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Analog computer study of unequally spaced antenna arrays

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ANALOG COMPUTER STUDY OF UNEQUALLY SPACED ANTENNA ARRAYS

BY

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A

THESIS

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UNIVERSITY OF MISSOURI AT ROLLA

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ABSTRACT

The problem undertaken in this thesis was to study the basic properties of unequally spaced antenna arrays. The study was done on a trial and error basis by simulating the expression for the far field pattern of a general array on an analog computer. Many patterns were obtained from the analog computer for the purpose of comparison.

The patterns of two spacing schemes that are typical of unequally spaced arrays are presented for the purpose of illustration.

It was concluded that non-symmetric arrays compared favorably with symmetric arrays. In general, the unequally spaced array gives a narrower beam width with fewer antennas at the expense of a higher sidelobe level and increased aperture than does the equally spaced array. The unequally spaced array also presents a means of obtaining special patterns for special applications.

A plot of beam width versus number of elements was made for three different spacing schemes. This figure illustrates the improvement in the beam width of an unequally spaced array when compared to an equally spaced array with the same number of elements.
ACKNOWLEDGEMENTS

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CHAPTER I

INTRODUCTION

A. Statement of the Problem

Until approximately five years ago, antenna arrays were designed with equal spacing between the individual elements of the array. The phase of the elements of the array is usually employed to determine the direction of the main beam or to electrically scan the array. The amplitude of the feed of the individual elements of the array is adjusted so as to obtain an optimum pattern with respect to beam width and sidelobe level.

Removing the restriction of equal spacing between elements broadens the field of array design and could possibly lead to an improvement over the types of arrays presently being used.

The purpose of this study was to establish the basic properties of unequally spaced antenna arrays. An attempt was made to determine trends in beam width, sidelobe level and the amount of variation in the sidelobes. Due to the complexity of the problem only one dimensional broadside arrays were considered. All elements of the array were fed in phase. The parameters of such an array become spacing and amplitude distribution.

The mathematics of unequally spaced arrays does not readily lend itself to analysis. Due to a lack of direct mathematical relationships an analog computer was used throughout the study. Broadside arrays with either equal or unequal spacing between elements can easily be simulated on an analog computer. The spacing and amplitude distribution of the elements can be fed into the computer by simple potentiometer
adjustments. A given array can be simulated on the computer and the pattern viewed on an oscillograph. A permanent plot can then be obtained with the aid of an X-Y plotter. The main advantage of the analog computer is that for a given number of elements the amplitude and position of the elements can be varied manually while the change in the pattern was being observed.

B. Significance of the Study

From a study of array theory it was apparent that, for a given type of array, a decrease in sidelobe level is obtained at the expense of a larger beam width. However, one array may be better than another type in all respects. The unequally spaced array has some properties that are more favorable than the equally spaced array. Usually these properties are obtained at a sacrifice. For a given number of elements an unequally spaced array tends to have a narrower beam width at the expense of greater aperture and higher sidelobe level.

The problem of finding an optimum unequally spaced array would be extremely difficult due to the complicated mathematics involved and to the infinite possible combinations of spacing and amplitude distribution. A further complexity is that an optimum for one application may not be optimum for a different application.

One approach to the problem was to determine the properties of unequally spaced arrays in general and then study arrays with a particular spacing scheme. Once the spacing scheme has been decided upon the array may be made optimum with respect to amplitude distribution. Making the amplitude distribution optimum is still a complicated problem in general and may require a trial and error type solution.
The basic properties and trends of unequally spaced arrays present a starting point for array synthesis. They may tell whether or not an unequally spaced array could be a solution to a particular array problem.

C. Reasons for the Study

The investigation of unequally spaced antenna arrays was undertaken by the author due to an interest in antenna theory. The study and application of unequally spaced antenna arrays is relatively new to the field of antenna theory. This thesis was undertaken to establish the basic properties and trends of unequally spaced arrays. Two types of arrays with different spacing schemes were analyzed for the purpose of illustration.
CHAPTER II

REVIEW OF THE LITERATURE

Most of the literature on unequally spaced antenna arrays was written during the last five years. Before this time unequally spaced antenna arrays were generally not considered a solution to an array problem. The main reason for the lack of interest was probably due to the extremely complex form of the mathematics involved.

Research in this area began by removing the restriction of equal spacing between the elements of an array. Removing this restriction gives the engineer one more degree of mathematical freedom in designing antenna arrays. The work done on this subject can be divided into two classes, the first being a trial and error type solution and the second is an approximation type solution.

The trial and error type solution consists of deciding on a spacing scheme and then calculating the pattern. This process is repeated several times. The patterns obtained are then compared with each other and then compared with those of conventional arrays. The approximation method consists of representing a given pattern by either a finite or infinite series and then adjusting the elements of the array so as to equal or approximate the series.

The solutions obtained so far are not unique. It is pointed out in the literature that unequally spaced arrays have certain definite advantages over equally spaced arrays. The main advantage is a narrow beam width with a decrease in the number of elements in the array. One disadvantage is an increase in the total length of the array. The total length of an array is called the aperture and is defined as the distance
between the extreme elements of the array.

A short paper by Unz (1)* in 1960 suggests the use of unequally spaced arrays on the basis of achieving one more mathematical degree of freedom. This paper does nothing more than introduce the subject.

A paper based on a trial and error procedure was written by King, Packard and Thomas (2) in 1960. They calculated the patterns for arrays where the relative spacings were logarithmic, proportional to prime numbers, and proportional to an arithmetic progression. Their results had the characteristic narrow beam width and high sidelobe level.

Sandler (3) in 1960 suggests an equivalence between equally and unequally spaced arrays. His method of synthesis consists of choosing a spacing scheme for an unequally spaced array and then expanding each term of the unequally spaced array in a Fourier cosine series. Once this has been done the Fourier cosine series is then made to approximate the expression for an equally spaced array. This method is not unique and is extremely difficult to apply.

Another approximation method is given by Lo (4) in a paper published in 1962. His method is based on an infinite series expansion of a given pattern. The series expansion is obtained by an application of Lesbegue-Stieltjes integrals and mechanical quadrature. This method, as well as all of the approximation methods, may give as a solution an array with extremely small spacing between some of the elements.

In a paper in 1962, Maffett (5) used mechanical integration and the trapezoidal rule to approximate a continuous aperture distribution by an unequally spaced array.

*Numbers in parenthesis designate references in the bibliography
A trial and error approach was employed by Andreason (6) in 1962. He simulated an antenna array on an analog computer and manually varied the spacing and amplitude while observing the resulting pattern. His work shows what might in general be expected from unequally spaced arrays.

Ishimaru (7), in 1962, used Fourier series and Poisson's sum formula to approximate a continuous aperture distribution by an unequally spaced array. He applied his work mainly to arrays of two and three dimensions. In a paper in 1964 (8) he discusses, in a general manner, the recent developments in the field.

Skolnik, Sherman and Ogg (9) in 1964, calculated the patterns for arrays whose density of elements was proportional to the amplitude distribution of given arrays. The elements of their arrays were fed with equal amplitude and constant phase.

It may be pointed out that none of the approaches to the problem gives a unique solution. This chapter gives a brief summary of what has been done in the area of unequally spaced arrays. The approximation methods are lengthy and often require the use of a computer. All of the work found in the literature is applicable only to arrays that are symmetric about the center point.
The antenna arrays considered in this thesis consist of in-line elements that radiate equally in all directions. The far field pattern of such an array is referred to as the universal pattern. An element that radiates equally in all directions is purely a mathematical concept and is referred to as an isotropic source. This concept is quite useful due to the fact that the far field pattern of an array of similar elements is given by the product of the pattern of one of the elements and the universal pattern.

For easy reference, the elements of an array will be numbered from left to right with the element on the extreme left designated as number one. The element on the extreme left will also be the reference element. In all diagrams and figures the elements of an array will be designated by a circle with the number of the element in the circle. The following figure defines some of the symbols that will be used throughout the thesis.

![Coordinate system for general array](image)

Fig. 3.1 Coordinate system for general array
The general expression for the universal pattern of an array of n elements is a complex polynomial containing n terms and is given by

\[ F(\theta) = \left| \sum_{r=1}^{n} A_r \ e^{j(d_r \cos \theta + \phi_r)} \right|. \]  \hspace{1cm} (3.1)

The terms in equation (3.1) are defined as follows:

- \( A_r \) is the amplitude of the r\(^{th} \) element
- \( d_r = \frac{2\pi D_r}{\lambda} \) (radians)
- \( D_r \) is the distance of the r\(^{th} \) element from the reference (meters)
- \( \phi_r \) is the phase of the r\(^{th} \) element referred to the reference (radians)
- \( \theta \) is the physical angle defined in Fig. 3.1 (radians)
- \( \lambda \) is the wavelength (meters).

The right side of equation (3.1), in its complete form, is multiplied by a phase term. This phase term is of no importance in the work to follow and for this reason will be discarded.

The amplitude, phase and position of each element, except the reference element, may be adjusted to obtain the desired pattern. Thus for an array of n elements there are 3(n-1) parameters to be determined. It is obvious that, even for small values of n, it is an extremely difficult problem to determine the best spacing, phase and position for each element of the array.

For the case of the strictly broadside array, all of the elements are in phase. Under this condition, all of the \( \phi_r \) terms will equal zero. There are two reasons for using constant phase. First it is a
much needed simplification of the problem. Second a strictly broadside pattern is desired.

Under this condition, equation (3.1) reduces to

\[ F(\theta) = \left| \sum_{r=1}^{n} A_r e^{jd_r \cos \theta} \right|. \]  

(3.2)

To further simplify the equation it is convenient to make a change in the variables. Let \( \Psi = d_n \cos \theta \) and \( B_r = D_r / D_n \). The subscript \( n \) refers to the element on the extreme right. With these substitutions equation (3.2) becomes

\[ F(\Psi) = \left| \sum_{r=1}^{n} A_r e^{jB_r \Psi} \right|. \]  

(3.3)

Where \( B_1 = 0 \) and \( B_n = 1 \). \( D_n \) is the total aperture.

Once the \( A_r \)'s and \( B_r \)'s are chosen the pattern may be plotted versus \( \Psi \). All that is needed to plot the pattern is the relative spacing between the elements and not the total aperture. The best aperture is readily obtained from the pattern plot.

Plots of equation (3.2) and (3.3) are similar in all respects. Maxima and minima of one correspond to the maxima and minima of the other. The only difference between the plot of equation (3.2) and that of (3.3) is a nonlinearity between the \( \Psi \) and the \( \theta \) axis. One plot contains as much information as the other. \( \theta = 90^\circ \) and \( \Psi = 0^\circ \) are equivalent points of the two equations and they correspond to the broadside direction. Certain properties of broadside arrays can be drawn from the form of these two equations.
By the use of equation (3.2) it can be shown that the pattern is symmetric about the axis of the array. This can easily be seen with the aid of the fact that \( \cos \theta = \cos(-\theta) \). Equation (3.3) is used to show that the pattern is also symmetric about the broadside direction. Changing \( \psi \) to \( -\psi \) gives the complex conjugate of the expression. The fact that the absolute value of a complex number is equal to the absolute value of the complex conjugate of the number, completes the proof.

Since the last proof was made without any consideration of the symmetry of the array it holds whether or not the array is symmetric about the center point. This is in agreement with the fact that to the far field the array appears as a point and not as distributed sources on a line. This brings up one question. Is the restriction that an array be symmetric about the center point justifiable? An attempt will be made in Chapter V to show that there is no valid basis for this assumption. The symmetry property does help to simplify the mathematics. For a symmetric array equation (3.3) reduces to a sum of cosine terms.

Equation (3.3), depending on the \( B_r \)'s, may or may not be periodic in \( \psi \). If the \( B_r \)'s are rational fractions the expression will be periodic and the period will be determined by the lowest common denominator. The expression will not be periodic if one or more of the \( B_r \)'s are irrational fractions. The pattern, for all practical purposes, will always appear to be periodic. For example, consider \( 1/\sqrt{2} \) for a value of \( B_r \). To four significant figures \( 1/\sqrt{2} = 1/1.414 = 1000/1414 \), the last number being a rational fraction. Thus, the pattern will always be periodic since the \( B_r \)'s can always be represented by a rational fraction.
It can be seen from equation (3.3) that the relative position of the elements in the array determine the length of the period of the pattern plotted versus $\Psi$. The main beam corresponds to values of $\Psi$ for which the terms in the expression add in phase. Since the terms can add in phase only once in a period, the main beam will be contained only once in a period and all other lobes in a period will be smaller than the main beam. The term sidelobes refers to the lobes of the pattern excluding that of the main beam and the sidelobe level is taken as the maximum value of the sidelobes.

The roots of equation (3.3) correspond to the zeros or nulls of the pattern. Because there are a finite number of roots in a period of the expression, the sidelobe level cannot be made equal to zero over a range of the variable $\Psi$. The most that can be expected is to minimize the value of the sidelobes over the desired range of $\Psi$.

For certain application, it may be desired that there be no nulls in the pattern. The nulls correspond to the roots of the equation and may be removed by manipulating either or both the amplitude and position of the elements. This manipulation forces $\Psi$ to take on meaningless values. An example of a meaningless value of $\Psi$ is a complex or imaginary number. By definition, $\Psi$ will always be a real number.

The pattern of $F(\Psi)$ is the pattern that will be referred to throughout the remainder of this thesis. The designer must decide on the range of the values of $\Psi$. This decision determines the total aperture of the array. If the range of $\Psi$ is made too large the main beam will also be produced in directions other than the broadside direction.
The preceding is all that can readily be derived from the form of the expression for \( F(\Psi) \). It is appropriate at this point to illustrate, in detail, the use of the plot of \( F(\Psi) \). The following figure will be used for this purpose.

![Diagram of \( F(\Psi) \)](image)

Fig. 3.2 General plot of \( F(\Psi) \)

One period of the function \( F(\Psi) \) is all that is needed. The function is not shown for negative values of the argument because the function is symmetric about the origin. From \( \Psi = d \cos \Theta \) it is seen that \( \Psi \) starts at its maximum value, decreases through zero to the negative of its maximum value and back to its maximum as \( \Theta \) varies from 0 to 360 degrees. Point A corresponds to \( x \), the maximum value of \( \Psi \). Point A is chosen so as to obtain a combination of minimum beam width and minimum sidelobe level. The beam width is the distance between half-power points of the main beam, in degrees. Once the point A is determined, it is a simple problem to calculate the total aperture. The \( B_r \)'s are then used to determine the position of the
intermediate elements. The same expression used to calculate the aperture can be used to calculate the beam width. The sidelobe level is readily obtained from the plot without further calculation.

It is almost impossible to design an array from equation (3.3). The problem is one of too many variables. For each element of the array, except the reference element, the relative amplitude and position must be determined. This becomes extremely difficult for arrays of more than three elements. The expression does not give an indication of either sidelobe level or beam width.

One approach to the problem is the approximation method. In general this method begins with the desired pattern, a continuous aperture distribution or an equally spaced array. By mathematical manipulation an expression equivalent to equation (3.3) is obtained. As pointed out in the Survey of the Literature, this result is not unique and may not be physically practical. Since there are several of these methods in existence, the approximation method will not be considered further.

A good approach would be one that is analogous to the equally spaced Dolph-Tchebyscheff array (10). Dolph found that the expression for an equally spaced array could be equated to a Tchebyscheff polynomial. The ideal characteristics of the Tchebyscheff polynomials gives an optimum condition for an equally spaced array.

A complex polynomial with similar characteristics that fits equation (3.3) would be needed for an unequally spaced array. It would be required that the polynomial have the property that it could be extended to an array with any number of elements. It is doubtful that such a polynomial exists, and if it does it has not, as of this time, been discovered.
Because of the difficulties just described, a trial and error type solution was the method of approach taken in this thesis. Equation (3.3) was simulated on an analog computer and a large number of spacing schemes were studied. This differs from Andreason's (6) work in that equation (3.3) is general and does not require any restriction such as symmetry. This thesis is also a more detailed study.
CHAPTER IV

SIMULATION ON THE ANALOG COMPUTER

For the purpose of simulation on the analog computer it is convenient to change the form of equation (3.3). This is done by applying Euler's equation;

\[ e^{jB_r \Psi} = \cos B_r \Psi + j\sin B_r \Psi. \]

The magnitude is then given by the square root of the sum of the square of the sum of the real terms plus the square of the sum of the imaginary terms. Equation (3.3) may then be expressed in the following form;

\[ F(\Psi) = \sqrt{\left(\sum A_r \cos B_r \Psi\right)^2 + \left(\sum A_r \sin B_r \Psi\right)^2}. \]  \hspace{1cm} (4.1)

This expression can readily be simulated on an analog computer. On the analog computer the \( \Psi \) variable becomes the time variable. The sine and cosine functions are generated by solving the second order differential equation,

\[ y'' + B_r y = 0 \]

where the derivatives are taken with respect to time. Such a circuit is needed for each antenna except the reference. The reference antenna is represented by a constant because \( B_0 \) which corresponds to this antenna equals zero.
Fig. 4.1 Schematic diagram of analog circuit
The constant plus the cosine terms are added together and then squared. The sum of the sine terms is squared and then the two squares are added. Taking the square root of the last sum completes the simulation. A schematic diagram of the analog circuit is given in Fig. 4.1.

One potentiometer for each antenna was needed to control the amplitude and two potentiometers were needed to control the position of each antenna. The pattern could be continuously observed on an oscillograph while the parameters of the array were varied. A permanent copy of any pattern could be made with the aid of an X-Y plotter.

The computer used was the EIA TR-48 analog computer made by Electronic Associates Inc. Fig. 4.2 is a photograph of the analog computer as it was used throughout the study. Due to its size, the computer was limited to arrays consisting of nine antennas. A digital computer could have been used instead of an analog computer, however, for this purpose the analog computer is much faster.

With the aid of the computer, two spacing schemes were found to have interesting properties. One of the two spacing schemes has the property of a narrow beam width and will be referred to as the narrow-beam-width scheme. The spacing scheme was an arithmetic progression with one unit between the first and second element, two units between the second and third element, three units between the third and fourth element, etc. All elements were fed with the same amplitude. Many spacing schemes were found which had a narrower beam width, but in all cases the sidelobe level was higher.

The other spacing scheme gave an approximation to a pattern with a constant sidelobe level and will be referred to as the constant-
sidelobe-level scheme. This spacing scheme is related to that of the arithmetic progression in that each number is repeated twice. For example, the spacing is proportional to the progression; 1, 1, 2, 2, 3, 3, 4, 4, etc. The amplitudes of the elements of the array were adjusted on the computer to obtain the minimum variation in sidelobe level.

The patterns corresponding to these spacing schemes are presented in the next chapter.
CHAPTER V

RESULTS OF COMPUTER STUDY

When beam width, sidelobe level, gain, aperture and number of antennas are considered simultaneously the idea of an optimum array has little meaning. An improvement in one of these quantities, in general, involves a deterioration in one or more of the others. The word optimum must be defined to mean the best array for a particular problem. Many times the engineer may not find the best array and will have to settle for a good solution to his problem.

A good example is the problem of obtaining a pattern which has equal radiation in all directions in a given plane. It appears that this could be accomplished with a single element, however, power handling requirements and electrical breakdown may require more than one element. The problem then becomes one of determining the array with the least number of elements that gives a good solution to the problem.

One factor may dominate the problem. In aircraft systems the aperture may be the dominating factor while in radio astronomy, narrow beam width may be the most important factor. In another instance it may be desired to save on the number of elements in the array at the expense of a larger aperture.

In this study over one hundred patterns, with different spacing schemes, were recorded by the X-Y plotter for the purpose of comparison. Many more than this were observed on the oscillograph, but were not recorded because they had either a larger beam width or sidelobe level.

All patterns of unequally spaced arrays had a tendency toward
a narrower beam width and a higher sidelobe level than that of an equally spaced array with the same number of antennas. The unequally spaced array had a much larger aperture than the equally spaced array. In some cases, the aperture of the unequally spaced array was more than five times greater than that of the equally spaced array. The aperture was determined so as to obtain a minimum beam width without producing a lobe equal to the major lobe at some angle other than in the broadside direction. This method of determining the aperture is described in Chapter III.

The result of high sidelobe level and narrow beam width is in agreement with the fact that the aperture is a major factor in determining the beam width while the density of the elements is a major factor in determining the sidelobe level. It is important to point out the fact that any beam width or sidelobe level can be obtained with just two elements. However, both of these properties cannot be obtained at the same time with two elements. The aperture and number of elements must be considered to obtain a combination of narrow beam width and low sidelobe level.

A comparison between symmetric and non-symmetric arrays showed that the non-symmetric array compared favorably to the symmetric array and in some cases gave a better pattern. For this reason and the fact that only symmetric arrays were studied in the literature, a large part of this study deals with non-symmetric arrays. A non-symmetric array is an array in which the elements are not spaced symmetrically about the center point of the array. The symmetry property of an array depends on the position of elements and not the amplitude distribution.
The following figures contain the patterns for arrays of three through nine elements with the narrow-beam-width spacing scheme and the constant-sidelobe-level spacing scheme. The patterns of three through nine element arrays with equal spacing and amplitude are included for the purpose of comparison. These patterns were taken directly from the analog computer with the aid of the X-Y plotter. All patterns were normalized so as to have a value of unity in the broadside direction. This was done by controlling the gain of the output amplifier of the computer. Measurements were made from these plots with drafting instruments.

When not otherwise given, the spacing and amplitude distribution are given in the upper right corner. The elements are represented by circles with the number of the element in the circle. The numbers between two circles is the spacing between the corresponding elements. This spacing is normalized so that the spacing between closest elements is unity. The numbers above the circles give the relative amplitude (which is not normalized) of the corresponding element.

The pattern of a nine element array with the narrow-beam-width spacing scheme is shown in Fig. 5.22. This pattern has a maximum sidelobe of .577 and a beam width of 1.2°. The Dolph-Tchebyscheff array with the same number of elements and same sidelobe level is shown in Fig. 5.23. It has a beam width of 4.6°. Calculation of Dolph-Tchebyscheff arrays is given in the Appendix. The aperture of this array is approximately four times that of the corresponding equally spaced array.

Fig. 5.20 is the pattern of a nine element array with the constant-sidelobe-level spacing scheme. The pattern has a beam width of 3.2° and a maximum sidelobe of .361. Fig. 5.21 is the pattern of a
Fig. 5.1 Three antennas with equal amplitude and spacing
Fig. 5.2 Three antennas with small variation in sidelobes
Fig. 5.3 Three antennas with narrow beam width and equal amplitude
Fig. 5.4 Four antennas with equal amplitude and spacing
Fig. 5.5 Four antennas with small variation in sidelobes
Fig. 5.6 Four antennas with narrow beam width and equal amplitude
Fig. 5.7 Five antennas with equal amplitude and spacing
Fig. 5.8 Five antennas with small variation in sidelobes
Fig. 5.9 Five antennas with narrow beam width and equal amplitude
Fig. 5.10 Six antennas with equal amplitude and spacing
Fig. 5.11 Six antennas with small variation in sidelobes
Fig. 5.13 Seven antennas with equal amplitude and spacing
Fig. 5.14 Seven antennas with small variation in sidelobes
Fig. 5.15 Seven antennas with narrow beam width and equal amplitude
Fig. 5.16 Eight antennas with equal amplitude and spacing
Fig. 5.17 Eight antennas with small variation in sidelobes
Fig. 5.18 Eight antennas with narrow beam width and equal amplitude
Fig. 5.19 Nine antennas with equal amplitude and spacing
Fig. 5.20 Nine antennas with small variation in sidelobes
Fig. 5.21 Dolph-Tchebyscheff array corresponding to Fig. 5.20
Fig. 5.22 Nine antennas with narrow beam width and equal amplitude
Fig. 5.23 Dolph-Tchebyscheff array corresponding to Fig. 5.22
An example of an application of an array with constant sidelobe level is an array designed to track approaching aircraft. Such an array would not have to continuously scan in search of aircraft. Since there are no nulls in the pattern and the sidelobe level is fairly constant, an aircraft would be detected regardless of its direction of approach. Once the aircraft has been detected, scanning could be initiated to lock the aircraft in the main beam.

Table I gives a comparison between the two arrays just discussed and an array with equal spacing and amplitude. Either unequally spaced type of array gave a narrower beam width than an equally spaced array at the expense of a higher sidelobe level.

Fig. 5.24 is a plot of beam width versus the number of elements for the two arrays just discussed and the equally spaced array. It is interesting to note that all three curves have approximately the same shape. The main difference is the displacement of the curves. It should be pointed out that the number of elements and the spacing scheme determine the aperture. If the beam width had been plotted versus aperture, the difference in the displacement of the curves would have been much smaller. This is due to the fact that the unequally spaced array has a larger aperture than the equally spaced array with the same number of elements. If an equally spaced array is made to have the same number of elements and aperture as an unequally spaced array, the beam widths of the two arrays will be approximately the same. However, under this condition, the pattern of the equally spaced array will have the major beam repeated in directions other than the broadside. Achieving a narrow beam width without reproducing the major
## TABLE I

COMPARISON OF DIFFERENT ARRAYS

<table>
<thead>
<tr>
<th>Equally spaced array</th>
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</thead>
<tbody>
<tr>
<td>Number of antennas</td>
<td>Beam width in degrees</td>
<td>Maximum sidelobe</td>
</tr>
<tr>
<td>3</td>
<td>23</td>
<td>.324</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
<td>.264</td>
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<td>.247</td>
</tr>
<tr>
<td>7</td>
<td>8.2</td>
<td>.236</td>
</tr>
<tr>
<td>8</td>
<td>7.2</td>
<td>.225</td>
</tr>
<tr>
<td>9</td>
<td>6.2</td>
<td>.220</td>
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</table>

<table>
<thead>
<tr>
<th>Constant sidelobe level array</th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of antennas</td>
<td>Beam width in degrees</td>
<td>Maximum sidelobe</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>.523</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>.357</td>
</tr>
<tr>
<td>5</td>
<td>9.6</td>
<td>.363</td>
</tr>
<tr>
<td>6</td>
<td>6.8</td>
<td>.435</td>
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<td>4.6</td>
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<tr>
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<td>.396</td>
</tr>
<tr>
<td>9</td>
<td>3.2</td>
<td>.361</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Array with narrow beam width</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of antennas</td>
<td>Beam width in degrees</td>
<td>Maximum sidelobe</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>.594</td>
</tr>
<tr>
<td>4</td>
<td>6.8</td>
<td>.698</td>
</tr>
<tr>
<td>5</td>
<td>4.2</td>
<td>.605</td>
</tr>
<tr>
<td>6</td>
<td>2.8</td>
<td>.566</td>
</tr>
<tr>
<td>7</td>
<td>2.0</td>
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<td>.575</td>
</tr>
<tr>
<td>9</td>
<td>1.2</td>
<td>.577</td>
</tr>
</tbody>
</table>
beam in directions other than the broadside is a major property of unequally spaced arrays. A plot of the sidelobe level versus number of elements was not made because the points oscillated about a given value and could not be fitted to a smooth curve.

Every type of array studied that had a large period of $F(\Phi)$, had a narrow beam width. The general rule was that the beam width became smaller as the period was increased. The decrease in beam width was accompanied by an increase in the sidelobe level. The aperture, also, increased as the beam width decreased. Decreasing the aperture for a given spacing scheme increases the beam width but does not change the sidelobe level. This can be seen from the discussion of Fig. 3.2 in Chapter III. From equation (3.3), it can be seen that once the relative spacing between elements has been decided upon, a decrease in aperture does not alter the sidelobe level. All of the arrays studied had a larger average spacing between elements than that of the equally spaced array. The smallest spacing between elements was close to one wavelength in most cases.

One of the main properties of the unequally spaced array is that it gives a smaller beam width with the same number of elements, without reproducing the major lobe at some angle other than the broadside direction. This decrease in beam width is achieved at the expense of an increased sidelobe level and a larger aperture. If equally spaced and unequally spaced arrays are compared on the basis of having the same aperture, they both have about the same beam width but the equally spaced array will have a higher sidelobe level due to the reproduction of the main beam in directions other than the broadside.

An equal amplitude distribution gave the lowest sidelobe level
for arrays with small beam width. In general, varying the amplitude
distribution from that of an equal amplitude distribution did not
improve the sidelobe level. This statement applies to arrays with
spacing schemes designed to produce small beam width.

Narrow beam width is not the only application of unequally
spaced arrays. Another application is arrays that produce special
patterns. An example of this is the array with small variation in
sidelobe level. Although it was designed to have a small variation
in sidelobe level, this array had a smaller beam width than the
Dolph-Tschebyscheff array with the same number of elements and
sidelobe level. Many patterns were found on the analog computer that
could be the solution to some special problem.

Another type of spacing scheme studied consisted of a combination
of two equally spaced arrays. One half of the elements of the array
were equally spaced. The other half of the array was also equally
spaced, but the distance between the elements of one half of the
array was twice that of the other half. Adjusting the amplitude of
the elements of this array gave a pattern which had a smaller beam
width than the equally spaced array and a sidelobe level that was
slightly greater. The following figure illustrates the spacing scheme
for an array of seven elements.

```
1 1 2 1 3 1 4 2 5 2 6 2 7
```

![Fig. 5.25 Combination of two equally spaced arrays](image)

A symmetric array of nine antennas with spacings proportional
to the progression; 1, 1, 2, 1, 1, 2, 1, 1, had three large lobes
centered about the broadside direction. The sidelobe level was
lower than .05 at points other than the three main lobes.
CHAPTER VI

CONCLUSIONS

This study does not begin to answer all the problems of unequally spaced arrays. However, it was broad enough to show the general properties of unequally spaced arrays. Most of the ideas discussed in this thesis are general and apply to all unequally spaced arrays and not to just particular spacing schemes.

The unequally spaced array has, in general, a narrower beam width than the equally spaced array with the same number of elements. The narrow beam width is achieved at the expense of a larger sidelobe level and an increase in aperture. If compared on the basis of the number of elements in the array, the unequally spaced array requires fewer elements to produce the same beam width as a given equally spaced array with the same sidelobe level.

Many unequally spaced arrays produce a narrower beam width than the Dolph-Tchebyscheff array with the same sidelobe level and number of elements. The inherent high sidelobe level of the unequally spaced array is not as bad as it might appear to be. Using directive elements for the actual physical elements of the array would greatly improve the sidelobe level and at the same time produce a further decrease in beam width.

It was shown that the non-symmetric array compares favorably with the symmetric array. By favorably, it is meant that both symmetric and non-symmetric arrays produce approximately the same beam width and sidelobe level with the same number of elements. The relatively large spacing between the elements of unequally spaced arrays reduces the effects of mutual coupling.
Because of the larger number of variables involved, the unequally spaced array has a greater potential for producing special patterns. Examples of special patterns are cited in this thesis. It was concluded that the aperture is the controlling factor in determining the beam width. The sidelobe level is primarily determined by the amplitude distribution and number of elements in the array. The patterns used for illustration are typical of those of unequally spaced arrays.

There are many possibilities of other research on the subject of unequally spaced arrays. The work in this thesis could be extended to include the phase of the elements of the array. This would include the study of end-fire arrays. Another possibility is the investigation of the electrical scanning properties of unequally spaced arrays. A study of the gain and directivity of unequally spaced arrays would also be of value.


APPENDIX

CALCULATION OF DOLPH-TCHEBYSCHEFF ARRAYS

This appendix presents an example of the calculation of Dolph-Tchebyscheff arrays. An array of nine elements will be considered. If the center of the array is taken as the reference, the expression for the pattern of an equally spaced array of nine elements is given by,

\[
F(\psi) = 2A_0 + 2A_1 \cos 2\psi + 2A_2 \cos 4\psi + 2A_3 \cos 6\psi + 2A_4 \cos 8\psi. \quad (A-1)
\]

The term \(2A_0\) is the amplitude of the center element. \(A_1\) is the amplitude of the two antennas immediately to the right and left of the center element. \(A_2\), the amplitude of the next two antennas to the left and right, etc., with \(A_4\) the amplitude of the extreme elements. Since only relative amplitudes are needed, the factor 2 will be dropped.

Equation (A-1) is then equated to the Tchebyscheff polynomial (10) of proper order. For this case the polynomial is,

\[
T_8(z) = 128z^8 - 256z^6 + 160z^4 - 32z^2 + 1 \quad (A-2)
\]

where \(z = z_0 \cos \psi\).

The desired range of the variable \(z\) is determined from the equation,

\[
z_0 = 1/2 \left| \left( R + \sqrt{R^2 - 1} \right)^{1/8} + \left( R - \sqrt{R^2 - 1} \right)^{1/8} \right|. \quad (A-3)
\]
\[ R = \frac{\text{main-lobe maximum}}{\text{sidelobe level}} \]

\( z_0 \) is the maximum value assumed by the variable \( z \).

Once \( z_0 \) has been determined, the cosine terms in equation (A-1) are replaced by the Tchebyscheff polynomial of proper order. After this is done equation (A-1) becomes,

\[
\]

Equating this expression to equation (A-2) gives the following equations for the amplitudes;

\[
A_4 = z_0^8
\]
\[-256A_4 + 32A_3 = -256z_0^6\]
\[160A_4 - 48A_3 + 8A_2 = 160z_0^4\]
\[-32A_4 + 18A_3 - 8A_2 + 2A_1 = -32z_0^2\]
\[A_4 - A_3 + A_2 - A_1 + A_0 = 1\]

These equations are then solved for the amplitudes.

The amplitudes for the two Dolph-Tchebyscheff arrays cited in this thesis are given as follows;

For a sidelobe level of .361, \( A_4 = 1.190, A_3 = .407 \)
\( A_2 = .450, A_1 = .478, A_0 = .244 \)

For a sidelobe level of .577, \( A_4 = 1.089, A_3 = .183 \)
\( A_2 = .193, A_1 = .199, A_0 = .101. \)
VITA

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In September, 1959, the author entered the University of Missouri at Rolla where he received the degree of Bachelor of Science in Electrical Engineering in January 1964.

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The author is a member of Tau Beta Pi and Eta Kappa Nu. On January 25, 1964 he was commissioned Second Lieutenant in the United States Army Corps of Engineers.