Efficiency and loss mechanism of the electro-fluid dynamic momentum transfer process applied to propulsion systems using atmospheric air as the working fluid

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EFFICIENCY AND LOSS MECHANISM OF
THE ELECTRO-FLUID DYNAMIC MOMENTUM
TRANSFER PROCESS APPLIED TO PROPULSION
SYSTEMS USING ATMOSPHERIC AIR AS THE WORKING FLUID

BY

ROGER COY CRITES, 1943-

A
THESIS
submitted to the faculty of
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ABSTRACT

The general equations governing EFD (electro-fluid dynamic) flow are established, simplified, and solved in closed form to provide a mathematical model of an ideal EFD propulsion system. This mathematical model, in conjunction with an analysis of ion-neutral and neutral-neutral molecular collision phenomena, indicates that the cause of inefficiency in the EFD propulsion system is the system entropy increase associated with the diffusion of ion momentum through the neutral fluid. This loss mechanism is shown to be analogous to resistive heating losses in a constant voltage linear electrical network and can be minimized by obtaining minimum ion mobility. It is shown that lifting forces of 10 lb_f per unit lifting area (for a 10 stage accelerator) may be achieved with an efficiency of 50% or greater by using microscopic water droplets as charge carriers. A hypothetical 20,000 lb_f gross weight flight vehicle is shown to obtain 50,000 lb_f of lift and be capable of attaining a 12,000 ft. ceiling, but requires a low weight 1.5 megawatt power supply. It is concluded that the development of suitable high voltage, light weight power supplies is necessary for the practical application of EFD propulsion to flight vehicles.
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LIST OF SYMBOLS

A - Cross-sectional area
a_i - Ion acceleration
B, B_1, B_2, B_3 - Constants of integration
C - Capacitance; constant
C_1, C_2 - Constants
D - Needle point diameter
E - Electric field
e - Elementary charge
F - Thrust; force
F() - Probability function
f - Frequency
F - Friction factor
f() - Probability density function
G - Volume flow rate
g_1, g_2, g_3 - Constants
H - Magnetic field
I - Current
J - Current density
K - Thermal conductivity
K_1, K_2 - Constants
k - Boltzmann's constant
L - EFD gap length
m - Molecular mass
m_i - Ion mass
LIST OF SYMBOLS CONT'D

\( \dot{m} \) - Mass flow rate

\( N \) - Number density

\( N_L \) - Loschmidt number

\( P_0 \) - EFD output power

\( P() \) - Probability

\( p \) - Pressure

\( p' \) - Hydrodynamic pressure loss

\( q \) - Net charge

\( q_i \) - Ion Charge

\( R \) - Radius; electrical resistance; specific gas constant

\( r, \theta, \psi \) - Spherical coordinates

\( S \) - Surface area

\( T \) - Absolute temperature

\( T_p \) - Pulse period

\( t \) - Time

\( t' \) - Momentum diffusion time

\( t' \) - Free time

\( \bar{t} \) - Mean free time

\( U \) - Potential energy

\( \vec{U} \) - Fluid velocity vector

\( u \) - Bulk fluid velocity

\( u_c \) - Net charge velocity

\( \bar{u}_i \) - Mean ion velocity
LIST OF SYMBOLS CONT'D

\( u_{ir} \) - Relative ion velocity
\( V \) - Electrical potential
\( V_f \) - Firing voltage
\( V_L \) - Collector potential
\( \Delta V_L \) - Voltage across EFD gap
\( \Delta V_L^* \) - Emission starting potential drop
\( V_o \) - Emitter potential
\( v \) - Thermal velocity
\( v_i \) - Ion velocity
\( v_{mp} \) - Most probable thermal velocity
\( v_r \) - Relative molecular velocity
\( (x,y,z) \) - Cartesian coordinates
\( Z_f \) - Fluid impedance
\( \beta \) - Ion mobility
\( \Gamma \) - Neutral-neutral to ion-neutral collision ratio
\( \epsilon \) - Electric permittivity
\( \epsilon^* \) - Maximum collision energy potential
\( \eta \) - Power efficiency
\( (\eta,\theta,\psi) \) - Prolate speroidal coordinates
\( \lambda \) - Free path
\( \bar{\lambda} \) - Mean free path
\( \lambda_e \) - Effective mean free path
\( \mu \) - Viscosity
LIST OF SYMBOLS CONT'D

$(\mu, \nu, \psi)$ - Parabolic coordinates

$\nu$ - Magnetic permeability

$\rho$ - Fluid density

$\rho_c$ - Charge density

$\rho_i$ - Ion mass density

$\sigma$ - Collision diameter

$\sigma_e$ - Effective collision diameter

$\Phi_e$ - Electromagnetic energy function

$\Phi_\mu$ - Viscous dissipation function

$\phi$ - Fluid parameter
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I. INTRODUCTION

Electro-Fluid Dynamics is that body of knowledge pertaining to the dynamic interaction of a fluid and an electric field. Of particular interest in Electro-Fluid Dynamics is the energy conversion phenomena, whereby energy in the form of an electrostatic field is transformed into kinetic energy of fluid flow. Also of some interest is the reverse interaction where the fluid kinetic energy is transformed into high voltage electrical energy. Conversion of electrical energy into fluid kinetic energy and vice-versa is accomplished on a molecular scale by non-random collision phenomena. As a result, devices which employ electro-fluid dynamic energy conversion may be mechanically simple, involving no moving parts, inherently inexpensive, and extremely rugged and reliable. An electro-fluid dynamic propulsion unit for atmospheric flight is an example of such a conversion device.

This kind of propulsion scheme would involve imparting unidirectional momentum to the surrounding air by direct interaction with an electric field, thereby obtaining thrust equal in magnitude to the net time rate of change of total imparted momentum. Envision an aircraft without wings which could hover, fly forward, backward, or sideways, powered by a propulsion system which produces lift and thrust
without noise or harmful combustion products. Such a vehicle would be electrically powered, possibly by a small advanced nuclear fission system, and would not require large expensive runways or sophisticated ground support equipment. The advantages of such a vehicle over conventional aircraft are obvious.

Interest in the possible application of Electro-Fluid Dynamics to atmospheric flight lead to initial research into the feasibility of a practical propulsion scheme in 1965 at the Ohio State University. This work was completed in fulfillment of a research requirement for the Bachelor of Aeronautical and Astronautical Engineering Degree. The objective of this effort was to determine empirically the feasibility of a practical propulsion scheme utilizing air at standard atmospheric conditions as the working fluid. The extent of the work included designing and fabricating a test facility to provide simulation of an EFD (Electro-Fluid Dynamic) propulsion system. Results indicated that EFD propulsion in air at atmospheric conditions was impractical. Specifically, the efficiency of the conversion process was about one percent; much too low to permit practical application. Other researchers have verified this conclusion, both analytically and experimentally as discussed in Section II.
Even though analytical models and experimental results reported in the literature verify that at normal atmospheric conditions efficiency is too low to permit practical application, no satisfactory explanation of the cause of this low efficiency has been offered. Curiosity concerning the nature of the loss mechanism responsible for this low efficiency provided the incentive to pursue this topic as the subject of a thesis research study.

The prime objective of this research effort was the description of the loss mechanisms limiting the efficiency of EFD propulsion in atmospheric flight. During this thesis research, a one-dimensional mathematical model of an ideal EFD propulsion unit was developed from basic conservation laws and solved in closed form. This provided a continuum flow model for the process. The continuum model is not descriptive of the molecular power loss mechanism, but is descriptive of the parametric dependence of this loss mechanism. It thereby provided a knowledge of which EFD parameters to examine on a molecular scale to obtain an understanding of the loss process. All available experimental data concerning EFD flow in atmospheric air was generated from test set-ups using needle point emitters as ion sources. The general
characteristics of needle point emission is discussed in Appendix A to help relate the theoretical results obtained to available experimental data.

Following a short discussion of prior research bearing upon the subject of EFD propulsion, the methods, results and conclusions of this thesis research are discussed in detail.
II. BACKGROUND

A. The EFD Propulsion Concept

The EFD propulsion concept is most easily explained in terms of the schematic in Figure 1.

A high voltage DC supply drives an ion emitter, and establishes an intense electrostatic field across an ion acceleration gap. The ion emitter may be a rake of needles producing positive or negative ions in air depending upon electrical polarity or it may be a more sophisticated mechanism. In any case, ions are produced at the emitter grid and accelerated by the intense electrostatic field toward the collector grid. As a result an ordered, unidirectional velocity component is superimposed upon the random thermal
velocity of each ion. In this way ions in the gap obtain a net unidirectional momentum in the direction of the electric field. As the accelerated ions make the trip from emitter to collector each ion suffers about $10^9$ collisions per second with neutral or uncharged air molecules. As a result, some of the unidirectional momentum obtained by the ions from the electric field is transferred by the collision process to the neutral air. When the ions reach the collector grid they are neutralized and the neutral accelerated air flows out through the collector providing thrust.

On a macroscopic scale this process must be governed by the continuum laws of fluid mechanics in conjunction with the laws of electrodynamics. A fairly general body of governing equations can readily be obtained, and even though the body of governing equations can not be easily solved, they provide a common starting place for any physical EFD mechanism that might be imagined. If, for instance, time variant operation, application of magnetic fields, or some other such scheme is entertained, the general governing equations provide a sure beginning from which applicable special cases may be generated. All of these analytical models must have in common that they are contained as special cases in this more general model.
B. The General Equations

The equations governing the electromagnetic behavior in any system under almost any conditions, as obtained from Reference 1, are:

Lorentz Force,
\[ \mathbf{F}_l = q\mathbf{E} + q(\mathbf{v}_c \times \mathbf{H}) \]  \hspace{1cm} (1)

Conservation of charge,
\[ \nabla \cdot \mathbf{J} + \frac{\partial \rho_c}{\partial t} = 0 \] \hspace{1cm} (2)

Gauss's Law,
\[ \nabla \cdot (\varepsilon \mathbf{E}) = \rho_c \] \hspace{1cm} (3)

Nonexistence of free magnetic poles,
\[ \nabla \cdot (\nu \mathbf{H}) = 0 \] \hspace{1cm} (4)

Maxwell's Equations
\[ \nabla \times \mathbf{H} = \frac{\partial}{\partial t} (\varepsilon \mathbf{E}) + \mathbf{J} \] \hspace{1cm} (5)

\[ \nabla \times \mathbf{E} = -\frac{\partial (\nu \mathbf{H})}{\partial t} \] \hspace{1cm} (6)

Generalized Ohm's Law,
\[ \mathbf{J} = f(\mathbf{E}, \mathbf{H}) \] \hspace{1cm} (7)

The equations governing the dynamic behavior of a fluid under fairly general conditions are

Conservation of mass,
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \] \hspace{1cm} (8)

Conservation of momentum,
\begin{equation}
\rho \frac{D\mathbf{U}}{Dt} = \mathbf{F}_B - \nabla p + \mu \nabla^2 \mathbf{U}
\end{equation}

Conservation of energy

\begin{equation}
\frac{D}{Dt} \left[ C_p T + \frac{1}{2} (\mathbf{U} \cdot \mathbf{U}) \right] = \Phi_e - \nabla \cdot (k \nabla T) - \Phi_\mu
\end{equation}

and when the fluid is a perfect gas, the

Equation of state,

\begin{equation}
p = \rho RT
\end{equation}

The fluid-dynamic and electromagnetic equations are coupled by the general body force \( F_B \) in the momentum equation, which is a function of the Lorentz force, and by the electromagnetic energy factor \( \Phi_e \) in the energy equation.

This set of governing equations serve as a convenient touchstone for any of the continuum analytical models proposed in the literature. However, before discussing related work from the literature, previous research in EFD propulsion by the author will be briefly summarized, (Reference 2).

C. Previous Research

Previous electro-fluid dynamics research by the author was completed in 1965 at Ohio State University. The extent of the work included design and construction of a test rig which simulated an EFD propulsion engine.
The basic configuration consisted of a needle array for an ion emitter, and a hand woven screen for an ion collector. The experimental variables were the number of needles, the geometrical needle array pattern on the emitter, and the gap length between the needle points and the collector grid. Subsequent data analysis indicated that the conversion of electrical power into propulsive power was accomplished with an efficiency of about 1%. Because of this very low efficiency, EFD propulsion using normal atmospheric air as a working fluid appeared impractical.

In an attempt to mathematically describe EFD propulsion performance, an extremely simplified theoretical analysis was attempted. Conservation of energy was invoked by considering the difference between the energy of collisionless ions arriving at the collector, and ions which have lost momentum in collisions with neutral gas molecules. It was assumed that in an equilibrium state each ion lost, during collision, all of the momentum gained from the electric field during its traverse of the mean free path $\lambda$. No loss mechanism was provided for and it was assumed that the mean flow velocity in the acceleration gap was negligible compared to the ion velocity. With no losses the energy difference between free ions in a vacuum and ions in
neutral gas is equal to the kinetic energy gained by
the neutral gas from the ions.

Assuming that the electric field in the gap is
countant and that no magnetic fields are present, the
magnitude of the Lorentz force per unit volume was
obtained from Equation 1 as

\[ F_1 = q_i N_i E \]  \hspace{1cm} (12)

The charge per ion, \( q_i \), combines with the ion number
density, \( N_i \), to yield the charge per unit volume.

The energy per unit volume gained by the fluid
was simply taken as the difference between \( F_1L \), the
energy per unit volume of ions accelerating freely
across the entire gap, and \( F_1\lambda \), the energy density of
ions which accelerate freely for the mean free path
and then lose all kinetic energy in a collision with
a neutral fluid molecule. It was assumed that \( \lambda \) is
negligible with respect to \( L \), and noted that the
relation between current and ion number density is

\[ I = q_i N_i u_i A \]  \hspace{1cm} (13)

where the ion velocity \( u_i \) is the product of the ion
mobility and the electric field, i.e.,

\[ u_i \approx u_{ir} = E \]  \hspace{1cm} (14)

These relations were combined to obtain the
fluid kinetic energy per unit volume \( W_k \), which is
$$W_k = \frac{IL}{A\beta} \quad (15)$$

It was noted that $W_k$ is actually $\int F_B \, dx$ where $F_B$ is the body force in the momentum equation, Equation 9.

Assumptions of one-dimensional inviscid, steady state, constant pressure flow were made so that Equation 9 in conjunction with Equation 15 yielded an expression which could be solved for the fluid velocity $U$. The result, after noting that $U = u$ in a one-dimensional system, is

$$u = (2IL/\rho \beta A)^{1/2} \quad (16)$$

For the conditions specified the conservation of mass (Equation 8) requires that the mass flow rate $\dot{m}$ be constant, i.e.,

$$\dot{m} = \rho u A = \text{constant} \quad (17)$$

This relation was combined with Equation 16 to yield a mass flux of

$$\dot{m} = (2IL/\rho A/\beta)^{1/2} \quad (18)$$

The thrust force generated is equal to the time rate of change of momentum which, for the assumed system, is $\dot{m}u$, or from Equation 16 and 18,

$$F = 2IL/\beta \quad (19)$$

Propulsive power ($\dot{m}u^2$) and efficiency were obtained as

$$P_o = (IL/\beta)^{3/2}(2/\rho A)^{1/2} \quad (20)$$

and

$$\eta = (1/\Delta V)(2IL/\rho A)^{1/2}(L/\beta)^{3/2} \quad (21)$$

where $\Delta V$ is the potential drop across the EFD gap.
These analytical results were found to agree qualitatively with the experimental data. No quantitative correlation was attempted. The conclusions drawn from this work may be summarized as follows.

1) The efficiency of the process is about 1%, which is too low to be competitive with conventional propulsion systems.

2) The process is fairly insensitive to needle density and array geometry.

3) The use of positive needles producing positive ions is more efficient due to the lower mobility of positive ions. However for gaps less than 0.5 inch, the negative needle negative ion process was more efficient. The reason is that ultra-violet radiation emitted near the needle points is more pronounced with positive needles, and with the collector grid close, photo-ionization on the negative collector produced sufficient quantity of negative ions and free electrons that recombination in the gap occurred.

No conclusions concerning the cause of inefficiency were reached, except for the observation that
the simple mathematical model obtained indicated that efficiency was inversely related to the ion mobility.

All analytical and/or semi-empirical efforts reported in the literature also reveal an inverse functional relation between efficiency and ion mobility, although the form of the function varies from author to author.

D. Pertinent Research Reported in the Literature

Interest in EFD flow phenomena has existed for quite some time. Arrhenius, circa 1897, carried out extensive experimental investigations of the forces generated by highly charged needles in various gases. The prime gases employed as working fluids were air, hydrogen, and carbonic acid. Arrhenius attributed the force on a needle to the electrostatic repulsion between the charged needle and the space charge of ions created by the needle. His published conclusions (Reference 3) were:

1) With a positively charged needle bleeding constant current into air, the needle force is proportional to the gas pressure.

2) For a positively charged needle bleeding constant current into various gases at constant pressure, the force varies as the square root of the molecular weight of the gas.
3) The needle force with an equal current for a negatively charged needle is less than for a positively charged needle, and the difference is inversely proportional to the pressure. The ratio of force on a negatively charged needle to that on a positively charged needle at the same current in air is 1:1.9 at 13.5 psi, 1:2.6 at 7.72 psi, 1:3.2 at 3.86 psi, 1:7 at 1.99 psi and 1:15 at 0.98 psi.

J. J. Thomson and G. P. Thomson, Reference 4, cite the work done by Arrhenius and attempt an analytical description of the thrust force experienced by a highly charged needle in a gas. The model used is a charged needle pointing in the x-direction, and emitting current by local ionization. It was reasoned that the net force on the ions in the x-direction would be

$$F_{\text{ions}} = \int \int \int E \rho_c \, dx \, dy \, dz$$

(22)

where $\rho_c$ is the charge density any place in the field $E$. This equation follows directly from the Lorentz force equation (Equation 1) for a system with no magnetic fields, and with the charge $q$ distributed in space.

By applying the definition of ion mobility,

$$\beta = \frac{u_i r}{E}$$

(23)
which was experimentally shown to be independent of the field strength, but dependent upon fluid density, and the definition of current

$$I = \int \int u_{ir} \rho d\gamma dz$$

(24)

to Equation 22, it was deduced that for constant current operation the thrust force acting on the needle is

$$F = (\text{const.}) \frac{I}{\beta} \int dx$$

(25)

From this analytical result Thomson and Thomson concluded that:

1) For constant current, $F$ varies inversely with $\beta$ and $\beta$ varies inversely with pressure, so that $F$ is directly proportional to pressure. This is in agreement with the data produced by Arrhenius.

2) It is probable that the large difference between force exerted for positive ions and negative ions observed by Arrhenius occurs because the negative charges may exist for significant periods of time as free electrons at lower pressures. Electron mobility is much higher than ion mobility so that the effective mobility $\beta$ in Equation 25 is abnormally large.

Following Thomson and Thomson, O. M. Stuetzer, in Reference 5, presents an analytical treatment of
static pressure generation in gases and liquids by the action of an electrostatic field upon a non-flowing fluid which contains unipolar ions. Stuetzer reasons that in a static system the electrostatic body force on the fluid must be exactly balanced by an increase in static fluid pressure. That is, for a one-dimensional system

\[ \frac{dp}{dx} = \rho_c E \]  (26)

This relation is easily shown to originate from a consideration of the conservation of momentum (Equation 9) and Lorentz force (Equation 1). When \( U = 0 \), as in a static system, Equation 9 reduces immediately to

\[ \vec{V}_p = \vec{F}_B \]  (27)

In this case, the body force \( \vec{F}_B \) is equal in magnitude to the Lorentz Force (Equation 1) per unit volume, and the gradient of pressure exists only in the x-direction. That is,

\[ |\vec{F}_B| = |\vec{F}_1| = \int \int \int dx dy dz = \rho_c E \]  \hspace{1cm} (28)

and

\[ |\vec{V}_p| = \frac{dp}{dx} \]  \hspace{1cm} (29)

Equations 27, 28, and 29 can be combined to obtain Equation 26.

Stuetzer relates local potential \( V \) to the charge density \( \rho_c \) by Poisson's equation,

\[ \nabla \cdot (\vec{V}) = -\frac{\rho_c}{E} \]  \hspace{1cm} (30)
This equation is obtained from Gauss's law (Equation 3) in conjunction with one of Maxwell's equations (Equation 6) and the fact that a potential gradient may be used to represent any vector whose curl is identically zero.

The current density was obtained as

\[ J \equiv I/A = \rho_c \beta E \]  

(31)

This equation follows for steady state operation which is a necessary condition for a true static fluid, from the conservation of charge law (Equation 2) and from the definition of ion mobility (Equation 23).

Equations 26, 30, and 31 served Stuetzer as a simplified governing set of equations for EFD pressure generation in a static fluid. After combining Equations 26 and 31, he integrated to get

\[ p(x) = \left( \frac{I}{2A\beta} \right) \]  

(32)

where the limits of integration are from \( p = 0 \) at \( x = 0 \) to \( p(x) \) at \( x \).

Combining Equations 30 and 31, integrating, and combining the result with Equation 32 an expression relating pressure to electric field strength was obtained,

\[ p(x) = \left( \frac{\epsilon}{2} \right) E^2(x) \]  

(33)

Again it has been assumed that \( p(0) = 0 \). Combining Equations 30 and 31, and integrating twice results in the voltage-current characteristic for the EFD gap as obtained by Stuetzer,
where $\Delta V_L$ is the potential drop across the EFD gap $L$, and $\Delta V^*$, which is introduced artificially, is the potential at which needle point emission starts. This result was combined with Equation 32 to obtain the pressure generated across an EFD gap of $L$ in a static fluid, i.e.,

$$p_L = (9/8) \varepsilon [\frac{(\Delta V_L - \Delta V^*)}{L}]^2 \quad (35)$$

Stuetzer also obtained experimental data using needle point discharge for an ion source and heavy insulating oils as a working fluid. The experimental results qualitatively verified the analytical model obtained.

The conclusions drawn are summarized below.

1) Pressures of about 3 psi (about $2(10^4)N/m^2$) can be generated in heavy insulating oils, such as castor oil, by a single stage consisting of needle emitters and a collector ring.

2) Pressure generation in gas would be about two orders of magnitude less than in insulating oils because, as seen in Equation 33, pressure generated is proportional to electric field strength squared, and gases breakdown at field strengths about an order of magnitude less than insulating oils.
In a subsequent publication, Reference 6, Stuetzer extended his work to cover flowing systems, but restricted it to one-dimensional systems using heavy insulating liquids for working fluids. His mathematical model is constructed from the ideal one-dimensional flow of liquids through a pipe of radius $R$, and plane parallel electrodes separated by a gap $L$. As in the static case, it is reasoned that the electric body force must be balanced by a static pressure increase

$$\frac{dp}{dx} = \rho CE \quad (36)$$

Strictly speaking this assumption is erroneous in application to flowing systems unless it is specifically assumed that the velocity will be maintained constant in $L$. As seen in the momentum equation (Equation 9) where $\vec{U} \neq 0$ the one-dimensional momentum equation is

$$\rho \frac{du}{dt} + \rho u \frac{du}{dx} = F_B - \frac{dp}{dx} + \mu \frac{\partial^2 u}{\partial x^2} \quad (37)$$

and even for inviscid, steady state flow the momentum equation is

$$\rho u \frac{du}{dx} = F_B - \frac{dp}{dx} \quad (38)$$

Although Stuetzer does not explicitly state the assumption of constant velocity it is inherent in his mathematical formulation. If $u = \text{constant}$, Equation 38
reduces to

\[ \frac{dp}{dx} = F_B \]  \hspace{1cm} (39)

Equation 39 can be combined with Lorentz force per unit volume, as previously discussed, to verify Equation 36.

All other governing equations for this simplified model are identical with the static (no flow) case except the definition of current density. In a no flow condition,

\[ J = \rho_c \beta E \]  \hspace{1cm} (40)

which simply states that the current density is equal to the space charge multiplied by the ion drift velocity. If, as in the case of liquids, the fluid velocity and the ion drift velocity are of the same order of magnitude then the current density becomes

\[ J = \rho_c (\beta E + u) \]  \hspace{1cm} (41)

The rest of Stuetzer's governing equations, although unchanged from his previous work, will be listed for convenience. These are

\[ \frac{dE}{dx} = \frac{\rho_c}{\varepsilon} \]  \hspace{1cm} (42)

and

\[ E = -\frac{dV}{dx} \]  \hspace{1cm} (43)

They may be obtained from the general electromagnetic governing equations as previously discussed.
Stuetzer eliminates $\rho_c$ in Equation 41 by substituting from Equation 42 to get,

\[
\frac{J}{\epsilon \beta} = (E + u/\beta) \frac{dE}{dx}
\]  \hspace{1cm} (44)

Integrating this expression from $E = 0$ at $x = 0$ to $E(x)$ at $x$ gives,

\[
E(x) = \left[ (2Jx/\epsilon \beta) + (u/\beta)^2 \right]^{\frac{1}{2}} - \frac{u}{\beta}
\]  \hspace{1cm} (45)

This relation may be differentiated according to Equation 42 to obtain the space charge distribution

\[
\rho_c(x) = \frac{J}{\epsilon \beta} \left[ (2Jx/\epsilon \beta) + (u/\beta)^2 \right]^{\frac{1}{2}} - \frac{u}{\beta}
\]  \hspace{1cm} (46)

which is combined with Equation 36 to obtain

\[
dp/dx = \frac{J}{\epsilon \beta} - \frac{u}{\beta} \left[ (2Jx/\epsilon \beta) + (u/\beta)^2 \right]^{\frac{1}{2}} - \frac{u}{\beta}
\]  \hspace{1cm} (47)

Integrating from $x = 0$ to $x = L$ results in the pumping pressure generated by the electric field, i.e.,

\[
P_L = (\epsilon / 2) \left[ \left( (2JL/\epsilon \beta) + (u/\beta)^2 \right) \right]^{\frac{1}{2}} - \left( \frac{u}{\beta} \right)^2
\]  \hspace{1cm} (48)

To obtain the potential drop across the EFD channel Equation 45 is integrated, after substituting from Equation 43 that

\[
dV/dx = -E(x)
\]  \hspace{1cm} (49)

The result is,

\[
\Delta V_L - \Delta V_L^* = \frac{2L}{3} \left\{ \left( \frac{2JL}{\epsilon \beta} \right) + \left( \frac{u}{\beta} \right)^2 \right\}^{3/2} - \left( \frac{u}{\beta} \right)^3 - \frac{3u}{2}
\]  \hspace{1cm} (50)

This expression is the current-voltage characteristic or generalized Ohm's law (Equation 7) for the system. It may be substituted into Equation 48 to obtain the pumping pressure $P_L$ in terms of potential drop. It
should be noted that $\Delta V_L^*$ is subtracted to account for the voltage required to initiate the charge density limited ion emission process.

The power input is

$$P = \Delta V_L I = \Delta V_L JA$$  \hspace{1cm} (51)

However, Stuetzer reasons that if $\Delta V_L^*$ is lost due to the ionization process, the power available for the pumping process is

$$P = \frac{\Delta V_L - \Delta V_L^*}{\Delta V_L} \Delta V_L JA$$  \hspace{1cm} (52)

or more simply

$$P = (\Delta V_L - \Delta V_L^*)JA$$  \hspace{1cm} (53)

By substituting Equation 50 into 48 and solving for $(\Delta V_L - \Delta V_L^*)$, and solving for $J$ directly from Equation 48, Equation 53 may be written as

$$P = \frac{(1/3)\epsilon\beta A(2p_L/\epsilon)}{(2p_L/\epsilon)^{3/2} + (3u/2\beta)}$$  \hspace{1cm} (54)

Stuetzer uses these derived relations in conjunction with hydrodynamic losses in a simple one-dimensional closed circuit ion pump, as illustrated in Figure 2, to predict the potential performance of a pump which works into a load. The load is simulated by a constriction in the flow channel, and the impedance of the pumping section is also simulated by an area constriction. The impedance is obtained from the usual electrical analogy, i.e.,
Figure 2 - Closed Circuit Ion Pump

Driving potential: $p' \sim \Delta V$ \hfill (55)
Response: $\dot{m}/\rho \sim I$ \hfill (56)
Ohm's Law: $\Delta V = IR$ \hfill (57)

\[ p' = (\dot{m}/\rho)Z_f \] \hfill (58)

so that fluid impedance $Z_f$ is

\[ Z_f = \frac{p'}{(\dot{m}/\rho)} \] \hfill (59)

where $p'$ is the pressure drop, and is analogous to electrical potential drop across a resistance. The volume flow rate $(\dot{m}/\rho)$ is analogous to current, and $Z_f$ is the analog of electrical resistance.
Stuetzer says, "If there were no friction losses in our system, the energy law would link the pressure drop \([p']\) with the flow rate \([\dot{m}/\rho]\) through the well-known equation."

\[
(\dot{m}/\rho) = A(2p'/\rho)^{\frac{1}{2}} \tag{60}
\]

Stuetzer is in error here; Equation 60 does not derive from the energy equation, but results from a consideration of the conservation of momentum (Equation 9) as follows. For steady one-dimensional flow with no friction Equation 9 becomes

\[
\rho u \frac{du}{dx} = F_B - \frac{dp}{dx} \tag{61}
\]

Stuetzer effectively ignored the body force in this part which will produce

\[
\rho u du = -dp \tag{62}
\]

Integration of this equation from the system baseline pressure and velocity, which is taken as zero, to the pressure and velocity in the constriction which represent the pump stage impedance, results in

\[
(1/2)\rho u^2 = p' \tag{63}
\]

which can be written in the form of Equation 60 to obtain volume flow rate, by multiplying both sides by the flow area \(A\), and noting that \(\dot{m}/\rho = uA\).

What Stuetzer has done is to generate a hypothetical pressure that would be developed by an EFD gap with
constant velocity and no resistance, and then correct
the system by subtracting hypothetical hydrodynamic
back pressures. Obviously, in a closed system (Figure
2) the velocity will increase until the decelerating
back pressures equal the EFD pumping pressure. If no
pressure losses occurred around the system the velocity
would be boosted with each circuit, and become infinite.
This is analogous to a simple resistive electrical
circuit; the sum of voltage drops around the circuit
must be zero. Thus Stuetzer writes
\[ p_L = p_1' + p_2' \] (64)
where \( p_L \) is the pumping pressure created across the
EFD gap, \( p_1' \) is the back pressure of the pump itself
and \( p_2' \) is the back pressure representing the load.

After correcting Equation 60 for viscous loss by
using an empirical pipe flow expression and combining
with Equation 64, Stuetzer obtains
\[ p_L = \frac{\rho}{2} \left( \frac{\dot{m}/\rho}{(A_1F)^2} \right)^2 \left[ 1 + \left( \frac{A_1}{A_2} \right)^2 \right] \] (65)
where \( F \) is the empirical friction factor.

Combining Equations 48 and 50 and solving for the
flow velocity results, after simplification, in
\[ u = \left( \frac{V_L - V_L^*}{(2/3)L} \right) \left( \frac{\beta}{C_f C_g} \right) \left( \frac{2 + C_f C_g}{(3/2) + C_f C_g} \right) \] (66)
where \( C_f \) is a fluid properties factor defined as
\[
C_f = \frac{\beta}{T}(\rho/\epsilon)^{\frac{1}{2}}
\] (67)
and \( C_g \) is a geometry factor defined as
\[
C_g = \left[ 1 + \left( \frac{A_1/A_2}{2} \right)^2 \right]^{\frac{1}{2}}
\] (68)

Noting that \( u = (\dot{m}/\rho)/A \), and combining Equation 66 with Equation 65 results in

\[
p_L = \frac{\epsilon}{2} \left[ \frac{\Delta V_L}{(2/3)L} - \Delta V_L^* \right]^2 \left[ \frac{2 + C_f C_g}{(3/2) + C_f C_g} \right] \] (69)

The pressure at the load is \( p_L - p_1^* \) or
\[
p_2^* = p_L \left( C_g^2 - 1 \right)/C_g^2
\] (70)

Combining Equations 69 and 70 with Equation 48 results in the actual current drawn, i.e.
\[
I = \frac{A\epsilon\beta}{2L} \left[ \frac{\Delta V_L}{(2/3)L} - \Delta V_L^* \right]^2 \left[ \frac{2 + C_f C_g}{(3/2) + C_f C_g} \right] \left( 1 + \frac{2}{C_f C_g} \right)
\] (71)

which multiplied by \( (\Delta V_L - \Delta V_L^*) \) gives the system input power. The output power is simply \( A_{up2} \), so

Equations 70 and 71 may be combined with the definition of power (Equation 53) to yield the power efficiency.

Defining \( \eta \) as
\[
\eta = \frac{P_{in}}{P_{out}}
\] (72)

Stuetzer obtains efficiency as
\[
\eta = \left( \frac{\Delta V_L - \Delta V_L^*}{\Delta V_L} \right) \left( 1 - \frac{p_1^*}{p_L} \right) \left[ 1 + (2/3)C_f(p_L/p_1^*)^{\frac{1}{2}} \right]^{-1}
\] (73)
These semi-empirical relations were shown to predict with acceptable accuracy both qualitative and quantitative performance of experimental systems pumping liquids such as kerosene and freon. Pertinent conclusions drawn from this work are:

1) The effect of flow is to increase the current and decrease the efficiency from their static values.

2) The mathematical description applies with acceptable accuracy to fairly complicated pumping arrangements so long as heavy insulating liquids are used as a working fluid and needle point emission is utilized for ion emitters.

3) Efficiencies of better than 20% have been experimentally demonstrated.

4) Paralleling or cascading several pumping units to increase the flow rate or pressure, if the units are properly matched, is analogous to connecting electrical cells in parallel or series to increase current or voltage.

Robinson, Reference 7, turned from dielectric liquids to study atmospheric air as the working fluid. The object of this work was to investigate the practicality of using EFD fans to pump air. A semi-empirical mathematical model was developed and correlated to
experimental data obtained using needle point emitters and screens in EFD channels.

The mathematical treatment follows that of Stuetzer in assuming a one-dimensional flow with parallel emitters and collectors. The fluid pressure is assumed to balance the electrostatic bulk force.

\[ \frac{dp}{dx} = \rho_c E \]  \hspace{1cm} (74)

It was noted that this expression is true for a non-flowing system. It can also be shown to be true for a closed circuit of incompressible fluid moving at constant velocity. However, the application here is intended to describe a fan with the fluid far ahead and behind the fan at rest. The velocity through the fan cannot be a constant in this case, and Equation 74 cannot be derived from the momentum equation (Equation 9). The proper relation will be derived in Section III.

Following Stuetzer's lead, Robinson obtains expressions relating current, velocity and voltage. The electric field is written as the negative gradient of the electrical potential

\[ E = -\frac{dv}{dx} \]  \hspace{1cm} (75)

which, as previously noted, may be derived from Maxwell's equation (Equation 6). This derivation will also be shown in detail in Section III. The current
density and velocity are related by the expression

\[ J = \epsilon (\frac{dE}{dx})(u + \beta E) \quad (76) \]

which is obtained from Gauss's law (Equation 3) and the definition of current density. \( E \) is the ion drift velocity in the EFD gap and \( u \) is the fluid velocity. For the static, no flow case the fluid velocity was omitted. For the flow of heavy liquids Stuetzer found the velocity of the ions and that of the flow to be the same order of magnitude. Robinson cites experimental data which indicates that \( u \ll \beta E \) and therefore reduces Equation 76 to

\[ J = \epsilon (\frac{dE}{dx}) \rho_c \beta \quad (77) \]

At this point Robinson, in need of a velocity relation because of the erroneous use of the momentum equation, defines the fluid pressure in Equation 74 to be an "electrical pressure" which is matched by the "aerodynamic back pressure", and he writes

\[ p = K(\rho u^2/2) \quad (78) \]

with \( K \) being a geometrical loss factor which is "almost independent of velocity". The effect is the same, ignoring the empirical constant, if it had initially been assumed that no static pressure increase is obtained across the fan. By integrating Equation 77 to obtain \( E(x) \), and combining the result with Equations 74 and 78 Robinson solves for the fluid velocity leaving
the EFD collector. He writes the result as,

\[ u = g_1 (I/\rho \beta K) \]  

(79)

\( g_1 \) is an empirical constant added to the analytical development to account for geometric effects.

The current-voltage relationship is taken to be

\[ I = g_2 \epsilon \beta \Delta V_L (\Delta V_L - \Delta V_L^*) \]  

(80)

where \( g_2 \) is yet another empirical geometric factor. Replacing current in Equation 79 with this relation, the following expressions for velocity, power, and efficiency are obtained.

\[ u = g_1 \left[ (g_2 \epsilon / \rho K) \Delta V_L (\Delta V_L - \Delta V_L^*) \right]^{1/2} \]  

(81)

\[ P = \rho^{-1/2} \left[ \frac{(g_1 g_2^{1/2})^{3/2} \epsilon}{K} \left\{ \Delta V_L (\Delta V_L - \Delta V_L^*) \right\} \right]^{3/2} \]  

(82)

\[ \eta = \frac{g_1 g_2^{1/2} A}{2 \beta} \left[ \frac{\epsilon}{\rho K^2} \left( \frac{\Delta V_L - \Delta V_L^*}{\Delta V_L} \right) \right]^{1/2} \]  

(83)

Experimental data verifies that these relations will adequately fit data, if the constants \( K, g_1 \) and \( g_2 \) are evaluated empirically. In a sense Robinson has not presented an analytical model, but rather a set of semi-empirical curve fitting relations.

Robinson's conclusions are summarized below.

1) The efficiency of an EFD fan is about 1%.

2) If mobility can be reduced, efficiency will increase. Dust or colloidal particles will attach electrons and/or ions, and act as...
massive ions with very low mobilities. Their use should be investigated.

Although most researchers returned to the EFD pumping of liquids since the much higher efficiency held greater promise of productivity, A. F. DeSeversky in 1964 patented the concept of an aircraft which utilized EFD propulsion to obtain lift and thrust, Reference 8. As a result, interest in the 200 year old knowledge of the "electric wind" revived. A. Maciulaitis in Reference 9 reports the results of an extensive computer study of EFD propulsion in earth's atmosphere.

Maciulaitis assumes a model which has plane electrodes in a constant area circular cross-section channel. It is assumed that all properties except the space charge induced electric field vary only with x, the coordinate along the EFD channel axis. In contrast to previous investigators, two-dimensional variation of the field is permitted. Effects of viscosity, heat conduction, and concentration diffusion are ignored. It is assumed that a complete neutralization of charge occurs at the emitter, i.e., space charge limited emission. It is also assumed that ion mobility varies inversely with density.

The governing equations, chosen by Maciulaitis are
\[ P = \rho RT \quad (84) \]
\[ \rho u = C_1 / A \quad (85) \]

\[ \rho u \frac{du}{dx} = \rho C E_x - \frac{dP}{dx} \quad (86) \]

\[ \frac{d}{dx} \left[ \rho u \left( \frac{\gamma RT}{\gamma - 1} + \frac{1}{2} u^2 \right) \right] = J \cdot E \quad (87) \]

\[ J_x = \rho C (u + \frac{\beta E_x}{\rho} E_x) \quad (88) \]

\[ I = J_x A \quad (89) \]

\[ E_{\text{applied}} + E_{\text{space}} = \overrightarrow{E} \quad \text{charge} \quad (90) \]

\[ \nabla \cdot E = \frac{1}{r} \frac{\partial}{\partial r} (rE_r) + \frac{\partial E_x}{\partial x} = \frac{\rho C}{\epsilon} \quad (91) \]

Equation 84 is the equation of state (Equation 11), and 85 is the one-dimensional steady state continuity equation, which may be obtained from the more general Equation 8. Equation 86 is recognized as the one-dimensional steady state momentum equation, which can be obtained from Equation 9 by using Equation 1 (Lorentz Force) to deduct the body force term. Equation 87 is the simplified energy equation. It may be derived from the more general Equation 10 after noting that the electromagnetic energy function in this case is simply \( J \cdot E \). Equation 88 defines the current density, 89 relates charge conservation (Equation 2), and 90 is the assumed superposition of electric fields.
Equation 91 is Gauss's law (Equation 3) written for this system.

These equations were programed and solved simultaneously on a digital computer for a variety of fluid conditions. The results may be summarized as follows:

1) For radius to length ratios greater than 5 the one-dimensional electric field is justified. The actual field remains nearly linear for ratios as low as 1.0, but the one-dimensional approach results in an error of 40% in magnitude.

2) For zero mobility power efficiency is essentially 100%.

3) For air, efficiency was extremely low at sea level pressure, but increased with increasing freestream velocity.

4) Since supersonic velocities are required to yield usable efficiency, and since turbojets are much more practical, Maciulaitis sees little future for EFD propulsion in the earth's atmosphere.

5) Even using colloidal ions in air, with the resultant low mobility and high efficiency the amount of power than can be added to the stream is low because of the limited electric field strength that air will support.
Christenson and Moller, in Reference 10, report a simplified mathematical analysis, and the fabrication and testing of a free boundary experimental apparatus. The analytical approach assumes an unbounded plane electrode arrangement where the ratio of acceleration channel length to the electrode diameter is less than 0.5.

Starting with the momentum balance through a control volume which is bounded by the collector and bounded very far upstream of the emitter, the momentum balance for the assumed one-dimensional steady state system was obtained as

\[ \rho u^2 = \left( \frac{\epsilon}{2} \right) E^2 \]  \hspace{1cm} (92)

where E is the field at the collector. This resulted from assuming that in an unbounded system the static pressure would not change due to unidirectional momentum addition, i.e., \( \frac{dp}{dx} = 0 \). This equation may be obtained from the general momentum equation (Equation 9) by using the Lorentz force relation (Equation 1) to obtain the body force \( \rho_c E \), and then eliminate \( \rho_c \) by substitution from Gauss's law (Equation 3). The current density relation is invoked in the usual manner. Unlike Robinson, Christenson and Moller do not drop the fluid velocity \( u \), from the definition of current density, but retained it throughout. The current
voltage characteristic is assumed to be

\[ I = C \Delta V_L (\Delta V_L - \Delta V_L^*) \]  \hspace{1cm} (93)

where \( C \) is a constant to be determined from experimental data.

From this starting point expressions were derived for flow velocity, thrust per unit input power, and power efficiency. These are respectively,

\[ u = \frac{\{\Delta V_L (\Delta V_L - \Delta V_L^*)\}^{\frac{1}{3}}}{L} \phi \beta \frac{1}{1 + [1/3(1 + \phi)]} \]  \hspace{1cm} (94)

where \( \phi = (2 \varepsilon/\rho \beta^2)^{\frac{1}{3}} \)

\[ \frac{F}{\Delta V_L} = (L/\Delta V_L) \frac{1}{\beta(1 + \beta)} \]  \hspace{1cm} (95)

\[ \eta = \frac{1}{2} \left[ \frac{\Delta V_L - \Delta V_L^*}{\Delta V_L} \right]^{\frac{1}{3}} \frac{1}{1 + (4/3 \phi)} \]  \hspace{1cm} (96)

Christenson and Moller point out that as \( \phi \) goes to infinity (i.e. as \( \beta \to 0 \)) efficiency increases, and note, as did Robinson that as \( \Delta V \) becomes very large optimum voltage is reached. It is pointed out that Equation 96 indicates an upper bound to efficiency of 50%.

During the experimental program a hot wire anemometer was used to survey the flow leaving the collector and calculate the momentum. It is significant that surveys made 6 inches and 11 inches below the collector indicated momentum values that agreed within 0.7%. This indicates that the assumption of \( dp/dx \approx 0 \) is justified because very little spreading or entrainment resulted.
The conclusions drawn were:

1) Efficiency is about 1% for atmospheric air at standard conditions.

2) Low efficiency is believed to be due to heating of the air.

3) Ion mobility, $\beta$, is the controlling factor governing efficiency.

Christenson and Moller suggest that kinetic energy of impact of ions on neutral molecules must govern the loss mechanism. They suggest pulsed voltage to the EFD gap in the hope that as a pulse rises and falls many ions will not achieve their full relative velocity before impact. They believe the lower mean impact velocity would cause an increase in efficiency.

The work of Christenson and Moller concludes the description of previous pertinent research, and brings the discussion of EFD propulsion up to date. A brief summary which relates to the efficiency and/or power loss in EFD systems is in order.

While References 2 thru 10 agree that ion mobility is the prime parameter controlling efficiency, in general the mathematical models presented exhibited considerable variation. Those authors with closed form solutions for the most part biased their results with empirical constants used to force their analytical
relationships into agreement with experimental data obtained from variously configured test set-ups. The various efficiency equations obtained performed oddly in the limit as the ion mobility decreased to zero. Some expressions went to infinity while others found plausible limits in the vicinity of 0.5. Maciulaitis (Reference 9) being the sole exception, maintained an adequately rigorous approach. He found that in the limit as the ion mobility approached zero the efficiency essentially became 100%. Unfortunately his results were numerical, and were obtained for complex situations involving an EFD engine in motion at elevated densities and temperatures. He presented no data for a simple hovering situation at standard atmospheric conditions.

The efficiency of the EFD process in atmospheric air was estimated and/or measured to be about 1% by all, but no one undertook an investigation of the mechanism causing this low efficiency. Most authors simply point out that 1% power efficiency is too low for practical application. Robinson (Reference 7) suggests that the ion mobility could be decreased, thereby increasing efficiency, by seeding air with heavy charged colloidal particles. Christenson and Moller (Reference 10) suggest that the loss mechanism may be due to high ion collision velocities, and suggest pulsed voltage to reduce the mean ion collision
velocity. Maciulaitis (Reference 9) maintains that even with 100% efficiency the EFD momentum exchange process is limited to very low power applications, making it unsuitable for flight vehicle propulsion.

Due to this diversity of results, the literature does not offer a single mathematical model of the EFD momentum exchange phenomena which can be accepted as a "touchstone" for the macroscopic or continuum performance of the process. Before starting a detailed evaluation of the interaction processes responsible for the inefficient energy conversion in EFD engines, it was felt desirable to have a fairly realistic closed form theoretical model of an EFD propulsion mechanism which would be traceable directly to the general body of laws governing fluid dynamics and electrodynamics (Equations 1 thru 11). This was necessary to provide guidance for the investigation of the EFD power loss mechanism, and since nothing available was acceptable, such a mathematical model had to be developed.
The objective in the development of this mathematical model was to describe the first order effects as completely as possible, and yet maintain a sufficiently simple approach to permit a clear appreciation of the interrelationships of the EFD parameters in a closed form solution.

Consider the assumptions universally made in the literature, i.e., plane parallel electrodes for emitter and collector with ions of either positive or negative charge. The flow will be assumed inviscid, incompressible, steady state, with negligible thermal conduction. Also it is assumed that radial gradients are negligible. No magnetic field will be intentionally imposed, and current will be low enough that inherent magnetic fields will be neglected, i.e., $H = 0$.

Under these conditions the Lorentz Force relation becomes

$$F_x = qE_x$$  \hspace{1cm} (97)

This represents the force on net charge $q$ due to the local value of the electric field $E_x$.

The mean total Lorentz Force acting upon the charge, $q$, in the acceleration gap between the emitter and collector is
Now $\int \int \int dx dy dz$ is a constant equal to the volume between the electrodes. As such it may be divided into the integrand in the numerator of Equation 98, where it will be recognized that

$$q/\int \int \int dx dy dz = \rho_c$$  \hspace{1cm} (99)

Making this substitution, the mean Lorentz Force may be written as

$$\overline{F_1} = \int \int \int \rho_c E dx dy dz$$  \hspace{1cm} (100)

Note that this expression is identical to that obtained by Arrhenius (Equation 22). Recalling that properties have been assumed to vary only in the $x$-direction, Equation 100 becomes

$$\overline{F_1} = A \int \rho_c E dx$$  \hspace{1cm} (101)

where $A$ is the cross-sectional area.

The mean Lorentz Force must be balanced by an equal and opposite force exerted by the bulk fluid upon the net charge. This reaction force in turn must be equal and opposite to the force exerted upon the fluid. (i.e., the integral of $F_B$ over the volume in the gap.) The only body force considered is the electrostatic one because without the electric field the fluid would remain at rest.

Therefore,

$$\int \int \int F_B dx dy dz = \overline{F_1}$$  \hspace{1cm} (102)
Applying the one-dimensional assumption and combining Equations 100 and 102 yield

\[ A \int F_B \, dx = A \int \rho_c E \, dx \]

or

\[ \int F_B \, dx = \int \rho_c E \, dx \]  \hspace{1cm} (103)

Differentiating both sides of this expression with respect to \( x \) gives the relation for the body force \( F_B \).

\[ F_B = \rho_c E \]  \hspace{1cm} (104)

Equation 9 under the assumptions made becomes,

\[ \rho u \frac{du}{dx} = \rho_c E - \frac{dp}{dx} \]  \hspace{1cm} (105)

This is identical to the form of the momentum equation used by 
Maciulatis in Equation 86. As previously noted, if \( u = 0 \) or if \( u \) = constant, this equation reduces to the momentum relation used by Stuetzer (Equations 26 and 36) and erroneously by Robinson (Equation 74).

Gauss's Law, Equation 3, under the present assumptions may be simplified to

\[ \frac{d}{dx}(\epsilon E) = \rho_c \]  \hspace{1cm} (106)

Adding the assumption that the permittivity \( \epsilon \) is a constant function of the fluid, a further simplification is possible, i.e.,

\[ \epsilon \frac{dE}{dx} = \rho_c \]  \hspace{1cm} (107)

Substituting Equation 107 into Equation 105 yields
\[ \rho u \frac{du}{dx} = \varepsilon E \frac{dE}{dx} = \frac{dp}{dx} \] (108)

Constant pressure operation will be specified since no flow boundaries exist, and Christenson and Moller, as previously mentioned, experimentally verified no beam spreading, or flow entrainment for such a system. Equation 108 reduces to

\[ \rho u \frac{du}{dx} = \varepsilon E \frac{dE}{dx} \] (109)

which may be integrated from \(-\infty\), where \(u = 0\) and \(E = 0\), and some point \(x\), in the acceleration gap to obtain

\[ u(x) = \left(\frac{\varepsilon}{\rho}\right)^{\frac{1}{2}} E(x) \] (110)

Unlike Christenson and Moller's approach, velocity is permitted to be a function of \(x\), and the problem of defining fluid velocity is a matter of properly evaluating the field distribution \(E(x)\).

The conservation of charge requirement (Equation 2) insures that the current anywhere in the channel remains constant so long as the charge density is invariant with time, i.e., \(\Delta \cdot \vec{J} = 0\) and \(\vec{J} = I/A\) so that,

\[ I = \text{Constant} \] (111)

The current \(I\), in coulombs per second, is the charge flow rate. The charge is carried by the ions with the net ionic velocity, \(u_1\). The current is then given by the relation,
As explained by Darrow in Reference 11, the ions have a drift velocity of $\beta E$ imparted to them by the electric field. This drift velocity is with respect to a coordinate system fixed to the neutral fluid. If the neutral fluid has a velocity of $u$, the total ion velocity will be

$$u_i = u + \beta E \quad (113)$$

which is the form of the current relationship used by Stuetzer (Reference 6) for flowing liquids. If $u = 0$ or $\beta E \gg u$, this equation reduces to the form used by Stuetzer for static fluids (Reference 5) and Robinson for air (Reference 7). Substituting Equations 113 into 112 gives the relation

$$I = \rho u_i A \quad (114)$$

Substituting from Equation 107 for $\rho$ results in

$$I = A \frac{dE}{dx} (u + \beta E) \quad (115)$$

The conservation of mass (Equation 8) may be simplified, for one-dimensional, steady state assumptions to

$$\frac{\partial}{\partial x} (\rho u A) = 0 \quad (116)$$

Assuming that changes in fluid temperature are very small and negligible, constant pressure operation insures constant fluid density. Equation 116 may
then be integrated to yield

\[
\text{uA = G = Constant} \quad (117)
\]

Combining Equation 110, 115, and 117 results in

\[
G \left[ 1 + \frac{\beta}{\left( \epsilon/\rho \right)^{1/3}} \right] \frac{\text{dE}}{\text{dx}} = \frac{I}{\epsilon} \quad (118)
\]

Integrating from \( x = 0, E = 0 \), to \( x, E(x) \) and solving for the field \( E(x) \) results in

\[
E(x) = \frac{K I x}{\epsilon} \quad (119)
\]

where

\[
K = (\epsilon/\rho)^{1/3} G^{-1} \left[ \beta + (\epsilon/\rho)^{1/3} \right]^{-1} \quad (120)
\]

Equation 119 may be combined with Equation 110 to give the velocity distribution in terms of current.

\[
u(x) = K \left( \frac{\epsilon}{\rho} \right)^{1/3} \frac{I x}{\epsilon} \quad (121)
\]

In order to establish a relation between current and applied voltage consider the Maxwell equation, Equation 6, simplified by the assumption of no magnetic fields,

\[
\nabla \times \mathbf{E} = 0 \quad (122)
\]

It is known that when the curl of a vector field is zero the vector may be expressed as a gradient of a scalar field. The scalar field can only be the electrical potential and therefore

\[
\mathbf{E} = -\nabla \Phi \quad (123)
\]

This expression may be further simplified by imposing the assumptions of the one-dimensional model.
developed to this point, with the result,

\[
d\frac{V}{dx} = -E(x)
\]  

(124)

Combining Equations 119 and 124 and integrating from the emitter to the collector gives the voltage-current relation,

\[
\nabla V_L = K \frac{IL^2}{2\epsilon}
\]  

(125)

where \( \nabla V_L \) is the potential drop across the acceleration gap. The apparent linear resistive impedance suggested by Equation 125 is misleading. \( K \) may be evaluated by combining

\[
G = u_{IAL}
\]  

(126)

and Equation 121 in conjunction with the definition of \( K \), Equation 120.

\[
K = \epsilon^{\frac{1}{2}} \left[ IL_{IA} \left\{ \beta + \left( \frac{\epsilon}{\rho} \right)^{\frac{1}{2}} \right\} \right]^{\frac{1}{2}}
\]  

(127)

Substituting this relation into Equation 125 reveals the square law characteristic exhibited by the experimental data.

\[
\Delta V_L = \frac{1}{2} \left[ \frac{IL^3}{A_L \epsilon \left\{ \beta + \left( \frac{\epsilon}{\rho} \right)^{\frac{1}{2}} \right\}} \right]^{\frac{1}{2}}
\]  

(128)

Combining Equation 125 and 121 evaluated at the collector, gives the velocity of fluid flow leaving the EFD mechanism in terms of applied voltage.

\[
u_L = \frac{2}{L} \left( \frac{\epsilon}{\rho} \right)^{\frac{1}{2}} \Delta V_L
\]  

(129)
It is seen that exit velocity is directly proportional to applied voltage and inversely proportional to gap length. Also note that velocity is independent of the efficiency controlling ion mobility. Stuetzer (Reference 6) obtained the velocity for a liquid in a closed loop channel. From Equations 66 and 67 it is seen that even for this system Stuetzer found velocity to be directly proportional to applied voltage (ignoring the starting potential $\Delta V_L^*$) and inversely proportional to EFD gap length. Robinson (Equation 81) also verifies this qualitative fact, as does Christenson and Moller (see Equation 94). However both Stuetzer's and Robinson's expressions are clouded with empirical constants and Christenson and Moller obtain an analytical relationship which is dependent on ion mobility. A detailed comparison of Equation 129 and applicable data is given in Section V.

The mass flow rate exiting the system ($\rho u_L A_L$) is

$$\dot{m} = (2\rho A_L/L)(\epsilon/\rho)^{1\over 2} \Delta V_L$$

(130)

and the thrust developed by the EFD mechanism ($\ddot{m} u_L$) is

$$F = (4A_L/L^2)\epsilon(\Delta V_L)^2$$

(131)

Thrust is directly proportional to the square of the applied voltage, inversely proportional to the square of the EFD gap length and directly proportional to the cross-sectional area.
The fluid kinetic power developed by the EFD mechanism \((\frac{1}{2} \mu u_L^2)\) is

\[
P_o = (4A_L/L^3)(\epsilon/\rho)^{\frac{1}{2}} \epsilon (\Delta V_L)^3
\]  

(132)

\(P_i\), electrical power entering the EFD mechanism is \(\Delta V_L I\) and may be obtained in terms \(\Delta V_L\) from Equation 128.

\[
P_i = (4A_L/L^3) \left[ \beta + (\epsilon/\rho)^{\frac{1}{2}} \epsilon (\Delta V_L)^3 \right]
\]  

(133)

The power efficiency, defined as the ratio of power out to power in, may then be derived as

\[
\eta = \frac{(\epsilon/\rho)^{\frac{1}{2}}}{\beta + (\epsilon/\rho)^{\frac{1}{2}}}
\]  

(134)

The result is relatively simple in comparison to the expressions offered in the literature (see Equations 73, 83, and 96). It will be recalled that the numerical results of Maciuulaitis indicated that when \(\beta\) goes to zero, the efficiency of the EFD process approaches unity. Equations 73, 83, and 96 all approach values of between 0.50 and 0.70 as \(\beta\) goes to zero. However Equation 134, in agreement with Maciuulaitis's numerical data, approaches unity as \(\beta\) goes to zero. Correlation of Equation 134 with existing data is shown in Section V.

The prime purpose of this simplified analysis was to obtain Equation 134, i.e., the relationship between power efficiency and the operating parameters for the EFD momentum transfer process.
The results apply to the energy conversion process itself and are not coupled to the ion production mechanism. This is necessary to avoid masking momentum exchange processes with ion production processes or empirical constants. The objective is a description of the EFD momentum exchange loss mechanism, not the system performance.
IV. SOME STATISTICAL CONSIDERATIONS

A. Objectives

Consider the implications of Equation 134.

\[ \eta = \frac{(\epsilon/\rho)^{\frac{1}{2}}}{\beta + (\epsilon/\rho)^{\frac{1}{2}}} \]

Recall that ion mobility is defined as ion relative velocity to electric field strength,

\[ \beta = \frac{u_{ir}}{E} \quad (135) \]

where \( u_{ir} \) is the ion velocity relative to the neutral fluid. Also recall Equation 110, which may be rewritten to read

\[ \sqrt{\frac{\epsilon}{\rho}} = \frac{u}{E} \quad (136) \]

Substituting these relations into the efficiency equation results in

\[ \eta = \frac{u}{u_{ir} + u} \quad (137) \]

It is apparent that efficiency is determined by the magnitude of relative ion velocity with respect to the neutral gas velocity.

In light of Equation 137 it was decided to investigate the mechanisms governing the magnitude of the velocity parameter defined as
\[ \eta^* = \frac{u}{u_{ir}} \]  

In cases where the efficiency is very low, such as experienced in air, \( u_{ir} \) is much larger than \( u \) and \( \eta \) and \( \eta^* \) will agree closely. By obtaining the molecular parameters involved in this ratio it was hoped that the physical loss mechanism could be deduced.

Before investigating the relationship of \( u_{ir} \) to \( u \), the ratio of field induced velocity to random thermal velocity is considered and discussed. This yields an improved understanding of the general nature of EFD-momentum exchange mechanics prior to an investigation of the nature of the parameter \( \eta^* \). It should be pointed out that in these statistical investigations the object is a qualitative understanding of the physical processes, not a rigorous mathematical treatment. No attempt has been made to obtain an elegant presentation.

**B. Velocity and Mean Free Path Considerations**

Consider the working fluid to be idealized air. It will be a one component gas with a molecular weight of 29, and will be assumed to consist of elastic spheres. With no ions present the thermal velocity distribution may be obtained as indicated in Reference 12,
\[ \frac{dN_v}{Ndv} = \frac{4}{\sqrt{\pi} v_{mp}} \left( \frac{v}{v_{mp}} \right)^2 \exp \left[ -\left( \frac{v}{v_{mp}} \right)^2 \right] \]  

(139)

It is noted here that \( \frac{dN_v}{Ndv} \) is actually the probability density function of \( v \), the thermal velocity component. It will be denoted as

\[ \frac{dN_v}{Ndv} \equiv f(v) \]  

(140)

where \( dN_v \) is the number density of molecules with velocity increment \( dv \), and \( N \) is the total number density. The most probable velocity can be shown to be

\[ v_{mp} = (2kT/m)^{\frac{1}{2}} \]  

(141)

where \( k \) is Boltzmann's constant, \( T \) is the absolute temperature and \( m \) is the mass of the idealized molecule. Thermal motion of gas is therefore known and it is necessary to consider the case of ions in an electric field.

Assume that a small quantity of unipolar ions are injected into a volume of gas whose velocity distribution is given by Equation 139. The ions will be evenly distributed throughout the volume. Further, assume that these ions are identical to the gas molecules, except that they carry a net electric charge. For the moment consider an elastic collision model with no intermolecular forces.
If a uniform electrostatic field is applied to the gas volume the ions will experience electric forces which superimpose a velocity upon their thermal velocities.

The force on an ion in free space is given by the Lorentz Force law (Equation 1). For the case of no magnetic fields

$$\overrightarrow{F_i} = q_i \overrightarrow{E} \quad (142)$$

The acceleration is therefore obtained as,

$$\overrightarrow{a_i} = \left(\frac{q_i}{m_i}\right) \overrightarrow{E} \quad (143)$$

The ion will continue to accelerate until it collides with another molecule. For the assumptions made the mean free path for the ions will be the same as the mean free path for the neutral molecules. Reference 13, derives the mean free path in such a situation as

$$\overline{\lambda} = \left[ \frac{1}{2} \pi \sigma^2 N \right]^{-1} \quad (144)$$

where $\sigma$ is the molecular collision diameter. Reference 13 also gives the distribution of free paths in such a gas as

$$\left(\frac{N_\lambda}{N}\right) = \exp\left(-\frac{\lambda}{\overline{\lambda}}\right) \quad (145)$$

where $N_\lambda$ is the number density traveling a distance without a collision. This expression may be written in the form of a probability density function by considering that
\[ P(\text{free path} \leq \lambda) \equiv F(\lambda) = 1 - \exp(-\lambda/\bar{\lambda}) \quad (146) \]

where \( F(\lambda) \) is the probability function of \( \lambda \). The probability density function may be obtained in the usual fashion

\[ f(\lambda) = \frac{dF(\lambda)}{d\lambda} \quad (147) \]

or

\[ f(\lambda) = \frac{1}{\lambda} \exp(-\lambda/\bar{\lambda}) \quad (148) \]

The mean free time is generally derived as

\[ \bar{t}_f = \frac{\bar{\lambda}}{\bar{v}} \quad (149) \]

where \( \bar{t}_f \) is the mean free time and \( \bar{v} \) is the mean speed. In the case of an ion, consider that the acceleration \( a_i \) due to an applied field adds to the thermal mean velocity.

By considering that

\[ \int_0^{\bar{t}_{fi}} (\bar{v} + a_i t) dt = \bar{\lambda} \]

the mean free ion time can be derived as

\[ \bar{t}_{fi} = -\frac{\bar{v}}{a_i} + \sqrt{\left(\frac{\bar{v}}{a_i}\right)^2 + \frac{2 \bar{\lambda}}{a_i}} \quad (150) \]

where \( \bar{t}_{fi} \) is the mean free ion time, and \( a_i \) is the ion acceleration due to the electric field.

The mean ion velocity may be defined as

\[ \bar{v}_i = \bar{v} + \bar{v}_{ie} \quad (151) \]
where $\bar{v}_{ie}$ is the mean contribution of the electric field. It is easily shown that

$$\bar{v}_{ie} = a_i \bar{t}_i \tag{152}$$

By combining Equations 143, 150, and 151 these relations may be written as

$$\bar{v}_{ie} = -\bar{v} + \left[ \frac{v^2}{\left(2Eq_i/m_i\right)} \right]^{\frac{1}{2}} \tag{153}$$

and

$$v_i = \left[ v^2 + \left(2Eq_i/m_i\right) \right]^{\frac{1}{2}} \tag{154}$$

In general, by the same reasoning

$$v_{ie} = -v + \left[ v^2 + \left(2Eq_i/m_i\right) \right]^{\frac{1}{2}} \tag{155}$$

and

$$v_i = \left[ v^2 + \left(2Eq_i/m_i\right) \right]^{\frac{1}{2}} \tag{156}$$

Assume that at constant temperature $\lambda$ and $v$ are independent variables. The ion velocity being a function of these two independent variables will have an amplitude distribution governed by the joint probability density function in accordance with the relation

$$P(v_{ic} \leq b) = \iiint_{S(v_{ic} \leq b)} f(\lambda, v) d\lambda dv \tag{157}$$

That is, the probability that the ion collision velocity has a value of $b$ or less is equal to the surface integral of the joint density function $f(\lambda, v)$ over the velocity surface defined by $v_i \leq b$. As noted in References 14 and 15, the joint density function of any number independent random variables can be demonstrated to consist of the product of the individual
density functions. Therefore $f(\lambda, v)$ is easily found as

$$f(\lambda, v) = f(\lambda)f(v) \quad (158)$$

Also the velocity surface can be defined from Equation 155 as

$$\sqrt{v^2 + 2a_i \lambda} \leq b \quad (159)$$

Equation 156 may therefore be written,

$$P(v_{ic} \leq b) = \int\int f(\lambda)f(v) d\lambda dv \quad (160)$$

where $S = (v^2 + 2a_i \lambda)^{\frac{1}{2}} \leq b$.

This relation can be used to find the maximum ion collision velocity from the actual distributions of thermal velocity and free path by considering that

$$P(v_{ic} \geq b) = 1 - P(v_{ic} \leq b) \quad (161)$$

For air at normal atmospheric conditions and an applied field of $10^6$ volts per meter the distributions of velocity and free path, given by Equations 140 and 148, are shown in Figure 3. Free path probability density has been plotted against $\sqrt{2a_i \lambda}$.

Figure 4 depicts the velocity surface in relation to the joint probability density function $f(\lambda, v)$. The density function has not been normalized by the projection of $d\lambda$ and $dv$ upon the surface, so the volume bounded by the joint density function and the velocity surface will not be unity. As long as it is recog-
nized that the total volume obtained by integrating over the entire surface corresponds to a probability $F(v_{ic})$ of unity, the maximum ion collision velocity may still be obtained.

Figure 3 - Velocity and Free Path in Air,

(S.T.P & $E = 10^6$ v/m)
The length of the vector from the origin to the boundary where the joint distribution function becomes negligible, \((f(\lambda,v)/f(\lambda,v) \text{ max.} \approx 0)\), is the maximum ion velocity. This is easily deduced from the fact

\[
\iint_{S_{v_i} \leq b} f(\lambda,v) d\lambda dv = \text{Volume Under } f(\lambda,v)
\]

If \(b\) is the boundary where \(f(\lambda,v) \approx 0\), then

\[
\mathbb{P}(v_i \leq b) = \int \int_{(S_{v_i} \leq b)} f(\lambda,v) d\lambda dv \approx \int_{0}^{\infty} f(v_i) dv_i \equiv 1.0
\]
Therefore the probability of an ion collision at a higher velocity would be, from Equation 161,

\[ P(v_1 \leq b) = 1 - P(v_1 < b) = 0 \]

The normal projection of the ion velocity boundary upon the \( v, \sqrt{2a_i} \lambda \) plane is shown in Figure 5. Again the magnitude of the vector to any point within this boundary represents a possible ion velocity.

\[ \sqrt{2a_i} \lambda \times 10^{-12} \text{m/sec} \]

\[ v \times 10^{-2} \text{m/sec} \]

Figure 5 - Ion Velocity Distribution
For the case evaluated, standard sea level atmosphere with a potential gradient of $10^6$ volts per meter, the maximum ion collision velocity is about 1355 m/sec, and the mean collision velocity is 807 m/sec. If the field is cut in half, maximum velocity becomes 1045 m/sec, with a mean value of 657, and if the field is doubled the maximum and mean values are 1885 m/sec and 1042 m/sec, respectively. Although a field strength of 2 million volts per meter is possible under some conditions, the value of 1 million volts per meter is much more easily sustained, and corresponds to a practical working value.

The probability of any value ion velocity must be determined by evaluation of the joint probability density function and will not be considered here.

C. The Loss Mechanism

An investigation of losses in the ionization process, surface emission of electrons, and surface losses at the collector had negative results. All of these losses lower system performance by about 1%. That leaves 98% to account for by some mechanism in the momentum transfer process.

The result of the continuum approach to the prediction of efficiency indicated that the parameter $\eta^*$ defined in Equation 138 should be investigated on a
molecular scale to obtain the prime mechanism limiting system performance. Writing this relation again for convenience,

\[ \eta^* = \frac{u}{u_{ir}} \]

where \( u \) is the mean velocity of the bulk fluid and \( u_{ir} \) is the mean velocity of the ions or bulk charge with respect to a reference system moving with velocity \( u \).

This ratio is easily shown to be independent of the electric field strength. Recall Equation 110,

\[ u(x) = \left( \frac{\epsilon}{\rho} \right) \frac{1}{2} E(x) \]

and the definition of ion mobility

\[ u_{ir} \equiv \beta E \]

Combining these Equations with Equation 138 results in

\[ \eta^* = \left( \frac{\epsilon}{\rho} \right) \frac{1}{2} \frac{E(x)}{\beta E(x)} = \left( \frac{\epsilon}{\rho} \right) \frac{1}{2} \frac{1}{\beta} \]

The fluid parameter \( (\epsilon/\rho) \) is not a function of electric field, and \( \beta \) has been demonstrated experimentally (References 11, 13, 16, 17 and 18) to be independent of the magnitude of the Electric field, except for very high field strengths. Therefore \( \eta^* \) is invariant with \( E \), i.e.,

\[ \eta^* = \left( \frac{\epsilon}{\rho} \right) \frac{1}{2} / \beta \neq f(E) \]

The efficiency with very low fields will be no better than that at the more usual gradients of about \( 10^5 \) to \( 10^6 \) volts per meter commonly employed in experimental
EFD research. This fact leads to the conclusion that the loss mechanism does not involve loss of ion kinetic energy to the vibration or electronic energy modes because of energetic collisions due to high \( u_{ir} \) velocity. As the field becomes very small the ion velocity collapses into the Maxwellian velocity distribution, as can be seen from a study of Figure 4. That is, as \( E \) becomes small, the \( \sqrt{2a_i\lambda} \) axis in velocity space becomes so small that the possible ion velocity vectors on the plane have values with very little difference from the thermal velocity axis.

Since the loss mechanism must be effective at very low ion drift velocities, an extremely simplified approach is permissible. By assuming a very low field strength, thereby keeping \( u_{ir} \) low, the classic rigid sphere elastic collision model of ion motion in a neutral gas is directly applicable to the molecular scale momentum exchange process. An analysis of this model is presented in Reference 13. Here it is assumed that the ion has the same mass and collision cross-section as the neutral gas molecules. The only difference permitted between ion and neutral is the charge of one electron. All inter-molecular forces and forces between ions and neutral molecules are assumed negligible. It is further assumed that the velocity increment delivered to the ion by the electric field in time \( t_i \) (mean free
time) is small compared to the thermal molecular motion of the gas.

These assumptions produce a microscopic model of ions undergoing random thermal motion, where the mean free paths are straight line segments, and all collisions are of the rigid sphere-elastic category. The ions and the neutral molecules therefore share the same mean free path magnitude, and the same thermal mean velocity.

It is reasoned, however, that the electric field will cause a small drift velocity to be superimposed upon the random thermal motion which is aligned with the field.

If \( q_i \) is the ionic charge and \( E \) is the electric field, the ions will experience a force of \( Eq_i \) and an acceleration of

\[
\dot{a}_i = \frac{Eq_i}{m_i}
\]

(164)

This acceleration may be derived from Lorentz Force (Equation 1) and Newton's law \( F = ma \). It acts on the ion during its motion along a free path. Assuming the field to be aligned with the \( x \)-axis of a Cartesian coordinate system it is seen that during each interval between collisions the ion moves a small distance \( s_x \) along the \( x \)-axis. This distance is related to the acceleration and the interval between collisions or
mean free time by the familiar relation from Newtonian mechanics,

\[ S = \frac{1}{2}at^2 \]  

(165)

where \( a \) will be given by Equation 164, \( t \) will be \( t_f \), the interval between collisions, and \( S \) will be \( S_x \), the small increment caused by the field. Thus Equation 165 may be written as

\[ S_x = \frac{1}{2} \frac{Eq_i}{m_i} \frac{t_f^2}{t_f} \]  

(166)

The average drift velocity of the ions is simply

\[ \overline{v_i} = \frac{S_x}{t_f} \]  

(167)

so that

\[ \overline{v_i} = \frac{1}{2} \frac{Eq_i}{m_i} \frac{t_f}{t_f} \]  

(168)

Now \( t_f \) is on the average given by the relation

\[ t_f = \frac{1}{\sqrt{v}} \]  

(169)

Combining Equations 168 and 169 result in

\[ \overline{v_i} = \frac{1}{2} \frac{Eq_i}{m_i} \frac{\lambda}{\overline{v}} \]  

(170)

Recognizing that \( \overline{v_i} = u_{ir} \), Equation 170 becomes

\[ u_{ir} = \frac{1}{2} \frac{Eq_i}{m_i} \frac{\lambda}{\overline{v}} \]  

(171)

This equation predicts ion drift velocities which are in disagreement with observation by an order of magnitude. Successive refinements of the theory lead
to no substantial improvement. The persistence of velocity of the ions along their original paths due to random collision angles accounted for a 6% improvement in correlation between theory and experimental data. Taking into account the true distribution of free paths actually decreased the correlation between experimental and theoretical ion mobilities. With these refinements the theoretical value for the mobility of the $O_2^+$ ion in air was obtained as $11.7 \times 10^{-4} \text{m}^2/\text{v} \cdot \text{sec}$, and the experimental determination of this quantity yielded a most probable value of $1.36 \times 10^{-4} \text{m}^2/\text{v} \cdot \text{sec}$ (Reference 16, page 123).

Several conclusions were drawn from the negative results of this investigation. An ideal mathematical model which was rigorously developed from the premise of a rigid sphere, elastic collision process with straight line free paths failed completely to predict ion mobility; ergo, the premise is false. The motion of an ion through a neutral fluid under the influence of an external electric field is not a rigid sphere elastic collision process with a straight line free path. Also, since it has been shown that efficiency and ion mobility are independent of electric field strength, it can be deduced that the cause of the non-ideal collision behavior is not field induced
velocity $v_{ie}$ (see Equation 156). If the effect of the external field is not the cause, the charge carried by the ion itself must be the cause, for it is the only factor which has not been eliminated. Without the charge, an $O_2^+$ ion would just be a discrete constituent of air, which at atmospheric temperatures and pressure could be approximated by ideal rigid sphere collision theory (Reference 12). Therefore, the charge carried by the ions themselves must be locally altering molecular free paths and velocities to such an extent that the ideal rigid sphere treatment is not applicable.

When it is realized that the electric field strength at 3A from a single electronic charge unit is about $1.6 \times 10^{10} \text{v/m}$ in comparison with about $10^6 \text{v/m}$ from the external field, it is not difficult to believe that the ionic charge significantly effects the collision process by induction of, and interaction with, dipoles in the neutral fluid molecules.

Reference 19, page 989 gives the energy of attraction between an ion and a neutral non-polar molecule as

$$\phi = - \frac{q_i^2 \alpha_n}{2r^4} - \frac{3 \alpha_n \alpha_i \hbar \nu \nu_0}{4r^6} \quad (172)$$

where $q_i$ is the ionic charge, and $\alpha_n$ and $\alpha_i$ are the polarizabilities of the neutral molecule and the ion, respec-
tively. $r$ is the distance between molecular centers, $h$ is Planck's constant, and $\nu_o$ is the frequency associated with the average effective energy difference as defined in Reference 19, page 967. Reference 19, page 32 also shows the intermolecular potential function for a non-polar pair, i.e., the Lennard-Jones (6-12) potential

$$\phi = 4\epsilon^* \left[ (\sigma/r)^{12} - (\sigma/r)^6 \right]$$

(173)

where $\epsilon^*$ is the maximum energy of attraction (depth of the potential well).

It is significant to note that in addition to the inverse 6th power attractive force experienced by a non-polar pair of neutral molecules, the ion-neutral potential exhibits an extremely strong inverse 4th power attractive force which should dominate the interaction. Reference 19, page 989 shows that at a value of $r$ corresponding to the collision diameter, the ionic induction energy may easily be an order of magnitude larger than the London dispersion energy (the $1/6$ power attraction energy).

It was found that the classic rigid sphere elastic collision approach failed to obtain realistic results for the prediction of $\mu_{ir}$ or mobility. From this failure it was possible to deduce that the intense local field about the ions completely changes the characteristics of the collision process. Reference 13
reports a theoretical treatment (by Wellisch) of the effects of an ion's electric field upon the collision process in a non-polar gas. The results show that due to the electrostatic forces generated between an ion and an induced dipole in a non-polar molecule, the probability of a collision is greatly increased. The resultant shortening of the mean free path is reflected as an apparent increase in the collision diameter.

The increased or effective collision diameter may be written as

\[
\frac{\sigma_e}{\sigma} = \left[ 1 + \left( 2 \frac{|U|}{\sigma m v^2} \right) \right]^{\frac{1}{2}}
\]

(174)

where \(\sigma_e\) is the effective ion-neutral collision diameter, \(\sigma\) is the "classic" collision diameter, \(|U|\) is the absolute value of the potential energy between an ion and neutral molecule evaluated at a separation of one "classic" collision diameter, \(m\) is the mass of the neutral molecule, and \(v^2\) is the mean square thermal velocity based on a Maxwellian velocity distribution.

The potential energy was obtained as (Reference 13).

\[
U = - \left( \epsilon - 1 \right) q_1 \frac{2}{\sqrt{8\pi N_L r^4}}
\]

(175)

where \(N_L\) is the Loschmidt number and all other symbols are as previously defined. Note that if \((\epsilon - 1)/4\pi N_L\) is taken as the polarizability \(\alpha_n\), Equation 175 is identical to the induction term in Equation 172.
By combining Equation 174 and 175 and evaluating the potential at \( r = \sigma \), the effective collision diameter is

\[
\left( \frac{\sigma_e}{\sigma} \right) = \left[ 1 + \left\{ (\epsilon - 1)q_i^2/4\pi m v^2 N_L \sigma^4 \right\} \right]^{-1} \tag{176}
\]

Note that as the ionic charge goes to zero, the right side of Equation 176 collapses to unity, and the effective collision diameter becomes equal to the "classic" collision diameter.

In order to apply this correction to the rigid sphere elastic collision model (Equation 171) derived for ion drift velocity, the mean free path must be related to the effective collision diameter. Reference 12, page 323 gives

\[
\lambda = \left[ 2^{\frac{1}{2}} N_\pi \sigma^2 \right]^{-1} \tag{177}
\]

Defining an effective mean free path \( \lambda_e \), which will take into account the effective collision diameter, it can be shown from Equation 177 that

\[
\left( \frac{\lambda_e}{\lambda} \right) = \left( \frac{\sigma}{\sigma_e} \right)^2 \tag{178}
\]

This can be written as

\[
\lambda_e = \lambda \left( \frac{\sigma_e}{\sigma} \right)^{-2} \tag{179}
\]

Combining Equations 176 and 179 results in

\[
\lambda_e = \lambda \left[ 1 + (\epsilon - 1)q_i^2/4\pi m v^2 N_L \sigma^4 \right]^{-1} \tag{180}
\]

Now substituting \( \lambda_e \) from Equation 180 into Equation 171 to account for the intense inverse 4th power energy potential of the ions gives
\[ u_{ir} = \frac{1}{2} \left( \frac{\text{Eq}i}{m_i} \right) \left( \frac{\Lambda}{\nu} \right) \left[ 1 + \left( \epsilon - 1 \right) q_i^2 / 4 \pi m v^2 N_L \sigma^4 \right] \] (181)

Evaluation of this equation for ions in air indicates a mobility \( u_{ir}/E \) of \( 2.85 \times 10^{-4} \text{ m}^2/\text{v} \cdot \text{sec} \). This equation yields the right order of magnitude for ion drift velocities. However, there is still considerable discrepancy with the experimental value of \( 1.36 \times 10^{-4} \text{ m}^2/\text{v} \cdot \text{sec} \) (for \( O_2^+ \)).

The improvement in the prediction of ion drift through a neutral gas under the influence of an external electric field, by considering the ionic charge induction contribution to the collision potential function, will be considered as verification of the conceptual ion-neutral collision model, i.e., increased effective collision diameter. Equation 181 will therefore be offered as a description of the essential parametric dependence of the ion relative velocity.

In order to establish the molecular scale dependence of \( \eta^* \), an expression for the mean induced fluid velocity \( u \) must be obtained.

Reference 12, page 323 gives the collision frequency for molecules with mean thermal velocity \( \overline{v} \) as

\[ f_c = 1.276 \pi \sigma^2 \overline{v} \] (182)

This expression will describe neutral-neutral collisions as is, i.e.,

\[ f_{cn} = 1.276 \pi \sigma^2 \overline{v} \] (183)
Assuming a low external electric field strength, the ion may be considered (as discussed in the derivation of Equation 181) to have thermal velocity \( \tilde{v} \) if the ionic induction forces are corrected for by the use of the effective collision diameter, ergo

\[
\frac{f_{ci}}{f_{cn}} = 1.276 \frac{N \pi \sigma_e^2 \tilde{v}}{} \tag{184}
\]

In this case the collision ratio \( \Gamma \), defined as the number neutral-neutral collisions per ion-neutral collision, is simply,

\[
\Gamma = \frac{f_{cn}}{f_{ci}} \frac{N_n}{N_i} \tag{185}
\]

which, from Equations 183 and 184, may be written as

\[
\Gamma = \left( \frac{\sigma^2}{\sigma_e^2} \right) \frac{N_n}{N_i} \tag{186}
\]

Assume that at a given location \( x \) in the EFD gap, both bulk velocity \( u \), and ion relative velocity \( u_{ir} \) are constant and the fluid on a continuum scale is being uniformly accelerated. Since there are not enough ions to collide with each neutral molecule at every point, the neutral molecules that do collide with ions must pass a portion of the momentum gained in the collision on to other neutral molecules in neutral-neutral collision. On the average the momentum passed from an ion to a neutral molecule must be shared in \( \Gamma \) neutral-neutral collisions before that molecule is likely to collide with another ion. The process is straightforward, but very difficult to
treat in explicit mathematical terms. Therefore, an over simplified approach will be used to obtain a crude description of major characteristics.

Consider a single ion introduced (at \( t = 0 \)) into \( N_n/N_i \) neutral molecules. After an elapsed time of \( t \) the molecules struck by the ion will have suffered the following number of collisions.

- 1st molecule: \( f_{cn} t - \frac{1}{f_{ci}} \)
- 2nd molecule: \( f_{cn} \left( t - \frac{1}{f_{ci}} \right) - 1 \)
- 3rd molecule: \( f_{cn} \left( t - \frac{2}{f_{ci}} \right) - 1 \)

or in general the total number of collisions resulting from \( i \) collisions of the ion which may convey \( x \)-momentum from the electric field is given by

\[
N_{n}^x = \sum_{i=1}^{n} 2 f_{cn} \left( t - \frac{i-1}{f_{ci}} \right) \quad (187)
\]

It must be noted that Equation 187 implies that each neutral-neutral collision imparting net \( x \)-directed momentum involves a molecule which has, through a chain of collisions, obtained net \( x \)-momentum, and a molecule undergoing random thermal motion. This neglects the fact that as the total number of ions having net \( x \)-momentum increases, the probability of the neutral-neutral collisions in which one molecule has not yet received any \( x \)-momentum decreases sharply. Therefore, the approach utilized is not quantitatively accurate, but
should reveal the qualitative picture, and is much easier to apply. Since a continuum expression has already been obtained which accurately predicts the magnitude of the power loss (Equation 134) and since the objective here is an understanding of the molecular scale mechanisms responsible for the loss, the crude approach chosen is felt to be justified.

Continuing with the analysis, n in Equation 187 is the total number of collisions suffered by the ion in time t. Since the ion-neutral collision frequency is \( f_{ci} \), n may be written as

\[ n = f_{cit} \]  \hspace{1cm} (188)

Equation 187 becomes

\[ N_n^x = \sum_{i=1}^{N_n^x} f_{ci} t \left( i - \left( i - 1/f_{ci} \right) \right) \]  \hspace{1cm} (189)

As noted, there are a total of \( (N_n/N_i) \) neutral molecules, and they will all be effected (in this rough approach) by the ion in time \( t' \). Therefore, \( t' \) is sort of a momentum diffusion time. During this time the x-momentum imparted by the ion diffuses through the bulk fluid imparting a net x-velocity component of u.

If a marble is shot along the x-axis into a cluster of stationary marbles, the "shooter" will strike relatively few marbles, but the momentum is transferred from marble to marble until the imparted momentum has diffused through and the entire cluster
is in motion. If this simple experiment is tried it will be noticed that only a very slight x-directed motion results in the marble cluster. The most noticeable effect is that the cluster increases in size, with marbles moving away in every direction from the path of the incident "shooter". This situation is analogous to the case of the ion (with increased $\sigma$ to account for induction potential) moving through $(N_n/N_i)$ neutral ions.

As with the marbles, the reaction of the neutral molecules during momentum diffusion will be a small x-directed motion, but a random expansion is the prime result. In an EFD flow outward moving neutrals are quickly replaced (constant pressure operation) with other neutral molecules, and an overall equilibrium results. The momentum "scattering" which results merely serves to increase the system entropy, and is responsible for power loss.

To estimate the importance of this loss mechanism consider the details of a neutral-neutral (elastic spheres) collision. Let one molecule be motionless, and the other approach it will $v_r$ relative velocity. Let $\theta$ be the angle between $v_r$ and a line connecting the centers of the molecules at the instant of impact. If all angles are equally probable, the probability
density function for \( \theta \) is (Reference 13).

\[
f(\theta) = 2 \sin \theta \cos \theta \quad (190)
\]

The mathematical expectation of \( \theta \) can be obtained as

\[
\text{Exp}(\theta) = \int_{-\infty}^{\infty} \theta f(\theta) d\theta = \bar{\theta} \quad (191)
\]
or in this case

\[
\bar{\theta} = \int_{0}^{\pi/2} 2 \sin \theta \cos \theta \, d\theta \quad (192)
\]

The resultant integration gives

\[
\bar{\theta} = \pi/4 \quad (193)
\]

If on the average,

\[
v_r = u_i - u = u_{ir} \quad (194)
\]
it is clear that the molecule being stuck will receive (on the average) a velocity of \( (u_{ir})\cos \theta \) in the \( \theta = 0 \) direction. The tangent component \( (u_{ir})\sin \theta \) is not transmitted during the collision, but is retained by the initial molecule. The x and y velocity components of the struck molecule will be \( (u_{ir})\cos \theta \cos \theta \) and \( (u_{ir})\cos \theta \sin \theta \). With the expected (mean) value of being \( \pi/4 \), the result is

mean x-velocity \( \quad 0.5(u_{ir}) \)
mean y-velocity \( \quad 0.5(u_{ir}) \)

With a molecular mass of \( m \) this means that on the average \( 0.5(u_{ir})m \) x-momentum is transferred during each collision.
In a chain of collisions, such as the momentum diffusion process governed qualitatively by Equation 189, the momentum transfer per molecule is \( \left[ \frac{1}{2} (u_{ir} m) \right]^j \), where \( j \) is collision number or the position of the molecule in the collision chain. Since \( (\frac{1}{2})^{10} \) is less than \( .001 \) within 10 collisions, virtually all of the momentum \( (u_{ir}) m \) is lost, and the associated energy is lost to an increase in system entropy. The number of ion-neutral collisions in time \( t \) is \( N_{i}f_{c}t \), and the number of ion neutral collisions necessary to diffuse the x-momentum from the ions can be deduced from Equation 189. However, it is obvious that the \( (u_{ir}) m N_{i} \) momentum will be completely dissipated since the collision ratio is on the order of \( 10^{4} \) for most EFD conditions.

A direct closed form statistical thermodynamic solution could be obtained which would verify the continuum efficiency equation, but again it is pointed out that the sole objective of the statistical investigation was the deduction of the prime loss mechanism. The rigorous approach from this foundation to the overall system performance is much too difficult to attempt here. Instead, the fact that the loss mechanism crudely described is consistent with the continuum model will be shown through the use of an electrical analog. Consider the circuit shown in Figure 6.
The driving force in EFD flow is provided by the electric field $E$. For a simple resistive network its analog is the potential drop provided by a constant voltage power supply. The ion relative velocity $u_{ir}$ and fluid velocity $u$ are analogous to currents $I_1$, and $I_2$ respectively. To be consistent, the analogs of shunt resistance $R_\beta$ and load resistance $R_n$ must correspond to the real linear impedances relating ion and flow velocities to the driving function. The relationship between relative ion velocity $u_{ir}$ and the driving field $E$ was obtained in Equation 181. It is
seen that the relationship is indeed linear. Solving Equation 181 for the impedance to ion relative flow and substituting the definition for mobility gives

\[ \frac{E}{u_{ir}} = \frac{1}{\beta} \]  

(195)

No relationship between the electric field and \( u \) was obtained from statistical considerations due to the difficulty in expressing ion momentum diffusion through the neutral fluid in an accurate and usable form. The analysis was terminated when the mechanism of power loss was sufficiently clear. However, it can be rationalized that ion velocity has been demonstrated to be linearly related to the electric field (Equation 181), and since the fluid velocity \( u \) depends upon momentum transfer from the ions, the relationship of \( u \) to \( E \) is necessarily linear. With this justification of the continuum results, Equation 110 can be used to supply the needed fluid flow impedance, i.e.,

\[ \frac{E}{u} = (\epsilon/\rho)^{-\frac{1}{2}} \]  

(196)

The electrical analogy for an EFD propulsion system based upon this pseudo-statistical mechanics analysis is summarized in Table 1.

Within this analogy the circuit in Figure 6 describes the EFD system performance very well. Consider for instance that the current \( I_2 \) is independent of shunt resistance. This implies, in agreement with
continuum results (Equation 129, 130, 131 and 132), that velocity, mass flow, thrust, and thrust power are independent of mobility $\beta$. The effect of the shunt is to increase the total current and to dissipate $I_1^2 R_\beta$ as heat, while the load still only uses $I_2^2 R_n$. This is in agreement with the statistical analysis which indicated that $(u_{ir})N_{im}$ momentum per unit volume simply dissipated as increased random motion, i.e., an increase in entropy and bulk gas temperature. Finally, the power efficiency of the analogous circuit (Figure 6) is easily shown to be

$$\eta = \frac{I_2^2 R_n}{I_2^2 R_n + I_1^2 R_\beta}$$

(197)

With Ohms resistive law,

$$V = IR$$

(198)

and the analogous relations in Table 1, Equation 197 can be written in the identical form of the prime result of the continuum analysis, (Equation 134 or 137).

### Table - 1 EFD-Electric Circuit Analog

<table>
<thead>
<tr>
<th>Driving Potential</th>
<th>Impedance</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta V = E$</td>
<td>$R_n = (\epsilon/\rho)^{-\frac{1}{2}}$</td>
<td>$I_2 = u$</td>
</tr>
<tr>
<td></td>
<td>$R = (1/\beta)$</td>
<td>$I_1 = u_{ir}$</td>
</tr>
</tbody>
</table>
The significance of this result is that as long as ion velocity is greater than fluid velocity, efficiency will suffer and no other factor can be manipulated to offset this effect. The prime loss mechanism is entirely analogous to $I^2R$ heating losses in a resistive circuit. The resistance is $\beta^{-1}$ and the current is $u_{ir}$. For a given voltage, the only way to decrease power loss is to increase the resistance, i.e., to decrease $\beta$. 
V. COMPARISONS

The general governing equations for EFD flow were presented in Section II and followed by a discussion of work presented in the literature. When it became apparent that none of the mathematical models offered in the literature provided a suitable starting point for an investigation into power loss mechanisms, the general equations were simplified and solved in closed form in Section III to provide the needed starting point. This starting point was an efficiency equation (Equation 134) which lead to the conclusion that the parameter to analyse in detail for an understanding of power loss mechanisms was the ratio of fluid velocity to relative ion velocity, (Equation 138). A byproduct of the analysis in Section III was a set of equations which predict EFD propulsion system velocity, mass flow rate, thrust and thrust power, (Equations 129, 130, 131, and 132).

In Section IV the efficiency governing velocity ratio, Equation 138, obtained from the continuum analysis in Section III was analysed on a molecular scale to obtain the prime loss factor. The loss mechanism was qualitatively revealed to be an increase in system entropy associated with the diffusion of mo-
mentum associated with \( u_{ir} \). The loss mechanism was found analogous to resistive heating in an electrical system. To substantiate these findings an analogous electrical network was developed from pseudo-statistical arguments and shown to reduce to the exact results obtained from the continuum analysis in Section III. To further substantiate this work, it is necessary to correlate the mathematical model obtained in Section III with available pertinent data.

As pointed out in Section III, the equations obtained describing EFD system operation do not consider the ionization process. Since all applicable experimental data have been obtained using needle points as ion emitters, the effects of needle point emission was considered at some length in Appendix A. The results indicated that needle point current was really governed by the reduced voltage \( (\Delta V_L - \Delta V_L^*) \), where \( \Delta V_L^* \) is the voltage drop required to just maintain a conductive condition. A needle point geometry factor was also found to exist. Unfortunately the integration constants were not obtained and the best that could be done is a comparison to an empirical reference condition. In any case, since the exact needle geometry used in generating available data is not known, how to correct the data for these effects is a moot point.
The equations derived in Section III were changed to reflect needle point emission coupled with the EFD flow by simply replacing the potential drop $\Delta V_L$ with the reduced voltage indicated from the analysis in Appendix A. The resulting model, for purposes of correlation with data taken using needle emitters is given below.

$$u_L = (2/L)(\varepsilon/\rho)^{1/2}(\Delta V_L - \Delta V_L^*)$$  \hspace{1cm} (199)

$$\eta = \left[ \frac{(\varepsilon/\rho)^{1/2}}{\beta + (\varepsilon/\rho)^{1/2}} \right] \frac{\Delta V_L - \Delta V_L^*}{\Delta V_L}$$  \hspace{1cm} (200)

Performance correlation is limited to a consideration of exit velocity for the EFD system. This is justified because $u_L$, for constant pressure and geometry, uniquely determines the mass flow rate, thrust and power output of the system.

The correlation of velocity is difficult because of the $\Delta V_L^*$ factor. Consider the factor $Lu_L$, which is dependent only upon $(\Delta V_L - \Delta V_L^*)$ for a given fluid condition.

$$Lu_L = 2(\varepsilon/\rho)^{1/2}(\Delta V_L - \Delta V_L^*)$$  \hspace{1cm} (201)

Unless the $\Delta V_L^*$ term is known accurately considerable scatter may result in the plotting of data. $\Delta V_L^*$ is a function of needle tip field strength and therefore will be a function of needle point geometry.

If all the data were generated using geometrically
similar needles, and an empirical reference case was available, the results of Appendix A would permit an estimate. Since this is not the case, unless $\Delta V_L^*$ is reported in the literature along with the data, which it wasn't, the value of correlating is somewhat nebulous. If a sample of data is used to estimate $\Delta V_L^*$, a good fit of the data will result and nothing will be learned for all mathematical models offered agree that velocity is directly proportional to voltage. The only data available to the author where $\Delta V_L^*$ is known is that presented in Reference 2. This data is plotted against Equation 199 in Figure 7. The line of best agreement

![Diagram of velocity correlation](attachment:image.png)

Figure 7 - Velocity Correlation
(Reference 10), is plotted because the experimental set-up closely paralleled that employed in the generation of data in Reference 2.

In comparing the efficiency, Equation 200, with available data the ionization efficiency was dropped, and Equation 134 was plotted in Figure 8. This is justified in this case because at very low efficiencies, about 1%, a 10% or 20% change in that 1% leaves it still about 1%. Also at the higher efficiencies, where a 10% or 20% change would make a notable difference, the fluid (insulating oil) will support an order of magnitude greater field strength than at the low efficiencies and \( \Delta V_L \) is usually so much higher than \( \Delta V_L^* \) that the ionization efficiency term is very nearly unity.

It can be seen in Figure 8 that Equation 134 adequately describes the efficiency of the EFD momentum transfer process over the entire range from less than 1% to nearly 100%.

As for comparisons between mathematical models, suffice it to point out that in contrast to other closed form solutions offered, the voltage-current relation remains finite in the limit as \( \beta \) goes to zero. Also, the efficiency relation does not fine some artificial limit as the cause of the inefficiency is removed.
Figure 8 - Efficiency
VI. CONCLUSIONS AND COMMENTS

A. Conclusions

The prime objective of this research was the determination of the cause of low efficiency in the EFD propulsion scheme (described in Section II) when air at atmospheric temperature and pressure is the working fluid. Due to the lack of a suitable mathematical model upon which to base this effort, one was developed in Section III from the general governing equations, (Equations 1 thru 11). The result of this continuum approach was a set of performance equations (Equations 129 thru 133) and for the EFD momentum transfer process efficiency (Equation 134).

In Section IV Equation 134 was used to guide a molecular scale, statistical mechanics investigation into the cause of inefficiency. The result of this statistical investigation was then used to construct an analogous linear circuit, which was demonstrated to behave in agreement with the performance equations (Equations 129 thru 133) obtained from the continuum analysis in Section III. Moreover, it was shown that the power efficiency of the analogous circuit could be reduced to an identity with Equation 134. For further verification of the applicability of the results of
this research work the mathematical model was correlated with applicable available data in Section V. Qualitative and quantitative agreement was obtained. Because of this success the analysis, although admittedly incomplete in the statistical treatment of loss mechanisms, is offered as adequate for drawing the following conclusions with regard to efficiency:

1) The prime loss factor is the increase in system entropy caused by the loss of x-directed momentum (associated with the ion relative velocity $u_{ir}$) during x-directed momentum diffusion through the neutral gas as a consequence of random molecular collision phenomena. Specifically, x-directed momentum (associated with $u_{ir}$) delivered to a neutral molecule during a collision with an ion will be essentially lost to random motion within 10 subsequent collisions of the molecule with other neutral molecules.

2) This loss mechanism is analogous to resistive heating losses, where $u_{ir}$ is analogous to current, $(1/\beta)$ is the analogous resistance, and the electric field strength $E$ is analogous to the voltage drop across the resistance. For a constant voltage supply, the only way to cut down on $I^2R$ heating
losses is to increase $R$ thereby decreasing $I$. Likewise the only manner in which EFD losses may be curtailed is to increase the analogous resistance, i.e., decrease $\beta$.

B. Comments

The results of the efficiency investigation show that the only way to increase the efficiency of the process is to decrease the ion mobility. Since at moderate field strengths ion mobility is independent of the electric field, the pulsed voltage technique suggested by Christenson and Moller (Reference 10) would not increase efficiency at all. It would merely decrease the flow velocity, mass flow, etc., by lowering the effective field. Robinson (Reference 7) suggested colloidal particles suspended in air as charge carriers. Such particles (References 4, 11, 16, and 18) as dust, small water droplets, etc., have extremely low values of mobility. It is reasonable to expect that with proper study and experimentation, ions could be prepared from water or water suspensions which could yield efficiencies on the order of 70% to 80%. It appears therefore that the efficiency problem can be handled relatively well.

As pointed out in Section I, a flight vehicle with an EFD propulsion system appears very desirable.
However previous work indicated that efficiencies were much too low to be practical. With the benefit of this analysis, Robinson's suggestion can be affirmed. Colloidal particles (dust, water, etc.) used as charge carriers in the EFD process would raise the efficiency to a sufficiently high level. Assuming that this is done, consider the impact of the thrust relation (Equation 131) upon potential EFD design. Equation 131 rewritten for convenience is

\[ F = \left(\frac{4A_I}{L^2}\right)\epsilon(\Delta V_L)^2 \]

Optimum geometry will correspond to maximizing the quantity \(\frac{4A_I}{L^2}\). Maximum thrust will be obtained if \(\Delta V_L\) is as high as possible without general electrical breakdown and sparking in the EFD gap. If the electrodes are plane and parallel, the maximum applied voltage in air is nearly a linear function of the product of EFD gap length \(L\) and the pressure \(p\) (Reference 9, page 25). This relation may be approximated as

\[ (\Delta V_L)_{\text{max}} = 2.6(10^4)pL \quad (202) \]

where \(p\) is the ambient pressure in atmospheres, \(L\) is the gap length in centimeters and \((\Delta V_L)_{\text{max}}\) is the maximum applied voltage.

Note from Equation 202 that \((\Delta V_L)_{\text{max}}/L\) is a constant for a given pressure. Assuming \(p = 1\) atm. and \(L = 1\) cm., \((\Delta V_L)_{\text{max}} = 26,000\) volts. Applying these
values to the thrust equation, after dividing both sides by the EFD accelerator area \( A \), results in

\[
\frac{F}{AL} = 239 \frac{(N/m^2)}{-} = 5 \frac{lb_f}{ft^2}
\]

The maximum thrust per unit area per accelerator stage is about \( 5 \frac{lb_f}{ft^2} \). A practical accelerator using water droplets as the charge carrier should easily produce a working thrust per unit area per accelerator stage of about \( 1 \frac{lb_f}{ft^2} \).

Consider an aircraft of about 20,000 \( lb_f \) gross weight. A well designed 10 stage accelerator should be able to maintain a working thrust of \( 10 \frac{lb_f}{ft^2} \) per unit lifting area. If the cabin, power supply, and water tanks occupy a 30 foot diameter saucer shaped area surrounded by an additional 55 foot diameter accelerator (flying saucer style), about 5000 \( ft^2 \) of lifting area is obtained. At sea level this would correspond to a working lift force of about 50,000 \( lb_f \).

The ceiling is fixed by Equation 202 which indicates that \( (\Delta V_L)_{\text{max}}/L \) is directly proportional to pressure. As pressure falls with altitude the maximum voltage must decrease proportionately to avoid electrical breakdown. Thrust is proportional to \( [(\Delta V_L)_{\text{max}}/L]^2 \), (see Equation 131) and therefore maximum thrust falls off with the square of the pressure drop. This consideration establishes the ceiling for this hypothetical
vehicle at about 12,000 feet. During maximum power operation at sea level the power supply would have to put out about 30,000 volts for a power consumption (see Equation 133, with $\eta = 0.80$) of 1.53 megawatts.

The development of practical EFD powered aircraft depends upon the availability of a suitable high voltage power supply. It is doubtful if anything presently available meets the requirements of light weight and high power output. As such power supplies become available, EFD powered flight will become a potential reality. In the mean time there is much work to be done upon the ionization and droplet suspension system. It is recommended that future research in EFD propulsion be oriented toward providing an efficient droplet charging and dispersion system, and toward development of optimum multistage accelerators. If this needed research is provided, a practical accelerator-airframe design can be available by the time a suitable power supply is developed.
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IX. VITA

Roger Coy Crites was born on 10 January 1943 in Youngstown, Ohio. He received his primary and secondary education in Berlin Center and Mineral Ridge, Ohio. He received a Bachelor of Aeronautical and Astronautical Engineering from Ohio State University in June of 1966, and has been a full time employee of the McDonnell Aircraft Co. in St. Louis, Missouri since June of 1966. His prime responsibility is the development of advanced concepts in gas dynamics technology.

During his employment he has received divisional commendation, St. Louis Section AIAA Meritorious Technical Contribution, and cost reduction awards. He has authored several technical papers (AIAA, STA, and SVS) on topics including miniature inlets in hypersonic slip flow, high response pressure measurement techniques, analog computer technology, and statistical analysis of random data.

He has been enrolled in the Graduate School of the University of Missouri-Rolla since September, 1966.
APPENDIX A
SOME CONSIDERATIONS OF NEEDLE POINT EMISSION

All available experimental data concerning EFD flow have been obtained in EFD systems using needle point discharge at the emitter. It is therefore necessary to investigate the dependence of EFD system performance upon needle point emission effects.

The fundamental phenomena occurring at a charged needle point is rather complex and not too well understood. Some experimental data is available which is descriptive of needle point to plane electrode operation, but no descriptive theoretical approach was found in the literature. It is known (References 11, 17, 18) that the current from a needle point actually occurs as high frequency pulses. These pulses apparently occur in the following cycle:

1) The field at the tip of the needle is so intense that the fluid in this region is ionized locally, producing a burst of ions.
2) The space charge rises in the vicinity of the needle point reducing the field strength rapidly to the point where ionization stops.
3) The space charge at the needle point is cleared and as a result the field strength rises again until another ionization burst starts the cycle over.
This process could be explained in terms of the applicable general governing equations given in Section II, but the process is so complex that the resultant set of simplified equations could not be easily solved. Therefore, another technique is employed. This technique has often proven invaluable for engineering purposes by providing a working mathematical model of a complex physical phenomena from a simple analogous electrical network. The network chosen to represent the time dependent operation of a needle point in a corona discharge is shown in Figure A-1.

![Figure A-1 - Equivalent Network](image)
The analogy is most easily described by following the cycle through step by step.

1) The voltage across the capacitor C becomes so high that $V_f$, the firing voltage of the neon lamp is reached. The lamp fires, its resistance which was infinite becomes nearly zero and a burst of current flows from the capacitor through $R_2$.

2) As this burst of current passes through $R_2$ the voltage across the capacitor rapidly falls to the cut-off valve of $V_c$ at which the resistance of the neon lamp returns to infinity and the current $I_2$ goes to zero.

3) The voltage across the lamp rises again as current $I_1$ charges the capacitor until the firing voltage $V_f$ is obtained and the cycle starts over.

The firing of the neon lamp is analogous to the ion burst from a needle point. The discharge of the capacitor, and subsequent rapid decrease in $V_1$, is analogous to the build-up of local space charge and rapid fall of potential gradient at the point. The cut-off voltage $V_c$ corresponds to the value of potential gradient at which emission stops, and the recharging of the capacitor through $R_2$ corresponds to the clearing of the space charge. $V_0$ is the applied potential.
It will be specified that the charging current to the capacitor is small in comparison with the discharge current through the lamp. It should be noted that even though $I_2$ and $I_1$ are different, they must convey the same total charge in any given cycle. In this case the time required for a current pulse is given by

$$t_p = (R_2 C) \ln \left( \frac{V_f}{V_c} \right) \quad (A-1)$$

The time required for the circuit to recover from this pulse, regain its charge, and ready itself for another pulse is given by

$$t_c = (R_1 C) \ln \left( \frac{V_o - V_C}{V_o - V_f} \right) \quad (A-2)$$

The total time from the start of one pulse until the start of the next is the pulse period $T_p$, and is simply the sum of the pulse width and the circuit recovery time.

$$T_p = t_p + t_c \quad (A-3)$$

However, it has been assumed that,

$$I_1 << I_2 \quad (A-4)$$

and it can be shown that,

$$\left[ I_1 << I_2 \right] \subset \left[ t_p << t_c \right] \quad (A-5)$$

If it is also assumed that the Ne-2 bulb cut-off voltage is negligible with respect to the firing voltage, it follows that

$$T_p \approx t_c = R_1 C \ln \left( \frac{V_o}{V_o - V_f} \right) \quad (A-6)$$
or, since the frequency of pulsation is the inverse of the period

\[ f = \left[ R_1 C \ln \left( \frac{V_0}{V_0-V_f} \right) \right]^{-1} \]  
(A-7)

The pulse charge, \( q_p \), may be evaluated by considering the definition of capacitance, Reference 20

\[ \frac{dq}{dV} = C \]  
(A-8)

which may be integrated under the present assumptions to yield

\[ \int_{\text{pulse}} dq = C \int_{0}^{V_f} dV \]

or

\[ q_p = CV \]  
(A-9)

The current pulses are repetitive and therefore the mean value of current through the lamp may be calculated from

\[ I = \frac{q_p}{T} = f q_p \]  
(A-10)

This equation simply states that the mean current must be the total charge in a pulse \( q_p \), divided by the total time between pulses, which is the same as the charge per pulse multiplied by the frequency of the pulses.

Equations A-7, A-9, and A-10 may be combined to yield the analogous operating relations.

\[ q_p = CV^* \]  
(A-11)

\[ f = \left[ Z_f C \ln \left( \frac{V_o}{V_o-V^*} \right) \right]^{-1} \]  
(A-12)
and

\[ I = V^* \left[ Z_f \ln \left( \frac{V_o}{V_o - V^*} \right) \right]^{-1} \]  

(A-13)

where \( V^* \), the voltage where emission begins, replaces \( V_f \), the neon bulb firing voltage, and \( V_o \) becomes the applied voltage across the EFD gap. \( Z_f \), the impedance of the fluid in the EFD gap, replaces the resistance \( R_1 \). \( C \) represents the capacitance of the needle point.

These results describe a system which pulsates at a frequency which is nearly linear for values of \( V_o/V^* \) greater than about 1.5 and which ceases to operate at \( V_o/V^* \) of unity. Figure A-2 shows the shape of voltage-frequency characteristic.

![Figure A-2 - Frequency And Current Characteristic](image)
This is identical to the voltage-current characteristic since Equation A-11 indicates that the charge per pulse is constant for a given needle point geometry. \( V^* \), the emission starting voltage, would depend upon needle point geometry because geometry controls the electric field, and emission will occur when the local field strength becomes critical.

It is this geometric effect that causes the electric field to intensify as the point of a needle is approached, and which accounts for the ability of needles to ionize a gas at an applied potential which produces mean field strength well below the theoretical ionization value. It is apparent that these effects must be examined in more detail in order to gain insight into the dependence of the current voltage relation upon needle point geometry.

The field in the immediate vicinity of a curved needle point cannot be treated as a one-dimensional problem in a Cartesian coordinate system. A further complication is that the boundary conditions would be very difficult to implement. Because of these considerations it was decided to abandon the Cartesian coordinate system. The needle, regardless of how sharp it feels, appears blunt and rounded when viewed under a microscope. After careful examination of several needle points the mathematical idealization chosen was a pra-
boloid of revolution. By an appropriate selection of focal length the needle point model agrees very well in shape with many needles. Luckily, as pointed out in Reference 21, an orthogonal parabolic coordinate system permits separation of variables in the general Laplace equation. Because of this, and the relative ease with which the boundary conditions may be expressed for a paraboloid of revolution, this coordinate system was chosen for the needle point phenomena investigation. Figure A-3 is a sketch of the coordinate system that is employed in the following investigation.

![The Coordinate System](image)

Figure A-3 - The Coordinate System
The needle is represented by the paraboloid of revolution, \( \nu = \text{constant} \), where the Cartesian coordinate system shown has its origin at the focal point. This paraboloid is generated by rotating the parabola

\[
Z = \left( \frac{x^2}{4Z_f} \right) - Z_f
\]  

(A-14)

about the Z-axis.

The "diameter" of the needle will be defined somewhat arbitrarily as the diameter at the focal length of the paraboloid, which is simply

\[
d_f = 4Z_f
\]  

(A-15)

Assume that just prior to a pulse the space charge has been cleared so that in the immediate vicinity of the needle point the space charge density is essentially zero. Equation 3 reduces to

\[
\nabla^2 (\epsilon E) = 0
\]  

(A-16)

Recalling that \( \epsilon = \text{constant} \), and that

\[
E = -\nabla V
\]

Equation A-16 may be written as

\[
\nabla^2 V = 0
\]  

(A-17)

The form of the Laplacian in the parabolic coordinate system is

\[
\frac{1}{(\mu^2 + \nu^2)} \left[ \frac{\partial^2 V}{\partial \mu^2} + \frac{1}{\mu} \frac{\partial V}{\partial \mu} + \frac{\partial^2 V}{\partial \nu^2} + \frac{1}{\nu} \frac{\partial V}{\partial \nu} \right] + \frac{1}{\mu^2 \nu^2} \frac{\partial^2 V}{\partial \psi^2} = 0
\]  

(A-18)

Since the potential must be the same at every point on the surface of a conductor (Reference 20) \( V \) will be
a constant on the surface of the needle point. This makes \( V \) independent of \( \mu \) and \( \psi \) simultaneously. Equation A-18 simplifies to

\[
\frac{d^2 V}{d\nu^2} + \frac{1}{\nu} \frac{dV}{d\nu} = 0 \quad (A-19)
\]

The solution to this equation is

\[
V = A + b \ln \nu \quad (A-20)
\]

where \( A \) and \( b \) are constants.

Recalling that \( \vec{E} = -\nabla V \), the gradient of \( V \) is

\[
\nabla V = \frac{1}{(\mu^2 + \nu^2)^{1/2}} \left[ \frac{\mu}{\mu} \frac{\partial V}{\partial \mu} + \frac{\nu}{\nu} \frac{\partial V}{\partial \nu} \right] + \left( \frac{\psi}{\nu \mu} \right) \frac{\partial V}{\partial \psi}
\]

or where \( V \) is defined by Equation A-21, the electric field becomes

\[
\vec{E} = \frac{\frac{\hat{\nu}}{\mu^2 + \nu^2}^{1/2} \frac{B}{\nu}} \quad (A-21)
\]

At the surface of the needle the electric field strength is obtained by evaluating Equation A-21 at \( \nu = \nu_s \).

\[
\vec{E} = \frac{\frac{\hat{\nu}}{\mu^2 + \nu_s^2}^{1/2} \frac{B}{\nu_s}} \quad (A-22)
\]

This equation demonstrates clearly the geometrical effect upon electric field. It is seen that the maximum field intensity occurs at the tip of the needle, \( \mu = 0 \). Equation A-22 implies that the field
at the point of an entire family of needles, generated by \( \nu = \text{constant} \) paraboloids, can be determined by the relation,

\[
E(0, \nu_1) = E(0, \nu_r) \frac{\nu_r^2}{\nu_1^2} \quad (A-23)
\]

where \( E(0, \nu_1) \) is the field at the point of an arbitrary needle surface \( (\nu_1 = \text{constant}) \), \( E(0, \nu_r) \) is the point field of a known reference geometry \( (\nu_r = \text{constant}) \). Once \( E(0, \nu_1) \) has been obtained the complete distribution of field intensity over the arbitrary needle may be obtained from

\[
\frac{E(\mu, \nu_1)}{E(0, \nu_1)} = \frac{\nu_1}{(\mu^2 + \nu_1^2)^{1/2}} \quad (A-24)
\]

or, in terms of the reference needle, the general family is obtained from

\[
E(\mu, \nu_1) = E(0, \nu_r) \frac{\nu_r^2}{\nu_1^2} (\mu^2 + \nu_1^2)^{-1/2} \quad (A-25)
\]

In the region of the needle point \( \rho_C = 0 \) except on the surface of the needle. From Equation 3

\[
\nabla \cdot (\epsilon E) = \rho_C
\]

Therefore, the total charge contained in the needle is given by

\[
q = \iiint \rho_C \, dx \, dy \, dz = \iiint \nabla \cdot (\epsilon E) \, dx \, dy \, dz \quad (A-26)
\]

By application of the divergence theorem to Equation A-26

\[
q = \iint_s (\epsilon E) \cdot \hat{n} \, dS \quad (A-27)
\]

where \( \epsilon E \) is defined by Equation A-21 and \( \hat{n} \) is the
surface normal unit vector. This is accomplished by integrating over a $\nu = \text{constant}$ paraboloid containing $\nu = \nu_s$, the paraboloid needle point. Equation A-24 can be used to predict, for various size needle points, how far the electric field must be integrated to insure whatever theoretical accuracy is desired in computing the needle charge. Combining Equations A-22 and A-27

$$q = \frac{eB}{\nu_s} \int \int \frac{-\hat{n} \cdot \hat{n}}{(\mu^2 + \nu_s^2)^{1/2}} \ dS \quad \text{(A-28)}$$

but

$$\hat{n} = \hat{\nu}$$

and for the chosen coordinate system

$$dS = \mu \nu_s (\mu^2 + \nu_s^2)^{1/2} d\mu d\psi \quad \text{(A-29)}$$

so that Equation A-28 becomes simply

$$q = -2\pi eB \int_0^{\mu_1} \mu d\mu \quad \text{(A-30)}$$

Integration yields

$$q = -\pi eB \mu_1^2 \quad \text{(A-31)}$$

$\mu_1$ will remain an arbitrary bound on $\mu$ for the moment.

Since the applied voltage was specified as $V_0$, the needle capacitance may be evaluated as

$$C = q/V_0$$

or

$$C = \frac{-\pi eB \mu_1^2}{V_0} \quad \text{(A-32)}$$
For the sake of completeness, it was decided to investigate the electrostatic characteristics of the class of needles which have conical, rather than paraboloidal, points. For convenience a spherical coordinate system was used. The relations describing the spherical coordinate system are

\[
X = r \sin \theta \cos \psi \\
Y = r \sin \theta \sin \psi \\
Z = r \cos \theta
\]

where \( r \) is the radius to a point in the \((r, \theta, \psi)\) system, \( \theta \) is the angle between \( r \) and the \( Z \)-axis and \( \psi \) is the angle from the \( X \)-axis to the projection of \( r \) into the \( X-Y \) plane. This coordinate system is common and will not be sketched.

It was discovered that when a mathematically sharp point is allowed, the field at the point becomes infinite. A mathematically sharp point never occurs in reality. To obtain a more realistic cone point model, and to avoid the mathematically sharp point and its attendant theoretical problems, the analysis was also carried out in prolate spheroidal coordinates, \((\eta, \theta, \psi)\). The relations describing are

\[
x = a \sinh \eta \sin \theta \cos \psi \\
y = a \sinh \eta \sin \theta \sin \psi \\
z = a \cosh \eta \cos \theta
\]

where \( a, \eta, \theta, \) and \( \psi \) are defined by the sketch in Figure A-4.
The needle point is described in both spherical and prolate spheroidal coordinate systems by generating the $\theta = \theta_s = \text{constant}$ surface. The analysis of these two needle shapes follow closely that given for the paraboloidal needle point. The resultant field, field ratio, and capacitive charge equations for all three families of needles are given in summary form in Table A-1.

Figure A-4 - Prolate Spheroidal Coordinates
### Table A-1 - Summary of Needle Point Equations

<table>
<thead>
<tr>
<th></th>
<th>Conic Point</th>
<th>Hyperbolic Point</th>
<th>Parabolic Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinate System</td>
<td>Spherical ((r, \theta, \psi))</td>
<td>Prolate Spheroidal ((\eta, \theta, \psi))</td>
<td>Parabolic ((\mu, \nu, \psi))</td>
</tr>
<tr>
<td>Electric Field</td>
<td>(\overrightarrow{E} = \frac{B_1 \hat{\theta}}{r \sin \theta})</td>
<td>(\overrightarrow{E} = \frac{B_2 \hat{\theta}}{a \sin \theta (\sinh^2 \eta + \sin^2 \theta)^{1/2}})</td>
<td>(\overrightarrow{E} = \frac{B_3 \hat{\nu}}{\nu (\mu^2 + \nu^2)^{1/2}})</td>
</tr>
<tr>
<td>Field Ratio</td>
<td>(\frac{E(r, \theta_1)}{E(r, \theta_r)} = \frac{\sin \theta_r}{\sin \theta_1})</td>
<td>(\frac{E(0, \theta_1)}{E(0, \theta_r)} = \frac{\sin^2 \theta_r}{\sin^2 \theta_1})</td>
<td>(\frac{E(0, \nu_1)}{E(0, \nu_r)} = \frac{\nu_r^2}{\nu_1^2})</td>
</tr>
<tr>
<td>Capacitive Charge</td>
<td>(q = 2\pi \varepsilon B_1 r)</td>
<td>(q = 2\pi \varepsilon B_2 a \sin \theta \sin \eta)</td>
<td>(q = \pi \varepsilon B_3 \mu^2)</td>
</tr>
</tbody>
</table>

**Note:** Subscripts 1, 2, 3 on B indicate evaluation of the constant B from applicable boundary conditions in the coordinate system indicated.
Returning attention to the problem at hand, determination of the influence of needle geometry upon the generalized Ohm's law in an EFD channel employing needle emitters, consider again the pulse charge as given in Equation A-9.

\[ q_p = CV_f \]

This charge must be the analog of the charge developed at the needle point as obtained in Equation A-32

\[ q = -\pi \varepsilon B \mu \]

Based on the circuit analogy, the firing voltage of the lamp is a function of the gas inside the bulb, and is independent of applied voltage \( V_o \). The capacitance is also independent of \( V_o \), and therefore the conclusion that the pulse charge is independent of applied voltage is reached. For the needle point discharge, it is reasoned that the ionization pulse initiation is determined by the field strength near the surface of the point. \( V_f \) would be the potential with respect to ground when the line integral from needle point to retreating space charge became critical. Increasing the voltage applied across the EFD gap will cause the space charge to retreat faster causing the critical field to develop more quickly. Increased current is expected because of the increased
speed with which the space charge is cleared after each pulse. Since \( q_p \) is independent of \( V_0 \), this can only occur by increasing the frequency of pulses. The physical operation of the needle point and the assumed electrical analog are seen to be logically consistant, so long as the needle point geometry is maintained constant.

If however, the needle geometry is changed, the relation of field strength to applied voltage changes and the pulse charge will change with it. The effect of increased pulse frequency due to the increased velocity of the space charge is a function of the fluid and the mobility and is accounted for in the generalized Ohm's law derived as Equation 128. The dependence upon needle diameter found experimentally must be the charge pulse width effect, governed by needle point geometry. To determine the net effects of a change in needle diameter, which refers to the diameter at the focal length of the paraboloid, Equations A-23 and A-24 were evaluated for needle diameters ranging from 0.001 inch to 10 inches. The reference diameter of 0.10 inch was chosen. The results are shown in Tables A-2 and A-3 and plotted in Figures A-5 and A-6.
## Table A-2

**Needle Diameter And Maximum Field Strength**

<table>
<thead>
<tr>
<th>Diameter, D, in.</th>
<th>Focal Length, Zf, ft.</th>
<th>Shape Factor, ( v_s, (ft.)^{1/2} )</th>
<th>Field Strength, ( E(0,\nu_s)/E(0,\nu_f) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>2.0833( \times 10^{-5} )</td>
<td>6.4549( \times 10^{-3} )</td>
<td>100.0000</td>
</tr>
<tr>
<td>0.010</td>
<td>2.0833( \times 10^{-4} )</td>
<td>2.0412( \times 10^{-2} )</td>
<td>10.0000</td>
</tr>
<tr>
<td>0.050</td>
<td>1.0417( \times 10^{-3} )</td>
<td>4.5644( \times 10^{-2} )</td>
<td>2.0000</td>
</tr>
<tr>
<td>0.100</td>
<td>2.0833( \times 10^{-3} )</td>
<td>6.4549( \times 10^{-2} )</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.150</td>
<td>3.1250( \times 10^{-3} )</td>
<td>7.9057( \times 10^{-2} )</td>
<td>0.6667</td>
</tr>
<tr>
<td>0.200</td>
<td>4.1667( \times 10^{-3} )</td>
<td>9.1287( \times 10^{-2} )</td>
<td>0.5000</td>
</tr>
<tr>
<td>0.250</td>
<td>5.2083( \times 10^{-3} )</td>
<td>1.0206( \times 10^{-1} )</td>
<td>0.4000</td>
</tr>
<tr>
<td>0.300</td>
<td>6.2500( \times 10^{-3} )</td>
<td>1.1180( \times 10^{-1} )</td>
<td>0.3333</td>
</tr>
<tr>
<td>0.350</td>
<td>7.2917( \times 10^{-3} )</td>
<td>1.2076( \times 10^{-1} )</td>
<td>0.2857</td>
</tr>
<tr>
<td>0.400</td>
<td>8.3333( \times 10^{-3} )</td>
<td>1.2910( \times 10^{-1} )</td>
<td>0.2500</td>
</tr>
<tr>
<td>0.500</td>
<td>1.0417( \times 10^{-2} )</td>
<td>1.4434( \times 10^{-1} )</td>
<td>0.2000</td>
</tr>
<tr>
<td>1.000</td>
<td>2.0833( \times 10^{-2} )</td>
<td>2.0412( \times 10^{-1} )</td>
<td>0.1000</td>
</tr>
<tr>
<td>5.000</td>
<td>1.0417( \times 10^{-1} )</td>
<td>4.5644( \times 10^{-1} )</td>
<td>0.0200</td>
</tr>
<tr>
<td>10.000</td>
<td>2.0833( \times 10^{-1} )</td>
<td>6.4549( \times 10^{-1} )</td>
<td>0.0100</td>
</tr>
<tr>
<td>Axial Location</td>
<td>Surface Position</td>
<td>Field Strength Ratio</td>
<td></td>
</tr>
<tr>
<td>---------------</td>
<td>------------------</td>
<td>---------------------</td>
<td></td>
</tr>
<tr>
<td>$Z/Z_f$</td>
<td>$\mu/\nu_s$</td>
<td>$E(\mu, \nu_s)/E(0, \nu_s)$</td>
<td></td>
</tr>
<tr>
<td>-1.0</td>
<td>0.0</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>-0.99</td>
<td>0.1</td>
<td>0.9950</td>
<td></td>
</tr>
<tr>
<td>-0.96</td>
<td>0.2</td>
<td>0.9806</td>
<td></td>
</tr>
<tr>
<td>-0.91</td>
<td>0.3</td>
<td>0.9578</td>
<td></td>
</tr>
<tr>
<td>-0.84</td>
<td>0.4</td>
<td>0.9285</td>
<td></td>
</tr>
<tr>
<td>-0.75</td>
<td>0.5</td>
<td>0.8944</td>
<td></td>
</tr>
<tr>
<td>-0.64</td>
<td>0.6</td>
<td>0.8575</td>
<td></td>
</tr>
<tr>
<td>-0.51</td>
<td>0.7</td>
<td>0.8192</td>
<td></td>
</tr>
<tr>
<td>-0.36</td>
<td>0.8</td>
<td>0.7809</td>
<td></td>
</tr>
<tr>
<td>-0.19</td>
<td>0.9</td>
<td>0.7433</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>1.0</td>
<td>0.7071</td>
<td></td>
</tr>
<tr>
<td>0.44</td>
<td>1.2</td>
<td>0.6402</td>
<td></td>
</tr>
<tr>
<td>0.96</td>
<td>1.4</td>
<td>0.5812</td>
<td></td>
</tr>
<tr>
<td>1.56</td>
<td>1.6</td>
<td>0.5300</td>
<td></td>
</tr>
<tr>
<td>2.25</td>
<td>1.8</td>
<td>0.4856</td>
<td></td>
</tr>
<tr>
<td>3.00</td>
<td>2.0</td>
<td>0.4472</td>
<td></td>
</tr>
<tr>
<td>8.00</td>
<td>3.0</td>
<td>0.3162</td>
<td></td>
</tr>
<tr>
<td>15.00</td>
<td>4.0</td>
<td>0.2425</td>
<td></td>
</tr>
<tr>
<td>24.00</td>
<td>5.0</td>
<td>0.1961</td>
<td></td>
</tr>
<tr>
<td>99.00</td>
<td>10.0</td>
<td>0.0995</td>
<td></td>
</tr>
<tr>
<td>399.00</td>
<td>20.0</td>
<td>0.0499</td>
<td></td>
</tr>
</tbody>
</table>
Figure A-5 - Point Diameter Effect on Maximum Field Strength

Figure A-6 - Field Strength Surface Variation
It is apparent from Table A-2 and Figure A-5, that the variation of maximum field is linear with respect to needle diameter (at Zf) and may be expressed as

$$\frac{E(0, \nu_1)}{E(0, \nu_r)} = \frac{D_r}{D_i} \quad \text{(A-39)}$$

where $E(0, \nu_1)$ and $E(0, \nu_r)$ are the maximum field strengths for needles of diameter $D_i$ and $D_r$ respectively. The needle described by the surface $\nu_r$, with $D_r$ diameter may be considered a reference case from which the maximum field strengths of all geometrically similar needles may be predicted.

If the negative sign in the capacitive charge relation is included in the constant $B$ and the subscript 3 is added to $B$ to note this distinction and the coordinate system in which $B$ is to be evaluated, the capacitive charge becomes

$$q = \pi \epsilon B_3 \mu^2 \quad \text{(A-40)}$$

which is the expression given in Table A-1.

$\mu$ is the surface coordinate where the contribution of surface integral of electric charge becomes negligible. This point will vary with needle shape $\nu_s$, and would be a different value for each different $\nu_s$. To avoid this and generalize the charge equation, the variable $\mu$ will be replaced by the non-dimensional variable $(\mu/\nu_s)$ which, as seen in Table A-3 is a constant, for a given
surface field ratio.

\[ q = \pi \epsilon B_3 \nu_s^2 (\mu/\nu_s^2) \]  
(A-41)

The location at which the field is sufficiently attenuated so that q changes are negligible with further integration will be specified as

\[ (\mu_1/\nu_s) = \text{constant} \]

Therefore the charge ratio of an arbitrary needle geometry \( \nu_i \) to a reference needle \( \nu_r \) is given by

\[ \frac{q_i}{q_r} = \frac{\nu_i^2}{\nu_r^2} \]  
(A-42)

Converting from surface coordinates to Cartesian, the charge ratio is directly proportional to the diameter ratio.

\[ \frac{q_i}{q_r} = \frac{D_i}{D_r} \]  
(A-43)

It has been deduced that q is independent of applied voltage so long as that voltage exceeds the firing potential. It has also been reasoned that it is independent of the mean current. Equation A-43 would then provide the correct pulse charge relation for a series of geometrically similar needles where \( D_r \) and \( q_r \) are the known diameter and charge of a reference needle, regardless of the electrical state of the needle. The rationale leading to this conclusion depends upon the validity of the analog circuit chosen, and upon the correctness of applying the Laplacian of the potential in the relatively space charge clear
region just prior to a pulse. Although these applications are reasonable, the thread of logic leading to this concept of needle point operation is admittedly somewhat tenuous. Consolidation of the basic applicability of the derived needle point characteristics is essential so that this model may be used with greater confidence.

Figure A-7, Reference 17, is a trace from a photograph of current versus time taken from an oscilloscope trace of a needle corona discharge. A single pulse of current was captured in the photograph during an operating state where the mean current flow was 8 A. Pulse initiation has a rise time on the order of one microsecond or less. This agrees with the performance of the neon oscillator circuit chosen as an analog, for firing time of a neon bulb is very short. The pulse decay from the needle point is noted to be very much like the characteristic exponential decay of a discharging capacitor.

The fact that the frequency of oscillation depends upon needle diameter and current was noted by Cobine, Reference 17, and data extracted from Reference 18 demonstrates that frequency versus current appears to be a direct linear relation with the slope of the line dependent upon needle point diameter. Again, this agrees with the theoretical model developed.
Figure A-7 - Needle Point Discharge

Since applied voltage $V_0$ is varied to change the current and since for a fixed needle geometry the pulse frequency is linear with respect to current, the charge per pulse must necessarily be a constant for a given needle. Equation A-10 derived from the analog circuit analysis

$$I = f q_p$$

and Equation A-43 which resulted from the field analysis

$$\frac{q_i}{q_r} = \frac{D_i}{D_r}$$

were used in conjunction with the 1.5mm needle data (Reference 17, page 263) to calculate the theoretical frequency versus current lines for 0.5mm and 4.73mm needles. The results together with the data are plotted in Figure A-8. The results show that the theoretical needle point model yields predictions which agree with available experimental data qualitatively and quantitatively.
It has been deduced that the current emitted from needles in air occurs in pulses of constant charge width, at a frequency which varies linearly with current. The pulse width or charge per pulse determines the slope of the frequency current relation and is controlled by needle point geometry. Current is governed not by the applied voltage but by the difference between the applied voltage and firing voltage or the voltage at which emission begins.

Unfortunately, it was not possible to evaluate the integration constants in the equations which predict the effects of needle point geometry upon the voltage
current characteristic with reasonable effort, and the geometric effect must still be accounted for empirically. The basic equations would permit a numerical evaluation of these effects. The model obtained here would provide a firm base for such an investigation.