Ladder network transfer function characteristics

Thomtavanit Hatayodom

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LADDER NETWORK TRANSFER FUNCTION CHARACTERISTICS

BY

THOMTAVANIT HATAYODOM, 1947-

A THESIS

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Approved by

Eddie R. Fouke (Advisor) Max Englund

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This thesis investigates the network characteristics and synthesis procedure previously developed. As a result of this investigation, location of the absolute minimum values of Navot's method is determined; a procedure is given for obtaining more desirable component values for a given ladder network without changing its transfer function; a procedure is given for obtaining the source location in a ladder network that yields the desired numerator degree of its transfer function; and a computer program for helping to obtain the above result is presented.
ACKNOWLEDGEMENTS

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I. INTRODUCTION

A. Purpose

The purpose of this research is to investigate: (1) the characteristics of resulting ladder networks of previously determined state-space synthesis procedures; (2) a procedure to determine practices or procedures that will make the previously determined synthesis techniques useful to practicing electrical engineers; and (3) the source location in a ladder network that yields the desired numerator degree of its transfer function.

B. Previous Work in the Area

This thesis utilizes the work of Navot[1], Fowler[2], Karni[3], Herrero and Willoner[4], Valkenburg[5], and Fowler and Hua[6].

Navot in 1967 developed a procedure for obtaining a tridiagonal matrix from a given strictly Hurwitz polynomial that has eigenvalues which are equal to the roots of the polynomial.

Fowler in 1969 developed a procedure for obtaining the component values in a ladder network by using a transformation of Navot's matrix and a decomposition procedure.

Karni and Valkenburg have presented well known synthesis procedures which use an insertion loss factor for synthesizing transfer functions that yields LC ladder network with resistor terminations. The resulting topology is the same as in Fowler's synthesis but uses S-domain techniques as opposed to state space techniques.
Herrero and Willoner have very thoroughly presented analysis techniques of the above mentioned ladder networks. These techniques are used quite often in this thesis.

Fowler and Hua developed the computer program to obtain the ladder network from a given characteristics polynomial which is used frequently in the first part of this thesis.
II. SYNTHESIS OF THE CHARACTERISTIC POLYNOMIAL, $H_n(s)$

A. Introduction

The synthesis of transfer functions is considered by presenting a synthesis procedure which yields a ladder network, with resistor terminations and reactive components between the terminations whose natural frequencies are equal to the roots of the transfer function characteristic polynomial.

B. Synthesis Procedure

Consider a voltage transfer function which has a constant numerator and strictly Hurwitz denominator polynomial, such as

$$T(s) = \frac{K}{s^n + h_{n-1} s^{n-1} + h_{n-2} s^{n-2} + \ldots + h_1 s + h_0} \quad (1)$$

Synthesis of this transfer function is done in two steps. First Navot's[1] method is used to obtain a tridiagonal matrix whose eigenvalues are equal to the roots of the transfer function denominator polynomial. Second Fowler's[2] method is used to determine the values of each component in the network from this matrix.

The first step is now presented in detail. Consider the strictly Hurwitz monic characteristic polynomial of (1),

$$H_n(s) = s^n + h_{n-1} s^{n-1} + h_{n-2} s^{n-2} + \ldots + h_1 s + h_0 \quad (2)$$

from which a tridiagonal matrix is to be obtained. It is necessary first to generate a secondary polynomial from $H_n(s)$. This secondary polynomial, $G_n(s)$, has the form
and is obtained by the relation
\[ G_n(s)G_n(-s) = H_n(s)H_n(-s) - c \] (4)
where
\[ 0 < c < m \] (5)
and
\[ m = \min \{|H_n(j\omega)|^2; 0 \leq \omega < \infty\} \] (6)

After determining \( m \), as shown in (6), \( c \) is chosen such that it is between the limits of (5). Then (4) is used to obtain the \( g_i \) of (3). From (5) it is obvious that \( c \) will not be unique, which in turn implies that neither will \( g_i \) be unique.

Next determine \( W(s) \) as
\[ W(s) = \frac{H_n(s) - G_n(s)}{(h_{n-1} - g_{n-1})H_n(s)} \] (7)
which is used to obtain a continued fraction expansion of the form
\[ W(s) = \frac{1}{f_1 + s + \frac{1}{f_2 + \frac{1}{s + f_3 + \cdots + \frac{f_n}{s + f_{n+1}}}}} \] (8)
With the \( f_j \) constants of (8), construct the tridiagonal matrix

\[
K = \begin{bmatrix}
-f_1 & -f_2 & 0 & 0 & \cdots & 0 \\
1 & 0 & -f_3 & 0 & \cdots & 0 \\
0 & \ddots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & 0 & 0 & -f_n \\
0 & \cdots & 0 & -f_{n+1}
\end{bmatrix}
\]  

(9)

It has been shown [2] when the degree of \( D(s) \) is odd this \( K \) matrix can be transformed into the form

\[
K = \begin{bmatrix}
-f_1 & 0 & 0 & \cdots & 0 & 0 \\
-k_2 & k_3 & \ddots & \ddots & \ddots & \ddots \\
0 & \ddots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & 0 & 0 & -f_{n+1} \\
-k_{n-2} & k_{n-3} & \ddots & \ddots & \ddots & \ddots
\end{bmatrix}
\]  

(10)

where \( k_j = \sqrt{f_{j+1}} \); \( j = 1,2,3,\ldots,n-1 \)  

(11)
With this $K$-matrix, Fowler[2] has shown that the following $A$-matrix can be equated to it.

$$A = \begin{bmatrix}
-C_b & -\frac{1}{2}B_T & C_b & -\frac{1}{2}B_T & L & -\frac{1}{2} \\
-L_c & B & C_b & -\frac{1}{2}B_c & 32 & -\frac{1}{2}B_T \\
\end{bmatrix}$$

(12)

Then the ladder component values can be obtained as follows:

$$\begin{bmatrix}
-f_1 \\
0 \\
. \\
. \\
. \\
-f_{n+1}
\end{bmatrix} = \begin{bmatrix}
g_1(C'_1)^2 \\
0 \\
. \\
. \\
. \\
g_2(C'_m)^2
\end{bmatrix}$$

(13)

where

\begin{align*}
f_1 &= g_1(C'_1)^2 \\
g_1 &= 1/R_1 \\
R_1 &= (C'_1)^2/f_1 \\
f_{n+1} &= g_2(C'_m)^2 \\
g_2 &= 1/R_2 \\
R_2 &= (C'_m)^2/f_{n+1}
\end{align*}
\[
\begin{bmatrix}
k_1 \\
-k_2 & k_3 \\
& \ddots \\
& & \ddots \\
& & & -k_{n-3} & k_{n-2} \\
& & & & -k_{n-1}
\end{bmatrix}
\begin{bmatrix}
C'_{1\,1}' \\
-C'_{2\,1}' & C'_{2\,2}' \\
& \ddots \\
& & \ddots \\
& & & -C'_{m-1\,r-1}' & C'_{m-1\,r}' \\
& & & & -C'_{m\,r}'
\end{bmatrix}
\]

(16)

\[
\begin{align*}
  k_1 &= C'_{1\,1}' \\
  k_2 &= C'_{2\,1}' \\
  k_3 &= C'_{2\,2}' \\
  \quad & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad
The decomposition of the equations when the A-matrix is equated with the K-matrix gives the fundamental loop equations and from this the ladder network can be drawn directly as shown in Figure 1.

![Ladder Network](image)

Figure 1. The Ladder Network When n = odd.

m and r are the number of capacitors and inductors respectively in the ladder network and \( m + r = n \).

It can be seen that there are \((n + 3)\) equations and \((n + 4)\) unknowns. Therefore a solution for this set of equations can be obtained when one of the unknowns is assigned an arbitrary value, such as:

\[
C_1' = 1
\] (18)

This value with equations (14), (15) and (17) implies

\[
L_1' = k_1
\] (19)

\[
C_i' = \frac{k_2 k_4 \cdots k_{2(i-1)}}{k_1 k_3 \cdots k_{2i-3}} ; \ 1 < i \leq m
\] (20)

\[
L_i' = \frac{k_1 k_3 \cdots k_{2i-1}}{k_2 k_4 \cdots k_{2(i-1)}} ; \ 1 < i \leq r
\] (21)
where

\[ m = \frac{n+1}{2} \]

\[ r = \frac{n-1}{2} \]

The \( C_i \)'s and \( L_i \)'s of the synthesized network of Figure 1 can be calculated by

\[ C_i = (C'_i)^{-2} \] (22)

\[ L_i = (L'_i)^{-2} \] (23)

From equation (13) and (14), \( R_1 \) and \( R_2 \) are evaluated by

\[ R_1 = \frac{1}{f_1} \] (24)

and

\[ R_2 = \left( \frac{k_2 k_4 \ldots k_{n-1}}{k_1 k_3 \ldots k_{n-2}} \right)^2 \left( \frac{\text{f}_{n+1}}{f_1} \right) \] (25)

Note that this solution is not unique since \( C'_1 \) was arbitrarily chosen.

When the degree of \( D(s) \) is even, the ladder component values can be obtained by using a similar technique as when \( n \) is odd. Then

\[ R_1 = (C'_1)^2/f_1 \] (26)

\[ R_2 = f_{n+1}/(L'_r)^2 \] (27)
\[ k_1 = C'_1 L'_1 \]
\[ k_2 = C'_2 L'_1 \]
\[ \vdots \]
\[ k_{n-2} = C'_{m-r-1} L'_{m-r} \]
\[ k_{n-1} = C'_{m-r} L'_{m-r} \]

Again, by a similar procedure, the ladder network of Figure 2 results.

![Figure 2. The Ladder Network When n = even](image)

It can be seen that again there are \((n + 3)\) equations and \((n + 4)\) unknowns. Again a solution for this set of equations can be obtained when one of the unknowns is assigned an arbitrary value, such as:

\[ C'_1 = 1 \]  
(29)

This value with equation (28) implies that

\[ L'_1 = k_1 \]  
(30)

\[ C'_i = \frac{k_2 k_4 \cdots k_{2(i-1)}}{k_1 k_3 \cdots k_{2i-3}} ; \ 1 < i < m \]  
(31)
The C_i's and L_i's of the synthesized network of Figure 2 can be calculated as

\[ C_i = (C'_i)^{-2} \]  \hspace{1cm} (33)

\[ L_i = (L'_i)^{-2} \]  \hspace{1cm} (34)

From equation (26) and (28), \( R_1 \) and \( R_2 \) are evaluated by

\[ R_1 = \frac{1}{f_1} \]  \hspace{1cm} (35)

and

\[ R_2 = \left[ \frac{k_1 k_3 \cdots k_{n-1}}{k_2 k_4 \cdots k_{n-2}} \right]^2 f_{n+1} \]  \hspace{1cm} (36)

Note that this solution is not unique since \( C'_i \) was arbitrarily chosen.

To illustrate this synthesis procedure of realizing a characteristic polynomial, an example is now given when \( n = 5 \).

C. Example 1: Synthesis of T(s) with \( n = 5 \)

Given the strictly Hurwitz characteristic polynomial

\[ H_5(s) = s^5 + 0.11s^4 + 1.11s^3 + 1.22s^2 + 1.11s + 0.2 \]  \hspace{1cm} (37)
then applying Navot's method,

\[
|H_5(j\omega)|^2 = (0.2 - 1.221\omega^2 + 0.11\omega^4)^2 + (111\omega - 111.1\omega^3 + \omega^5)^2
\]  
and

\[
m = \min \{|H_5(j\omega)|^2 ; 0 < \omega < \infty\}
\]
yields

\[
m = 0.04
\]

therefore

\[
0 < c < 0.04
\]  
(39)

Arbitrarily choose \( c = 0.02 \) and then

\[
G_5(s)G_5(-s) = -s^{10} - 222.18s^8 - 12564.93s^6 - 24662.66s^4
\]

\[-12320.5ls^2 + 0.02
\]  
(40)

Using the computer program shown in Appendix A, the coefficients of \( G_5(s) \) and the continued fraction expansion below are obtained.

\[
G_5(s) = s^5 - 0.109s^4 + 111.099s^3 - 1.1553s^2 + 110.99s - 0.14
\]  
(41)

and

\[
W(s) = \frac{1}{0.109 + s + \frac{100.27}{s + 9.728}}
\]

\[s + 0.9312
\]

\[s + 0.16
\]

\[s + 0.0003
\]  
(42)
Then the $K$ matrix is obtained as follows:

$$K = \begin{bmatrix} -0.109 & -100.27 & 0 & 0 & 0 \\ 1 & 0 & -9.728 & 0 & 0 \\ 0 & 1 & 0 & -0.9312 & 0 \\ 0 & 0 & 1 & 0 & -0.16 \\ 0 & 0 & 0 & 1 & -0.0003 \end{bmatrix}$$

(43)

where

$$f_1 = 0.109$$

$$k_1 = \sqrt{100.27} = 10.013$$

$$k_2 = \sqrt{9.728} = 3.12$$

$$k_3 = \sqrt{0.9312} = 0.965$$

$$k_4 = \sqrt{0.16} = 0.4$$

$$f_6 = 0.0003$$

The values of each component are determined as follows:

An arbitrarily chosen $C_1 = 1$ Farad, which yields

$$C'_1 = 1 \ f$$

Using equations (17) through (23), obtain

$$L_1 = 0.01 \ h$$

$$C_2 = 10.3076 \ f$$

$$C_3 = 59.92 \ f$$

$$L_2 = 0.104 \ h$$
and

\[ R_1 = 9.14 \ \Omega \]

\[ R_2 = 52.71 \ \Omega \]

Figure 3. The Resulting Ladder Network of the Synthesized Characteristic Polynomial When \( n = 5 \).

The network topology has previously been determined and is shown in Figure 3 with the calculated component values. This network will yield the characteristic polynomial given in (37).
III. LOCATION OF THE ABSOLUTE MINIMUM

When using Navot's method to obtain a matrix whose eigenvalues are equal to the roots of a given strictly Hurwitz characteristic polynomial, one needs to calculate the $m$ of equation (6) where:

$$m = \min\{|H_n(j\omega)|^2; \ 0 \leq \omega < \infty\} \quad (44)$$

It will now be shown in general the procedure for locating where absolute minimum occurs. Consider

$$H_n(s) = s^n + a_{n-1}s^{n-1} + \ldots + a_1s + a_0 \quad (45)$$

which can always be factored into this form

$$H_n(s) = \prod_{i=1}^{k} (s + \sigma_i) \prod_{j=1}^{m} (s^2 + 2\zeta_j\omega_n js + \omega_n^2) \quad (46)$$

Since $H_n(s)$ is strictly Hurwitz then $\sigma_i$, $\zeta_j$ and $\omega_n$ are real and positive. Rewrite (46) as:

$$H_n(s) = \prod_{i=1}^{k} H_1(s) \prod_{j=1}^{m} H_2(s) \quad (47)$$

Equation (47) will be considered in two separate parts.

First, consider $H_1(s)$ defined by

$$H_1(s) = s + \sigma_i \quad (48)$$

$$|H_1(j\omega)|^2 = \sigma_i^2 + \omega^2; 0 \leq \omega < \infty \quad (49)$$

as $\omega \to 0$

$$|H_1(j0)|^2 = \sigma_i^2 \quad (50)$$
as \( \omega \to \omega' \) where \( \omega' > 0 \) then

\[
|H_1(j\omega')|^2 = \sigma^2 + \gamma
\]  

(51)

where \( \gamma \) = positive number.

And thus

\[
|H_1(j\omega')|^2 > |H_1(j0)|^2 \quad \omega' > 0
\]  

(52)

Therefore the absolute minimum of \( |H_1(j\omega)|^2 \) always occurs at \( \omega = 0 \).

Second, consider \( H_2(s) \) defined by

\[
H_2(s) = s^2 + 2\zeta_j\omega_njs + \omega_n^2
\]  

(53)

\[
|H_2(j\omega)|^2 = (\omega_n^2 - \omega^2)^2 + (2\zeta_j\omega_n\omega)^2
\]  

(54)

The absolute minimum of equation (54) can be located by considering the BODE PLOT of Figure 5. It is obvious that the absolute minimum occurs at \( \omega_n \) if \( \zeta < 0.707 \), otherwise it occurs at \( \omega = 0 \). This implies that one does not need to differentiate equation (54) to find the values of \( \omega \) that give the relative minima. Therefore we only have to compare \( |H_n(j\omega)|^2 \) at each of the \( m' \) values of \( \omega_n \) and at \( \omega = 0 \).

\[
m' = (n - \text{realroot})/2
\]  

(55)

The constraint on \( \omega_n, \zeta, \) and \( \sigma \) for \( |H_3(j\omega)|^2 \) have an absolute minimum at \( \omega = 0 \) is now developed. The product of equation (49) and (54) can be written as,

\[
f(j\omega) = (\sigma^2 + \omega^2)[(\omega_n^2 - \omega)^2 + (2\zeta_\omega_n\omega)^2]
\]  

(56)

now when \( \omega = 0 \)
\[ f(j0) = \sigma^2 \omega_n^2 \] (57)

and when \( \omega = \omega_n \)

\[ f(j\omega_n) = (\sigma^2 + \omega_n^2)(2\zeta\omega_n^2)^2 \] (58)

The characteristic of Figure 5 yields

\[ (\sigma^2 + \omega_n^2)(2\zeta\omega_n^2) > \sigma^2 \omega_n^2 \] (59)

\[ 4\zeta^2 \omega_n^2 (\sigma^2 + \omega_n^2) > \sigma^2 \] (60)

From equation (60) and the characteristics of Figure 4, \( \zeta \) can not be too small. Since if there are roots nearly on \( j\omega \) axis it tells us that values of resistors in the network must be approximately zero which is not a practical design procedure.

This procedure is also good for practical design of RLC ladder network. Since Butterworth and odd Chebyshev polynomials can be shown to have absolute minima at \( \omega=0 \) by using the above method.
Figure 4. Root Location

Figure 5. Bode Plot
IV. VARIATION OF THE NETWORK COMPONENT VALUES

A. Variation Resulting From the Choice of $c$ and $C_1$

This section is to show how the K-matrix and component equations resulting from the previous synthesis procedure will vary by different choices of $c$ and $C_1$ in equation (5) and (18) respectively.

It will be easier to show how the synthesized network component values change as the value of $c$ is varied by plotting the graph of each component value versus $c$.

Using Example 1 and the results from the computer program in Appendix A, graphs and data can be obtained and discussed as follows.

Figure 6, curve F, shows how varying $c$ affects the value of $R_1$. It can be seen that as $c$ varies over its entire range, that the change of $R_1$ is small enough to be assumed a constant. This also can be shown from the calculations of $R_1$ in equation (14).

First $C_1$ is assumed to be constant and $f_1$ is the value in the first column and first row of the K-matrix of (9), which can be seen in (4), (7), and (8) to change very little as $c$ is varied. Therefore $R_1$ will remain nearly constant, as $c$ is varied.

Varying the value of $C_1$ results in magnitude scaling[3] of the other network components as shown below.

\[ R_1 = \frac{(C_1')^2}{f_1} \]  \hspace{1cm} (61)

but

\[ (C_1')^2 = (C_1)^{-1} \]  \hspace{1cm} (62)

\[ R_1 = (C_1 f_1)^{-1} \]  \hspace{1cm} (63)
Since $f_1$ is constant, decreasing the value of $C_1$ will make $R_1$ larger. Then this can be put in the form:

$$R_1^\# = R_1 k$$

where

- $R_1^\#$ = new value of $R_1$
- $k$ = magnitude scale factor = $1/C_1$
- $R_1$ = previous value of $R_1$

Figure 7, curve F., shows how varying $c$ affects the value of $R_2$. It can be seen that $R_2$ decreases when $c$ increases. When $c$ is very small, $R_2$ will be very large. As seen from (15) and (20), $R_2$ can be written as:

$$R_2 = \frac{f_3 f_5 / f_2 f_4 f_6}{54}$$

(64)

Note that from the continued fraction (8) when $c$ is very small $f_6$ is extremely small compared to the rest of the values of $f_j$, while $f_2, f_3, f_4$ and $f_5$ stay almost constant. This will make $R_2$ large. But when increasing $c$, $f_6$ increases and again the changes of $f_2, f_3, f_4$, and $f_5$ are very small. Then $R_2$ will decrease as $c$ increases.

It can be seen from (64) that $R_2$ stays constant while $C_1$ varies.

Figure 8, curve F., shows that varying $c$ does not affect the value of $L_1$. The change of $L_1$ is so small that it can be neglected. This can be shown from the calculations in (11), (17), (22), and (23) to give

$$L_1 = (C_1 f_2)^{-1}$$

(65)
First assume $C_1$ to be constant. Varying $c$ does not change $f_2$ much. Therefore $f_2$ can be assumed to be constant. Then $L_1$ must be constant. Again $C_1$ is a magnitude scaling factor[3].

Figure 9, curve $F_1$, shows that varying $c$ does not appreciably affect the value of $L_2$. This can be shown by the calculations from (11), (17), (21), (22), and (23) to give,

$$L_2 = f_3/C_1 f_2 f_4$$

Note that as $c$ varies the changes of $f_2$, $f_3$, and $f_4$ are very small and are assumed to be constant. Since $C_1$ is constant, $L_2$ is constant. Again $C_1$ is a magnitude scaling factor[3].

Figure 10, Curve $F_1$, shows that varying $c$ does not appreciably affect the value of $C_2$. This can be shown by the calculations from (11), (17), (20), and (22) to give,

$$C_2 = f_2 C_1 / f_3$$

Note that as $c$ varies the changes of $f_2$ and $f_3$ are very small and are assumed to be constant. Since $C_1$ is constant, $C_2$ is also constant. Again $C_1$ is a magnitude scaling factor[3].

Figure 11, curve $F_1$, shows that $C_3$ is more dependent upon the value of $c$ than $C_2$ is. Increasing $c$ increases $C_3$ as shown by (11), (17), (20), and (22);

$$C_3 = f_2 f_4 C_1 / f_3 f_5$$

Note from the continued fraction (8) $f_2$, $f_3$, $f_4$, and $f_5$ do not change much by varying $c$. But the change of $f_2$ is bigger than $f_3$, $f_4$, and $f_5$. Since $C_1$, $f_3$, $f_4$, $f_5$ are assumed to be constant, $C_3$ is changed by the percentage change of $f_2$. Again, $C_1$ is a magnitude scaling factor[3].
Figure 6. $R_1$ vs $c$

Figure 7. $R_2$ vs $c$
Figure 8. $L_1$ vs $c$

$F_a$

$F_j$

Figure 9. $L_2$ vs $c$

$F_a$

$F_j$
# TABLE I

**TABULATED VALUES FOR n = 5**

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<th>$c_1$</th>
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<th>$c_3$</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$c$</th>
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<td>12.5997</td>
<td>0.039</td>
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B. Variation Resulting From Changing the \( f_j \) in K-matrix

In the K-matrix of equation (9) the \( f_j \) can be changed around without changing the eigenvalues. In other words, the notation \( j \) is made to vary from \( (n + 1) \) to \( l \). This will result in all of the component values being very different from those obtained when using the \( f_j \) ordered as shown in equation (9).

The new \( f_a \) can be written in this form

\[
 f_a = f_j
\]

where

\[
a = 1, \ldots, n + 1 \text{ and } j = n + 1, \ldots, l.
\]

Then

\[
 K_a = \begin{bmatrix}
 -f_1 & -f_2 & & & \\
 1 & 0 & -f_3 & & \\
 & \ddots & \ddots & \ddots & \\
 & & 1 & 0 & -f_n \\
 & & & 1 & -f_{n+1}
\end{bmatrix}
\]

and

\[
 K_j = \begin{bmatrix}
 -f_{n+1} & -f_n & & & \\
 1 & 0 & -f_{n-1} & & \\
 & \ddots & \ddots & \ddots & \\
 & & 1 & 0 & -f_2 \\
 & & & 1 & -f_1
\end{bmatrix}
\]
Then by using the same equations as given previously, one can calculate all of the component values.

To illustrate these results, Example 1 was recalculated with the \( f_j \) changed around and the results shown as curves \( F_a \) in Figures 6 through 11.

C. Example 2: Synthesis of \( T(s) \) with \( n = 6 \)

This example is to show how the network component values change as \( c \) is varied for \( n = 6 \) case.

Given the strictly Hurwitz monic polynomial of

\[
H_6(s) = s^6 + 1000s^5 + 112.1s^4 + 11110.1s^3 + 123.2s^2 + 1110.1s + 2
\]

(71)

As per the section III, \( \omega = 0 \) yields the absolute minimum \( H_6(j\omega)^2 \) and thus

\[
m = H_6(j\omega)^2 = 4
\]

which implies that

\[
0 < c \leq 4
\]

Arbitrarily choose \( c = 2 \), then by using Appendix A solve for \( G_6(s) \) which yields

\[
G_6(s) = s^6 - 999.99s^5 + 111.558s^4 - 11110.011s^3 + 116.196s^2
\]

\[
- 1110.03s + 1.4142
\]
Then obtain the continued fraction expansion as shown below:

\[
\frac{H_6(s) - G_6(s)}{[h_5 - g_5][H_6(s) - G_6(s)]]} = \frac{1}{999.99 + s + \frac{100.7234}{s + 9.9327 + \frac{10673}{s + 0.0942 + \frac{0.0159}{s + 0.0003}}}} \tag{72}
\]

This results in

\[
\begin{align*}
    f_1 &= \sqrt{999.99} \\
    k_1 &= \sqrt{100.72} \\
    k_2 &= \sqrt{9.93} \\
    k_3 &= \sqrt{1.067} \\
    k_4 &= \sqrt{0.09} \\
    k_5 &= \sqrt{0.015} \\
    f_7 &= 0.0003
\end{align*}
\tag{73}
\]

Now the values of each component can be found by using equations (29) through (36). The computer program was used to obtain the values of components and they are

\[
\begin{align*}
    R_1 &= 0.0001 \ \Omega \\
    R_2 &= 0.0002 \ \Omega \\
    C_1 &= 1 \ \mu f \\
    C_2 &= 10.1407 \ \mu f \\
    C_3 &= 114.93 \ \mu f \\
    L_1 &= 0.0099 \ \mu h
\end{align*}
\tag{74}
\]
It is known that the resulting ladder network will be as shown in Figure 12.

\[
\begin{align*}
L_2 &= 0.09 \quad \text{h} \\
L_3 &= 0.547 \quad \text{h}
\end{align*}
\]

Figure 12. The Result Ladder Network of the Synthesized Characteristic Polynomial When \( n = 6 \).

For \( n = 6 \) all the component values do not vary significantly as \( c \) varies as shown by the \( F_j \) curves of Figures 13 through 19.

Figure 13 and Figure 14 show the values of \( R_1 \) and \( R_2 \) to be too small for practical purposes.

Figure 18 and Figure 19 show the values of \( C_2 \) and \( C_3 \) to be too large for practical purposes. However, magnitude and frequency scaling[3] can be used to resolve these problems.

Arbitrarily choose a magnitude scaling factor, \( k = 10^4 \), and a frequency scaling factor, \( f = 10^3 \text{h} \). Then


\[ R_1' = kR_1 \Omega \]
\[ R_2' = kR_2 = 2 \Omega \]
\[ C_1' = C_1/kf = 0.1 \, \text{f} \]
\[ C_2' = C_2/kf = 1.0147 \, \text{f} \]
\[ C_3' = C_3/kf = 11.493 \, \text{f} \]
\[ L_1' = (k/f)L_1 = 0.099 \, \text{h} \]
\[ L_2' = (k/f)L_2 = 0.9 \, \text{h} \]
\[ L_3' = (k/f)L_3 = 5.47 \, \text{h} \]

(75)

By having done this magnitude and frequency scaling in the circuit, the above practical values of each component can be obtained.

By changing the \( f_j \) around in the K-matrix (9) for \( n = 6 \), all the components are changed as shown graphically by the \( F_a \) curves in Figures 13 through 19. Note that this affords a range of values for \( R_1 \) and \( R_2 \) that the previous arrangement of the \( f_j \)'s did not.
Figure 13. $R_1$ vs $c$

Figure 14. $R_2$ vs $c$
Figure 15. $L_1$ vs $c$

Figure 16. $L_2$ vs $c$
Figure 17. $L_3$ vs $c$

Figure 18. $C_2$ vs $c$
Figure 19. $C_3$ vs $c$
D. Scaling the Parameters by Synthesis Procedure

This section is to show how to change the values of each component of a given ladder network to more desirable values without affecting the transfer function. A procedure is given here as to how this might be accomplished.

1. Synthesis of $T(s)$ With $n = 5$, Example 3

Given a ladder network as shown

\[ \text{Figure 20. The Ladder Network When } n = 5. \]

whose component values are

\[ R_1 = 10^3 \Omega; \quad R_2 = 10^3 \Omega; \quad C_1 = 10^{-3} \text{f}; \quad C_2 = 10^{-4} \text{f}; \]
\[ C_3 = 10^{-3} \text{f}; \quad L_1 = 10^{-2} \text{h}; \quad L_2 = 10^{-3} \text{h} \]

It is convenient to use a frequency scaling to yield the coefficient values of the characteristic polynomial that are more convenient for computation.
Arbitrarily choose \( f = 10^{-3} \)

\[
R_1' = R_2' = 10^3 \Omega; \quad C_1' = 1 \ \mu f; \quad C_2' = 0.1 \ \mu f; \quad C_3' = 0.01 \ \mu f;
\]

\[
L_1' = 10 \ \mu h; \quad L_2' = 1 \ \mu h
\]

Then the voltage transfer function can be obtained by using the Cumulant's method[4].

\[
T(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{(R_1 + R_2)} = \frac{N(s)}{D(s)} \quad (76)
\]

where

\[
N(s) = 10^{-4}
\]

and

\[
D(s) = s^5 + 0.11s^4 + 111.1s^3 + 1.22s^2 + 11s + 02 \quad (78)
\]

This monic polynomial equation (8) is the same as used in Example 1 so as to add to previous results. By using the same procedure as in Example 1 the new values of each component can be obtained, as shown below.

\[
R_1 = 9.14 \Omega; \quad R_2 = 52.71 \Omega; \quad C_1 = 0.001 \mu f; \quad C_2 = 0.00103 \mu f;
\]

\[
C_3 = 0.0599 \mu f; \quad L_1 = 0.01 \ \mu h
\]

By using this procedure it can be seen that more desirable component values might be obtained without altering the transfer function.
V. DETERMINATION OF THE SOURCE LOCATIONS IN LADDER NETWORKS

A. Introduction

To obtain the short circuit transfer admittance function, \( y_{12}(s) \), there are two classifications to be considered. The first is when \( n \) is odd and the other when \( n \) is even. \( n \) is the highest degree of the transfer function characteristic polynomial. Also, \( n \) is equal to the number of capacitances and inductances in the network being considered.

Figure 21. The Ladder Network When \( n = \text{odd} \)

Figure 22. The Ladder Network When \( n = \text{even} \)
B. Obtaining Short Circuit Transfer Function, $y_{12}(s)$, and the Locations of Source $V_1$ and $V_2$ when $n$ is odd

In this case the number of capacitors are greater than the number of inductors by one.

$$x = \frac{n + 1}{2}$$
$$y = \frac{n - 1}{2}$$

(79)

where $x$ is the number of capacitors and $y$ is the number of inductors.

When $n$ is odd there are three classifications to be considered.

1) Only $V_1$ is moved.
2) Only $V_2$ is moved.
3) $V_1$ and $V_2$ are both moved.

$$y_{12}(s) = -\frac{N(s)}{D(s)}$$

(80)

where

$$N(s) = a_m s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0$$

(81)

and

$$D(s) = b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0$$

(82)

1. If $V_1$ is moved only, we must consider the numerator polynomial $N(s)$. It will be shown that $m$, the degree of the numerator polynomial, is always odd. When $V_1$ is moved from mesh 1 to mesh 2 there is only one reactive element $C_1$ in parallel with $R_1$, and the degree of the numerator is one.
This can be shown by using Cumulant's method[4] to obtain the numerator polynomial $N(s)$.

$$N_{1-2}(s) = (R_1 \to sC_1) = \begin{bmatrix} R_1 & 1 \\ -1 & sC_1 \end{bmatrix} = sR_1C_1 + 1 \quad (83)$$

$m_{1-2}$ indicates the degree of $N(s)$ where $V_1$ is moved from mesh 1 to mesh 2. $m_{1-2} = 1$.

When $V_1$ is moved from mesh 2 to mesh 3, $m_{2-3}$ will be 2, since there are two more reactive elements, $L_1$ and $C_2$, being added.

Again by using Cumulant's method[4], the numerator polynomial will be
\[ N_{1-3}(s) = (R_1 \overline{sC_2}) = \begin{bmatrix} R_1 & 0 & 0 \\ -1 & sC_1 & 0 \\ 0 & -1 & sL_1 \\ 0 & 0 & -1 & sC_2 \end{bmatrix} \]

\[ N_{1-3}(s) = s^3 R_1 C_1 C_2 L_2 + s^2 C_2 L_1 + sR_1 (C_1 + C_2) + 1 \] (85)

\( m_{1-3} \) indicates the degree of \( N(s) \) where \( V_1 \) is moved from mesh 1 to mesh 3. \( m_{1-3} = 3 \).

Thus when \( V_1 \) is moved from mesh 2 to mesh 3 the degree is increased by 2 since \( m_{1-3} - m_{1-2} = 3 - 1 = 2 \). Therefore no matter where \( V_1 \) is moved, \( m \) is always odd. This is because \( V_1 \) moved from mesh 1 to mesh 2 gives an odd degree, and from mesh 2 to mesh 3 gives an even degree. Therefore odd plus even will always be odd.

This can be applied in general when given a short circuit transfer admittance and it is required to know where \( V_1 \) should be placed to yield the numerator polynomial of specified degree \( m \).

Let

\[ m = i + j \]
\[ i = j + 1 \]
\[ i = (m + 1)/2; i = 1, 2, 3, \ldots \]
\[ j = (m - 1)/2; j = 1, 2, 3, \ldots \]
\[ L = (m + 3)/2; L = 2, 3, 4, \ldots, (a - 1) \] (86)

where \( i \) is the number of capacitances to the left of source \( V_1 \) and \( j \) is the number of inductances to the left of source \( V_1 \) and \( L \) is the mesh in which \( V_1 \) should be placed.
Figure 25. The Ladder Network When \( V_1 \) is Moved to Mesh L.

Then

\[
y_{12}(s) = - \frac{|K|}{|A|} \tag{87}
\]

where

\[
|K| = (R_1 - sC_1) = \\
\begin{bmatrix}
R_1 & 1 \\
-1 sC_1 & 1 \\
\vdots & \vdots \\
-1 sL_{j-1} & 1 \\
-1 sL_j & 1 \\
-1 sC_i & 1
\end{bmatrix} \tag{88}
\]

and

\[
|A| = (R_1 - R_2) = \\
\begin{bmatrix}
R_1 & 1 \\
-1 sC_1 & 1 \\
\vdots & \vdots \\
-1 sL_1 & 1 \\
\vdots & \vdots \\
-1 sL_i & 1 \\
-1 sC_1 & 1 \\
-1 sC_x & 1 \\
-1 R_2 & 1
\end{bmatrix} \tag{89}
\]
2. If \( V_2 \) is moved only, it is the same as \( V_1 \) being moved, since the network is symmetrical about its center mesh. Therefore the presentation of part 1 applies in this case also.

3. If \( V_1 \) and \( V_2 \) are both moved, the numerator transfer admittance function can be obtained by the product of \( |K| \) in part 1 and \( |K'| \) in part 2 and the denominator is still the same \( |A| \).

\[
y_{12}(s) = -\frac{|K|}{|K'|/|A|} = -\frac{a_m^n s^{m''} + \cdots + a_{m''-1} s^{m''-1} + 1}{a_n^n + \cdots + a_0}
\]

(90)

\( m'' \) is the degree of the numerator admittance transfer function is always even. Since \( m \) is always odd so is \( m' \). Then \( m'' = m + m' \) will always be even.

Again, for this kind of admittance transfer function, it can be determined where \( V_1 \) and \( V_2 \) should be placed. \( m''/2 \) will be the number of capacitances and inductances which are to the left of \( V_1 \) and \( V_2 \) respectively.

The number of capacitors to the left of \( V_1 \) is \( (m''+2)/4 \), the number of inductors to the left of \( V_1 \) is \( (m''-2)/4 \), the number of capacitors to the right of \( V_2 \) is \( (m''+2)/4 \), and the number of inductors to the right of \( V_2 \) is \( (m''-2)/4 \). Notice that \( m'' = (2, 6, 10, 14, 18, ...) \).
But again for $m''$ even, it can be considered that the source $V_1$ is moved $p$ times more than $V_2$. Since it is known that by having either $V_1$ or $V_2$ moved 1 mesh starting from mesh 1 to mesh 2 or mesh (a) to mesh (a-1) the degree of the numerator admittance will increase by 2 at a time. Therefore if $V_1$ is moved $p$ times more than $V_2$ is moved, $(m''-2p)/2$ will be the number of capacitances and inductances which are to the right of source $V_2$.

The number of capacitors which are to the right of $V_2$ is $(m''-2p+2)/4$, the number of inductors which are to the right of $V_2$ is $(m''-2p-2)/4$, the number of capacitors which are to the left of $V_1$ is $(m''+2p+2)/4$, and the number of inductors which are to the left of $V_1$ is $(m''+2p-2)/4$. 
This will be the same procedure when \( V_2 \) is moved \( p \) times more than \( V_1 \).

The number of capacitors which are to the right of \( V_2 \) is
\[
\frac{(m''+2p+2)}{4},
\]
the number of inductors which are to the right of \( V_2 \) is
\[
\frac{(m''+2p-2)}{4},
\]
the number of capacitors which are to the left of \( V_1 \) is
\[
\frac{(m''-2p+2)}{4},
\]
and the number of inductors which are to the left of \( V_1 \) is
\[
\frac{(m''-2p-2)}{4}.
\]

C. Obtaining Short Circuit Transfer Function, \( y_{12}(s) \), and Location of Source \( V_1 \) and \( V_2 \) When \( n \) is Even.

In this case the number of capacitors are equal to the number of inductors in the network.

\[
\text{number of capacitors} = \text{number of inductors} = x = \frac{n}{2}
\]

Again there are three classifications to be considered:

1) Only \( V_1 \) is moved.

2) Only \( V_2 \) is moved

3) \( V_1 \) and \( V_2 \) are both moved.

\[
y_{12}(s) = -\frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + 1}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_o}
\]  \hspace{1cm} (91)

1. This will be the same as shown in section B-1. \( m \) the degree of numerator admittance transfer function is always odd, and \( m \) is always increased by 2 from moving one mesh to another. By using the same procedure to obtain admittance transfer function
\[ y_{12}(s) = -\frac{(R_1 - sC_1)}{(R_1 - sL_1 + R_2)} = -|K|/|A'| \]  

(92)

It is noted that \(|A|\) is not equal to \(|A'|\), since the last term \(R_2\) is added by the last inductance \(L_x\) which are in series. Therefore, \(|A'|\) will be obtained in this form:

\[
|A'| = (R_1 - sL_x + R_2)
\]

(93)

\[
\begin{bmatrix}
R_1 & 1 \\
-1 & sC_1 & 1 \\
-1 & sL_1 & 1 \\
& & . & . & . \\
& & & & \ddots & . & . & . \\
& & & & & & -1 & sC_x & 1 \\
& & & & & & -1 & sL_x + R_2
\end{bmatrix}
\]

Figure 27. The Ladder Network When \(V_1\) is Moved Only.

\[ m = i + j; \quad i = (m+1)/2; \quad j = (m-1)/2; \quad i = j + 1 \]

\(i\) = number of capacitances which are to the left of \(V_1\).

\(j\) = number of inductances which are to the left of \(V_1\).
\[ y_{12}(s) = -\frac{a_{m's^{m'}} + a_{m'-1}s^{m'-1} + \ldots + 1}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_0} \] (94)

2. \( m' \) the degree of the numerator admittance transfer function is always even. It is because when \( V_2 \) is moved from mesh (a) to mesh (a-1) there are two active elements \( L_x \) and \( C_x \) which are to the right of source \( V_2 \). This will cause \( m' \) to begin by two instead of one. This again can be shown by using Cumulant's method[4].

![Figure 28. The Ladder Network for \( V_2 \) is Moved by 1 Mesh.](image)

\[(R_2 + sL_x - sC_x) = \begin{bmatrix} R_2 + sL_x & 1 \\ -1 & sC_x \end{bmatrix} = s^2L_xC_x + sR_2C_x + 1 \] (95)

It is already shown that if \( V_2 \) is moved from one mesh to the other, the degree \( m' \) will increase by two. Therefore, since the first movement of \( V_2 \) gives the degree of even, \( m' \) will always be even.
This again can be applied for a given admittance transfer function whose $m'$ and $n$ the degree of numerator and denominator respectively are even, and it is required to know where $V_2$ will be placed in this kind of ladder network. It can be determined immediately since $m'$ and $n$ are odd, $n$ will determine how many capacitances and inductances there are altogether in the network, and $m'$ will be the number of capacitances and inductances which are to the right of $V_2$.

$$m' = i' + j'$$

$$i' = j'$$

$$i' = m'/2 = j$$

$i'$ = number of capacitance to the right of $V_2$

$j'$ = number of inductance to the right of $V_2$.

Now one must consider the admittance transfer function when $V_2$ is moved to mesh $L$ ($L = a-1, a-2...$).

$$y_{12}(s) = - |K'|/|A'|$$
3. The numerator admittance transfer function can be obtained by the product of two determinants \(|K|\) and \(|K'|\). But the denominator admittance transfer function is still the same as \(|A'|\).

\[
y_{12}(s) = - \frac{|K| |K'|/|A'|}{-l sC_{x-1} \cdots -l sL_{x-1}}
\]

(97)

where

\[m'' = m + m'
\]

Since \(m\) is odd but \(m'\) is even, therefore \(m''\) is always odd.
This can be applied for a given admittance transfer function, and it is required to determine where \( V_1 \) and \( V_2 \) will be placed. It is known that \( m' \) is greater than \( m \) by one. Therefore,

\[
m' = m + 1
\]
\[
m'' = m + m + 1
\]
\[
m = (m'' - 1)/2
\]

and

\[
m' = (m'' + 1)/2
\]

Since \( m \) and \( m' \) are calculated then one can determine the number of capacitance and inductance which are to the right and left of \( V_1 \) and \( V_2 \) respectively.

Figure 29. Circuit Diagram When Both \( V_1 \) and \( V_2 \) Are Moved

The number of capacitance to the left of \( V_1 \) is \((m'' + 1)/4\), the number of inductance to the left of \( V_1 \) is \((m'' - 3)/4\), the number of capacitance to the right of \( V_2 \) is \((m'' + 1)/4\), and the number of inductance to the right of \( V_2 \) is \((m'' + 1)/4\).
It is known that when $V_1$ and $V_2$ are moved by the same amount of mesh the number of capacitance to the left of $V_1$ is equal to the number of capacitance to the right of $V_2$. But the number of inductance to the left of $V_1$ is less than the number of inductance to the right of $V_2$.

Now if $V_1$ is moved $p$ times more than $V_2$, this will be different from the showing above. In this case $m$ is greater than $m'$ and $m$ is found out to be $2p+1$.

$$m = 2p+1$$

$$m' = m'' - m = m''-2p-1$$

The number of capacitance to the left of $V_1$ is $p+1$, the number of inductance to the left of $V_1$ is $p$, the number of capacitance to the right of $V_2$ is $(m''-2p-1)/2$, and the number of inductance to the right of $V_2$ is $(m''-2p-1)/2$.

Now if $V_2$ is moved $p$ times more than $V_1$ $m'$ will be $2(p+1)$.

$$m'' = m + m'$$

$$m = m''-2(p+1)$$

The number of capacitance to the left of $V_1$ is $(m''-2p-1)/2$, the number of inductance to the left of $V_1$ is $(m''-2p-3)/2$, the number of capacitance to the right of $V_2$ is $(2p+1)/2$, and the number of inductance to the right of $V_2$ is $(2p+1)/2$. 
VI. CONCLUSIONS

This study presents a synthesis procedure for transfer functions, determines the location of \( \omega \) which yields the absolute minimum value in Navot's method, shows the network component value affect for various choices of \( c \) and \( C_1 \), shows the network component value affect for changing the \( f_j \) in the K-matrix, shows how network component value scaling may be effected by the synthesis procedure, and gives a procedure for determining the locations of the sources in ladder networks for a given transfer function.

The synthesis procedures in Section II yield a portless ladder network with two terminal resistors, and \( n \) reactive elements. This realization uses a tridiagonal matrix and a transformation of this matrix with a decomposition procedure.

Section III determines the location of \( \omega \) which yields the absolute minimum of Navot's method. The results of this section show that for a given strictly Hurwitz polynomial the absolute minimum value always occurs at either \( \omega = 0 \) or at one of the \( m \) values of \( \omega_{nj} \), if the accompanying \( \xi_j \) is less than 0.707. This implies that it is a rather simple matter to construct a digital computer program for evaluating \( m \).

Section IV shows that the change of \( c \) affects the component values in the ladder network. The changing of the LC component values is very small. \( R_1 \) and \( R_2 \) varies significantly. Varying the values of \( C_1 \) acts as a magnitude scaling factor. Changing the \( f_j \)
in the K-matrix significantly changes the resistor component values in the ladder network. Example 3 shows that the synthesis procedures can be used to obtain more desirable component values for a given ladder network whose component values are not practical or desirable.

Section V presents a systematic procedure that determines an acceptable port location in given ladder networks to yield all possible transfer function numerator polynomial degrees.


VITA

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APPENDIX A

FLOWCHART AND COMPUTER PROGRAM FOR
SYNTHESIS OF THE VOLTAGE TRANSFER FUNCTION
This computer program is based on Dr. Fowler's Dissertation and can be used to synthesize RLC ladder networks from a given strictly Hurwitz polynomial.

POLRT Subroutine and Subroutine PMPY are used to find the roots of polynomial and the multiplications of two polynomials respectively. The final results of this program yield the component values in the network as required.

The following presents a simplified block diagram of the program and the program itself.
START

READ N, H(J)
WRITE N, H(J)

FIND COEFF. OF H(S)*H(-S) BY SUBROUTINE PMPY

OBTAIN COEFF. OF G(S)*G(-S) AND FIND THE ROOTS BY SUBROUTINE POLRT

CHOOSE POSITIVE ROOTS FOR G(S)

COMPUTE COEFF. OF G(S) FROM ABOVE ROOTS

COMPUTE CONTINUED FRACTION EXPANSION

CALCULATE CKT. ELEMENTS RLC

STOP
SUBROUTINE PMPY

Description of parameters:

Z: vector of resultant coefficients, ordered from smallest to largest degree
IDIMZ: dimension of Z (calculated)
X: vector of coefficients for first polynomial, ordered from smallest to largest degree
IDIMX: dimension of X (degree = IDIMX-1)
Y: vector of coefficients for second polynomial, ordered from smallest to largest degree.
IDIMY: dimension of Y (degree = IDIMY-1)

METHOD: Dimension of Z is calculated as IDIMX+IDIMY-1. The coefficients of Z are calculated as the sum of products of coefficients of X and Y, whose components are added to the corresponding exponent of Z.

BLOCK DIAGRAM

ENTER

DEFINE IDIMZ

ADD PRODUCTS OF X(I)*Y(J) TO FORM Z(K)

EXIT
Block diagram for obtaining coeff of G(s)

Description of parameters:
IR = number of real roots of G(s)
JX = number of complex roots.
BLOCK DIAGRAM FOR CONTINUED FRACTION EXPANSION

1. **H(I) = H(I) * HN1**
   **G(I) = G(I) * GN1**

2. **COMPUTE P(I)**
   **I = 1, N - 1**

3. **COMPUTE F(I)**

4. **N < 2**
   **YES**
   **COMPUTE F(2), F(3)**
   **NO**
   **N = 2**
   **GOTO 140**

5. **COMPUTE Q(J)**
   **J = 1, N - 1**

6. **F(2) = Q(N - 1)**

7. **COMPUTE Q(K) = Q(K) / F(2)**

8. **N < 3**
   **YES**
   **COMPUTE F(N), F(N - 1)**
   **NO**
   **N = 3**
   **GOTO 140**

9. **M - 1**
   **ML = N - 1 - M**

10. **COMPUTE R(L)**
    **L = 1, ML**

11. **F(M + 2) = R(ML)**

12. **COMPUTE R(I) + R(I) / R(ML)**

13. **P(J) = Q(J)**
    **Q(J) = R(J)**

14. **M = M + 1**

15. **M < N - 3**
    **YES**
    **GOTO 140**
    **NO**
    **F(N) = F(I)**
    **F(N + 1) = Q(L)**

16. **WRITE F(I)**

17. **CALCULATIONS OF R, L, C**

18. **GOTO 140**
BLOCK DIAGRAM FOR SUBROUTINE POLRT

ENTER
DIMENSION
N=M+1
I=1,N
B(I)=A(I)
CALL POLRT

IS IER=0?
NO
GO TO 10
YES
II=0
JJ=0
ICT=0
ICT=ICT+1

IS (N-ICT)≥0?
NO
GO TO 10
YES
IS ABS(ICT)>.00001?
NO
RETURN
YES
GO TO 2

JJ=JJ+1
REAL(JJ)=RR(ICT)
AIMJ=ABS(ICT)

ICT=ICT+1

II=II+1
REROOT(II)=RR(ICT)

PRINT IER
C  \( H(S) = H(N+1)S^N + H(N)S^{N-1} + H(N-1)S^{N-2} + \ldots + H(2)S + H(1) \)

DIMENSION GG(30),Z(30)
DIMENSION H(30),W(30),HH(30), AN(30)
DIMENSION BB(30),REROOT(30),REAL(30),AIMA(30)
DIMENSION S(30),SA(30),SB(30),X(30),Y(30)
DIMENSION C(30),G(30),P(30),Q(30),R(30),F(30)
DIMENSION FF(30)
500 FORMAT (F14.4)
501 FORMAT (I2)
502 FORMAT (' DEGREE OF H(S) = 'I4, ' COEFF OF H(S) IN DECREASING POWERS OF S'
503 FORMAT (F14.4)
504 FORMAT (' COEFF OF H(S)*H(-S) ',/(F14.4))
505 FORMAT (' COEFF OF G(S) IN DECREASING POWERS'/)
511 FORMAT (' COEFF OF THE CONTINUED FRACTION'/)
512 FORMAT (' VALUES OF THE RESISTORS IN OHMS'/10X,'R1=' F10.4,//
513 FORMAT (' VALUES OF INDUCTORS IN HENRY'//(F14.4))
514 FORMAT (' VALUES OF CAPACITORS IN FARAD'//(F14.4))
516 FORMAT (' WHEN C1='F10.4)
517 FORMAT (' THE ROOTS FOR G(S) ARE '/)
518 FORMAT (/(F10.4,'+ J', F10.4,/F10.4,'- J', F10.4/)
519 FORMAT (/' THE ACTUAL VALUES OF THE CIRCUIT ELEMENTS'/)

C

C READ DEGREE N
READ (1,501) N
N1=N+1
C READ COEFF IN DECREASING POWERS
DO 5 I=1,N1
   J=N1+1-I
   READ (1,500) H(J)
   5 WRITE (3,503) H(J)
H(N1)=H(N1)
C
C COEFF OF H(-S)
6 DO 10 I=2,N1,2
   10 Y(I)=-H(I)
   DO 15 J=1,N1,2
   15 Y(J)=H(J)
C
C COEFF OF H(S)*H(-S)
WRITE (3,504)
IDIMH=N1
IDIMY=N1
IDIMW=N1
CALL PMFY (HH,IDIMHH,H,IDIMH,Y,IDIMY)
DO 20 I=1,IDIMHH
   K=2*I+N+2-I
   20 WRITE (3,503) HH(K)
NN=2*N
**ABSMIN=H(1)\*H(1)**

C

**COEFF OF G(S)\*G(-S)**

DO 190 IC=1,12,1

TIC=IC/100.

AM=(ABSMIN+1.\*(TIC))/4.

GG(1)=HH(1)-AM

WRITE (3,515) AM

WRITE (3,508)

NN2=NN+1

DO 47 J=2,NN2

GG(J)=HH(J)

DO 48 I=1,NN2

K=NN2+1-I

WRITE (3,503) GG(K)

NGG=NN

CALL BAISTO(NGG,GG,II,REROOT, JJ, REAL, AIMA)

C

**CHOOSE ROOTS FOR G(S)**

WRITE (3,517)

IR=0

JX=0

IF (II .EQ. 0) GO TO 61

DO 60 K=1,II

IF (REROOT(K)) 50,50,60

50 IR=IR+1

S(IR)=REROOT(K)

REROOT(K)=REROOT(K)*(-1.)

WRITE (3,503) REROOT(K)

60 CONTINUE

61 IF (JJ .EQ. 0) GO TO 66

DO 63 L=1,JJ

IF (REAL(L)) 62,62,63

62 JX=JX+1

SA(JX)=2.*REAL(L)

REAL(L)=-REAL(L)

SB(JX)=REAL(L)**2+AIMA(L)**2

WRITE (3,518) REAL(L),AIMA(L),REAL(L),AIMA(L)

63 CONTINUE

C

**COEFF OF G(S)**

66 WRITE (3,509)

IF (IR-1) 82,64,65

64 Y(1)=S(1)

Y(2)=1.

IDIMY=2

GO TO 85

65 IDIMX=2

Y(1)=S(1)

Y(2)=1.

X(1)=S(2)

X(2)=1.
CALL PMPY (Z,IDIMZ,X,IDIMX,Y,IDIMY)

IF (IR-2) 85,80,70
70 II2=II-2
   DO 76 K1=1,II2
   DO 75 I=1,IDIMZ
75 Y(I)=Z(I)
   X(2)=1.
   X(1)=S(K1+2)
   IDIMX=2
   IDIMY=IDIMZ
   CALL PMPY (Z,IDIMZ,X,IDIMX,Y,IDIMY)
76 CONTINUE
80 DO 81 I=1,IDIMZ
81 Y(I)=Z(I)
   IDIMY=IDIMZ
   GO TO 85
82 Y(I)=1.
   IDIMY=1
85 IF (JX-1) 100,86,86
86 X(3)=1.
   X(2)=SA(1)
   X(1)=SB(1)
   IDIMX=3
   CALL PMPY (Z,IDIMZ,X,IDIMX,Y,IDIMY)
   IF (JX-1) 100,100,87
87 JX1=JX-1
   DO 90 J1=1,JX1
   DO 88 I=1,IDIMZ
88 Y(I)=Z(I)
   IDIMY=IDIMZ
   X(3)=1.
   X(2)=SA(J1+1)
   X(1)=SB(J1+1)
   IDIMX=3
   CALL PMPY (Z,IDIMZ,X,IDIMX,Y,IDIMY)
90 CONTINUE
100 K=IDIMZ+1-J
   WRITE (3,503) Z(K)
C
C EVALUATION OF THE CONTINUED FRACTION
WRITE (3,511)
   DO 101 I=1,IDIMZ
101 G(I)=Z(I)
   IF(HN1 .EQ. 1.) GO TO 104
   DO 103 J=1,N1
   G(J)=G(J)*HN1
103 H(J)=H(J)*HN1
104 D=H(N)-G(N)
   M1=N-1
   DO 105 I=1,M1
105 P(I)=(H(I)-G(I))/D
F(1)=H(N)-P(N-1)
IF (N-2) 135,135,108
108 DO 110 J=2,M1
110 Q(J)=H(J)-P(J-1)-P(J)*F(1)
Q(1)=H(1)-P(1)*F(1)
F(2)=Q(N-1)
M2=N-2
DO 115 K=1,M2
115 Q(K)=Q(K)/F(2)
M3=N-3
IF (M3) 132,132,116
116 DO 131 M=1,M3
ML=N-1-M
DO 120 L=2,ML
120 R(L)=P(L)-Q(L-1)
R(1)=P(1)
F(M+2)=R(ML)
MI=ML-1
DO 125 I=1,MI
125 R(I)=R(I)/R(ML)
DO 130 J=1,ML
P(J)=Q(J)
Q(J)=R(J)
130 CONTINUE
132 F(N)=F(1)
F(N+1)=Q(1)
GO TO 140
135 F(2)=H(1)-P(1)*F(1)
F(3)=F(1)
GO TO 140
140 I1=N+1
WRITE (3,503) (F(I), I-I1)

C
DO 144 I=1,20
C(I)=0.
144 W(I)=0.
DO 145 I=2,N
145 F(I)=SQRT(F(I))
DO 193 NC1=1,5,1
C(1)=10.0**(-1*(NC1-1))
WRITE (3,516) C(1)
C(1)=1./SQRT(C(1))
146 W(1)=F(2)/C(1)
R1=C(1)**(2)/F(1)
IF (N-2*(N/2)) 160,160,150
150 M=(N+1)/2
MR=(N-1)/2
G2=F(3)/F(2)
IF (N-3) 158,158,155
155 DO 156 I=2,MR
156 G2=G2**F(I*2+1)/F(I*2)
G2 = G2**(-2)*F(N+1)

R2 = 1./G2

J1 = MR - 1

IF (N-3) 159, 159, 154

DO 157 J = 1, J1

W(J+1) = W(J)*F(2*J+2)/F(2*J+1)

DO 161 J = 1, MR

C(J+1) = C(J)*F(2*J+1)/F(2*J)

GO TO 175

M = N/2

MR = N/2

R2 = F(2)

IF (N-2) 164, 164, 165

R2 = R2**(-2)*F(N+1)

GO TO 175

DO 166 K = 2, M

R2 = R2*F(2*K)/F(2*K-1)

R2 = R2**(-2)*F(N+1)

IF (N-2) 175, 175, 168

II = MR - 1

DO 170 I = 1, II

W(I+1) = W(I)*F(2*I+2)/F(2*I+1)

C(I+1) = C(I)*F(2*I+1)/F(2*I)

DO 180 I = 1, M

C(I) = C(I)**(-2)

DO 185 I = 1, MR

W(I) = W(I)**(-2)

WRITE (3, 519)

WRITE (3, 512) R1, R2

WRITE (3, 513) (W(I), I = 1, MR)

WRITE (3, 514) (C(I), I = 1, M)

CONTINUE

CONTINUE

STOP

END

SUBROUTINE BAISTO (M, A, II, REROOT, JJ, REAL, AIMA)
REAL A(1), REROOT(1), REAL(1), AIMA(1), RR(30), RI(30), B(30)

N = M + 1

DO 11 I = 1, N

B(I) = A(I)

PRINT 102, N, (B(I), I = 1, N)

102 FORMAT (I12, (6E16.6))

CALL POLRT (B, REAL, M, RR, RI, IER)

IF (IER.NE.0) GO TO 10

II = 0

JJ = 0

ICT = 0

1 ICT = ICT + 1

IF (N-ICT.LE.0) RETURN

IF (ABS(RI(ICT)).LT.(.00001)) GO TO 2

JJ = JJ + 1

REAL(JJ) = RR(ICT)
AIMA(JJ) = ABS(RI(ICT))
ICT = ICT + 1
GO TO 1
2
II = II + 1
REROOT(II) = RR(ICT)
GO TO 1
10
PRINT 101, IER, (B(I), I = 1, N)
RETURN
101
FORMAT (/ 'POLRT HAD PROBLEMS '/' IER = ', I6/(6E16.6))
END
SUBROUTINE PMPY (Z, IDIMZ, X, IDIMX, Y, IDIMY)
C
MULTIPLICATION OF TWO POLYNOMIALS
DIMENSION Z(30), X(30), Y(30)
IDIMZ = IDIMX + IDIMY - 1
DO 30 I = 1, IDIMZ
30
Z(I) = 0.
DO 40 I = 1, IDIMX
DO 40 J = 1, IDIMY
K = I + J - 1
40
Z(K) = X(I)*Y(J) + Z(K)
RETURN
END
//DATA DD *