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Maximum present value calculations with digital computers

James Boyd Robison

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ABSTRACT

This paper discusses maximum present value calculations for a mineral property using digital computers. A semi­quantitative mathematical model to optimize operating conditions is considered. The model is based on a modified cash flow analysis. Many variables affect the solution, and their effect upon present value is described. Selected data (mining rate, costs, reserves, metal sales price, and others) for a hypothetical open pit copper mine are used in a set of solutions for present value. The results are discussed in detail. It is concluded that: (1) Digital computer methods are rapid and useful, (2) Changes in the mining rate, average ore grade, metal sales price and others do affect present value, often severely, (3) Compared to a standard, a higher percentage rate of return may be associated with a lower present value, (4) Time dependent changes in costs may severely affect the valuation model, and (5) More work seems warranted for this type of analysis.
INTRODUCTION

Statement of the Problem

The main concern of this study is a discussion and results of some semi-quantitative mathematical models of mineral property valuations and the type and kind of mine financial analysis problems. The discussion is oriented towards present value calculations, applying digital computer methods to the analyses.

The two parts of Section II of this paper review the available literature on mineral property valuation methods (including digital computer methods) and operational variables. Section III establishes the assumptions for the study, and introduces the computer programming method. Section IV, using selected data, illustrates the results of the method and discusses the results. Characteristics of the results and areas which need further study are also discussed. Possible methods for future study are suggested. Section V, Conclusions, mentions the major conclusions of the investigation. Actual data and the computer program are included in the Appendix.

Importance of the Problem

The objective of a mineral property valuation is to obtain a true representation of the geologic character and economic worth of a mineral property. Any valuation scheme which does not give a true representation is undesirable. It must be noted, however, that there is presently no truly representative mineral property valuation method.
Many writers have discussed economic and geologic valuations, and there is general agreement that the methods are not perfect. These imperfections have been excused, justifiably, because many complex and inter-related variables affect the valuations. Methods for considering these variables have not been available. No attempt is made in this study to quantify geological models of mineral deposits.

Many ore deposits of the past, even when mined at less than optimum conditions of profitability, have produced very satisfactory profits. Such mines did so under rule-of-thumb operating standards which were easily established. With costs and expenses now consuming a larger share of total income, profits for these same properties are decreasing. As a result of these factors the control of mining rates, cut-off grades, and costs needs much more critical consideration.

A general rule of the past might have been stated: "Mine as much as you can, as fast as possible, at the lowest possible cut-off and sell at the highest possible price". Yet, while such guidelines may have increased the annual profits, they did not necessarily increase the total profits. Since increased annual profits correlate with disproportionate increases in taxes (up to a maximum of 50 per cent of total profits), total profit could have been lowered over the mine life.

To illustrate the complex character of a valuation, ten different mining rates per day, two possible total number of
working days per year, five possible average mining grades, three percentage recovery rates, three different selling prices, and two depreciation rates suggest a total of 1800 different operational conditions, if each combination is examined. Even with this large number, many other variables and possibilities exist for the named factors.

Amortization tonnage figures may be derived mathematically to determine present value for a known reserve and mining rate. By comparing several mining rates and grades a maximum set of profit making conditions can be established. Even when the best choice is made under these conditions, the mine may still operate at less than optimum profitability.

The examination of all controllable and non-controllable variables is generally not undertaken. Some variables may appear to have little effect on total profitability, but an examination of all information may reveal a critical dependency upon selected variables.

Valuation calculations take time, and often time is not available for a careful consideration of all variables. Digital computers have made it possible to evaluate many mining cost variables and arrive at a set of optimum operating conditions. A decided increase in the speed of solving the problems, and often an increase in the accuracy of the solutions is possible.

The most nearly correct solution of any problem is dependent upon the accuracy of the original information and the correct statement of the problem. Many have recognized
that accurate quantitative descriptions of mineral property data and operational goals are not available. This paper notes in detail why quantitative descriptions are often difficult to obtain.

Reasons for Selection of the Problem

The author has been interested in mineral property valuation problems for several years, especially with regards to digital computer solution methods. He has been encouraged in this interest by various members of the academic staff at the University of Missouri at Rolla, where he completed his undergraduate work. A logical thesis problem area for graduate work was thus valuation problems.

The facilities of the Computer Science Center of the University of Missouri at Rolla make it possible to use digital computer studies in the solution of various engineering problems. The availability of the Center facilities was another deciding factor in the selection of the problem.

Because the writer has done some work in open pit copper mines, he selected such a mine to consider present value estimations of profitability. While the method arrived at in this study is semi-quantitative, it is adaptable to quantitative solutions based on exact mathematical models. Such exact models were not available for this study.
Definitions

Several definitions of terms used are required to permit the reader to clearly understand the discussion. These include: ore, income, costs, expenses, standard costs, and cash flow.

"Ore is a natural aggregation of minerals from which a metal or metallic compounds can be recovered with profit on a large scale" (36). More recently, the term ore has also been considered to include some non-metallic minerals.

Income is the total amount of cash, or other assets, received for a valuable mineral product, or a service.

Costs and expenses are distinguished. Costs, or direct expenses, depend on labor and materials usage, and the total amount of material mined and processed. Costs, which are not dependent upon total income, include mining, comminution, beneficiation, smelting and refining, and others.

Indirect expenses, here called expenses, are affected not only by the amount of material mined and processed, but also by the excess of income over costs. Typical expenses include depreciation, depletion, and taxes.

Standard costs are expected costs under normal operating conditions. Changes from normal have a measurable effect.

Cash flow refers to the time dependent return of profits and capital.
Acknowledgements

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Dr. Billy Gillette, of the Computer Science Center of the University of Missouri at Rolla, assisted in solving some of the technical problems encountered in preparing the digital computer programs. The assistance of other members of the Computer Science Center staff is also noted.
II. LITERATURE REVIEW

Many articles have been written recently describing mine valuation methods through use of digital computers. These articles are reviewed in this section. In addition, many of the factors involved in mineral deposit valuation are considered, together with comments from authors who have investigated them and their effect.

Methods for Mineral Property Valuation

Mineral property valuations may be made during the exploration stage, the development stage, or the exploitation stage. Mathematical models and valuation methods may be considered for each.

During the exploration stage it is desirable to determine if an amortization tonnage is available. Sainsbury (38) and others (30, p. 449) have investigated this problem.

If an amortization tonnage is known to exist the mineral property may enter the development stage. During this period the mineral property is prepared for exploitation. A mining and processing plant must be built, and stripping of waste rock begins. Such development may be undertaken even though a complete knowledge of the mineral reserves may not be available.

The exploitation phase of a mine is a successful culmination of the exploration and development stages. From this phase will come the pay out of capital investments and profits for the venture.
MATHEMATICAL MODELS

Recently considerable effort has been devoted to mathematically modeling mineral deposits and describing distribution of valuable minerals by grade and total amount in them. New digital computer programs permit calculation of the total tonnages of mineralized rock and even subdivide these by grade and proposed mining level \(4, 5, 9, 13, 18, 19, 29, 40, 42\). The United States Bureau of Mines has been notably active in the statistical study of mineral deposits. Methods are now being developed from exploration data to simulate statistical models of mineral deposits \(20, 21, 22\). The purpose of each of these systems is to develop a more accurate mine valuation and plan.

When a mine is in the exploitation stage the economic goal is maximum profitability. Restrictions imposed upon the mine plan often limit operational processes. Certain stripping has taken place, and processing and mining units of a fixed size exist. Increased profitability can be created by: (1) lowering costs of mining and processing, (2) changing the average grade of mined and processed ore, (3) increasing or sometimes decreasing processing rates, (4) changing total reserves of ore, or others, so long as previous capital investments can be completely recovered. While the effect of one change may be commonly closely predicted, the net effect of several changes greatly increases the difficulty of accurate predictions. Digital computer evaluation systems have been suggested (and evaluated) by
many authors (4,8,11,14,15,16,17,20,37), although the systems do not generally completely describe all variables. An article by Hewlett (20) probably comes closest to an accurate listing of all variables, their effects and methods of evaluating the variables.

VALUATION METHODS

Many excellent discussions of mineral property valuation methods are available (2,6,30,32,33,35-a,39,41). The complexity of the various methods varies.

H. M. Callaway (6) derives a series of "basic break-even formulas" for exploration amortization decisions, and considers net-worth to be a deciding criterion for valuation. Evan Just (6) in a reply to the article agrees that for most purposes complex valuation methods are declining and "such points as interest to stockholders . . . are rather academic". Just does note that for mines such as the porphyry coppers the complexity may be necessary. It is important to note that these discussions were in 1958, before the wide-spread use of high-speed digital computers.

E. L. Vickers (41) feels that a more complex valuation system is necessary for operating companies. He proposes a system of "marginal analysis" to determine cut-off grade. Here the governing criterion is maximum total profits, although present value is mentioned as another possibility.

Soderberg (39) and others (5,11,35-a) have discussed open pit mine planning, generally using maximum present value as the optimizing criterion. These authors agree that the
major variables affecting mineral property planning include: total tonnage of ore at various cut-off and average grades, stripping ratios, mining and processing costs, and metal sales value. The authors also agree (35-c, p. 396) that depreciation, depletion, and taxation, while important, should not be the deciding variables which affect profit optimizing solutions. Soderberg, for example, believes that costs, not market prices, are the basic controllable variables in open pit mine design. Market prices eventually affect depletion and taxation, and Soderberg is in effect saying depletion and taxation are not controllable factors.

Variables of Mineral Property Valuation

Many variables affect valuation optimizing solutions. The following paragraphs discuss some of these variables and their effect upon valuation methods.

ACCOUNTING BASIS

Any financial analysis must be based upon some consistent and realistic accounting standard. Generally, in mineral property studies, an expenditure or value per ton of ore is used as the standard (35-c, p. 401).

In practice, total expenditures are allocated on the basis of total tonnage of ore mined, and these per ton figures are used for comparison or analysis. The per ton figures must not only mathematically represent all costs, expenses, and profits, they must also realistically represent costs, expenses, and profits.
ACCOUNTING SYSTEM

The accounting system used within a particular company will dictate the form of cost, expense, and profit per ton figures. Since a financial analysis is no more accurate than the data used in the analysis, it is apparent that the accounting system may affect the form of profit optimizing solutions (35-c, p. 395). With some accounting systems the necessary data, i.e., standard costs, may not be available.

Companies commonly carry two or more sets of books representing the financial status of the company (35-c, p. 395). One of these books will involve depreciation, depletion, and taxation transactions according to Internal Revenue Service regulations. It represents the true financial history of the company. Occasionally, two sets of depletion books are used. One book carries depletion at cost and shows the book value of all assets. The other books carries depletion either at cost, or on the basis of net or gross income percentages, and represents the depletion figure used for taxation calculations. It is imperative that cost model data be based upon the latter. The actual depreciation, depletion, and taxation can severely affect the life and ultimate profitability of any mineral property.

COST-VOLUME RELATIONSHIPS

Cost-volume relationships have been considered by several writers (23, p. 322, 41). In mining, semi-variable relations are noted, i.e., in addition to fixed costs there are also variable costs.
A typical semi-variable cost-volume curve, on a total production basis, is shown in A, Figure 1. When this curve is related to costs/ton, and tons/day (week, month, year), the new curve appears as in B, Figure 1.

![Cost-volume curves](image)

Figure 1. Typical cost-volume curves. (A), Semi-variable curve on a total production basis. (B), Semi-variable curve on the basis of tons per operating period.

There is at least one point on the "tons per operating period" curve where costs/ton are lowest. In mining operations this low point is commonly not coincident for mining, milling and concentrating, or refining rates. A selection of a mining rate based only on mining costs may not represent the least expensive rate of mining and processing.

The cost-volume curves usually consider the efficiency which may result from higher processing rates. The value of this efficiency must be balanced against increased expenditure of capital in the property and the necessarily higher depreciation and depletion recovery needs. Such information is not always given directly in the curves.

**INCOME**

No matter how efficient a mining operation may be, if no product is sold and there is no income of cash or other assets, that mine is a failure. Income is dependent upon
the total quantity of material sold, and the selling price of the material. The unit of sale may be tons or pounds of ore, concentrates, or refined metal; and the value or selling price may be dollars, or other fiscal units. On some occasions (35-c, p. 404), special sources of income must be considered. Their effect upon a mathematical model of income must also be considered.

Several factors complicate a straight-forward estimation of ore value. These include percentage recovery, inventories, selling price, total amount of material (reserves), and by-products in the ore.

Percentage Recovery

Percentage recovery, or some other recovery term (30, p. 463, 32, p. 120) is used to specify the amount of valuable material actually recovered, against the amount present in the ore. The two amounts are commonly not the same. If increased recovery can be effected, it is usually connected with increased costs of processing. The latter may exceed the value of the extra material recovered.

Inventories

An inventory time lag must be considered (23, p. 222-240). Because processing does take time (mine face to market time may exceed 6 months), any changes in the processing method or rate will not usually immediately affect income. Most valuation schemes assume that inventory stocks do not have any influence upon income.
Sales Price

The selling price of the material must also be considered. Most valuation methods assume constant selling prices. Yet, prices may be expected to fluctuate widely, and often wildly, especially over long periods of time (30, p. 465, 34, p. 257). The price may go either up or down.

A general rule of mining has been to cut production when prices are low, and raise production when prices are high. The market usually dictates these conditions (32, p. 139). The amount of reduction or increase is usually only a guess. While the control of marketing and prices is not considered here, it should be noted that any change in sales value usually dictates the need for a critical re-evaluation of the ultimate profitability of a mine.

Total Amount of Valuable Material

The amount of valuable material is ultimately dependent upon the total tonnage of ore (reserves), and the average grade or percentage of metal or mineral contained in the ore. Recovery rate, as noted, is another such factor.

A characteristic of most low-grade metallic mineral deposits is a rather disproportionate increase in total tons of ore as the average grade, or a cut-off grade, is lowered. Figure 2 illustrates a plot of average grade versus total tonnage of ore for a low-grade metallic mineral deposit.
Lasky (28, p. 85) points out that an average for the porphyry copper deposits gives the equation: \( G = 12.9 - 1.4 \log T \). \( G \) is the average grade of copper, and \( T \) represents the total tons of ore in the deposit of that grade. Total profits may also be dependent upon the average grade of ore which is actually mined and processed (7, 26, 27, 41).

**By-Products**

One or more valuable by-products may be associated with the major metal. The value of these by-products, if they are recovered, may critically affect the profitability of a mining operation. All of the factors which influence income for the major metal also affect by-product value.

**MINING EXPENDITURES**

A discussion of the costs and expenses of mining and processing is of primary importance for digital computer studies. Previous paragraphs have pointed out the need for recording expenditures on a per ton basis. The following discussion is related to this base.

It is possible to divide all expenditures into cost of labor and materials. For financial studies costs are classified into mining or processing categories.
Labor and Materials

Ultimately the cost of extracting or processing ore may be divided into expenditures for labor or materials. Some writers (35-c, p. 400) would list services as a third category.

Labor includes the wages paid directly to hourly workers and salaried employees, both directly and as some fringe benefits. Materials, in this definition, include all physical supplies: explosives, chemicals for processing, fuel, water, power, services, and others. Some writers suggest on-site transportation expenditures should be listed separately. These latter expenditures can be divided into labor and materials as defined here.

The cost-volume curves noted generally do not take into account the increased costs of labor overtime. This factor is especially important if more than standard labor hours are used (on a standard cost basis), since an increase in productive capacity for a fixed size of processing unit can usually be most easily effected by increasing the total number of processing days per week or year.

The relative proportion of labor and material expenditures varies (35-b, p. 200). In most open pit mines labor now accounts for about forty percent of all costs and materials the remaining sixty percent. It is important to note (34, p. 12) that average labor expenditures are not only increasing faster than material expenditures on a percentage basis, but are also assuming a higher proportion of total
expenditures. This suggests that a valuation method which assumes constant costs over the life of a mineral property is at best only approximating realistic conditions. Perhaps the assumption justifying this approximation is that hoped for future efficiencies or metal price increases will balance increased expenditures. Such an assumption may have no justification (35-b, p. 200).

Classification of Costs

Labor and materials may be used as the basis for a financial analysis. Most valuation methods utilize a different classification of costs. Total costs are subdivided into categories for exploration, development, mining, concentrating, smelting and refining, overhead, and others. These cost areas include both labor and materials. Each of these categories has a characteristic cost curve. As noted, the actual cost/ton and the low point on the various curves is not coincident for all categories.

Exploration and development costs might properly be listed as expenses in some studies. These two categories are listed as costs, rather than expenses, in this paper so the reader will maintain a continuity of thought in the total financial analysis method.

A brief discussion of the major cost categories and the variables affecting them follows.

Exploration. The cost of exploration depends upon the amount of geological, geochemical, geophysical, drilling, and other
physical explorations and assay work on the property. Governmental laws permit some of these costs to be written off profits before taxes. The remaining costs must either be capitalized in the same manner as expenses, or paid off as a direct cost during the year of origin. These laws limit the amount of write-off of exploration costs ($400,000), and the time in which it may be done to only four years (35-d, pp. 476-479).

The recovery of exploration costs during the early life of a mine may severely affect mine valuations. Extremely large direct write-offs of exploration costs during the early life of a mine, when most exploration occurs, may show a book profit lower than the investor will accept.

The problems of extending ore reserves, or finding new bodies of mineralized rock, are also a necessary part of exploration valuation models.

Development. Several factors influence development expenditures. These expenditures may total millions of dollars and may be categorized into: (a) mineral deposit preparation expenditures (primarily stripping for open pits), and (b) mining and processing plant capital expenditures.

Stripping costs may begin before exploitation and are often coincident with actual exploitation of the deposit. Some method of accounting for the time differential must be considered. If major developmental features are completed before exploitation of the deposit commences the related
expenditure may be capitalized and recovered by depletion allowances. Or, the expenditure may be treated as a deferred expense and recovered in a manner similar to depreciation.

Stripping expenditures are a function of the total amount of rock to be removed, slope stability, ore body geometry, initial stripping location, mineral distribution, and others.

The total amount of waste rock to be removed is a function of the slope stability of the rock at the pit site and the geometry of the ore body. The total amount of stripping is usually dependent upon the cut-off grade (39), which in turn is directly influenced by mine profits. Profits are a function of the mining rate, the costs of mining, and other variables.

Initial location of the stripping operation is important. Commonly mineral distribution decreases away from a pit center. Generally the higher grade rock would be mined and processed first to return the highest possible immediate income. Yet, it may not be desirable or practical to strip from the pit center outward. Thus, in some cases, lower average grade rock must be mined first. Finally, taxation on higher profits may also dictate the desirability of a lower initial profit, and influence the stripping location.

Development expenditures for processing plants are commonly completed before actual exploitation of the deposit starts. These capital expenditures have been discussed by
several writers (10,35-b).

The amount of capital necessary for mine and plant construction (concentrator, smelter, etc.) and the necessary working capital varies. The size of the ore body, average grade and mineral character, the annual processing rate, the expected mineral recovery, and other factors are important.

All capital expenditures influence later processing costs, although the method of accounting for the two factors is different.

The amount of capital necessary for mine site equipment is determined by the size, type, and amount of such necessary equipment. Capital expenditures for comminution equipment are primarily governed by rock strength and the desired grain size reduction of the ore. Concentrator capital expenditures are affected by the processing rate and the type of mineralization which in turn determines the type of recovery equipment. Finally, interest on capital may be a critical factor together with the availability of such capital.

**Mining Costs.** Mining costs are a function of: (a) the height of a bench and the stability of the bench slope, (b) the cost of maintenance (up to 35% or more of all mining costs), (c) cost of drilling and blasting, and (d) mine to mill transportation.

Maintenance, drilling, and blasting costs are a function of the strength and structure of the rock. The latter two
Mine to mill transportation costs (up to 50% or more of the total mining cost) are dependent upon the vertical and horizontal distribution of mineralization and the haulage distance. Mining and transportation of waste rock is also a function of mineral distribution, but the total cost is more directly determined by the total tonnage of waste rock. The latter is dependent upon cut-off grade. Costs and profits determine this variable.

A knowledge of mineral distribution is a function of drill hole assay data, which in turn may be a function of exploration costs.

**Comminution.** Comminution, or crushing and grinding, costs are dependent upon the toughness and abrasiveness of the rock type processed, and the maintenance costs which in turn are a function of total tonnage processed. A lower cut-off grade may be related to a mineralogically different rock. This could affect the comminution costs. These costs are also related to the size reduction necessary to recover the metal(s). Ore or waste size is often determined by the rock type and strength, explosives used in the pit, expected percentage recovery, and the cost of energy needed for size reduction.

**Concentration.** Concentration costs vary with reagent usage, availability and cost of water, energy consumption, total tonnage processed, and maintenance. They also vary with
percentage recovery. This depends upon mineralogy, sizing, interlocking mineral characteristics, reagent usage, and the type of concentrating equipment. By-product recovery is also related to these same factors. The nature of the mineralization is often a function of the grade of the ore, which influences the mine cut-off. Costs of disposing of tailings and costs of water recovery are primarily affected by total tonnage of rock processed, particle size, climate, cost of land, and pollution problems.

Smelting and Refining. Smelting and refining costs are affected by: energy costs (fuels, electricity, and oxygen), total energy use, and smelter fluxing agent costs. These are influenced by the nature of the concentrate and its grade. Previous mining and processing activity determine these. Energy consumption is affected by recovery rate, and the amount of water contained in the concentrates. Maintenance costs are also important.

Finished Product Transportation Costs. Costs related to transporting the finished product of the above processes are defined as finished product transportation costs. These do not include on-site moving expenditures which are included in mining, concentrating, and smelting costs. While it may be less expensive to ship concentrates rather than refined metal, taxes and royalty restrictions may dictate the need for on-site smelting and refining. The distance from markets or established processing units, the availability of processing
energy, or the availability of transportation may be critical factors in these costs.

Overhead Costs. Overhead costs are influenced by the physical size of the operation and particularly the number of personnel in the operation. Insurance expenditures, some fringe benefits, and interest on capital may be listed here. In some accounting systems these are assigned to other costs. The method of assigning these costs used in a financial model must correspond with the method followed in practice.

Cost Summary. All costs are closely related. An accurate cost model should describe these relationships. The model should ideally be set up using actual cost information as far as available. Obviously, in a new mineral property such information would exist only as estimates. Models for many of the variables noted have been described (1,12,25,31,43,44). Reference should be made to these writers for more detailed information.

Classification of Expenses

Certain of the expenditures in a mining operation might better be classified as indirect expenses, or expenses as used here. These include royalties, depreciation of fixed assets, depletion and taxes. Such expenses are often dependent on the difference between costs and income.

Some expenses, taxes in particular, need not be paid if there is no excess after costs are deducted from income. This
also implies, however, that there is no profit.

Royalties. Royalties (32, p. 261) may be paid on a per ton of ore basis or be pro-rated according to profits. Often they are paid on both bases or in other ways. Royalties adversely affect total profits, especially if unreasonable rates are imposed. Unfavorable rates may be imposed if material is not processed within a geographic area. The royalty may also be influenced by processing costs or methods, the grade of the ore, or unfavorable political climates.

Depreciation. The need for recovery of investments in fixed assets through depreciation is widely recognized (23, p. 194-212). Many methods of depreciation may be used, each with a characteristic cash flow pattern. Commonly, for valuations, a straight line or unit depreciation is used, even though actual accounting may use other methods. It is imperative that a mine financial model accurately reflect the depreciation method to be used, even though it may be complex. Present value calculations involve a comparison of time and cash flow patterns, and are influenced by the depreciation method.

Depletion. By federal legislative grace (35-d, p. 460-476) depletion allowances are available for the recovery of certain development expenditures. Such allowances are necessary to completely recover all investments in a mineral property, and to recover the value of a depleted asset.
Depletion allowances do not always accomplish this purpose. Depletable investments are commonly capitalized in a manner similar to depreciable assets. The actual depletion allowance selected may be either on this cost or unit basis, or on a percentage of gross income. If the percentage basis is higher the allowance is limited to a set percentage of net income. The accounting method utilized establishes the net income, and thus may at times affect allowable depletion.

The unit depletion allowance may only be credited if there is some book value to be depleted. The gross income percentage allowance may be claimed indefinitely. While only one depreciation method may generally be used over the life of an asset, either of the two depletion allowances may be used from year to year, with the maximum allowable to be the governing criterion. The possible depletion allowances, and the maximum allowable according to law, are listed in Table I.

Table I. Possible depletion allowances and allowable de­pletion according to law (35-d, p. 473). Net is net taxable income. Gross is gross income. Cost is a cost or unit basis on total reserves.

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It should be noted that under certain favorable circumstances it is possible to recover more than the normal cost of the depletable asset. This apparent creation of assets must be considered in the mine valuation model.

Taxes. One of the major expenses of any business is taxes; either local, state, or federal. Some taxes are on total tonnage of ore mined, others on material produced, and still others on profits. While no income tax (on profits) is paid if there is no excess of income over costs, other taxes must usually still be paid. All taxes seriously affect the life and ultimate profitability of a mine. Some taxes may be included in the previously mentioned overhead costs. A model representing the effect of each tax must be established to consider this possibility.

The various local, state, and federal tax agencies encourage the use of every legitimate tax advantage possible to minimize the tax actually paid. With present value solutions a more rapid return of cash, which in theory increases the present value, may have the reverse effect, due to severe tax burdens on higher profits. Under some conditions higher income may still accrue when higher taxes are paid. The net effect of many cash flow variables must be considered.

Ultimately, if all other factors remain constant, for a given mining rate the amount of taxation will directly influence the profitability and ultimate life of the mineral property, using present value calculations.
PROFITS

There is a difference between income and profits. Profits are the cash or other assets left after the costs and expenses of mining and processing have been deducted from total income. Profits are thus affected by depreciation, depletion, and taxation.

Recently Evan Just has discussed profits (24), noting that mining profits are generally lower than profits for other industries.

A more detailed description of profits is included below under valuation methods, profit goals, and cash flow considerations.

VALUATION METHODS

Many methods for estimating the value of a mineral property have been proposed (6, 20, 32, 33, 39, 41). The two most common methods involve a study of net profit or the present value of future profits.

Net profit is equal to the sum of all future annual profits. This criterion may be important for valuation studies but is not considered in this study.

For most mining ventures present value is the optimizing criterion (32). The methods used to calculate this present value are numerous and there is no general conclusion as to the best method. Some mining firms do not attempt to maximize present value. An integrated limestone quarry for a steel mill will not increase production above the mill requirements to increase present value if there is no need
for the increased production. Here the minimization of total costs may be the best valuation criterion. In addition, needs for mineral conservation or other considerations may affect the valuation method.

Present value calculations attempt to relate anticipated future profits, including depreciation and depletion allowances, to the present. The present value of the property is the difference between this amount and the amount of capital necessary to build and develop the mineral property. Usually, a straight-forward solution of present value is desired.

Present value is defined as the value in dollars, at the present time, of a series of future annual payments. Present value is affected by (a) time, and (b) the rate of interest applied to the annual payments.

Time is equal numerically to the total reserves of ore divided by the annual production rate of ore. Time is thus influenced by the mining rate per day, and the number of operating days per year.

A general assumption in most present value solutions is that the grade of ore is constant from year to year. However, a lowering of the average grade of ore mined increases the total tonnage or ore, and at a constant mining rate there is an increase in total mine life(39). Many operating mines do change the average grade of ore mined and processed, and the resulting change in mine life will affect the present value of all future earnings.

A change in the average grade of ore mined may increase
the total value of the mine using present value calculations. A decrease in value per ton coupled with an increase in total tons of ore may indicate a higher present value.

An increase in the annual production rate, which may be associated with a decrease in processing cost per ton, will also change the mine life. For the same, or changed reserves, the change in extraction rate will change the total present value. There are many other possibilities which might be considered.

A serious deficiency of all present value methods is the lack of influence given to profits which are returned after 15 to 20 years. Many open pit mines have a life in excess of 20 years. Maximizing solutions based only on present value methods may not illustrate the best profitability conditions.

Knowledge of a pay-off period is desirable, and the length of this period is often a restriction upon maximizing solutions.

Interest rates or return on capital also influence present value solutions. Various interest rates are possible and the different present value methods (Hoskold, Morkill, and others) are based on these rates. There are numerous objections to these methods.

A cash flow rate may be used (3,24). Net profits plus allowances for depreciation and depletion are divided by the corporate net worth, to give a cash flow rate of return.

Increased mine life and increased processing rates are
usually associated with a need for increased capital. The final interest rate and mine life may indicate an increase in present value. Often they do not. Under some conditions, a lower rate of return for a shorter period of time may indicate a higher present value, even though net profits may be less.

PROFIT GOALS

Profit goals are ultimately established by stockholders or others not immediately concerned with the optimum mining conditions. When metal prices increase, these people usually expect more profit. A present value formula may suggest a lowered production rate at higher prices for profit maximization. The stockholders still expect higher immediate profits. The holding of an established market position at a high production rate may be the over-riding criterion in establishing profit goals (24). Any present value maximizing technique which does not realistically consider these factors and others of a similar nature is not satisfactory.

CASH FLOW CONSIDERATIONS

The distribution of profits and the annual return of profits are important variables in present value solutions. In corporate systems part of the profit is usually disinvested each year as a return per unit of corporate stock. The remaining profit is reinvested in the business.

With a wasting asset some provision must be made for returning all capital invested to the investor. The various
present value formulas generally assume this is done. However, most large mining companies may be assumed to have a perpetual life even though individual properties have a finite life. Under these conditions most present value methods may not be applicable, and a different maximizing criterion must be considered.

Horizontal integration of mineral properties or the vertical integration of processing units may affect profit distribution and ultimate profit goals. The rate of return of profits from all corporate levels may be an important consideration. The total corporate profit system must then be studied to determine the best operational conditions for particular properties or units. The total profit may be at a maximum even though some of the properties appear to be operating at less than optimum conditions, if considered separately.
III. FINANCIAL ANALYSIS MODEL OF HYPOTHETICAL COMPANY AND MINE

In the earlier sections some of the variables which affect financial valuations of mineral properties have been discussed. Most studies establish restrictions either upon the range of the variables, or the nature of these variables. This section will discuss the restrictions and assumptions in this study, and will illustrate the mathematical form of the present value analysis model. All of the restrictions are applied to a hypothetical open pit copper mine synthesized by the author.

Restrictions and Assumptions

In a financial valuation it is necessary to consider more than expected or present income and cost figures. The mining company, type of mine, geometry and distribution of minerals in the ore body, and other factors should be evaluated. The following paragraphs outline the assumptions used to establish the hypothetical mine used for this study.

MINING COMPANY

The mining company considered has no geographic distribution of mining operations, i.e., no other mines, and there is no vertical integration of processing units beyond the concentrating stage. The company has no smelting and refining units, and all concentrates produced are sold to external smelting firms.
the latter life of the mine. This assumption, commonly used in mine financial valuations, is critical, especially when present value calculations are used. A decrease in the average grade of ore processed with time, under constant extraction rates, results in a variable flow of cash, at constant sales value.

FINAL PRODUCT

As previously noted, the company produces copper concentrates to be sold to external smelters. A smelter contract for expected income from the sale of concentrates is included in the Appendix. An important assumption is that the company can sell any amount of concentrate produced within a year. The quantity sold does not affect eventual selling prices for the refined metal. Another assumption is that all of the product processed within a fiscal year is sold during that year. There is no product inventory. Further, there is no interest charge on capital and working costs for products in the processing stage.

CAPITAL EXPENDITURES

The necessary capital expenditures for the mine can be accurately related to proposed mining rates, especially in terms of tons of ore mined and processed per day. A fixed percentage of capital expenditures represents investments in mining and processing units, and the remaining capital is used to develop the ore body for mining. The percentage for development remains constant no matter what stripping ratio
is used to expose and develop the ore body. In this study the thickness of overburden is assumed to be constant and independent of ore grade. Normally, the stripping ratio varies with the average grade of ore to be exposed and processed.

A major assumption is that all development work and processing plant construction takes place instantaneously. There is no interest cost for delayed capital expenditures which are made before commercial exploitation begins and income accrues to the mining company. Commonly this cost, and interest on unredeemed capital during exploitation, must be considered. There is no allowance for working capital during the life of the mineral property.

COSTS

The direct cost of mining and processing, on a per ton basis, can be accurately described. Such costs are assumed to remain constant for any mine life. In this study only the costs of mining, milling (comminution and concentration), and overhead are considered. The costs for these three factors, in terms of a processing rate, can be accurately read from a cost-volume curve. The data used in the study are summarized in the Appendix.

All costs are on the basis of a 300 day working year. This length working year will exist over any possible mine life.

Once exploitation of the property begins the cost of
waste removal is a direct cost and is not in any manner capitalized. The ratio of waste to ore remains constant at 1.5/l, and the cost of mining and transporting a ton of waste is the same as the cost of mining and transporting a ton of ore. Further, the tons per day of ore mined is numerically equal to the tons per day of ore processed.

DEPRECIATION AND DEPLETION

Depreciation and depletion allowances as established by Internal Revenue Service codes are used to recover the invested capital.

A straight line (here unit depreciation) allowance is used. The depreciable capital is redeemed completely by the end of the mine life, with no salvage value remaining. All capital expenditures are made only once, and they are independent of the mine life.

Either of the two possible depletion allowances (unit or percentage of gross income) may be used, subject to the restrictions previously noted.

Most present value calculations attempt to describe the financial history of a mine by considering conditions to be constant for each year of the life of the mine. If an allowance for depreciation and depletion is included in the present value formula, such as the Hoskold, this may logically be done. There are, however, objections to the method of considering depreciation and depletion in these formulas.
If depreciation is considered on an actual accounting basis, then unit depreciation may be used only as long as there is some depreciable asset value. After this time, no depreciation allowance may be used. When depreciation is assumed to be over the life of the mine, as here, conditions will be constant each year if the processing rate is also constant.

When depletion is considered on an accounting basis, the problem is more severe. Under some conditions percentage of gross may exceed the unit basis, and the excess depletion is considered to be a depletion overplus. The unit method may be used only as long as there is some depletable book value, after which time only the percentage method may be used. Thus, all of the book value of the depletable asset may be recovered before the property is completely mined out. Any percentage depletion now allowed exceeds a unit value of zero, and the entire figure is the overplus. Further, the basis for unit depletion changes from year to year, and the depletion overplus also changes from year to year.

The overplus may be considered as a profit. If the overplus is included in the return on the investment, as a profit logically should be, then the return after taxes, and the percent return may change from year to year. If the overplus is constant from year to year, then profits will remain uniform. But, as noted, the annual overplus does vary and some of the overplus recovered before the unit basis is zero will actually be a depletion return, and not
a profit. The total amount recovered for depletion will not change, but the time of recovery will vary. Time and the percent used in a present value calculation affect the final answer. For this reason, any present value method which uses actual federal depletion allowances based on a unit and percentage method must evaluate the present value of each year of the life of the mine.

The year by year calculation is tedious, and takes considerable time even on a digital computer. The accurate description of present value demands this type of analysis. As an approximation, this study assumed that the unit basis could be used for each year of the life of the mine, and a depletion overplus was calculated on this basis. For this reason some of the results in this paper are not realistic. The magnitude of the effect of this assumption will be discussed in Section IV.

ROYALTIES AND TAXES

No royalties are to be paid at this mine. All taxes other than federal income tax are included in overhead. The federal income tax is calculated on the basis of 22% of the first $25,000 of profits, with a surtax of 26% on all profits in excess of $25,000.

CONSTANT CONDITIONS

Direct costs, depreciation, depletion, and taxation are assumed to be constant over the life of the mine. The income from sale of concentrates, and the grade of the
concentrates are constant.

There is a uniform annual return of income for a given mining rate, average grade of ore, and sales value. Changes in costs, grade of ore, sales value, and other variables are assumed to be negligible over the mine life. These restrictions are placed upon most valuation schemes. Section IV of the paper will note the effect of some of these assumptions.

**SUMMARY**

Obviously the restrictions noted suggest an unrealistic mine. Certainly many of the restrictions can not be made for practical valuation problems. Many of these conditions do not remain constant over any appreciable length of time, nor is it likely that a complete knowledge of the nature of any ore body is possible.

Yet, these restrictions are commonly used. They may also be utilized as standard conditions, as the basis for future studies. Understand that the model based on the restrictions in this discussion is not reliable nor truly representative. It is possible, however, to modify parts of the model to represent actual problems or to use other data.

**Numerical Form of the Model**

After restrictions and assumptions are imposed on a valuation study, a mathematical model can be prepared. The mathematical form of the model used in this study is described in the following paragraphs.
The mathematical model is used to evaluate the present value of profits which will be realized if a known reserve of ore, of constant average grade, is processed at a constant rate. The product of the mine is sold at a constant price. Allowances for depreciation, depletion, and taxation are included. All costs, expenses, and profits are based on a per ton standard. All tonnages in the model are based on a 2000 pound net ton. Factors discussed include income, costs, expenses, profit and finally present present value.

INCOME

The only income received from the mine operation is from the sale of copper concentrates based on an established smelter contract. The equation for the value of one ton of ore (VT) is:

\[ VT = (V) (G) (R) (2000 \text{ pound/ton}) \]

where,

- \( VT \) is the value per ton of ore, expressed in dollars/ton,
- \( V \) is the value per pound of contained copper, in decimal dollars,
- \( G \) is the average grade of the ore, expressed as a decimal, and,
- \( R \) is the total percentage recovery figure for the mine, expressed as a decimal.

The value per pound of copper is based on the smelter return and not on the open market price of refined metal.

COSTS

Direct costs for mining, milling (including concentrating), and overhead are read from a table. Mining costs are first multiplied by 2.5 (waste/ore is 1.5/1) to account for waste and ore totals. The actual data are included in the
Appendix to this report. The expression for direct costs on a per ton basis is:

\[ \text{Costs} = 2.5(\text{Mining}) + \text{Milling} + \text{Overhead}. \]

**EXPENSES**

Some of the following expenses can be calculated only if there is some excess of income over costs. In this model a calculation is completed through the taxation stage, to note the total loss which would result if the mine were to operate. Expenses include depreciation, depletion, and taxes on income. No royalties are included in this model.

**Depreciation**

Total capital is a function of the mining rate. The depreciable capital is equal to a fixed per cent (here 60%) of total capital and all depreciable assets are purchased only once. Depreciation is on a unit basis. The model assumes that the total reserves of ore are known at the start of mining. Depreciation is over the exact life of the mine. The annual mining rate is constant, and straight line depreciation is used assuming no asset salvage value. Unit depreciation (DEP) is thus:

\[ \text{DEP} = (\text{Mining rate}) \left( \frac{\text{Capital expenditures function}}{\text{Total tons of reserves}} \right) \times (0.60) \]

**Depletion**

Depletion is calculated either on the basis of the unit or cost method, or on the percentage of gross method which is limited to 50% of net income. In this study gross income
includes all income and is the value per ton of ore, calculated on a smelter return basis. Net income includes deductions for all costs and depreciation from gross income. Depletion (DPL) is thus:

\( DPL = \frac{(\text{Mining rate})(\text{Capital expenditures function})}{\text{Total tons of reserves}} \times 0.40 \)

or

\( DPL = (0.15)(\text{Value per ton}), \% \text{ of gross income, or} \)

a limiting factor of \((0.50)\) (net income).

If the allowable depletion exceeds the unit basis, the excess is considered to be a depletion overplus.

**Taxes**

After calculation of the depletion factor, the net taxable income is:

\( \text{Value/ton - Costs - Depreciation - Depletion.} \)

This per ton figure is multiplied by the total tonnage mined during a year (300 days/year times tons/day) to give a total net taxable income. If there is no net taxable income there is no federal income tax. If the net taxable income is less than (or equal to) $25,000 the net tax is \((0.22)(\text{net taxable})\). If taxable income is over $25,000 the tax is:

\[ \text{TAX} = (0.22)(25,000) + (0.48)(\text{net taxable} - 25,000). \]

The actual tax is then divided by tons per year, to give a tax per ton. It is assumed in these calculations that there are no other taxes on income, all other taxes are included in the cost of overhead.
PROFITS

Profits are equal to the excess of income over all of the previously listed costs and expenses. If there is any depletion overplus it is added to profit to evaluate the return on the investment. The total profit, including the overplus, is equal to the per ton profit multiplied by the annual tonnage processed.

As noted previously, the assignment of the depletion overplus to profits may not always be justified. It must be emphasized that the method used in a particular model must correspond to the method used within an operating company.

PRESENT VALUE

The present value of all future profits is a function of the profits per year, the total number of years, and the rate of interest applied to anticipated profits. Total profit is discussed in the preceding paragraphs.

Time is numerically equal to the total tons of ore reserves divided by the number of tons of ore mined per year, assuming a constant extraction rate. It is assumed in these calculations that the time in years, even if it should be an uneven fraction, is used directly in the calculations. Parks (32, p. 188) points out that such an assumption will not withstand a rigorous mathematical analysis. If the life of the mine exceeds 10 years, as most of the large open pits do, the net error associated with the use of an inexact time
probably does not exceed 2%. Most of the input data is at best no more accurate than this, and the assumption appears to be justified. Later discussions note that when the mine life exceeds ten years the use of present value formulas as the only basis for valuation is probably not justified.

Interest figures, in present value calculations, can be of many forms. Here interest is considered to be the net rate of return of profits on invested capital, on a modified cash flow basis. It is assumed that capital investments will remain constant over the life of the mine, and capitalization is approximately a perpetuity. The depletion overplus is added to profits, as previously noted, and disinvested from the business. The equation for interest (I), where I is a decimal, is:

\[ I = \frac{\text{(Net profits, including overplus) \times (Total tons per year)}}{\text{Total capital investment}}. \]

This is not a true cash flow rate, because allowances for unit depreciation and unit depletion are not included in the total return.

Present value is calculated using the formula for the present value of an immediate annuity, at compound interest. The use of Hoskold type present value formulas is not considered in this study, and present value is calculated on the basis of only one interest rate. It is necessary to add the annual depreciation and depletion returns to the annual profits, to calculate the present value of all returns. The formula used in the calculations is noted in Parks (32, p. 172) and is:
Present Value = \( A \frac{(R^N - 1)}{(R^N)(r)} \)

\( A \) is the value of one year's return, including profits, depreciation, and depletion. The depletion overplus is not considered here, as it is included in the depletion return.

\( R \) is the amount of $1.00 at one year's interest, \( r \).

\( N \) is the number of years. As noted previously \( N \) is usually expressed in terms of even years or interest periods, and not as an inexact decimal as used here.

Finally, the present value figure used for comparison of different operational conditions is numerically equal to the above present value less the immediate value of all capital investment for the given mining rate.

**TYPICAL PRESENT VALUE SAMPLE PROBLEM**

A typical present value calculation is shown in Table II. This information is slightly modified from actual input data and the results. The table was prepared by hand calculations. Round-off of numbers gives a different answer than the actual computer program utilized.

**COMPUTER PROGRAM**

A digital computer program was prepared for this study to complete the calculations listed in the above paragraphs. The program is written in FORTRAN II and was processed on an IBM 1620 Model II digital computer with associated card read and processing equipment. On-line print out capacity of 240 lines per minute was available. Random access disc storage eliminated the need for concern about memory storage capacity.
Table II. SAMPLE PROBLEM, VALUATION METHOD

A. NET PROFIT

The conditions for this valuation are: mine and process 60,000 tons/day; 300 days/year; total capital, $81,999,600; 250 million tons of ore, average 0.80% copper; 90% total recovery of copper; value of $.25 per pound of contained copper on smelter return basis. Mining ore plus waste is 2.5/1.0, with costs for mining multiplied by this factor. All costs as noted in Appendix for this mining rate. All values on a per ton basis.

INCOME (.25) (.0080) (2000) (.90)  $3.600

COSTS
   Mining (2.5) (.449)  1.122
   Milling                   .663
   Overhead                  .745
   _____________________  2.530
                        1.070

DEPRECIATION
   (.60) (81,999,600) .196
   ________________  .874
   250,000,000

DEPLETION
   (.40) (81,999,600) .130
   ________________  .874
   250,000,000

   or, (.15) (3.600) .540
   or, (.50) (0.874) .437

   Use .437
   ________________  .437

INCOME TAX
   Taxable income, (.437) (300) (60,000)
   = $7,866,000
   Tax, $5500 + (.48) ($7,866,000 - $25,000)
   = $3,749,180

   Tax per ton  $3,749,180 = .208
   (300)(60,000)

   NET PROFIT PER TON*
   ________________  .229

   *This figure does not include the overplus.

This example of the valuation method utilizes data from the actual computer valuation program. Because of round-off differences, some figures may differ slightly from the actual computer solution.
Table II (concluded). SAMPLE PROBLEM, VALUATION METHOD

B. CALCULATION OF PRESENT VALUE

<table>
<thead>
<tr>
<th>Depletion overplus</th>
<th>(0.437 - 0.130)</th>
<th>$0.307</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net return</td>
<td>(0.229 + 0.307)</td>
<td>$0.536/ton</td>
</tr>
</tbody>
</table>
| Percent return     | \[
\frac{(0.536)(18,000,000)}{81,999,600} \]
|                    | 11.7%           |
| Time               | 250,000,000 tons| 18,000,000 tons/year | 13.89 years |
| Net annual return  | $(0.229 + 0.196 + 0.437) | $0.862 |
| Or,                | $(0.862)(18,000,000) | $15,516,000 |
| Present value      | \[
\frac{(15,516,000)((1 + 0.117)^{13.89} - 1)}{(1.117)^{13.89}(0.117)} \]
|                    | $104,112,360    |

| PRESENT VALUE       | $104,112,360 |
| LESS CAPITAL        | 81,999,600   |
| NET PRESENT VALUE   | $22,112,760  |
The program can evaluate up to 40 different processing rates, 10 metal sales values, and 10 grades of ore for a given present value solution comparison. All other input data are assumed to remain constant.

A modification of the program is available which evaluates changes in present value if basic input data is changed. A third program, more complex, calculates the effect of changing costs and income over a mine life.

A copy of the original program, and a legend, is included in the Appendix. More detailed information on data input and output, and flow charts for all programs, can be obtained from the Department of Geological Engineering or the Department of Mining Engineering of the University of Missouri at Rolla.

A typical solution for one mining rate, grade of ore, and sales value involves approximately 2.0 seconds of computer time on the computer utilized for this study. In the modified program which calculates the effect of cost and income changes with time, a solution for one year of the life of the mine also involves 2.0 seconds. However, for a mine with a life of 15 years the first program would involve only 2.0 seconds of computer time, while the second would require 30.0 seconds of computer time.

In either event, the calculation time is considerably less than the time for a typical hand calculation which takes up to 30 minutes for a given mining rate, ore grade, and sales value.
Machine round-off errors in the program are practically negligible. All per ton figures are carried to the nearest mil, and the machine carries all numbers to eight significant digits. The primary restriction upon accuracy is dependent upon the accuracy of the original input data.
IV. RESULTS OF THE STUDY

Digital computer results of the solution of the model described in Section III are presented below with an orientation towards an accurate description of present value for the hypothetical mineral property. Four major areas are considered: (1) Standard conditions for the valuation model, (2) Standard conditions and effect of changes in the mining rate, grade of ore, and metal sales value (sales prices), (3) Effect of changes in cost and sales values with time, and (4) Areas and methods for further study.

**Standard Conditions**

Certain standards are assumed in a financial valuation. Present value of the mineral property is a standard. Then changes in the mining rate, average ore grade, or others are made and a new present value is computed. Generally an increase in present value indicates the new conditions are more desirable and a decrease indicates the changes are not desirable.

Before these changes are made it is necessary to consider the effect of changes in standard conditions. These changes may be due to errors in the original assumptions or new information.

The standard conditions and standard present value are listed in Table III. Operational variables considered in this study include: reserves at a constant grade, capitalization, development expenditures, percentage recovery,
standard costs, the waste to ore ratio, tax rates, number of operating days per year, and present value interest rates. In all instances only one change is considered at a time. Mining rate, average ore grade, and metal sales value changes are discussed in the next part of this section.

Table III. Standard conditions and standard present value for hypothetical mine.

| Mining rate: 60,000 tons/day and 300 days/year. |
| Total capitalization: $81,999,600 (depreciation 60% and depletion 40% of total). |
| Total reserves: 250 million tons of 0.80% average grade. |
| Percentage recovery: 90% of all copper. |
| Sales value of copper: $0.25 per pound, ($3.60 per ton of ore). |
| Standard costs: Mining, $0.449; Milling, $0.663, and Overhead, $0.745 per ton of ore. |
| Waste to ore ratio: 1.5/1 |
| Tax rate: 22% base tax with 26% surtax rate. |
| Standard interest rate: 11.7%. |
| Present value: $21,969,890 |

Results of the study are given in Tables: IV, V, VI, VII, VIII, IX, X, XI, and XII. Each table has the same format as follows:

1) Standard conditions and present value.
2) Two changes for the standard and resultant new present values.
3) Normally influenced-those factors which normally change when a standard variable changes.
4) Model result changes—depreciation, depletion, depletion overplus, taxes, profits, present value interest rates, and mine life if these change from standard. The amount of change is not given.

5) Footnote explanation of necessary details. Note that if present value increases most of the results listed in (4) also increase.

All changes in the program are calculated on a per ton basis whenever possible. Depletion on a unit basis is assumed to remain constant for each year of the mine life.

Table IV. Effect of changes in standard ore reserves.

<table>
<thead>
<tr>
<th>Standard reserves:</th>
<th>250 million tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present value ........</td>
<td>$21,969,890</td>
</tr>
<tr>
<td>230 million tons, present value</td>
<td>$21,403,980</td>
</tr>
<tr>
<td>270 million tons, present value</td>
<td>$22,252,970</td>
</tr>
</tbody>
</table>

Normally influenced: basis for unit depreciation and depletion, total capital expenditures, and mine life.

Model result changes: depreciation, depletion, overplus, taxes, profit, interest rate, and mine life.

Table V.* Effect of changes in standard capitalization.

<table>
<thead>
<tr>
<th>Standard capitalization:</th>
<th>$81,999,600</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present value ........</td>
<td>$21,969,890</td>
</tr>
<tr>
<td>$75,000,000 capital, present value</td>
<td>$20,194,164</td>
</tr>
<tr>
<td>$87,000,000 capital, present value</td>
<td>$23,062,560</td>
</tr>
</tbody>
</table>

Normally influenced: basis for unit depreciation and depletion, recovery methods, mine life, and others.

Model result changes: depreciation, depletion, overplus, taxes, profit, and interest rate.

* Results of Table V are misleading. Logically a lower capitalization should, and does, give a lower present value. However, the interest rate is actually higher for the lower capitalization.
Table VI. Effect of changes in standard amount of development expenditures.

<table>
<thead>
<tr>
<th>Standard development percentage</th>
<th>40%</th>
<th>Present value</th>
<th>$21,969,890</th>
</tr>
</thead>
<tbody>
<tr>
<td>30% development, present value</td>
<td>$21,995,480</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50% development, present value</td>
<td>$21,940,580</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Normally influenced: amount of development waste stripping, total reserves, mining waste/ore ratio, and unit basis for depletion.

Model result changes: depreciation, depletion, overplus, taxes, profits, and rate of return.

Table VII*. Effect of changes in standard percentage recovery rate.

<table>
<thead>
<tr>
<th>Standard percentage recovery</th>
<th>90%</th>
<th>Present value</th>
<th>$21,969,890</th>
</tr>
</thead>
<tbody>
<tr>
<td>85% recovery, present value</td>
<td>$20,573,880</td>
<td></td>
<td></td>
</tr>
<tr>
<td>95% recovery, present value</td>
<td>$21,936,400</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Normally influenced: per ton value of ore, standard costs, and capitalization.

Model result changes: per ton ore value, depletion, overplus, taxes, profit, and interest rate.

* For a higher per ton value the present value should increase. This increase is noted between 85 and 90 percent recoveries. Present value then decreases for a higher interest rate at 95% recovery.
Table VIII. Effect of changes in standard operating costs.

Standard costs and present value summarized below:

<table>
<thead>
<tr>
<th>Mining</th>
<th>Milling</th>
<th>Overhead</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.449</td>
<td>$0.663</td>
<td>$0.745 (std.)</td>
<td>$21,969,890</td>
</tr>
<tr>
<td>$0.424</td>
<td>$0.633</td>
<td>$0.710</td>
<td>$22,061,170</td>
</tr>
<tr>
<td>$0.474</td>
<td>$0.693</td>
<td>$0.780</td>
<td>$21,308,210</td>
</tr>
</tbody>
</table>

Normally influenced: changes in recovery rate, waste to ore ratio, annual mining rate, and others.

Model result changes: depletion, overplus, taxes, profits, and rate of interest.

Table IX. Effect of changes in standard waste/ore ratio.

<table>
<thead>
<tr>
<th>Standard waste/ore ratio:</th>
<th>1.5/1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present value ........</td>
<td>$21,969,890</td>
</tr>
</tbody>
</table>

1/1 ratio, present value, $21,887,130
2/1 ratio, present value, $20,252,010

Normally influenced: ore reserves, standard costs, total capitalization, percentage recovery, and mine life.

Model result changes: depletion, overplus, taxes, profits, and interest rate.
Table X. Effect of changes in standard federal income tax rate.

<table>
<thead>
<tr>
<th>Standard tax rate,*</th>
<th>Present value, $21,969,890</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified tax rate,**</td>
<td>Present value, $21,899,440</td>
</tr>
</tbody>
</table>

Normally influenced: tax rates, final profit, and interest rate.

Model result changes: taxes, profits, and rate of return.

* Federal tax rate (1965): 22% base tax, 26% surtax above $25,000.
** Federal tax rate (prior 1965): 30% base tax, 22% surtax above $25,000.

Table XI*. Effect of changes in standard number of operating days per year.

<table>
<thead>
<tr>
<th>Standard number of operating days:</th>
<th>Present value ..........</th>
<th>$21,969,890</th>
</tr>
</thead>
<tbody>
<tr>
<td>275 days, present value, $22,165,070</td>
<td></td>
<td></td>
</tr>
<tr>
<td>325 days, present value, $21,775,710</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Normally influenced: standard costs, annual mining rate, total mine life, and annual return of depreciation and depletion.

Model result changes: interest rate and mine life.

* In Table XI the highest present value is noted for the lowest interest rate and longest mine life (mine life inversely proportional to annual mining rate).
Table XII*. Effect of changes in standard interest rate compared to two rate present value formulas.

<table>
<thead>
<tr>
<th>Standard interest rate:</th>
<th>11.7%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present value ..........</td>
<td>$21,969,890</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Two rate formulas:</th>
<th>present value,</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.7%, and 9%</td>
<td>$28,115,940</td>
</tr>
<tr>
<td>11.7%, and 6%</td>
<td>$36,930,130</td>
</tr>
</tbody>
</table>

Normally influenced: present value.

Model result changes: present value.

* Many present value calculations are made with a two rate formula. One interest rate is applied to the redemption of depletable capital. Here no allowance is made for depletion on a federal basis. Some writers have attempted to apply a two rate formula to models which also include federal depletion allowances.

In examples above standard interest rate in the two rate formulas (these are not Hoskold formulas) applies only to profits (including the overplus). An arbitrary rate of 6 or 9 percent is applied to the calculated annual allowances for depreciation and depletion (less the overplus).

Note the substantial change in the present value. The latter tends to increase for a decrease in the interest rate. This apparently anomalous result is discussed in the second part of this section.

The importance of the interest rate is often overlooked. An arbitrary interest rate in a present value calculation will allow the selection of optimum conditions for that interest rate. But optimum present value for all conditions may not be at the specified rate. In practice it seems more desirable to use an interest rate based on the accounting system for a particular company rather than an arbitrary 6%, or 9%, or other annual interest rate.
Operational Variable Changes

The effect of changes in present value is considered when the daily mining rate, average grade of ore, and metal sales value (price) change. The range of these variables is noted in Table XIII.

Table XIII. Summary of range of operational variable changes for model valuation.

| Daily mining rate: 35,000 to 75,000 tons per day at intervals of 2500 tons per day. |
| Average grade of ore: 0.85, 0.80, 0.75, and 0.70 percent copper (see Appendix for reserves). |
| Metal sales value: $0.23, 0.25, 0.27, and 0.29 per pound of copper. |

The seventeen different mining rates, four ore grades, and four metal sales values suggest a total of 272 different operational conditions. All of these were evaluated with a digital computer program. Results are summarized in Table XIV, and Figures 3, 4, 5, and 6. All figures were prepared using a digital computer on-line plotter (Calcomp Model 566). Negative present values (indicating a loss) were plotted as zero values.

Computed present value varies widely, from some negative values to a maximum of about 29 million dollars. Input costs are semi-variable and the low point on the cost curves varies from about 55,000 to 65,000 tons/day. A maximum present value is expected in this range. The expected result is noted in Figure 3. In some instances (see figures) the maximum present value correlates with a
<table>
<thead>
<tr>
<th>GRADE-%</th>
<th>VALUE-$/LB.</th>
<th>TONS/DAY</th>
<th>PERCENT RETURN</th>
<th>MINE LIFE, YEARS</th>
<th>PRESENT VALUE IN DOLLARS</th>
<th>MAXIMUM % RETURN</th>
<th>MINE LIFE, YEARS</th>
<th>TONS/DAY FOR MAX. RETURN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>0.23</td>
<td>55,000</td>
<td>6.6</td>
<td>9.09</td>
<td>13,314,206</td>
<td>6.6</td>
<td>9.09</td>
<td>55,000</td>
</tr>
<tr>
<td>0.85</td>
<td>0.25</td>
<td>60,000</td>
<td>11.3</td>
<td>8.33</td>
<td>17,816,205</td>
<td>11.7</td>
<td>9.09</td>
<td>55,000</td>
</tr>
<tr>
<td>0.85</td>
<td>0.27</td>
<td>65,000</td>
<td>15.2</td>
<td>7.69</td>
<td>20,367,900</td>
<td>16.5</td>
<td>9.52</td>
<td>52,500</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.85</td>
<td>0.29</td>
<td>72,500</td>
<td>16.8</td>
<td>6.89</td>
<td>22,009,260</td>
<td>20.5</td>
<td>9.09</td>
<td>55,000</td>
</tr>
<tr>
<td>0.80</td>
<td>0.23</td>
<td>60,000</td>
<td>6.8</td>
<td>13.88</td>
<td>19,214,420</td>
<td>7.0</td>
<td>14.49</td>
<td>57,500</td>
</tr>
<tr>
<td>0.80</td>
<td>0.25</td>
<td>67,500</td>
<td>9.9</td>
<td>12.34</td>
<td>23,025,930</td>
<td>11.8</td>
<td>14.49</td>
<td>57,500</td>
</tr>
<tr>
<td>0.80</td>
<td>0.27</td>
<td>75,000</td>
<td>11.4</td>
<td>11.11</td>
<td>25,296,980</td>
<td>16.6</td>
<td>14.49</td>
<td>57,500</td>
</tr>
<tr>
<td>0.80</td>
<td>0.29</td>
<td>75,000</td>
<td>16.3</td>
<td>11.11</td>
<td>26,485,660</td>
<td>20.3</td>
<td>15.15</td>
<td>55,000</td>
</tr>
</tbody>
</table>

Table XIV. Summary of present value calculations for variable grade of ore, value of copper, and mining rate. The present value is the maximum for each average grade and sales value. Maximum rate of return and related mine life, and tons/day are also listed. Mine life in total years.
<table>
<thead>
<tr>
<th>GRADE-%</th>
<th>VALUE-$/LB.</th>
<th>TONS/DAY</th>
<th>PERCENT RETURN</th>
<th>MINE LIFE, YEARS</th>
<th>PRESENT VALUE IN DOLLARS</th>
<th>MAXIMUM % RETURN</th>
<th>MINE LIFE, YEARS</th>
<th>TONS/DAY FOR MAX. RETURN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>0.23</td>
<td>62,500</td>
<td>5.5</td>
<td>21.33</td>
<td>22,252,710</td>
<td>5.8</td>
<td>23.18</td>
<td>57,500</td>
</tr>
<tr>
<td>0.75</td>
<td>0.25</td>
<td>70,000</td>
<td>7.8</td>
<td>19.04</td>
<td>25,863,570</td>
<td>10.3</td>
<td>23.18</td>
<td>57,500</td>
</tr>
<tr>
<td>0.75</td>
<td>0.27</td>
<td>75,000</td>
<td>10.2</td>
<td>17.77</td>
<td>27,826,360</td>
<td>14.8</td>
<td>23.18</td>
<td>57,500</td>
</tr>
<tr>
<td>0.75</td>
<td>0.29</td>
<td>75,000</td>
<td>14.7</td>
<td>17.77</td>
<td>26,408,820</td>
<td>18.8</td>
<td>23.18</td>
<td>57,500</td>
</tr>
<tr>
<td>0.70</td>
<td>0.23</td>
<td>62,500</td>
<td>3.3</td>
<td>32.00</td>
<td>22,168,120</td>
<td>3.6</td>
<td>34.78</td>
<td>57,500</td>
</tr>
<tr>
<td>0.70</td>
<td>0.25</td>
<td>70,000</td>
<td>5.5</td>
<td>28.57</td>
<td>26,718,470</td>
<td>7.8</td>
<td>34.78</td>
<td>57,500</td>
</tr>
<tr>
<td>0.70</td>
<td>0.27</td>
<td>75,000</td>
<td>7.7</td>
<td>26.66</td>
<td>28,409,680</td>
<td>12.0</td>
<td>34.78</td>
<td>57,500</td>
</tr>
<tr>
<td>0.70</td>
<td>0.29</td>
<td>75,000</td>
<td>11.9</td>
<td>26.66</td>
<td>25,130,940</td>
<td>16.1</td>
<td>34.78</td>
<td>57,500</td>
</tr>
</tbody>
</table>

Table XIV, (Concluded).
mining rate of up to 75,000 tons/day.

Figure 3 also illustrates that while there is a linear increase in metal sales value the increase in present value is not linear. In fact, for a processing rate of 40,000 tons/day a higher sales value ($0.29) is associated with a present value lower than that for the $0.27 per pound value. Net profits, however, are higher for the $0.29 sales value.

In Figures 4, 5, and 6, the results become more pronounced in each succeeding figure. Note also that the present value curve is concave downward in Figure 3. This curve straightens out and becomes concave upward in Figures 4, 5, and 6 for certain metal sales values and production rates.

These results suggest that the optimum processing conditions (indicated by a higher present value) may be noted when the metal sales value is lowered (at least under conditions of Figures 5 and 6). Common sense indicates that profits are lowered, as is the net annual return, when less income is received.

This apparently anomalous result may be explained by an inspection of the interest term (IT) used in the present value calculations:

\[
IT = \frac{(1+r)^N - 1}{(1+r)^N \cdot r}
\]

In this term, for the same \( N \), an increase in \( r \), the rate of return, is associated with a decrease in the magnitude of the term. In some instances the amount of decrease of
Figure 3. Present value versus the extraction rate, when average ore grade is 0.85%, and total reserves are 150 million tons. Sales values of copper of $0.23, 0.25, 0.27, and 0.29 per pound are illustrated.
Figure 4. Present value versus the extraction rate, when average ore grade is 0.80%, and total reserves are 250 million tons. Sales values of copper of $0.23, 0.25, 0.27, and 0.29 per pound are illustrated.
Figure 5. Present value versus the extraction rate, when average ore grade is 0.75%, and total reserves are 400 million tons. Sales values of copper of $0.23, 0.25, 0.27, and 0.29 per pound are illustrated.
Figure 6. Present value versus the extraction rate, when average ore grade is 0.70%, and total reserves are 600 million tons. Sales values of copper of $0.23, 0.25, 0.27, and 0.29 per pound are illustrated.
the interest term is sufficient to offset increased annual profits.

In the model the effect may be directly traced to the method of accounting for the depletion overplus. The overplus is used in the modified cash flow rate. For a constant mining rate an increase in the metal sales value almost directly increases the depletion overplus. Recall that capitalization for the same mining rate is constant and independent of mine life and ore reserves. In addition, capitalization is independent of ore grade and reserves. The basis for unit depreciation and depletion decreases as reserves increase. (Normally, as mine life increases the capitalization also increases.) Since the unit depreciation and depletion allowances are included in the present value calculation a decreasing unit basis tends to lower present value.

There appears to be an upper bound on present value (compare figures and the table). It is commonly agreed that the present value of profits returned after 15-20 years is essentially negligible. This observation is confirmed in the results.

Finally the production rate for a maximum rate of return (no present value) in consistently between 55,000 and 60,000 tons/day (Table XIV). In only a few instances is present value also at a maximum in this range. The desired rate of return may be a limiting bound on the operational variations.
It is important to note that present value calculations alone may not illustrate the best operational profit making conditions, i.e., lower present value at higher rates of return. Still, the use of present value as a valuation criterion is desirable and the results indicate a possible need for new or different present value formulas.

**Time Variable Changes**

Changes in mining costs and sales value with time for processing rates of 55,000, 60,000, and 65,000 tons/day are considered along with changes in average ore grade and processing rates with time. The latter are not evaluated.

Most valuation models assume no changes in operational variables with time. Yet, costs and selling prices may be expected to increase or decrease over the life of a mine. Average ore grade processed may also change annually and will usually decrease. Finally, many mines increase the annual processing rate from year to year.

The effect of any or all of these changes on present value may be severe. Calculation of such changes is usually not undertaken because the mine must be valued for each year of the mine life and such calculations are tedious. In addition, there is usually some uncertainty about the amount of annual change to be expected.

A brief discussion of the effects of changes in ore grade and mining rate is considered. Then time variable depletion accounting is discussed. The results of several
calculations are summarized. Finally, the effect of changes in costs and sales values with time and results for several rates of change are briefly described and analyzed.

CHANGES IN ORE GRADE AND MINING RATE

The relation between total reserves and average ore grade is non-linear. A change in the average grade processed results in a change in total reserves and mine life. Since depreciation and depletion on a unit basis depend on the total reserves these also change.

Any change in the annual mining rate will affect the mine life. Depreciation and depletion on a unit basis do not change. The annual return of these allowances does change and a year by year summary is required.

TIME VARIABLE DEPLETION CALCULATIONS

Although a unit depletion allowance may be used, most open pit mines use percentage depletion because the allowance is higher. A problem arises if a depletion overplus is calculated on the difference between unit and percentage depletion. Unit depletion may be calculated only if there is some depletable book value. If percentage depletion is higher than the unit value the book value reaches zero before the end of the mine life. The basis for unit depletion decreases each year, and reserves are also lowered annually. The depletion overplus thus varies from year to year, and hence varies the rate of return used in the present value calculation.
Two present value calculations follow. The first assumes unit depletion is on a uniform annual basis. The assumption was used in the first two parts of this section. The second calculation assumes that the depletion basis changes from year to year. Both are for standard conditions.

1) Uniform return of depletion, present value $21,969,890
2) Variable depletion return, present value $10,248,462

For both of these calculations the total return of profits, depreciation, and depletion is constant and equal. Because the depletion overplus does vary in the second calculation a different present value is noted. In this calculation the unit depletion allowance was zero after about five years. The interest rate changed annually for these years. After five years the interest rate remained essentially constant. The maximum difference in the interest rate for the two calculations was about 3 percent. The total present value varies by about 100 percent.

A further problem arises in the method of accounting for depletion. If the overplus for the first few years is immediately put in a depletion fund, as it might be since all unit depletion is recovered in this time, the overplus is zero. The interest rate is lowered and again the present value changes. This problem is not considered in the model.

For time variable changes the variable depletion return is utilized. It is assumed that the unit depletion varies from year to year and the percent return also varies. These considerations follow.
COSTS AND SALES VALUE CHANGES WITH TIME

The effect of increases in the cost of labor and materials and the metal sales value (selling price) is now considered. Decreases are not considered, although metal sales value especially may decrease with time. All of these calculations are based on the time variable depletion basis described above.

As a standard it is assumed that labor is 40% and materials 60% of all costs. The average cost of labor is assumed to increase at rates of either 3 or 5 percent a year. Materials and sales value are assumed to increase at average rates of either 1.5 or 2.5 percent a year.

Time dependent changes are calculated for labor and materials of 35 and 65 percent and 45 and 55 percent respectively (see Table XV). In the calculations it is assumed that labor assumes a higher proportion of total costs each year. The rate of increase is either 0.3 or 0.5 percent annually.

The present value for each year of the mine life is calculated and the sum of these values gives a net present value. The results of these calculations are summarized in Table XV. The observations below are based on these results.

For no annual change the present value is highest at a mining rate of 60,000 tons/day. This is also noted here when there are time variable changes.
<table>
<thead>
<tr>
<th>% LABOR</th>
<th>% MATERIALS</th>
<th>% ANNUAL CHANGE</th>
<th>% CHANGE - MATERIALS &amp; SALES VALUE</th>
<th>% CHANGE LABOR</th>
<th>NET PRESENT VALUE IN DOLLARS at daily mining rates of:</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>65</td>
<td>0.3</td>
<td>1.5</td>
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<td>2.5</td>
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<td>10,733,828</td>
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</tbody>
</table>

Table XV. Present value with time variable costs and sales value at three mining rates. All annual changes are positive. Percent annual change notes change in ratio of labor to materials. Annual change in materials equivalent to annual metal sales price increase. Average ore grade constant at 0.80%. Initial sales value constant at $0.25/pound of copper.
For the same labor-materials ratios present value tends to increase with annual changes in some instances and decreases in others.

For the same processing rate the present value may be higher than standard in some instances and lower in others. Other trends can be noted.

Obviously a valuation method which does not consider the possibility of time dependent changes is not desirable. The assumption that metal sales value increases will match increased costs is not satisfactory and may give misleading results.

Finally, the previously mentioned changes in total reserves or average ore grade, and annual processing rates may compound the problem of considering changing costs.

**Areas for Further Study and Methods of Study**

Certain areas of study appear desirable: (1) A more accurate knowledge of ore reserves, (2) Relationship between processing rates and standard data, (3) The possible use of different depreciation methods, (4) The effect of different valuation criteria, and (5) Time dependent changes in data.

Mathematical models for these areas will be complex. Many involve non-linear relationships. Iterative calculations such as the ones in this study are extremely tedious and methods of dynamic programming may be useful. Statistical simulation models may probably be utilized with success in some of the problems. Finally, nomograms might be useful.
V. CONCLUSIONS

Many writers have considered mineral property financial valuations. Accurate mineral property financial models are necessary if accurate valuations are to be made. These models must clearly define the optimizing goals for the valuation. Variables which affect the model (including the accounting system, statements of income and costs, and others) must be accurately described. Their effects upon the valuation method must be considered.

For any valuation model certain restrictions and assumptions must be accepted as a basis for study. The importance of changes in these assumptions can not be overlooked.

For the present value model described in this study the following conclusions apply. (1) A digital computer method provides a means for rapid evaluation of many sets of operating conditions. These conditions are usually not considered because of time limitations on other methods of evaluation. (2) Changes in so-called standard data do change the present value, although for many changes the effect is not severe. (3) When present value calculations are based only on changes in the daily mining rate, grade of ore, and metal sales price the present value may vary widely. Often a higher rate of return is associated with a lower present value. If present value is the only valuation criterion a false impression of the best
operational conditions may be noted. (4) The effect of time dependent changes in operational conditions on present value may be critical. In some instances an optimum processing rate when no changes are considered may not be the optimum rate when changes with time are considered.

(5) There is no completely descriptive valuation method presently available. Further work in these areas does seem to be desirable. In particular this work might include the derivation of a present value formula which realistically describes a mineral property when cash flow interest rates are utilized.
BIBLIOGRAPHY


   a.) Valuation of mineral property, Ch. 4, L. C. Raymond, pp. 131-162.
   b.) Cost of acquiring and operating mineral properties, Ch. 5, pt. 1, Paul M. Tyler, pp. 163-219.
   c.) Accounting for the extractive industries, Ch. 9, pt. 1, Maurice E. Peloubet, pp. 393-433.
   d.) Taxation of mineral properties, Ch. 10, Granville S. Borden, pp. 451-495.


APPENDICES
APPENDIX I
COMPUTER PROGRAM DATA

The data for the hypothetical mining company used in this paper is not the data for any operating mining company. All data are synthetic and were generated by the author. Some of the data are modified from information in Parsons (34) and the annual reports of various non-ferrous mining companies. The information available was adjusted to fit the restrictions imposed on the hypothetical mining company. The mine does not produce any by-products and costs are of necessity lowered to account for a lower income per ton of ore than normally would be expected.

Copper mineralization, reserves, smelter contract and calculation of metal value, costs, and capitalization are noted below.

**Copper Mineralization**

No copper oxides, or sulfides other than chalcopyrite and chalcocite occur in the ore body or the concentrate. The ratio of the two copper minerals in the ore and concentrates is constant. The percentage of each mineral is:

- Chalcopyrite $\text{CuFeS}_2$ 90% of total mineralization (copper).
- Chalcocite $\text{CuS}_2$ 10% of total mineralization (copper).

**Reserves**

The reserves of the hypothetical mine are shown below. The log-log plot of these reserves is approximately a straight line. For each average grade of ore a tonnage
APPENDIX I (Continued)

increment and cumulative tonnage figure are given.

<table>
<thead>
<tr>
<th>Average Grade</th>
<th>Tonnage Increment</th>
<th>Cumulative Tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.05 %</td>
<td>10 Million</td>
<td>10 Million</td>
</tr>
<tr>
<td>1.00 %</td>
<td>15 &quot;</td>
<td>25 &quot;</td>
</tr>
<tr>
<td>0.95 %</td>
<td>22 &quot;</td>
<td>47 &quot;</td>
</tr>
<tr>
<td>0.90 %</td>
<td>38 &quot;</td>
<td>85 &quot;</td>
</tr>
<tr>
<td>0.85 %</td>
<td>65 &quot;</td>
<td>150 &quot;</td>
</tr>
<tr>
<td>0.80 %</td>
<td>100 &quot;</td>
<td>250 &quot;</td>
</tr>
<tr>
<td>0.75 %</td>
<td>150 &quot;</td>
<td>400 &quot;</td>
</tr>
<tr>
<td>0.70 %</td>
<td>200 &quot;</td>
<td>600 &quot;</td>
</tr>
</tbody>
</table>

Smelter Contract and Calculation of Metal Sales Value

Metal sales value for copper in the ore is calculated on the basis of a smelter contract for concentrates. This calculation is:

- **Chalcopyrite** 34.5% Cu \((34.5)(.90)\) = 31.05 units of Cu per 100 units concentrate
- **Chalcocite** 79.8% Cu \((79.8)(.10)\) = 7.98 units Cu total

One unit equals 20 pounds, so there are \((20)(39.03)\) or approximately 780 pounds of copper per ton of concentrate.

Assume New York market price is $0.26 per pound of copper.

Gross value of concentrate is \((780)(.26) = 202.80\). If the gross value exceeds $55.00, smelter base charge is $8.50.

Payment on the basis of 100% of wet assay less total of 15 pounds of copper per ton of concentrate, at New York price less $0.03 per pound of copper.

Payment is:

\[
(780 - 15)(0.26 - 0.03) - 8.50 = 167.45
\]

Value of copper contained in ore is thus: \(167.45/780 = .214\)

The New York market price is thus approximately $0.05 higher per pound of copper. Obviously, for the reserves noted in a preceding paragraph, the company will do its own smelting,
APPENDIX I (Continued)

but this information summarizes expected income from the sale of copper. The program evaluated a New York price range from $.28 to $.34 per pound of copper. As an approximation, the $0.05 differential between New York and smelter values was used. The sales value used in the calculations was summarized in increments of $0.02 per pound from $0.23 to $0.29 per pound of copper.

Costs

For this study only the costs of mining, milling (including concentrating), and overhead were considered. For each cost area several per ton values were plotted on a graph. A smooth curve was passed through the points. The data summarized below were read directly from these curves, to the nearest mil. These are synthetic data, and are adjusted to the hypothetical mine.

<table>
<thead>
<tr>
<th>MINING RATE</th>
<th>MINING</th>
<th>MILLING</th>
<th>OVERHEAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>35,000</td>
<td>.595</td>
<td>.795</td>
<td>.850</td>
</tr>
<tr>
<td>37,500</td>
<td>.562</td>
<td>.764</td>
<td>.833</td>
</tr>
<tr>
<td>40,000</td>
<td>.535</td>
<td>.737</td>
<td>.819</td>
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<tr>
<td>42,500</td>
<td>.514</td>
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<td>.500</td>
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</tr>
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<td>.745</td>
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<td>.744</td>
<td>.755</td>
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<tr>
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<td>.765</td>
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<td>72,500</td>
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<td>.775</td>
</tr>
<tr>
<td>75,000</td>
<td>.439</td>
<td>.875</td>
<td>.785</td>
</tr>
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</table>
APPENDIX I (Concluded)

Mining rate is in terms of tons per day, and all costs are expressed in dollars per ton of ore.

Capitalization

Total capitalization is assumed to be a direct function of the daily mining and processing rate. Important factors such as the total life of the mine, fixed asset replacement with time, plant expansion requirements, and salvage value are not considered.

The mine reserves have been discussed in a previous paragraph. For these reserves, mining rates were considered which would give a total mine life ranging from about 5 to 60 years when the mine operates 300 days per year. On this basis the mining rates varied from 35,000 to 75,000 tons/day. The costs associated with these rates have been discussed.

For a mining rate of 35,000 tons/day a total capitalization of 45 million dollars was selected. For the 75,000 ton/day rate a capitalization of 100 million dollars was selected. These points were plotted and a straight line was drawn through them. The equation for this line, in terms of daily mining and processing rates, is the capitalization function noted in the computer program. This equation is:

\[ \text{Total Capital} = (\text{Daily rate})(1266.66) + 6,000,000 \]

The result is expressed in dollars.
C***48398GEX004 JAMES B ROBISON 04/23/65 FORTRAN 2
C COSTING PROFIT, FOR PRESENT VALUE.
C GRADE, RESERVES, VALUE, TONNAGE RATE VARIABLE.
READ 20,N,MM,MMM
DIMENSION T(40),CMN(40),CML(40),COH(40),VP(40)
DIMENSION GRDE(10),RSRV(10),VALUE(10)
DO 1 L=1,MM
1 READ 22,GRDE(L),RSRV(L)
DO 2 K=1,N
2 READ 21,T(K),CMN(K),CML(K),COH(K)
DO 3 I=1,MMM
3 READ 29,VALUE(I)
DO 19 L=1,MM
DO 19 I=1,MMM
VT=VALUE(I)*GRDE(L)*2000.*0.9
PUNCH 27
PUNCH 23,VALUE(I),VT,GRDE(L),RSRV(L)
DO 18 J=1,N
TCAP=T(J)*1266.66+6000000.
CAP=0.60*TCAP
C SALVAGE=0.0, DEPRECIABLE CAPITAL = TOTAL CAPITAL*0.6
DEP=CAP/RSRV(L)
SUM=VT-2.5*CMN(J)-CML(J)-COH(J)-DEP
C DEPLETION CALCULATION, BASED ON COST, 15 PRCNT GROSS, 50 PRCNT NET
DPLA=0.4*TCAP/RSRV(L)
DPLB=0.15*VT
DPLC=0.50*SUM
C SELECTION OF PROPER DEPLETION ALLOWANCE
IF(DPLA-DPLB)31,32,32
32 DPL=DPLA
CAPRI=0.0
GO TO 17
31 IF(DPLA-DPLC)35,32,32
35 IF(DPLB-DPLC)37,38,38
38 DPL=DPLC
GO TO 16
37 DPL=DPLB
16 CAPRI=DPL-DPLA
17 CONTINUE

C DEPLETION FACTOR NOW SELECTED
SSUM=SUM-DPL
TT=T(J)*300.

IF(SSUM)11,11,12
11 PRCNT=SSUM*TT/TCAP
TAXT=0.0
PNO=SSUM-TAXT+CAPRI
GO TO 13

12 TAXI=SSUM*TT

IF(TAXI-25000.)8,8,9
8 TAXT=(TAXI*.22)/TT
GO TO 10

9 TAXT=(5500.+(.48*(TAXI-25000.)))/TT
10 PNO=SSUM-TAXT+CAPRI
PRCNT=PNO*TT/TCAP

13 TIME=RSRV(L)/TT
YPNO=(SSUM-TAXT+DEP+DPL)*TT
TF=(1.+PRCNT)**TIME

C VP(J)=(YPNO*(TF-1.))/(TF*PRCNT)-TCAP
C OUTPUT LISTS TONS/DAY, PRESENT VALUE, DEPREC, DEPLET, DPLOVRPLUS,
C TAX, NET PROFIT, ALL IN TERMS OF TONS/DAY. ALSO LISTS PER CENT

C RETURN ON INVESTMENT, AND LIFE OF MINE IN YEARS.

C SELECTION OF MAXIMUM AND MINIMUM PRESENT VALUE, LISTING TONS NO.

18 PUNCH 28,T(J),VP(J),DEP,DPL,CAPRI,TAXT,PNO,PRCNT,TIME

C VPMX=VP(1)
C VPMN=VP(1)
DO 43 M=1,N

41 IF(VPMX-VP(M))41,42,42
41 VPMX=VP(M)
JJ=M

42 IF(VPMN-VP(M))43,44,44
44 VPMN=VP(M)
KK=M

APPENDIX II. (Continued)
Plus, tax, profit, percent return, and time.
followed by present value, depreciation, depopulation, over-
per ton of ore, grade and reserves, mining rate per day
sales price. Results list value/pound of copper and value
reserves, mining rate and associated costs, and metal
Averages includes operational dimension average, average, and
data.

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<th>MAX PRES VALUE</th>
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<td>6.0000</td>
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APPENDIX III

Summary of Terms in Computer Program

The following list describes each of the variables in the computer program. All variables noted "dollars/ton" are expressed as decimals in the program.

GRDE - Grade of copper in the ore, expressed as a decimal.

RSRV - Total reserve of ore in tons for a given grade.

T - Mining and processing rate in tons/day of ore.

CMN - Cost of mining in dollars/ton of ore.

CML - Cost of milling (including concentrating) in dollars/ton of ore.

COH - Cost of overhead in dollars/ton of ore.

VALUE - Value, or sales price, of copper in dollars/pound.

VT - Value of ore in dollars/ton.

TCAP - Total capital investment, including stripping, in millions of dollars.

CAP - Capital in fixed depreciable assets, in millions of dollars.

DEP - Unit depreciation in dollars/ton of ore.

SUM - Net income before deduction for depletion, in dollars/ton of ore.

DPLA - Depletion per ton, unit basis, in dollars/ton of ore.

DPLB - Depletion per ton, % of gross income, in dollars/ton of ore.

DPLC - Depletion per ton, % of net income, in dollars/ton of ore.

DPL - Depletion factor selected, in dollars/ton of ore.

CAPRI - Capital reinvestment factor, or depletion overplus. This factor, in dollars/ton, may be added to profit as a depletion overplus, or reinvested in the business.

SSUM - Net taxable income, in dollars/ton of ore.
APPENDIX III (Concluded)

TT - Total tons per year of mined and processed ore.
TAXI - Net taxable income in dollars.
TAXT - Income tax in dollars/ton of ore.
PNO - Net operating profit in dollars per ton of ore.
CAPRI is included in PNO if applicable.
PRCNT- Rate of return on investment (TCAP), expressed as a decimal. Equivalent to percentage return of PNO.
TIME - Life of mine in total years.
YPNO - Yearly net profit and return on investment in total dollars. This term includes DEP and DPL. CAPRI is not included in the term, since it is included in
DPL.
TF - Time factor used in present value calculation, expressed as a decimal.
VP - Present value expressed in total dollars, after TCAP has been deducted.
VPMX - Maximum present value in total dollars.
VPMN - Minimum present value in total dollars.
VITA

James Boyd Robison was born September 30, 1941, in Sioux City, Iowa. He is the son of a Lutheran clergyman. His undergraduate training was at the Missouri School of Mines, where he received the Bachelor of Science degree in Mining Engineering in 1963.

Mr. Robison is a member of Tau Beta Pi, Phi Kappa Phi, Sigma Gamma Epsilon, and is an Associate Member of the Society of the Sigma Xi. He is a member of the American Institute of Mining Engineers.

Mr. Robison has worked for the Phelps Dodge Corporation and the Baroid Division of National Lead Company. He was a Graduate Assistant in the Department of Geological Engineering and Geology at the University of Missouri at Rolla while doing graduate work. After graduation Mr. Robison will move to Salt Lake City, Utah, where he has accepted a position with the Kennecott Copper Corporation.

He is married and has one child.