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An investigation of transient negative resistance in a cadmium sulfide device

John Irvin Giem

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AN INVESTIGATION OF TRANSIENT NEGATIVE RESISTANCE
IN A CADMIUM SULFIDE DEVICE

BY

JOHN IRVIN GIEM

A

THESIS

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ABSTRACT

An analysis has been made of a CdS device which exhibits a transient negative resistance at room temperature and low field strengths whereas previously reported cases of negative resistance in CdS were observed at low temperatures and moderate to high electric fields. The CdS bar was illuminated on one end in the vicinity of an ohmic In contact. Contact was made to the other end of the device with a rectifying Ag contact and the device was reverse biased.

Experimental evidence and the analytical results both indicated that it is necessary that the material used to make the negative resistance devices must have a relatively low carrier mobility, high trap density, and a low regeneration rate for trapped carriers.

As the light induced carriers reach the rectifying contact and raise the carrier concentration, the voltage across the contact decreases due to the dependence of the voltage on carrier concentration. This induces a transient negative resistance if the transit time of carriers from the illuminated section of the crystal is relatively long.
ACKNOWLEDGMENTS

The author wishes to express his appreciation to Dr. N. G. Dillman for his assistance and guidance throughout the investigation, to Dr. R. S. Carson and Larry Stoddard for their many helpful discussions with the author, and to the Electronics Division of Eagle-Picher Industries for supplying the CdS crystals used in the preliminary phases of investigation. This investigation was funded by the Space Sciences Research Center of the University of Missouri.
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I. INTRODUCTION

The purpose of this document is to describe a special two terminal cadmium sulfide (CdS) device which exhibits a transient negative resistance at room temperature and low voltages and to provide an explanation of its operation.

The device was originally proposed by Dillman\textsuperscript{1} and was later built by Mohr\textsuperscript{2}.

The device was studied by Mohr to determine the necessary conditions for operation and the different modes of performance were investigated. Several possible explanations for the operation of the device were presented.

The author in an attempt to explain the operation of the device tried several methods of analysis. This paper presents, along with a description of the device, the simplification necessary to analyze the device, the analysis, and the resulting conclusions as to its operation.
II. REVIEW OF LITERATURE

Negative resistance in CdS has been observed and predicted by other researchers. Smith\(^3\) observed a change in current level at high voltages which was interpreted as the onset of double injection of carriers, whereas low temperature was used by Litton and Reynolds\(^4\) to observe negative resistance and current oscillations, which were explained by hole injection at an Ag electrode and an increase in hole lifetime due to the filling of traps.

Tap crystals, produced by doping CdS with sodium, exhibited a negative resistance at 4.2\(^\circ\)K at moderate field strengths, which depended upon the intensity and duration of prior stimulation by light. Park and Litton\(^5\) explained the phenomenon through the filling of hole traps by the illumination and double injection of carriers upon application of the voltage.

Current oscillations were observed by McLeod and Hayes\(^6\) at high field strengths in nonuniformly illuminated CdS with ohmic In electrodes but they were unable to explain the phenomenon.

It is noted that in the above listed references, and in others\(^7,8,9,10,11,12,13\) the negative resistance occurred at room temperature in the form of current oscillations at high field strengths or could be observed as a static negative resistance at low temperatures with moderate to high field strengths. None of these cases are adequate
for the explanation of the device characteristics described in this paper.
III. OBSERVED CHARACTERISTICS OF THE DEVICE

The devices were made from selected single crystals of CdS approximately 2 mm square in cross section and 10 mm long, the sides of which were either cleaved or polished surfaces, with the c-axis of the crystal parallel to the longitudinal axis of the devices. After abrading the ends of the device, an ohmic contact was formed on one end by vacuum deposition of In and a rectifying Ag contact was made to the opposite end through the use of Silver Print. A mask was placed over the half of the crystal with the Ag contact to exclude all visible light. The rest of the device could be illuminated by an incandescent light source as illustrated in Figure 1.

It was found that in order to observe a negative resistance it was necessary to subject the crystal to total darkness with the contacts shorted for approximately three minutes before each measurement cycle. To observe the negative resistance, the crystal was exposed to light and a triangular current source was connected such that the Ag contact was negative with respect to the In contact.

The triangular current source was approximated by using a high impedance in series with a triangular voltage source. A curve which is representative of the results (except for noise due to the high impedances involved) is presented in Figure 2. The plot starts with zero voltage and current at point A and increases along the upper curve until point B is reached. Then the device enters into the
negative resistance region and the voltage decreases rapidly until point C is attained. The slope of this segment is limited by high series resistance used to approximate the current source. The true shape of the negative resistance region was unattainable due to the high impedance of the device under test, typically between 50 MΩ and 500 MΩ. From point C the voltage increases again until point D is reached at which time the source starts to return to zero. During the return sweep and all subsequent sweeps the lower curve is followed between points ACD until the source is removed and the device is placed in darkness as described above.

This phenomenon differs from previously reported negative resistance in CdS in that it is a low-voltage transient event occurring at room temperature with the sweep speed between 40 and 100 seconds. Variations in the sweeping period and changes in light intensity would cause a change in the onset of the negative resistance.

Other voltage and light combinations were also investigated but the above described configuration with the Ag electrode negative biased and masked from the light was the only one that produced a negative resistance. Many crystals were tested, subject to the above conditions, and those crystals exhibiting the negative resistance could be characterized as being photosensitive with a high trap density (for example, 440 ppm Zn)².
Figure 1. Schematic Representation of the Cadmium Sulfide Device

Figure 2. Typical Characteristics of the Device
IV. ANALYSIS OF THE DEVICE

In order to determine the nature of the electron density at the Ag contact, and the resulting voltage current relationships, it will be necessary to examine the continuity equations and related equations pertaining to the desired solution. The equations for the one dimensional case with electron traps are as follows:\textsuperscript{15}

\[ \frac{\partial p}{\partial t} = -\frac{p - p_0}{\tau_p} + g_{vt} - r_{tv} + f - \frac{\partial}{\partial x} \frac{J_p}{q} \quad (1) \]

\[ \frac{\partial n}{\partial t} = -\frac{n - n_0}{\tau_n} + g_{tc} - r_{ct} + f + \frac{\partial}{\partial x} \frac{J_n}{q} \quad (2) \]

\[ \frac{\partial n_t}{\partial t} = r_{ct} + g_{vt} - g_{tc} - r_{tv} \quad (3) \]

\[ J_p = q \mu_p pE - qD_p \frac{\partial p}{\partial x} \quad (4) \]

\[ J_n = q \mu_n nE + qD_n \frac{\partial n}{\partial x} \quad (5) \]

\[ J = J_p + J_n \quad (6) \]

\[ \frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon} \quad (7) \]

The first three equations are the continuity equations for holes, free electrons, and trapped electrons, respectively. In the first two equations the first term on the right of each represents the rate of disappearance of free carriers due to recombination. The generation rate of
holes, by electrons being excited from the conduction band into traps, is given by

\[ Q_{vt} = (N_t - n_t)N v S_p \exp\left(\frac{E_G - E_t}{kT}\right) \]

while \( Q_{tc} \) represents the rate of electrons going into the conduction band after being freed from trapping centers and is expressed by

\[ Q_{tc} = n_t N_c v S_n \exp\left(-\frac{E_t}{kT}\right) \]

In the above equations \( n_t \) represents the density of trapped electrons, \( N_t \) the density of trapping centers at a depth of \( E_t \) from the conduction band with a capture cross section of \( S_p \) for a free hole when occupied by an electron and a capture cross section for \( S_n \) for a free electron when unoccupied. \( E_G \) is the bandgap of the material, \( v \) is the thermal velocity of a free electron and \( N_c \) and \( N_v \) are the effective densities of states in the conduction and valence bands, respectively. The recombination rate of free holes with trapped electrons is represented by \( r_{tv} \) and is given by

\[ r_{tv} = p n_t v S_p \]

and \( r_{ct} \) represents the rate at which free electrons are captured by traps and is expressed by

\[ r_{ct} = n(N_t - n_t) v S_n \]

The fourth term in each of the first two equations, \( f \), represents the generation rate of free carriers by external
excitation such as light, and the last terms represent the rate of change in the hole and electron currents with respect to distance, $x$.

The fourth and fifth equations represent the hole and electron current densities, respectively. The first term in each equation represents the drift current density due to an electric field and the second term represents a diffusion current density due to a concentration gradient of the respective carrier type.

The sixth equation states that the total current density consists of the sum of the electron current density and the hole current density.

The last equation of the set is Poisson's equation relating the electric field to the net charge density.

The rectifying Ag contact can be described by the equation

$$I_f = \frac{1}{4} n_1 qvA \exp\left(-\frac{\phi_m - \chi_S}{kT}\right) \left(\exp\left(\frac{qV_f}{kT}\right) - 1\right)$$

In the above equation, $v$ is the thermal velocity of the electrons, $A$ is the cross sectional area, $\phi_m$ is the work function of the silver, $\chi_S$ is the electron affinity of the CdS. $I_f$ and $V_f$ are the forward current and voltage at the Ag/CdS junction. $I = -I_f$ and $V = -V_f$ will be used in the remainder of the paper since the junction is always reverse-biased.

It was found that a complete solution utilizing the above equations was too complicated to solve, necessitating the following assumptions:
1. The majority of the voltage drop occurs across the reverse-biased Ag/CdS junction.

2. The conduction in the illuminated half of the device is due to a drift current of a relatively large number of electrons in a very low field.

3. The electron concentration in the illuminated portion is essentially constant.

4. The device is in a quasi-steady-state condition due to the long period of the applied signal.

5. Conduction in the darkened region is due to diffusion of electrons.

6. All of the trapping effects can be accounted for by a change in the diffusion length, $L_n$, which can be accounted for by a change in the effective lifetime, $\tau_n$, of the electrons and/or in the diffusion constant $D_n$, $(L_n = (\tau_n D_n)^{1/2})$.

7. The current is due to electron conduction only since the hole lifetime and mobility are small compared to the corresponding values for electrons.

8. Once a trap has been filled, it will remain so during the remaining time of interest.

The above assumptions will allow us to reduce the analysis to three basic equations. They are the steady-state continuity equation, the diffusion current equation and the equation describing the rectifying contact.
\[ 0 = -\frac{(n - n_0)}{\tau_n} + f + \frac{1}{q} \frac{\partial J}{\partial x} \]  \hspace{1cm} \text{(Continuity)}

\[ J_n = qD_n \frac{\partial n}{\partial x} \]  \hspace{1cm} \text{(Diffusion)}

\[ I_f = \frac{1}{4} n_1 qvA \exp\left(-\frac{\phi_m - \chi_s}{kT}\right)\left(\exp\left(\frac{qV_f}{kT}\right) - 1\right) \]  \hspace{1cm} \text{(Contact)}

The continuity and the current density equations can be combined (See Appendix B for detailed analysis).

\[ 0 = -\frac{(n - n_0)}{\tau_n} + f + D_n \frac{d^2 n}{dx^2} \]

In the illuminated region between \( x = L/2 \) and \( x = L \), where \( L \) is the length of the device, the second derivative of \( n \) is zero since the concentration is constant. Hence the solution of the resulting equation yields

\[ n = n_0 + f \tau_{n2} \text{ for } L/2 \leq x \leq L \]

where \( \tau_{n2} \) is the lifetime of the electrons in the illuminated region.

In the dark region (\( 0 \leq x \leq L/2 \)), the boundary conditions imposed on the electron concentration are

\[ n = n_0 + f \tau_{n2} \text{ at } x = L/2 \]

\[ n = n_1 \text{ at } x = 0 \]

where \( n_1 \) is the electron concentration at the Ag contact. Since this half of the crystal is masked from the light, \( f \) is equal to zero. The solution of the continuity equation yields
In order to obtain a relationship for \( n_1 \), \( n \) was substituted into the equation

\[
\frac{d \ln n}{L/2L_n} - I = A J_n = q A D_n \frac{dn}{dx} x=0
\]

and the equation was solved for \( n_1 \).

\[
n_1 = n_o + \frac{f \tau n_2}{\cosh(L/2L_n)} + \frac{L_n}{D_n} \frac{I}{A q} \tanh \left( \frac{L}{2L_n} \right)
\]

Using typical values of \( L = 1 \text{ cm}, 10^{-6} \text{ sec} < \tau_{n1} < 10^{-3} \text{ sec} \) and \( D_n \) is \( 2.6 \text{ cm}^2/\text{sec} \), the equation is approximately;

\[
n_1 = n_o + \frac{f \tau n_2}{\cosh(L^2/4\tau n_1 D_n)} \frac{I}{A q} + \left( \tau_{n1}/D_n \right)^{1/2} \frac{I}{A q}
\]

where \( L_n = (\tau_{n1} D_n)^{1/2} \).

As can be seen from the above equation \( n_1 \) will increase with an increase in light intensity, current, or the carrier lifetime in the darkened region. To verify that \( n_1 \) has the desired characteristics necessary for negative resistance, the equation describing the rectifying contact is solved for the voltage, and the first and second derivatives are examined (see Appendix C).

The equation describing the rectifying contact can be simplified by combining all the constant terms,
\[ I = n_1C \left(1 - \exp\left(-\frac{qV}{kT}\right)\right) \]

where \( c = \frac{1}{4} qvA \exp(-\frac{\phi_{m} - \chi_{s}}{kT}) \)

Solving for \( V \)

\[ V = \frac{kT}{q} \ln \left(\frac{n_1C}{n_1C - I}\right) \]

If a ramp current generator is applied \( (I = Kt) \), then

\[ V = \frac{kT}{q} \ln \left(\frac{n_1C}{n_1C - Kt}\right) \]

Since the term in the logarithm must be positive, a lower limit on \( n_1 \) is established.

\[ n_1 > \frac{Kt}{C} \]

Forming

\[ \frac{dV}{dt} = \frac{kTK(n_1 - t(dn_1/dt))}{qn_1 (n_1C - Kt)} \]

Noting that the derivative of \( V \) is zero at \( t_1 \) and \( t_2 \), then

\[ \frac{dn_1}{dt} = \frac{n_1}{t} \quad \text{at} \quad t = t_1 \quad \text{and} \quad t = t_2 \]

At times other than at \( t_1 \) and \( t_2 \) it is known that

\[ \frac{dV}{dt} > 0 \quad \text{for} \quad t < t_1, \quad \text{and} \quad t > t_2 \]

and \( \frac{dV}{dt} < 0 \quad \text{for} \quad t_1 < t < t_2 \)

These yield the following limitations:

\[ \frac{dn_1}{dt} < \frac{n_1}{t} \quad \text{for} \quad t < t_1 \quad \text{and} \quad t > t_2 \]

and \( \frac{dn_1}{dt} > \frac{n_1}{t} \quad \text{for} \quad t_1 < t < t_2 \)
The second derivative of $V$ when evaluated at $t_1$ and $t_2$ provides additional information concerning the derivatives of $n_1$ because

$$\frac{d^2V}{dt^2} < 0 \quad \text{at } t = t_1, \quad \text{and} \quad \frac{d^2V}{dt^2} > 0 \quad \text{at } t = t_2$$

yielding

$$\frac{d^2n_1}{dt^2} > 0 \quad \text{at } t = t_1, \quad \text{and} \quad \frac{d^2n_1}{dt} < 0 \quad \text{at } t = t_2$$
V. CONCLUSIONS

From the above information, the operating principles of the device can be deduced. The initial placement of the device in a darkened condition with shorted contacts is necessary to empty the electron traps in the crystal, thereby reducing $\tau_{n1}$ and $\tau_{n2}$ to the order of $10^{-6}$ seconds. When the device is illuminated and the source is connected, the traps in the lighted half of the device are rapidly filled, increasing the lifetime $\tau_{n2}$ to approximately $10^{-3}$ seconds. As time progresses, the carriers from the illuminated region start to diffuse into the darkened portion, filling the traps, and thereby increasing the effective lifetime of the carriers. This increases the diffusion length of the carriers until the diffusion length becomes comparable with the length of the masked region, Figure 3, at which time the electron concentration $n_1$ at the Ag contact starts to increase, while still meeting the requirement $dn_1/dt < n_1/t$. The concentration $n_1$ will increase rapidly for $t_1 < t < t_2$ because $dn_1/dt > n_1/t$. The accelerated filling of traps produces an increasing lifetime, $\tau_{n1}$, resulting in a type of positive feedback.

This time of rapid filling of traps and rapidly increasing $n_1$ corresponds to the period between $t_1$ and $t_2$, for which $dn_1/dt > n_1/t$. The passing of the device from the negative resistance mode back into the positive resistance mode occurs when the majority of the traps are
Figure 3. Hypothetical Electron Distribution
filled and any further increase in $n_1$ will be caused by increasing current densities due to the driving source.

Examination of the second derivative at $t_1$ indicates that $dn_1/dt$ is increasing in magnitude which corresponds to the start of a rapid increase in $n_1$. At $t_2$ the negative values of $(d^2n_1/dt^2)$ indicates that even though $(dn_1/dt)$ is positive, its magnitude is decreasing and will approach a limit imposed upon it by the driving source.

During the retrace and subsequent traces the plot will not exhibit the negative resistance but will continue to trace out the lower portion of the curve as shown in Fig. 2 since the traps are already filled and cannot influence the electron concentration at the rectifying contact. The importance of high trap densities was verified experimentally.
BIBLIOGRAPHY


APPENDICES
APPENDIX A

Properties of Cadmium Sulfide

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<th>Symbol</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_p$</td>
<td>Hole Mobility</td>
<td>$10$ to $18 , \frac{\text{cm}^2}{\text{volt-sec}}$</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>Hole lifetime</td>
<td>$10^{-7}$ to $10^{-6}$ sec</td>
</tr>
<tr>
<td>$v$</td>
<td>Thermal velocity of a free electron</td>
<td>$10^7 , \frac{\text{cm}}{\text{sec}}$</td>
</tr>
<tr>
<td>$S_n$</td>
<td>Capture cross section for electrons</td>
<td>$10^{-21}$ to $10^{-19}$ cm$^2$</td>
</tr>
<tr>
<td>$\mu_n$</td>
<td>Electron Mobility</td>
<td>$40$ to $340 , \frac{\text{cm}^2}{\text{volt-sec}}$</td>
</tr>
<tr>
<td>$\tau_n$</td>
<td>Electron lifetime</td>
<td>$5 \times 10^{-4}$ to $5.5 \times 10^{-4}$ sec</td>
</tr>
<tr>
<td>$X_s$</td>
<td>Electron affinity</td>
<td>$4.5$ ev</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>Relative dielectric constant</td>
<td>$9.0$ to $10.3$</td>
</tr>
<tr>
<td>$E_G$</td>
<td>Energy gap (at $300^\circ$K)</td>
<td>$2.42$ ev</td>
</tr>
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</table>
APPENDIX B

Derivation of the Expression for \( n_1 \)

Using the same eight assumptions as stated in the text, the derivation is based on three equations, the steady-state continuity equation without trapping for electrons, the current diffusion equation and the rectifying contact equation.

\[
0 = -\frac{(n - n_o)}{\tau_n} + f + \frac{1}{q} \frac{\partial j_n}{\partial x}
\]  

\( \text{(Continuity)} \)

\[
j_n = q D_n \frac{\partial n}{\partial x}
\]  

\( \text{(Diffusion)} \)

\[
I_f = \frac{1}{4} n_1 q v A \exp(-\frac{\phi m}{kT}) (\exp(\frac{qV_f}{kT}) - 1)
\]  

\( \text{(Contact)} \)

Combining the continuity and diffusion equations

\[
0 = \frac{-(n-n_o)}{\tau_n} + f + D_n \frac{\partial^2 n}{\partial x^2}
\]

In the region \( \frac{L}{2} \leq x \leq L \), \( n \) will be a constant hence \( \frac{\partial^2 n}{\partial x^2} = 0. \)

Therefore

\[
0 = \frac{-(n-n_o)}{\tau_{n2}} + f
\]

Hence

\[
n = n_o + f \tau_{n2} \text{ for } \frac{L}{2} \leq x \leq L
\]

This produces one of the boundary conditions for the solution of \( n \) for \( 0 \leq x \leq \frac{L}{2} \). The complete boundary conditions for this situation are:
\[ n = n_0 + f \tau_n^2 \quad \text{for} \ x = \frac{L}{2} \]
\[ n = n_1 \quad \text{for} \ x = 0. \]

In the region between 0 and \( \frac{L}{2} \) the generation of carriers due to light is negligible thereby reducing \( f \) to zero.

Hence
\[ 0 = \frac{-(n-n_0)}{\tau_n} + D \frac{\partial^2 n}{n_0 \partial x^2} \]

or
\[ \frac{\partial^2 n}{\partial x^2} - \frac{n}{L_n^2} = \frac{n_0}{L_n^2} \]
where \( L_n = \sqrt{\tau_n} D_n \)

This equation has the general solution
\[ n = B \exp\left(\frac{X}{L_n}\right) + C \exp\left(-\frac{X}{L_n}\right) + n_0 \]

Use the appropriate boundary conditions:
\[ n_1 = B + C + n_0, \text{ at } x = 0 \]
\[ n_0 + f \tau_n^2 = B \exp\left(\frac{L_n}{2L_n}\right) + C \exp\left(-\frac{L_n}{2L_n}\right) + n_0, \text{ at } x = \frac{L}{2} \]

Solving the above equations for \( B \) and \( C \) yields
\[ n = \frac{[f \tau_n^2 - (n_1 - n_0) \exp\left(\frac{L_n}{2L_n}\right)]}{2 \sinh\left(\frac{L_n}{2L_n}\right)} \exp\left(\frac{X}{L_n}\right) \]

\[ \frac{\exp\left(-\frac{X}{L_n}\right)}{2 \sinh\left(\frac{L_n}{2L_n}\right)} [\exp\left(\frac{L_n}{2L_n}\right) - f \tau_n^2] \]

A relationship between the reverse current \( I \) and \( n_1 \) can be obtained by using the diffusion equation
\[-I = I_f = J_n A = Aq D_n \frac{\partial n}{\partial x} \]

\[
\begin{align*}
\frac{AqD_n}{L_n} \left[ \frac{f \tau n_2 - (n_1 - n_0) \exp\left(\frac{-L}{2L_n}\right)}{2 \sinh\left(\frac{L}{2L_n}\right)} - \frac{(n_1 - n_0) \exp\left(\frac{L}{2L_n}\right) - f \tau n_2}{2 \sinh\left(\frac{L}{2L_n}\right)} \right] = & \quad x = 0 \\
= & \quad Aq D_n \frac{f \tau n_2 - (n_1 - n_0) \cosh\left(\frac{L}{2L_n}\right)}{\sinh\left(\frac{L}{2L_n}\right)}
\end{align*}
\]

from which is obtained

\[
(n_1 - n_0) \cosh\left(\frac{L}{2L_n}\right) = f \tau n_2 - \frac{L_n I \sinh\left(\frac{L}{2L_n}\right)}{Aq D_n}
\]

Hence, solving for \(n_1\):

\[
n_1 = n_0 + \frac{f \tau n_2}{\cosh\left(\frac{L}{2L_n}\right)} + \frac{L_n I \sinh\left(\frac{L}{2L_n}\right)}{D_n Aq \cosh\left(\frac{L}{2L_n}\right)}
\]

which reduces to

\[
n_1 = n_0 + \frac{f \tau n_2}{\cosh\left(\frac{L}{2L_n}\right)} + \left(\frac{n}{D_n} \right)^2 \frac{I}{Aq} \tanh\left(\frac{L}{2L_n}\right)
\]
APPENDIX C

Derivation of Restrictions and Requirements on \( n_1 \)

From the first and second derivatives of the reverse voltage \( V \), restrictions can be placed on \( n_1 \) and its first and second derivatives so that a transient negative resistance can exist. If the boundaries of the negative resistance region are denoted by \( t_1 \) and \( t_2 \) as illustrated in Figure 2, then the following relationships concerning the voltage can be stated.

\[
V > 0 \text{ for } t > 0
\]

\[
\frac{dV}{dt} \bigg|_{t=t_1} = \frac{dV}{dt} \bigg|_{t=t_2} = 0
\]

\[
\frac{dV}{dt} > 0 \text{ for } 0 < t < t_1 \text{ and } t > t_2
\]

\[
\frac{dV}{dt} < 0 \text{ for } t_1 < t < t_2
\]

\[
\frac{d^2V}{dt^2} < 0 \text{ for } t = t_1
\]

\[
\frac{d^2V}{dt^2} > 0 \text{ for } t = t_2
\]

Since the majority of the voltage will be dropped across the reverse biased silver contact, the equation describing the rectifying contact will be investigated to determine the restrictions and requirements imposed upon \( n_1 \).
\[-I = AJ = n_1 A \frac{1}{4} qv \exp\left(\frac{\phi_{m} - \chi_{s}}{kt}\right) \left[\exp\left(\frac{qv}{kt}\right) - 1\right]\]

\[I = n_1 C \left(1 - \exp\left(-\frac{qv}{kt}\right)\right)\]

where \(C = A \frac{1}{4} qv \exp\left(-\frac{\phi_{m} - \chi_{s}}{kt}\right)\)

Solving for \(V\)

\[V = \frac{kT}{q} \ln \left(\frac{n_1 C}{n_1 C - I}\right)\]

Since the device under consideration is being tested using a linearly increasing current let

\[I = Kt\]

where \(K\) is the slope of the current ramp. Hence

\[V_r = \frac{kT}{q} \ln \left(\frac{n_1 C}{n_1 C - Kt}\right)\]

This requires that

\[n_1 > \frac{Kt}{C}\]

Restrictions on the derivatives of \(n_1\) may be obtained by investigating the derivatives of the reverse voltage.

\[\frac{dV}{dt} = \frac{kT}{q} \left(\frac{n_1 C - Kt}{n_1 C}\right) \frac{d}{dt}\left(\frac{n_1 C}{n_1 C - Kt}\right)\]

\[\frac{dV}{dt} = \frac{kT}{q} \frac{K(n_1 - t \frac{dn_1}{dt})}{n_1 (n_1 C - Kt)}\]

Since the derivative of the voltage is zero at time \(t_1\) and \(t_2\), then

\[n_1 - t \frac{dn_1}{dt} = 0 \quad \text{at} \quad t = t_1 \quad \text{and} \quad t = t_2\]
Hence
\[ \frac{dn_1}{dt} = \frac{n_1}{t_1} \quad \text{at} \quad t = t_1 \]
and
\[ \frac{dn_1}{dt} = \frac{n_1}{t_2} \quad \text{at} \quad t = t_2 \]

For \(0 < t < t_1\) and \(t > t_2\), the derivative of \(V\) is positive, hence
\[ \frac{dn_1}{dt} < \frac{n_1}{t} \quad \text{for} \quad 0 < t < t_1 \text{ and } t > t_2 \]
The derivative of \(V\) is negative between \(t_1\) and \(t_2\) which requires that
\[ \frac{dn_1}{dt} > \frac{n_1}{t} \quad \text{for} \quad t_1 < t < t_2. \]

Information concerning the second derivative of \(n_1\) can be obtained from the following:
\[
\frac{d^2V}{dt^2} = \frac{-kT}{q} \frac{d^2n_1}{dt^2} + \frac{(n_1 C - Kt)}{n_1} \left( \frac{dn_1}{dt} - \frac{n_1}{t} \right) \left( \frac{dn_1}{dt} \left( 2n_1 C - Kt - Kn_1 \right) \right)
\]

It is necessary to examine this equation at \(t_1\) and \(t_2\). The second term in the numerator reduces to zero since
\[ \left. \frac{dn_1}{dt} \right|_{t=t_1,t_2} = \frac{n_1}{t} \]
Hence
\[ (t - \frac{dn_1}{dt} - n_1) = (t \frac{n_1}{t} - n_1) = 0 \]
In the first part of the numerator the term \((n_1 C - Kt)\) is always positive indicating that the sign of the second derivative of the voltage will be determined by the sign of the second derivative of \(n_1\) at time \(t_1\) and \(t_2\).

Hence,
\[
\frac{d^2 v}{dt^2} \bigg|_{t=t_1} < 0
\]
which requires
\[
\frac{d^2 n_1}{dt^2} \bigg|_{t=t_1} > 0
\]
and
\[
\frac{d^2 v}{dt^2} \bigg|_{t=t_2} > 0
\]
requires
\[
\frac{d^2 n_1}{dt^2} \bigg|_{t=t_2} < 0
\]

In summary the conditions imposed upon \(n_1\) are as follows.
\[
n_1 > \frac{Kt}{C} \text{ for } t > 0
\]
\[
\frac{dn_1}{dt} = \frac{n_1}{t} \text{ for } t=t_1 \text{ and } t=t_2
\]
\[
\frac{dn_1}{dt} < \frac{n_1}{t} \text{ for } t<t_1 \text{ and } t>t_2
\]
\[ \frac{dn_1}{dt} > \frac{n_1}{t} \text{ for } t_1 < t < t_2 \]

\[ \frac{d^2 n_1}{dt^2} > 0 \text{ for } t = t_1 \]

\[ \frac{d^2 n_1}{dt^2} < 0 \text{ for } t = t_2 \]
VITA

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