A study of ball mills

Robert Lee Kidd

Follow this and additional works at: http://scholarsmine.mst.edu/masters_theses

Part of the Physical Sciences and Mathematics Commons

Recommended Citation

This Thesis - Open Access is brought to you for free and open access by the Student Research & Creative Works at Scholars' Mine. It has been accepted for inclusion in Masters' Theses by an authorized administrator of Scholars' Mine. For more information, please contact weaverjr@mst.edu.
A STUDY OF BALL MILLS.

BY

ROBERT LEE KIDD.

A

THESIS

submitted to the faculty of the
SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI

in partial fulfillment of the work required for the
Degree of

MASTER OF SCIENCE

Rolla, Mo.

1929.

Approved by

[Signature]

Supervising Engineer,

Mississippi Valley Experiment Station

United States Bureau of Mines.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td>V</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>V</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>History</td>
<td>2</td>
</tr>
<tr>
<td>Ball and Tube Mills</td>
<td>2</td>
</tr>
<tr>
<td>Mills Containing More Than One Grinding Compartment</td>
<td>19</td>
</tr>
<tr>
<td>Double Discharge and Single Discharge Two-Compartment Ball Mills</td>
<td>25</td>
</tr>
<tr>
<td>P-ball Mills</td>
<td>38</td>
</tr>
<tr>
<td>Counter-Balanced Mills</td>
<td>52</td>
</tr>
<tr>
<td>Ball Mill Feeders</td>
<td>40</td>
</tr>
<tr>
<td>Grinding Media</td>
<td>43</td>
</tr>
<tr>
<td>Liners</td>
<td>45</td>
</tr>
<tr>
<td>THEORY</td>
<td>47</td>
</tr>
<tr>
<td>Review of the Literature on the Theory of the Ball Mill</td>
<td>47</td>
</tr>
<tr>
<td>Fischer's Theory</td>
<td>47</td>
</tr>
<tr>
<td>Davis' Theory</td>
<td>52</td>
</tr>
<tr>
<td>Action of the Charge at Slow Speed</td>
<td>53</td>
</tr>
<tr>
<td>Action of the Charge at Higher Speed</td>
<td>54</td>
</tr>
<tr>
<td>Early Comparisons of Observed and Calculated Curves</td>
<td>58</td>
</tr>
</tbody>
</table>

---

I
Discussion by Haultain and Dyer

Conclusions arrived at by Haultain and Dyer

The Author's Investigation of the Parabolic Path

Derivation of the Dean Equation

Application of the New Theory

Conclusions
ILLUSTRATIONS

| Figure 1. | Type of Ball Mill Used in the Dye Industry — 4 |
| Figure 2. | Krupp Dry Grindin Ball Mill —— 9 |
| Figure 3. | Hardinge Conical Mill —— 11 |
| Figure 4. | Marcy Ball Mill —— 14 |
| Figure 5. | First Ball Mill to Have Screening Section Separate From Grinding Section —— 15 |
| Figure 6. | Williamson Ball Mill —— 18 |
| Figure 7. | First Two-Compartment Dry Grinding Mill —— 20 |
| Figure 8. | Allis Chalmers Dry Grinding "Compel" Mill —— 23 |
| Figure 9. | Fairchild "Double Discharge" Ball Mill —— 26 |
| Figure 10. | The Marathon Mill —— 29 |
| Figure 11. | Hardinge Conical Cylindrical Rod Mill —— 30 |
| Figure 12. | Counter-Balanced Mill Introduced in the Seventies —— 33 |
| Figure 13. | Forrester-Hendy Rod Mill —— 35 |
| Figure 14. | Forrester-Raymer Mill —— 37 |
| Figure 15. | Location of Center of Gravity of Rod Charges in Four Compartment Mill —— 39 |
| Figure 16. | Diagram Showing Action of Tube Mill —— 50 |
| Figure 17. | Shows Effect of Centrifugal Force —— 56 |
| Figure 18. | Paths of Travel of Particles in an 8-ft. Mill Making 24. R. F. M. —— 59 |

III
Figure 19. View of Mills with 36-inch Mill Equipped
as Squirrel Cage --------------------- 66
Figure 19-A ---------------------------------- 67
Figure 19-B ---------------------------------- 68
Figure 19-C ---------------------------------- 69
Figure 19-D ---------------------------------- 70
Figure 20. ----------------------------------- 75
Figure 21. ----------------------------------- 78
Figure 22. ----------------------------------- 83
Figure 23. Comparison of Old Orthodox Parabolic Paths
with the Paths Calculated by the New
Theory -------------------------------------- 89
Figure 24. Paths of Concentric Layers of 1.25-Inch
balls at Various Distances from the Center
of the 5-ft. Mill young Turning at 60%
of its Critical Speed. ---------------------- 90

Plate I. Photographs Showing Positions of Various
Ball Charges at Different Speeds ---------- 71.

Plate II. Sketches Comparing the Actual Observed
Paths for Different Charges at
Various Speeds with the New
Theoretical Curve ---------------------- 76
This thesis is presented to the Faculty of the Missouri School of Mines and Metallurgy of the University of Missouri in partial fulfillment of the work required for the degree of Master of Science.

The results of the investigation embodied in this thesis were obtained from work carried on at the Mississippi Valley Experiment Station of the United States Bureau of Mines, Department of Commerce, in cooperation with the Missouri School of Mines and Metallurgy of the University of Missouri at Rolla, Missouri.

ACKNOWLEDGMENTS

Acknowledgments are due to Mr. Will H. Coghill, Supervising Engineer of the Mississippi Valley Experiment Station of the United States Bureau of Mines, for supervision of the investigation and his helpful suggestions; to Mr. A. E. Campbell, Junior Metallurgist, United States Bureau of Mines; Mr. A. A. Gow, Assistant Metallurgist, Missouri School of Mines Cooperative Staff; and to Professor C. H. Dean, Head Department of Mathematics, Missouri School of Mines and Metallurgy of the University of Missouri for his guidance in the Mathematics of this thesis.
A STUDY OF BALL MILLS.

By

Robert Lee Kidd.

INTRODUCTION.

Since its advent as a disintegrator in the cement and metallurgical industry, the ball mill has been the subject of considerable thought and discussion.

The principal aim in modern milling is to maintain quality and reduce production costs. Improvements which show lower power consumption and maintenance, with the same or increased efficiency otherwise, are of considerable interest to the operator who wishes to show as great a margin as possible between the cost and selling price of the product.

Although the design and construction of ball mills continue to show marked improvements, the problem of establishing a basis for measuring the results accomplished in crushing is as yet complex. However, it is generally conceded that the work of crushing is shown by a surface relation; work varies as the amount of new surface produced.
The study of the ball mill is very complicated, due to the varying conditions existing in the mill. Some of these conditions are: size of grinding media, size of feed, moisture content, and speed with its effect on the paths of the grinding media.

A brief history of fine grinding will first be outlined, after which some of the theories of ball milling will be stated. Then the paths of the grinding media will be given some attention, and a new point of view about the trajectory will be brought out.

**HISTORY.**

**Ball and Tube Mills.**

Progress in the development of ball and tube mills which have been known in one form or another, for approximately ninety years, has been more marked during the last decade than during the entire history of this type of grinding machinery prior to 1920, as will be seen by reviewing the literature on these machines.

The history of fine grinding is more or less scattered through the literature. L. Sell, in 1897, published an article\(^1\)


in which he gives a very complete history of fine grinding prior to
that date. Little effort has been made in recent years to bring
the history, in complete form, up to date. In 1927 W. T. 
Millar* published a complete history of crushing machines but did

*Eng. & Min. Jour., Vol. 124, 1927; pp. 490, 410, 131, 290, 336,
330, and 636.

not include fine grinding machines.

It is the object of this review to bring up to date in
brief form the history of fine grinding machines.

The ball mill, consisting of a cylindrical drum of
wood or sheet iron and rotating around a horizontal axis, prob-
ably found its first use in the manufacture of powder, essen,
indigo, gypsum, etc.,* especially in France. Figure 1 gives an

*Bell, L., loc. cit.

idea of the type of mill used in the dye industry. Beginning
about the middle of the Nineteenth century they were recommend-
ed for grinding ores in connection with amalgamation, especially
gold bearing ores. The final results of these operations were
not very satisfactory at this time. Their failure was due
principally to design and construction features.

- 3 -
Mill inclined at 45° containing cast iron balls to submit the indigo to a first dry grinding.

Fig. 1.


Figure 1.

Type of Ball Mill used in the Dye Industry.
Cement was invented in 1828 and nine years later, 1834, Hult built an "improved" ball mill, especially designed for grinding gypsum, from which the greatest results were expected. This mill has a cylindrical, horizontal drum of wood, about 4.5 feet in diameter and 25 inches long, which rotated about a shaft extending through the mill and coinciding with its horizontal axis. In this drum the grinding was performed by eight iron balls, six large ones of 8 pounds each, and two smaller ones of 5 pounds each; the mill was not completely closed during the grinding process. On the shell 42 openings about 100 mm. long and 25 mm. wide were provided and covered with wire screens in order to discharge the feed as soon as the required fineness had been accomplished. The charging of the feed was done by means of a door in one end of the mill. The grinding process was dry and periodic. The efficiency of the mill was claimed as 20 to 25 sacks of gypsum of 30 kgs. each per hour. Unfortunately no note of the fineness of the discharged gypsum is given.

In 1865 a ball mill, which operated continuously and crushed dry, was installed at Black Hawk, Gilpin County, Colo. 4


At this time Black Hawk was six hundred miles from any railroad.
so that the mill was constructed from scrap material. Two circular cast-iron discs, four feet in diameter, were used as heads. They were mounted three feet apart on a steel shaft. Brought iron bars 0.5 by 2.5 inches were used as staves for the drum. Each bar was spread by forging at the ends and in the middle, for the length of an inch, so as to measure 2-5/8 inches at those points. They were bound on the heads and at the middle by three hoops of iron. Thus, there were openings of 1/8 inch between the staves. One end had a circular opening around the shaft through which the one could be fed. The other was closed. A wooden frame was provided with a wooden casing to confine the dust. A gear wheel on the drum shaft, and a pinion mounted on a counter-shaft with a pulley furnished means to drive the drum at approximately 24 r.p.m. Chilled cast-iron balls were first tried, but did not prove satisfactory. Finally forged steel balls were used; these were made from old shafting. The capacity of the mill, using 1000 pounds of balls, was one ton of gold-bearing ore per hour, crushed through 30 mesh.

In the Seventies several new designs appear which showed considerable improvement over the old mills. One of these mills, built by Gebrüder Sachenberg*, was put into opera-

---

*Sell, L., loc. cit.
tion in the summer of 1876 at the Gottes-Goldhungs-Coaster of
the Minsfelder Gewerkschaft for the disintegrating of copper
matte. According to Sachemberg, this mill in 24 hours produced
10,000 kgs. of finished product, while a Chilean mill in the
same time with half the power consumption, not including the
power required for screws, etc., produced only 2,000 kgs. of
small grains, which had to be screened in order to produce a
finished product. 7

7 Notizblatt des Eisen-und Cement Vereins, pp. 10-11, 1876.

The Sachemberg-Brueschner ball mill differs from the
earlier ball mills principally in that continuous operation is
possible. The product, before being discharged from the mill,
is subjected to a sizing process by screens which are a part of
the mill itself. Another type introduced at this time was a wind
swept mill in which the fine material was removed by circulating
air through the mill; this mill failed, due to mechanical de-
fects.

A patent was granted in 1879 in England for a tube
mill. 8 The dimensions of this mill were approximately 12 feet in

8 The first tube mill in metallurgy; Eng. & Min. Jour. Vol. 61
and Vol. 82, 1906; p. 1151, and 187, respectively.

length and 2.5 feet in diameter. It was used on the Red River
Tin Streams, Cornwall, England, for crushing sand to slime. The
grinding media consisted of scrap iron.
In 1898 an airing pulverizer was installed at a property near Niles, Montana. This mill was eight feet in length and five feet in diameter. It weighed 16,000 pounds, was lined with porcelain and charged with pebbles. The discharge opening was larger than the feed opening, and the operation of the mill was continuous. Later installations were 15 feet long, lined with chilled-iron and charged with chilled-iron balls; later chrome steel was substituted.

A tube mill installation, designed for wet grinding, was made in 1896 by Messrs. A. I. Orr & G. Cruse, work at the Hamman's Star Gold Mines, Ltd., Western Australia.

This installation supplied considerable impetus to the use of ball mills for it was not long before many other similar installations were being made. Prior to 1899 Krupp manufactured ball mills which were used for dry crushing. These mills, Figure 2, had a screen at the periphery outside of the liners, the latter being so arranged that when the ore was partly ground it would emerge through the openings which were somewhat larger than the screen openings that determined the size of the final product. That portion of the material that did not pass through the outside screen re-entered the grinding portion of the mill.
Although ball and tube mills were first used about 1924, they did not make much progress in ore dressing until about 1965. However, the tube mill was quite prominent in the cement industry. From 1965 to 1984 constituted a period of development for the working details and elimination of defects.

The year 1904 marks the beginning of large scale use of ball and tube mills for wet grinding in cyanide practice. This large scale practice was first introduced on the land.12


The next radical development was the Hardinge conical mill,12 which made its appearance in 1907. This mill instead of having a cylindrical grinding chamber, is composed of two cones, Figure 5, with their bases attached to a short cylinder. The cone at the feed end is shorter than the one through which the pulp discharges. A high efficiency was claimed for this mill, due to the supposedly selective action of the pebbles or balls used for grinding, whereby the larger balls or pebbles remain in the drum portion of the mill and grade to smaller sizes toward the discharge. As the ore particles tend to arrange themselves in similar manner, the larger balls or pebb-

- 10 -
The effect of the classifying action of the cone arranges the balls or pebbles that grind the material as shown here. The force required to do the crushing is roughly proportioned to the sizes of particles being reduced.

Figure 3.

Hardinge Chemical Mill
bles work on the larger pieces of ore with intermediate predi-
tions down to the smallest pebbles and smallest particles of ore.
Thus it was sought to apply energy as needed in accordance with
the size of the ore particles.

About 1910 marks the beginning of extensive flotation
practice. This practice called for a very fine product, in most
cases passing 120 mesh. This practice necessitated fine grind-
ing for which purpose the tube mill was well adapted.

The short ball mill was developed to take feed up to
two or four inches in diameter and reduce it to, say, 10 to
50 mesh in one operation, the product being screened or classi-
fied outside the mill and the oversize returned with the original
feed. The grinding media consisted of steel balls and the
lining was of such form as to provide a series of steps or
corrugations, which lifted the charge as the mill revolved.
Early experience showed such mills to be efficient grinders, but
when used with trunnion or overflow discharge or practically the
same level as the feed they possessed one bad feature, namely,
the heavier mineral remained in the mill longer than was neces-
sary and was consequently ground too fine. To eliminate this
condition, discharging at a lower level than that of the feed
was introduced.
The Marcy mill, a mill of this type, was introduced in 1913. It consists of a cylinder of greater diameter than length, equipped with the usual spiral feeder and loaded with pebbles or balls as may be required. The drum is divided into two sections, Figure 4,—the crushing section, equipped with lifters to insure the elevation and dropping of the balls to secure maximum crushing efficiency, and a small section at the end of the mill, separated from the crushing space by means of a manganese-steel plate and having perforations about one-eighth inch wide. In this end section there are radial screens and lifters, the lifters forming the bottom of a box, the top of which is formed by the screen itself. These are the finishing screens, and material which will pass through them is considered as being of finished size. The material passing through the screens is diverted by arrangement of the compartment so that it will discharge through the central orifice; while the oversize—material which remains on the screen—is diverted back into the mill through a proper opening.

The action of the lifters, in addition to forming the bottom of the box and assisting in the screening operation, is to maintain a low pulp level in the mill. By constantly removing the solution and pulp from the bottom of the crushing
drum, a removal of the material already sufficiently ground is accomplished rapidly and no energy is lost in doing work where it is not required. The coarser material is left in a comparatively dry state upon the balls or pebbles, and the crushing and grinding action of the mill can have maximum effect upon it.

A mill of similar design but operating dry was introduced by H. Reverbng of Colonge14 about 1890. This mill was

14Sell, L., loc. cit.

mounted on a hollow shaft. A screw conveyor, Figure 5, extended through the feed end a short distance into the mill. This conveyor was used for feeding the ore to the mill. From the other end extended another screw conveyor for returning the oversize from the screen to the feed end of the grinding section. The mill is divided into two sections, the grinding section and a small section at the discharge end of the mill, separated from the grinding space by means of a perforated steel plate. The screening section contains a cone-shaped screen, the small diameter being next to and equal to the diameter of the grinding section. The screen is connected directly to the grinding section while the screen housing is separate and stationary. The oversize from the screen is returned to the feed end of the grinding section, while the fines pass through a discharge pipe in the bottom of the screen housing.
It can hardly be said that this type of mill is practical, due to the extreme length of the grinding section unless some other advantage could be made to offset this disadvantage.

A comparatively new development in ball mill practice, represented by the Williamson ball mill,19 was introduced in 1924. It has a series of step-like or wedge-shaped extensions in place of the usual end liners, Figure 6; these projections at the discharge end being perforated. The principal feature claimed for this mill is its ball action, resulting in equivalent grinding for less power, less ball consumption, and less liner wear. Due to the end construction, the descending of the pulp and balls is toward the center of the mill and downward until approaching the lower part of the course, when the direction of flow is reversed away from the center and toward the ends of the mill. The result is a more compact ball stream, more thorough mixing of the charge, and a greater number of effective contacts.
Hulls Containing More Than One Grinding Compartment.

In the early Nineties, Jean Heinstein of Heidelberg secured a patent on a dry grinding, 2-compartment ball mill.\[18\]

\[18\] Sell, L., loc. cit.

This mill was divided into two compartments by means of a perforated steel plate. The first compartment, Figure 7, known as the coarse grinding compartment, contained a single large ball, while the second or fine grinding compartment contained a large number of small balls. The discharge end of the fine grinding compartment was fitted with a screen for screening the finished product. That material which, on account of its physical properties, could not be screened easily was blown through by means of an air current.

Theoretically, the idea of using different grinding media in the coarse crushing compartment from that used in the fine grinding compartment is very good. In such case the economic advantages to be gained by performing in a single machine, at the same time, two different operations—one of coarse crushing and the other, fine grinding—should be compared with those when using separate machines for the two operations. It is obvious, to mention only the most vital point, that coarse crushing and fine grinding can be accomplished at the same rate only under very limited conditions.
Fig. 19.
Kammerkugelmühle von Heinzein.

Figure 7.
First Two-Compartment Dry Grinding Mill.
In large commercial plants a separation of the two operations is to be recommended, while in small scale plants the combined machine may prove economical.

Long mills, divided into compartments, are used successfully in the dry grinding of cement and clinker, but have not made much progress in the grinding of ores.

A mill, divided into two compartments, using rolls in the coarse crushing compartment and rods in the fine grinding compartment was put on the market in 1923. Between the two compartments is a grizzly of manganese steel with tapering holes. The rolls, it is said, will handle 4-inch material. When the rock is reduced to one-half inch size it passes through the grizzly into the rod compartment, where the fine grinding is done. The pulp then passes through the end grizzly onto a revolving screen, where it is classified and the oversize returned to the mill to be re-ground.

Another type of mill, manufactured by the Taylors Company made its appearance at this time. It is divided into three compartments with the largest balls in the first compartment and the smallest in the third. Partitions with central openings are used to separate the compartments. By placing a
sawyer-shaped grating with radial slots in each compartment just before the dividing partition a considerable increase in capacity and reduction of power per unit ground is claimed for this type of mill.

In 1896, mill patrons introduced the wet grinding "comped" mill.\textsuperscript{13} Prior to this date the same company had intro-

duced a "comped" mill for dry grinding, which gained considerable success in the cement industry; it is shown in Figure 8. Both of these mills might be considered as 2-compartment tube mills.

In the wet grinding comped mill the feed enters at one end into a short compartment with large balls or pebbles and passes from this compartment into one compartment of a double sump below the mill, then to a classifier which removes the finished fines and delivers the coarse to the other compartment of the sump, from which it is automatically picked up and fed by a scoop-feeder into the second compartment of the mill, which has smaller grinding media, and finally discharges through the other end of the mill. Both compartments have quick-discharge gratings and the division head between the two compartments is provided with a circular ten-cap screen for returning coarse particles to the first compartment.
Fig. 6. Sectional View of Allis-Chalmers Dry Grinding "Consol" Mill

Figure 6.

Allis Chalmers Dry Grinding "Consol" Mill.
Coarse crushing and fine grinding compartments may be arranged concentrically. In this case the coarse crushing would be performed in the inner compartment and the fine grinding in the ring-shaped outer compartment. A mill of this type was introduced, in the Nineties, by Julius Seise \textsuperscript{2c} of Rossleu.

\textsuperscript{2c} Sell, L., loc. cit.

The liner of the inner section, no shell being required, was made up of heavy iron bars or staves, presenting the appearance of a grizzly. This arrangement permitted the free passage of the crushed product to the outer or fine grinding compartment.

The liner of the outer compartment consisted of perforated plates, while the shell consisted of a screen. The oversize from this screen being returned to the inner section.

Grinding media for the inner compartment consisted of large iron balls, while that of the outer compartment was made up of smaller balls.
Double Discharge and Single Discharge Two-Compartment Ball Mills.

The Fairchild "double discharge" ball mill\(^1\), Figure 9, was introduced in 1924, for which increased capacity and greater flexibility was claimed.

This mill consists of a horizontal cylinder which may be mounted either on trunnions or on tires and rollers. This cylinder is divided midway into two separate cylindrical compartments, each containing its own ball charge. Surrounding the center of the mill is a feed drum with scoops and spirals, which supply equal amounts of feed to both compartments. The pulp flows horizontally in both directions from the center of the mill toward the ends, where it passes through quick-discharge gratings with lifters and passes through both trunnions. Thus, the Fairchild mill is really two separate mills, end to end, with a common feed and a common mounting.

The two separate grinding compartments in the mill not only produces the effect of two mills, but saves floor space and reduces power, ball, and liner costs, and, with the center feed feature, permit a more efficient and economical arrangement with other equipment that has heretofore been impossible.

---

\(^1\) Developments in grinding machinery, Eng. & Min. Jour., Vol. 117, p. 905, 1924.
Fig. 3—Sketch illustrating mill and classifiers in closed circuit, with one, two, and three classifiers.

C. & M. J. Vol. 117, p. 206 (1924)

Figure 9.

Fairchild "Double Discharge" Ball Mill.
A patent was granted in 1924 to S. B. Spain for a **U. S. Patent No. 1,513,932.**

2-compartment ball mill. The principle is very similar to the Fairchild "double discharge" ball mill, the difference being in the direction of flow of the pulp, which is opposite to that in the Fairchild mill.

In this mill, two perforated plates set a short distance apart, in the center of the mill divide it into two equal parts. The discharge being through vents in the shell between the two partitions. The feeding being accomplished through the ends of the mill. Thus, this mill instead of having a common feed, as is the case with the Fairchild mill, has a common discharge for the two compartments.
Rod Mills.

The idea of using a rod or number of bars extending from end to end in a drum type of mill was first introduced in the seventies, when a German by the name of Marinko obtained a patent on a mill of this type.

Very little was done on the development of the rod mill until 1912. Three years later a number of rod mills were installed by the Phelps Dodge Corporation. Figure 10 shows the original design. Nineteen hundred seventeen saw the introduction of the Marcy mill, a rod mill somewhat different in design. Some of the features of this mill were an easily opened head for the removal of worn rods with the minimum inconvenience, and a large discharge opening which would give a low discharge level, resulting in a minimum quantity of pulp retained in the mill. The lining of the mill was slightly tapered toward the discharge end to retain the rods, this later proved a mistake in design.

In 1927 Hardinge introduced a conical-ended rod mill.

This mill is shown in Figure 11. The outstanding features claimed for this design is the conical ends, which are responsible
Figure 10.

The Marathon Mill — Original Design.
Eng. & Min. Jl. Vol. XIII,
p. 598.
Rod mill with discharge ports open

Position in conical-ended rod mill after starting showing the open spaces at both feed and discharge ends

Showing how the discharge level can be varied to meet different requirements

Figure 11.
Hardinge Conical Ended Rod Mill
for the alignment of the rods within the mill during the grinding process. This feature is said to eliminate the heavy pounding on the heads, and consequent high linear wear. The large space in front of the rods facilitates access of feed to the whole mass of rods, and likewise the space at the discharge end, it is claimed, increases the discharge rate, as there is no obstruction from the rod ends to hinder the free flow of the product. A variable discharge level is provided for by discharge ports, which can be opened or closed to suit the particular grinding problem.
Counter-balanced Mills.

The counter-balanced mill was first introduced in the Seventies by Hugo Gropel of Budapest. This mill was relatively complicated, due to the fact that it consisted of four grinding compartments included in a single shell, Figure 12, and the grinding media consisted of iron balls. The purpose of these several grinding compartments was not to divide the grinding process into different steps, since a complete crushing cycle is performed in each compartment, but to decrease the power consumption required for operation of the mill. With this object in view the compartments are arranged symmetrically around a horizontal axis so that one of the grinding compartments is descending while another is ascending. Thus, the amount of energy required to raise one compartment and its contents, in order to do crushing on the downward path, is counterbalanced by the downward movement of the descending compartment and its contents. By this arrangement the unnecessary power consumption required for special construction is avoided. This condition obtains not only when the compartments are arranged symmetrically to the axis, but also the contents of the compartments must, at all times, remain in a position symmetrical to the axis of the mill, which condition, of course, never occurs. Nevertheless the purpose to save power must be recognized.
However, criticism of the practical importance of this mill, and especially whether that of power economy is great enough to justify such complicated construction, can be made only after accurate practical tests have been made. No such tests are available or are not known—1897.

The arrangement of the chambers is shown in Figure 12. The beams \( L_1 \) - \( L_2 \), which drop the balls at a definite position of the compartment shell in order to disintegrate the charge by impact, are not necessary.

Charging of the compartments takes place through an opening in one end of each compartment. The discharge of the finished product is accomplished by means of perforated plates \( L_1 \).

The screening of the product is accomplished by means of a screen shell which surrounds all of the grinding compartments. From this screen the oversize is returned to the grinding compartments by means of sheet-iron baffles.

Nothing more was done on this type of mill until 1921, when the Forrester-Hendy rod mill,\(^7\) shown in Figure 13, was

\(^7\) Robie, E. H., loc. cit.

given large scale tests. The mill is divided into six cylindrical compartments, each containing a separate charge of rods. The axis of rotation is not within the space enclosing the rods. Great claims were made for power economy with this mill, the
theory being that the rods in the descending compartments of
the mill would tend to counterbalance those on the ascending
side.

The most recent development along these lines, the
Forrester-Hexxen mill, is a distinct novelty; extensive con-

Allen, A. W., Peripheral-discharge cylindrical mills; Eng.

rect between grinding media and liner is avoided, and it is
possible to arrange the screens around the periphery of the mill
with the knowledge that the classification of the crushed ore
will not be hindered by the presence of the grinding media. Rods
are used, bunched or cradled in the manner shown in Figure 14.
An additional advantage of cradling results in a significant de-
crease in power consumption, a balanced unit being produced.

The feed, of the usual ball- or rod-mill size, is
picked up by a trunnion-extension scoop. The pulp then passes,
by means of a delivery tube with spiral feeder, to the center
of the mill—to a chamber with outlets to four compartments.
The ore pulp is free to pass into any part of the interior of the
mill, but the crushing rods are separated into four distinct
bunches by a spider protected with lizards.
Fig. 7—Forrester-Rexman mill, showing arrangement of grinding rods

Figure 14.
Forrester-Rexman Mill.
The pulp discharges from the distributing chamber onto and between the falling rods, the revolution of the mill insuring fracture and abrasion in the ordinary manner. The ore as crushed passes to the bottom of the mill, where it comes in contact with the course screen, the undersize passing through and the oversize returned to the center of the mill.

With the rod load divided as illustrated in Figure 15, the amount of power required to drive a given weight is claimed to be only about 25 per cent of that of the ordinary mill.\(^5\)
\[^5\] Power consumption in single and multiple compartment rod mills; Eng. & Min. Jour., Vol. 119, p. 171, 1925.

On the other hand, it is very probable that the crushing effect per rod is less.\(^6\) The mill has not had sufficient use in metal-

...urgical plants to furnish reliable data as to performance.
Location of Center of Gravity of Rod Charges in Four Compartment Mill.

Figure 15.
In the original mills the feeding was accomplished by means of openings in either end or on the shell of the mill. The mill had to be stopped when the charging and discharging took place, both being accomplished through the same opening; after these operations were completed the opening was covered by means of a plate which had to be fastened on by bolts or otherwise.

With the introduction of the peripheral discharge mill the charging was done through an opening in one end, around the horizontal shaft. The feed being discharged from a hopper into the mill, through a spout, the end of which was flush with the end of the mill. This arrangement was unsatisfactory due to the radial arms extending from the shaft to the shell, at either end of the mill. These arms prevented the spout from extending into the mill. This method was improved upon by making a recess in that portion of the arms exposed in the opening and connecting the hopper to the mill by means of an elbow, which covered the entire opening, the shaft extending through the center of one arm of the elbow, for which a suitable bearing was provided. The end of the elbow extending into the mill was free from the mill and did not interfere with its operation.
The next radical improvement was the hollow shaft, or in some cases the mill was mounted by means of hollow trunnions, the feeding taking place through the hollow shaft or the trunnions. The screw conveyor was the outgrowth of the hollow shaft.

In the earlier mills the position of the feed with respect to the feed opening was not considered of vital importance. As this factor became more important, especially in the design of mills for large scale operations, the type of feeder had to be given serious consideration. This led to the development of three general types, namely, one-way scoop, three-way scoop, and drum. Occasionally a two-way scoop is used.

The one-way scoop consists of a single spiral with open end and central side opening for delivering into the feed trunion. The three-way scoop contains three spirals of but one-third turn each, and is designed to give greater capacity than a single scoop. It is not made of as great a radius as the larger single scoop. Both types revolve with the mill dipping in a rectangular feed box from which they pick up a certain portion of the material at each revolution. Spiral feeders serve to elevate the feed as well as introduce it into the mill and, therefore, make possible a lower feed-delivery point.
Drum feeders consist of a cylindro-conical drum, open at both ends, with inside helix to transfer material from the feed side into the mill trunnion. It is more positive and less subject to breakage than the spiral type of feeder.

A combination drum-an-scoop feeder is used when the original feed is coarse and can be delivered at the height of the mill axis, and when sand oversize is delivered from a mechanical classifier.
Grinding Media.

Iron balls were the first grinding media used, but in grinding cement it was soon found that the fine iron altered the physical properties of the cement, so flint pebbles were substituted for the iron balls in this industry.\(^\text{12}\)

\(^{12}\) Sell, I. E., loc. cit.

Balls with chains were also tried as grinding media in the old periodic mills but did not meet with success.

Iron balls, usually an alloy, and pebbles are still the principal grinding media. The only change being in the size and shape of the iron balls. Iron rods were introduced in the seventies, but did not prove of commercial value until 1912.

Balls with concavities, spherical shapes,\(^{18}\) and slugs have been recommended in recent years for dry grinding, but in wet grinding the deformed sphere is superior. Balls of egg or parabolic shape are recommended.

Cubes were given much publicity in 1924\(^{19}\), but do not seem to have given universal satisfaction.

Short pieces of old drill steel, or any pieces of small iron mixed with regular balls, are used.¹¹ The same company tried cubes, and other shapes, but found that pentagonal dodecahedrons gave the best results. More recently hexagonal rods²² have been tried but the results are not favorable.

Rubber-coated spheres have been suggested, but no definite tests have been published. Rubber-covered rods have been tried in laboratory machines for certain types of ores and have proven satisfactory.

¹¹News; Quincy finds marked advantage in angular grinding media; Eng. & Min. Jour., Vol. 119, p. 775, 1925.

Liners.

Cast-iron liners" were used in the original period

Bell, E., loc. cit.

mills. With the introduction of ball mills in which the discharge of the ground material took place through the perforated shell, perforated steel plates were into use. Porcelain linings set on wooden bases, the latter resting upon iron shells, were used to some extent in the early mills, when the discharge of the ground product took place at one end.

Wooden blocks were also tried", but their life was too short; in most cases lasting only a few days. Silica liners were introduced at the Treasury Mine, on the land, in 1902, and are still used by some operators in preference to other types.

Today there are many types of metal liners in use."

Miller, W. H. M., Ball and tube mill liners; Eng. & Min. Jour., Vol. 118, p. 613, 1884.

Some have corrugated wearing surfaces, others are fitted with lifters or projecting ribs, all of which have the same object in view, that is, to prevent slippage.
The El Oro lining consists of plates designed with deep ribs running longitudinally in such a way that the pebbles or pieces of hard rock used as grinding media become wedged into the troughs between the thin taper ribs on the liners and even project above them, so that the wear is taken in large measure on the pebble or rock surface.

Rubber liners for tube and ball mills were first tried in 1921\textsuperscript{2} and proved satisfactory from the start, but their adoption has not been very extensive.

REVIEW OF THE LITERATURE ON THE THEORY OF THE BALL MILL

The theory of the ball mill has been worked out and discussed at intervals since 1904. The leading articles are by: (1) Hermann Fischer, (2) E. W. White, and (3) H. E. T. Haultain and F. C. Dyer.

FISCHER'S THEORY

Prior to 1904 the popular belief regarding the working of ball and tube mills was as follows:

\[ \text{The revolving of the drum and, accordingly, the} \]
rising of one of its sides, sets the balls rolling down the incline, which movement communicates itself to the ore. The latter is ground between the balls and slowly transformed into dust. The inlet of the ore being situated higher than the outlet, caused, as was generally believed, the ore to move slowly towards the latter.

Fried Krupp, of Magdeburg-Buckau, decided to investigate the matter in a thorough fashion, and for this purpose special ball mills were constructed. Some of these were made of glass, while others had movable gratings, which allowed the interior to be continuously watched while the mills were at work. As a result, photographs could be taken at all stages of the work, and these pointed to the fact that in a tube-mill the balls do not grind the ore, but crush it by being thrown in an inclined direction from the side of the tube to its bottom, and that the higher position of the inlet had nothing to do with the movement of the ore towards the outlet.

The diameter of the drum was about 3 ft. 4 in. Flint stone balls were used as grinding media. At 21:23 R. P. M. the balls nearest the center rolled slowly down the incline, while the bulk of the balls advanced higher up the side of the drum. When rotating at 26, 30, 32 R. P. M., the movement of the
rolling balls became more lively, and a marked increase in
the spaces between the balls which lay next to the center of
became apparent. The bulk of them had advanced higher; it
will be seen that these balls which are close to the walls
of the drum and touching it apparently adhere to it and
move up the side of the drum until they reach a certain
height, when they are thrown to the bottom of the drum,
describing an arc in their line of flight, Fig. 16.

By raising the speed to 35 R. P. M., the formation
of the balls gets looser still, consequently the height they
attain in the drum is again increased. The arc described by
the balls in their line of flight can be clearly seen, as
even the balls which lie closest to the center are seen to
detach themselves from the heap in series to take up the
movement. At the bottom of the drum the dropped balls rejoin
the rest which are creeping up the side, so that a free
space appears between the line of flight of the falling
balls and those resting against the walls of the drum. This
space resembles a kidney in its shape. Furthermore, the
falling balls do not come down in clusters, but in series,
the balls of each row keeping to their own line of flight
according to their distance from the wall of the drum while
mounting up. This movement was so marked that the spaces
Fig. 176. Diagram of action of tube mill.

Fig. 18

Diagram Showing Action of Tube Mill
between the different series could be actually counted, while
the balls when mounting seemed comparatively closely packed
together.

A further increase in speed resulted in the balls
adhering during the whole of the circuit to the side of the
drum.

It appears that the correct number of revolutions
lies between 32 and 36 R. P. M., and that the speed should
never exceed 42 R. P. M.

Further tests were made with the same drum, but
this time a quantity of crushing material of a kind emitting
no dust was thrown into the drum. The ore behaved in exactly
the same way as the balls. While mounting the side of the
drum it filled the spaces between them, and when dropping
described the same arc with them. When it had reached the
bottom of the tube, which in this case represents the point
of impact of a stamping mill, it met with the balls which
were remounting the side of the drum, and between these and
those balls, which had followed it in its line of flight
and now dropped on it, was crushed and splashed in all
directions. The line of flight may be computed for each
ball, if we omit the resistance of air and the friction
causd by other balls and the crushing material.
A ball will drop from the side of the drum at the moment that the part of the ball which is nearest to the center and limited by the line of velocity, \( mg \sin \gamma \), equals the rotary force \( \frac{mv^2}{r} \), see fig. 16. The arc line of flight may be reckoned by this formula:

\[
y = x \frac{\cos \theta}{\sin \gamma} - \frac{x^2}{2v^2 \sin^2 \gamma}
\]

The vertical force of the movement is reduced until the ball reaches the apex. Of course, a reduction of speed in the direction of the line of flight results, and the balls approach each other at the apex. It follows that the balls at this point push the crushing material aside, and at the same time influence each other's flight. But if we omit this disturbance, we shall see the ball continue in its flight and the distance between it and its followers increase. At this point the crushing work is really carried out.

From this it results that to get the maximum work out of a ball mill the mill must be set to work so that it affords the best line of flight to the balls.

**Davis' Theory.**

Mr. E. W. Davis, in 1819, arrived at the following

---

\( \)\(^4\) Davis, E. W., Lec. cit.
conclusions:

(1) Action of Charge at Slow Speed.

In a mill, revolving at a very low speed, the charge is tilted until the critical angle is reached, after which the balls simply roll down the slope to the lower side of the mill. This critical angle is affected but slightly by a change in the speed of the mill, up to a certain point; the increase in speed simply increases the rapidity with which the charge is raised to the top of the incline. In this condition the balls are in contact with one another except as they may bounce in rolling down the slope of the charge; also, the balls must roll down the incline at the same rate, pounds per hour, at which they are raised to the top. Then with a mill half full of balls, any particular ball will roll down the incline something less than twice per revolution of the mill.

As the speed of the mill is slowly increased, the time required to bring the ball back to the top of the pile is diminished, but the time required by it in rolling down remains practically the same. It would seem that the whole problem of crushing would resolve itself into getting the balls to the top of the heap fast enough. This would be true if it were not for centrifugal force and inertia. As
the speed of the mill is increased these two forces grow very rapidly in importance.

(2) Action of Charge at Higher Speeds.

Davis continues by considering the forces acting
on a particle $p$, Fig. 17, in contact with the lining of the mill. The centrifugal force $c$ acts to press it against the lining while $W_1$, a component of the weight $W$, acts to pull it away from the lining. Then if $\theta_1$ is the angle between the vertical axis and the radius $op$, $W_1 = W \cos \theta_1$. It is possible for $c$ to be greater than, equal to, or less than $W_1$ or $W \cos \theta_1$, for as $\theta_1$ decreases $W_1$ increases. Then

\[ c = W \cos \theta_1 = f_1 \]

and $f_1$ may be positive, negative or zero.

If $f_1$ is positive the particle will be held against the lining of the mill. As $\theta_1$ decreases $f_1$ decreases with it, when $\theta_1$ is zero, $f_1$ is still positive, it is evident that the particle will maintain contact with the mill lining throughout a complete revolution.

If $f_1$ becomes zero for some value of $\theta_1$ the particles below $p$, having a greater angle $\theta$, will be held against the lining of the mill by a positive force $f$. In other words, as the mill rotates, the force with which a particle $p$ is held in position decreases until it reaches zero. At this point the particle is being pushed on by the particles below it and is free to move in a path governed
by this initial velocity and gravity. The path it takes will, of course, be parabolic. Then when the angle \( \alpha \) is such that \( f \) is zero, the particle \( p \) will leave off contact with the mill and start on a parabolic path. In this position

\[ c = \frac{v}{\cos \alpha} \]

The centrifugal force \( c = \frac{v^2}{r} \), where \( w \) = weight of particle; \( v \) = initial velocity; \( r \) = radius; and \( g \) = 32.2 ft. per sec. per sec. Also the initial velocity of the particle is its velocity in the circular path, or \( v = 2 \pi n \) in which \( n \) = speed of mill in revolutions per second. Then by substitution in the formula \( c = w \cos \alpha \),

\[ \frac{w^2}{r} = v \cos \alpha \quad \text{or} \quad \frac{w(2\pi n)^2}{r} = v \cos \alpha \]

\[ \cos \alpha = \frac{4\pi^2 n^2}{c} \]

From this equation it is evident that an increase in either the speed of the mill or its radius will cause a decrease in the angle \( \alpha \) and the parabolic path will not start until the particle is carried farther around in the direction of rotation. At any speed \( n \),

\[ \cos \alpha = k \pi \]

where \( k = 1.226 \) \( n^3 \). Then at constant speed, the particle \( p \) nearer the center of the circle than \( p_0 \) will start on its parabolic path from a larger angle \( \alpha \), or as \( r \) decreases \( \alpha \) increases, the relation between \( r \) and \( \alpha \) given in equation
(2) always holding true when \( \alpha \) is the angle at which the parabolic path starts. Equation (2), then, is really the equation of the curve above which all particles are following the parabolic path and below which all particles are following the circular path.

From equation (1) \( n = \frac{r \cdot \cos \alpha}{g \cdot r^2} \) and if the radius is considered as the constant, \( \alpha \) must decrease as \( n \) increases. Since the maximum value of \( \alpha \) is zero, when \( \cos \alpha = 1 \), the speed has reached a point above which \( c \) is always greater than \( \alpha \cdot \cos \alpha \) (Fig. 17), and the particle will cling to the lining of the mill throughout the complete cycle.

Then the speed at which any particle of radius \( r \) will cling is given by the equation \( n = \frac{r}{g \cdot r^2} \), in which \( n \) is in revolutions per second and \( r \) is in feet. If the speed is in revolutions per minute the equation will become

\[
n^2 = 54.19 \frac{r}{r} \tag{3}
\]

This equation shows the critical speed \( n^2 \) at which the particle of radius \( r \) will cling to the lining of the mill or to the next outer layer of particles of radius greater than \( r \). If \( n^2 \) is sufficiently large, \( r \) will be sufficiently small to include all of the balls in the mill and the mill will rotate as a flywheel with no relative motion between the particles in the charge.

Below the critical speed given in equation (3),
the particle at radius \( r \) will reach the critical angle \( \alpha \) and will then start on its parabolic path. The equation of the parabola, with origin at \( P_1 \), is
\[
y = x \tan \alpha_1 - \frac{g x^2}{2 v_1^2 \cos^2 \alpha_1}
\]

(3) Early Comparison of Observed and Calculated Curves.

Davis concludes as follows: It is evident that the perpendicular to the radius at \( p \) would intersect the perpendicular to the radius at \( a \) if it were produced far enough, Fig. 12. If the two particles considered are closer together than \( a \) and \( p \) on the curve \( a-b \), the intersection between the lines of initial direction are closer to the points considered. If the two points are adjacent on the curve \( a-b \), it is apparent that the intersection will be very close to the points and will in fact, take the form of a slight crowding action between the particles. The results of this will be a slight deformation of the curve through which the particle travels. This will be just as prominent in a small charge as in a large one, although the results will be more apparent in the small charge.

It may also be shown that when the size of the charge exceeds 0.4 the volume of the mill, there will be a tendency for the particles near the center of the mass to
Fig. 18.
Paths of Travel of Particles in an S-St.

Mill Making by R. F. W.
crowd one another. This is, however, not at all serious and could probably never be detected in operation, but, when the mill is filled beyond 0.64 of its volume, the interference becomes quite important and the mill probably could not be made to operate efficiently when more than 0.6 full.

It is very important to prevent slipping between the charge and the lining of the mill; the tendency to slip is much smaller with the large charges than with the small ones. If the friction between the charge and the lining of the mill is not great enough to carry the particles up to the curve a-b, the efficiency of the mill will be very greatly reduced and the lining of the mill will be rapidly worn away. Flat sides will also appear on the balls and the cycle of the charge will be slow and irregular. In an open-trunmion discharge mill, the pulp will not flow from the mill regularly but will come in pulsations. Lifter or roughened liners are therefore desirable, as they insure a greater coefficient of friction.

DISCUSSION BY HAULTAIN AND DYER.

Although the entire philosophy of the ball mill is summed up in the equations given by Davis, Haultain and Dyer.\(^5\)

---

questions the foundation upon which this philosophy is based.

The following is a brief abstract of the above mentioned paper:

Many runs were made with a great variety of materials, including crushed marble, seeds of many kinds, and many mixtures (some wet and some dry). In a test with small, hard, white peas, two of the photographs show the Davis effect at the Davis speed, but the authors fail to get any such effects with disks or balls at these speeds.

Experiments were then conducted with mills 6 in. and 5 in. in diameter. The mills were of wood, with the surface roughened by coarse sandpaper; the material was crushed marble, about 30 mesh, with the fines screened out. Many experiments were made, but the Davis effect was not obtained with the Davis speed. When sand, galena, and shot were used, the sand and galena gave identical curves; the speed (N.F.R.) varies inversely as the square root of the diameter of the mill.

When the 4-in. mills contained 0.0, 0.5, and 0.2 load, the Davis effect was obtained at speeds considerably higher than the Davis. When the mill was run at Davis speed with marble crushed to 60, 120, and 200 mesh, the Davis effect was obtained with the 120-mesh.
When the 6-in. mill lined with corrugated linoleum is run at the Davis speed with 50-mesh marble, the grains carried upward by the corrugations travel the Davis path for that speed but the others do not. Evidently, the path of the grain depends on the amount of slip.

In none of the experiments do we get away from slip, except with certain selected fine-grained material, so fine that the particles adhere rather than slide. When shot was used with the roughened, but not stepped or corrugated, lining, the slip was evidently very great, but when a corrugated lining was used, a lining that would definitely lift a layer of shot, the Davis effect was approached at the Davis speed.

If there were no slip between layers, the other balls would follow close behind and there would be a hole in the center as in the Davis diagram. Davis explains the absence of the hole in his sand pictures as due to the effect of interference. The shot shows that it is clearly due to slip. Considering layer by layer of balls, the outer layer cannot slip on account of the corrugations, but the next layer slips somewhat on the first layer. The slip between each succeeding pair of layers increases still more until we get to a neutral layer that is not sure whether it is rising or falling or stationary. No matter what may be the
nature of the steps or corrugations, or whether there is
only a light load in a 6-in. mill or the heavy load in a
6-ft. mill, there must be slip or adjustment between balls
in the circular rising paths. The line joining the centers
of adjacent balls passes through an angle of more than 90°
in reference to the line of action of gravity. The corre-
gations may hold the outer layer of balls without slip, but
there will be increasing slip with the other layers. Any
slip or adjustment in the outer layer will be augmented in
the others.

Conclusions arrived at by Haultain and Dyer:
(1) Next to speed, the most important factor in
determining the paths of balls or rods in tube-mills is slip.

(2) The amount of slip depends on several vari-
ables (the size of balls, the nature of the lining, the
character of the pulp, and other factors) but slip is always
present in practice.

(3) In some mills (in many mills), the effect of
slip may be so great as to eliminate all free or parabolic
fall at ordinary mill speeds.

(4) There is always a continuous change occurring
in the relative positions of the balls in the upward or
circular path.
(5) At the speeds adopted in practice, there is more of the cascade effect than of the free-fall or parabolic-path effect.

(6) In tube-mills, the paths of the balls and of the ore particles are affected by segregation. This seems to be due entirely to size and is but little affected by differences in specific gravity. The action is practically the same as that taking place in a talus or rock dump where the smaller pieces settle through the spaces between the larger, and the large pieces roll.

(7) At low speeds, the small sizes are the middle; at high speeds they tend to go to the periphery.

(8) The best mixing seems to be at the highest speed attainable without the parabolic path. If there is much of the parabolic path some present there may be distinct segregation of the larger balls toward the center and of the smaller balls and of the ore towards the periphery.
The belief has generally been held that the balls follow a parabolic path beginning at the instant they leave the liner on their upward flight, and continuing until they strike it again on their downward flight.

To study the ball paths by visual examination, a mill, 36 inches in diameter by six inches long, was especially equipped with one-inch screens on the ends. This "squirrel cage" may be seen in Figures 19, 19-a, 19-b, 19-c, and 19-d, which also show the laboratory equipment for ball mill tests. The shortness of the mill reduced slippage to a minimum. By the use of the squirrel cage the behavior of the balls at various speeds and different charges could be studied at first.

Plate I represents a series of photographs of the 36-inch squirrel cage mill, revolving at 10, 20, 30 per cent, and so on, and including the critical speed, with ball charges equivalent to 10, 20, 30, 50 and 50 per cent of the mill volume. The balls were 1.25 inches in diameter.

If a ball charge equivalent to 10 per cent of the mill volume is used and the mill rotated, it will be seen from the photographs that as the speed increases the balls climb higher up the side of the mill. However, slippage is so great that they never reach the velocity necessary to throw them out into space. At the predetermined critical speed none of the
Figure 19.

View of Mills With 36-inch Mill Equipped as Squirrel Cage.
balls are carried over in contact with the liner as implied by the term critical speed.

When the charge is increased to 20 per cent of the mill volume it is raised somewhat higher and there is some free falling of the balls. There is still considerable slippage, and at the critical speed none of the balls are carried over. With this charge a considerable toe is formed by the balls in the bottom of the mill, preventing the balls from striking directly against the liner.

With the ball charge increased to 30 per cent of the mill volume, the toe is better developed and the balls are thrown further out from the charge and strike the liner above the toe at speeds near the critical. None of the balls maintain contact with the liner throughout a complete revolution of the mill at its "critical speed", although the outside layer does not leave the liner until it reaches a point well up toward the highest point of the mill.

When the ball charge is equivalent to 40 per cent of the mill volume the outside layer of balls will maintain contact with the liner throughout a complete revolution of the mill when revolving at its critical speed. These observations prove that the slippage between the outside layer and the liner decreases as the ball charge increases, and is entirely eliminated at the critical when the ball charge reaches a value be-
tween 35 and 40 per cent of the mill volume; it also appears to be eliminated at slower speeds. The toe is so developed that the mill can be driven at approximately 75 per cent of its critical speed without the balls striking directly against the liner.

If the ball charge is increased to 50 per cent of the mill volume, the behavior of the charge will be similar to that for 40 per cent of the mill volume. The paths of the free falling balls reach out a greater distance from the charge as the volume of the charge increases, indicating that crowding increases with the ball charge.

A series of sketches was made showing the outside layer of various ball charges in the 36-inch mill, with the mill rotating at different percentages of its critical speed. These sketches are shown in Plate II. The solid lines represent the paths of the outside layer, while the broken lines represent the theoretical paths to be referred to later.

An application of the old parabolic curve to these sketches will show that the balls extend much farther than the old curve had indicated. The difference is best shown in Figure 30 where the departure of the ball charge is shown by photograph. On it is drawn in broken line the old parabolic curve to correspond with the given speed and diameter. The balls are seen to be thrown far beyond the old parabola.
PLATE II

Data:
- Mill: 16"
- Length of mill: 16'
- Dia. of mill: 16.5'
- Critical speed: 75.0 RPM
- Type of screen for steel screen
- Observed paths for different ball charges at different speeds

Solid curves represent observed paths for different ball charges at different speeds.

Broken lines represent theoretical curves.

Table of Observed Paths for Different Ball Charges:

<table>
<thead>
<tr>
<th>Speed</th>
<th>Ball Charge</th>
<th>Observed Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>40%</td>
<td>30%</td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>40%</td>
<td></td>
</tr>
<tr>
<td>60%</td>
<td>50%</td>
<td></td>
</tr>
<tr>
<td>70%</td>
<td>60%</td>
<td></td>
</tr>
<tr>
<td>80%</td>
<td>70%</td>
<td></td>
</tr>
</tbody>
</table>

Plate II.
Figure 20.

Comparison of Actual Path of Outside Layer of Balls in 3-FT.

With the Old and New Theoretical Paths.
anticipation of the discussion to be made later it may be stated that the heavy line in the picture is the path according to the newly developed equation, to be discussed in its turn.

The aforementioned observations led to the conclusion that the balls do not act independently of each other after they leave the circular path. They are in contact when they leave the rim, and as they proceed they tend to travel with a retarded velocity in accord with the old parabolic equation because they are traveling upward, but this retardation is impossible, due to the fact that the balls behind are pushing at a constant speed. Hence they must travel at a constant speed, that is, at the speed of the rim, until they reach the top of their flight. Then the balls reach the top they possess an initial horizontal velocity equal to that of the rim of the mill. This velocity and the force of gravity are the only two factors governing the action of the balls through the remainder of their theoretical paths. The theoretical downward path is parabolic. In this way the balls do not act independently until they have passed the highest points in their respective paths. Then they become free falling bodies except for crowding or interference to be discussed later.
As already stated it was observed that when the ball charge was sufficiently large to overcome slippage, the balls were "thrown" out further than a calculated parabolic curve would indicate. This has been clearly shown in Figure 20, where the broken line is the old parabola. The following is the mathematics describing the behavior of the balls in second with what has been stated. The old theory of the parabola will first be mathematically examined.

Consider the 35-inch mill containing a charge of 1.25-inch balls, of sufficient weight to prevent slippage and rotating at 50 per cent of its critical speed, or 27.02 r.p.m., the critical speed of the mill being 45.03 r.p.m.

Referring to Figure 21, let B be the point at which the balls start on their "parabolic" path. Then when Ball No. 1 starts on its parabolic path, Ball No. 2 will continue to follow the circular path until it reaches point B, and its velocity will be the same as that of the mill, or 4.086 feet per second. In the meantime, by the old theory, Ball No. 1 has reached a point C on the parabolic path with a gradual retardation of its velocity.

Since the diameter of the balls is 1.25 inches, the chord A-B (Figure 21) = 1.25 inches, or 0.1042 feet.

Distance from center of mill to center of ball = \( \frac{O-A}{2} = 1.5 - \frac{0.1042}{2} = 1.448 \) feet.
\[
\sin \frac{\theta}{2} = \frac{0.05203}{1.449} = 0.03587
\]

\[
\frac{\theta}{2} = 2.063^\circ
\]

\[
\theta = 4.126^\circ
\]

\[
\text{arc } AB = (1.449) \left(\frac{4.126}{57.30}\right) = 0.1043 \text{ feet.}
\]

Time required for ball to travel over the arc \( AB \) is

\[
\frac{0.1043}{4.065} = 0.02552 \text{ second.}
\]

Now let us find the position of Ball No. 1 on the parabolic path at the end of 0.02552 second, the time being measured from point \( B \).

Take point \( B \) as the intersection of the \( x \) and \( y \) axis. Then, from the formula of the parabola, at any time \( t \),

\[
x = (V \cos \alpha) \, t
\]

and

\[
y = Vt \sin \alpha - 0.5 gt^2
\]

where \( V \) is the initial velocity, \( \alpha \) the angle the initial velocity makes with the horizontal, and \( t \) the time.
Since the 36-inch mill is traveling at 60 per cent of its critical speed, \( a = 60.04 \) and \( t \), from above, is 0.2552 second. Then from the above equations:

\[
x = 0.03751 \text{ feet}
\]
\[
y = 0.0667 \text{ feet}
\]

Therefore \( x - y = 0.0292 \text{ feet} \).

But \( x - y \) must be equal to or greater than \( 2R \), since \( 2R \) is the diameter of the ball, or 0.1042 feet. It is evident that the velocity of Ball No. 1 along the parabolic path must be the same as that of Ball No. 2 along the circular path, and will be constant until it reaches the highest point of the curve.

For a ball to follow a parabolic curve the horizontal component of the actual velocity must remain constant throughout the flight of the ball. This means that the velocity along the path must decrease from a maximum, at its starting point, to a minimum, equal to the horizontal component, at the highest point in the curve. The velocity may then increase on the downward flight.

In the ball mill the balls are being pushed by those behind until they reach the highest point in their path. This means that the horizontal component of the velocity along the path, instead of remaining constant, decreases from a
MINIMUM at the point B, to a MAXIMUM at the highest point in
the path equal to the initial velocity or the constant veloci-
ity along the curve. From this point the balls start on their
downward flight and will follow a parabolic path.

While it is evident from the foregoing, that the equa-
tion of the parabola describes the path CD, the path BC must
be determined by some equation other than that of the parabola.
The derivation of this equation is as follows:

6 Private communication, Prof. G. R. Dean, Head, Department of
Mathematics, Missouri School of Mines & Metallurgy, Rolla, Mo.
Derivation of the Dean Equation.

To find the equation of the path, in a vertical plane, followed by a particle whose velocity along the path is constant and whose direction of travel is affected by gravity, Figure 22.

Velocity along curve is constant = \( v \)

Mass of particle = \( m \)

Radius of Curvature = \( \rho \)

Normal acceleration = \( \frac{v^2}{\rho} \)

Force acting outward, normal to curve = \( \frac{mv^2}{\rho} \)

Force acting inward, normal to curve = \(- mg \cos \beta \)

These two forces must be equal, therefore

\[
\frac{mv^2}{\rho} = - mg \cos \beta
\]

or

\[
\rho \cos \beta = - \frac{v^2}{g}
\]  

(1)

The equation for the radius of curvature is

\[
\rho = \left[ 1 + \left( \frac{5y}{8x} \right)^2 \right]^{\frac{5}{2}} \frac{b^2 y}{8 x^3}
\]
Since
\[
\cos \beta = \frac{\delta x}{\delta s} = \frac{1}{\left[1 + \left(\frac{\delta x}{\delta s}\right)^2\right]^{\frac{1}{2}}}
\]

The last statement is true since
\[
\frac{\delta s}{\delta x} = \left[1 + \left(\frac{\delta x}{\delta s}\right)^2\right]^{\frac{1}{2}}
\]

Substituting these values in equation (1) we have:
\[
\left[\frac{\left[1 + \left(\frac{\delta x}{\delta s}\right)^2\right]^{\frac{1}{2}}}{\frac{\delta^2 y}{\delta x^2}}\right] \left[\frac{1}{1 + \left(\frac{\delta y}{\delta x}\right)^2}\right]^{\frac{1}{2}} = -\frac{v^2}{g}
\]

Simplifying
\[
1 + \left(\frac{\delta y}{\delta x}\right)^2 = -\frac{v^2}{g} \ldots \ldots \ldots \ldots (2)
\]

or
\[
\frac{\delta^2 y}{\delta x^2} = -\frac{g}{v^2} \ldots \ldots \ldots \ldots (3a)
\]
Integrating
\[ \tan^{-1} \frac{\delta y}{\delta x} = -\frac{\varepsilon x}{v^2} + C \]

In the above equation \( \frac{\varepsilon x}{v^2} \) must be expressed in degrees, so multiplying by 57.29 we have:

\[ \tan^{-1} \frac{\delta y}{\delta x} = -57.29 \frac{\varepsilon x}{v^2} + C \]

or

\[ \frac{\delta y}{\delta x} = \tan \left[ -57.29 \frac{\varepsilon x}{v^2} + C \right] \]

When \( x = 0 \), \( y = 0 \), and \( \frac{\delta y}{\delta x} = \tan \beta \)

Hence:
\[ C = \beta \]

Then equation (3) becomes:

\[ \frac{\delta y}{\delta x} = \tan \left[ \beta - 57.29 \frac{\varepsilon x}{v^2} \right] \]

or

\[ \delta y = \tan \left[ \beta - 57.29 \frac{\varepsilon x}{v^2} \right] \delta x \]

\[ y = \int \tan \left[ \beta - 57.29 \frac{\varepsilon x}{v^2} \right] \delta x \]

\[ = \frac{v^2}{\varepsilon} \log \cos \left( \beta - 57.29 \frac{\varepsilon x}{v^2} \right) + c' \]

When \( x = 0 \), \( y = 0 \), and \( c' = 0 \), then we have:
\[ y = \frac{v^2}{g} \left[ \log_e \cos(\beta - 57.29 \frac{g x}{v^2}) - \log_e \cos \beta \right] \] 

Equation (4) is for balls of infinitesimal size.

Changing equation (4) from the base "e" to the base "10", we have:

\[ y = 2.3025 \frac{v^2}{g} \left[ \log_{10} \cos \left( \beta - 57.29 \frac{g x}{v^2} \right) - \log_{10} \cos \beta \right] \] 

or

\[ y = 2.3025 \frac{v^2}{g} \left[ \log_{10} \left( \frac{\cos(\beta - 57.29 \frac{g x}{v^2})}{\cos \beta} \right) \right] \] 

The highest point in the curve is where \( \frac{\delta y}{\delta x} = 0 \).

Substituting in equation (5-a) we have:

\[ x = \frac{v^2 \beta}{g}, \text{ where } \beta \text{ is expressed in radians} \] 

Then

\[ y_{\text{max}} = 2.3025 \frac{v^2}{g} \log_{10} \cos \beta \]
Equation (5) is for balls infinitesimal in size, and will be referred to hereafter as the [EQUATION]. Although the equation for balls of finite size can be derived, it is too complicated for practical purposes.

The equation representing the points of intersection of the ball paths with the liner is too complicated for practical use, so the paths must be plotted on coordinate paper and these points located by measurement.
Application of New Theory.

In Figure 23 is shown the old orthodox parabolic paths versus the paths calculated by the new formula for 1.25-inch balls leaving the liner of a 56-inch mill at different speeds. The broken and the solid lines are the old and the new curves, respectively.

According to the new theory the 50 per cent curve intersects the liner a short distance below the horizontal diameter of the mill, and by the old theory, at a point well down the side of the mill. If one compares the new 60 per cent curve with the old 80 per cent curve, one sees that they intersect the liner very near the same point. This is of vital importance in milling, because the balls strike much higher on the liner of the mill than was formerly supposed.

The paths of the concentric layers of balls at various distances from the center of a mill are shown in Figure 24. The mill is 56 inches in diameter and rotates at 60 per cent of its critical speed, or 27.02 r.p.m. (The calculation of the respective paths has not taken into consideration the ball charge on either side of the layer considered).

The curve 0-0 passes through the ends of the circular paths. This curve also marks the beginning of the paths represented by the new Dean equation, already described. The curve 0-0 passes through the highest points in the paths and represents
Figure 24.
Paths of Concentric Layers of 1.25-inch balls at Various Distances From the Center of the 8-ft. Mill when Turning at 60% of its Critical Speed.
the end of the path described by the Dean equation. After the
balls pass these points there are no forces tending to counter-
act the force of gravity, so the balls will follow parabola
paths.

Assume one layer of 1.25-inch balls ascending with
the liner of the 36-inch mill, no slippage between the ball layer
and the liner, and the mill revolving at some speed below the
critical, say 60 per cent of the critical. After leaving the
liner the balls will follow a path represented according to the
new theory, see Figure 24. Now, if, the ball charge is in-
creased sufficiently to have a condition represented by Bells
1 to 10 inclusive,—all having line contact along the curve \( q-b \)—
these balls should occupy their respective positions \( 1', 2', 3', ...
\), along the curve \( q-c \), when they each the highest points \( 1 \)
in their respective paths. This is based on the assumption that
the path of each ball is traced separately. The shortest dis-
tances between the ball centers along the line \( q-c \) are as fol-
lows:

- 91 -
From No. 1' to 2' ........ 0.113 feet
2' to 3' ........ 0.110 feet
3' to 4' ........ 0.107 feet
4' to 5' ........ 0.105 feet
5' to 6' ........ 0.102 feet
6' to 7' ........ 0.099 feet
7' to 8' ........ 0.096 feet
8' to 9' ........ 0.094 feet
9' to 10' ....... 0.092 feet.

Since the diameter of the balls is 0.104 feet it will be seen that there is no crowding in the first five layers of balls between the curves O-Q and Q-Q. However, there is a gradual decrease in the distance between adjacent ball centers along the curve O-Q, as the distance from the center of the mill becomes less. Crowding begins with the sixth layer where the "shortest distance" is less than the diameter of the balls, and it increases with the number of layers, causing the actual path of the outside layer to move upward and further away from the theoretical curve. Similarly this crowding effect causes a distortion of the inside layers. This phenomenon becomes more and more marked as the number of layers increases, finally resulting in a kidney-shaped area in which there is no unconfined motion.
If a charge of 4.35-inch balls in the 36-inch mill be of such size that not more than five layers of balls are traveling in the circular paths, there will be no crowding, and the kidney-shaped area will be vacant, providing the outside layer is traveling with the velocity of the mill.

Further reference may now be made to Plate II where it will be seen that for various ball charges at 40 per cent of the critical speed, the outside layer of balls is even thrown out further than indicated by the new theoretical curve, and that this distance is the greatest with the largest ball charge. With the same charges, but with mill speed increased to 50 per cent of the critical the observed paths are not so far from the theoretical. At 50 per cent of the critical the observed paths are grouped closely, and the mean coincides very closely with the theoretical.

The great difference between the new and the old equation has been shown in Figure 25, and it has been shown in Plate II that the new equation is in accord with the observed paths.

Further confirmation may be obtained by referring back to Figure 21. The broken line represents the old parabolic path and the solid line the path described in accord with the new theory. Both curves are for a velocity equivalent to 50 per cent of the critical speed of the mill. It will be noticed that the outside layer of balls follow very closely the theoretical path described by the new theory. Hence a consideration of the new equation is advocated.
CONCLUSIONS

Conclusions arrived at by the author in his investigation of the parabolic path are:

(1) By observation of the balls in a 36-inch squirrel cage mill it was noted that they do not follow a parabolic path when they leave the liner, and that they are thrown much further than indicated by the old parabolic theory.

(2) As the ball charge increases the slippage decreases.

It is entirely eliminated in the 36-inch mill at the critical speed, when the ball charge reaches a value between 35 and 40 per cent of the mill volume.

(3) At the working range a toe is formed by the balls in the bottom of the mill preventing the balls from striking directly against the liner.

(4) This toe permits driving the 36-inch mill at approximately 75 per cent of its critical speed without the balls striking directly against the liner. The ball charge being between 35 and 40 per cent of the mill volume.

(5) As the ball charge is increased from a minimum, crowding increases, causing the paths of the free falling balls to reach out a greater distance from the charge.

(6) Sketches of the paths of the balls show that the part of the path between the highest point in the path and the point where the balls start forming the toe is parabolic in nature.
(7) The balls do not act independently of each other until they have passed the highest points in their respective paths.

(8) The balls travel with a constant velocity throughout their upward flight.

(9) It is shown, mathematically, that the balls cannot slow up after leaving their circular path.

(10) The Dean equation describes the behavior of the balls from the time they leave the liner until they reach the highest point in the path. This description is in accord with actual observations.

(11) There is considerable discrepancy, as shown in Figure 23, between the old orthodox parabolic paths and the paths calculated by the new formula. In all cases the paths calculated by the new theory are far removed from their corresponding parabolic paths.

(12) With the 56-inch mill revolving at 60 per cent of its critical speed it was observed that the paths, representing various ball charges from 30 to 50 per cent of the mill volume, were grouped very closely about the theoretical 60 per cent curve.