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Effect of single stirrups on reinforced concrete beams

Vinodbhai T. Patel

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EFFECT OF SINGLE STIRRUPS ON REINFORCED CONCRETE BEAMS

BY

VINODESHAI T. PATEL

A THESIS

submitted to the faculty of the UNIVERSITY OF MISSOURI AT ROLLA

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1965

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ABSTRACT

This study was made to determine the location of vertical stirrups and also to determine the amount of stirrup-steel required to prevent diagonal cracking. The study is limited to rectangular, simply supported, reinforced concrete beams subjected to a concentrated point load at the center of the span. Five sets, each containing three beams were tested. Two sets were cast without stirrups and the results were used to locate the stirrups for the remaining tests. The other three sets were cast with one stirrup on either side of the central load. Stirrups were of the "U" type and formed from No. 9 and No. 11 steel wire and 3/16 inch diameter bar. Results indicated only a small increase in moment carrying capacity between the first two sets and the latter three. Some inconsistencies between observed and theoretical behavior are noted.
ACKNOWLEDGEMENT

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The author is also thankful to Dr. William A. Andrews and the staff of the Civil Engineering for their advice and assistance in preparing this thesis. The author wishes to express his grateful appreciation to Mr. Raman A. Patel and Mr. Vipin R. Shah for their help during the experiment.

The author is also indebted to the several members of the Mechanics Department for their assistance on the use of the testing machine.
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LIST OF SYMBOLS

The symbols used are defined where they first occur in the text and are listed here in alphabetical order for convenience.

\[ a = \text{shear arm in inches.} \]

\[ A_0 = \text{cross-sectional area of bent-up bar crossing the diagonal crack in square inches.} \]

\[ A_s = \text{cross-sectional area of tensile steel in square inches.} \]

\[ A_v = \text{cross-sectional area of vertical stirrups in square inches.} \]

\[ b = \text{width of beam section in inches.} \]

\[ d = \text{effective depth of beam in inches.} \]

\[ f'_c = \text{compressive strength of concrete in pounds per square inch at fourteen days.} \]

\[ f_v = \text{allowable stress in vertical stirrups in pounds per square inch.} \]

\[ f_y = \text{yield strength of steel in pounds per square inch.} \]

\[ I = \text{moment of inertia of the cross-section with respect to neutral axis in inch}^4. \]

\[ j_d = \text{internal moment arm in inches.} \]

\[ M = \text{bending moment in pound-inches.} \]

\[ M_{cr} = \text{critical bending moment in pound inches.} \]

\[ M_{fl} = \text{flexural bending moment in pound inches.} \]

\[ \Sigma = \text{sum perimeters of all effective bars crossing the section on tension side in square-inches.} \]

\[ P = \text{axial compressive load in pounds.} \]

\[ P_f = \text{load at failure in pounds.} \]

\[ P_u = \text{ultimate load in pounds.} \]

\[ p_w = \text{percentage of tensile steel.} \]
Q = first moment with respect to neutral axis of cross-sectional area cut off at a distance y from neutral axis in inch$^3$.

S = spacing of vertical stirrups in inches.

U = bond stress in pounds per square inch.

V = total vertical shear at a section in pounds.

$V_c$ = shear resisted by the concrete compression zone.

v = unit shear stress in pounds per square inch.

$\alpha$ = inclination of bent up bar with respect to axis of beam.

$\phi$ = capacity reduction factor, $\phi = 0.85$ for diagonal tension, bond and anchorage as per ACI 318-63.

$\sigma$ = direct stress in pounds per square inch

$\tau$ = shear stress in pounds per square inch
I INTRODUCTION

Generally reinforced concrete beams are designed for maximum moment, but in order to develop this moment capacity it is necessary to insure that the beam will not fail in some other manner than bending. Thus the section must be checked for resistance to a diagonal failure, and where the capacity of the unreinforced web is deemed inadequate, stirrups are employed to provide sufficient strength. Although a great amount of work has been expended on the problem of shear and diagonal tension since the very beginning of rational design of reinforced concrete, renewed interest has been shown in this problem during the last decade. The American Concrete Institute has strongly recommended that further research be carried out to determine the mechanisms which control shear failure of reinforced concrete members so that it will be possible to develop fully rational design procedures. With this in mind, the present investigation was undertaken to determine the location of diagonal cracks and the effect of a single vertical stirrup on the behavior of a simply supported rectangular reinforced concrete beam, subjected to a centrally located concentrated load.
II REVIEW OF LITERATURE

As discussed in ACI-ASCE Joint Committee 326 report "Shear and Diagonal Tension" (1) early developments in the design of reinforced concrete before the year 1900 were influenced by two main ideas regarding the mechanisms of shear failure in reinforced concrete members. One belief was that the horizontal shear was the main cause of shear failure. This seemed reasonable at a time when engineers were familiar with the action of web rivets in steel girders and shear keys in wooden beams for which shearing stresses were computed using the classical equation derived for elastic materials.

\[ v = \frac{V Q}{I b} \]

where,

- \( v \) = unit horizontal shearing stress at a distance \( y \) from the neutral axis.
- \( V \) = total vertical shear at the section.
- \( Q \) = first moment with respect to the neutral axis of the cross sectional area cut off at a distance \( y \) from neutral axis.
- \( I \) = moment of inertia of the cross section with respect to neutral axis.
- \( b \) = width of the cross section at a distance \( y \) from neutral axis.
Reinforced concrete beams were treated as an extension of the older materials assuming that the concrete alone could resist low horizontal shearing stresses, and that vertical stirrups acted as shear keys for higher shearing stresses.

The second belief, accepted by nearly all engineers today, considered diagonal tension as the basic cause of shear failures. The origin of the concept of diagonal tension is uncertain, but a clear explanation was presented by W. Ritter in the year 1899. He stated that stirrups resisted tension, not horizontal shear, and suggested design of vertical stirrups by the equation,

\[ V = \frac{A_v f_v j d}{s} \]

where,
- \( A_v \) = total cross sectional area of vertical stirrups.
- \( f_v \) = allowable stress in the stirrups.
- \( j d \) = internal moment arm.
- \( s \) = spacing of stirrups in the direction of the axis of the member.

Ritter's ideas were not widely accepted at the time, but his design expression for vertical stirrups is similar to that appearing in the design specifications of most countries today.

The discussions among engineers of the two different beliefs continued for nearly a decade. In the year 1906, E. Morsch of Germany pointed out that, if a state of pure
shear exists, then a tensile stress of equal magnitude must exist at a 45 degree plane, and he developed the equation for nominal shearing stress widely used today,

\[ v = \frac{V}{bd} \]

Morsch's data, supported by tests by F. Von Emperger and E. Probst, supported the argument for diagonal tension. By 1910, a general acceptance of Ritter's viewpoint of diagonal tension had been achieved although the concept of horizontal shear has reappeared periodically in the literature even in recent years. Today, however, most design codes and specifications throughout the world predicate their design procedures on the concept of diagonal tension.

The analysis of stirrup action known as the truss analogy was presented by the University of Illinois Engineering Experiment Station in June 1927 in complete form and then with simplifying assumptions. In the simplified form it is assumed that the action of a reinforced concrete beam with stirrups may be represented as that of a truss in which the concrete compression zone is the top chord, the tension reinforcement is the bottom chord, the stirrups or bent-up bars are the tension web members, and portions of the concrete web of beam are compression members.

In the year 1908, the National Association of Cement Users recommend that the shearing strength of concrete should be assumed at 200 psi and if the shearing stress ex-
ceed this limit, a sufficient amount of steel should be introduced in such sections to overcome the deficiency. In 1910, the recommendations were changed such that in calculating web reinforcement the concrete should be considered to carry 40 psi and any excess over this should be resisted by means of reinforcement in tension. In 1913, the First Joint Committee of ACI-ASCE retained these general methods with modification of allowable stresses. The allowable shearing stress for beams with horizontal bars only, was set at 0.02 $f'_{c}$ with a maximum value of 66 psi. Similarly, the allowable shearing stresses for beams thoroughly reinforced for shear was set at 0.06 $f'_{c}$ with a maximum value of 198 psi. From 1917 to 1950, the general trend was toward the use of less and less web reinforcement, and the ceiling value of shear stress was raised to 0.075 $f'_{c}$. In 1951, the American Concrete Institute recommended that all plain bars must be hooked and all deformed bars must meet the requirements of ASTM specification A-305. A maximum shearing stress of 0.03 $f'_{c}$ was specified for all beams without web reinforcement and a ceiling of 0.12 $f'_{c}$ was specified for beams with web reinforcement.

In Germany, the maximum allowable stress was set at 64 psi for members without web reinforcement in 1904. This value could be exceeded by 20 percent if web reinforcement was provided. In the USSR the formula for shear strength of a diagonal section given in the "Standards and Technical Specifications for the Design of Plain and Reinforced Concrete Structure"
of 1955, was

\[ V = m \ m_n \ m_s \ f_V (\Sigma A_\alpha \sin \alpha + \Sigma A_v) + V_c \]

where,

- \( V \) = design shear force at the section.
- \( V_c \) = shear force resisted by concrete compression zone.
- \( \Sigma A_v \) = total cross sectional area of vertical stirrups crossing diagonal section.
- \( \Sigma A_\alpha \sin \alpha \) = sum of cross sectional area of bent-up bars crossing diagonal section.
- \( f_V \) = design stress of web reinforcement.
- \( m \) = coefficient depending on various conditions; the usual value is \( m = 1 \).
- \( m_s \) = coefficient depending on uniformity of steel, e.g. for hot rolled deformed bars \( m_s = 0.9 \).
- \( m_n \) = coefficient introduced to take into account the possibility that web reinforcement does not always yield prior to failure.

The strength in shear of an inclined diagonal section, determined from the above equation, depends on its angle of inclination. When vertical stirrups are used without bent-up bars, the projected length of the critical diagonal section corresponds to a minimum value of the expression \( (m_n m_s f_V \Sigma A_v + V_c) \).

The National Building Code of Canada of 1953 specifies that the maximum allowable shear stress is 0.03 \( f_c' \) for
members without web reinforcement and 0.12 $f_{c}^{'}$ for members with proper web reinforcement.

The British Standard Code of Practice CP 114 of 1957 follows the basic principles of the German code of 1916. Where the shear stress exceeds the permissible shear stress for concrete, all shear must be resisted by the web reinforcement alone.

The American Concrete Institute Standards of 1963 (2), specifies that the shear stress permitted on a web without reinforcement should not exceed $1.1 \sqrt{f_{c}^{'}}$ for working stress design at a distance "d" from the support unless a more detailed analysis is made. The shear stresses at sections between the face of the support and the section a distance "d" therefrom should not be considered critical. With a more detailed analysis the shear stress permitted on an unreinforced web shall not exceed that given by,

$$v_{c} = \sqrt{f_{c}^{'}} + 1300 \frac{p_{w} V d}{M}$$

but not to exceed 1.75 $f_{c}^{'}$ where,

- $v_{c}$ = shear stress carried by concrete.
- $p_{w} = A_{s}/bd$
- $A_{s}$ = area of steel.
- $V$ = total shear.
- $M$ = bending moment.

A general and direct comparison of specifications of the four major countries; U.S.A., Germany, Britain and the
The specification of the U.S.S.R. is entirely based on ultimate strength design, while the shear design methods of the other three countries are based on working load and allowable stress.

During the past decade the mechanism of shear failure has commanded the renewed interest of many research workers in different countries. The first clear realization of the important parameters involved came with the experiments of Clark (3). He put forward a diagram showing that the shear at failure depends on the shear arm ratio $a/d$. He also showed that for the same beams, the shear stress at failure changes considerably with a little change in the $a/d$ ratio.

B. Broms, in his paper (4), wrote that the flexural cracks which form in reinforced concrete beams cause a redistribution of stresses which results in secondary shear and lateral tensile stresses. High secondary shear stresses probably contribute to the development of diagonal tension cracks, and it is possible that the horizontal cracks which result from these lateral tensile stresses affect the failure mechanism of the beam.

Bresler and Pister pointed out (5) that the strength of concrete is a function of the state of stress and cannot be predicted by limitations of tensile, compressive and shearing stresses independently of each other. For example,
concrete having pure compressive strength, $f_c'$, and shearing strength of $0.08 f_c'$, would fail under a compressive stress of $0.5 f_c'$ with the shearing stress increased to approximately $0.2 f_c'$. Therefore the strength of structural elements can be properly determined only by considering the interaction of the various components of the state of stress. Such conditions as shrinkage, restraint to contraction or expansion, foundation settlements, duration of loading and previous stress history, may have an important effect on the state of stress at the critical section, and must be carefully evaluated, particularly with respect to possible reduction of strength.

Dr. Kani pointed out (6) that under increasing load a reinforced concrete beam is transformed into a comb-like structure. The analysis of this structural system has disclosed that there are two different mechanisms of structural behavior possible. In the tensile zone the flexural cracks form vertical concrete teeth and the compression zone becomes the back-bone of the structure. As long as the capacity of the concrete teeth is not exceeded, beam-like behavior governs; but after the resistance of the concrete teeth has been destroyed, a tied arch action remains. Figure 1 shows the comparision of test results obtained from experiments conducted by Dr. Kani and calculated data. The transformation of the reinforced concrete beam into a tied arch weakens the comb-like structure, but there will not be a sudden collapse of the comb-like structure, if a/d is less than the value at
Figure 1 Comparison of Theoretical and Test Results by Dr. Kani
point A, since the capacity of the remaining arch is higher than the capacity of the concrete teeth. Therefore, under gradually increasing loads, the transformation of the beam into an arch occurs gradually and the structure fails when the capacity of the arch is exceeded. In the region between points A and B, the capacity of the arch is lower than the capacity of the concrete teeth; thus when the capacity of the concrete teeth is exceeded, there will be a sudden collapse of the beam since the capacity of the remaining arch is lower. In the region beyond point B only flexural failure is possible.
III EQUIPMENT AND MATERIALS

A. Laboratory Equipment

1. Mixer

The concrete mixer used was a stationary, nontilting, electrically operated mixer, having a capacity of three cubic feet. It is manufactured by Lancaster Iron Works Inc., Lancaster, Pennsylvania. The interior surface was kept very clean, dry and free from any foreign materials before use.

2. Testing Machines

Two types of Riehle Universal Testing Machines were used. The first, used for compression test of cylinders, had a range of 300,000 pounds, graduated in 1000 pound increments and was hydraulically operated. It is located in the Structural Laboratory of the Civil Engineering Department, Rolla, Missouri.

The second, used to determine the tensile strength of reinforcing steel, has a load range of 60,000 pounds, graduated in 200 pound increments and is also hydraulically operated. It is located in the Material Testing Laboratory, Mechanics Department, Rolla, Missouri.

A Tinius Olsen Testing machine was used for testing beams for flexural strength. It is mechanically operated and has a load capacity ranging from zero to 10,000 pounds, graduated in 10 pound increments. It is located in the Structural Laboratory, Civil Engineering Department, Rolla, Missouri. It is manufactured by Tinius Olsen Testing Machine
3. Forms

Two types of forms were used, one for casting beams and the other for cylinders. The beam forms were made from 3/4 inch plywood and had inside dimension of 3.75 x 6.5 x 40.0 inches. They were kept thoroughly clean and well oiled before pouring of the concrete. The cylinders were standard 6 inches by 12 inches forms of waxed cardboard with a smooth thin metal bottom.

B. Materials

1. Cement

Type I cement, manufactured by the Alpha Portland Cement Company, Lemay, Missouri, was used throughout the experiment.

2. Fine aggregate

The fine aggregate used was a river sand from Meramec River near Pacific, Missouri, containing mostly chert and quartz. A sieve analysis was made using a Rotap machine in accordance with the "Standard Method of Test for sieve Analysis for Fine Aggregate," ASTM Designation: C-136-46. The results were plotted as shown in Figure 16 in the Appendix. The fineness modulus was calculated as 2.5.

3. Coarse aggregate

The coarse aggregate used was crushed limestone obtained from the Springfield, Missouri area. The sieve analysis was made in accordance with the "Tentative Specifications for Coarse Aggregate", ASTM Designation: C-33-55 and results were plotted as shown in Figure 17 in the Appendix.
4. Reinforcing bar and Stirrups

The reinforcing steel used was an intermediate grade steel with a yield strength of approximately 42,000 psi. The stress-strain curves are shown in Figures 18 and 19 in the Appendix. The stirrups used were No. 9 and No. 11 galvanized steel wire with cross-sectional areas of 0.0172 and 0.0123 square inches, respectively, and a 3/16 inch intermediate grade steel bar with a cross-sectional area of 0.0275 square inches. Ultimate strengths were determined for the stirrup materials and are shown in Table 2.

C. Preparation of Specimens

The concrete mix was designed to have a slump of approximately 2.75 inches and compressive strength approximately equal to 3500 psi. at 14 days. The mix was designed using the Portland Cement Association's "Design and Control of Concrete Mixtures", (7). The amount of cement, fine aggregate, coarse aggregate and water used for each batch of concrete mix is shown in Table 1.

Table 1. Concrete Mix Content

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Weight in Pounds</th>
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<tbody>
<tr>
<td>Cement</td>
<td>56.4</td>
</tr>
<tr>
<td>Fine Aggregate</td>
<td>174.0</td>
</tr>
<tr>
<td>Coarse Aggregate</td>
<td>183.0</td>
</tr>
<tr>
<td>Water</td>
<td>34.98</td>
</tr>
</tbody>
</table>

The beams were cast according to the "Standard Method of Making and Curing Concrete Compression and Flexure Test
Specimens in the Laboratory, "ASTM Designation: C-192-55. Three cylinders were cast from each batch, cured in the moist room at 100 percent relative humidity and an average temperature of 72 degree F. Each batch produced about three cubic feet of concrete which was sufficient to make three beams and three cylinders.

All the materials were weighed on a Toledo Scale accurate to one fourth of a pound. In preparing concrete, one third of the water and all of the fine aggregate and coarse aggregate was placed in the mixer. After mixing for about three minutes, all the cement and the rest of the water was added and mixed for an additional 2.5 minutes. A slump test was made on each batch immediately after mixing and before placing concrete in the forms. The slump test was conducted in accordance with ASTM Designation: 143-39.

The beam forms were thoroughly oiled and the reinforcing bar was properly positioned with steel rebar chairs. The reinforcing bar was clean and free from rust and grease. The concrete was placed in two layers, rodded fifty times per square foot per layer and the top surface trowelled smooth. The cylinders were filled in three layers, each layer being rodded twenty-five times.

The three series with stirrups were cast in a similar manner. For the No. 9 and No. 11 wire stirrups series, the stirrups were anchored by providing a concrete block on the
top of the beams, Figure 2, while for the 3/16 inch bar series, the stirrups were anchored by providing a steel plate across the top of the beam to which the stirrups were welded, Figure 2.

The beams were kept for twenty-four hours outside the curing room to prevent swelling of the wooden forms due to moisture. A period of twenty-four hours was considered to be the earliest time that the forms could be removed without damaging the beams.

The cylinder forms were removed after curing had progressed for twenty-four hours outside the moist room; then marked and placed in the curing room.
Figure 2 Test Set Up for Beams and Anchorage Details
IV TEST PROCEDURE

A. Beams

To determine the positions of tension cracks, two series of beams without stirrups were cast as mentioned in Chapter III. In Set A of the beams, a No. 6 deformed bar was used as a tensile reinforcement, while in Set B, a No. 5 deformed bar was used. After fourteen days curing the beams were tested in accordance with the ASTM Designation: C-79-49. The beams were simply supported with a clear span of 36 inches, and a point load was applied at the center of the span. Aluminum plates of 1.5 x 3.75 x 5/16 inch dimensions, were grouted at the point of load and at the supports, using plaster of paris. These plates were used to minimize any failure in bearing. One dial gage, graduated in 0.0001 of an inch, was installed at one fourth span as shown in Figure 2. An initial load of 500 pounds was placed on the specimen and then released in order to set the zero reading. The load was then applied gradually and deflection at intervals of 500 pounds was recorded and the test was continued until failure occurred. Crack propagation was observed during the test and marked with the aid of a pencil as shown in Figures 4 and 5. Time required for the above test varied from about 10 to 15 minutes.

To locate the position of vertical stirrups, points "a" and "b" were determined as shown in Figure 3. Point "a" is
near the top surface where the slope of the crack changes, and point "b" is at the bottom surface where an extension, of the crack meets the bottom surface. From the six beams tested, the cracking zone was determined by plotting these "a" and "b" points as shown in Figure 6, and from this information the position of the vertical stirrups was located. In addition to this, a load and deflection curve was plotted as shown in Figure 7. By mistake Set A was tested with the knife edge improperly located; however, since the results did not vary much from Set B, Set A results were utilized for fixing the position of the vertical stirrups.

![Figure 3 Determination of Points "a" and "b"](image)

After the location of the stirrups had been determined as explained above, three identical sets of beams were cast, each from one batch of concrete. Each set contained the three different stirrups as shown in Table 2. The stirrups were positioned at 8.75 inches from the center line of the beam as determined from Sets A and B. After fourteen days of curing the beams were tested as explained for Sets A and B and the crack patterns are shown in Figures 8, 9 and 10. The load and deflection curves were plotted for all the beams
as shown in Figures 11, 12 and 13.

B. Cylinders

The cylinders were capped with a sulfur compound at a minimum of one day before the test was conducted. They were tested in accordance with the "Standard Method of Test for Compression Strength of Model Concrete Cylinders", ASTM Designation: C-39-49. The results are indicated in Table 2.
Table 2
Properties of Beams and Test Results

<table>
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<tr>
<th>Beam</th>
<th>Tensile reinforcement</th>
<th>Type of web reinforcement</th>
<th>( f'_c ) in psi</th>
<th>( f_y ) psi</th>
<th>Ultimate load of stirrups in lbs. ( P_u )</th>
<th>Load at failure ( P_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set A</td>
<td>6 bar</td>
<td>None</td>
<td>3360</td>
<td>64000</td>
<td></td>
<td>7500</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6500</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8000</td>
</tr>
<tr>
<td>Set B</td>
<td>5 bar</td>
<td>None</td>
<td>3740</td>
<td>42000</td>
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<td>8200</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>7450</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7800</td>
</tr>
<tr>
<td>Set C</td>
<td>6 bar</td>
<td>11 wire</td>
<td>3530</td>
<td>43000</td>
<td>750</td>
<td>7900</td>
</tr>
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<td></td>
<td></td>
<td>9 wire</td>
<td></td>
<td></td>
<td>1050</td>
<td>8800</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3/16&quot; bar</td>
<td></td>
<td></td>
<td>1540</td>
<td>8450</td>
</tr>
<tr>
<td>Set D</td>
<td>6 bar</td>
<td>11 wire</td>
<td>3790</td>
<td>43000</td>
<td>750</td>
<td>8200</td>
</tr>
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<td></td>
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<td>9 wire</td>
<td></td>
<td></td>
<td>1050</td>
<td>8900</td>
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<tr>
<td></td>
<td></td>
<td>3/16&quot; bar</td>
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<td></td>
<td>1540</td>
<td>8420</td>
</tr>
<tr>
<td>Set E</td>
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<td>11 wire</td>
<td>3830</td>
<td>43000</td>
<td>750</td>
<td>7800</td>
</tr>
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<td></td>
<td></td>
<td>1050</td>
<td>8700</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3/16&quot; bar</td>
<td></td>
<td></td>
<td>1540</td>
<td>8600</td>
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Figure 4  Crack Pattern at Failure for Set A
Figure 5  Crack Pattern at Failure for Set B
Figure 6 Cracking Zone

Note:  
△ point a in Figure 4 and 5
○ point b in Figure 4 and 5
Figure 7 Load-Deflection Curve for Beam Set B
Figure 8  Crack Pattern at Failure for Set C
Figure 9  Crack Pattern at Failure for Set D
Figure 10  Crack Pattern at Failure for Set E
Figure 11 Load-Deflection Curves for Beam Set C
Figure 12 Load-Deflection Curves for Beam
Set D
Figure 13 Load-Deflection Curves for Beam Set E
DISCUSSION OF RESULTS

As mentioned previously, the ultimate moment capacities determined from Set A of the beams was considered to be in error and was discarded since the knife edge of the testing machine was not resting in the proper location. However, since there was little disagreement between the A and B series, in the location of diagonal cracks, these results were used to locate the position of the stirrups for the succeeding tests.

Although beams Sets A and B had been designed on the basis of Kani's work to produce a failure below the calculated ultimate moment capacity, see Figure 1, it is clear from the results that this was not the case. Only a small increase in moment capacity was produced by adding the stirrups. However the failure of the Set B, at 7.9 K without stirrups was actually some 20% higher than the calculated ultimate load of 6.5 K for No. 5 tensile steel. With stirrups and No. 6 tensile steel the failure load of about 8.3 K agreed quite well with the calculated value of 8.7 K. No. explanation of the behavior of the unreinforced section can be presented.

The a/d ratio of 3.55 used in these experiments should have caused premature failure of the unreinforced web section with respect to moment capacity. It is not too surprising that the premature failure did not occur since many of the values necessary in Kani's equations are pure guess-work at this stage of development. The results do indicate, however,
Figure 14 Average Load Deflection Curves
that in order to achieve a premature failure, which is necessary in order to measure the effect of stirrups, it will be necessary to use a smaller a/d ratio for this section.

It will be noticed from Figure 14 that the addition of stirrups to the section did have an effect upon the stiffness of the beam. There was a consistent decrease of deflection with the addition of stirrups. In addition, the stirrups seem to have controlled the horizontal failure at the ends which was evident in Sets A and B. Thus, it may be concluded from this meager evidence, that even for sections with adequate shear capacity it may be advantageous to use a nominal number of stirrups.

With "U" type stirrups used, the ultimate load capacity in tension for the stirrups were

\[
\begin{align*}
\text{No. 11 wire} &= 2 \times 750 = 1500 \text{ lbs.} \\
\text{No. 9 wire} &= 2 \times 1050 = 2100 \text{ lbs.} \\
3/16 \text{ inch bar} &= 2 \times 1540 = 3080 \text{ lbs.}
\end{align*}
\]

Since none of the stirrups used broke, it can be said that at the location of the stirrups the concrete must have carried a shear force greater than

\[
3.6 K \equiv \left(\frac{8.3}{2} - 1.5\right) K
\]

According to the ACI Building Code (2), the ultimate shear force that the concrete at this section would be allowed to carry would be (Art. 1701-c)
\[ V_u = 2\phi \sqrt{f'_c} b d \]
\[ = 2 \times 0.85 \sqrt{3500} \times 3.75 \times 5.1 \]
\[ = 2 \times 0.85 \times 59.25 \times 3.75 \times 5.1 \]
\[ = 1.92 \text{ K} \]

Thus the ACI Code seems to be quite conservative, at least for this particular section.

The calculated ultimate bond stress was,
\[ U = \frac{V_u}{\phi \Sigma_0 j d} \]
\[ = \frac{8.3/2}{0.85 \times 1.963 \times 4.56} \]
\[ = 547 \text{ psi} \]
as compared to an allowable of 750 psi, by the ACI Building Code. This would indicate a satisfactory design condition, however, the horizontal failure along the line of the tensile reinforcement would indicate either excessively high bond stresses or dowel action. The cover of 1.1 inches may have been inadequate although it is within the 3/4" allowed for unexposed surfaces.

In comparing the failure patterns of Sets A and B with Sets C, D and E Figures 4, 5, 8, 9, 10, it can be seen that the addition of stirrups has effected the patterns and in general, has cause the cracks to fall within the bounds of the stirrups. Comparing the results of Sets C, D and E, there appears to be some effects on the crack pattern caused by stirrup size and concrete strength. It appears that the larger stirrups tend to pull the sloping part of the crack closer to the top surface. However, this observation is
strictly tentative since the evidence is far from sufficient. With respect to concrete, there is an increasing order of strength through Sets C, D, and E (Table 2) and there also seems to be a consistent increase in the angle of the crack with the beam axis. This would seem to be consistent with a simplified analysis of the combined stress condition in the compression zone. Using a constant shear stress at a point in the compression zone, the Mohrs' circle solution is shown in Figure 15.

![Mohr's Circle for Combined Stress](image)

**Figure 15** Mohr's Circle for Combined Stress

If tension produces cracking, $\frac{1}{2}\theta$ indicates the angle the crack makes with the axis of the beam. Since $\theta_B > \theta_A$, a higher compressive strength leads to an increase in the slope of the failure crack. Another inconsistency which cannot be explained in the increase in failure load of the No. 9 wire stirrups over the larger 3/16 inch bar stirrups.
VI CONCLUSIONS

1. The addition of stirrups confined the diagonal failure cracks within the limits of the stirrups although there is no large increase in the ultimate moment capacity of the beams.

2. The addition of stirrups did cause an increase of the stiffness of the beam system as compared to the beams without stirrups.

3. There is some evidence that the use of a nominal number of stirrups, even in a beam which is satisfactory with respect to a diagonal tension failure, might be necessary in order to control horizontal cracking at the level of tensile reinforcement.

4. There is some indication, both experimentally and theoretically that higher concrete strength increases the slope of the failure line with respect to the axis of the beam.
VII RECOMMENDATIONS

In order to effectively measure the results of a single stirrup, it will be first necessary to obtain a beam with an appreciable reduction in moment capacity. The section to be obtained should have a reduction in moment capacity of about 60 percent.

The effect of different a/d ratios may be checked for the same stirrup positions and also for different locations of the stirrups.
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Figure 16 Sieve Analysis for Fine Aggregate

Figure 17 Sieve Analysis for Coarse Aggregate
Figure 18 Stress-Strain Curve for No. 6 Bar
Figure 19  Stress-Strain Curve for No. 5 Bar
VITA

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He received his primary and secondary education in Sarsa, India. He received his college education from Vitthalbhai Patel College of Science and Birla College of Engineering, Vallabhbh Bihari Vidyanagar, Anand, Gujarat, India. He received his Bachelor of Engineering Degree in Civil Engineering from Sadar Vallabhbhai Vidyapetth (University) in June 1962.

From September 1962 to January 1964, he worked as a Junior Engineer, in Public Works Department, Gujarat State, India. He was appointed on the Kakarapar Irrigation Project as Site Engineer.

He enrolled at the Missouri School of Mines and Metallurgy at Rolla in February 1964 to work towards his Master of Science Degree in Civil Engineering.