1965

Effects of internal pressure upon the buckling of thin circular cylindrical shells under axial compression

LeRoy E. Thompson

Follow this and additional works at: http://scholarsmine.mst.edu/masters_theses

Part of the Civil Engineering Commons

Recommended Citation

This Thesis - Open Access is brought to you for free and open access by the Student Research & Creative Works at Scholars' Mine. It has been accepted for inclusion in Masters Theses by an authorized administrator of Scholars' Mine. For more information, please contact weaverjr@mst.edu.
EFFECTS OF INTERNAL PRESSURE UPON THE
BUCKLING OF THIN CIRCULAR CYLINDRICAL
SHELLS UNDER AXIAL COMPRESSION

BY

LEROY B. THOMPSON

A

THESIS

submitted to the faculty of the
UNIVERSITY OF MISSOURI AT ROLLA
in partial fulfillment of the requirements for the
Degree of

MASTER OF SCIENCE IN CIVIL ENGINEERING

Rolla, Missouri

1965

Approved by

Joseph H. Smith (advisor) James W. Joiner

R. A. Schaffer

James E. Spooner
ABSTRACT

An experimental investigation was made of the effects of internal pressure upon the buckling of thin circular cylindrical shells under axial compression. A comparison was made of the experimental and theoretical results; the latter were derived from the large-deflection theory. Results showed that the internal pressure had an appreciable strengthening effect upon the buckling of the cylindrical shell. The comparison showed that the experimental results were lower than the theoretical results for a given internal pressure. The discrepancy between the results can be expected since there were factors not included in the theory: imperfections, end conditions, lengths, and material irregularities.
ACKNOWLEDGEMENT

The author wishes to express his appreciation to Dr. Joseph H. Senne, Jr., Professor of Civil Engineering, University of Missouri at Rolla, who gave constructive suggestions and guidance on the preparation of this paper. He would also like to thank Professor R. F. Davidson, Chairman of the Department of Mechanics, for his permission to use the facilities of its laboratory. The author also wishes to thank J. E. Spooner, Assistant Professor of Civil Engineering, who read the paper and made helpful suggestions.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>vii</td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. REVIEW OF LITERATURE</td>
<td>4</td>
</tr>
<tr>
<td>III. THEORY</td>
<td>9</td>
</tr>
<tr>
<td>IV. DISCUSSION</td>
<td>16</td>
</tr>
<tr>
<td>A. Description of Apparatus</td>
<td>16</td>
</tr>
<tr>
<td>1. Test Specimens</td>
<td>16</td>
</tr>
<tr>
<td>2. Material Specimens</td>
<td>20</td>
</tr>
<tr>
<td>3. Instrumentation</td>
<td>22</td>
</tr>
<tr>
<td>4. Equipment</td>
<td>25</td>
</tr>
<tr>
<td>B. Experimental Procedures</td>
<td>25</td>
</tr>
<tr>
<td>C. Experimental Results</td>
<td>27</td>
</tr>
<tr>
<td>V. CONCLUSIONS</td>
<td>34</td>
</tr>
<tr>
<td>VI. RECOMMENDATIONS</td>
<td>36</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>37</td>
</tr>
<tr>
<td>VITA</td>
<td>40</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Bodies of the cylindrical shell specimens.</td>
<td>17</td>
</tr>
<tr>
<td>2.</td>
<td>Bottom and top ends of specimen A.</td>
<td>18</td>
</tr>
<tr>
<td>3.</td>
<td>Bottom end of specimen B.</td>
<td>21</td>
</tr>
<tr>
<td>4.</td>
<td>Top end of specimen B.</td>
<td>21</td>
</tr>
<tr>
<td>5.</td>
<td>Strain gage positions.</td>
<td>23</td>
</tr>
<tr>
<td>6.</td>
<td>Switching units and strain indicators.</td>
<td>24</td>
</tr>
<tr>
<td>7.</td>
<td>Testing setup.</td>
<td>26</td>
</tr>
<tr>
<td>8.</td>
<td>Experimental results of the buckling stress</td>
<td>29</td>
</tr>
<tr>
<td>9.</td>
<td>Experimental values of the buckling load at various internal pressures</td>
<td>30</td>
</tr>
<tr>
<td>10.</td>
<td>Post-buckling failure of specimen A.</td>
<td>33</td>
</tr>
<tr>
<td>11.</td>
<td>Comparison of theoretical and experimental results showing the increment of</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>buckling stress due to internal pressure</td>
<td></td>
</tr>
</tbody>
</table>
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>Theoretical buckling stresses for various internal pressures</td>
<td>15</td>
</tr>
<tr>
<td>II.</td>
<td>Experimental buckling stresses for various internal pressures</td>
<td>28</td>
</tr>
</tbody>
</table>
Symbols:

- \( f_0, f_1, f_2 \): parameters used in the deflection function
- \( m \): number of waves in longitudinal direction within length equal to circumference of cylindrical shell
- \( n \): number of waves in circumferential direction
- \( p \): internal pressure, psig
- \( t \): thickness of cylindrical shell wall, inches
- \( \sigma \): average compressive stress, psi
- \( \beta = \frac{m}{n} \): aspect ratio of buckled waves
- \( \mu \): Poisson's ratio
- \( E \): Young's modulus, psi
- \( R \): radius of cylindrical shell, inches
- \( B_1, B_2, \ldots, B_6 \) \( B_1', B_2', \ldots, B_6' \): functions of \( \beta \)
- \( D_1, D_2, \ldots, D_5 \): functions of \( \rho \) and \( \beta \)
- \( (D)_\rho = \frac{\partial D}{\partial \rho} \): \( (D) \) represents the functions \( D_1, D_2, \ldots, D_5 \)
- \( (D)_\beta = \beta \frac{\partial D}{\partial \beta} \): \( (D) \) represents the functions \( D_1, D_2, \ldots, D_5 \)
- \( L \): length of cylindrical shell, inches

Nondimensional parameters:

- \( \rho = \frac{f_2}{f_1} \)
- \( \varsigma = \frac{R}{f_1 t} \)
- \( \eta = n^2 \frac{t}{R} \)
\[ \sigma = \frac{\sigma R}{t} \]

\[ \Delta \sigma_{cr} = \sigma_{cr} - (\sigma_{cr})_p = 0 \]

\[ \bar{p} = \frac{p (R/t)^2}{E} \]

\[ \bar{\sigma}' = (\bar{\sigma} - \frac{1}{\beta^2 \rho}) \gamma D_1 \]

\( \bar{\omega} \) = work or energy parameter

Subscripts:

- \( u \) unbuckled state just prior to buckling

- \( cr \) buckling condition
I

INTRODUCTION

The reason for conducting research in this particular area is to study the increased efficiency obtained from a thin circular cylindrical shell under axial load by including an additional loading of internal pressure. The structure is efficient if its material is used to the maximum limit of its permissible working stresses. When a minimum of dead weight of the structure is the critical design condition, the most structurally efficient design will in all probability yield the most economical structure. In modern spacecraft, aircraft, and missiles, each pound of dead weight saved will yield many other additional advantages. One such advantage is that an additional pound may be included into the payload of the particular vehicle. This payload may be either cargo, warhead, test equipment, or surveillance systems. Another advantage is that the range of the vehicle can be increased when the weight saved may be used for additional propulsion fuel. As is already known, the dead weight can be minimized by selecting a workable material with low specific weight and yet the highest permissible working stresses for the given unit weight; but this same objective can be obtained also by the extensive use of pressurized cylindrical shell structures. Many times, pressurization may be dictated by circumstances other than
for structural purposes, such as the case where internal systems must be pressurized. Also, if the structure is sealed and is launched into a less dense atmosphere or into space, the external pressure will decrease while the pressure internally is maintained near the initial condition at the time of launching; consequently this will yield the internal pressure loading upon the structure. Therefore, even though some shells may not be pressurized for structural purposes the structure may have this type of loading already, and it can be utilized in addition for a structural design use. This internal pressure will materially increase the stress at which buckling will occur in a circular cylindrical shell. The increase in the buckling stress then definitely achieves a higher structural efficiency and thus a considerable weight saving.

Many experiments have been conducted (3,5,11,12,13, 24,25) on thin pressurized circular cylindrical shells under axial compressive load, but the results are far from being complete. It has been found that the theoretical investigations cannot be relied upon for design purposes due to the discrepancies found between experimental and theoretical buckling loads. The reason for these discrepancies is that the deflection theory which most accurately describes the problem does not account for all the exact numerical values of the parametric coefficients which experimentation have found
to be critical. In this paper, the experimentation will be extended to find some of the numerical coefficients of shells with different magnitudes of parameters and the results compared to existing theory.
II
REVIEW OF LITERATURE

The first theoretical investigation of the buckling of thin-walled cylindrical shells under axial compression and lateral pressure was done by Flugge (23) in 1932. Flugge's conclusion was that the internal pressure effects on the buckling load were negligible. This conclusion was contradictory to the experimental and theoretical work which followed in the years thereafter. This early theoretical analysis differs because Flugge's analysis was based upon the small, rather than the large, deflection theory. The small-deflection theory was found to be applicable only when the deflections are small with respect to the wall thickness.

Since cylindrical shells can have very large deflections without reaching yield stress, the large-deflection theory is the most accurate theoretical approach. This theory neglects the local bending stresses with respect to the membrane stresses. It was assumed that the external forces, uniformly distributed along the edge of the shell, were tangent to the meridians (1, 2, 4, 6, 7).

The use of large-deflection theory for shells under axial compression was first advanced by Von Karman and Tsien (8) in an attempt to explain the discrepancies between the buckling loads predicted by theory and those
obtained from experimental results. The results indicated that cylindrical shells can be maintained in equilibrium in the buckled state by a compressive load considerably lower than that predicted by theory. Therefore, the cylindrical shells designed by the theoretical method might possibly fail. The treatment of Von Karman and Tsien was found later to be incomplete since the total potential energy was not differentiated with respect to all the physical parameters involved.

Donnell (10) first introduced a set of simplifying assumptions which are now commonly used. From these assumptions Donnell initiated the generalized equations for the new large-deflection theory.

Based upon the observed contradictions of experimental results with Flugge's conclusion, Lo, Crate, and Schwartz (11) devised a revised large-deflection theory, using the method of Von Karman and Tsien. The results showed that the buckling load increases with increasing values of internal pressure up to a limit where it remains nearly constant. The large-deflection theoretical buckling loads for the large values of internal pressures were then comparable to the small-deflection theory.

The tests by Dow and Peterson (26) showed buckling loads that were considerably above the average from other experiments. This was at least partly due to the fact that their test cylinders were pressurized by oil, so
that buckling was accompanied by a rise in internal pressure. Another factor was found to be non-uniformity of loading around the shell circumference which caused a large amount of scattered data in the test results.

The photoelastic experimental work conducted by Tennyson (22) indicated that buckling loads were within ten percent of the classically predicted values and all the tested shells behaved completely elastically, thus permitting repeatable tests.

Harris, Suer, Skene, and Benjamin (13) developed a semi-empirical procedure which permitted an axial compressive loading and internal pressure buckling analysis of cylindrical shells with a knowledge of the cylinder geometry only. This analysis was achieved by correlating experimental data statistically with theoretical parameters. Fung and Sechler (12) proposed a design method which gave slightly conservative values for the buckling stress of axial compression and internal pressure loadings on a cylindrical shell.

Sergev (15,16) and Walton (15) conducted experimental tests on the strength of pressurized columns. The research indicated that the instability criterion was entirely different for the two cases of pressurized columns with open and with closed ends. The results also indicated that nothing was gained in the critical buckling load by pressurizing the columns; although the stress state in the column wall was very different from the
unpressurized case. This was due to the fact that their columns had large slenderness ratios and small radius to thickness ratios.

Almroth and Brush (24) worked with the upper and lower bounds of a buckling load which were not sensitive to chance variation for pressurized cylinders and for cylinders filled with a soft elastic core. It was found that these bounds converged with increasing internal pressure or core stiffness.

The results of the analysis by Siede (25) indicated that shearing stresses between the elastic core and the cylindrical shell were assumed to be negligible, so that restraint was offered only against normal displacements. A closed-form solution for the elastic support offered by the core was given and comparisons were made with experimental data.

Donnell and Wan (9) studied theoretically the effects of the initial imperfections of a cylindrical shell upon the critical buckling loads. It was found that the imperfections caused the buckling to occur prematurely. This remains a purely static theory containing idealized assumptions which cannot be verified practically.

Lee (18) made both an analytical and experimental study of inelastic instability of initially imperfect cylindrical shells subjected to axial compression. The comparisons of experimental with theoretical results indicated that the application of the deformation theory
provided a fairly accurate prediction of buckling strength, but failed to yield a correct description of the post-buckling behavior. The buckling strength was over-estimated by using an incremental theory with the initial imperfections being considered.

Almroth, Holmes, and Brush (29) concluded that minimization of initial imperfections in axially compressed cylindrical shells greatly increased the buckling load, and the magnitude of the minimum post-buckling equilibrium load was relatively insensitive to initial imperfections.

It has been found that many factors combine to determine the buckling load of a particular cylinder. These include care in fabrication, experience of the investigator, end conditions of the test specimen, initial deformation, limits of strain regions, functions of different cylindrical shell parameters, etc. The nature of the shell problem seems to be that design values for buckling load and internal pressure must be found by experimental methods since current theories are not sufficient to determine exact numerical values of the parametric coefficients.
III
THEORY

The existing procedure for the computation of the buckling stress by large-deflection theory, which involves the solution of four simultaneous nonlinear equations for each pressure loading, was advanced by Von Karman and Tsien (8). Improvements in these four equations were made by other investigators in which the potential energy was properly differentiated with respect to all physical parameters and the effects of internal pressure (11) were included.

The solution of the four simultaneous equations shown below (Eqs. 1, 2, 3, and 4) required a tedious and lengthy numerical process. Lo, Crate, and Schwartz (11) improved this numerical process by the introduction of one more equation (Eq. 5) which defined the condition at buckling. These five simultaneous equations enabled the solution to be found for the particular condition at buckling. Where previously four simultaneous equations had to be solved several times in order to get sufficient results to plot a stress-strain curve relationship in order that the critical condition at buckling could be detected.

\[
\begin{align*}
\bar{\sigma}' &= (\eta S)^2 (2D_2) - (\eta S)D_3 + 2 \eta D_5 \quad \text{(Eq. 1)} \\
\bar{\sigma}' &= (\eta S)^2 (2D_2) - (\eta S)(1.5D_3) + D_4 + \eta^2 D_5 \quad \text{(Eq. 2)} \\
\bar{\sigma}' &= \left[ (\eta S)^2 (D_2) - (\eta S)(D_3) + (D_4) + \eta^2 (D_5) \right] \frac{D_1}{(D_1)^1} \quad \text{(Eq. 3)}
\end{align*}
\]
The nomenclature of the terms are as follows:

\[ \bar{\sigma}' = (\bar{\sigma}' - \frac{1}{\beta^2 \bar{p}}) \eta D_1 \]

\[ \bar{\sigma} = \frac{\sigma R}{E t} \]

\( \sigma \) = average compressive stress  
\( E \) = Young's modulus  
\( R \) = radius of cylindrical shell  
\( t \) = thickness of cylindrical shell wall  
\( \bar{p} = \frac{p (R / t)^2}{E} \)

\( p \) = internal pressure  
\( \beta = \frac{m}{n} \) = aspect ratio of buckled waves  
\( m \) = number of waves in longitudinal direction within length equal to circumference of cylindrical shell  
\( n \) = number of waves in circumferential direction  
\( \eta = \frac{n^2 t}{R} \)

\[ D_1 = \frac{1}{2} \beta^2 \left( \frac{3}{4} + \rho + \rho^2 \right) \]

\[ \rho = \frac{f_2}{f_1} \]

\( f_0, f_1, f_2 \) = parameters used in deflection function
\[ 5 = R_{lt} \]

\[ D_2 = B_1 + B_2 \theta + B_3 \theta^2 + B_4 \theta^3 + \frac{1}{2} B_4 \theta^4 \]

\[ D_3 = (2B_4 + \frac{1}{64})(1 + 2\theta) \]

\[ D_4 = (2B_4 + \frac{1}{32}) + \frac{1}{8}(\theta + \theta^2) \]

\[ D_5 = B_5 + B_6 (\theta + \theta^2) \]

\[ B_1 = \frac{1}{64} \left[ \frac{1}{8} \theta^4 + \frac{17}{4} \frac{\theta^4}{(1 + \theta^2)^2} + \frac{\theta^4}{(1 + 9\theta^2)^2} + \frac{\theta^4}{(9 + \theta^2)^2} \right] \]

\[ B_2 = \frac{1}{16} \left[ \frac{9}{2} \frac{\theta^4}{(1 + \theta^2)^2} + \frac{\theta^4}{(1 + 9\theta^2)^2} + \frac{\theta^4}{(9 + \theta^2)^2} \right] \]

\[ B_3 = \frac{1}{16} \left[ \frac{11}{2} \frac{\theta^4}{(1 + \theta^2)^2} + \frac{\theta^4}{(1 + 9\theta^2)^2} + \frac{\theta^4}{(9 + \theta^2)^2} \right] \]

\[ B_4 = \frac{1}{8} \frac{\theta^4}{(1 + \theta^2)^2} \]

\[ B_5 = \frac{1}{6(1 - \mu^2)} \left[ \frac{1}{8}(1 + \theta^2)^2 + \frac{1}{4}(1 + \theta^4) \right] \]

\[ B_6 = \frac{1}{6(1 - \mu^2)}(1 + \theta^4) \]

\[ \mu = \text{Poisson's ratio} \]

\[ (D_1)_{\theta} = \frac{1}{2} \theta \theta (1 + 2\theta) \]

\[ (D_2)_{\theta} = B_2 + 2B_3 \theta + 3B_4 \theta^2 + 2B_4 \theta^3 \]

\[ (D_3)_{\theta} = 2(B_4 + \frac{1}{64}) \]

\[ (D_4)_{\theta} = \frac{1}{8}(1 + 2\theta) \]

\[ (D_5)_{\theta} = B_6 (1 + 2\theta) \]

\[ (D_1)_{\theta} = 2D_1 \]
The four equations (Eqs. 1, 2, 3, and 4) were derived in the following manner. The expressions for the energy due to elastic extensional, bending, applied compressive load, and the internal pressure were found. The summation of these four energy expressions was the equation for the total potential energy. The total potential energy equation was differentiated with respect to the initial deflection function parameter \((f_0)\) and the derivative was set equal to zero. The resulting expression was substituted into the four energy expressions, which in turn were expressed in terms of the nondimensional
parameters \( \bar{\sigma}, \bar{p}, \bar{p}, \eta, \bar{S}, \) and \( \bar{W} \). The term \( \bar{W} \) is the nondimensional parameter for energy. The summation of the four nondimensional parameters of energy formed the nondimensional total potential energy parameter in the buckled state. The equilibrium positions of the cylindrical shell in the buckled state were obtained by differentiating the nondimensional total potential energy with respect to each parameter \( \eta, \bar{S}, \bar{p}, \) and \( \beta \) and by setting the derivatives equal to zero. The results were four simultaneous nonlinear equations. The D and B terms for the certain functions of \( \beta \) and \( \bar{p} \) as well as the nondimensional parameter of average compressive stress \( \bar{\sigma}' \) were then substituted into the above four simultaneous nonlinear equations, which yielded Eqs. 1, 2, 3, and 4.

Equation 5 was derived in the following manner. An expression was written for the work done by the pressure during buckling. The strain energy expression for the buckled state was written as the sum of the strain energy in the unbuckled state just prior to buckling and the work done by the pressure during buckling. A relationship of end shortening and average compressive stress was written for the buckled and unbuckled states. The fact was then recognized that the end shortening remains unchanged during buckling from the unbuckled state to the buckled state, provided the loading machine is assumed to be rigid. Therefore, the relationships for the buckled and unbuckled states could be equated and a relationship among
the nondimensional parameters of average compressive stresses could then be determined. This relation as well as the nondimensional parameter of average compressive stress $\bar{\sigma}'$ was then substituted into the strain energy expression. The nonlinear equation (Eq. 5) was thus obtained for the buckling criterion.

The solution of the five equations (Eqs. 1, 2, 3, 4, and 5) gives the buckling stress for a given internal pressure. The method for the solution of these five nonlinear simultaneous equations was computed in the following manner (11). Equations 2 and 5 were equated, thus determining $\eta_5$. In like manner, $\eta^2$ was obtained using Equations 1 and 2. For a preassigned value of $\beta$, various values of $\bar{\sigma}$ were assumed. Then $\eta_5$ and $\eta^2$ were computed, which were substituted into Equations 1 and 3 to obtain the $\sigma'$ for each of the two equations, respectively. The resulting two $\sigma'$ values were plotted against the assumed various values of $\bar{\sigma}$. The pair of curves would intersect at a common value of both $\sigma'$ and $\bar{\sigma}$. The corresponding values of $\eta_5$ and $\eta^2$ were computed and substituted in Equation 4 so the nondimensional parameter of pressure $\bar{p}$ could be determined. For each assigned value of $\beta$, the corresponding values of $\sigma'$ and $\bar{p}$ were thus obtained. The relationship between $\sigma'$ and $\bar{p}$ was known, so the corresponding values of $\bar{\sigma}$ and $\bar{p}$ were thus obtained (Table I). When the factor $\eta_5 = 0$, the classical
critical buckling stress of 0.605 occurred as the cut-off buckling stress which was independent of pressure (11).

Table I *

Theoretical buckling stresses for various internal pressures

<table>
<thead>
<tr>
<th>( \bar{p} )</th>
<th>( \bar{\sigma} )</th>
<th>( \Delta \bar{\sigma}_{cr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.376</td>
<td>0.000</td>
</tr>
<tr>
<td>0.020</td>
<td>0.444</td>
<td>0.068</td>
</tr>
<tr>
<td>0.040</td>
<td>0.480</td>
<td>0.104</td>
</tr>
<tr>
<td>0.060</td>
<td>0.506</td>
<td>0.130</td>
</tr>
<tr>
<td>0.080</td>
<td>0.528</td>
<td>0.152</td>
</tr>
<tr>
<td>0.100</td>
<td>0.547</td>
<td>0.171</td>
</tr>
<tr>
<td>0.120</td>
<td>0.565</td>
<td>0.189</td>
</tr>
<tr>
<td>0.140</td>
<td>0.581</td>
<td>0.205</td>
</tr>
<tr>
<td>0.168</td>
<td>0.605</td>
<td>0.229</td>
</tr>
</tbody>
</table>

* Reference (11), page 23.
IV
DISCUSSION

A. Description of Apparatus.

1. Test Specimens.

The two specimens used for the experimental tests are hereby described as specimen A and specimen B. Specimen B was used for the internal pressure-bending moment interaction tests conducted in reference (27).

Test Specimen A: The specimen used for the tests was a cylindrical shell, 30 inches long with a 15 inch diameter, made of No. 28 U.S. standard gage galvanized sheet steel. This gage is equivalent to a nominal thickness of 0.021 inch.

The butt-joint of the two longitudinal edges was covered by inside and outside splice straps of the dimensions 0.021 inch thick and 1\(\frac{1}{8}\) inches wide. The straps were riveted with 1/8 inch diameter rivets spaced in a staggered arrangement 3/8 inch on center along the total length of the cylinder (Fig. 1). Soldering was done around the rivet heads and edges of straps in order to properly seal the butt-joint.

Both the top and the bottom ends (Fig. 2) were made of an 18 inch square by 3/8 inch thick plywood with a mounted 15 inch diameter disk of 3/4 inch thick plywood to act as an inside shoulder support.
Fig. 1. Bodies of the cylindrical shell specimens.
Fig. 2. Bottom and top ends of specimen A.
for the shell. There was a piece of rubber, 18 inches square by 1/8 inch thick, placed between the two pieces of plywood. The plywood ends were then bolted to an 18 inch square by 3/4 inch thick A-7 steel plates. Steel balls were tack-welded to the steel plates in order to apply a concentric concentrated compression load. Strain gage wire outlets and the air pressure inlet were provided on the top end. They were sealed with an epoxy cement of the compound Epibond 104 with its hardener 951. The plywood ends were also sealed to the shell by the epoxy cement after the disks were force fitted within the inside shell diameter.

**Test Specimen B:** The specimen used for the tests was a cylindrical shell, 27 inches long with a 15 inch diameter, made of No. 28 U.S. standard gage galvanized sheet steel. The gage is equivalent to a nominal thickness of 0.021 inch.

The butt-joint of the two longitudinal edges was covered by a lapped splice strip with the dimensions of 0.021 inch thick and 1 1/2 inches wide. The splice was riveted with 1/8 inch diameter rivets spaced in a staggered arrangement 3/8 inch on center along the total length of the cylinder (Fig. 1). Soldering was done around the rivet heads and edges of splice in order to properly seal the butt-joint.
Both the top and the bottom ends (Figs. 3, 4) were made of \( \frac{1}{4} \) inch A-7 steel plates and then welded to 15 inch outside diameter steel rings \( \frac{1}{4} \) inch thick and 1 inch in height. The rings acted as an inside shoulder support for the shell. Strain gage wire outlets and the air pressure inlet were provided on the top end. They were sealed by Smooth-on No. 1 iron cement. The joints between the cylindrical shell and ends were arc-welded using nickel rods. The quality of welding was not good due mainly to the difference in thickness of materials. As a result there was minor leakage of air found along these joints.


The modulus of elasticity, yield strength, and ultimate strength of the galvanized sheet steel were experimentally determined. Five strips of a nominal width of 0.750 inch by a thickness of 0.021 inch were tested for the above mentioned properties. The strips were instrumented with SR-4 electric strain gages type A-1 in order to record the tensile strains. The results were found to be \( 29.3 \times 10^6 \) psi. for the modulus of elasticity, 42000 psi. for the yield strength, and 52000 psi. for the ultimate strength.
Fig. 3. Bottom end of specimen B.

Fig. 4. Top end of specimen B.
3. Instrumentation.

The two cylindrical specimens were instrumented with the same pattern of strain gages. There were 12 SR-4 electric strain gages equally spaced along the outside circumference of the cylindrical shell at mid-length, and directly opposite them on the inside were 12 additional gages. On specimen A of these 24 gages, 16 were type A-1 strain gages used to measure the strains along the longitudinal direction of the shell, and the remaining 8 were AR-1 rosette strain gages used to measure both the longitudinal and circumferential strains (Fig. 5). On specimen B of these 24 gages, 16 were type A-3 strain gages used to measure the strains along the longitudinal direction of the shell, and the remaining 8 were AR-1 rosette strain gages used to measure both the longitudinal and circumferential strains (Fig. 5).

The strain gages were connected to two switching units, a Baldwin-Lima-Hamilton 20 channel unit and a Budd 10 channel unit. The switching units were then connected to a Baldwin SR-4 strain indicator and a Budd strain indicator respectively (Fig. 6). The strains could be read to a micro-inch per inch.
Arabic numerals denote gages measuring longitudinal strains.

Roman numerals denote gages measuring circumferential strains.

△ Type A-1 strain gage, specimen A.

△ Type A-3 strain gage, specimen B.

○ Type AR-1 rosette strain gage, specimens A and B.

Fig. 5. Strain gage positions.
Fig. 6. Switching units and strain indicators.
4. Equipment.

A Tinius-Olsen 60,000 pound universal testing machine was used. A booster-storage tank of 60 psi capacity, manufactured by Soiltest Incorporated, Model K-670, was used as a stand-by air source. The tank had standard equipment of a source pressure gage as well as an air regulator with a pressure gage connected to the air line outlet. The testing machine could be read within 100 pounds. The air pressure gage could be read within one psi.

B. Experimental Procedures.

The specimens were subjected to a concentric axial compressive load in the testing machine. Compressed air was used to produce internal pressure, which was maintained at any desired constant value by an air pressure regulator. The air source with a regulator was needed to the utmost since the leakages were appreciable in specimen A and minor in specimen B.

The switching units were adjusted so that all the initial gage readings could be recorded. Then the cylindrical shell was preloaded with an axial load slightly greater than that which would be the total end load created by the internal pressure. Compressed air was next let into the cylindrical shell from the storage tank until the desired internal pressure was obtained. The desired internal pressure was controlled by the air
Fig. 7. Testing setup.
regulator. The axial compressive load was increased in increments until the initial buckling was observed. At each increment of axial load all gage readings were recorded. The internal pressure was maintained at a constant level during the test procedure. The axial compressive load was then reduced and the internal pressure was changed to another desired value. As each value of internal pressure was changed, the same test procedure was repeated.

C. Experimental Results.

The experimental results are tabulated in Table II and plotted in Fig. 8 and Fig. 9. The Fig. 8 shows the nondimensional critical buckling stress plotted against the nondimensional internal pressure. The curves illustrate an increase in buckling stress with an increase of internal pressure. The rate of change for the increase in buckling stress decreased for an increase of internal pressure. The data for specimen A is not as complete as specimen B because specimen A could not retain the high internal pressures due to the excessive pressure losses through its ends. The term \( \sigma_{ucr} \) is the average compressive stress when in the unbuckled state just prior to buckling. The stress \( \sigma_{ucr} \) is found by dividing the critical buckling load \( P_{cr} \) by the cross-sectional end area of the cylindrical shell. The critical buckling load \( P_{cr} \) occurred when the strain maintained a constant value for a small increase in compressive
Table II

Experimental buckling stresses for various internal pressures

<table>
<thead>
<tr>
<th>p</th>
<th>( \bar{p} )</th>
<th>( P_{cr} )</th>
<th>( \bar{f}_{cr} )</th>
<th>( \Delta \bar{f}_{cr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>psi.</td>
<td></td>
<td>lbs.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Specimen A

0   | 0.00000       | 12,200      | 0.151           | 0.000           |
1   | 0.00435       | 13,200      | 0.163           | 0.012           |
3   | 0.01305       | 14,700      | 0.181           | 0.030           |
5   | 0.02175       | 15,300      | 0.188           | 0.037           |

Specimen B

0   | 0.00000       | 18,300      | 0.225           | 0.000           |
1   | 0.00435       | 19,700      | 0.243           | 0.018           |
2   | 0.00870       | 20,400      | 0.252           | 0.027           |
3   | 0.01305       | 21,800      | 0.269           | 0.044           |
4   | 0.01740       | 22,300      | 0.275           | 0.050           |
5   | 0.02175       | 23,000      | 0.283           | 0.058           |
8   | 0.03480       | 24,200      | 0.298           | 0.073           |
12  | 0.05220       | 25,500      | 0.314           | 0.089           |
16  | 0.06960       | 27,000      | 0.333           | 0.108           |
20  | 0.08700       | 27,800      | 0.343           | 0.118           |
Fig. 8. Experimental results of the buckling stress at various pressures.
Fig. 9. Experimental values of the buckling load at various internal pressures.
load. Usually, no visible evidence of buckling could be observed during the occurrence of this phenomenon. The buckling occurred locally and not simultaneously at all the strain gages. In Fig. 9 the critical buckling load is plotted against the internal pressure. The curves illustrate an increase in buckling load with an increase of internal pressure.

The magnitudes of buckling stress and buckling load are greater for specimen B than specimen A at a given internal pressure. The reasons for this difference can be two-fold. The specimen A was 30 inches in length, where the specimen B was 27 inches in length. The ends of the specimen A were constructed of plywood (Fig. 2), where the ends of the specimen B were constructed of steel (Figs. 3 and 4). The major reason for the differences in the magnitudes is believed to be due to the construction of the ends. The non-uniformity of loading around the shell circumference would be more appreciable for the plywood ends than for the steel ends. The test results indicated this in the strain readings by having more variation in strains for specimen A than specimen B. The difference in specimen lengths is believed to be of minor significance, since the change in the slenderness of the specimens was small.

Strains in the circumferential direction of the cylindrical shell increased as pressurization took place. The variations of the circumferential strains were small
during the testing at a given internal pressure.

The longitudinal strains were found to vary linearly with the compressive load.

The specimen A was failed under a post-buckling load at zero internal pressure after the pressurization tests were completed (Fig. 10). The post-buckling load was 12,950 lbs.
Fig. 10. Post-buckling failure of specimen A.
V

CONCLUSIONS

From the theoretical and experimental results shown in Fig. 11, the internal pressure is seen to have an appreciable strengthening effect on the cylindrical shell. In Fig. 11 the increment of buckling stress $\Delta \sigma_{cr}$ due to the presence of internal pressure is plotted against the internal pressure. The increment of buckling stress $\Delta \sigma_{cr}$ is the difference between the buckling stress with the pressure $\sigma_{cr}$ and that without the pressure $(\sigma_{cr})_0 = 0$. The theory gives a fairly good prediction of the increase of compressive buckling stress that may be expected as a result of internal pressure. The curves for the specimens indicated the common trend of an increase in buckling stress for an increase in internal pressure. The discrepancies between the theoretical curve and the experimental curves of Fig. 11 is believed to be caused by such factors as manufacturing imperfections in the specimens, end condition effects, length of specimens, and material irregularities, which were not included in the theory.

The results of the strains in the circumferential direction are mainly a function of internal pressure and do not depend primarily upon the compressive load.
Fig. 11. Comparison of theoretical and experimental results showing the increment of buckling stress due to internal pressure.
VI
RECOMMENDATIONS

An extensive experimental program should be conducted under relatively constant conditions. This is required in order to describe more precisely the effects of radius-thickness ratio and length-radius ratio. Curves of $\sigma_{cr}$ plotted against $L/R$ for various $R/t$ ratios would illustrate these relationships. The experimental program should involve many identical specimens to give a reasonable statistical sample for each combination of values of $L/R$ and $R/t$. Care should also be taken to eliminate the possibilities of yielding at the cylinder ends, or over-all plastic buckling; and separate determinations of these effects should be made with many different materials.

Since the experimental data was limited to values of $\bar{\sigma}$ less than 0.09, additional experimental data is needed to check the theoretical values for $\bar{\sigma}$ greater than 0.09 and beyond the cut-off point of $\bar{\sigma} = 0.168$.

Future theoretical work should include a better large-deflection analysis, including the effects of finite length, end conditions, and plasticity.


27. CHU, C.P. (1964) Effect of internal pressure on compressive buckling stress of a cylindrical shell under eccentric loading. M.S. Thesis, Univ. of Mo. at Rolla, Mo.


VITA

The author, LeRoy Earl Thompson, son of Mr. and Mrs. Henry H. Thompson, was born on May 22, 1934, in Lithium, Missouri. He received his primary education in McBride, Missouri and his secondary education at the Perryville Public High, Perryville, Missouri. He received a Bachelor of Science degree in Civil Engineering from the School of Mines and Metallurgy of the University of Missouri, Rolla, Missouri in May 1956.

He held the position of an associate structural engineer with the McDonnell Aircraft Corporation, St. Louis, Missouri, for the periods from June 1956 to April 1957, and October 1957 to September 1960. During the period of April 1957 to October 1957 he was on a leave of absence from the McDonnell Aircraft Corporation while serving a tour of military service with the U.S. Army Corps of Engineers. He has held the position of Instructor in Civil Engineering at the School of Mines and Metallurgy of the University of Missouri, Rolla, Missouri since September 1960.

He has been enrolled in the Graduate School of the School of Mines and Metallurgy of the University of Missouri, Rolla, Missouri on a part-time basis since September 1960.

He was married to Freida Joanne Medley in February 1958, and their children are Julie Kay, Karen Lynne, and Michael Lee.