight sand, lime or a gas reservoir, may exhibit this infinite behavior for a long period of time.

Equation (23) can be rewritten as follows:

$$p_w = p_o - q_o \frac{\mu_o \beta_o}{4\pi k_o h} \ln \left( \frac{\Delta t}{t_0 + \Delta t} \right)$$

which in practical field units, becomes

$$p_w = p_o \cdot 70.62 \frac{q_o \mu_o \beta_o}{k_o h} \ln \left( \frac{\Delta t}{t_0 + \Delta t} \right)$$
or, using common logarithms,

\[ p_w = p_o + 162.6 \frac{q_o \mu_o \rho_o}{k_o h} \log \left( \frac{\Delta t}{t_o + \Delta t} \right). \]  

(26)

From equation (26), the slope of the time plot is

\[ m = \frac{d}{d \ln \left( \frac{\Delta t}{t_o + \Delta t} \right)} = 162.6 \frac{q_o \mu_o \rho_o}{k_o h}. \]  

(27)

The slope of a \( \Delta t \) plot may be obtained by partially differentiating \( p_w \) with respect to \( \ln(\Delta t) \) for a constant \( t_o \). This will give

\[ m(\Delta t) = \frac{p_w}{\ln(\Delta t)} = m \left[ \frac{t_o}{t_o + \Delta t} \right]. \]  

(28)

It is seen from the above equation that equation (27) is modified by multiplication by the factor \( \left( \frac{t_o}{t_o + \Delta t} \right) \). It indicates that the commonly used \( p_w \) vs. \( \Delta t \) plot is not generally linear, nor does it have the same slope as the time ratio plot except at \( \Delta t = 0 \). The slope of equation (28) will be constantly decreasing in value until \( \Delta t \) approaches infinity.

If the flowing time is long, (i.e. \( t_o > \Delta t \)), the \( \Delta t \) vs. \( p_w \) plot will be linear for a period of time. However, after extended shut-in times, it will take on curvature, and the \( \Delta t \) at which significant curvature begins will be a function of \( t_o \).

By taking a second derivative of \( p_w \) with respect to \( \ln(\Delta t) \) of equation (28), it is evident that the slope of equation (28) is constantly changing:

\[ \frac{\partial}{\partial \Delta t} \left[ \frac{\partial p_w}{\partial \ln(\Delta t)} \right] = \frac{\partial m(\Delta t)}{\partial \ln(\Delta t)} = m \left[ -\frac{t_o \Delta t}{(t_o + \Delta t)^2} \right]. \]  

(29)
The rate of change of the slope of a straight line is zero. Hence, from equation (28), the rate of change of slope is seen to be zero only at \( \Delta t = 0 \) and \( \Delta t = \infty \). These time interval values correspond to those in equation (28) in which \( m(\Delta t) \) is the same as \( m \) of the time ratio plot, when \( \Delta t = 0 \).

Between limits \( \Delta t = 0 \) and \( \Delta t = \infty \), equation (29) has a finite value and the \( \Delta t \) plot is not linear. However, as indicated previously, where \( t \) is long, the changing slope of \( m(\Delta t) \) may be very small during the first portion or perhaps all of a build-up period allowed by economic considerations (generally 72 hours or less).

Equations (27) and (28) can be used to develop an expression which indicates the magnitude of the difference in these two common plotting methods. For a given time \( t_0 \), the ratio of the correct slope to that of a \( \Delta t \) plot at any point \( \Delta t \) will be

\[
\frac{m}{m(\Delta t)} = \frac{m}{m(t_0 + \Delta t)} = \left[ \frac{t_0 + \Delta t}{t_0} \right].
\]

If the ratio indicated by equation (30) is significantly greater than unity, the slope value from a \( \Delta t \) plot will be unacceptable. Or for a given time \( t_0 \), the percent error in the slope from a \( \Delta t \) plot, at any point \( \Delta t \), in terms of the correct time ratio slope is:

\[
\text{percent error} = 100 \left( \frac{m - m}{m} \right) = 100 \left[ \frac{t_0 + \Delta t - t_0}{t_0 + \Delta t} \right] = 100 \left[ \frac{\Delta t}{t_0 + \Delta t} \right].
\]
Equation (31) involves $t_0$ and $\Delta t$, so values of $t_0$ and $\Delta t$ should provide a good simple check as to whether a $\Delta t$ plot can be used or not.

Figure II relates the superimposed production rates for build-up analysis. There are two constant-rate transients which are involved

![Diagram](image-url)

Fig. II - Superimposed Production Rate for Pressure Build-up Analysis
These two transients are: drawdown transients and build-up transients respectively. For example, a well flows at a constant rate \( q \) for time \( t_0 \). At shut-in the rate is zero, but in terms of net pressure effect, rate \( q \) may be considered to be extending from time zero forward through the shut-in time, superimposing a \((-q)\) rate throughout the shut-in time period \( \Delta t \).

For short flowing times, the drawdown transient is still producing a measurable pressure drop at the well-bore during the shut-in period. Thus, net pressure changes experienced at the well and as recorded by pressure bomb are due to both the build-up transient during \( \Delta t \) and the drawdown transient operating during \( (t_0 + \Delta t) \).

If the well has been flowing for a relatively long time, well-bore pressure changes with time prior to shut-in will be small. In this event, recorded net pressure change will be due mostly to the \((\Delta t)\) transient and a plot of \( p_w \) vs. \( \ln(\Delta t) \) will be linear for a considerable range of \( (\Delta t) \). Later portions of the plot will eventually take on curvature due to the well-bore pressures approaching the static reservoir pressure asymptotically.

**Gas Well Analysis:**

The equations shown above have been developed for the purpose of analysing pressure build-up tendencies in oil wells. Equations for a gas well can be derived from the equation (14).

The following changes are necessary in equation (14) to be applicable to compressible fluid flow:

1.) \( p_w \) in the equation is replaced by \( p_w^2 \) and \( p_0 \) by \( p_0^2 \).

2.) In the denominator of the coefficient of the bracket 4 is replaced by 2.
3.) \( q_0 \beta_0 \) is replaced by \( q_g z^{\frac{n}{Ta}} \).

4.) \( \mu_0 \) is replaced by \( \mu_g \).

5.) The compressibility factor \( c \) is replaced by the reciprocal of the static pressure, \( p_r \).

Derivation of this equation is presented in the appendix.

By making the above substitutions, the following equation for gas flow pressure determinations is obtained.

\[
p_o^2 - p_w^2 = \frac{q_g \mu_g T z}{2 \pi k_g h Ta} \left[ \ln \frac{k_a + p_r}{\mu r_w z} + 0.809 + 2 S \right]
\]

Where 

\( T_a = \) Base temperature °R 
\( T = \) flowing temperature °R 
\( z = \) deviation factor 
\( p_r = \) average reservoir pressure.

The analysis of the pressure build-up curves for gas flow allows the same parameter to be plotted as described previously with the exception that the \( p_w \) is replaced by \( p_w^2 \) and \( p_w^2 \) is plotted versus log of shut-in time \( \Delta t \) or \( \log(\frac{\Delta t}{t_0} + \Delta t) \). The slope analysis is identical to that of the slightly compressible flow previously discussed.

In practical engineering units, the slope \( m \) is:

\[
m = 1637 \frac{q_g \mu_g k h}{T z} \]

**Variable Production rate before Closing in**

Equation (23) was derived for a well which produces uniformly at rate \( q \) from time zero to time \( t \) and is then closed in. However, such conditions do not normally exist, and so some correction must be applied to take account of the varying rates at which a well will have produced
during its history.

To illustrate the methods used for corrections, suppose that the production history of the well was as shown by the broken line in Figure III. The production history can be approximated by superimposing a series of steps (as shown in Figure III) and then the relationship

Fig. III - Illustration for Correcting Variable Production Rate.
given by equation (23) is used in the modified form:

\[
p_w = p_0 - \left\{ q \frac{\ln \frac{t_0 + \Delta t}{t_0 + \Delta t - t_1}}{t_0 + \Delta t - t_1} + q_1 \frac{\ln \frac{t_0 + \Delta t - t_1}{t_0 + \Delta t - t_2}}{\Delta t} + q_2 \frac{\ln \frac{t_0 + \Delta t - t_2}{t_0 + \Delta t - t_3}}{\Delta t} + q_3 \frac{\ln \frac{t_0 + \Delta t - t_3}{\Delta t}}{\Delta t} \right\}.
\]

The quantities \( t \) and \( q \), as indicated in Figure III, and are so approximated that they represent the same total production as the well actually produced.

\[
p_w \text{ is plotted versus } q_0 \frac{\ln \frac{t + \Delta t}{t + \Delta t - t_1}}{t + \Delta t - t_1} + q_1 \frac{\ln \frac{t + \Delta t - t_1}{t + \Delta t - t_2}}{\Delta t} + q_2 \frac{\ln \frac{t + \Delta t - t_2}{t + \Delta t - t_3}}{\Delta t} + q_3 \frac{\ln \frac{t + \Delta t - t_3}{\Delta t}}{\Delta t} \text{ etc.}
\]

The above equation has only theoretical importance. Instead of making this correction equation (24) can be modified by introducing a so-called correct time \( t_c \),

\[
p_w = p_0 - q \frac{\mu}{4 \pi k h} \ln \frac{t_c + \Delta t}{\Delta t}.
\]

Where \( q \) is calculated from the last established production rate before closing in the well \( t_c \) is obtained by dividing the total cumulative production of the well by the last established production rate,

\[
t_c = \frac{N_p}{q}
\]

**Well at the Center of a finite Circular Reservoir**

Exact solutions to the flow equation:
\[
\frac{\partial^2 P}{\partial r^2} + \frac{1}{r} \frac{\partial P}{\partial r} \approx \frac{f c \mu}{k} \frac{\partial P}{\partial t}
\]

are available for the case of a single well in the center of a finite circular reservoir. (21)

\[
P_w = p_o + \frac{q \mu}{2 \pi k h} \left\{ \frac{3}{4} + \ln \frac{r_w}{r_e} - \frac{\frac{r_w^2}{r_e^2} - \frac{2k t}{f \mu c r_e^2}}{\frac{2 \sum_{n=1}^{\infty} J_n \left( \frac{x_n r_w}{r_e} \right) - \frac{x_n^2 k t}{f \mu c r_e^2}} \right\}
\]

where: \(x_n (n = 1, 2, \ldots)\) are the roots of \(J_1(x) = 0\).

The above solution is complicated for general engineering use. Hornor derived an approximating equation to equation (36).

\[
P_w = p_o - \frac{q \mu}{4 \pi k h} \left\{ \ln \frac{t + \Delta t}{\Delta t} + \frac{r_e^2 f \mu c}{4 k (t + \Delta t)} \right\}
\]

where the function \(v\) is defined by:

\[
y(u) = Ei(-u) + \frac{1}{u} e^{-u}.
\]

Until \(\Delta t\) is very large, the second \(y\) function will be almost zero and the first will be nearly constant.

If we define \(u_1\) as \(\frac{r_e^2 f \mu c}{4 k t}\),

then

\[
P_w = p_o - \frac{q \mu}{4 \pi k h} \left\{ \ln \frac{t + \Delta t}{\Delta t} + y(u_1) \right\}.
\]
Figure IV shows the comparison between the exact equation (36) and the approximated equation (37).
Fig. IV - Comparison between Theoretical and Approximated values [Reproduced from Horner (10)]
SUGGESTED PROCEDURE FOR BUILD-UP ANALYSIS

The obtaining of reliable pressure build-up data on an oil or gas well depends on the proper analysis of well conditions existing prior to initiation of the pressure build-up test and on proper well preparation. The well conditions should be analyzed and the type of flow ascertained.

A. Well preparation:

1.) From fluid-pressure data determine whether single or two-phase flow exists.

2.) Calculate the time required to establish a steady-state flow condition thus allowing the determination of a stabilized production rate prior to shut-in. The approximate time required is given by equation (3) or (4) depending on whether it is a single-phase or two-phase fluid flowing. As values of \( k, r_e \) and \( c_t \) must be estimated from the best data available, it is advisable to allow a greater time for stabilization than the calculated value.

3.) Set the well on as low a flow rate as possible and maintain this flow rate for the stabilization period calculated in part A-2.

B. Performing the test:

1.) Measure the bottom-hole pressure prior to the time of shutting the well in.

2.) Close the well in and continue measuring the pressure as a function of time.

3.) If it is desired to obtain all possible information from the well, it should be left shut in for a length of time...
given by equations (5) or (6).

C. Analyzing the Results:

1.) Plot the bottom-hole pressure as a function of the

\[ \log \frac{\Delta t}{t_o + \Delta t} \]

where

\[ t_o = \frac{\text{Cumulative Production from well}}{\text{Stabilized Flow Rate}} \]

\[ \Delta t = \text{the elapsed time since the well was closed in.} \]

D. Determination of Permeability:

The slope of the straight line is used to calculate the effective permeability of the formation such that

\[ k_o = \frac{162.6 \ \alpha_o \mu_o \phi_o}{m \ h} \]

\[ k_g = \frac{1637 \ \alpha_g \mu_g \mu_o}{m \ h} \]

If any doubt exists as to the selection of the correct portion of the curve, the following test should be applied

\[ \frac{t}{t_o} = \frac{0.0002637 \ k \ t_o}{f \ c \ \mu \ r_w^2} \]

If the above calculated value falls in the range of \( 10^{-2} \) to \( 10^{-1} \), the proper straight line section was used and the permeability estimation is valid. \( (10) \)
E. Skin Effect Estimation:

The skin effect is calculated using equation (15-A) or (15-B).

F. Determining Static Reservoir Pressure:

The forward extrapolation of the straight line portion of I (infinite \( \Delta t \)) gives the value of pressure \( p^* \). Using this value of pressure \( p^* \) so obtained the value of the final closed in pressure \( (p_o) \) is obtained as follows:

1.) Classify the wells drainage area, and approximate the position of the well in the drainage area, i.e. a square, rectangular, circular etc.

2.) Calculate the value of \( t_D \) represented by the average total previous producing time

\[
\frac{0.00633 k t_o}{f \mu c A}
\]

where \( A \) is the area of the wells drainage area and \( t_o \) is total average producing time in days.

3.) Read number "\( N \)" from approximate curve of figure 2 to 8 from Mathews\(^{(12)}\). The curve selected should represent the type of drainage area as determined from the best available information.

The final closed-in pressure is then given by:

\[
p_o = p^* - N \left( \frac{m}{2.303} \right).
\]

G. Estimating Condition Ratio:

Condition ratio for an oil well is calculated by using the
following equation:

\[ cr = \frac{2m \log \left( \frac{r_e}{r_w} \right)}{P_o - P_f}. \]

The condition ratio for a gas well is calculated by using the following equation:

\[ cr = \frac{2m \log \left( \frac{r_e}{r_w} \right)}{P_o^2 - P_f^2}. \]
The pressure build-up data on wells A, B, C, and D have been analyzed using the suggested procedure. Well A is an oil well, while wells B, C and D are gas wells.

Calculation of the Skin Effect on well "A" shows that the permeability near the well-bore is practically the same as the permeability at the interwell area, which is again supported by the calculation of Condition Ratio.

Calculation of the Skin Effect on well "B" shows that the permeability near the well-bore is slightly greater than the permeability of the interwell area, which is again supported by the calculation of Condition Ratio.

Wells C and D have been analyzed using both the cumulative production since completion and the cumulative production since the last test. Most of the wells are tested once a year and the basic production quota assigned to the well is based on this test; and the well will be produced for the following year accordingly. Depending on the market demand, there is always a possibility that the well will be produced at a constant rate until a new test is required at the end of a year's production.

Analysis of wells C and D shows that appreciable error is introduced in the calculation of permeability using the incremental cumulative production, figures, but a negligible error is introduced in the calculation of static pressure.
EXAMPLE PROBLEMS

WELL A

A pressure build-up test on Kansas oil well yielded the following data:

<table>
<thead>
<tr>
<th>Shut-in time (hours)</th>
<th>Well pressure $P_w$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1,534</td>
</tr>
<tr>
<td>1</td>
<td>1,941</td>
</tr>
<tr>
<td>2</td>
<td>1,967</td>
</tr>
<tr>
<td>4</td>
<td>1,993</td>
</tr>
<tr>
<td>6</td>
<td>2,005</td>
</tr>
<tr>
<td>8</td>
<td>2,015</td>
</tr>
<tr>
<td>10</td>
<td>2,024</td>
</tr>
<tr>
<td>14</td>
<td>2,035</td>
</tr>
<tr>
<td>20</td>
<td>2,048</td>
</tr>
<tr>
<td>30</td>
<td>2,062</td>
</tr>
<tr>
<td>40</td>
<td>2,073</td>
</tr>
<tr>
<td>50</td>
<td>2,080</td>
</tr>
<tr>
<td>80</td>
<td>2,082</td>
</tr>
</tbody>
</table>

Net sand thickness, $h = 9.6$ ft.

Porosity, $f = 16\%$.

Average oil compressibility, $c = 50 \times 10^6 \text{ vol/vol/psi}$.

Well radius, $r_w = 3$ in.

Stabilized oil production rate prior to shut-in, $q_0 = 95$ STB/D

Cumulative oil production since completion, $N_p = 3040$ bbl.

Oil formation volume factor, $B_o = 1.29 \text{ vol/vol}$

Reservoir oil viscosity, $\mu_o = 0.7$ cp.

Well is drilled on 160 acre spacing.

$$t_o = \frac{24N_p}{q_o} = \frac{(24)(3040)}{(95)} = 768 \text{ hours}.$$
FIGURE V

WELL A

KANSAS

Pressure Build-up

\( p^* = 2180 \)

Slope \( m = 83 \text{ psia/cycle} \)

\( p_w \text{ 1 hr.} = 1942 \)
<table>
<thead>
<tr>
<th>Shut-in time (hours)</th>
<th>$\Delta t$</th>
<th>Well pressure $P_0$ (Psia)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t_0 + \Delta t$</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>0</td>
<td>1.535</td>
<td>1,535</td>
</tr>
<tr>
<td>1</td>
<td>1.941</td>
<td>1,941</td>
</tr>
<tr>
<td>2</td>
<td>1.967</td>
<td>1,967</td>
</tr>
<tr>
<td>4</td>
<td>1.993</td>
<td>1,993</td>
</tr>
<tr>
<td>6</td>
<td>2,005</td>
<td>2,005</td>
</tr>
<tr>
<td>8</td>
<td>2,015</td>
<td>2,015</td>
</tr>
<tr>
<td>10</td>
<td>2,024</td>
<td>2,024</td>
</tr>
<tr>
<td>14</td>
<td>2,035</td>
<td>2,035</td>
</tr>
<tr>
<td>20</td>
<td>2,048</td>
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<td>30</td>
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<td>2,062</td>
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<tr>
<td>40</td>
<td>2,073</td>
<td>2,073</td>
</tr>
<tr>
<td>50</td>
<td>2,080</td>
<td>2,080</td>
</tr>
<tr>
<td>80</td>
<td>2,082</td>
<td>2,082</td>
</tr>
</tbody>
</table>

CALCULATIONS:

**Permeability**

\[ m = \text{slope} = 83 \text{ psia/cycle} \quad \text{(Fig. - V)} \]

\[ k_0 = \frac{162.6 \ q_o \ P_o \ \rho_o}{m \ h} \]

\[ k_0 = \frac{(162.6)(95)(0.70)(1.29)}{(83)(9.6)} \]

\[ k_0 = 17.5 \text{ md} \]

**Skin Effect**

\[ S = 1.151 \ \frac{P_w \ 1 \text{ hr}}{m} - \frac{P_f}{1.151 \log \left( \frac{q_o \ \rho_o}{10.4 \text{ m h f c r}_w^2} \right)} \]

\[ S = 1.151 \ \frac{1942 - 1534}{83} - 1.151 \log \left( \frac{(95)(1.29)}{(10.4)(83)(9.6)(0.16)(50 \times 10^6)} \right) \]

\[ S = 0.512 \]
Static Reservoir pressure

\[ p^* = 2180 \text{ psia} \quad \text{(Fig. V)} \]

\[ t_D = \frac{0.00633 k_{o} t_{o}}{f u c A} \]

\[ t_D = \frac{(0.00633)(17.5)(32)}{(0.16)(0.70)(50 \times 10^{-6})(160 \times 43560)} \]

\[ t_D = 0.0908 \]

\[ N = 1.08 \quad \text{From Mathews (12), for a well in the center of circular drainage area.} \]

\[ p_o = 2180 - 1.08 \left( \frac{83}{2.303} \right) \]

\[ p_o = 2141 \text{ psia} \quad \]

Condition ratio

\[ r_e = \sqrt{\frac{A \times 43560}{\pi}} \]

\[ r_e = \sqrt{160 \times 43560 \over 3.14} \]

\[ r_e = 1490 \]

\[ c r = \frac{2 m \log \left( \frac{r_e}{r_w} \right)}{p_o - p_f} \]

\[ c r = \frac{(2)(83) \log \left( \frac{1490}{0.25} \right)}{2141 - 1534} \]

\[ c r = 1.032 \]
### Well B

A pressure build-up test on an Oklahoma Panhandle area on gas well yielded the following data:

<table>
<thead>
<tr>
<th>Shut-in time ( \Delta t ), (hours)</th>
<th>Well pressure ( P_w ), (psia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1,693</td>
</tr>
<tr>
<td>1</td>
<td>1,025</td>
</tr>
<tr>
<td>2</td>
<td>2,165</td>
</tr>
<tr>
<td>5</td>
<td>2,215</td>
</tr>
<tr>
<td>13.5</td>
<td>2,260</td>
</tr>
<tr>
<td>22</td>
<td>2,281</td>
</tr>
<tr>
<td>35</td>
<td>2,301</td>
</tr>
<tr>
<td>45</td>
<td>2,312</td>
</tr>
<tr>
<td>60</td>
<td>2,323</td>
</tr>
<tr>
<td>72</td>
<td>2,330</td>
</tr>
</tbody>
</table>

Other data for this well:

- **Net sand thickness**, \( h = 65 \) ft.
- **Porosity**, \( f = 15.1 \) %.
- **Average reservoir pressure**, \( p_r = 2375 \) psia.
- Stabilized gas production rate prior to shut-in, \( q_g = 5000 \) MScF/day.
- **Reservoir temperature**, \( T = 635 \) °R.
- **Reservoir gas viscosity**, \( \mu_g = 0.02 \) cp.
- **Gas deviation factor at reservoir condition**, \( z = 0.83 \).
- **Radius of drainage**, \( r_e = 2,100 \) ft., 320 acre well spacing.
- **Well-bore radius**, \( r_w = 4 \) in.
- **Cumulative production since completion**, \( G_p = 900,000 \) MScF.

\[
t_0 = \frac{24G_p}{q_g} = \frac{(24)(900,000)}{5,000} = 4,320 \text{ hours.}
\]
<table>
<thead>
<tr>
<th>Shut-in time $\Delta t$, (hours)</th>
<th>$\Delta t$</th>
<th>Well pressure $P_w$, (psia)</th>
<th>$P_w^2 \times 10^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>1.693</td>
<td>2.8562</td>
</tr>
<tr>
<td>1</td>
<td>2.3143</td>
<td>1.925</td>
<td>3.7056</td>
</tr>
<tr>
<td>2</td>
<td>4.6275</td>
<td>2.165</td>
<td>4.6872</td>
</tr>
<tr>
<td>5</td>
<td>11.5610</td>
<td>2.215</td>
<td>4.9062</td>
</tr>
<tr>
<td>13.5</td>
<td>31.152</td>
<td>2.260</td>
<td>5.1076</td>
</tr>
<tr>
<td>22</td>
<td>50.6781</td>
<td>2.281</td>
<td>5.2030</td>
</tr>
<tr>
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<td>72</td>
<td>163.9344</td>
<td>2.330</td>
<td>5.4289</td>
</tr>
</tbody>
</table>
FIGURE VI
WELL B
OKLAHOMA PANHANDLE
Pressure Build-up

$p_w^2 = 6.25 \times 10^6$

Slope $m = 0.500 \times 10^6$ psia$^2$/cycle

$p_w^2 \ 1 \text{ hr.} = 4.58 \times 10^6$

\[
\frac{\Delta t}{t_o + \Delta t}
\]
CALCULATIONS:

Permeability

\[ m = \text{Slope} = 500,000 \text{ psia}^2/\text{cycle} \] (FIG.-6)

\[ k_g = \frac{1.637 q \mu_p zT}{\text{mh}} \]

\[ k_g = \frac{(1.637)(5,000)(.02)(0.83)(635)}{(500,000)(65)} \]

\[ k_g = 2.65 \text{ md.} \]

Skin Effect

\[ S = 1.151 \frac{p_w^2(1hr)-p_r^2}{m} - 1.151 \log \frac{q_t T_p p_r}{1.033 \text{ mfh} r_w^2} \]

\[ S = 1.151 \frac{(4.58 \times 10^6) - (2.866 \times 10^6)}{(500,000)} \]

\[ - 1.151 \log \frac{(5000)(635)(0.83)(2375)}{(1.033)(500,000)(65)(0.151)(0.33)^2} \]

\[ S = -0.721 \]

Static Reservoir Pressure

\[ p_{o}^2 = 6.25 \times 10^6 = 2500 \text{ psia} \] (FIG.-6)

\[ t_D = \frac{0.00633 \kappa t \sigma p_r}{j \mu A} \]

\[ t_D = \frac{(0.00633)(2.65)(180)(2375)}{(0.151)(0.02)(320 \times 43560)} \]

\[ t_D = 0.1703 \]

\[ N = 1.68 \] (From Mathews—for a well in center of circular drainage area.)

\[ p_{o}^2 = p_{r}^2 - N \left( \frac{M}{2.303} \right) \]

\[ p_{o}^2 = 6.25 \times 10^6 - 1.68 \left( \frac{500,000}{2.303} \right) \]
\[ p_o^2 = 5.89 \times 10^6 \]
\[ p_o = 2428. \]

\[ c_r = \frac{2m (\log \frac{r_a}{r_b})}{p_o^2 - p_f^2} \]

\[ c_r = \frac{(2)(500,000)(\log \frac{2100}{\pi^{0.33}})}{5.890 \times 10^6 - 2.866 \times 10^6} \]

\[ c_r = 1.272 . \]
WELL C

A pressure build-up test on an Oklahoma (Oswego formation) gas well yielded the following data:

<table>
<thead>
<tr>
<th>Shut-in time (Δt, hours)</th>
<th>Well pressure (p_w, psia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
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<tr>
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<tr>
<td>54</td>
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<tr>
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<tr>
<td>66</td>
<td>3993</td>
</tr>
<tr>
<td>72</td>
<td>3994</td>
</tr>
</tbody>
</table>

Other data for this well:

Net sand thickness, h = 45 ft.
Porosity, f = 13.2 %.
Reservoir temperature, T = 645 °R.
Reservoir gas viscosity, \( \mu_g = 0.015 \) cp.
Gas deviation factor at reservoir condition, z = 0.911.
Cumulative production since completion, \( C_p = 243,142 \) MCF.
Stabilized production rate prior to shut-in, \( q_g = 2300 \) MCF/day.
Average reservoir pressure, \( p_r = 4100 \) psia.
Well is drilled on 640 acre spacing.
1.) Using Cumulative production, since well completion.

\[ t_o = \frac{(243, 142)(24)}{(2300)} \]

\[ t_o = 2537 \text{ hr.} \]

<table>
<thead>
<tr>
<th>Shut-in time ( \Delta t ) (hours)</th>
<th>Well pressure ( p_w ) (psia)</th>
<th>( p_w^2 \times 10^6 )</th>
<th>( \frac{\Delta t}{t_o \times \Delta t} \times 10^{-6} )</th>
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<td>15.949</td>
<td>253.5516</td>
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<tr>
<td>72</td>
<td>3994</td>
<td>15.957</td>
<td>275.9978</td>
</tr>
</tbody>
</table>

CALCULATIONS

\[ m = \text{Slope} = 0.300 \times 10^6 \text{ psia/cycle} \] (Fig. VII-A)

\[ k_g = \frac{1637 q_t h_g z T}{m h} \]

\[ k_g = \frac{(1637)(2300)(0.015)(0.011)(648)}{(0.300 \times 10^6)(45)} \]

\[ k_g = 2.469 \text{ md.} \]
FIGURE VII-A

WELL C (USING CUMULATIVE PRODUCTION)
OKLAHOMA (OSWEGO FORMATION)

Pressure Build-up

$p^2 = 16.42 (10^6)$

Slope $m = 0.300 \times 10^6 \text{ psia}^2/\text{cycle}$
**Static Reservoir Pressure**

\[ p^* = 16.42 \times 10^6 \text{ psia}^2 \]  
(Fig. VII-A)

\[ t_D = \frac{0.00633 k_t t_0 p_r}{f \mu A} \]

\[ t_D = \frac{(0.00633)(2.469)(105.7)(4100)}{(0.132)(0.015)(640 \times 43560)} \]

\[ t_D = 0.128. \]

\[ N = 1.40 \]  
(12) From Mathews - well in a center of a circular drainage area.

\[ p_0^2 = p^* - N \left( \frac{m}{2.303} \right) \]

\[ p_0^2 = 16.42 \times 10^6 - 1.40 \left( \frac{0.300 \times 10^6}{2.303} \right) \]

\[ p_0^2 = 16.24 \times 10^6 \text{ psia}^2 \]

\[ p_0 = 4030 \text{ psia}. \]

2.) Using incremental production, since last test:

Incremental production = 87,000 Mcf

\[ t_0 = \frac{(87,000)(24)}{2300} \]

\[ t_0 = 907 \text{ hrs.} \]
Shut-in time  

| Δt (hours) | t₀ (psia) | p_w² x 10⁶ | Δt 

<table>
<thead>
<tr>
<th></th>
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<th></th>
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<tbody>
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<td>3993</td>
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<tr>
<td>72</td>
<td>3994</td>
<td>15.957</td>
<td>735.4443</td>
</tr>
</tbody>
</table>

Slope = 0.360 x 10⁶ psia²/cycle.

**Permeability**

\[ k_g = \frac{1637 \cdot \delta \cdot \mu_g \cdot z \cdot T}{m \cdot h} \]

\[ k_g = \frac{(1637)(2300)(0.015)(0.911)(648)}{(0.360 \times 10^6)(45)} \]

\[ k_g = 2.058 \text{ md.} \]

**Static reservoir pressure**

\[ p_r^2 = 16.38 \times 10^6 \text{ (Fig. VII-B)} \]

\[ t_D = \frac{0.00633 \cdot k_g \cdot t_0 \cdot p_r}{f \cdot \mu \cdot A} \]

\[ t_D = \frac{(0.00633)(2.058)(37.8)(4100)}{(0.132)(0.015)(640 \times 43560)} \]
FIGURE VII-B

WELL C [USING INCREMENTAL PRODUCTION]

OKLAHOMA (GOWEGO FORMATION)

Pressure Build-up

\[ p^{*2} = 16.38 \times 10^6 \]

Slope \( m = 0.360 \times 10^6 \) psi^2/cycle
\( t_D = 0.0365 \)

\[ N = 0.45 \quad \text{(12)} \]

From Mathews, well in a center of a circular drainage area.

\[ p_o^2 = p^2 - N \left( \frac{m}{2.303} \right) \]

\[ p_o^2 = 16.38 \times 10^6 - 0.45 \left( \frac{0.360 \times 10^6}{2.303} \right) \]

\[ p_o^2 = 16.284 \times 10^6 \text{ psia}^2 \]

\[ p_o = 4040 \text{ psia}. \]
WELL D

A pressure build-up test on an Oklahoma (Oswego formation) gas well yielded the following data:

<table>
<thead>
<tr>
<th>Shut-in time (Δt, hours)</th>
<th>Well pressure (P_w, psia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3860</td>
</tr>
<tr>
<td>2</td>
<td>3887</td>
</tr>
<tr>
<td>4</td>
<td>3911</td>
</tr>
<tr>
<td>6</td>
<td>3924</td>
</tr>
<tr>
<td>9</td>
<td>3935</td>
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<tr>
<td>12</td>
<td>3941</td>
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<td>3948</td>
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<tr>
<td>24</td>
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<td>3964</td>
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<tr>
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<td>3969</td>
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<td>48</td>
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<td>54</td>
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<td>65</td>
<td>3977</td>
</tr>
<tr>
<td>72</td>
<td>3978</td>
</tr>
</tbody>
</table>

Other data for well:

Net sand thickness, h = 40 ft.

Porosity, f = 13.1 %.

Reservoir temperature, T = 645 °R.

Reservoir gas viscosity, \( \mu_g = 0.015 \) cp.

Gas deviation factor at reservoir condition, \( z = 0.905 \).

Stabilized production rate prior to shut-in, \( q_g = 2100 \) MSCF/day.

Average reservoir pressure, \( p_r = 4000 \) psia.

Well drilled on 640 acre spacing.

Cumulative production since completion, \( G_p = 191,706 \) MCF.
1. Using cumulative production:

\[ t_o = \frac{(191,706)(24)}{2100} \]

\[ t_o = 2191 \text{ hrs.} \]

<table>
<thead>
<tr>
<th>( \Delta t ) (hours)</th>
<th>Wall pressure ( P_w ) (psia)</th>
<th>( P_w^2 \times 10^6 )</th>
<th>( \left( \frac{\Delta t}{t_o + \Delta t} \right) \times 10^{-4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>72</td>
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</tbody>
</table>

**CALCULATIONS**

Slope = \( 0.360 \times 10^6 \) psia\(^2\)/cycle. (Fig.-VIII-A)

**Permeability**

\[ k_g = \frac{1637 \, q_g \, \rho_g \, z \, T}{m \, h} \]

\[ k_g = \frac{(1637)(2100)(0.015)(0.905)}{(0.360 \times 10^6)(40)} \]

\[ k_g = 3.240 \text{ md} \]
FIGURE VIII-A

WELL D (USING CUMULATIVE PRODUCTION)
OKLAHOMA (OSWEGO FORMATION)
Pressure Build-up

\[ P_{\infty} = 16,38 \times 10^6 \] psi

Slope \( m = 0.360 \times 10^6 \) psi\(^2\)/cycle
Static Reservoir Pressure:

\[ p^2 = 16.38 \times 10^6 \text{ psia}^2 \quad \text{(Fig. VIII-A)} \]

\[ t_D = \frac{0.00633 \, \text{kg} \, t \, \text{to PR}}{f \, \mu_A} \]

\[ t_D = \frac{(0.00633)(3.240)(91.2)(4000)}{(0.131)(0.015)(640 \times 43560)} \]

\[ t_D = 0.136 \quad \text{(12)} \]

\[ N = 1.46 \]

From Mathews - for well in a center of a circular drainage area.

\[ p_o^2 = p^2 - N \left( \frac{m}{2.303} \right) \]

\[ p_o^2 = 16.38 \times 10^6 - 1.46 \left( \frac{0.360 \times 10^6}{2.303} \right) \]

\[ p_o^2 = 16.15 \times 10^6 \text{ psia}^2 \]

\[ p_o = 4018 \text{ psia} \]

2.) Using incremental production:

Incremental production since last test = 70,706 Mscf

\[ t_o = \frac{(70,706)(24)}{2100} \]

\[ t_o = 808 \text{ hrs.} \]
FIGURE VIII-B
WELL D (USING INCREMENTAL PRODUCTION)
OKLAHOMA (GEORGO FORMATION)
Pressure Build-up

\[ p^{*2} = 16.26 \times 10^6 \text{ psia}^2 \]

Slope \( m = 0.000 \times 10^6 \text{ psia}^2/\text{cycle} \)
<table>
<thead>
<tr>
<th>Shut-in time (hours)</th>
<th>Well pressure (psia)</th>
<th>( \frac{p_w^2}{10^6} )</th>
<th>( \frac{\Delta t}{t_o + \Delta t} \times 10^{-4} )</th>
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<td>72</td>
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<td>818.1818</td>
</tr>
</tbody>
</table>

Slope \( \approx 0.400 \times 10^6 \text{ psia}^2/\text{cycle} \) (Fig. VIII-B)

Permeability

\[
k_g = \frac{1637 \, \alpha_\text{g} \, \rho_\text{g} \, z \, T}{m \, h}
\]

\[
k_g = \frac{(1637)(2100)(0.015)(0.905)}{(0.400 \times 10^6)(40)}
\]

\[k_g = 2.916 \text{ md}.
\]

Static Reservoir Pressure

\[p^* = 16.26 \times 10^6 \text{ psia}^2\] (Fig. VIII-B)

\[
t_D = \frac{0.00633 \, k_g \, t_o \, P_r}{f \, \rho \, A}
\]

\[
t_D = \frac{(0.00633)(2.916)(33.7)(4000)}{(0.131)(0.015)(640)(43560)}
\]
\( t_0 = 0.045 \)

\( N = 0.55 \)

From Mathews (12) for well in a center of the circular drainage area.

\[
p_o^2 = p^*^2 - N \left( \frac{m}{2.303} \right)
\]

\[
p_o^2 = 16.26 \times 10^6 - 0.55 \left( \frac{0.400 \times 10^6}{2.303} \right)
\]

\[
p_o^2 = 16.16 \times 10^6 \text{ psi}^2
\]

\[
p_o = 4020 \text{ psi}.
\]
### COMPARISON OF RESULTS

#### WELL C

<table>
<thead>
<tr>
<th></th>
<th>Using Cumulative Production</th>
<th>Using Incremental Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_g$</td>
<td>2.469</td>
<td>2.058</td>
</tr>
<tr>
<td>$p_o$</td>
<td>4030</td>
<td>4040</td>
</tr>
</tbody>
</table>

#### WELL D

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<th>Using Incremental Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_g$</td>
<td>3.240</td>
<td>2.91b</td>
</tr>
<tr>
<td>$p_o$</td>
<td>4018</td>
<td>4020</td>
</tr>
</tbody>
</table>

#### % Error in Using Incremental Production

<table>
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<th></th>
<th>$k_g$</th>
<th>$p_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WELL C</td>
<td>-16.63</td>
<td>+0.25</td>
</tr>
<tr>
<td>WELL D</td>
<td>-10.00</td>
<td>+0.05</td>
</tr>
</tbody>
</table>
CONCLUSIONS

Engineering analysis of the significance of pressure build-up characteristics of producing wells is based on a solution of the equation of diffusivity. The particular form of the solution of the diffusivity equation depends on the assumed reservoir boundary and initial conditions.

Analysis of the pressure build-up curves on the example problems showed that a small error is introduced into the determination of the static reservoir pressure by using the more accessible cumulative production since the last test than the cumulative production since completion of the well. A considerable error is introduced into the determination of the permeability of the formation by using the more accessible cumulative production since the last test than the cumulative production since completion of the well.

It has been shown that the generally-used method of plotting the logarithm of the shut-in time versus the well-bore pressure will result in erroneous determinations of desired reservoir properties.
NOMENCLATURE

\( \beta_o \) = formation volume factor, reservoir volume per stock tank volume

\( c \) = compressibility, psi\(^{-1}\)

\( cr \) = condition ratio, dimensionless

\( D \) = damage factor, dimensionless

\( f \) = effective porosity, fraction

\( h \) = net reservoir thickness, feet

\( J_0 \) = bessel function of the first kind of order zero

\( J_1 \) = bessel function of the first kind of order unity

\( J \) = productivity index, stock tank bbl. per day, per psi

\( k_e \) = effective permeability, millidarcy

\( k_{he} \) = effective capacity, millidarcy-feet

\( \ln \) = logarithm to base e

\( \log \) = logarithm to base 10

\( m \) = slope of pressure build-up curve, psia per cycle

\( p^* \) = extrapolated straight-line pressure at infinite shut-in time, psia

\( P_0 \) = static reservoir pressure, psia

\( P_w \) = pressure in well-bore during build-up, psia

\( P_f \) = bottom-hole flowing pressure for stabilized production rate, psia

\( P_{1\, \text{hr}} \) = extrapolated straight-line pressure one hour after shut-in, psia

\( P_r \) = average reservoir pressure, psia

\( N_p \) = cumulative production since completion (oil), barrels

\( q \) = stabilized production rate prior to shut-in, barrels

\( R \) = gas-oil ratio, std. cubic foot per stock tank bbl.

\( r_e \) = drainage radius, feet

\( r_w \) = well-bore radius, feet
$S = \text{skin effect, psi}$

$T = \text{flowing temperature, } ^\circ\text{R}$

$T_a = \text{base temperature, } ^\circ\text{R}$

$t_c = \text{pseudo cumulative production time, hours}$

$t_{si} = \text{time required to obtain satisfactory data, hours}$

$t_s = \text{stabilized time for steady-state condition, days}$

$t_o = \text{cumulative production time, hours}$

$\Delta t = \text{shut-in time, hours}$

$t = \text{dimensionless time}^{(10)} \text{ used for identifying correct part of the straight-line which will be used for calculating the slope}$

$t_D = \text{dimensionless time}^{(12)} \text{ used for reading } \mu \text{ from Matthews curve}$

$z = \text{gas deviation factor, dimensionless}$

$\gamma = \text{density, pound per cubic foot}$

$\mu = \text{viscosity, centipoise}$

$G_D = \text{cumulative production since completion (gas) M.C.}$

SUBSCRIPTS:

$g = \text{gas}$

$T = \text{total system, oil and gas}$

$o = \text{oil}$

$D = \text{dimensionless}$
Appendix A

Derivation of the equation for condition ratio, for a compressible fluid flow.

From the Darcy equation for radial flow of a compressible fluid:

\[
q_g = \frac{0.000704 (p_o^2 - p_f^2) k_g h}{n_g T z \ln \frac{r_e}{r_w}} \quad \text{MSCF/day @ 14.65 psia 60° F.}
\]

from which, the capacity is given by:

\[
k_g h = \frac{q_g n_g T z \ln \frac{r_e}{r_w}}{0.000704 (p_o^2 - p_f^2)} \quad \text{(A)}
\]

The slope, \( m \), is given by:

\[
m = \frac{1637 q_g n_g T z}{k_g h} \quad \text{(Equation 33)}
\]

from which

\[
k_g h = \frac{1637 q_g n_g T z}{m} \quad \text{(B)}
\]

By definition, the condition ratio is

\[
cr = \frac{A}{B}
\]

or

\[
cr = \frac{q_g n_g T z m \frac{r_e}{r_w}}{0.000704 (p_o^2 - p_f^2)} \cdot \frac{1637 q_g n_g T z}{m}
\]

\[
cr = m \ln \frac{r_e}{r_w} \frac{1.15 (p_o^2 - p_f^2)}{1.15 (p_o^2 - p_f^2)}
\]

\[
cr = m 2.303 \log \frac{r_e}{r_w} \frac{1.15 (p_o^2 - p_f^2)}{1.15 (p_o^2 - p_f^2)}
\]

hence

\[
cr = \frac{2 m \log \frac{r_e}{r_w}}{(p_o^2 - p_f^2)}
\]
Appendix B

Derivation of the equation for condition ratio, for a slightly compressible fluid flow.

From the Darcy equation for radial flow of a compressible fluid:

\[ q_o = \frac{0.00708 (p_o - p_f) k_o h}{\beta_o \mu_o \ln \frac{r_e}{r_w}} \]  

STE/day

from which, the capacity is given by:

\[ k_o h = \frac{q_o \beta_o \mu_o \ln \frac{r_e}{r_w}}{0.00708 (p_o - p_f)} \quad (A) \]

The slope, \( m \), is given by

\[ m = \frac{162.6 q_o \mu_o \beta_o}{k_o h} \quad (Equation \ 27) \]

from which

\[ k_o h = \frac{162.6 q_o \mu_o \beta_o}{m} \quad (B) \]

By definition, the condition ratio is

\[ c_r = \frac{A}{B} \]

or

\[ c_r = \frac{q_o \beta_o \mu_o \ln \frac{r_e}{r_w}}{0.00708 (p_o - p_f)} \]

\[ c_r = 1.15 \frac{m \ln \left(\frac{r_e}{r_w}\right)}{1.15 (p_o - p_f)} \]

\[ c_r = \frac{m 2.303 \log \left(\frac{r_e}{r_w}\right)}{1.15 (p_o - p_f)} \]
hence

\[ c r = \frac{2 m \log \left( \frac{r_2}{r_o} \right)}{(p_0 - p_f)}. \]
Appendix C

Derivation of pressure build-up equation (32) for a compressible fluid flow, from the equation (14) which is for a slightly compressible fluid flow.

The pressure differential, for radial flow of a slightly compressible fluid is

\[ p_o - p_w = \frac{q_o \mu_o \beta_o}{4 \pi k h} \left[ \ln \frac{k t}{f \mu c r_w^2} + 0.809 + 2 \xi \right] \]  

(14)

where

\( q_o \beta_o \) is a liquid flow rate at reservoir condition.

By definition: \( q_g \) = gas flow rate at base conditions.

Then:

\[ q_g \times \frac{1}{\left[ \frac{p_o + p_w}{2} \right]} \times \frac{T z}{T_a} = q_o \beta_o \]  

(A)

Where

\( T = \) flowing temperature °R.

\( T_a = \) base temperature °R.

\( z = \) compressibility factor:

Substituting (A) in equation (14), yields

\[ p_o - p_w = \frac{q_g}{4 \pi k h} \left( \frac{p_o + p_w}{2} \right) \frac{T z}{T_a} \mu_o \left[ \ln \frac{k t}{f \mu c r_w^2} + 0.809 + 2 \xi \right] \]  

(B)

or

\[ p_o^2 - p_w^2 = \frac{q_g}{2 \pi k h T_a} \left[ \ln \frac{k t}{f \mu c r_w^2} + 0.809 + 2 \xi \right]. \]  

(C)
Compressibility of gas is approximated by \( \frac{1}{p_r} \), where \( p_r \) is the reservoir pressure. Then equation (C) becomes

\[
p_o^2 - p_w^2 = \frac{q_o T^2 u_o}{\pi k h T_a} \left[ \ln \frac{k t p_r}{f u r_w^{2}} + 0.809 + 2 S \right]
\]

(D)

which is the pressure differential for a gas flowing radially in a porous reservoir.
DEFINITIONS

**Damage factor:**

Because of the drilling and completion technique and perhaps due to production practices, an additional resistance to the flow of the fluids is concentrated around the well-bore. This additional resistance is defined as the "Damage factor", or "damage".

**Slightly compressible liquid:**

When fluid flows through a porous medium, the flow rate into an element of volume of the porous medium may not be the same as the flow rate out of that element and the fluid content of the porous medium changes with time. Such fluid is known as compressible fluid. When $c(D^2/R)^2$ is small and negligible, the fluid is known as a slightly compressible fluid since the product of the square of the pressure differences with radius changes times the fluid compressibility reflects the alteration of a compressible fluid to one which is slightly compressible.

**Undersaturated:**

A liquid or vapor capable of holding additional gaseous or liquid components in solution at the specified pressure and temperature.

**Afterflow:**

When a well is shut-in at the surface, rather than at the sand face, flow will continue into the well-bore after shut-in, due to the compressibility of the fluids in the bore. Within the formation itself, flow will also continue from high permeability to low permeability. This is commonly referred to as the "afterflow" effect of pressure build-up testing.
Ei (-x) function:

Values of Ei (-x) are defined by the equation \( Ei (-x) = -\int_{-x}^{0} \frac{e^{-u}}{u} \, du \).

For values of \( x \) smaller than 0.01, the equation is approximated by

\[ Ei (-x) \approx \ln x + 0.5772. \]

Skin Effect:

When the fluid is flowing into the well-bore, the pressure drop in the well per unit rate of flow is controlled by the resistance of the formation, the viscosity of the fluid and the additional resistance concentrated around the well-bore resulting from the drilling and completion technique and, perhaps, from the production practices. The pressure drop caused by this additional resistance is defined by Van Everdingen as a "skin effect". The zone in which this skin effect is concentrated is defined as "skin". Mathematically, this factor is given by Equation (15 A-B).
BIBLIOGRAPHY


V I T A

Hansraj J. Patel

Born - June 15, 1940, at Uttersanda, India

Married - One child

Graduate of Wilson College, Bombay University, India. B. S. Degree, Chemistry, 1961.

Graduate of Missouri School of Mines and Metallurgy. B. S. Degree, Mining Engineering - Petroleum Engineering Option, 1964.

Junior member AIME.

Professional Record: