An analysis of an audio amplifier utilizing an operational amplifier and negative feedback

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AN ANALYSIS OF AN AUDIO AMPLIFIER UTILIZING AN OPERATIONAL AMPLIFIER AND NEGATIVE FEEDBACK

BY

ANTHONY FRANCIS LEZA

A

THESIS

submitted to the faculty of
THE UNIVERSITY OF MISSOURI - ROLLA
in partial fulfillment of the requirements for the
Degree of
MASTER OF SCIENCE IN ELECTRICAL ENGINEERING
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Approved by

[Signatures]
An audio amplifier utilizing a monolithic operational amplifier is analyzed. The analysis is a block diagram approach where the amplifier is divided into three parts; operational amplifier, power amplifier, and feedback network. A Fourier series analysis is used to describe the distortion and signal components of the amplifier output. The results of this analysis demonstrate the advantage of large negative feedback on frequency response and harmonic distortion. Using relatively few passive components a high quality audio amplifier is constructed with an output power of 15 watts RMS power and negligible harmonic distortion.
ACKNOWLEDGEMENTS

The author wishes to express his gratitude to his advisor, Dr. Ralph S. Carson, Professor of Electrical Engineering, for his guidance and assistance in this Master's thesis.

Thanks are also due to Mr. Patrick Vennari for his programming assistance.
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LIST OF SYMBOLS

$e_i$ ... input signal to audio amplifier (sinusoid)
$E_i$ ... maximum value of $e_i$
$e_o$ ... output signal from audio amplifier
$E_o$ ... maximum value of $e_o$
$e_1$ ... input signal to power amplifier (sinusoid)
$E_1$ ... maximum value of $e_1$
$e_2$ ... output signal from power amplifier
$E_2$ ... maximum value of $e_2$
$\beta$ ... feedback ratio
$Q_1$ ... transistor 2N 3904
$Q_2$ ... transistor 2N 3906
$Q_3$ ... transistor 2N 3791
$Q_4$ ... transistor 2N 3715
$V_{cc}$ ... DC power supply voltage
$R_B$ ... base limiting resistor
$C_B$ ... base lead capacitor
$R_L$ ... load resistor
$R_i$ ... input resistor
$R_f$ ... feedback resistor
$C_f$ ... feedback capacitor
$I_0$ ... reverse bias saturation current for an ideal diode
$I_b$ ... transistor base current
$K$ ... empirically found constant describing transistor input characteristic
$V_\beta$ ... transistor turn on voltage
$V_{be}$ ... voltage across ideal diode in transistor model
\( V_{\text{be}} \) ... voltage from base to emitter in transistor model

\( I_c \) ... transistor collector current

\( V_{\text{ce}} \) ... voltage from collector to emitter

\( h_{\text{FE}} \) ... h parameter DC current gain

\( h_{\text{oe}} \) ... h parameter output conductance

\( V_1 \) ... voltage at node 1 in power amplifier

\( V_2 \) ... voltage at node 2 in power amplifier

\( V_3 \) ... voltage at node 3 in power amplifier

\( V_4 \) ... voltage at node 4 in power amplifier

\( V_5 \) ... voltage at node 5 in power amplifier

\( T \) ... period of signal

\( \gamma \) ... amount of time that power amplifier output signal is near zero - delay time

\( \omega \) ... radian frequency of input or output signal

\( \omega' \) ... radian frequency of positive and negative sinusoidal pulse for the approximate waveform of crossover distortion

\( A_2 \) ... gain of power amplifier

\( B_k \) ... coefficients of Fourier series representing crossover distortion

\( d_0 \) ... theoretical distortion signal

\( D_{ok} \) ... coefficients of Fourier series which represents \( d_0 \)

\( d_c \) ... theoretical distortion in output of audio amplifier

\( D_{ck} \) ... coefficients of Fourier series which represents \( d_c \)

\( E_{ok} \) ... coefficients of Fourier series which represents output of audio amplifier.

\( Z_{\text{in}} \) ... input impedance of operational amplifier

\( Z_{\text{out}} \) ... output impedance of operational amplifier.
$A_{VOL}$ ... mid-band gain of operational amplifier

$A_1$ ... frequency dependent gain of operational amplifier

$\omega_c$ ... upper cutoff radian frequency for operational amplifier

$P_{LOAD}$ ... power delivered to load

$D_{TOT}$ ... per cent total harmonic distortion
I. Introduction

A. The Integrated Monolithic Operational Amplifier

The recent development of the monolithic integrated operational amplifier has caused a great change in circuit design theory. It is now possible to design circuits around the operational amplifier as a basic building block and use relatively few additional passive components. The operational amplifier characteristically is a high gain device. Because of this, negative feedback can be employed to provide good stability and increased bandwidth.

The techniques used for the manufacture of these integrated circuits are repeated operations of masking, photo etching, and dopant diffusion on a single wafer of silicon. Because of this type of fabrication, it is actually easier to make a transistor or diode than it is a resistor or capacitor. So it is desirable for the ratio of active to passive components to be much higher in an integrated circuit as compared to a discrete circuit of similar function. This fact results in the use of differential amplifiers in integrated circuits. Coupling capacitors are not needed because the differential amplifier is a DC amplifier. With this type of design a high ratio of active to passive components can be achieved. A more detailed description of the operational amplifier circuitry will be discussed in Section III.
The use of differential amplification offers a number of advantages, such as, DC amplification, good stability, immunity to interference signals, and wide versatility to mention a few. However, these advantages are dependent on how well the two devices are matched. Since the transistors are made from the same silicon chip and are physically close to one another in the integrated circuit, they are very closely matched in performance and temperature. Therefore with these characteristics, the integrated operational amplifier is attractive for circuit design. Also, as manufacturing techniques are perfected, the cost will be lowered making the integrated circuit economically attractive.

B. Audio Amplifier Using The Operational Amplifier

The circuit which this thesis will analyze is an audio amplifier using the type of operational amplifier discussed above. The audio amplifier has three basic functional divisions: operational amplifier, power amplifier, and feedback network. The simplified block diagram is given in figure 1.

The operational amplifier is a Motorola MC 1433 monolithic integrated operational amplifier. The power amplifier is a complementary class B amplifier using two complementary silicon driver transistors and two complementary silicon power transistors. The function of this
Figure 1. Simplified Block Diagram of Audio Amplifier
circuit is to give the necessary power gain to drive the load. The audio amplifier will produce 15 watts (RMS) power into a four ohm load. This circuit will be discussed more fully in Section II. The feedback network is a R-C network which determines the amount of feedback from output to input.

There are many advantages to this type of audio amplifier design. One, there are fewer discrete components than a similar audio amplifier of the same power rating. The operational amplifier replaces many front end stages of standard transistor design, where each of these stages require many passive elements for biasing and signal coupling. This causes simpler layout and fewer connections resulting in higher reliability. Two, the load is directly coupled to the output of the amplifier. Normally an output transformer would be used, which is often a limiting factor in an audio amplifier's fidelity. Three, the frequency response is DC to beyond the audio range because of the operational amplifier's differential amplification and wide bandwidth. The power amplifier uses wide bandwidth transistors which do not limit the frequency response. Therefore, the upper cutoff frequency is adjustable by selection of the passive elements used. Four, the amplifier has high efficiency because the power amplifier is class B. Also the operational amplifier's power requirement is very small. Five, the amplifier has a very low level of distortion. This is probably the most favorable advantage
of this type of audio amplifier design. Because of the operational amplifier's high gain a large amount of negative feedback is used. As will be shown in Section IV, this feedback results in a reduction in harmonic distortion by approximately a factor equal to the gain of the operational amplifier. In a high-quality audio amplifier the harmonic distortion must be kept to very low level (less than 1 percent).

C. Method Of Solution

The audio amplifier will be studied by analyzing each of the three functional blocks listed above separately and then combining the results to describe the entire amplifier.

The power amplifier will be analyzed first. The most prominent source of this distortion in the entire amplifier is crossover distortion in this stage due to the class B operation.

To analyze this circuit a transistor model is formulated and used to describe the response of the power amplifier. The digital computer is used to solve a set of five simultaneous non-linear equations. With this information a block diagram representation of the power amplifier can be made consisting of an ideal amplifier in series with a distortion signal generator. This model description of the power amplifier makes a description of the entire amplifier possible.
Next the operational amplifier will be studied. An expression which describes the gain as a function of frequency will be found. Biasing and frequency stabilization by use of external circuitry will be discussed. With this information it is known how the operational amplifier will function in the audio amplifier.

By using the information concerning the operational amplifier, a description of the feedback network is determined. The usual symbol for the transfer function describing how much of the output is returned to the input is $\beta$. So a description of the feedback network is actually a determination of $\beta$.

Using the information of the above three sections the entire audio amplifier can be described. This description is derived from a Fourier series of the distortion in the power amplifier. Knowing the distortion introduced, the gain of the operational and power amplifiers, and $\beta$, the per cent harmonic distortion in the output signal can be determined. By varying the input signal level (and thereby the output power) and frequency, an evaluation of the audio amplifier can be made.

Finally, experimental data of the same factors above will be taken and correlation of theoretical and experimental results will be discussed.
II. Description Of Power Amplifier

A. Transistor Model Used For Power Amplifier Simulation

Figure 2 is the schematic diagram of the power amplifier used in the audio amplifier. As was discussed earlier, this is a class B complementary power amplifier. All the transistors are silicon. Q1 and Q2 are 210 mw complementary driver transistors. Q3 and Q4 are 150 w complementary transistors.

In order to evaluate the response of this circuit, a transistor model was formulated. Since this is a class B amplifier with no bias, the models must be accurate in the cutoff region because the point of operation moves from cutoff into the active region and back again as the input signal varies around zero. The majority of the distortion in the power amplifier is due to crossover distortion which is caused by operation in the nonlinear region of cutoff.

The model used for this purpose is a "modified $h$ parameter" model. The output circuit for the transistor model is a dependent current generator whose current is equal to $h_{FE} I_b$. In parallel with the generator is a conductance $h_{oe}$. This, so far, is identical to a standard $h$ parameter equivalent circuit. The input circuit is composed of an ideal diode in series with a "battery" equal to the turn-on voltage, $V_\phi$. The current through
Figure 2. Schematic Diagram of Power Amplifier

- **Q1**: 2N3904
- **Q2**: 2N3906
- **Q3**: 2N3791
- **Q4**: 2N3715
- **C_B**: 0.1 mfd
- **R_B**: 20 ohms
- **V_{cc}**: 15 volts
the diode is described by the diode equation:

\[ I_d = I_o \left( e^{K V_{be}} - 1 \right) \]

where:  
- \( I_d \) = diode current  
- \( I_o \) = saturation current at reverse bias  
- \( K \) = empirically determined constant  
- \( V_{be} \) = actual voltage across ideal diode

Figure 3 shows the transistor equivalent circuit described above. Notice in the equivalent circuit the use of a battery, \( V_y \). It should be stressed that \( V_y \) is not an actual battery in the equivalent circuit but simply a transposition factor. In order to fit the actual input characteristics with the diode equation the starting point on the horizontal base voltage axis must be shifted to the right (increasing voltage) by an amount \( V_y \). Thus \( V_y \) is a transposition factor of \( V_{be} \). Therefore, in figure 4 for the model input characteristics, the exponential equation starts at \( V_y \).

In order to find the values of \( V_y \), \( K \), \( h_{FE} \), and \( h_{oe} \) for a particular transistor a photograph of the input (\( I_b \) vs \( V_{be} \)) and output (\( I_c \) vs \( V_{ce} \)) characteristics is taken. From the output characteristic \( h_{FE} \) and \( h_{oe} \) are determined and \( K \) and \( V_y \) is obtained from the input characteristic. \( I_o \) is found by measuring the current with the base emitter junction reversed biased.

Measurement of one of the four transistors used in the audio amplifier is demonstrated in figure 4. The definitions of the parameters are:
where:

\[ I_b = I_o \exp(KV_{b'e}) - 1 \]
\[ V_{be'} = V_{be} + V_f \]
\[ I_b = I_o \exp(K(V_{be} - V_f)) - 1 \]

Figure 3. Equivalent Model Of Transistor
Input Characteristics

Actual (2N3906)

Output Characteristics

Figure 4. Actual and Model Transistor Characteristics
\[ h_{FE} = \frac{I_c}{I_b}, \quad V_{ce} = \text{constant} \quad (\text{DC current gain}) \]
\[ h_{oe} = \frac{I_c}{V_{ce}}, \quad I_b = \text{constant} \quad (\text{output admittance}) \]

\[ V = \text{turn on voltage} = \text{voltage at which } I_b \]
\[ \text{increases such that transistor goes into} \]
\[ \text{the active region.} \]

\[ K = \text{empirically determined constant from input} \]
\[ \text{characteristic data using a least square} \]
\[ \text{fit. (Refer to Appendix I)} \]

\[ I_o = \text{current with base-emitter junction reversed} \]
\[ \text{biased.} \]

As can be seen from figure 4, a point must be chosen on the output characteristic where the \( h_{FE} \) and \( h_{oe} \) can be measured. If the point of measurement is changed, the values of \( h_{FE} \) and \( h_{oe} \) will also change. Therefore, an approximation must be made. The points of measurement are taken approximately in the center of the region of operation. This is a compromise between the extreme regions of saturation and cutoff. Table I lists all of these measured constants and points of measurements.

Once these constants are known, a set of idealized model characteristics can be formed. This is also done in figure 4. In the left column, the actual photo characteristics are given, in the right column are the idealized characteristics obtained from the measured parameters. Measurement in this fashion of all the transistors yields
<table>
<thead>
<tr>
<th>Type</th>
<th>( h_{\text{FE}} )</th>
<th>( h_{\text{oe}} )</th>
<th>( K )</th>
<th>( V_\gamma )</th>
<th>( I_\circ )</th>
<th>Points of Measurement</th>
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<tr>
<td>Q1</td>
<td>2N3904</td>
<td>167.0</td>
<td>( 2 \times 10^{-4} )</td>
<td>28.18078</td>
<td>0.5</td>
<td>( h_{\text{FE}} ) at ( V_{\text{be}} = 5.0 ) volts</td>
</tr>
<tr>
<td>Q2</td>
<td>2N3906</td>
<td>165.0</td>
<td>( 5 \times 10^{-4} )</td>
<td>30.94029</td>
<td>0.5</td>
<td>( h_{\text{oe}} ) at ( V_{\text{ce}} = 5.0 ) volts</td>
</tr>
<tr>
<td>Q3</td>
<td>2N3791</td>
<td>66.7</td>
<td>( 0.25 )</td>
<td>11.94721</td>
<td>0.0</td>
<td>( h_{\text{FE}} ) at ( V_{\text{be}} = 10.0 ) volts</td>
</tr>
<tr>
<td>Q4</td>
<td>2N3715</td>
<td>56.7</td>
<td>( 0.020 )</td>
<td>12.21675</td>
<td>0.0</td>
<td>( h_{\text{oe}} ) at ( V_{\text{ce}} = 10.0 ) volts</td>
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Table 1. Transistor Model Parameters
Table I which lists all the parameters. With these parameters and the transistor model the power amplifier can be analyzed.

B. Simplifying Assumptions

Refering to figure 2, notice the capacitors from Q3 and Q4 bases to ground. The assumption will now be made that we can neglect the $C_B$ capacitors in this analysis.

The justification stems from the considerations that $C_B$ is a 0.1 mfd capacitor designed only to attenuate high frequency (much higher than the audio spectrum). Therefore, $C_B$ at audio frequencies does not present a low enough impedance to attenuate the audio signal. Also notice that the transistor models have no capacitance in the equivalent circuits. The upper frequency cutoff from the transistors themselves was neglected because the gain-bandwidth products of the power transistors is 4 MHZ minimum and 250 MHZ for the driver transistors. With these gain-bandwidth products it is easily shown that the transistors are not frequency limited in the audio range.

This assumption is verified experimentally later in this section. For the time being, by using these assumptions, the power amplifier analysis can be greatly simplified.
C. Node Voltage Equations For Power Amplifier

With the above assumptions, the power amplifier of figure 2 can be redrawn using the equivalent transistor models just discussed but neglecting the effect of C_B. Figure 5 is the power amplifier circuit using the equivalent models.

The transistor parameters V_T, K, h_FE and h_oe are subscripted to match the number of the transistor Q1, Q2, Q3, or Q4. Also the nodes of the circuit are numbered V1, V2, V3, V4, and V5. It should be noted here that the transistor model is a representation for frequencies from DC to the upper limit on the audio range (20,000 KHZ). Therefore the Q3 and Q4 emitters are connected to the power supplies and not AC ground since this is a DC model.

The circuit can be solved by writing the node voltage equations. The base currents, which are exponential functions of the base-emitter voltages, can be expressed by substituting the voltage difference between the proper two nodes for V_be. In this fashion the node voltage equations can be written expressing the base currents as exponential functions of the node voltages.

With these considerations we can now write the equations for all five nodes.
e_1 = input signal \( (E_1 \sin \omega t) \)
e_2 = output signal

Figure 5. Power Amplifier Using Transistor Models
For node 1:
\[ I_{01} (h_{FE1} + 1) (\exp(K_1 V_{be1}) - 1) \]
\[ -I_{02} (h_{FE2} + 1) (\exp(K_2 V_{be2}) - 1) \]
\[ +I_{03} (h_{FE3} + 1) (\exp(K_3 V_{be3}) - 1) \]
\[ -I_{04} (h_{FE4} + 1) (\exp(K_3 V_{be4}) - 1) \]
\[ +(V_2 - V_1) h_{oe1} -(V_1 - V_3) h_{oe2} + (V_{cc} - V_1) h_{oe3} \]
\[ -(V_1 + V_{cc}) h_{oe4} - (V_1 / R_L) = 0 \]  \hspace{1cm} (1)

For node 2:
\[ R_B h_{FE1} I_{01} (\exp(K_1 V_{be1}) - 1) + R_B (V_2 - V_1) h_{oe1} \]
\[ -V_4 + V_2 = 0 \] \hspace{1cm} (2)

For node 3:
\[ R_B h_{FE2} I_{02} (\exp(K_2 V_{be2}) - 1) + R_B (V_1 - V_3) h_{oe2} \]
\[ -V_3 + V_5 = 0 \] \hspace{1cm} (3)

For node 4:
\[ R_B I_{03} (\exp(K_3 V_{be3}) - 1) -V_4 + V_2 = 0 \] \hspace{1cm} (4)

For node 5:
\[ R_B I_{04} (\exp(K_4 V_{be4}) - 1) - V_3 + V_5 = 0 \] \hspace{1cm} (5)

where:
\[ V_{be1} = e_1 - V_1 - V \gamma_1 \]
\[ V_{be2} = V_1 - e_1 - V \gamma_2 \]
\[ V_{be3} = V_{cc} - V_4 - V \gamma_3 \]
\[ V_{be4} = V_5 - V_{cc} - V \gamma_4 \]

These equations describe the response of the power amplifier using the equivalent transistor model discussed in Section II - A.
D. Solution of Equations

In order to solve the five simultaneous non-linear equations, the well known Newton-Raphson interactive technique was used. (Refer to Appendix II) A program was written to solve the equations assuming a sinusoidal input voltage. The reason for a sinusoidal input will become apparent later in this discussion.

The period of the sinusoid is broken into 200 segments and the equations solved for each of these segments. Knowing the node voltages, the base currents, collector currents, base-emitter voltages, and collector-emitter voltages can be found by substituting back into the proper equations. At each time increment the computer calculates all of the above variables. Appendix III lists the power amplifier analysis program, flow chart, the actual program, and a sample of the output.

The only input variable into the program is the maximum value of the input sinusoid, $E_1$. From the previous assumptions the output waveform from the computer should describe the actual circuit's waveform for any frequency in the audio range.

To verify this, figure 6 compares the theoretical waveform from the equivalent model circuit to the actual response of the power amplifier for three different frequencies. Figure 6-a is the theoretical response plotted by the computer. Figure 6-b is the actual response
Theoretical Response Using Transistor Models

INPUT ($e_1$)

$$\text{input}(e_1) = 10 \sin \omega t$$ (volts)

Figure 6-a. Verification of Power Amplifier Frequency Independence (Theoretical)
Actual Power Amplifier Response

All vertical scales 5 volts/division

Input \( e_1 = 10 \sin \omega t \) for all cases

Time (10 ms/division)
Frequency = 20 KHz

Time (0.2 ms/division)
Frequency = 1 KHz

Time (10 s/division)
Frequency = 20 KHz

Figure 6-b. Verification of Power Amplifier Frequency Independence (Actual)
at various frequencies. In all cases an input voltage peak of 10 volts was used. From the observation of figure 6 it can be seen that the output signal appears to represent a sinusoid except for a "flat" region near the zero voltage point. This is commonly called crossover distortion. As can be seen from figure 6 the theoretical computer description is a good approximation at low frequencies (up to 10 KHZ), but deviates at 20 KHZ. This deviation, for now, will be neglected and discussed in Section VI.

E. Block Diagram Of The Power Amplifier

Thus far, the response of the power amplifier has been obtained by use of the transistor models. Now, in order to use this information in an overall analysis of the audio amplifier it must be placed into a usable form, such as a Fourier series.

Therefore, let us derive the Fourier series for a general waveform of crossover distortion. The first step is to make an approximate waveform of crossover distortion so that the form of the Fourier series is reasonable. Refering to figure 7, an approximate waveform is described. The positive and negative sides of the waveform are assumed to be sinusoids of a reduced frequency of the overall periodic waveform with the zero regions at the crossover point.
Figure 7. Approximate Waveform of Crossover Distortion
Mathematically let

\( T = \) period of waveform

\( \gamma = \) "delay" time

Then the approximate signal is

\[
f(t) = \begin{cases} 
  0 & 0 < t < \frac{\gamma}{2} \\
  \sin \omega t & \frac{\gamma}{2} < t < \frac{T - \gamma}{2} \\
  0 & \frac{T - \gamma}{2} < t < \frac{T + \gamma}{2} \\
  \sin \omega t & \frac{T + \gamma}{2} < t < \frac{T - \gamma}{2} \\
  0 & T - \frac{\gamma}{2} < t < T 
\end{cases}
\]  

(6)

where: \( \omega' = \frac{2\pi}{T - 2\gamma} \)

For this signal the Fourier series is:

\[
f(t) = \sum_{k=1}^{\infty} B_k \sin(2\pi kt/T) 
\]

(7)

where:

\[
B_k = (4/T) \cos(\frac{\omega' \gamma}{2}) \left[ \frac{\sin(b(\frac{T - \gamma}{2}))}{2b} - \frac{\sin(a(\frac{T - \gamma}{2}))}{2a} \\
- \sin(b(\frac{\gamma}{2})) + \frac{\sin(a(\frac{\gamma}{2}))}{2a} \right] \\
+ (4/T) \sin(\frac{\omega' \gamma}{2}) \left[ \frac{\cos(b(\frac{T - \gamma}{2}))}{2b} + \frac{\cos(a(\frac{T - \gamma}{2}))}{2a} \\
- \cos(b(\frac{\gamma}{2})) - \frac{\cos(a(\frac{\gamma}{2}))}{2a} \right]
\]  

(8)

(See Appendix IV for derivation)

\[
a = \frac{2\pi(T(k+1) - 2k \gamma)}{T(T-2)} 
\]

(9)

\[
b = \frac{2\pi(T(k-1) - 2k \gamma)}{T(T-2)} 
\]

(10)
Figure 7 is actually a computer summation using the above Fourier series with the series truncated at 50 harmonics. This explains the rounded edges near the zero values. Notice that the peak value is normalized to a peak value of one. Therefore multiplying all of the Fourier coefficients by a gain factor will adjust the output peak to the proper level.

To find the Fourier series of the output waveform of the power amplifier all that need to be known are the average maximum value of the waveform, \( \gamma \) and \( T \). Since the frequency is not a variable in the power amplifier, \( T \) and \( \gamma \) cannot be specified, therefore the ratio of \( \gamma \) to \( T \) is used in their place.

At this point the output waveshape and Fourier series is known. This information can be used to form an equivalent block diagram representation of the power amplifier shown in figure 8.

Figure 8 represents the power amplifier by an ideal amplifier with a gain \( A_2 \) and a distortion signal, \( d_0 \), being added to the output. If the input is assumed to be a sinusoid and the Fourier series of the output is known, the form of \( d_0 \) can easily be found.

First assume

1) \( e_1 = E_1 \sin \omega t \)

2) normalized Fourier series representing crossover distortion = \( \sum_{k=1}^{\infty} B_k \sin k t \)
Equivalent Block Diagram of the Power Amplifier

Figure 8. Block Diagram of Power Amplifier
Then the output is, \( e_2 = E_1A_2 \sum_{k=1}^{\infty} B_k \sin k\omega t \) (11)

where: \( A_2 = \frac{\text{peak to peak value of output}}{\text{peak to peak value of input}} \)

As defined by figure 8:
\[
A_2e_1 + d_0 = e_2
\] (12)

And
\[
d_0 = e_2 - e_1A_2 = E_1A_2 \sum_{k=1}^{\infty} B_k \sin k\omega t - A_2E_1 \sin \omega t \tag{13}
\]

Distortion \( d_0 \) can also be represented by a Fourier series
\[
d_0 = \sum_{k=1}^{\infty} D_{ok} \sin k\omega t \tag{14}
\]

Then for the first harmonic:
\[
D_{01} = A_2E_1(B_1-1) \tag{15}
\]

And for the remaining harmonics:
\[
D_{ok} = E_1A_2 B_k \tag{16}
\]

where:
\[
k = 2,3,4,5,\ldots
\]

Combining the two expressions 15 and 16, \( d_0 \) can be expressed as:
\[
d_0 = A_2E_1(B_1-1) \sin \omega t + E_1A_2 \sum_{k=2}^{\infty} B_k \sin k\omega t \tag{17}
\]

Using the information from the computer analysis of the power amplifier and the Fourier series for crossover distortion, \( d_0 \) and \( e_2 \) can be computed. Figure 9 shows \( d_0 \) and \( e_2 \). Figure 9-a is the theoretical computer results and figure 9-b is actual photograph response, when \( e_1 \) has a peak value of 10 volts. \( d_0 \) was photographed by filtering out the fundamental component of the output signal. Again, since the theoretical result, figure 9-a uses only the first 50 terms in the Fourier series and is calculated at
Figure 9-a. Waveform of Distortion Signal, $d_0$ (Theoretical)
Actual Power Amplifier Response

\[ \text{input}(e_1) = 10 \sin \omega t \]

upper channel: output \( (e_2) \) vs time

vertical scale: 5 volts/division

*The vertical scale is not given because the distortion analyzer, used to filter out the fundamental, has a non-calibrated output. This photograph is given primarily for a qualitative insight and not for measurement purposes.*

Figure 9-b. Waveform of the Distortion Signal, \( d_0 \)

(Actual)
discrete increments of time, the distortion signal will not be a perfectly smooth line.

In summary, this section has analyzed the power amplifier by use of equivalent transistor models. From these models, node voltage equations could be written and solved by numerical techniques. The non-dependence on frequency in the audio range was assumed and experimentally verified. Then the Fourier series of amplifier output was derived for the general case of any input signal level. With this information, the power amplifier was represented by an ideal amplifier with a distortion signal being added to the output.

The next step is to investigate the performance of the operational amplifier.
III. Adaptation And Description Of Operational Amplifier

The next functional block of the audio amplifier is the operational amplifier. In this section the operational amplifier will be investigated as to how it functions in this particular configuration. The operational amplifier's gain as a function of frequency, frequency stability, and biasing will be discussed in that order.

One of the characteristic features of an operational amplifier is its very high open-loop gain. It is because of this high gain that feedback can be used and all the benefits of feedback obtained.

The device used in this circuit is the MC 1433 Motorola integrated operational amplifier. Figure 10 gives the schematic and equivalent circuit of the operational amplifier as given in the Motorola specification sheet. Note that this circuit is not a circuit of discrete elements but is fabricated on a silicon chip by planar methods of photo etching and dopant diffusing. Figure 10 is shown to give the reader a better idea of the operational amplifier. No attempt will be made to analyze this circuit, but rather the amplifier will be viewed simply as a high-gain amplifier as depicted in the equivalent circuit. Also in figure 10 are some of the pertinent parameters describing the operational amplifier. The function and use of the various leads will be explained as we proceed.
Motorola MC 1433 Monolithic Operational Amplifier

CIRCUIT SCHEMATICS

(taken from Motorola specification sheet)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>min</th>
<th>typ</th>
<th>max</th>
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<td>input impedance</td>
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<td>600</td>
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</tr>
<tr>
<td>$Z_{out}$</td>
<td>output impedance</td>
<td>-</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>$A_{vol}$</td>
<td>open loop gain</td>
<td>20K</td>
<td>50K</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 10. Operational Amplifier Schematic and Equivalent Circuit
A. Open-Loop Gain

The first specification of an operational amplifier is its open-loop gain, \( A_{\text{VOL}} \). Normally, the larger this number is the better the operational amplifier performs. If the open-loop gain is large, more feedback can be applied and distortion is reduced and the frequency response extended. For the operational amplifier in this discussion, the minimum value of 20,000 will be used for gain. The minimum value is used because our calculations should then reflect the worst case.

The operational amplifier does have an upper cutoff point. This point is essentially determined by the external circuitry used for frequency stability. This circuitry will be given later. For now, let us say that the upper cutoff frequency, \( f_o \), for the case of interest is approximately 500 HZ. Figure 11 is the Bode plot of the operational amplifier as given in the specification sheet for the case of interest. The gain is given in decibels.

![Figure 11 Frequency Response of Operational Amplifier](image)
From figure 11 it can be seen that the gain expression can simply be expressed as:

\[ A_1 = \frac{A_{\text{VOL}}}{1 + j \frac{\omega}{\omega_0}} \]

where: \( \omega_0 \) = upper cutoff frequency of the operational amplifier (open-loop)

Then

\[ |A_1| = \frac{A_{\text{VOL}}}{\sqrt{1 + (\omega/\omega_0)^2}} \]  \hspace{1cm} (18)

Therefore we describe the open-loop gain magnitude of the operational amplifier by equation 18.

B. Frequency Compensation

Now consideration will be given to the frequency stability of the operational amplifier. By frequency stability it is meant that the operational amplifier does not become unstable over all of the frequencies of interest. In other words, the phase shift is restricted so that at high frequencies negative feedback does not turn into positive feedback and cause oscillations. Again, by use of the specifications sheet, values for these compensation networks are given.*

* For a detailed analysis of these networks see "A High Voltage Monolithic Operational Amplifier"; Wisseman, L.L. Motorola Application Note - AN - 248.
It is beyond the scope of this paper to do an analysis of these frequency compensation networks because it requires considerable investigation of the operational amplifier itself. Figure 12 shows the operational amplifier with all of its supporting circuitry.

Network $R_1$ and $C_1$ is an intermediate stage frequency compensation network which couples two stages inside the operational amplifier. Network $R_2$, $C_2$, and $C_3$ is also a frequency stabilization network which serves as a path for the output signal to be returned to the internal circuit of the operational amplifier. These networks determine the cutoff frequency of the operational amplifier and keep it stable over the audio range. Generally speaking $f_0$ can be extended greater than 500 HZ but at the cost of less stability. These networks perform well and essentially eliminate high frequency instability.

C. Biasing

The operational amplifier also has biasing requirements. From figure 12 it can be seen that the operational amplifier has two inputs, one inverting and one non-inverting. It is desirable to have the DC bias currents flowing into these two inputs as equal to one another as possible. This is necessary to maintain the balance of the differential amplifier stage in the operational amplifier. Without this balance the output voltage has a non-zero
Figure 12. External Circuitry of Operational Amplifier

- $R_i = 11\, k\Omega$
- $R_f = 100\, k\Omega$
- $C_f = 39\, pf$
- $R_1 = 10\, \Omega$
- $R_2 = 910\, \Omega$
- $R_3 = 10\, k\Omega$
- $C_1 = 0.1\, mfd$
- $C_2 = 10\, pf$
- $C_3 = 200\, pf$
- $C_4 = 0.1\, mfd$
- $V_{cc} = 15\, volts$
value for a zero voltage input.

To accomplish this, resistor $R_3$ is included. It can be shown that if

$$R_3 = \frac{R_i R_f}{R_i + R_f}$$

the input offset current will be minimized. * Input offset current is defined as the difference between the bias currents in each of the inputs.

The capacitor $C_4$ acts as an AC short which places the non-inverting lead (A) at ground potential. The input voltage then is applied between the inverting lead (B) and ground.

D. Feedback Network

The final functional block for analysis is the feedback loop. Let us define $\beta$ as the feedback ratio, which is the ratio of how much of the output is fed back into the input. $\beta$, as will be shown, essentially determines the gain of the entire amplifier as long as the gain of the operational amplifier is large. Therefore this quantity is quite important and is found in all of the gain and distortion equations in Section IV.

To derive $\beta$, two assumptions must be made. First we shall assume that the input impedance of the operational amplifier is very high, so that we can neglect the input current of the operational amplifier. Refering to figure 10, * Blair, K. "Getting More Value Out Of An Integrated operational Amplifier Data Sheet" ; Motorola Application Note - AN - 273.
it shows a typical value of $Z_{in} = 600 \, \text{K}\Omega$ which justifies our assumption. For the second assumption refer to figure 13. Since the open-loop gain $(A_1, A_2)$ is very large, the value of $e_g$ will be essentially zero in comparison to $e_i$ or $e_o$. Note that since we are primarily interested in the signal only, the distortion from the power amplifier was temporarily neglected.

Using these two assumptions $\beta$ can be found. Apply Kirchhoff's current law at node $e_g$.

$$I_{in} = I_f \text{ from assumption 1}$$

$$\frac{e_i - e_g}{R_i} = \frac{e_g - e_o}{R_f/(1 + j\omega C_f R_f)}$$

Since $e_o = e_g A_1 A_2$
then $e_g = \frac{e_o}{A_1 A_2} \approx 0 \text{ by assumption 2}$

Therefore by neglecting $e_g$:

$$\frac{e_o}{e_i} = \frac{R_f}{R_i} \frac{1}{(1 + j\omega R_f C_f)} \quad (19)$$

To relate equation 19 to standard feedback theory refer to figure 14 which shows in block diagram form the basic feedback amplifier. If $A\beta > 1$ the gain expression is simply $-\frac{1}{\beta}$. Now equate equations (19) to equation (20).

$$-\frac{1}{\beta} = \frac{R_f}{R_i} \frac{1}{(1 + j\omega C_f R_f)}$$

or for magnitude

$$|\beta| = \frac{R_i}{R_f} \sqrt{1 + \omega^2 R_f^2 C_f^2} \quad (21)$$
Assume:

1) \( Z_{\text{in}} \to \infty \)
so that \( I_b \to 0 \)

2) \( A_1 A_2 \to 0 \)
so that \( \frac{e_0}{A_1 A_2} = e_g \approx 0 \)

Figure 13. Feedback Network
\[ \frac{e_o}{e_i} = \frac{A}{1 - A_\beta} \approx \frac{1}{\beta} \] (20)

assume: \( A_\beta \gg 1 \)

Figure 14. Basic Feedback Amplifier
Equation 21 is then, the expression for $|\beta|$. The reason why this particular feedback network was used will become clear in the next section.
IV. Computation Of Amplifier Characteristics

Now that the three basic parts of the audio amplifier are mathematically described, a qualitative investigation can begin.

First the audio amplifier will be assembled using the equivalent block diagrams of the power amplifier, operational amplifier and feedback network. Then the form of the output signal and output distortion will be derived. With these equations the theoretical response of the amplifier will be fully described.

A. Block Diagram Of The Audio Amplifier

Figure 15-a is the total audio amplifier schematic with the power amplifier fully drawn, all the operational amplifier circuitry and the feedback network included. Figure 15-b is the same audio amplifier using (1) the power amplifier equivalent block diagram, (2) the operational amplifier with the gain expression of equation 18 and (3) the block diagram of the feedback network, β. Let us now derive the expressions for the output signal and output distortion.

B. Output Signal And Output Distortion

Refering to figure 15-b we can write the output
Figure 15-a. Schematic and Block Diagram of Audio Amplifier (Schematic)
Figure 15-b. Schematic and Block Diagram of Audio Amplifier (Block Diagram)

\[ e_i = E_i \sin \omega t \]
\[ e_o = E_o \sin \omega t \]
expressions. First the assumption will be made that since there is distortion, \(d_0\), inside the loop, there will be distortion, \(d_c\), at the output. \(d_0\) represents open-loop distortion and \(d_c\) represents closed-loop distortion. Also for the analysis \(e_i\) and \(e_o\) are assumed to be sinusoids and the distortion signals are described by a Fourier series.

The two signals at the output can now be equated. The signal component will be separated from the distortion component.

\[
A_1A_2e_i + \beta A_1A_2e_o + \beta A_1A_2d_c + d_o = e_o + d_c \tag{22}
\]

\[
A_1A_2e_i = e_o(1 - \beta A_1A_2) \quad d_o = d_c(1 - \beta A_1A_2) \tag{23}
\]

\[
\frac{e_o}{e_i} = \frac{A_1A_2}{1 - \beta A_1A_2} \tag{24}
\]

\[
\frac{d_o}{d_c} = \frac{d_o}{1 - \beta A_1A_2} \tag{25}
\]

where: \(A_1 = \frac{A_{\text{VOL}}}{1 + j(\omega/\omega_o)} \tag{18}\)

\[
\beta = -\frac{R_i}{R_f} (1 + j\omega C_f R_f) \tag{21}
\]

\(A_2 = \text{gain of power amplifier determined from computer analysis (} A_2 \text{ is not a function of frequency)} \)

Now substitute equation 18 into 23 and 24.

\[
\frac{e_o}{e_i} = \frac{A_{\text{VOL}} A_2}{1 - \beta A_2 A_{\text{VOL}} + j(\omega/\omega_o)} \tag{25}
\]

\[
d_c = \frac{d_o}{1 - \beta A_2 A_{\text{VOL}} / (1 + j(\omega/\omega_o))} \tag{26}
\]
From 25 it can be seen that at low frequencies if 
$A_2A_{VOL} \gg 1$ the gain expression reduces to $-1/\beta$, which was
demonstrated before (equation 20). Also one of the ben­
efits of feedback can be seen in that the upper cutoff
frequency of the amplifier with feedback has been extend­
ed. The cutoff frequency of the close loop amplifier, $\omega_{CL}$, 
now equals: $(1-A_2A_{VOL})\omega_0$. Therefore the frequency
response of the audio amplifier is greatly increased by feedback.

The other advantage of feedback can be recognized in
equation 26 where the distortion in the output is equal to
the distortion in the loop, $d_o$, reduced by a factor of
$\beta A_2A_{VOL}$. Thus, since the factor is indeed large ( $2 \times 10^3$),
the distortion in the output is greatly reduced. This is
very desirable in audio amplification.

Again it should be stressed that $e_i$ and $e_o$ represent
sinusoid signals and $d_o$ and $d_c$ represent non-sinusoidal
distortion signals described by a Fourier series.

From equation 14:

$$d_o = \sum_{k=1}^{\infty} D_{ok} \sin k\omega t$$

d_c can be similiarly be expressed as:

$$d_c = \sum_{k=1}^{\infty} D_{ck} \sin k\omega t$$ (27)

where:

$$D_{ck} = \frac{D_{ok}}{1 - A_2A_1(k\omega) \beta(k\omega)} \quad k = 1,2,3...$$
Since the frequency of the fundamental component 
\(k=1\) of the distortion signal equals the frequency of \(e_i\) or \(e_o\), the Fourier series representing the output can be expressed as:

\[ e_o + d_c = \sum_{k=1}^{\infty} E_{ok} \sin k\omega t \]  

(28)

where: \( e_o = E_o \sin \omega t \)

and:

\[ d_c = \sum_{k=1}^{\infty} D_{ck} \sin k\omega t \]

so that: \( E_{o1} = E_o + D_{c1} \); \( E_{ok} = D_{ck} \) \( k = 2,3,4... \)

Expression 28 is useful in determining the harmonic content of the final output signal.

C. Adaptation Of Power Amplifier Analysis

In order to find the distortion signal, \(d_o\), information from the power amplifier analysis must be obtained first. Specifically this information is in the form of \(A_2\) (power amplifier gain) and \(\frac{\gamma}{T}\) (ratio of delay time to waveform period). When \(A_2\) and \(\frac{\gamma}{T}\) are computed a specific input voltage \(e_1\) must be given. (\(e_1 = \) input voltage to power amplifier). Therefore in order to use the analysis of power amplifier and the analysis of the entire audio amplifier together there must be a relation between \(e_1\) in figure 8 and \(e_1\) in figure 15-b.

To find this relationship it will be necessary to refer
to both figure 8 and figure 15-b. For the first step the two output voltages will be equated.

\[ e_2 = e_o + d_c \]  \hspace{1cm} (29)

\( e_2 \) is not a sinusoid, but is represented by a Fourier series. From equation 28 the Fourier series of the right side of the equation is known. Since the purpose of this derivation is to determine the relationship of \( e_1 \), which is a sinusoid, only the fundamental of the output will be investigated. All of the harmonics (2nd, 3rd, 4th, 5th, etc.) are greatly reduced due to the feedback, but the fundamental is passed through the amplifier without reduction, but actually gain. Therefore we shall rewrite equation 29 considering only the fundamental harmonic.

\[ e_2 \big|_{1st \ harmonic} = E_{2_{1}} \sin \omega t = (E_o + D_{cl}) \sin \omega t \]  \hspace{1cm} (30)

where: \( E_{2_{1}} = \text{peak to peak value of 1st harmonic of } e_2 \).

Now the assumption must be made that \( \text{Del} \) can be neglected in comparison to \( E_0 \). We know that:

\[ D_{cl} = \frac{D_{o1}}{1 - \beta A_1 A_2} \]  \hspace{1cm} \text{from equation 24}

or

\[ D_{cl} = \frac{A_2 E_1 (B_1 - 1)}{1 - \beta A_1 A_2} \]  \hspace{1cm} \text{from equation 15}

where \( B_1 \) is the first harmonic coefficient for normalized crossover distortion.

Since \( (B_1 - 1) \) is very near to zero and this is further reduced by the factor of the open-loop gain, the assumption is valid.
Now, rewriting equation 30 we have
\[ E_{21} \sin \omega t = E_0 \sin \omega t \]  
(31)
inserting equation 11 for \( E_{21} \) we have
\[ E_1 A_2 B_1 = E_0 \]
or rearranging
\[ E_1 = \frac{E_0}{A_2 B_1} \]  
(32)

Therefore it has been demonstrated that the fundamental of the power amplifier output is essentially equal to \( E_0 \) and that the input and output of the power amplifier are related by expression (32).

Normally the characteristic of the audio amplifier (frequency response, per cent distortion in the output signal, etc.) are evaluated at some particular power output. The assumption will be made that the distortion components in the output are of such low level that their contribution to the total power delivery to the load is negligible. The power, then, can be simply found by
\[ P_{\text{LOAD RMS}} = \frac{E_0^2}{2R_L} \]  
(33)
where \( E_0 \) is peak value of the output sinusoid \( (e_0 = E_0 \sin \omega t) \). Or we can use the fundamental of the output of the power amplifier as described by equation (31).
\[ P_{\text{LOAD RMS}} = \frac{(E_{21})^2}{2R_L} \]  
(34)

To find \( E_{21} \) we can use the Fourier series of cross-over distortion. If the average peak value and ratio of
If $\gamma/T$ is known, $B_1$ (magnitude of the fundamental) can be found.

Mathematically:

$$E_{21} = E_2 B_1$$

where:

$$E_2 = \frac{\text{peak to peak value of } e_2}{2}$$

$$B_1 = \text{normalized magnitude of fundamental for } e_2$$

($\gamma/T$ must be known)

Substituting 35 into 34:

$$P_{\text{LOAD RMS}} = (E_2)^2 \left( \frac{B_1^2}{2 R_L} \right) \quad (36)$$

The quantity $(B_1^2 / 2 R_L)$, as a function of $\gamma/T$, has been calculated and listed in table 2. (See equation 8 for equation of $B_k$.) Using this table and the power amplifier analysis program, the fundamental power content can be found. Given an input voltage level to the power amplifier, $e_1$, the output $e_2$ will have an average peak value $E_2$ and a $\gamma/T$. Squaring $E_2$ and using $\gamma/T$ in table 2, the power in the fundamental can be found by using equation (36). To find a particular power level this process is repeated until the level is found to the desired accuracy.

The output power levels which will be used are 15, 12.5, 10, 7.5, 5, 2.5, and 1 watt. By using the process described above, table 3 was generated by using the power amplifier gain of the power amplifier, $A_2$.

At this point the necessary information about the power
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<thead>
<tr>
<th>$\frac{\tau}{T}$</th>
<th>$B_1$</th>
<th>$\frac{B_1^2}{2R_L}$</th>
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<td>.050</td>
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<td>0.1113</td>
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Table 2. Fundamental Power Content of Power Amplifier Output
<table>
<thead>
<tr>
<th>$E_1$ Peak (Volts)</th>
<th>$E_{2AV}$ (Volts)</th>
<th>$E_{2AV}^2$ (Volts)</th>
<th>$\frac{I}{T}$</th>
<th>$\frac{B_1}{8^2}$</th>
<th>$A_2 = \frac{E_2}{E_1}$</th>
<th>Fundamental Power Into 4 Ohms (Watts)</th>
<th>Nominal Power (Watts)</th>
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<td>.1170</td>
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<tr>
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<td>122.5</td>
<td>.010</td>
<td>.1224</td>
<td>.9380</td>
<td>14.996</td>
<td>15.0</td>
</tr>
</tbody>
</table>

Table 3. Results Of Power Amplifier Analysis Program
amplifier has been obtained in tabular form in table 3. And since the relation between $e_2$ and $e_0$ is known, we can proceed to find the characteristics of the entire audio amplifier.

D. Audio Amplifier Analysis Program

From equations (25) and (26) the audio amplifier gain and distortion are known. To find $d_c$, $d_o$ must be known. By using table 3 and equation 17, $d_o$ can be calculated. To perform these calculations a program was written. Again, the audio amplifier will be examined at the constant power levels of 15, 12.5, 10, 7.5, 5, 2.5, and 1 watt.

So that these power levels are maintained, the output voltage, $e_o$, is a fixed value in the program, adjusted so that it yields the desired power output. From the gain expression equation (25), the input voltage, $e_i$, can be found. Also by using equation (26) the output distortion can be calculated. These results will take the form of a Fourier series describing the output, $e_o + d_c$. This series is used to calculate the per cent harmonic distortion in the output. To do this, all of the harmonic amplitudes must be normalized with respect to the fundamental. Then for per cent distortion for a particular harmonic multiply by 100. This process is shown below.

$$A_1 = \text{amplitude of fundamental of output signal}$$
\[ A_2 = \text{amplitude of 2nd harmonic} \]
\[ A_3 = \text{amplitude of 3rd harmonic} \]
\[ A_{50} = \text{amplitude of 50th harmonic} \]
\[ \frac{A_2}{A_1} \times 100 = D_2 = \text{per cent 2nd harmonic distortion} \]
\[ \frac{A_3}{A_1} \times 100 = D_3 = \text{per cent 3rd harmonic distortion} \]
\[ \vdots \]
\[ \frac{A_{50}}{A_1} \times 100 = D_{50} = \text{per cent 50th harmonic distortion} \]

To find the per cent total harmonic distortion, the square root of the sum of the squares of the individual percentages is found as follows:

per cent total harmonic distortion = \[ \sqrt{D_2^2 + D_3^2 \ldots D_{50}^2} \]

Note that the Fourier series was truncated at 50 terms because the magnitudes beyond the 50th harmonic are negligible.

Since the gain and distortion expressions are functions of frequency through \( A_1 \) and \( \theta \) (see equations (18) and (21)) the characteristics of the audio amplifier must be calculated at various frequencies. Therefore, there are two independent variables, power output and frequency; and two dependent variables, per cent total harmonic distortion and voltage gain. How these quantities will be plotted is shown below.

voltage gain vs frequency at specified power
per cent total harmonic distortion vs frequency at specified power
per cent total harmonic distortion vs power
at specified frequency

The audio amplifier analysis program is written so that for a particular output voltage, (thereby fixing the output power level) the voltage gain and per cent harmonic distortion is calculated for a range of frequencies over the entire audio spectrum (10 HK to 50 KHZ). Table 4 gives the results from this program. The program itself and flow chart is described more fully in Appendix V.

With the tabulation of the theoretical results in table 4, plots describing the performance of the audio amplifier can be drawn. This is done in figures 16, 17, and 18.

In summary, this section had described the entire audio amplifier in a usable block diagram form. Then using standard feedback theory, the expressions for output gain and distortion were derived. To find the distortion signal, $d_o$, the operation of the power amplifier had to be described both outside and inside the audio amplifier. This meant, specifically, relating $e_2$ to $(e_o + d_c)$. This essentially integrated the two programs into one. Tables 3 and 4 give the results at specific output power levels. The three figures 16, 17, and 18, which sum up these results in graphical form are the theoretical response of the audio amplifier.
<table>
<thead>
<tr>
<th>Output Power</th>
<th>15.0 Watts</th>
<th>12.5 Watts</th>
<th>10.0 Watts</th>
<th>7.5 Watts</th>
<th>5.0 Watts</th>
<th>2.5 Watts</th>
<th>1.0 Watts</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.12</td>
<td>9.08 /19.16</td>
<td>0.010</td>
<td>9.08 /19.16</td>
<td>0.015</td>
<td>9.08 /19.16</td>
<td>0.016</td>
<td>9.08 /19.16</td>
</tr>
<tr>
<td>25.7</td>
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<td>0.011</td>
<td>1.01</td>
<td>0.0016</td>
<td>1.01</td>
<td>0.0017</td>
<td>1.01</td>
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<td>0.0100</td>
<td>0.0073</td>
<td>0.0100</td>
<td>0.0073</td>
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<td>0.0228</td>
<td>0.014</td>
<td>0.0228</td>
<td>0.014</td>
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<td>0.013</td>
<td>0.0458</td>
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Table 4. Results of Audio Amplifier Analysis Program
Figure 16. Theoretical Frequency Response Curve
Figure 17. Theoretical Plot of Per Cent Total Harmonic Distortion versus Frequency
Figure 18. Theoretical Plot of Per Cent Total Harmonic Distortion versus Output Power
V. Experimental Verification

A. Objectives

In order to evaluate the validity of the theoretical description above, experimental data must be taken and compared to the theoretical results. The form of this comparison will be essentially the same as the plots of figures 16, 17, and 18.

Hopefully, there will be a correspondence between the theoretical and experimental data. However, due to equipment limitations and simplifying assumptions throughout the analysis, we can expect variation between the experimental and calculated data.

B. Equipment

Figure 19 gives a block diagram of the test setup to measure the response of the audio amplifier.

The measurements can be divided into frequency response (gain vs frequency) and per cent total harmonic distortion verses power or frequency. For the frequency response measurements no equipment limitations exist. The voltage gain is the ratio of the input and output voltages read on the VTVM's. The desired output power can be obtained by maintaining the output voltage at the desired level.

A problem does exist in the measurement of distortion.
Figure 19. Pictorial Diagram of Test Setup and Equipment
Full-scale deflection on the General Radio distortion meter at its lowest scale is 0.3 per cent. When the audio oscillator output is directly fed into the distortion analyzer a reading of approximately 0.15 per cent is measured. Therefore, the audio oscillator itself is introducing distortion. From our theoretical results the distortion in the output of the audio amplifier (0.01 per cent) is far below the level introduced by the oscillator. To measure this level of distortion is practically impossible. Since the generally accepted maximum level of distortion (2.0 per cent)\(^*\) is well above the values discussed here, measurement and calculations of distortion at such low levels is purely academic. However, to reduce the distortion from the audio oscillator, an adjustable band pass filter is used as shown in figure 19. With this filter the per cent distortion from the oscillator with the filter is approximately 0.06 per cent. This is as close to a pure sinusoid as can be expected. Therefore we can expect the distortion level measurement to be never less than approximately 0.06 per cent because this is the level introduced by the oscillator.

C. Results

Using the setup of figure 19 and following the data outline in table 4 (theoretical results), the experimental

results were obtained. The results are tabulated in table 5.

And again in the same fashion of figures 16, 17, and 18, the experimental results of table 5 are plotted in figures 20, 21, and 22.

From these six plots (figures 16, 17, 18, 20, 21, and 22) a comparison between the theoretical and experimental data can be made.
<table>
<thead>
<tr>
<th>Power (Watts)</th>
<th>15.0 Watts (Nominal)</th>
<th>12.5 Watts (Nominal)</th>
<th>10.0 Watts (Nominal)</th>
<th>7.5 Watts (Nominal)</th>
<th>5.0 Watts (Nominal)</th>
<th>2.5 Watts (Nominal)</th>
<th>1.0 Watts (Nominal)</th>
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<td>14.81</td>
<td>---</td>
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</table>

* db = 20 \log_{10}(A_v)

Table 5. Results of Experimental Analysis of Audio Amplifier
Figure 20. Experimental Frequency Response Curve
Figure 21. Experimental Plot of Per Cent Total Harmonic Distortion versus Frequency
Figure 22. Experimental Plot of Per Cent Total Harmonic Distortion versus Output Power
VI. Discussion And Conclusions

A comparison will now be made between the theoretical and experimental data. This data has been plotted as described in Sections IV and V. Ideally, the two sets of plots should exactly agree but in the actual case there are discrepancies. An attempt will be made to explain the sources of these errors.

First let us examine the frequency response plots. The theoretical data (figure 16) yields a low-frequency gain of 19.16 db and the upper cutoff frequency of approximately 40 KHZ for all of the output power levels. For the experimental data (figure 20) the low frequency and 3 db point is 19.17 db and approximately 40 KHZ for 1 watt output power; 19.50 db and approximately 35 KHZ for 15 watts. For all cases, experimental and theoretical, the gain is down approximately 1 db at 20 KHZ. This demonstrates that the amplifier is well within the bandwidth requirements for high quality audio amplifier (gain deviation ± 3 db from 20 Hz to 20 KHZ). The one watt level agrees very closely with the theoretical results. At 15 watts the measured gain is approximately ± 0.3 db from the theoretical value and the 3 db frequency is 35 KHZ rather than 40 KHZ. These small differences can be ignored and the theoretical results accepted as a close description of the actual case.

Now let us examine the plots of per cent total harmonic distortion versus frequency. From the theoretical data,
figure 17, the per cent distortion is reduced. This is quite a unique characteristic. Usually the distortion is greatest at maximum power output. This characteristic can be accounted for by examining the response of the power amplifier which is the source of the distortion present. The power amplifier analysis is summarized in table 3. Notice that the ratio $\gamma/T$ is greatest for small input voltage swings. Physically this means that if the input voltage is small, a considerable section of the output signal will be in the crossover point, near zero volts, until the driver transistor turns on. Since the turn-on voltage of the driver transistors in the equivalent model circuit is 0.5 volts, the output will not move from zero until the input signal is greater than 0.5 volts. The larger the ratio $\gamma/T$, the more distortion is contained in the output. Therefore, as the input voltage increases, $\gamma/T$ decreases, thus reducing the distortion in the output.

Another feature of figure 17 is the general increase of the per cent distortion as frequency increases. This is very easily explained by considering equation 24 which is the expression for the output distortion, $d_\circ$. Both $A_1$ and $\beta$ are functions of frequency described by equations 18 and 21 respectively. If we consider $d_\circ$ fixed for a fixed output power, by substitution of equations 18 and 21 into equation 24, it can be shown that the output distortion, $d_\circ$ increases as frequency increases. Or more simply, the product $\beta A_1$ decreases as frequency increases.
Figure 21 is the experimental measurement of percent distortion versus frequency and should be the same as figure 17. However, upon investigation there is considerable deviation between the two (note the vertical scale change). To compare the two plots let us first discuss the low frequency portion of figure 21. From 50 HZ to 1 KHZ the distortion levels are relatively constant. Note that the curve starts at 50 HZ because that is the low frequency limit on the distortion analyzer. Also, it can be seen that just as in the theoretical plot, the 1 watt output power has the highest distortion and the 15 watt has the lowest distortion. This then agrees with theory; the higher the power, the lower the distortion. However, the magnitude of the percent distortion is a full order of magnitude less in the experimental data than was predicted. This at first, seems to indicate that theory is in error, but there is an explanation. Recalling from Section V, that the audio oscillator with the bandpass filter has a distortion level of approximately 0.06 per cent, we see that the input test sinusoid has a distortion level of 0.06 per cent. Also from the manufacturer's specifications of the distortion analyzer the full scale accuracy is ± 5 per cent + the residual distortion of the analyzer itself. This residual distortion is 0.05 per cent below 7500 Hz and 0.10 per cent above 7500 Hz. (See figure 19 for the full distortion analyzer specifications). Therefore, even without the amplifier considered, there are two sources
of distortion; the audio oscillator and the distortion analyzer.

With these considerations the level of distortion measured seems to be consistent with the predicted values. There simply are no instruments available that can measure distortion levels on the order of .005 per cent.

Next the section of the plot from 1 KHZ to 19 KHZ will be examined. It is in this section that the most serious deviation from the expected values takes place. Also note that figure 21 stops at 19 KHZ, this is because the upper frequency limit on the distortion analyzer is 19 KHZ. The theoretical data, figure 17, however evaluates the percent distortion up to 50 KHZ.

First let us compare the distortion curves for one watt. At 19 KHZ, theory predicts 0.316 per cent and the measured value is 0.42 per cent. Again, considering the extraneous distortion introduced (oscillator and distortion analyzer) as discussed above, this seems to be a tolerable deviation.

From theory, if the power level increases we can expect a drop in distortion. However, just the reverse happens in the actual experimental data. For 5, 10, and 15 watts output power, the measured distortion level is 0.43 per cent, 0.56 per cent, and 0.73 per cent at 19 KHZ respectively. These deviations cannot be explained by the above considerations.

To justify these results we must examine more closely
the simplifying assumptions made previously. First, in the audio amplifier analysis in Section IV, the method of analysis assumed that if the input was a pure sinusoid, the output would be composed of a fundamental signal and many harmonics. These harmonics were considered to be the distortion in the output signal. The magnitudes of these harmonics are small compared to the magnitude of the fundamental. Therefore, the assumption was made that the harmonics themselves would not generate further distortion, since the output is fed back to the input through $\theta$. While this is generally a good assumption, there actually is further distortion generated by the original distortion signals at the output. Second, the operational amplifier was assumed to be distortion free and actually is not. To analyze this source of distortion is beyond the scope of this paper and will just be mentioned. Third, the power amplifier was analyzed on the basis of transistor models which were a good approximation of the transistors used. However as can be seen from figure 4, the actual transistor characteristics can change considerably as the point of operation changes. Thus, the value of $h_{FE}$, for instance, might be 100 near cutoff and 50 near saturation. Although median values were chosen, it can be seen that the theoretical power amplifier response will deviate from the actual case. And, to analyze the waveform of crossover distortion, the Fourier series of an approximate waveform was found. Figures 6 and 7 show the actual waveform photographs and approximated
signal. As can be seen, the approximation is close except at the higher frequencies. This then, would seem to be another cause of the resulting deviation between the measured and predicted values of distortion.

While all of the above sources of error contribute to discrepancy found between figures 17 and 21, they do not explain why there is such a large difference (e.g., at 15 watts, 19 KHZ: theoretical - .069 per cent; experimental 0.73 per cent).

The author has concluded that high frequency oscillations are the primary cause of the deviation from the predicted values. Since the operational amplifier is a very high gain device it is very susceptible to these high frequency oscillations. These oscillations are caused from excessive lead inductance, excessive power supply impedance, and grounding loops. To reduce these effects, the circuit layout used leads as short as possible and two 1.0 mfd bypass capacitors were placed from the supply leads to ground. For best results the circuit should be fabricated on a printed circuit board to get lead lengths to a minimum. The experimental tests used the amplifier constructed on standard vector board with the two power transistors on a heat sink. Also during the distortion measurements, diagramed in figure 19, the distortion analyzer, VTVM, and oscilloscope were connected across the load. This added approximately 100 pf across the load making it still more susceptible to oscillation. During the
Experimental measurements these high frequency oscillations could be observed on the oscilloscope as a broadening of the trace near the waveform peaks. This effect was more noticeable at higher power levels which was demonstrated in the distortion measurements. To avoid pickup of noise signals and 60 HZ hum, shielded cable was used for the input and output leads.

The frequency compensation networks, $C_f$ and $C_B$, all contribute to reduce these oscillations. Their values had to be chosen on two criteria; frequency response and per cent distortion. The distortion could be reduced, but only at the cost of poorer frequency response. The values chosen were a compromise between reasonable distortion levels and good frequency response. As given before, as long as the distortion is less than 2.0 per cent, the amplifier is considered to be a high quality audio amplifier.

Returning to figure 21, the effect of a rise in distortion for a rise in power at high frequency, can be explained on basis of these oscillations. At low power levels, the voltages and currents are also at low levels and the oscillation is at a minimum. However the reverse happens as output power is increased. These effects therefore, are not shown in the theoretical results simply because the model system does not include these operational difficulties (power supply impedance, lead inductance, etc.).

Considering all of the factors above, the theoretical model describes the distortion levels for audio amplifier
accurately only for low frequencies. It does however, give a general approximation at higher frequencies when the limitations discussed above are considered.

Figures 18 and 22 contain no new information but simply hold frequency constant and vary the output power level. The same considerations for figures 17 and 21 apply here.

Considering this last section, a general evaluation of this analysis can be made. First it has been demonstrated that a high quality audio amplifier can be constructed using fewer components than previously possible before the advent of the monolithic operational amplifier. Second the block diagram analysis is a straightforward approach utilizing common Fourier series analysis and transistor modeling. The equations describing the amplifier are very similar to those found in standard feedback theory. Third, the use of the operational amplifier as a "block gain" is shown in this analysis. This type of design is very favorable because the response can be determined quickly by just analyzing the feedback network. Fourth, the theoretical results of the analysis are reasonably accurate for low frequency and a good approximation for high frequency when the high frequency oscillations are considered.

To make the analysis more accurate would entail considerably more detail which would lose the present simplicity. Therefore the audio amplifier and analysis described, can
be considered to be very useful.
BIBLIOGRAPHY


APPENDIX I

LEAST SQUARE CURVE FIT OF THE FORM  \( y = y_0 (\exp(kx) - 1) \)

Given a known constant, \( y_0 \), and a set of data \((y_1, x_1; y_2, x_2; y_3, x_3; \ldots; y_n, x_n)\), let us derive the expression for \( k \) using the least square method. First, the assumption will be made that for the values of interest, the above equation can be approximated by \( y = y_0 (\exp(kx)) \). Using this equation greatly simplifies the analysis.

The next step is to take the natural logarithm of both sides of the simplified equation thusly.

\[ \ln(y) = \ln(y_0) + kx \]

Now the square of the difference factor, \( S \), is found.

\[ S = \sum_{i=1}^{n} (\ln(y_i) - \ln(y_0) - kx_i)^2 \]

To find the minimum of \( S \), the derivative with respect to \( k \) is found.

\[ \frac{\partial S}{\partial k} = 2 \sum_{i=1}^{n} (\ln(y_i) - \ln(y_0) - kx_i) x_i = 0 \]

Solving for \( k \), the result is:

\[ \sum_{i=1}^{n} (\ln(y_i))(x_i) = \ln(y_0) \sum_{i=1}^{n} x_i + k \sum_{i=1}^{n} (x_i)^2 \]

\[ k = \frac{\sum_{i=1}^{n} (\ln(y_i))(x_i) - \ln(y_0) \sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} (x_i)^2} \]

This, then, is the equation for \( k \).

The program to calculate \( k \) is now listed. The variable conversion into Fortran is given in table form. Notice that the value of \( V_{be} (x_i) \) is converted to \( V_{be} \) by subtracting \( V_x \).
Also, once the value of \( k \) is determined it is inserted into the exact equation originally being fitted and \( y \) calculated for a range of \( x \) so that the resulting curve can be compared to the actual data.

### Variable Conversion Table

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<th>least square derivation</th>
<th>Section II variables</th>
<th>Fortran variables</th>
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</tr>
<tr>
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<td></td>
<td>( V_y )</td>
<td>( VGAMA )</td>
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Variable Conversion Table

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1 DATA = 12.32, TIME=0.2, PAGES=150
2 DIMENSION Y(2), X(20)
3 DO 1 I=1,4
4 X(I) = (1,10)**N
5 SC = Y(I), VGAMA
6 DO 7 I=1,N
7 Y(I) = Y(I) - VGAMA
8 WRITE (2,20)
9 WRITE (2,20) (Y(I),X(I),I=1,N)
10 SUMYY = 0
11 SUMX = 0
12 SUMZX = 0
13 DO 2 I=1,N
14 Y(I) = ALOC(Y(I))*X(I)
15 SUMYY = SUMYY + X(I)
16 DO 3 I=1,N
17 SUMX = SUMX + X(I)
18 DO 4 I=1,N
19 SUMZX = SUMZX + X(I)*Y(I)
20 R = (SUMXX = SUMYY - MNG(SC*SUMX, SUMY) / (SUMXX))
21 WRITE (2,20)
22 WRITE (2,20) SC,R, VGAMA
23 WRITE (2,20)
24 V = V + 1
25 CONTINUE
26 CALL EXIT
27 FORMAT (12)
28 FORMAT (2F15.8)
29 FORMAT (///,2, "BASE CURRENT", TP4, "BASE VOLTS")
30 FORMAT (1H1)
31 FORMAT (///, "SOLUTION OF THE EQUATION", I3 = 1, E18.8,
32 1EXP(\( F(I,5), * VBE \)), /* WITH VGAMA = 1E5.3")
33 FORMAT (///, "END TRANSISTOR Q", TP4)
34 FORMAT (///, 4, F10.4)
35 END
```
### FOR TRANSISTOR Q1

**SOLUTION OF THE FORM, $I_B = 0.30000000E+06 \times \exp(28.180709 \times V_{BE})$**

**Results using equation:**

$$I_B = I_O \left(\exp(K V_{BE}) - 1\right)$$

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<th>BASE VOLTS</th>
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</tbody>
</table>

**SAMPLE OUTPUT**

**Measured Data**
APPENDIX II

NEWTON-RAPHSON METHOD FOR A SYSTEM OF EQUATIONS

In order to solve the five simultaneous non-linear equations (Section II, equations 1, 2, 3, 4, and 5) the Newton-Raphson iterative technique was used.

The derivation for the approximation algorithm will now be derived. Let the five equations have the following notation.

\[
A(v_1, v_2, v_3, v_4, v_5) = I_{o1} (h_{FE1} + 1)(\exp(K_1 v_{be1}) - 1) - I_{o2} (h_{FE2} + 1)(\exp(K_2 v_{be2}) - 1) + I_{o3} (h_{FE3} + 1)(\exp(K_3 v_{be3}) - 1) - I_{o4} (h_{FE4} + 1)(\exp(K_4 v_{be4}) - 1) + (v_2 - v_1) h_{oe1} - (v_1 - v_3) h_{oe2} + (v_{cc} - v_1) h_{oe3} - (v_1 + v_{cc}) h_{oe4} - \frac{v_1}{R_L}
\]

\[
B(v_1, v_2, v_3, v_4, v_5) = R_B h_{FE1} I_{o1} (\exp(K_1 v_{be1}) - 1) + R_B (v_2 - v_1) h_{oe1} - v_4 + v_2
\]

\[
C(v_1, v_2, v_3, v_4, v_5) = R_B h_{FE2} I_{o2} (\exp(K_2 v_{be2}) - 1) + R_B (v_1 - v_3) h_{oe2} - v_3 + v_5
\]

\[
D(v_1, v_2, v_3, v_4, v_5) = R_B I_{o3} (\exp(K_3 v_{be3}) - 1) - v_4 + v_2
\]

\[
E(v_1, v_2, v_3, v_4, v_5) = R_B I_{o4} (\exp(K_4 v_{be4}) - 1) - v_3 + v_5
\]

where:

\[
v_{be1} = e_1 - v_1 - v_{11}
\]

\[
v_{be2} = v_1 - e_1 - v_{12}
\]

\[
v_{be3} = v_{cc} - v_4 - v_{33}
\]

\[
v_{be4} = v_5 - v_{cc} - v_{44}
\]
Let $V_{10}, V_{20}, V_{30}, V_{40}, V_{50}$ be an approximation to the solution of the five equations. Now expand the equations about the above approximation using the Taylor's series and neglecting the higher order terms since we make the assumption that the initial approximation is relatively close to the actual solution. To simplify notation let:

$$
A = A(V_1, V_2, V_3, V_4, V_5) \text{ etc}
$$

$$
\frac{\partial A}{\partial V_1} = A_1 ; \frac{\partial A}{\partial V_2} = A_2 \text{ etc}
$$

$$
A_0 = A \left| \begin{array}{cccc}
V_{10}, V_{20}, V_{30}, V_{40}, V_{50}
\end{array} \right| \text{ etc}
$$

$$
A_{10} = \frac{\partial A}{\partial V_1} \left| \begin{array}{cccc}
V_{10}, V_{20}, V_{30}, V_{40}, V_{50}
\end{array} \right| \text{ etc}
$$

The expansion is:

$$
A = A_0 + A_1(V_1 - V_{10}) + A_2(V_2 - V_{20}) + A_3(V_3 - V_{30}) + A_4(V_4 - V_{40}) + A_5(V_5 - V_{50}) \approx 0
$$

$$
B = B_0 + B_1(V_1 - V_{10}) + B_2(V_2 - V_{20}) + B_3(V_3 - V_{30}) + B_4(V_4 - V_{40}) + B_5(V_5 - V_{50}) \approx 0
$$

$$
C = C_0 + C_1(V_1 - V_{10}) + C_2(V_2 - V_{20}) + C_3(V_3 - V_{30}) + C_4(V_4 - V_{40}) + C_5(V_5 - V_{50}) \approx 0
$$

$$
D = D_0 + D_1(V_1 - V_{10}) + D_2(V_2 - V_{20}) + D_3(V_3 - V_{30}) + D_4(V_4 - V_{40}) + D_5(V_5 - V_{50}) \approx 0
$$

By transposing the first term on the left side of the equations to the right side, a system of equations is formed. To solve this system of equations Cramer's rule is used.
From this we can see that the next approximation is:

\[ V_1 = V_{10} + \frac{\text{Det} 1}{|\text{Jacobian}|} \]

This same process is repeated to obtain the next approximation for \( V_2, V_3, V_4, \) and \( V_5 \).

\[ V_2 = V_{20} + \frac{\text{Det} 2}{|\text{Jacobian}|} \]
These then, are the recursion formulas which generate successive approximations until the desired accuracy is obtained.

The actual equations for all of the partial derivatives and determinant expansions are listed in the program described in appendix III.
The function of this program is to find the response of the power amplifier by solving the five non-linear equations given in Section II (equations 1, 2, 3, 4, and 5). The procedure by which these equations will be solved is the Newton-Raphson iterative method. The recursion formulas are given in Appendix II. In order to meet the requirements for Fortran coding there must be a variable name change. This is given below in tabular form.

**SYMBOL**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Fortran Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_1, K_2, K_3, K_4$</td>
<td>$Q_1, Q_2, Q_3, Q_4$</td>
</tr>
<tr>
<td>$I_{o1}, I_{o2}, I_{o3}, I_{o4}$</td>
<td>$SC_1, SC_2, SC_3, SC_4$</td>
</tr>
<tr>
<td>$V_{y1}, V_{y2}, V_{y3}, V_{y4}$</td>
<td>$VGAMA1, VGAMA2, VGAMA3, VGAMA4$</td>
</tr>
<tr>
<td>$h_{FE1}, h_{FE2}, h_{FE3}, h_{FE4}$</td>
<td>$HFE1, HFE2, HFE3, HFE4$</td>
</tr>
<tr>
<td>$h_{oe1}, h_{oe2}, h_{oe3}, h_{oe4}$</td>
<td>$HOE1, HOE2, HOE3, HOE4$</td>
</tr>
<tr>
<td>$R_B$</td>
<td>$RB$</td>
</tr>
<tr>
<td>$R_L$</td>
<td>$RL$</td>
</tr>
<tr>
<td>$V_{cc}$</td>
<td>$VCC$</td>
</tr>
<tr>
<td>$E_1$</td>
<td>$E1$</td>
</tr>
<tr>
<td>$V_{be1}, V_{be2}, V_{be3}, V_{be4}$</td>
<td>$VBE1, VBE2, VBE3, VBE4$</td>
</tr>
<tr>
<td>$V_{be1}, V_{be2}, V_{be3}, V_{be4}$</td>
<td>$VBE5, VBE6, VBE7, VBE8$</td>
</tr>
<tr>
<td>$\frac{\partial A}{\partial V_1} = A_1$, etc</td>
<td>$A1$, etc</td>
</tr>
<tr>
<td>Det 1, etc</td>
<td>DET1, etc</td>
</tr>
<tr>
<td>Jacobian</td>
<td>AJAKE</td>
</tr>
<tr>
<td>$I_{b1}, I_{b2}, I_{b3}, I_{b4}$</td>
<td>$CB1, CB2, CB3, CB4$</td>
</tr>
</tbody>
</table>
As was shown in Appendix II, the evaluation of the five equations, twenty five derivatives and six determinants is necessary for each iteration. For clarity all of these equations are listed. Notice that the equations are in Fortran coding form.

\[ A = (SC1)*(HFE1+1)*(\exp(Q1*(VBE1))-1) \]
\[ - (SC2)*(HFE2+1)*(\exp(Q2*(VBE2))-1) \]
\[ + (SC3)*(HFE3+1)*(\exp(Q3*(VBE3))-1) \]
\[ - (SC4)*(HFE4+1)*(\exp(Q4*(VBE4))-1) \]
\[ + (V2-V1)*HOE1 - (V1-V3)*HOE2 + (VCC-V1)*HOE3 - (V1+VCC)*HOE4 - (V1/RL) \]

\[ A1 = (SC1)*(HFE1+1)*(-Q1)*(\exp(Q1*(VBE1))) \]
\[ - (SC2)*(HFE2+1)*(Q2)*(\exp(Q2*(VBE2))) \]
\[ - HOE1 - HOE2 - HOE3 - HOE4 - (1/RL) \]

\[ A2 = HOE1 \]
\[ A3 = HOE2 \]
\[ A4 = (SC3)*(HFE3)*(-Q3)*(\exp(Q3*(VBE3))) \]
\[ A5 = (-SC4)*(HFE4)*(Q4)*(\exp(Q4*(VBE4))) \]

\[ B = (RB*HFE1*SC1)*(\exp(Q1*(VBE1))-1) + RB(V2-V1) \]
\[ - V4 + V2 \]
\[ B1 = (RB*HFE1*SC1)*(-Q1)*(\exp(Q1*(VBE1))) - RB*HOE1 \]
\[ B2 = 1 + RB*HOE1 \]
\[ B3 = 0 \]
\[ B4 = -1 \]
\[ B5 = 0 \]

\[ C = (RB*HFE2*SC2)*(\exp(Q2*(VBE2))-1) + RB(V1-V3)*HOE2 - V3 + V5 \]
\[ C1 = (RB*HFE2*SC2)*(Q2)*(\exp(Q2*(VBE2)))+RB*HOE2 \]
\[ C2 = 0 \]
\[ C3 = -RB*HOE2 - 1 \]
\[ C4 = 0 \]
\[ C_5 = 1 \]
\[ D = (RB*SC3)*(\exp(Q3*(VBE3))-1)-V_4+V_2 \]
\[ D_1 = 0 \]
\[ D_2 = 1 \]
\[ D_3 = 0 \]
\[ D_4 = (SC3*RB)*(\exp(Q3*(VBE3)))*(-Q3)-1 \]
\[ D_5 = 0 \]
\[ E = (RB*SC4)*(\exp(Q4*(VBE4))-1)-V_3+V_5 \]
\[ E_1 = 0 \]
\[ E_2 = 0 \]
\[ E_3 = -1 \]
\[ E_4 = 0 \]
\[ E_5 = (RB*SC4)*(Q4)*(\exp(Q4*(VBE4)))+1 \]

Notice that many of the equations are equal to zero. This fact reduces the size of the determinant expansions considerably. These expansions are given below.

\[ \text{DET}_1 = B_2*D*A_4^*(C_3*E_5+1)-B_2*D_4*A^*(C_3*E_5+1)-B_2*D_4 \]
\[ E^*(A_3-C_3*A_5)+D*A_2^*(C_3*E_5+1)-A^*(C_3*E_5+1)+ \]
\[ C^*(A_3*E_5+A_5)-E^*(A_3-C_3*A_5)+B_2*D_4*C^*(A_3*E_5+A_5) \]
\[ +B*C_3*E_5^*(A_2*D_4-A_4)+B^*(A_2*D_4-A_4) \]

\[ \text{DET}_2 = -D*B_1*A_4^*(C_3*E_5+1)-D*A_1^*(C_3*E_5+1)+D*C_1^*(A_3^*E_5+A_5) \]
\[ +D_4*B_1*E^*(A_3-A_5*C_3)-D_4*B*A_1^*(C_3*E_5+1)+D_4*B \]
\[ +C_1^*(A_3*E_5+A_5) \]

\[ \text{DET}_3 = -E^*C_1*A_5^*(B_2*D_4+1)+E*A_1^*(B_2*D_4+1)-E*B_1^* \]
\[ (A_2*D_4-A_4)+E_5*C_1^*A_2^*(-B*D_4-D)-E_5*C_1^*B_2^* \]
\[ (-A_4*D_4+A_4)+E_5*C_1^*(A+B*A_4)-E_5*C*A_1^*(B_2 \]
\[ *D_4+1)+E_5*C*B_1^*(A_2*D_4-A_4) \]

\[ \text{DET}_4 = -A_5^*(-B_1*C+B*C_1)-(A_1*B+A*B_1)-D*A_5^*(-C_1*B_2) \]
\[ -D^*(A_1*B_2-A_2*B_1)+E*B_1^*(A_3-C_3*A_5)-E_5*A_3^*(-B_1 \]

DET5 = -C1*A2*(D+B*D4)+C1*B2*(-A4*D+A*D4)+C1*(A4
*B1*C3-E*(A1*C3-C1*A3)+(E*D4*Cl*B2*A3)-
(E*D4*C3)*(A1*B2-B1*A2)

+(E5*B1*A4*C3)+E5*(A1*C3-C1*A3)-E5*D4*B1
*A2*C3+E5*D4*B2*(A1*C3-C1*A3)

The flow chart of the program is now given. Since
this program uses standard techniques no further explain-
ation is given.

POWER AMPLIFIER ANALYSIS PROGRAM FLOW CHART
Now the actual program and output sample is given.
POWER AMPLIFIER ANALYSIS PROGRAM

TRANISTOR PARAMETERS

\[ \begin{align*}
Q_1 & = 50 \cdot 10^7 \\
Q_2 & = 70 \cdot 10^7 \\
Q_3 & = 11 \cdot 10^7 \\
Q_4 & = 12 \cdot 10^7 \\
SC_1 & = 1 \cdot 10^7 \\
SC_2 & = 2 \cdot 10^7 \\
SC_3 & = 3 \cdot 10^6 \\
SC_4 & = 4 \cdot 10^6 \\
VGA_A1 & = 1 \cdot 10^3 \\
VGA_A2 & = 2 \cdot 10^3 \\
VGA_A3 & = 3 \cdot 10^3 \\
IHFE & = 1 \cdot 10^4 \\
IHFE & = 2 \cdot 10^4 \\
IHFE & = 3 \cdot 10^4 \\
IHFE & = 4 \cdot 10^4 \\
HOF1 & = 2 \cdot 10^4 \\
HOF2 & = 3 \cdot 10^4 \\
HOF3 & = 4 \cdot 10^4 \\
HOF4 & = 5 \cdot 10^4
\end{align*} \]

CIRCUIT VALUES

\[ \begin{align*}
BP & = 5 \\
BI & = 4 \\
VCC & = 15 \\
V1 & = 10 \cdot 10^3 \\
V2 & = 14 \cdot 4 \\
V3 & = -14 \cdot 4 \\
V4 & = 14 \cdot 4 \\
V5 & = -14 \cdot 4
\end{align*} \]
c

MM=24
NN=MM+1
KTP1=53
KTP2=150
R=6.0
DC 7 I=1, NN
26 IF (((NN/4+1)-1) 26, 25, 25
27 V(1, I)=V(1, KTP1)
V(2, I)=V(2, KTP1)
V(3, I)=V(3, KTP1)
V(4, I)=V(4, KTP1)
V(5, I)=V(5, KTP1)
V(6, I)=V(6, KTP1)
ITP(I)=C
KTP1=KTP1-1
28 IF (((NN/4+1)-1) 29, 25, 25
29 V(1, I)=V(1, KTP2)
V(2, I)=V(2, KTP2)
V(3, I)=V(3, KTP2)
V(4, I)=V(4, KTP2)
V(5, I)=V(5, KTP2)
V(6, I)=V(6, KTP2)
ITP(I)=C
KTP2=KTP2-1
30 GO To 7
31 M=1  *(SIN((2.*D1*D)/(MM )))
32 IF (((NN/2+2)-1) 35, 32, 35
33 V1=0.0
V2=14.4
V3=-14.4
V4=14.4
V5=-14.4
35 N=C

NEWTON-CARPHSON METHOD

1 APPROX1=V1
APPROX2=V2
APPROX3=V3
APPROX4=V4
APPROX5=V5
VCF1= V1-V1-VSAMA1
VHF2=V1-V1-VSAMA2
VHF3=VCC-V4-VSAMA3
VBE4=V5+VCC-VSAMA4
V1 = V1 + (DET1 / AJAKE)
V2 = V2 + (DET2 / AJAKE)
V3 = V3 + (DET3 / AJAKE)
V4 = V4 + (DET4 / AJAKE)
V5 = V5 + (DET5 / AJAKE)
N = N + 1
IF (ABS(V1-APRX11-ERR01) > 2.1, I)
2 IF (ABS(V2-APRX2)-ERR02) > 3.1, I
3 IF (ABS(V3-APRX3)-ERR03) > 4.1, I
4 IF (ABS(V4-APRX4)-ERR04) > 6.1, I
5 IF (ABS(V5-APRX5)-ERR05) > 6.1, I
6 V(1,1) = VIN
V(2,1) = V1
V(3,1) = V2
V(4,1) = V3
V(5,1) = V4
V(6,1) = V5
ITP(1) = N
7 R = C + 1.

PRINT RESULTS

K = 0
DO 21 K = 1, N
DATA (3,110)
VIN = V(1,K)
V1 = V(2,K)
V2 = V(3,K)
V3 = V(4,K)
V4 = V(5,K)
V5 = V(6,K)
21 PRINT (K,V1,V2,V3,V4,V5)

VRE5 = VIN - V1
VRE6 = V1 - VIN
VRE7 = VRE5 - V4
VRE8 = VRE6 + VRE7
VRE9 = VRE8 - VRE7
VRE2 = VRE8 - VRE7
VRE3 = VRE8 - VRE7
VRE4 = VRE8 - VRE7
VREF(5,16) = KK
CR1 = (CC1) * (VYD(0#VRE1) - 1.)
CR2 = (CC2) * (VYD(0#VRE2) - 1.)
CC = (V4-V2)/(PR)
CC2 = (V2-V5)/(PR)
CRA = CC
CC2 = (HEE3) * (CR3) + (VCC - V1) * (HCE3)

CC4 = (HEE4) * (CR4) + (V1 + VCC) * (HCE4)

VCE1 = V2 - V1

VCE2 = V1 - V3

VCE3 = VCC - V1

VCE4 = V1 + VCC

WRITE (3, 20)

WRITE (3, 30) VIN, V1, V2, V3

WRITE (3, 40)

WRITE (3, 60) V4, V5

WRITE (3, 80)

WRITE (3, 100) VN5, VN6, VN7, VN8

WRITE (3, 120)

WRITE (3, 160) VCE1, VCE2, VCE3, VCE4

WRITE (3, 180)

WRITE (3, 200) C91, C92, C93, C94

WRITE (3, 220)

WRITE (3, 240) CC1, CC2, CC3, CC4

21 KK=KK+1

CALL EXIT

10 FORMAT ('//, ' ITERATIONS=', 14, ', ' TIME INCREMENT=', 14, ', ' NUMBER OF TIME STEPS=', 14)

20 FORMAT ('//, T16, 'VIN', T20, 'V1', T46, 'V2', T44, 'V3')

30 FORMAT (4E13.8)

40 FORMAT ('IF12', 4)

50 FORMAT ('//, TO, 'VN61', T27, 'VN62', T43, 'VN63', T63, 'VN64')

60 FORMAT ('//, TO, 'V21', T27, 'V22', T43, 'V23', T63, 'V24')

70 FORMAT ('//, TO, 'V31', T27, 'V32', T44, 'V33', T63, 'V34')

80 FORMAT ('//, TO, 'V41', T28, 'V42', T51, 'V43', T61')

90 FORMAT ('//, TO, 'V51', T28, 'V52', 'V53')

110 FORMAT ('//, T9, 'VCE1', T27, 'VCE2', T43, 'VCE3', T63, 'VCE4')

120 FORMAT ('//, T9, 'VCE1', T27, 'VCE2', T43, 'VCE3', T63, 'VCE4')

END
OUTPUT SAMPLE

ITERATIONS= 10
TIME INCREMENT= 25
NUMBER OF TIME STEPS= 200

\[
\begin{align*}
V_{\text{IN}} &= 0.70710666 \times 10^0 & V_1 &= 0.63564530 \times 10^0 & V_2 &= 0.13428010 \times 10^0 & V_3 &= -0.14042410 \times 10^0 \\
V_4 &= 0.14124620 \times 10^0 & V_5 &= -0.14245010 \times 10^0 \\
V_{\Phi 1} &= 0.72861340 \times 10^0 & V_{\Phi 2} &= -0.72861340 \times 10^0 & V_{\Phi 3} &= 0.87537380 \times 10^0 & V_{\Phi 4} &= 0.78498000 \times 10^0 \\
V_{CF 1} &= 0.70775666 \times 10^0 & V_{CF 2} &= 0.29328500 \times 10^0 & V_{CF 3} &= 0.86495460 \times 10^0 & V_{CF 4} &= 0.21350440 \times 10^0 \\
I_{R 1} &= 0.26390100 \times 10^{-3} & I_{R 2} &= -0.36590900 \times 10^{-3} & I_{R 3} &= 0.34830330 \times 10^{-3} & I_{R 4} &= 0.10130400 \times 10^{-1} \\
I_{C 1} &= 0.34830330 \times 10^{-3} & I_{C 2} &= 0.10130400 \times 10^{-1} & I_{C 3} &= 0.25304200 \times 10^{-1} & I_{C 4} &= 0.10614020 \times 10^{-1}
\end{align*}
\]
APPENDIX IV

DERIVATION OF FOURIER SERIES FOR WAVEFORM OF CROSSOVER DISTORTION

Using figure 7 and equation 6, the Fourier series can be found by using the definition of the Fourier series for an odd function.

For an odd function, \( f(t) \):

\[
f(t) = B_k \sin \left( \frac{2\pi kt}{T} \right)
\]

where: \( B_k = \frac{4}{T} \int_0^{T/2} f(t) \sin \left( \frac{2\pi kt}{T} \right) dt \)

Substituting for \( f(t) \):

\[
B_k = \left( \frac{4}{T} \right) \int_0^{T/2} f(t) \sin \left( \frac{2\pi kt}{T} \right) dt
\]

where: \( \omega' = \frac{2\pi}{T-\tau} \)

Since the value of \( f(t) = 0 \) for; \( 0 < t < \frac{\tau}{2} \) and \( \frac{T-\tau}{2} < t < \frac{T+\tau}{2} \), new limits can be used. Also \( f(t) \) can be expanded thusly.

\[
B_k = \left( \frac{4}{T} \right) \cos \left( \frac{\omega'\tau}{2} \right) \int_{T/2}^{(T-\tau)/2} \sin \omega' t \sin \left( \frac{2\pi kt}{T} \right) dt
\]

\[ - \left( \frac{4}{T} \right) \sin \left( \frac{\omega'\tau}{2} \right) \int_{\tau/2}^{(T-\tau)/2} \cos \omega' t \sin \left( \frac{2\pi kt}{T} \right) dt \]

Using integral forms:

\[
\int \sin(mt) \sin(nt) dt = \frac{\sin(m-n)t}{2(m-n)} - \frac{\sin(m+n)t}{2(m+n)}
\]

\[
\int \sin(mt) \cos(nt) dt = -\frac{\cos(m-n)t}{2(m-n)} - \frac{\cos(m+n)t}{2(m+n)}
\]

for both cases \( m^2 \neq n^2 \)

\( B_k \) becomes:
\[ B_k = \left( \frac{4}{T} \cos \left( \frac{\omega' \gamma}{2} \right) \left( \frac{\sin (bt)}{2b} - \frac{\sin (at)}{2a} \right) \right) \left( \frac{T - \gamma}{2} \right) \]

\[ - \left( \frac{4}{T} \sin \left( \frac{\omega' \gamma}{2} \right) \left( - \frac{\cos (bt)}{2b} - \frac{\cos (at)}{2a} \right) \right) \left( \frac{T - \gamma}{2} \right) \]

where: \( a = m+n = \frac{2\pi(T(k+1) - 2k\gamma)}{T(T-2\gamma)} \)

\( b = m-n = \frac{2\pi(T(k-1) - 2k\gamma)}{T(T-2\gamma)} \)

Now substitute limits:

\[ B_k = \left( \frac{4}{T} \cos \left( \frac{\omega' \gamma}{2} \right) \left( \frac{\sin (b(T - \gamma))}{2b} - \frac{\sin (a(T - \gamma))}{2a} \right) \right) \]

\[ - \frac{\sin (b(-\gamma))}{2b} + \frac{\sin (a(-\gamma))}{2a} \]

\[ + \left( \frac{4}{T} \sin \left( \frac{\omega' \gamma}{2} \right) \left( \frac{\cos (b(T - \gamma))}{2b} + \frac{\cos (a(T - \gamma))}{2a} \right) \right) \]

\[ - \frac{\cos (b(-\gamma))}{2b} - \frac{\cos (a(-\gamma))}{2a} \]

This then is the expression for the Fourier coefficients for the waveform of crossover distortion, which is the same as equation 8 in Section II.
To describe this program the first requirement is to give the conversion from the symbols used in this paper to the Fortran symbols. The definitions of the terms used in the program output are also given.

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>FORTRAN SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0$</td>
<td>$E0$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$A2$</td>
</tr>
<tr>
<td>$\gamma/T$</td>
<td>DELRAT</td>
</tr>
<tr>
<td>$R_f$</td>
<td>$RF$</td>
</tr>
<tr>
<td>$R_i$</td>
<td>$RI$</td>
</tr>
<tr>
<td>$C_f$</td>
<td>$CF$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$PI$</td>
</tr>
<tr>
<td>$A_{vol}$</td>
<td>$A1$</td>
</tr>
<tr>
<td>$f_o = \frac{\omega_o}{2\pi}$</td>
<td>FREQØ</td>
</tr>
<tr>
<td>$\omega'$</td>
<td>$\omegaGA$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$TAU$</td>
</tr>
<tr>
<td>$T$</td>
<td>$TI$</td>
</tr>
<tr>
<td>$f$</td>
<td>$FREQ$</td>
</tr>
<tr>
<td>$A_1(k\omega)$</td>
<td>$AV(I)$</td>
</tr>
<tr>
<td>$\beta(k\omega)$</td>
<td>$BETA(I)$</td>
</tr>
<tr>
<td>$B_k$</td>
<td>$COD(I)$</td>
</tr>
<tr>
<td>$\frac{A_1A_2}{1-\beta A_1A_2}$</td>
<td>GAIN(I)</td>
</tr>
</tbody>
</table>
Once the circuit parameters have been specified (eg: $R_1$, $R_f$, $C_f$, $A_{vol}$, etc.), the only input data is $E_0$, $A_2$, and $\gamma / \Gamma$. From this, the program will calculate the remaining variables.

So that the data points are evenly spaced on the log
frequency axis, the increments in frequency are obtained by repeated multiplication of the last frequency increment by a constant. To demonstrate this, if it is desired to have ten frequency increments from 10 HZ to 50 KHZ the multiplier is 2.57. It works like this:

<table>
<thead>
<tr>
<th>FREQUENCY INCREMENT</th>
<th>MULTIPLY BY</th>
</tr>
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<tbody>
<tr>
<td>1st</td>
<td>10 HZ</td>
</tr>
<tr>
<td>2nd</td>
<td>25.7 HZ</td>
</tr>
<tr>
<td>3rd</td>
<td>66.0 HZ</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>9th</td>
<td>19031.1 HZ</td>
</tr>
<tr>
<td>10th</td>
<td>48909.9 HZ</td>
</tr>
</tbody>
</table>

This yields the desired results of equal spacing on the log frequency axis, but has the disadvantage of having large gaps in the latter end of the scale; for example from the increment from nine to ten. To avoid this, the program is set up such that two frequencies can be chosen at will. Therefore the program gives results at twelve points in frequency; two of which are arbitrary and ten of which follow the equal increment frequency sweep. The two arbitrary frequency points are 30 KHZ and 40HKZ.

The program flow chart is given along with the actual program and output sample.
FLOW CHART

\[
\begin{align*}
\text{SN} &= 0.13 \times 2 \\
\text{INT} &= 0.02 \\
\text{DELINT} &= 0.15 \\
\text{SF} &= 1, F \\
\text{RI} &= 1000, \\
\text{CF} &= 30, F = 12 \\
\text{DI} &= 2, 141623 \\
\text{AI} &= -2000, \\
\text{FREQ} &= \text{FCC} \\
\text{FREQ} &= 2000, \\
L &= 1, 12 \\
\text{IF } L = 1, 12 \\
\text{TI} &= 1, \text{FREQ} \\
\text{TAU} &= \text{FREQ} \times \text{TI} \\
\text{OMCA} &= (2. \times \text{DI}) / (\text{TI} - 2. \times \text{TAU}) \\
\text{TI} &= (\text{TI} - 2. \times \text{TAU}) / 2. \\
\text{DI} &= \text{TAU} / 2. \\
\text{FREQ}, \text{TI}, \text{TAU}, \text{DF}, \text{RI}, \text{CF} \\
S &= 1. C \\
I &= 1, 5. \times \\
A &= (2. \times A) \times (A \times (S + 1)) - (2. \times S \times \text{TAU}) / ((\text{TI}) \times (\text{TI} - 2. \times \text{TAU})) \\
\beta &= (2. \times \beta) \times (\text{TI} \times (S - 1)) - (2. \times \text{TAU}) / ((\text{TI}) \times (\text{TI} - 2. \times \text{TAU})) \\
\text{COS} (A) &= (4. / \text{TI}) \times (\text{COS} (A \times \text{CAU}) / 2.) \times (\sin (9. \times \text{LI} / 2. \times \beta) - (\sin (4. \times \text{LI} / 2. \times \beta) - (\sin (A \times \text{LI} / 2. \times \beta) + (4. / \text{TI}) \times (\text{COS} (A \times \text{CAU}) / 2.) \times (\cos (A \times \text{LI} / 2. \times \beta) - (\cos (A \times \text{LI} / 2. \times \beta) - (\cos (A \times \text{LI} / 2. \times \beta) - (\cos (A \times \text{LI} / 2. \times \beta) + (S = S + 1. \\
X &= 1. \\
I &= 1, 5. \times \\
S = S + 1. \\
V = 1. \\
I &= 1, 5. \times \\
V = V + 1. \\
\text{SW} (J) &= (A1) / \text{SORT} (1. \times \text{FREQ} \times \text{FREQ} \times \text{X} + 1. \times \text{FREQ} \times \text{FREQ}))
\end{align*}
\]
CALL HARDIS (END, DIST1)
CALL HARDIS (END, DIST2)

DIST1
DIST2

5

$(L-2)$

$FRE0 = 40000$

$FRE0 = 10$

$FRE0 = FRE0 * 2.57$

CONTINUE

SUBROUTINE SUMUP block

$\pi = 3.14159$

$X = W / 100.$

$T(1) = X$

$L = 1.100$

$SUN = 0$

$Y = 1.0$

$I = 1, 50$

$L = V(I) * \sin((Y + 2 * \pi * T(I)) / W)$

$SUN = SUN + L$

$Y = Y + 1$

$W(I) = SUN$

$T(L + 1) = T(L) + Y$

RETURN

END
SUBROUTINE HASHDIS (SIGNAL, DIST)

SUMDIS = 2.0

T = 1.49

Y = ((SIGNAL(1+1)/SIGNAL(1)) * 100.)
   * ((SIGNAL(1+1))/SIGNAL(1)) * 100.)

SUMDIS = SUMDIS + X

DIST = SORT(SUMDIS)

RETURN

END
AUDIO AMPLIFIER ANALYSIS PROGRAM

DIMENSION COD(50), DISLP(50), DISO(50), SOUT(50), SCOD(100), ISDISLP(100), SDISO(100), SOUT(100), AV(50), BETA(50), GAIN(50)

SYSTEM VARIABLES

OUTPUT VOLTAGE (MAX)

A2 = 0.9276

DELAY (DELAY RATIO) = TAU/PERIOD

DELAY = 0.015

CIRCUIT VALUES

CF = 1.55

R1 = 11666

CF = 30 * R1

CONSTANTS

R1 = 3.141593

OPERATIONAL AMPLIFIER CONSTANTS - OPEN LOOP GAIN AND UPPER CROSSOVER FREQUENCY

M1 = -2000 C.

PE00 = E00:

RFO = 3 E00:

R = 5 R1 = 1.2

T1 = 1 / E00

PHASE DELAY = TAU

NCA = (2 * DI) / (TI - 2 * TAU)

TAU' = (TI - TAU) / 2.

WRITE (3, 45)

WRITE (3, 10) PE00, TI, TAU, R1, DI, CF

WRITE (3, 45)
GENERATE COEFFICIENTS OF FOURIER SERIES FOR CROSSTOVER DISTORTION

21  
22  
23  
24  
25  
26

S = 1.0

DO 1 I = 1, 50

A(i) = (2.0 * PI) * (T * (S + 1) - (2.0 * S * TAU)) / ((T) * (T - 2.0 * TAU))

DO 1 S = S + 1.

1 CONTINUE

GENERATE GAIN OF OPERATIONAL AMPLIFIER FOR 1ST 50 HARMONICS

27
28

X = 1.

DO 1 I = 1, 50

AV(i) = (A(i) / (SORT(1.0 * (FREQ * FREQ * XX) / (FREQ * FREQ))))

1 X = X + 1.

GENERATE BETA FOR 1ST 50 HARMONICS

31
32

Y = 1.

DO 12 I = 1, 50

BETA(I) = (PI / F) * SORT(1.0 * 4.0 * PI * PI * CF * CF * RF * RF * Y * Y * FREQ * FREQ)

12 Y = Y + 1.

GENERATE GAIN FACTOR FOR 1ST 50 HARMONICS

35
36

Z = 1.

DO 13 I = 1, 50

GAIN(I) = (AV(I) * A2) / (1.0 - AV(I) * A2 * BETA(I))

13 Z = Z + 1.

CALCULATE INPUT VOLTAGE(MAX), E1(MAX), POWER INTO 4 OHM LOAD,
VOLTAGE GAIN AND VOLTAGE GAIN DB

39
40
41
42
43
44
45
46
47
48

FIN = E0 / GAIN(1)

E1 = (FIN) / (A2 * COD(I))

POWER = (FIN * F0) / 9.

VGAIN = E0 / E1

DBGAIN = 20.0 * LOGIC(ARG(VGAIN))

WRITE (3, 120) FIN, E1, FIN, POWER, VGAIN, DBGAIN

WRITE (3, 120)

DO 14 K = 1, 50

ATER = 1.0 / BETA(K)

14 WRITE (3, 50) ATER, AV(K), GAIN(K), K
GENERATE DISTORTION INSIDE LOOP, DISLP

40 DISLP(I) = F1*A2*(COD(I) - 1)
50 DO 2 I = 2, 50
51 2 DISLP(I) = F1*A2*COD(I)
C GENERATE COMPONENTS OF OUTPUT SIGNAL

52 DO 3 I = 1, 50
53 3 DISO(I) = DISLP(I) / (1 - AV(I)*A2*RETA(I))
C GENERATE OUTPUT AND OUTPUT DISTORTION

54 EOUT(I) = ER + DISO(I)
55 DO 4 I = 2, 50
56 4 EOUT(I) = DISO(I)
57 WRITE(3, 40)
58 WRITE(3, 50)
59 DO 7 M = 1, 50
60 7 WRITE(3, 40) COD(M), DISLP(M), DISO(M), EOUT(M), M
61 IF (L - 2) 15, 15, 16
C FIND F(T) USING 1ST 50 HARMONICS

62 CALL SUMUP (COD, TI, SCOD)
63 CALL SUMUP (DISLP, TT, SDISLP)
64 CALL SUMUP (DISO, TI, SDISO)
65 CALL SUMUP (EOUT, TI, SOUT)
66 WRITE (3, 60)
67 WRITE (3, 70)
68 DO 8 I = 1, 50
69 8 IF (R- N) 75, 73
70 SFIN = FIN*SIN(2*PI*FREQ*TI*R)/100.
71 WRITE (3, 80) SCOD(N), SDISLP(N), SDISO(N), SOUT (N), SFIN, N
72 8 R = R + 1
73 16 CONTINUE
C FIND TOTAL HARMONIC DISTORTION WITH AND WITHOUT FEEDBACK

74 CALL HARDIS (COD, DIST1)
75 CALL HARDIS (EOUT, DIST2)
76 WRITE (3, 40)
77 WRITE (3, 100) DIST1
78 WRITE (3, 110) DIST2
79 WRITE (3, 40)
80 IF (L - 2) 18, 9, 17
GO TO 5

9 FREQ=10.

GO TO 5

17 FREQ=FREQ*2.57

5 CONTINUE

CALL EXIT

10 FORMAT(1' FREQUENCY=',F10.4,1', PERIOD(SEC)=',E18.8,1',
1' TIME DELAY=',E18.8,1', FEEDBACK RESISTANCE=',E18.8,1',
2' INPUT RESISTANCE=',E18.8,1', FEEDBACK CAPACITANCE=',E18.8,1'

20 FORMAT(///1', RECIPROCAL OF BETTA',T23,1' OP AMP GAIN ',T46,1'GAIN ',
1'T57,1' HARMONIC')

30 FORMAT (T3,F10.5,T23,F10.3,T42,F10.5,T60,T13)

40 FORMAT (///1)

50 FORMAT (T0,1'CON',T2P,1'DISLP',T46,1'DISO',T64,1'OUT',T75,1'HARMONIC')

60 FORMAT (4F18.8,T77,T13)

70 FORMAT (T4,1'SCON',T16,1'SDISLP',T2P,1'DISO',T40,1'OUT',T52,1'SFIN',
1'T72,1'TIME INCREMENT')

80 FORMAT (5E12.4,T77,T13)

90 FORMAT (5E12.4)

100 FORMAT (1' TOTAL HARMONIC DISTORTION WITHOUT FEEDBACK=',F10.4,1'X')

110 FORMAT (1' TOTAL HARMONIC DISTORTION WITH FEEDBACK=',F10.4,1'X')

120 FORMAT (///1', OUTPUT VOLTAGE (PEAK) =',F10.5,1', EL=',F10.5,1',
1' FOUT (PEAK) =',F10.5,1', POWER INTO 4 OHMS=',F10.5,1',
2' VOLTAGE GAIN=',F10.5,1', VOLTAGE GAIN IN OHM=',F10.5)

END

SUBROUTINE SUMUP(V,W,UI)

DIMENSION V(50),T(101),UI(100)

PI=3.14159

Y=W/10C.

T(1)=Y

DO 1 L=1,100

SUM=0.

Y=1.0

DO 2 J=1,50

Z=V(1)*SIN((Y+7.*PI*T(L))/(W))

SUM=SUM+Z

2 Y=Y+1.

10
1) $T[L+1]=T(L)+X$
2) RETURN
3) END

SUBROUTINE HARDIS (SIGNAL, DIST)
4) DIMENSION SIGNAL(50)
5) SUMDIS=0.0
6) DO 1 L=1,40
7) $Y=((SIGNAL(L+1)/SIGNAL(1))*100.)*((SIGNAL(L+1)/SIGNAL(1))*100.)$
8) SUMDIS=SUMDIS + Y
9) DIST=SORT(SUMDIS)
10) RETURN
11) END

DATA

OUTPUT SAMPLE

FREQUENCY = 40000,0000
PERIOD (SEC) = 0.24900080E-04
TIME DELAY = 0.37499978E-06
FEEDBACK RESISTANCE = 0.10000000E+06
INPUT RESISTANCE = 0.11000000E+05
FEEDBACK CAPACITANCE = 0.38999990E-10

OUTPUT VOLTAGE (PEAK) = 0.13020
F1 = 2.99992
EIN (PEAK) = -1.44969
POWER INTO 4 OHMS = 10.42007
VOLTAGE GAIN = -6.31544
VOLTAGE GAIN IN DB = 16.00807
<table>
<thead>
<tr>
<th>RECIPROCAL OF BETA</th>
<th>CP AND GAIN</th>
<th>GAIN</th>
<th>HARMONIC</th>
</tr>
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<td>6.40228</td>
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<tr>
<td>4.13026</td>
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<td>CAD</td>
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**Total Harmonic Distortion Without Feedback = 3.1176%**

**Total Harmonic Distortion With Feedback = 0.1150%**
VITA

Anthony Francis Lexa was born on July 19, 1945 in St. Louis, Missouri. He received his primary and secondary education also in St. Louis. He received his Bachelor of Science Degree in Electrical Engineering in June of 1967 from the University of Missouri at Rolla. He has been enrolled in the Graduate School of the University of Missouri at Rolla as a graduate assistant since September of 1967.