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Linvill's model as applied to P-N junction photo-diodes

Rudy Michael Chittenden

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LINVILL'S MODEL AS APPLIED TO P-N JUNCTION PHOTO-DIODES

BY

RUDY MICHAEL CHITTENDEN

A

THESIS

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Approved by

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ABSTRACT

The problem undertaken in this study is to present a performance evaluation of Linvill's model as it pertains to the p-n junction photo-diode. The evaluation is performed by comparing the distributed mathematical solutions, the solution obtained using Linvill's lumped model, and the experimental measurement of the transient voltage rise across a photo-diode due to a unit-step pulse of light incident on the junction.

Despite the absence of a suitable light source for use in the experimental measurement, it is concluded that Linvill's model provides an excellent means to determine the risetime of a photo-diode. This is provided that the photo-diode can conduct current in the forward direction to enable accurate measurement of the parameters of Linvill's model. If this provision is not satisfied, the results cannot be assumed accurate.
ACKNOWLEDGEMENTS

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CHAPTER I

INTRODUCTION

A. Statement of the Problem.

The purpose of this study is to evaluate the accuracy of the use of Linvill's model for the p-n junction photodiode in expressing the transient voltage waveform across the photo-diode due to a unit-step pulse of light incident on the junction. The evaluation is made through a comparison of results obtained from a distributed solution, from a solution using Linvill's model, and from an experimental measurement (the experimental measurement is later deleted due to the absence of a suitable source of modulated light).

The development of Linvill's model is presented and the assumptions made are pointed out and discussed in the final stage of this study.

In order to obtain an analytical expression for the resultant transient voltage, the carrier concentration is found both as a function of distance and time. From this distribution, the expression for the voltage is found.

The major difficulty in making the experimental measurements is producing a unit-step of light. Theoretically, of course, this is impossible, but no error is introduced as long as the risetime of the light pulse is short compared to the effective risetime of the photo-diode. In this study, this restriction called for a light pulse with a risetime of approximately one-tenth of a micro-second. Methods of obtaining
such a pulse of light are discussed (risetime for this study designates the time required for the transient to reach 99 per cent of its steady-state value).

B. **Significance of the Study.**

The application of models for circuit analysis and design cannot be exaggerated. The use of any one particular model is dependent, among other considerations, upon the relative ease and accuracy with which it can be applied.

The parameters of Linvill's model are easily measured and its use is fairly simple, therefore, it is of importance that an evaluation of the accuracy of this model be made and the value of its use be known.

C. **Reasons for the Investigation.**

An interest in the theory and application of models for solid state devices led the author to explore this area for a possible thesis topic.

The book by Linvill, studied in a course taken by the author, presented the development and application of models for transistors and diodes. Although Linvill had little to say about photo-sensitive devices, it was believed that his model would apply in this area.

---

Numbers in parenthesis refer to references listed in the bibliography.
CHAPTER II

REVIEW OF THE LITERATURE

The model discussed here was first presented in a graduate subject that has formed the basis for the book by Linvill and Gibbons(1). Later, the presentation in the aforementioned text was worked into a more compact form by Linvill(2).

Papers prepared by Chang(3), Gossick(4), Bhar(5), Holmes and Feucht(6), and Ritter(7), were helpful in explaining the theory of the p-n junction and the distribution of carrier concentration.

Methods for solving the partial differential equations by Laplace transforms are given by Churchill(9) and by Carslaw and Jaeger(10).

Other background material was obtained from Shockley(8), Hunter(11), and Lange(12).

Methods of modulating light at extreme frequencies are discussed by Jones(13) and Draper(14), and the theory concerning the determination of the absorption coefficient is discussed in a book by Bauman(15).

Material on the operation and characteristics of the nitrobenzene-filled Kerr cell was obtained from Cronemeyer and Spanberger(16).
CHAPTER III

REVIEW OF SEMICONDUCTOR THEORY

A. Introduction and Explanation.

The purpose of this chapter is to review the theory of semiconductors in general, and the mechanics of the operation of photo-diodes in particular.

This chapter supplies the base on which the entire study is built, in that the characteristics of semiconductors pertinent to this study are summarized and their contributions are assessed. The most important feature of the basis for this study is the discussion of carriers of current in semiconductors and their modes of flow, i.e., drift and diffusion. The above-mentioned discussion and the discussion on the specifics of photo-diode operations constitute the bulk of the theoretical concepts supporting this thesis.

B. General.

Materials which are intermediate in conductivity between insulators and the metallic conductors are classified as semiconductors. The most familiar of semiconductors in transistor and diode applications are germanium and silicon, though compounds formed from elements of Group III and Group V of the periodic table show great promise for use in semiconductor devices. The discussion given here will concentrate on germanium and silicon, since the diode to be tested is of one of these, and also, knowledge of these two types of semiconductors is more complete than knowledge of other semiconductors.
Germanium and silicon exhibit a crystalline structure in which each atom is equally spaced from four neighboring atoms. Each atom, having four valence electrons of its own, shares an electron with each of the four neighboring atoms to complete its valence structure. The net charge of each atom is zero, the negative charge of orbital electrons just balancing the positive charge of the nucleus.

The electrons moving about the nucleus are found with only certain permitted energies, the magnitude of which varies with the separation of the electron from the nucleus. The allowed energy states fall into ranges or bands separated by a forbidden band. The region of energy states which falls below the forbidden band is called the valence band and the region of energy states above the forbidden band is called the conduction band.

The amount of energy required to move an electron from the valence band to the conduction band, which corresponds to separating an electron from its nucleus, is dependent on the width of the forbidden band. This amount of energy is 0.7 ev in germanium and 1.0 ev in silicon.

At room temperature a small fraction of the electrons have a sufficient amount of energy to pass from the valence band into the conduction band and are free to move about. These freed electrons are referred to as "conduction electrons" or more simply as just "electrons," since we are not interested in valence electrons in the flow of electricity. The sites
from which these electrons have escaped exhibit a net positive charge and are called "holes." Holes and electrons constitute the two carriers of electricity in semiconductors.

The thermal generation of electron-hole pairs as described above operates continuously. However, due to recombination of electrons and holes the population of electrons and holes fluctuates randomly about some average value and does not increase indefinitely.

If the density of electrons in an intrinsic or undoped material is \( n_i \) cm\(^{-3}\), and the corresponding hole density is \( p_i \) cm\(^{-3}\), the equilibrium density of holes and electrons as a function of temperature is given by:

\[
n_i = p_i = CT^{3/2} \exp(-qE_g/2kT)
\]

where: \( C \) = constant of proportionality dependent upon the material
\( T \) = absolute temperature
\( E_g \) = size of the forbidden gap, ev
\( q \) = charge on the electron, 1.69x10\(^{-19}\) coul
\( k \) = Boltzmann's constant, 1.38x10\(^{-23}\) joule/\(^{\circ}\)K

The significance of Eq. (3.1) is its demonstration of the variation of the equilibrium density of holes and electrons with temperature. To demonstrate this variation, we take the following differential of the most significant part
of Eq. (3.1), the exponential factor and obtain:

\[
d\left[\exp\left(-\frac{qE_g}{2kT}\right)\right] = \left[\exp\left(-\frac{qE_g}{2kT}\right)\right] \frac{qE_g}{2kT} \frac{dT}{T} \quad (3.2)
\]

The fractional change in density is found as follows;

\[
\frac{d\left[\exp\left(-\frac{qE_g}{2kT}\right)\right]}{\exp\left(-\frac{qE_g}{2kT}\right)} = \frac{qE_g}{2kT} \frac{1}{T} \, dT \quad (3.3)
\]

Eq. (3.3) tells us that for germanium an increase of one degree centigrade above room temperature will increase the density of holes and electrons by approximately 5 per cent and for silicon the increase will be about 7 per cent per degree rise in temperature. This increase in carrier density with temperature accounts for the characteristic decrease in resistivity of semiconductors with rising temperature.

The density of hole-electron pairs in germanium at room temperature is approximately \(2.5 \times 10^{13} \text{ cm}^{-3}\) or about one-billionth of the atom density which is about \(4.5 \times 10^{22} \text{ cm}^{-3}\). In silicon, the density of hole-electron pairs at room temperature is about \(6.8 \times 10^{10} \text{ cm}^{-3}\), compared to the atom density which is \(5.0 \times 10^{22} \text{ cm}^{-3}\). These values are seen to correlate with the relative sizes of the forbidden gaps of germanium and silicon, the larger hole-electron pair density corresponding to the smaller forbidden gap and vice versa.

Up to this point we have been discussing densities of intrinsic or undoped materials. We are now prepared to discuss the more interesting and more important (to this study)
properties of impure or doped semiconductors.

At some time during or after the growth of a crystal, a small fraction of impurities from the third or fifth column of the periodic table may be added to the crystal of germanium or silicon. Impurities from the fifth are called donor impurities, because they have one extra valence electron (or a total of five electrons) which is free to conduct electricity. Impurities from the third column are called acceptor impurities due to their lack of a valence electron which causes them to accept valence electrons of neighboring atoms. Since it is found that practically all of the impurities are ionized, donor impurities contribute electrons to the material, now called n-type material and acceptor impurities accept electrons or correspondingly contribute holes to the material, now called p-type material.

The reason why the addition of impurities to a material makes it either p-type or n-type is that the addition of impurities in typical densities (from $10^{13}$ to $10^{19}$ cm$^{-3}$) does not significantly alter the rate at which hole-electron pairs are generated thermally. So, for example, in a donor-doped material, the electron density is increased by the addition of more electrons, while the hole density is decreased due to the increased rate of recombination. The result is that the recombination process dominates until the equilibrium density of holes is down to value such that:

$$pn = n_i^2 = C^2T^3 \exp\left(-\frac{qE_g}{kT}\right) \quad (3.4)$$
It is usually assumed that the density of electrons in n-type material and the density of holes in p-type material equal approximately their corresponding doping densities. Thus, in n-type material the equilibrium density of electrons, designated \( n_n \), is given by:

\[
n_n = N_d \quad (3.5)
\]

where \( N_d \) is the density of donor atoms.

The equilibrium density of holes in n-type material, designated \( p_n \), from Eqs. (3.4) and (3.5) is:

\[
p_n = n_i^2/N_d \quad (3.6)
\]

Similar relations for the equilibrium densities in a p-type material are:

\[
p_p = N_a \quad (3.7)
\]

and

\[
n_p = n_i^2/N_a \quad (3.8)
\]

where \( p_p \) is the equilibrium density of holes in the p-type material, \( n_p \) is the equilibrium density of electrons in the p-type material, and \( N_a \) is the density of acceptor atoms.
C. Current Carriers and Their Modes of Flow.

There are two means of transport which are significant in the flow of current carriers. These are the diffusion of carriers from regions of high density of carriers to regions of low density of carriers and the drift of carriers in an electric field. The case of transport of carriers by diffusion will be considered first.

![Diagram of semiconductor material](image)

Fig. 3.1 Block of semiconductor material.

In a semiconductor material with a uniform density of holes and electrons, the net current passing through an intersecting plane is zero, since the holes and electrons are in random motion due to their thermal velocities. The net current is not zero however if the density is not uniform.

Consider the rectangular block of semiconductor of uniform cross-section $A$, as shown in Fig. 3.1. It is assumed that the regions $r_1$ and $r_2$ are sufficiently small that the variation of carrier density (holes) across them is small.
With this assumption and the knowledge that the net flow of current is proportional to the gradient of density, the current of holes between regions \( r_1 \) and \( r_2 \) by diffusion is given by:

\[
    i_{PD}(r_1, r_2) = \frac{(p_{r_1} - p_{r_2})}{\Delta x} AqD_p
\]

(3.9)

Which, for \( \Delta x \) approaching zero, becomes:

\[
    i_{PD} = AqD_p (-\delta p/\delta x)
\]

(3.10)

In Eqs. (3.9) and (3.10), the symbol \( D_p \) is the diffusion constant. For germanium and silicon, this constant has the values:

\[
    D_p(\text{Ge}) = 49.3 \text{ cm}^2\text{sec}^{-1} \\
    D_p(\text{Si}) = 12.4 \text{ cm}^2\text{sec}^{-1}
\]

The values for this constant in n-type material are:

\[
    D_n(\text{Ge}) = 101 \text{ cm}^2\text{sec}^{-1} \\
    D_n(\text{Si}) = 35 \text{ cm}^2\text{sec}^{-1}
\]

An electric field in a semiconductor material adds a drift component of velocity to the random motion of holes and electrons. The velocity of carriers in an electric field is directly proportional to the strength of the field, the constant
of proportionality being called the mobility of carriers, designated by \( \mu \). Thus, the drift velocity of holes, designated \( v_p \) can be written as:

\[
v_p = E\mu_p
\]  

(3.11)

and the drift velocity of electrons, designated \( v_n \), can be written as:

\[
v_n = E\mu_n
\]  

(3.12)

The following are values for the mobility of holes and electrons in germanium and silicon.

\[
\begin{align*}
\mu_p(\text{Ge}) &= 1,900 \text{ cm}^2\text{volt}^{-1}\text{sec}^{-1} \\
\mu_p(\text{Si}) &= 430 \text{ cm}^2\text{volt}^{-1}\text{sec}^{-1} \\
\mu_n(\text{Ge}) &= 3,900 \text{ cm}^2\text{volt}^{-1}\text{sec}^{-1} \\
\mu_n(\text{Si}) &= 1,350 \text{ cm}^2\text{volt}^{-1}\text{sec}^{-1}
\end{align*}
\]

To write the expression for the drift current, we again refer to Fig. 3.1. The electric field is described as the difference in potential between center points of the two regions.

The current density in a unit cross-sectional area aligned with the electric field is numerically equal to the
charge, or holes, enclosed in an imaginary tube of unit cross section and length \( v_p \), or:

\[
    j_p = p v_p q = pE\mu_p q
\]  

(3.13)

Thus, for Fig. 3.1:

\[
    i_p (r_1, r_2) = \frac{p r_1 - p r_2}{2} \frac{v r_1 - v r_2}{\Delta x} A q \mu_p
\]  

(3.14)

where the density enclosed is related to the average of the average densities of the two regions. It should be noted from Eq. (3.14), that the flow of carriers by drift is a nonlinear phenomenon.

The relationship between the flow of current by diffusion and the flow of current by drift in terms of their constants of proportionality has been derived by Einstein:

\[
    \eta_p / D_p = \eta_n / D_n = q / kT
\]  

(3.15)

D. The Approximation of Space-charge Neutrality.

To begin at this point to discuss analytically the particulars of diode and transistor mechanics could only result in confusion. We would have to determine the movements of two types of current carriers, each with two distinct types or modes of movement.

The approximation of space-charge neutrality relieves this situation without reducing the validity of even the
most rigorous examination. The analysis is reduced to consideration of only the movements of the minority carriers by assuming that any movement of minority carriers is immediately accompanied by the flow of majority carriers. The actual lapse of time involved has been estimated by Linvill (2) to be in the order of micromicroseconds for a typically doped segment of n-type germanium.

Thus, if the distribution of holes in an n-type material has been determined as \( p(x,t) \), the distribution of electrons can be found from:

\[
n(x,t) = p(x,t) + N_d \quad (3.16)
\]

E. Basics of Photo-Diode Operation.

The semiconductor photo-diode is a device used to convert radiant energy into electrical energy. Its primary use to date, however, is not as a source of energy, but as a control or measuring device, such as the photographic light meter. It is feasible, however, that the search for means to tap the sun as a source of energy may find new and more profitable uses for this device.

The biasing arrangement for the photo-diode is shown in Fig. 3.2. The voltage applied to the diode is the reverse direction so as to prohibit the flow of majority carriers. The current flowing with no light incident on the
junction is therefore due entirely to minority carriers. Since, as is the case in most photo-diodes, the p-region is more highly doped than the n-region, the minority concentration in the p-region is substantially less than the minority concentration in n-region, and the reverse current is solely determined by \( p_n \), the equilibrium density of holes in the n-region. Typical values for the concentrations in each region are given in Fig. 3.2(b).

When the light is made to shine on the junction, hole-electron pairs are generated according to the quantum nature of light, the number of pairs produced being proportional to the intensity of the light. In both regions of the semiconductor, the percent increase of majority carriers relative to the percent increase in minority carriers is quite small, and the net current resulting from the illumination is in the reverse direction. The illumination current then is composed mainly of minority carriers.

In this study, the photo-diode is to be pulsed with a unit step of light, and the transient rise in voltage across the entire diode will be analysed. Any voltage drop across the p-type emitter region will be neglected due to the high conductivity of this region. The determination of the distribution of the minority carriers in the n-type region will provide all the information necessary to predict the waveform of the desired transient.

The holes in the n-region will distribute across the region according to the laws of the flow of carriers in
semiconductors as derived earlier in this chapter. Two methods for determining this distribution of carriers for a given diode are demonstrated in Chapters 4 and 5.

![Diagram](image)

\[
\begin{align*}
\text{Fig. 3.2 a)} & \quad \text{Biasing arrangement.} \\
\text{b)} & \quad \text{Typical density levels.}
\end{align*}
\]
CHAPTER IV
LINVILL’S MODEL

A. Introduction.

A model of a physical device is a mathematical entity with precise laws governing the relationship of its variables. The use of models is quite common, and often the process or device is confused with the model, especially if the model is a very good approximation of the process. For example, a resistor, as shown in Fig. 4.1, is a parameter used to relate voltage and current. The R, which is the symbol for resistance, does not represent exactly any physical device, since no physical element can behave as purely resistive. Thus, the symbol in Fig. 4.1 is a model and a good one, but is too often confused with the physical element it represents.

![Fig. 4.1 A resistor.](image)

Depending on the relative ease with which they can be applied, models of diodes and transistors may either be distributed or lumped in nature. In either case, carrier flow from volume element to volume element and changes in the density of carriers within an element of volume are the main considerations. If these volume elements are very small, a differential equation may be written for carrier flow and
integration may be performed over the entire volume to determine the populations. If these equations, together with the appropriate initial and boundary conditions, can be solved, very accurate results may be obtained for the distributions of the current carriers. A model employing this rigorous type of solution would be referred to as a distributed model.

It is often found that the intricate boundary conditions associated with the analysis of semiconductor behavior make the use of the distributed model impractical and sometimes, impossible. It is for this reason that a great number of equivalent circuits and lumped models have been derived for use in semiconductor analysis. The lumped models are simplifications of the distributed models, employing finite volume elements instead of very small volume elements. The differential equations become difference equations, and the solution is much more straightforward.

B. Derivation of Linvill's Model.

The model derived by Linvill(1) was a lumped model. The initial assumptions made in deriving the model are as follows (for an n-type semiconductor):

a) Space-charge neutrality is assumed.

b) Minority carriers flow only by diffusion in one-dimensional flow according to the equation:

\[ j_p = -qD \frac{\partial p}{\partial x} \]  

(4.1)
c) The minority-carrier densities obey the linear continuity equation:

\[
\frac{\partial \rho}{\partial t} = \frac{p-p_n}{\tau_p} - \frac{1}{q} \text{grad } j_p \tag{4.2}
\]

d) The minority-carrier densities in the vicinity of the junction obey Shockley's injection relation:

\[
p(x=0) = p_n \exp \left( \frac{qV_j}{kT} \right) \tag{4.3}
\]

Substituting Eq.(4.1) into Eq.(4.2):

\[
\frac{\partial \rho}{\partial t} = -\frac{p-p_n}{\tau_p} + D_p \frac{\partial^2 \rho}{\partial x^2} \tag{4.4}
\]

Linvill in deriving his model used the solution for Eq.(4.4). The solution will not be repeated here since it is straightforward and the solution of a similar equation more appropriate to this study is presented in Chapter 5.

The boundary conditions used by Linvill were \( p(\infty, t) = p_n \) and Eq.(4.3).

The solutions found by Linvill for the minority carrier distribution and the current at the junction are as follows(1):

\[
p(x,t) - p_n = p_0(0) \exp \left( -\frac{x}{L_p} \right) + p_1(0) \exp \left( \frac{-x(1+s\tau_p)^{1/2}}{L_p} \right) \exp(st) \tag{4.5}
\]

\[
i_p = I_p \frac{p_0(0)}{p_n} + I_p \frac{p_1(0)}{n(1 + s\tau_p)^{1/2}} \exp(st) \tag{4.6}
\]
where; \( p_0 \) = steady-state density
\[ p_1 = \text{time-dependent density} \]

The expression for the hole current in Eq.(4.6) was found by substituting Eq.(4.5) into Eq.(4.1).

At this point, Linvill introduced the concept of "current-density immittances." The solutions found in Eqs.(4.5) and (4.6) are actually solutions for a distributed model. The parameters of Linvill's lumped model are found by requiring that its current-density immittance should be a reasonable approximation to that of the distributed model. The current-density immittance is defined as the ratio of the complex current component and the complex density component. Thus, from Eqs.(4.5) and (4.6), the current-density driving-point immittance for holes is:

\[
\frac{I_p}{p_n} \frac{p_1(0)}{p_1(0) \exp(st)} = \frac{I_p}{p_n} \frac{p_1(0)}{p_1(0) \exp(st)} \exp(st) = \frac{I_p}{p_n} (1 + s\tau_p)^{1/2}
\]

(4.7)

where; \( I_p = AqD_p p_n / L_p \)

It would be possible, using procedures developed for the approximation problem of network synthesis, to develop a lumped network that would closely approximate the driving-point immittance of Eq.(4.7). For a diode in the reverse-bias
condition, however, the model must be limited to only one lump due to the effect of the capacitance of the junction which is neglected in Eq. (4.7).

The capacitance of the junction is a function of the applied voltage. There exists in the vicinity of the junction a region called the transition region in which the densities are changing toward their equilibrium values. When a negative voltage is applied across the junction, a charge of holes must move from the edge of the transition region in the p-region and a corresponding charge of electrons must move from the edge of the transition region in the n-region. This movement of charge corresponds to a change of the capacitance required to store the charge. This capacitance appears in parallel with the immittance defined by Eq. (4.7). Its effects are negligible when the junction is forward-biased, since due to its small size (10 μf) the currents through it are small compared to other currents elsewhere. This is no longer true when the junction is reverse-biased.

A one-lump structure which has an immittance approximating the function given in Eq. (4.7) can be found by making a binomial expansion of the square root as follows:

\[ (1 + s\tau_p)^{1/2} \approx 1 + \frac{s\tau_p}{2} \]  (4.8)

The structure found by Linvill to have an immittance equal to that of Eq. (4.8) is shown in Fig. 4.2. This structure is valid only for low frequencies such that \( s\tau_p \ll 1 \).
The symbols $S_p$ and $H_c$ are called storance and combinance, respectively. Their relation to current and minority-carrier density are:

\[ i_p = S_p \frac{dp}{dt} \quad (4.9) \]

\[ i_H = H_c p \quad (4.10) \]

where $S_p$ has the dimensions, coulomb-cm$^3$ and $H_c$ has the dimensions amp-cm$^3$.

Fig. 4.2 One-lump approximation for the immittance of a diode.

The complete model as derived by Linvill is shown in Fig. 4.3. The symbol $C_j$ represents the capacitance of the junction. It is assumed that the density of minority carriers in the vicinity of the junction obeys the requirements of Eq. (4.3) and that recombination in the transition region is negligible, or the hole current entering the junction is equal to the hole current leaving the junction.
The method used to determine the parameters of the model in Fig. 4.3 is discussed in Chapter 6. The storage effects of the junction capacitance will be included in the value of the storage.

\[ V_j \]

\[ C_j \]

\[ S_p \]

\[ H_c \]

\[ I_0 \]

Fig. 4.3 Linvill's model.

C. Solution for Transient Voltage Rise (Linvill's Model).

The value of illumination current \( I_o \) chosen for this solution is \( 3.93 \times 10^{-5} \)amp. The illumination current divides between the storance \( S_p \) and the combinance \( H_c \), therefore:

\[ \frac{dp_e(t)}{dt} + H_c p_e(t) = I_0 \]  \hspace{1cm} (4.11)

Taking the Laplace transform of Eq.(4.11) with respect to "t" yields; where \( \tilde{p}_e(s) \) denotes the Laplace transform of \( p_e(t) \):

\[ s S_p \tilde{p}_e(s) - S_p p_e(0) + H_c \tilde{p}_e(s) = I_o / s \]  \hspace{1cm} (4.12)

where \( p_e(0) = -p_n \).  

*\( I_o \) is equal to zero due to the fact that no transient current flows in the external circuit.
Solving for $P_e(s)$ we obtain:

$$P_e(s) = \frac{I_0/s - p_n S_p}{S_p(s + H_c/S_p)} \quad (4.13)$$

To return to the time domain we take the inverse Laplace of $P_e(s)$ to find:

$$p_e(t) = \frac{I_0}{H_c} - \frac{I_0}{H_c} + p_n \exp\left(-\frac{H_c t}{S_p}\right) \quad (4.14)$$

The expression for the voltage across the junction is found by substituting Eq. (4.14) into Eq. (4.3) and solving for $V_j$:

$$V_j = \frac{kT}{q} \ln\left[1 + \frac{p_e(t)}{p_n}\right] \quad (4.15)$$

where; $p_e = p + p_n$.

The variation of $V_j$ with respect to time for the values of $S_p$ and $H_c$ as determined in Chapter 6 is shown in Fig. 5.3.
A. Introduction.

In order to determine the validity and accuracy of Linvill's model in predicting the transient voltage rise across a photo-diode, a more rigorous analysis will be presented in this chapter.

The solution of the differential equation for distributed models can be obtained, if certain limitations are placed upon the range of magnitude of the signals applied to the photo-diode. These limitations will be discussed as they appear in the analysis.

B. General.

The voltage across the photo-diode is composed of two distinct types. They are the voltage across the junction proper, $V_j$, and the voltage drop across the bulk material, $V_b$. If the distribution of minority carriers in the region under consideration is known as a function of time and distance, the voltages $V_j$ and $V_b$ may be found.

The equations governing the behavior of injected minority carriers into an n-region are (7):

$$\frac{dp}{dt} = - (p-p_n) \sqrt{\frac{r}{p}} - q^{-1} \text{div} J_p$$  \hspace{1cm} (5.1)

$$J_p = q \mu_p p E - q D_p \text{grad}(p)$$  \hspace{1cm} (5.2)
The problem may be simplified by using the Shockley injection relation for small injection levels \(p/N_d \ll 1\) and assuming one-dimensional flow of carriers. The differential equation derived by Shockley is:

\[
\frac{\partial p}{\partial t} + \frac{(p-p_n)}{\tau_p} = D_p \frac{\partial^2 p}{\partial x^2}
\] (5.3)

The basic simplifications employed in Eq. (5.3) are the assumption of space-charge neutrality and the neglecting of the flow of carriers by drift in relation to greater flow of carriers by diffusion. The latter simplification confines the electric field within the semiconductor to zero. An electric field does exist, however, and its contribution to the total voltage across the diode will be included in a later section.

Ritter(7), in discussing the theory of a junction transistor, expounded on the validity range of Shockley's theory. He concluded that for a base of finite width \(w\), the injection level must be small, so that:

\[
p/N_d \ll (w/L_p)^2 \frac{b}{b-1}
\] (5.4)

where \(L_p\) is the diffusion length of holes in n-type semiconductors; and \(p\) is the injected hole density.

The restriction imposed by Eq. (5.4) is quite severe. For a typically-doped diode, the signal current would have
to be limited to a very few microamperes. Ritter went on to demonstrate, however, that if the recombination of carriers in the base region could be neglected, the following equation could be used for any arbitrary injection level.

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2}$$

(5.5)

It shall be assumed in this study that the length of the n-region is short compared to the diffusion length of holes, and recombination in this region will be neglected. Eq.(5.5), then, is the desired starting point for the derivation of the distribution of holes in the base region.

C. Solution of the Partial Differential Equation.

The first step in the solution is the determination of the correct initial and boundary conditions (the dimensions and references for the following discussion are defined in Fig. 5.1).

![Fig. 5.1](image-url)

Physical description and values for 1N85 photo-diode.

- **1N85 photo-diode**
  - \( W = 0.222 \text{ cm} \)
  - \( A = 33.8 \times 10^{-4} \text{cm}^2 \)
  - \( N_D = 0.890 \times 10^{15} \text{cm}^{-3} \)
  - \( p_n = 7.02 \times 10^{11} \text{cm}^{-3} \)
The generation of carriers by the light source must be included by altering Eq.(5.5) as follows:

\[ \frac{\partial p_e(x,t)}{\partial t} - D_p \frac{\partial^2 p_e(x,t)}{\partial t^2} = Qe^{-\alpha x} \]  

(5.6)

where; \( Q \) = the number of hole-electron pairs generated per second
\( \alpha \) = the optical absorbtion coefficient, cm\(^{-1}\)

The subscript "e" in \( p_e(x,t) \) of Eq.(5.6) is used to denote that the densities under discussion are excess densities. If \( p \) is the total hole density, the excess hole density \( p_e \) is given by:

\[ p_e = p - p_n \]  

(5.7)

Prior to the instant the light is turned on (\( t = 0^- \)), the junction is heavily reverse-biased. Therefore, the total hole density is zero, and the excess hole density is \(-p_n\), or:

\[ p_e(x,0) = -p_ne^{-\alpha x} \]  

(5.8)

The two boundary conditions to be used are:

\[ \lim_{x \to -\infty} p_e(x,t) = 0 \]  

(5.9)

\[ \frac{\partial p_e(x,t)}{\partial x} \bigg|_{x = 0} = -R \]  

(5.10)
where: \[ R = \frac{I_o}{qAD_p} \]

\[ I_o = \text{illumination current, amp} \]

\[ A = \text{cross-section area of the bulk material, cm}^2 \]

The condition defined by Eq. (5.9) is not exactly the precise one to use for a finite base region. The correct condition should be:

\[
\lim_{x \to W} p_e(x,t) = 0 \tag{5.11}
\]

However, using Eq. (5.10) in the place of Eq. (5.11) greatly simplifies the solution. The author first used Eq. (5.11) in attempting a solution. To obtain a final result, it became necessary to take the inverse Laplace transform of a form similar to:

\[
\frac{1}{s(s-\beta^2)} \cdot \frac{\sinh \sqrt{s}}{\cosh \sqrt{s}} \cdot \cosh \sqrt{s} \tag{5.12}
\]

The transform of (5.12) involves the product of infinite series.

Using Eq. (5.9) in place of Eq. (5.11) gives the solution in closed form. It will be shown later that the density drops by a factor of \(10^{-5}\) in a distance of less than \(0.05\,\text{cm}\). The value for \(W\) in this experiment is approximately \(0.2\,\text{cm}\), so the error involved in using Eq. (5.9) is extremely small.
The boundary condition defined by Eq. (5.10) is a result of the approximation of small level injection. It may be derived as follows:

\[ j_p = q\mu_p p E - qD_p \text{grad}(p) \tag{5.13} \]
\[ j_n = q\mu_n n E - qD_n \text{grad}(n) \]

and by the assumption of space-charge neutrality,

\[ p-n + N_d - N_a = 0 \tag{5.14} \]
\[ \text{grad}(p) = \text{grad}(n) \]

Substituting Eqs. (5.14) into Eqs. (5.13) and solving for \( E \) yields:

\[ E = \frac{j_p + qD_p \frac{dp}{dx}}{q\mu_p p} = \frac{j_n - qbD_p \frac{dp}{dx}}{qb\mu_p (p + N_d)} \tag{5.15} \]

where; \( b = \text{ratio of electron to hole mobility} \)

Solving for \(-j_p/qD_p\) in Eq. (5.15) yields:

\[ -I/AqD_p = \frac{2p + N_d}{p(I - j_n/bj_p) + N_d} \frac{dp}{dx} \tag{5.16} \]

where; \( I = j_p A \)

For small injection levels, \( p \ll N_d \), and neglect \( j_n/bj_p \), yields (7):

\[ \frac{dp}{dx} = -I/AqD_p \tag{5.17} \]
Evaluating Eq. (5.17) at the junction \( x=0 \) gives Eq. (5.10). The relation between \( I \) and \( I_o \) will be discussed in Chapter 6.

The required initial and boundary conditions are summarized below:

a) \( p_e(x,0) = -p_n \)

b) \( \lim_{x \to +\infty} p_e(x,t) = 0 \) \hspace{1cm} (5.18)

c) \( \frac{\partial p_e(x,t)}{\partial t} \bigg|_{x=0} = -R \)

The solution may now continue by taking the Laplace transform of Eq. (5.6) with respect to \( t \).

\[
sl \tilde{p}_e(x,s) - p_e(x,0) - L \left[ D_p \frac{\partial^2 p_e(x,t)}{\partial x^2} \right] = \frac{Q e^{-sx}}{s} \hspace{1cm} (5.19)
\]

Now,

\[
L \left[ D_p \frac{\partial^2 p_e(x,t)}{\partial x^2} \right] = \int_0^s D_p \frac{\partial^2}{\partial x^2} \left[ e^{st} p_e(x,t) \right] \, dt
\]

If \( e^{-st} p_e(x,t) \) is such a function that the interchange of the order of integration with respect to \( t \) and the second order differentiation with respect to \( x \) is valid, the Laplace transform of the bracketed term of Eq. (5.19) is:

\[
L \left[ D_p \frac{\partial^2 p_e(x,t)}{\partial x^2} \right] = D_p \frac{\partial^2 \tilde{p}_e(x,s)}{\partial x^2} \hspace{1cm} (5.20)
\]
Substituting Eqs. (5.20) and (5.8) into Eq. (5.19) and rearranging terms:

\[ D_p \frac{d^2}{dx^2} P_e(x,s) - s P_e(x,s) = - \frac{Q e^{-\alpha x}}{s} - n e^{-\alpha x} \]  

(5.21)

A solution for Eq. (5.21) is:

\[ P_e(x,s) = A e^{-\sqrt{s/D_p}} x + B e^{\sqrt{s/D_p}} x + \frac{Q e^{-\alpha x}}{s(s - \alpha^2 D_p)} + \frac{n e^{-\alpha x}}{(s - \alpha^2 D_p)} \]  

(5.22)

The constants A and B in Eq. (5.22) may be determined from application of the transform of the boundary conditions defined in Eqs. (5.18).

\[ \lim_{x \to \infty} P_e(x,s) = 0 \]  

(5.23)

\[ \frac{d P_e(x,s)}{dx} \bigg|_{x=0} = - \frac{P}{s} \]  

Substituting Eq. (5.22) into Eqs. (5.23) yields:

\[ B = 0 \]  

(5.24)

\[ - \frac{P}{s} = - \sqrt{s/D_p} A - \frac{Q}{s(s - \alpha^2 D_p)} - \frac{n}{(s - \alpha^2 D_p)} \]

or,

\[ A = \frac{P \sqrt{D_p}}{s^{3/2}} - \frac{\sqrt{D_p} Q}{s^{3/2}(s - \alpha^2 D_p)} - \frac{n \sqrt{D_p}}{s^{3/2}(s - \alpha^2 D_p)} \]
Substituting the values for $A$ and $B$ into Eq.(5.22):

$$
\bar{p}_e(x,s) = \frac{R \sqrt{D_p}}{s^{3/2}} e^{-\frac{2x}{\sqrt{D_p} s}} - \frac{\alpha \sqrt{D_p}}{s^{1/2}} \frac{Q e^{\frac{-x}{\sqrt{D_p} s}}}{(s - \kappa^2 D_p)} - \frac{\alpha \sqrt{D_p}}{s^{3/2}} \frac{e^{-\frac{2x}{\sqrt{D_p} s}}}{(s - \kappa^2 D_p)}
+ \frac{Q e^{-\frac{\kappa x}{s - \kappa^2 D_p}}}{s} + \frac{P e^{-\frac{\kappa x}{s - \kappa^2 D_p}}}{s}
$$

(5.25)

The inverse Laplace transform of Eq.(5.25) will give the desired expression for $p_e(x,t)$. The inverse transform of the first term and the last two terms may be obtained directly from available tables(9). The second and third terms require the use of the convolution integral.

$$
\mathcal{L}^{-1} \left[ F_1(s) F_2(s) \right] = \int_0^t f_1(\tau) f_2(t-\tau) \, d\tau
$$

(5.26)

We shall consider the second term of Eq.(5.25) first.

Let,

$$
F_1(s) = \frac{e^{-\frac{2x}{\sqrt{D_p} s}}}{s^{3/2}}
$$

and

$$
F_2(s) = \frac{\alpha \sqrt{D_p}}{s - \kappa^2 D_p}
$$

then,

$$
\mathcal{L}^{-1} \left[ F_1(s) \right] = 2 \sqrt{\frac{t}{\pi}} \exp \left( \frac{\kappa^2 D_p}{4} t \right) \exp \left( -\frac{\kappa^2 D_p}{4} t \right)
$$

and,

$$
\mathcal{L}^{-1} \left[ F_2(s) \right] = \alpha \sqrt{D_p} Q e^{\alpha^2 D_p t}
$$
Therefore,
\[
\int \left[ F_i(s) F_i(s) \right] = \frac{2 \sqrt{\text{DP}} Q e^{x^2 \text{DP}_p t}}{\sqrt{\pi}} \int_0^t e^{-\left( x^2 \text{DP}_p \tau + x^2 \frac{4 \text{DP}_p \tau}{4} \right)} \tau^{1/2} d \tau 
\]
\[+ \alpha x Q e^{x^2 \text{DP}_p t} \int_0^t e^{x^2 \text{DP}_p \tau} \text{erfc} \left( \frac{x}{2 \text{DP}_p t} \right) d \tau \quad (5.27)\]

The above expression cannot be simplified any further without making some gross approximations, so it will be used as is, to calculate \( p_e(x,t) \). This will prove to be the most accurate approach, since a numerical solution will be used to analyse the final results.

Proceeding as above with the third term of Eq.(5.25), we let:
\[
F_i(s) = \frac{e^{-\frac{x}{\text{DP}_p} s}}{s^{1/2}}
\]
and
\[
F_z(s) = \frac{\alpha P_n \sqrt{\text{DP}_p}}{(s - \alpha^2 \text{DP}_p)}
\]
Then,
\[
\int F_i(s) = \frac{1}{\sqrt{\pi t}} \exp \left( -\frac{x^2}{4 \text{DP}_p t} \right)
\]
and
\[
\int F_i(s) = \alpha P_n \sqrt{\text{DP}_p} e^{x^2 \text{DP}_p t}
\]

Therefore,
\[
\int \left[ F_i(s) F_i(s) \right] = \frac{\alpha P_n \sqrt{\text{DP}_p} e^{x^2 \text{DP}_p t}}{\sqrt{\pi}} \int_0^t e^{-\left( x^2 \frac{4 \text{DP}_p \tau}{4} + x^2 \text{DP}_p \tau \right)} \tau^{1/2} d \tau
\]

The above expression may be simplified by letting \( z^2 = \alpha^2 \text{DP}_p \).
and performing the indicated operations.

\[ F_1(s) F_2(s) = \frac{2 \rho_n e^{\alpha^2 D_0 t}}{\sqrt{\pi}} \int_0^{\sqrt{\alpha^2 + 4}} e^{-\left(\frac{\alpha x}{4} + \frac{x^2}{2} + z^2\right)} d\frac{x}{2} \]

The above integral can be written as:

\[ \int_0^{\sqrt{\alpha^2 + 4}} e^{-\left(\frac{\alpha x}{4} + \frac{x^2}{2} + z^2\right)} d\frac{x}{2} = \frac{1}{2} \int_0^{u_1} e^{-\left(\frac{\alpha x}{4} + \frac{x^2}{2} + z^2\right)} d(z + \frac{x}{2}) \]

\[ + \frac{1}{2} \int_{u_1}^{u_2} e^{-\left(\frac{\alpha x}{4} + \frac{x^2}{2} + z^2\right)} d(z - \frac{x}{2}) \]

\[ = e^{\frac{z}{2}} \int_0^{u_1} e^{-\left(\frac{\alpha x + z}{4} + z^2\right)} d(z + \frac{z}{2}) \]

\[ + e^{\frac{z}{2}} \int_{-u_1}^{0} e^{-\left(\frac{\alpha x - z}{4} + z^2\right)} d(z - \frac{x}{2}) \]

where:

\[ \beta = \frac{\alpha x}{2} \]

\[ u_1 = \alpha \sqrt{D_0 t} + \frac{x}{2} \sqrt{D_0 t} \]

\[ u_2 = \alpha \sqrt{D_0 t} - \frac{x}{2} \sqrt{D_0 t} \]

Now,

\[ F_1(s) F_2(s) = \frac{\rho_n e^{\alpha^2 D_0 t}}{2} \left[ \frac{2}{\sqrt{\pi}} \int_0^{u_1} e^{-\left(\frac{\alpha x + z}{4} + z^2\right)} d(z + \frac{x}{2}) \right] \]

\[ + \frac{2}{\sqrt{\pi}} \int_{-u_1}^{0} e^{-\left(\frac{\alpha x - z}{4} + z^2\right)} d(z - \frac{x}{2}) \]
The above integrals can be identified by the following relationships:

\[ \text{erf}c (\lambda) = \frac{2}{\sqrt{\pi}} \int_{0}^{\lambda} e^{-r^2} dr = -\frac{2}{\sqrt{\pi}} \int_{\lambda}^{\infty} e^{-r^2} dr \]

and,

\[ \frac{2}{\sqrt{\pi}} \int_{-\infty}^{\lambda} e^{-r^2} dr = \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-r^2} dr + \frac{2}{\sqrt{\pi}} \int_{\lambda}^{\infty} e^{-r^2} dr \]

\[ = 1 + \text{erf} (\lambda) = 2 - \text{erf}c (\lambda) \]

Therefore,

\[ F_1(s) F_2(s) = \frac{\rho_n e^{-\alpha_d \rho t}}{2} \left[ 2 e^{-\alpha x} - e^{-\alpha x} \text{erf}c \left( \alpha \sqrt{D_{p_t} - \frac{x^2}{4D_{p_t}}} \right) - e^{\alpha x} \text{erf}c \left( \alpha \sqrt{D_{p_t} + \frac{x^2}{2D_{p_t}}} \right) \right] \]

Combining Eqs. (5.27) and (5.28), and the inverse transforms of the first and last two terms gives:

\[ p_a(x,t) = 2 R \frac{\sqrt{D_{p_t}/\pi}}{t} \exp \left(-\frac{x^2}{4D_{p_t}}\right) - R \alpha x \text{erf}c \left( \frac{x^2}{2D_{p_t}} \right) \]

\[ - \frac{2\alpha Q \sqrt{D_{p_t}}}{\sqrt{\pi}} e^{\alpha x \sqrt{D_{p_t}}} \int_{0}^{t} \tau^{\frac{x}{2}} \exp \left(-\frac{x^2}{4D_{p_t}} - \alpha^2 D_{p_t} \tau \right) d\tau \]

\[ + \alpha Q x e^{\alpha x \sqrt{D_{p_t}}} \int_{0}^{t} e^{-\alpha^2 D_{p_t} \tau} \text{erfc} \left( \frac{x^2}{2D_{p_t}} \right) d\tau \]

\[ + \frac{\rho_n e^{\alpha x \sqrt{D_{p_t}}}}{2} \left[ e^{-\alpha x} \text{erf}c \left( \alpha \sqrt{D_{p_t} - \frac{x^2}{2D_{p_t}}} \right) + e^{\alpha x} \text{erf}c \left( \alpha \sqrt{D_{p_t} + \frac{x^2}{2D_{p_t}}} \right) \right] \]

\[ + \frac{Q e^{-\alpha x x}}{\alpha D_p} \left( e^{\alpha x \sqrt{D_{p_t}}} - 1 \right) \] (5.29)
D. **Solution for the Transient Voltage.**

The voltage $V_j$ across the junction can be found from the Shockley injection relation (8):

$$ V_j = \frac{kT}{q} \ln \left[ 1 + \frac{p_e(t)}{p_n} \right] \quad (5.30) $$

To find the transient voltage drop across the bulk of the semiconductor, we note that the current in this section of the semiconductor is composed of two parts: drift current and diffusion current, and the sum of these must equal the illumination current, $I$. Thus:

$$ I = A \sigma(x) E_B(x) + qAD \frac{\partial p_e(x,t)}{\partial x} - qAD \frac{\partial p_e(x,t)}{\partial x} - qAD \frac{\partial p_e(x,t)}{\partial x} \quad (5.31) $$

where $\sigma(x)$ and $E_B(x)$ are the variables, conductivity and electric field, respectively.

The last two terms in Eq. (5.31) represent the total diffusion current due to the excess density of both electrons and holes. Since the excess density of holes must equal the excess density of electrons, the value of $I$ can be written as follows:

$$ I = A \sigma(x) E_B(x) + qAD_p (1 - B) \frac{\partial p_e(x,t)}{\partial x} \quad (5.32) $$

where; $B = \mu_n / \mu_p$. 
Solving for $E_B(x)$ in Eq.(5.32) we obtain:

$$E_B(x) = \frac{I}{A_0(x)} + \frac{qD_p (1-B)}{\sigma(x)} \frac{\partial p_e(x,t)}{\partial x}$$

(5.33)

For reasons which will become evident later, let:

$$E_B(x) = E_R(x) + E_D(x)$$

(5.34)

where,

$$E_R(x) = \frac{I}{A_0(x)}$$

(5.35)

and

$$E_D(x) = \frac{qD_p (1-B)}{\sigma(x)} \frac{\partial p_e(x,t)}{\partial x}$$

(5.36)

The voltage $V_R$, corresponding to $E_R(x)$ in Eq.(3.35), is due to the IR drop through the bulk material, and $E_D(x)$ in Eq.(5.36) corresponds to the space-charge voltage $V_D$, due to the inequality of electron and hole mobilities. $V_R$ must be obtained by digital integration, but $V_D$ can be obtained by direct integration of Eq.(5.36).

The conductivity $\sigma(x)$ in the previous equations is related to the carrier densities as follows:

$$\sigma(x) = q_{\mu_p} \rho_p(x,t) + q_{\mu_n} n(x,t)$$

(5.37)

where, by use of the assumption of space-charge neutrality,
the densities \( p(x,t) \) and \( n(x,t) \) are:

\[
p(x,t) = p_e(x,t) + p_n
\]  
(5.38)

\[
n(x,t) = p_e(x,t) + N_d
\]
where \( N_d \) is the density of donor atoms. Thus, \( \varphi(x) \) can be written as:

\[
\varphi(x) = q_{\mu_p}(1+B)p_e(x,t) + q_{\mu_p}(p_n+B N_d)
\]  
(5.39)

Substituting Eq. (5.10) into Eq. (5.7) and multiplying through by \( dx \), we obtain:

\[
E_D(x)dx = \frac{D_p(l-B)}{\mu_p} \frac{dp_e(x,t)}{(p_n+B N_d) + (1+B)p_e(x,t)}
\]  
(5.40)

To obtain \( V_D \) we integrate Eq. (5.40) from \( x = 0, p_e(0,t) \), to \( x = W, p_e(W,t) = 0 \), resulting in:

\[
V_D = \frac{D_p(l-B)}{\mu_p(l+B)} \ln \left[ 1 + \frac{(B+1)p_e(0,t)}{p_n+B N_d} \right]
\]  
(5.41)

As mentioned previously, \( V_R \) must be obtained by digital integration.

\[
V_R = \int_0^W E_R(x) \, dx
\]  
(5.42)
The total voltage across the diode is the sum of the three voltages $V_j$, $V_R$, and $V_D$.

E. **Results.**

The numerical values for $p_e(x,t)$, $V_j$, $V_R$, and $V_D$ were obtained by programming the appropriate equations on a 1620 IBM digital computer. The variation of $p_e(x,t)$ over distance and time is shown in Fig. 5.2. This figure demonstrates the rapid decay of the carrier density as $x$ increases.

The waveform of the resultant voltage is shown in Fig. 5.3 together with the solution found in Chapter 4. It is evident from the figure that the diode used (1N85) is not typical of most photo-diodes in its speed of response. A light source to pulse this diode would have to have a risetime of only about 4 microseconds, but its pulse width would have to be in excess of 50 microseconds.
Fig. 5.2 Distribution of carriers across the base region.
Fig. 5.3 Resultant voltage waveform.
CHAPTER VI

EXPERIMENTAL TECHNIQUES AND RESULTS

A. Light Source.

The requirements placed upon the risetime and pulse width of the pulse of light to be shone on the junction of the photo-diode are quite severe. It is necessary that the risetime of the light pulse be short in comparison to the effective risetime of the photo-diode, and the pulse width should be large in comparison to the time required for the photo-diode to reach its steady state. An examination of the literature available concerning the transient response of photo-diodes and diodes in general indicated that the risetime of the light pulse must be on the order of 100 nanoseconds and the pulse width must be at least 10 microseconds (3,4,5,6).

Jones(13) has written a general discussion on the methods of modulating light. The method most suitable for the requirements described above involves the use of the Kerr cell. The Kerr cell operates as a light shutter, turning on or off the light as is desired.

The Kerr cell, as illustrated in Fig. 6.1, is essentially two parallel plates in a container filled with liquid nitrobenzene across which is connected a high direct-current voltage. The light from the source on the left is focused through polarizing plates on the center of the cell halfway between the plates. With no voltage on the Kerr cell, the
light continues through unchanged. The polarizing plate on the right of the cell is adjusted at this point, so that no light reaches the sample.

![Diagram of Kerr cell](image)

**Fig. 6.1** The Kerr cell.

When voltage is applied to the cell, the electric field between the plates alters the polarization of the light entering from the left, due to the changes in nitrobenzene under the influence of an electric field. The light no longer cancels at the second polarizing plate, and the light is allowed to shine on the sample. The only limitation on the risetime of the pulse of light is the speed at which the voltage can be applied to the plates.

The circuit shown in Fig. 6.2 was used to apply the voltage to the Kerr cell. The mercury-wetted relay applies the 250 volts of the power supply to the grid of the thyratron in approximately 50 nanoseconds ($5 \times 10^{-8}$). The thyratron,
activated by the pulse from the relay, fires and places the 5,100 volts across the Kerr cell. The speed at which this is accomplished is determined primarily by the risetime of the pulse on the grid of the thyratron, which in this instance will be in the neighborhood of 50 nanoseconds. This time will also correspond closely to the risetime of the pulse of light.

Fig. 6.2 Circuit to operate the Kerr cell.

The pulse width of the light can be regulated by adjusting the parameters in the circuit controlling the decay of the voltage on the Kerr cell. The cell itself acts as a capacitor of from 100 to 200 micromicrofarads. The cell discharges through the resistor $R_2$. By adjusting $R_2$, the decay time can be made long enough so that the magnitude of the light pulse stays close to its maximum magnitude for a time long enough to allow the photo-diode to reach its steady-state condition.
The capacitor $C$ in fig. 6.2 is where the charge is stored when the thyatron is off. The value of $C$ together with $R_1$ and $R_2$ determine the repetition rate of the light pulses. Ample time must be allotted for the voltage to build on the capacitor $C$, before the tube is fired the next time.

A major disadvantage of the circuit shown in Fig. 6.2 is the noise level. When the thyatrons fire, surges of up to 35 amperes flow through the circuit, producing high frequency noise. Since the signals involved in the photodiode are small, the output must be well shielded, and the intensity of the light source used must be as high as possible.

The author, in attempting to use the circuit of Fig. 6.2 to operate a Kerr cell with a distance between the plates of 3 mm., was unable to obtain enough light intensity to excite the photodiode. The insertion of the polarizing plates and the Kerr cell between the light source reduces the intensity of the light at the sample by a factor of approximately one hundred. The highest intensity light sources used were a mercury vapor lamp, an arc lamp and a 500 watt tungsten lamp with reflector. The Kerr cell, however, was observed to be operating, though no measurements could be made to determine the characteristics of the light pulse.

The failure to obtain results in the above instance does not in any way depreciate the value of the Kerr cell as a modulator of light. It is believed by this author that had
a Kerr cell with a greater aperture been available, the re-
sults would have been much more favorable. Kerr cells are
made with much larger apertures than the one used in this
experiment. The voltage required to fire the cell increases,
however, as the distance between the plates is increased.
Voltages on an order of 40 kilovolts (40,000 volts) are com-
monly used for plate separations of about 1 cm(13).

A larger Kerr cell would be more suitable as a means to
provide a fast-rising pulse of light due to the fact that
more light could be collected by the cell and the light in-
tensity would be diminished less by the cell. To operate
the cell correctly, the light being focused on the center of
the cell must not be allowed to strike either of the plates
as it enters. This would cause internal reflections in the
cell disrupting the plane of polarization and light would
pass through when the cell was off. For a small cell, this
restriction limits the size of the light beam entering the
cell and therefore limits the size of the lens to be used to
focus the beam and the amount of light the lens can capture.
For a larger cell, the beam can be larger and more intense,
permitting more intensity to reach the sample.

The amount of noise present will be increased due to
the higher voltage needed to operate the larger cell. If
necessary this noise can be reduced by shielding the thyra-
tron from the output equipment and by inserting a limiting
resistor in series with the thyatron of such a value that
will not seriously affect the desired risetime. The fastest
risetime attainable will be limited by the speed of response of the liquid surrounding the cell which for the most common liquid, nitrobenzene, is approximately one nanosecond (16). The size of the limiting resistor for a nitrobenzene-filled cell will be determined then by how long the desired risetime is compared to one nanosecond.

Another light source tested for possible use was an Amglo M6-LZ, xenon-filled flashtube. It was found that although the intensity of the tube was easily high enough, the fastest risetime attainable with low-inductance capacitors, resistors, and leads was greater than 5 microseconds. This tube was used later to measure the parameters of Linvill's model.

An article by Draper (14) suggested another possible light source, using the sweep trace of an oscilloscope to produce millimicrosecond pulses through a well-defined slit. It was found in this instance that the combination of low intensity and the photo-diode's relative insensitiveness to green light (common to most oscilloscopes) makes the method impractical.

The absence of a suitable light source to pulse the photo-diode prevented any experimental confirmation of the results of Chapters 4 and 5.

B. Determination of the Parameters for Linvill's Model.

Only two measurements are required to measure the parameters of Linvill's model. One is the reverse saturation
current, and the other is the charge removed from the diode following a pulse of light on the junction.(2)

![Fig. 6.3 Circuit used to measure $H_c$.](image)

The arrangement in Fig. 6.3 is used to measure the reverse saturation current. With the junction reverse biased, the excess density from Eq.(5.30) is $-p_n$. In the steady-state condition, current $I_s$ is given by:

$$I_s = p_n H_c$$  \hspace{1cm} (6.1)

The charge stored in the diode will be measured by comparing the waveform of the voltage across the photo-diode in Fig. 6.3, and the waveform of the light pulse.

The time "$t_o$" in Fig. 6.4, is the time at which the light is effectively turned off. The area under the curve following the time "$t_o$" in Fig. 6.4(a) represents the volume of charge stored in the diode, $Q_D$ (the phrase "volume of charge" means the quantity or amount of charge present).
If the current flowing in the diode (determined by $V_{D_{\text{MAX}}}/R_L$) is $I_0$, then the charge of holes on the hole storage, $Q_s$, is:

$$Q_s = (1 + \frac{I_0}{I_S})p_nS_p$$

where; $I_0$ is an incremental current.

Fig. 6.4 a) Waveform of the light pulse.

b) Waveform of the voltage across the diode.
It is evident that:

\[ Q_s = Q_D \quad (6.3) \]

Substituting Eq.(6.3) into Eq.(6.2) and solving for \( S_p \) yields:

\[ S_p = \frac{Q_D}{P_n(1 + \frac{T_o}{T_s})} \quad (6.4) \]

The light source used to pulse the photo-diode was the flashtube discussed in the previous section. The circuit used to flash the tube is shown in Fig. 6.5. The transformer

![Fig. 6.5 Circuit for firing flashtube.](Image)
in the circuit is an ignition coil of the type used in automobiles. Its purpose is to provide a high-voltage pulse to "tickle" or excite the tube causing it to flash. The waveform applied to the coil of the relay was obtained from the load resistor of a cathode-follower, whose grid was driven by a multivibrator oscillating at approximately 1 cps. The slow repetition rate allowed time for the power supply to recover to its initial state after the tube had fired. It was necessary to fire the tube periodically to facilitate synchronization of the oscilloscope with the signals to be recorded.

The photographs on the following page were taken on a Type 516 Tektronix oscilloscope of the waveforms of the light pulse and the voltage across the photo-diode. The light pulse was made to fall simultaneously on the photo-diode and the entrance slit of a monochrometer. The monochrometer was used to focus the light on a photomultiplier and also to reduce the intensity of the light to prevent driving the photomultiplier into saturation.

The output of the photomultiplier, which corresponds to the shape of the light pulse, was amplified through a Model 400 AB, Hewlett-Packard, vacuum-tube voltmeter with a gain of approximately 400 on the lowest scale and an upper 3 db frequency of 3 megacycles. All external connections and leads in the photomultiplier circuit were made with coaxial cables to prevent amplification of stray signals. The cables were terminated in the characteristic impedance to reduce the capacitance of the circuit and insure an accurate
Fig. 6.6 a) Waveform of light emitted by flashtube.
   b) Waveform of voltage across photo-diode.
description of the light pulse.

Before proceeding to the actual determination of the value of the storance, some comments are necessary on the accuracy of the experimental technique outlined above.

The assumptions made in performing a measurement of the type just discussed are that the light was incident only on the junction and did not strike any part of the bulk material, and that the carriers produced by the light were distributed evenly across the cross-sectional area of the junction. Naturally, any deviation of these approximations from perfection that occur during the experiment would result in incorrect values for the parameters of Linvill’s model. The model derived by Linvill represents physically an area only as large as the transition region. This means that for typical photodiodes the width of the light beam striking the junction should be less than $10^{-4}$ cm wide, a very difficult task at best.

The error involved in assuming the injected carriers are evenly distributed across the cross-sectional area of the junction is generally smaller, especially if the material under test is thin and also has a low absorption coefficient. This would insure that in the small distance between the two surfaces of the material, the density of injected carriers would not vary appreciably. Using the value obtained for the absorption coefficient in Section C of this chapter, and the thickness of the material tested (.038 cm), it was determined from Eq. (6.6) that the density decreased by only 20 per cent across the thickness of the material. The actual average
error due to the exponential shape of the distribution would be less than 10 per cent. It will be illustrated shortly that this amount of error is negligible compared to the amount of error present in the other approximation.

To determine the amount of error involved in assuming the light to be incident only on the junction, several measurements of the storance were made for varying widths of the incident beam of light. The results of these measurements, listed in Table I, showed that as the width of the beam was decreased the magnitude of the storance also decreased. The width of the beam was varied by placing an adjustable metal slit against the diode. The smallest width attainable was only .0013 in., which was not small enough to allow any "leveling-off" of the magnitude of the storance. This last value for the storance would have to be the value used for Linvill's model if no other method were available to determine a more accurate value, and in the absence of another method, no check could be performed experimentally to determine the accuracy of the value used. A rough estimate can be obtained, however, for the appropriate value of the storance from the following equation derived by Linvill(2):

\[ S_p = \frac{gA W}{2} \]  

(6.5)

where; \( A \) = cross-section area of the junction
\( W \) = length of the p-type material
TABLE I

VARIATION OF STORANCE WITH WIDTH OF LIGHT BEAM

<table>
<thead>
<tr>
<th>Width of light beam (inches)</th>
<th>Storance $S_p \times 10^{-23}$ (coul-cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1640</td>
<td>153.5</td>
</tr>
<tr>
<td>0.0810</td>
<td>142.2</td>
</tr>
<tr>
<td>0.0285</td>
<td>119.0</td>
</tr>
<tr>
<td>0.0130</td>
<td>85.0</td>
</tr>
<tr>
<td>0.0040</td>
<td>54.5</td>
</tr>
<tr>
<td>0.0013</td>
<td>27.2</td>
</tr>
</tbody>
</table>

From the foregoing discussion it would seem that Linvill's model could not be used for a photo-diode with any assurance of the accuracy of the results. There is another method to determine the parameters, however, if the photo-diode to be tested is designed for safe conduction of forward currents. This method, suggested by Linvill(2), is similar to the method described previously except that the diode is pulsed with a step of current instead of with a pulse of light. This method does not have the limitations of the previous method and therefore yields the correct value for the storance to be used in Linvill's model. The response of the photo-diode under consideration to a pulse of current is shown in Fig. 6.7.

The values for the storance calculated from Fig. 6.6 and Fig. 6.7, are $27.2 \times 10^{-23}$ coul-cm$^3$ and $17.3 \times 10^{-23}$ coul-cm$^3$, respectively. From these values, the error in the magnitude of the storance measured by pulsing the photo-diode with light was 36.4 per cent. These values were found by inserting the
value for \( Q_D \), determined by a graphical integration of the area under the curves, into Eq.(6.4). The value for \( I_0 \) in the latter determination of \( S_p \) is the magnitude of the current flowing in the forward direction through the diode prior to the time the pulse of current is applied. Otherwise, Eq.(6.4) applies equally well to either method.

\[ 20 \text{ volts/cm} \]
\[ 50 \mu\text{sec/cm} \]
\[ 5.7\mu\text{amp/cm} \]
\[ I_0 = 600\mu\text{amp} \]

**Fig. 6.7 Response of photo-diode to pulse of current.**

The reverse-saturation current \( I_s \) was measured to be \( 12.0 \times 10^{-6}\text{amp} \). Insertion of this value for \( I_s \) into Eq.(6.1) yielded the value for the combinance \( H_c \).

The values for Linvill's model used for the problem in Chapter 4 and as determined above are:

\[ H_c = 1.71 \times 10^{-17}\text{amp-cm}^3 \]
\[ S_p = 17.3 \times 10^{-23}\text{coul-cm}^3 \]
C. Determination of the Absorption Coefficient.

If the illumination of the junction of a photo-diode produces a current \( I_0 \), corresponding to the generation of \( \Delta p \) hole-electron pairs per second, the current leaving the diode is given by Lambert's Law (14):

\[
I = I_0 e^{-\alpha x}
\]

(6.7)

where; \( I \) = the current in the external circuit
\( \alpha \) = absorption coefficient
\( x \) = length of the n-region
\( I_0 \) = \( q\Delta p \)

Thus, to determine the absorption coefficient for a given light source, it becomes necessary to determine the absolute intensity of light falling on the junction from which can be found the number of hole-electron pairs being produced per second.

The light used for this measurement was a 375 watt tungsten lamp operating at a temperature of 2348°K. To determine its intensity at a given distance, an area of one square inch of water in a well-insulated container was exposed to the light. The change in temperature of the water was recorded for certain lengths of time. The relation used to calculate the amount of energy transferred to the water is as follows:

\[
Q = \frac{S_w W \Delta T}{(1-R_w)\Delta t} \text{ watt/in}^2
\]

(6.8)
where: $S_w = \text{heat capacity of water, } 4.18 \text{ joules/g°C}$

$W = \text{amount of water, grams}$

$\Delta T = \text{temperature change, °C}$

$\Delta t = \text{time elapsed, min}$

$R_w = \text{reflection coefficient of the water}$

Three runs were performed and the average of the three values obtained was determined as:

$$Q_{ave} = 8.43 \text{ watts/in}^2$$

The reflection coefficient of the water was measured with an optical pyrometer by a comparison of the total light incident on the water to light reflected by the water. The value obtained for $R_w$ was 0.650.

Knowing energy density of the light source at a certain distance, the number of carriers generated per second $\Delta p$ can be computed from the quantum nature of light, the total energy being equal to the average energy per photon (one photon produces one hole-electron pair) times the total number of photons. The equation is:

$$\Delta p = \rho_D(1-R_D)A_j Q_{ave}$$

where: $\rho_D = \text{spectral efficiency of the photo-diode } = 0.457$

$R_D = \text{reflection coefficient of the photo-diode } = .555$

$A_j = \text{area of the junction exposed to light, in}^2$

$\Delta \lambda = \text{range of wavelengths in the lamp’s spectrum}$
A comment is necessary here on the wavelength dependency of the absorption coefficient. The coefficient determined by the method described above is a total absorption coefficient for "white" light. The variation of the coefficient and the energy per photon with the wavelength of light was accounted for by a spectrum analysis of the light and the photodiode.

The spectral distribution of the light source was obtained by assuming black body radiation and plotting the relative energy density versus wavelength. The plot was obtained from Planck's radiation equation (15):

$$E_\lambda = \frac{8\pi h \lambda^{-5}}{(e^{h\lambda/\kappa T} - 1)}$$

(6.10)

The product of the above distribution with the spectral sensitivity of the photo-diode was obtained to determine $\rho_D$, the spectral efficiency of the photo-diode. The numerical value for $\rho_D$ was found from the ratio of the total area under the curve of the product of the two curves to the total area under the curve of Eq. (6.10). The value of $\rho_D$ represents the percentage of the total light energy incident on the photo-diode which is actually detected. (The curves and areas discussed above are shown in Figs. 6.8 and 6.9). It should be mentioned at this point that the value of $\alpha$ is only accurate for a lamp operating at the same temperature as the one used in the experiment.
The current produced by the illumination is equal to \( q\Delta p \). The value for \( \Delta p \) determined from Eq. (6.9) was: \( \Delta p = 0.302 \times 10^{-16} \, \text{sec}^{-1} \), giving an illumination current of \( I_0 = 0.510 \times 10^{-3} \, \text{amp} \). The current flowing in the circuit of the photo-diode placed the proper distance from the light source was measured as: \( I = 0.130 \times 10^{-3} \, \text{amp} \). Substituting these values into Eq. (6.7) yields:

\[
a = 15.55 \, \text{in}^{-1} = 6.13 \, \text{cm}^{-1}
\]

The experimental determination of the absorption coefficient represents one of the main sources of error present in the distributed solution. It is necessary therefore to determine the relative magnitude of this error and its effect on the final solution.

The error in this experiment is centered in the following measurements:

a) the square inch of water exposed to the light source
b) the measurements of the reflectivity of the surfaces involved
c) the measurement of the temperature change of the water
d) the measurement of the filament temperature

The error from the measurements of the temperature changes of the water due to loss of heat was made negligible by adequate insulation of the water from the surrounding air. By making the time of exposure of the water to the light long enough (15 minutes) so that large changes in temperature (4 to 5 degrees) occurred, the changes in temperature were easily
measured within 2 per cent of their actual values.

The measurements of the reflectivity and the filament temperature were accomplished with a Model 8B Pyro manufactured by the Pyrometer Instrument Company, Inc., which stated the accuracy of its instrument as one-half of one per cent.

The dimensions of the square hole through which the light was incident on the water were found to vary by no greater than one thirty-second of an inch or approximately 3 per cent of the designated value. The maximum error involved in assuming the area to be one square inch therefore is approximately 6 per cent.

The total error involved in determining the absorption coefficient is approximately 9.5 per cent, including three separate measurements with the pyrometer.
Fig. 6.8 Spectral curves of diode and lamp
Fig. 6.9 Effective spectral curve of photo-diode and lamp combined
CHAPTER VII

CONCLUSIONS

The resultant waveforms for the transient voltage across the diode found from the two solutions presented in this study are shown in Fig. 5.3. The times required for the voltage to reach its steady-state value for each solution compared very well. The time required for the distributed solution to reach its steady-state value was approximately 40 microseconds, while the corresponding time for the solution using Linvill's model calculated as four time constants or $4S_p/H_c$ was 39.5 microseconds. This is well within the experimental error involved in determining the absorption coefficient for the distributed solution.

The difference in magnitude of the steady-state voltages found by the two solutions suggests one drawback to the use of Linvill's model. Since Linvill's model is concerned with the material very close to the junction, relatively large concentrations of carriers and their contributions to the total voltage across the diode are neglected. In the case of this study, for example, Fig. 5.2 demonstrates that at a distance of approximately $10^{-5}$ cm, where Linvill's model ends, the concentration of carriers had not yet begun to decrease appreciably. Neglecting these carriers causes the steady-state voltage as predicted by Linvill's model to be much lower than it actually is.
The absence of a suitable light source prevented an experimental measurement which could be used as a second solution to compare with the solution obtained with Linvill's model and also as a check solution to evaluate the actual error made in the determination of the absorption coefficient. The value of Linvill's model as a means to determine the rise-time of a photo-diode can easily be seen, however, despite the absence of the experimental measurement.

There are several possibilities of studies which could be made to supplement and extend the material presented in this study. One possibility could be the development of a method to determine a value for the storance of Linvill's model independent of the junction capacitance. This, as discussed in Chapter 4, would enable the design of a more accurate model. Another possibility could be a study to develop a suitable and practical method to produce a pulse of light with the particular characteristics required for the testing of photo-diodes as described in this study.
BIBLIOGRAPHY


VITA

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In that same year, he was accepted as a cadet in the United States Military Academy at West Point, New York. He transferred from the Academy in 1962 to the University of Missouri at Rolla, where he received the degree of Bachelor of Science in Engineering in January, 1964.

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