Multiple-input multiple-output system on a spinning vehicle with unknown channel state information

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MULTIPLE-INPUT MULTIPLE-OUTPUT SYSTEM ON A SPINNING VEHICLE WITH UNKNOWN CHANNEL STATE INFORMATION

by

ADITYA MURALIDHAR

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Approved by

Dr. Kurt Kosbar, Advisor
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Dr. Maciej Zawodniok
ABSTRACT

This thesis work presents the investigations into the performance of a multiple-input multiple-output (MIMO) system with its transmitters on a spinning vehicle and no available channel state information (CSI) at the transmitter or the receiver. The linear least squares approach is used to estimate the channel and the estimation error is measured. The spinning is simulated based on a sample radiation pattern of a patch antenna. The antenna gains at different angles are obtained from this pattern and are used to model spinning. Spinning gives rise to a periodic component in the channel which can be estimated based on the spin rate relative to the data rate of the system. It is also determined that spinning causes the bit error rate of the system to degrade by a few dB.
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1. INTRODUCTION

Multiple-input multiple-output (MIMO) communication systems have more than one antenna at both the transmitter and the receiver. It has been theoretically proven that these systems can achieve higher performance in terms of capacity and bit error rate over traditional single-input single-output (SISO) systems if the path gains between different antenna pairs are independent [1].

One of the applications of MIMO is in the area of aerospace telemetry. The reason this technology can be beneficial is the geometry of the situation. Suppose an aircraft has only one transmitting antenna, then when it travels and turns at high altitudes there will be instances where the transmitted power towards the receiver is very low due to the transmitter pointing in a direction that is opposite to the receiver. Thus, the receiver loses connection with the transmitter for a certain amount of time which can prove to be a very critical problem. This problem can be overcome by placing another antenna on the aircraft in such a way that at any instant, at least one antenna will radiate signals towards the receiver. This will ensure that the receiver does not lose contact with the aircraft. The possible downside of this approach is that the signals from the two antennas might interact with each other destructively and cancel out before they reach the receiver. However, the probability of this happening is extremely small.

This thesis presents the investigations into the performance of an Alamouti space time coding scheme with its transmitters on a spinning vehicle and no available channel state information (CSI) at the transmitter or the receiver. These results are computed for BPSK and QPSK modulation. The channel is estimated based on the least-squares approach. The spinning vehicle can be thought of as an aircraft or a missile with its body spinning at a constant spin rate. The spinning frequency is assumed constant relative to the spin axis. The geometry of the problem is depicted by Figure 1.1. The transmitters are assumed to be mounted on a spinning vehicle, and the receivers located on ground. It is assumed that the transmitters and the receivers are placed sufficiently far apart, at least $\lambda_c/2$, where $\lambda_c$ is the wavelength of the carrier, so that the channel paths are independent [2], [3].
The Rayleigh channel model is considered in this problem. This model assumes a rich scattering environment in which no single component dominates over the rest.

In this work, first the baseline results for an Alamouti coded 2x2 MIMO system is generated and compared with known published results. The channel is estimated and the estimation error is measured. The effect of spinning on channel estimate and the overall system performance is studied. The results are generated for BPSK and QPSK modulated data. The performance metric used is the bit-error rate with respect to different signal to noise ratio (SNR). The system is evaluated at different spin rates through computer-based simulations, specifically using Matlab.

Since the channel is not known at the transmitter or the receiver, it must be estimated. There are two broad techniques to estimate the channel, namely blind channel estimation and pilot-symbol-based channel estimation. The pilot-symbol-based channel estimation, and in particular the linear least squares channel estimation is used in this work. This technique will be described further in Section 2.10.
Spinning can be simulated based on different antenna models. Initially a relatively simple model based on half-sine wave is used. Once the result for this model is obtained, a more practical case which uses a sample patch antenna at the transmitters is simulated. The gain values of patch antenna with respect to different angles are obtained from the radiation pattern of a patch antenna.

It is found that spinning gives rise to a periodic component in the channel based on the spin rate. Further, it is also determined that spinning causes the bit error rate of the system to degrade. It should be noted that the bit error rate of the system will vary with antennas having different radiation patterns.

This thesis is divided into five parts. The second part is dedicated to the background required for this thesis work and introduces the concept of spinning transmitters. This part includes the basics of MIMO communication systems, modulation schemes, multipath propagation channel models available, Alamouti coding and the linear least squares approach for channel estimation. The third part gives a description of the simulation procedures used and describes the modeling of spin of transmitters in detail. The fourth part discusses the results obtained and finally, the fifth part concludes the thesis.
2. BACKGROUND

2.1 WIRELESS COMMUNICATIONS

Wireless communications is a form of communication in which there is no wired connection between the transmitter and the receiver. A transmitter sends information through the wireless channel and the function of the receiver is to reproduce the original information. The data sent by the transmitter might get corrupted due to noise in the channel. Certain procedures are required to help increase the immunity of the signal against corruption from noise. Figure 2.1 shows the block diagram of a typical communication system.

![Figure 2.1: Typical Communication System](image)

The source (transmitter) generates analog or digital data. This is the source data that needs to be communicated to the receiver. It is passed through a source encoder which encodes the data into symbols and removes redundancy. Then controlled redundancy is added to the signal by a channel encoder. This helps in error correction and detection of the signal and improves the immunity of the system against noise. The signal is then modulated with a high frequency wave and is transmitted through the channel.
The spectrum allotted for transmission is usually in the range of very high frequencies (in the order of MHz or GHz). For the problem description described in this work, the frequency used could be in the range of a few GHz and the data rate considered is 1kbps. The exact inverse process is carried out at the receiver to recover the original data.

2.2 INTRODUCTION TO MIMO SYSTEMS

Multipath propagation is one of the main challenges in conventional wireless communication systems since it causes the signal to arrive at the receiver with different amplitudes, different phases and with different path delays. The different multipath propagation mechanisms are reflection, diffraction and scattering [4]. Figure 2.2 illustrates the different multipath propagation mechanisms.

![Figure 2.2: Multipath Propagation Mechanisms](image)

Reflection occurs when the signal encounters a surface that is much larger than the wavelength of the signal. Diffraction occurs when the signal travels through the edges of an impenetrable surface of size much larger than wavelength of the signal. Scattering
occurs when the waves collide with a surface of the same order of magnitude as the wavelength and it causes the energy to be radiated in many different directions. [5]

It can be very challenging to decode the received signal accurately in the presence of multipath. It is shown in [1] that MIMO uses multipath to its advantage. In a MIMO system with M transmit and N receive antennas, the capacity of the system increases by a factor of $\min\{M,N\}$.

Two main features of MIMO are spatial multiplexing and spatial diversity. Spatial multiplexing refers to transmission of individual data streams from each antenna. Spatial diversity refers to a technique used to combat fading by using multiple copies of the transmitted signal at the receiver. There is a tradeoff between spatial multiplexing and spatial diversity.

In most cases, channel information at the transmitter is required to achieve spatial diversity. Alamouti [6] proposed a method which can achieve transmit diversity without any knowledge of the channel at the transmitter and linear processing at the receiver.

### 2.3 GENERAL OUTLINE OF A 2X2 MIMO SYSTEM

Figure 2.3 shows the general structure of a 2x2 MIMO system. As the name suggests it has two transmitters and two receivers. It is assumed that the antennas are placed sufficiently apart so that all the paths are independent of each other and data transmitted from the two antennas are uncorrelated. There are four independent paths for the signal to travel from transmitter to the receiver in this case.

Consider a MIMO system with $N_t$ transmit antennas and $N_r$ receive antennas. When a continuous wave signal, $s$ is transmitted from the $j^{th}$ transmit antenna each of the $N_r$ receive antennas observes a complex weighted version of the transmitted signal. The signal received by the $i^{th}$ antenna can be given by $h_{ij}s$ where $h_{ij}$ is the complex channel gain between $j^{th}$ transmit and $i^{th}$ receive antenna. The MIMO channel matrix can be denoted as [7]
Figure 2.3: 2x2 MIMO System

\[ H = \begin{bmatrix} h_{11} & \cdots & h_{1N_t} \\ \vdots & \ddots & \vdots \\ h_{N_r1} & \cdots & h_{N_rN_t} \end{bmatrix} \]  (1)

where \( h_{ij} \) is the complex channel gain between the \( i^{\text{th}} \) receive antenna and the \( j^{\text{th}} \) transmit antenna. If a signal vector \( x = [x_1 \ x_2 \ \ldots \ x_{N_t}]^T \) is sent from the transmit antenna array, that is \( x_j \) is sent from the \( j^{\text{th}} \) transmit antenna the received signal \( y = [y_1 \ y_2 \ \ldots \ y_{N_r}]^T \) can be written as

\[ y = Hx + n \]  (2)

where \( n \) is the \( N_r \times 1 \) noise vector. The elements of the noise vector are independent complex Gaussian distributed with zero mean and variance \( \sigma^2 = N_0/2 \). The elements of the channel matrix \( H \) are assumed to be independent, zero mean, complex Gaussian random variables. Since the elements of \( H \) are complex, it can be written as

\[ h_{ij} = \alpha_{ij} e^{\theta_{ij}} \]  (3)

where \( \alpha_{ij} \) is the amplitude gain between the \( j^{\text{th}} \) transmitter and the \( i^{\text{th}} \) receiver and \( \theta_{ij} \) is the change in phase in those paths. The distribution of elements of \( H \) can be given as
2.4 BINARY PHASE SHIFT KEYING

In Binary Shift Phase Keying (BPSK) the bits from the source are mapped onto complex symbols with a phase difference of π radians. For digital signals, binary bit 1 is mapped onto +1 and 0 is mapped onto -1 in the complex domain assuming that the value of the constant A in equation (5) is set to 1. Figure 2.4 shows the signal space constellation diagram for BPSK. The equation for BPSK can be written as

\[
s(t) = \begin{cases} 
A \cos(2\pi f_c t) + 0j\sin(2\pi f_c t) & \text{binary 1} \\
-A \cos(2\pi f_c t) + 0j\sin(2\pi f_c t) & \text{binary 0} 
\end{cases}
\]

The equation for a normal distribution is:

\[
f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}
\]
2.5 QUADRATURE PHASE SHIFT KEYING

Quadrature Phase Shift Keying (QPSK) is a digital modulation technique used for wired and wireless communications. Every phase change in the transmitted signal represents a unique set of digital data. As the name suggests, it can undergo four separate phase changes and can take values of $\pi/4$, $3\pi/4$, $5\pi/4$ and $7\pi/4$. Figure 2.5 shows the signal space constellation diagram for QPSK. The equation for QPSK can be written as

$$s(t) = \begin{cases} 
A \cos(2\pi f_c t) \cos\left(\frac{\pi}{4}\right) + j A \sin(2\pi f_c t) \sin\left(\frac{\pi}{4}\right) & \text{data} = 00 \\
-A \cos(2\pi f_c t) \cos\left(\frac{3\pi}{4}\right) + j A \sin(2\pi f_c t) \sin\left(\frac{3\pi}{4}\right) & \text{data} = 10 \\
-A \cos(2\pi f_c t) \cos\left(\frac{5\pi}{4}\right) - j A \sin(2\pi f_c t) \sin\left(\frac{5\pi}{4}\right) & \text{data} = 11 \\
A \cos(2\pi f_c t) \cos\left(\frac{7\pi}{4}\right) - j A \sin(2\pi f_c t) \sin\left(\frac{7\pi}{4}\right) & \text{data} = 01
\end{cases}$$

(6)

Figure 2.5: QPSK Constellation
2.6 FREQUENCY-SELECTIVE AND FLAT FADING

One of the distinctions made with reference to the type of fading is flat fading and frequency-selective fading. This distinction is based on the coherence bandwidth of the channel. The coherence bandwidth is the range of frequencies over which multiple frequencies of a signal experience comparable or correlated fading. Thus, if the signal bandwidth is smaller than the coherence bandwidth all the frequency components of the signal will experience similar fading. This is called flat fading. On the other hand, if signal bandwidth is larger than the coherence bandwidth then some frequency components may experience deeper fade than others; this is called frequency-selective fading. The frequency which will be in deep fade depends on the geometrical and physical characteristics of the system. The flat fading model is considered in this work.

2.7 CHANNEL MODELS

2.7.1 Rayleigh Fading Model. The Rayleigh fading model is a statistical model which can be used to model multipath propagation with no direct line of sight component between the transmitter and the receiver. Due to mechanisms of scattering, reflection and diffraction the signal travels from transmitter to the receiver in multiple paths. If the number of paths is sufficiently large, then the channel response can be modeled as circularly symmetric complex Gaussian in accordance with the Central Limit Theorem. It is known that the envelope of the sum of quadrature Gaussian signals obeys a Rayleigh distribution [8].

A Rayleigh distributed envelope can be statistically described by its probability density function which is given by

\[
p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} & 0 \leq r \leq \infty \\ 0 & r < 0 \end{cases}
\]

where variance \( \sigma^2 = \text{E} \{ r^2 \} \), and \( \text{E} [ . ] \) denotes the expectation operator.
2.7.2 Rician Fading Model. The presence of a strong LOS component violates the assumption of the Rayleigh channel model. In such a scenario, the channel is said to follow Rician distribution statistics. It can be intuitively visualized as a Rayleigh channel with the addition of a strong LOS component. The strength of the LOS component is measured by a value called the K-factor. The Rician channel can be mathematically written as

\[ H = \sqrt{\frac{K}{K+1}} H_{Sp} + \sqrt{\frac{1}{K+1}} H_{Sc} \]  

where \( K \) is the Rician K-factor, \( H_{Sp} \) is the LOS component of channel and \( H_{Sc} \) is the scatter component of the channel. The Rician K factor is a measure of the ratio of power in the LOS component to the power in the scatter component. \( H_{Sp} \) depends on the antenna separation and the angle of arrival or departure of the LOS component. The elements of \( H_{Sc} \) are statistically independent Gaussian random variables [9]. It can be seen that when \( K \rightarrow 0 \), the channel is equivalent to a Rayleigh channel.

2.8 THE ALAMOUTI SCHEME FOR 2X2 SYSTEM

Alamouti proposed a simple scheme for a system with two transmit antennas that could achieve full diversity and had a simple decoding algorithm. In this scheme, the binary data bits are first modulated using an M-ary modulation scheme. A block of two symbols, \( s_0 \) and \( s_1 \) is transmitted from each antenna in the same time slot. In the next time slot, \(-s_1^*\) and \(s_0^*\) is transmitted where \((.)^*\) denotes the conjugate operation. Figure 2.6 illustrates the transmit chain in the Alamouti scheme. The transmit matrix can be mathematically written as

\[ S = \begin{bmatrix} s_0 & -s_1^* \\ s_1 & s_0^* \end{bmatrix} \]
The receiver chain consists of a channel estimator, a combiner and a maximum likelihood detector. Figure 2.7 shows a diagram of the Alamouti scheme. The Alamouti scheme assumes that the data stream generated by the two antennas is independent and there is no knowledge of CSI at the transmitter. This thesis work assumes that there is no available CSI at the receiver as well.
2.9 MATHEMATICAl DESCRIPTION OF ALAMOUTI SCHEME

From equation (9) the transmit matrix, $S$ is a 2x2 Alamouti coded matrix. Let the channel matrix be $H$ which is of order 2x2 since there are two receivers and two transmitters. A 2x2 system has four different paths as shown in Figure 2.3. Each path fades independently and will get corrupted due to noise. Let the 2x2 noise matrix be denoted by $N$. The noise in each of the paths is assumed to be Gaussian distributed with zero mean. The transmit equation for this system can be written as

$$ R = H \ast S + N $$

(10)

The receive matrix, $R$ is a combination of signals received from both the antennas at receiver. Let the elements of $R$ be represented by $r_{ij}$. At the receiver, the process of combining and maximum likelihood detection is carried out to detect the transmitted bits. These require the knowledge of the channel at the receiver. Let the estimated channel be given by $H_{est}$ which is the 2x2 estimated channel matrix. The receiver combining scheme can be written as

$$ s_0^\sim = h_{11}^* r_{11} + h_{12}^* r_{12}^* + h_{21}^* r_{21} + h_{22}^* r_{22}^* $$

(11)

$$ s_1^\sim = h_{12}^* r_{11} - h_{11}^* r_{12}^* + h_{22}^* r_{21} - h_{21}^* r_{22}^* $$

(12)

where $r_{ij}$ are elements of the receive matrix $R$, $h_{ij}$ are elements of the channel matrix $H_{est}$ and $s_i^\sim$ is the $i^{th}$ combined signal output.

Let the channel gain in each path be given by

$$ \alpha_1 = |h_{11}|, \alpha_2 = |h_{12}|, \alpha_3 = |h_{21}|, \alpha_4 = |h_{22}| $$

(13)

The combiner output and channel estimates are input to the maximum likelihood detector which uses the following decision criteria for decoding.
for $s_0$ choose $s_i$ if

$$
(a_1^2 + a_2^2 + a_3^2 + a_4^2 - 1)|s_i|^2 + d^2(s_0^\top, s_i) \leq (a_1^2 + a_2^2 + a_3^2 + a_4^2 - 1)|s_k|^2 + d^2(s_0^\top, s_k) \quad \forall i \neq k
$$

(14)

where $d^2(x,y) = (x-y) (x^* - y^*)$ which is the squared Euclidean distance between $x$ and $y$. Similarly, for $s_1$ choose $s_i$ if

$$
(a_1^2 + a_2^2 + a_3^2 + a_4^2 - 1)|s_i|^2 + d^2(s_1^\top, s_i) \leq (a_1^2 + a_2^2 + a_3^2 + a_4^2 - 1)|s_k|^2 + d^2(s_1^\top, s_k) \quad \forall i \neq k
$$

(15)

### 2.10 CHANNEL ESTIMATION: THE LEAST SQUARES APPROACH

In a practical scenario, usually there is no available channel information at the receiver. Thus, there is a need to estimate the channel. This can be performed in two ways: pilot-symbol-aided channel estimation and blind channel estimation. In pilot-symbol-aided channel estimation, a sequence of data known to the receiver called pilots is transmitted. Based on the received data and the known pilot data the channel estimate is calculated. In the blind channel estimation technique data is unknown to the receiver, but the receiver will have some knowledge about the statistics of the channel.

One of the pilot symbol aided techniques available is the linear least squares method. In this method, based on the transmitted and the received data the maximum likelihood estimate of the channel is calculated.

Suppose the channel matrix is denoted by $H$, the training transmit matrix is $X$ and the noise matrix is $N$, then the system equation can be written as (2). According to the LS algorithm, the channel matrix can be estimated as

$$
H_{ls} = SX^\dagger
$$

(16)

where $X^\dagger$ represents $X^H(XX^H)^{-1}$ and $(.)^H$ denotes the Hermitian transpose.
It is shown in [10] that if the power transmit constraint used is:

$$\|P\|^2 = P_t$$

where $\|\cdot\|$ is the Frobenius matrix norm and $P_t$ is a constant, then the estimate is given by

$$H_{ls} = H + \frac{N_t}{P_t}NX^H$$

with $N_t$ being the number of transmit antennas.

2.11 SPINNING OF VEHICLE

One of the assumptions for all the techniques described above is that the transmitters and the receivers are stationary. But, if the transmitters are mounted on a rotating vehicle there will be a few changes to the system. The most significant change would be in the channel. Due to rotation, the channel gain between the transmitter and the receiver will continuously change depending on the speed of rotation and the angle between the transmitter and the receiver. The performance of the system will depend not only on the spinning, but also the radiation pattern of the antennas. We will look at two different radiation models. The first will be a mathematically simple model of a half cycle of a sinusoid. Then we will examine a more realistic model, where a sample patch antenna radiation pattern is used.

2.11.1 Half-Sine Wave Model. One easy way to model spinning is the ‘half-sine wave model’. This assumes that full power is received by the receiver when the transmitter is facing the receiver, and as it moves away due to rotation the power received degrades as a sinusoidal function until it rotates about 90 degrees after which the receiver does not receive any power since the transmitter points in the opposite direction. As the transmitter rotates again towards the receiver, it will start receiving power based on the sine of the angle between the transmitter and the receiver. Figure 2.8 illustrates this concept. Since the system under consideration in this work contains two transmitting
antennas placed π radians apart, the pattern of the second antenna leads or lags the first antenna by a phase of π radians. Figures 2.9 and 2.10 illustrate the half-sine wave pattern of two antennas for a speed of 1 rotation per 1000 bits when the bit rate of the system is 1kbps.

Figure 2.8: Radiated Power Direction

Figure 2.9: Antenna Gain for Transmitter-1
In this case the actual channel for the system will be the channel for a stationary system multiplied by the values from Figures 2.9 and 2.10.

**2.11.2 Patch Antenna.** A more practical case for modeling the spin of the transmitters would be replacing the values obtained from the sine-wave model by sample radiation pattern of patch antennas. Figure 2.11 shows a sample radiation pattern of patch antennas in Cartesian coordinates. This particular radiation pattern was obtained from [11]. Patch antennas are antennas which radiate maximum power in a certain direction. The radiation pattern is a graph which depicts the amount of power radiated with respect to different angles. They can be shown either in polar or Cartesian coordinates. The radiation pattern curve is shown in Cartesian coordinate. If the three dimensions in polar coordinates are denoted by \((\rho, \theta, \phi)\), then the two curves represent the radiation pattern along \(\theta\) and \(\phi\) directions. It is assumed that the direction of \(\theta\) is parallel to the main axis of symmetry of the vehicle and the direction of \(\phi\) is perpendicular to it. However, for the problem described, for angles further than \(\pi/2\) radians the power received will be zero since the power radiated at those angles would be blocked by the body of the object before reaching the receiver. Figure 2.12 represents this graph.
Figure 2.11: Sample Radiation Pattern of Patch Antenna[11]

Figure 2.12: Radiation Pattern for Proposed Problem
3. SIMULATION DESCRIPTION

The evaluation of performance of a wireless communication system can be done through either field trials or computer-based simulations. Generally before the actual deployment of the system, computer-based simulations are preferred since they are cost effective and convenient. The system is built and evaluated using Matlab. Figure 3.1 shows the block diagram of the system which is simulated.

![Figure 3.1: Simulated System](image)

3.1 SIMULATION OF THE SYSTEM

3.1.1 Source Data Generator. The source data is generated using a uniform pseudorandom number generator. It is generated using the randn function in Matlab. It generates a matrix containing pseudorandom values drawn from a normal distribution having zero mean and unity standard deviation. The data generator is set such that equally likely binary bits [0,1] are produced.

3.1.2 BPSK Modulator. BPSK modulation is a robust form of modulation which is easy to implement and is relatively simpler to decode. As explained in Section 2.4, bit 0 is mapped to -1 and bit 1 is mapped to +1. BPSK data is generated using \( s = 2d - 1 \) where \( d=[0,1] \).
3.1.3 QPSK Modulator. QPSK is a form of modulation for satellite communications and sending data over cables. As explained in Section 2.5 the bits 00, 01, 10 or 11 are mapped onto four different data points. The four data points being \( \cos(n\pi/4) \pm j \sin(n\pi/4) \) where \( n \) takes on values of 1,3,5,7.

3.1.4 Alamouti Encoder and Channel. The modulated data is encoded into Alamouti form before transmission by using the Equation (9). A Rayleigh channel is a complex channel having real and imaginary parts which are both normally distributed with zero mean. The noise in the channel is assumed to be Additive White Gaussian Noise (AWGN) which is also complex and normally distributed. It is implemented without the use of awgn( ) function. Energy per bit, \( E_b \) is set to unity and the power of the noise, \( \sigma_n^2 \) is calculated based on the SNR (\( E_b/No \)). Normally distributed complex random variables with zero mean and variance \( \sigma_n^2 \) is generated to model noise. The effect of spinning on the channel was introduced in Section 2.11 and the model will be explained in Section 3.2.

3.1.5 Channel Estimation and Decoding. Channel estimation is performed using the linear least squares approach. The mathematical description of channel estimation is explained in Section 2.10. The pinv function is used in Matlab to perform the pseudo inverse operation required for channel estimation. The estimated channel and the received signal are sent to the combiner whose output is then sent to the maximum likelihood detector for decoding. The equations for the combiner and maximum likelihood detector are given by equations (11) through (15).

3.1.6 System Performance. The performance metric used to measure the performance of the system is bit-error rate with respect to different SNRs. When measuring bit-error rate it is useful to have error bars which indicate a confidence interval. The confidence interval used for the simulations in this work is 95%. The bit error rate can be measured using the nnz function or the berfit function in Matlab. The nnz function compares two vectors and counts the number of non-zero elements. In this case, the vectors compared are the decoded data and the transmitted data. The berfit function calculates the error based on the number of bits in error and the number of bits transmitted. Also, it performs curve fitting on the bit error rate curve. In case of the nnz function, the bit error rate can be found by dividing the number of bits in error by the
total number of transmitted bits. This step is not required if the berfit function is used. The confidence intervals are measured using the berconfint function.

3.2 MODELING OF SPIN

Spinning will cause the channel to change based on the spin velocity. Initially a half-sine wave model is considered and then readings from a patch antenna are obtained and used to generate the channel with rotating transmitters.

3.2.1 Rotation Based on Half-Sine Wave. A file is created which lists the gain of the channel for a particular antenna angle. This gain is based on the speed of rotation and the frequency of the sinusoidal wave used. The Rayleigh channel will then be multiplied by these gain values. Once the gain pattern for one transmit antenna is created, it is shifted by a phase of $\pi$ radians to obtain the gain pattern for the second transmit antenna. This is because the transmitters are mounted $\pi$ radians away from each other. Figures 3.2 through 3.5 show the gain pattern of the transmit antennas for slow and fast rotations.

![Figure 3.2: Antenna Gain of Transmitter-1 (Slow Rotations)](image-url)
Figure 3.3: Antenna Gain of Transmitter-2 (Slow Rotations)

Figure 3.4: Antenna Gain for Transmitter-1 (Fast Rotations)
3.2.2 Rotation Based on Patch Antenna Radiation Patterns. The readings from the half-sine wave model is replaced by readings from the radiation pattern of a patch antenna. A file is created which contains the gain pattern with respect to the angle of the antenna. Readings of the gain value are taken every $\pi/12$ radians and interpolated to the required resolution using the interp1 function. For the purposes of this work, the power radiated by the antenna when the antenna turns away from the receiver is set to zero although in practice there will be very little amount of power radiated. Further, the pattern is normalized such that the maximum gain value is set to 1. It is modeled such that the system can be tested for any radiation pattern without any additional increase in software complexity. Figures 3.6 and 3.7 show the gain patterns when patch antennas are used for a rotation speed of 5 rot/1000 bits.
Figure 3.6: Antenna Gain for Transmitter-1 (Patch Antenna)

Figure 3.7: Antenna Gain for Transmitter-2 (Patch Antenna)
4. DESCRIPTION OF RESULTS

Initially results were obtained to verify the implementation of the channel and system. These results were compared with existing results. The results shown below are applicable to BPSK modulation. The results for QPSK will be illustrated later.

4.1 BIT ERROR RATE (BER) CURVES

4.1.1 BER of 2x2 System with Known CSI at Receiver. Figure 4.1 shows the bit error rate of a 2x2 system which assumes stationary transmitters and perfect channel information at receiver. It matches well with the results obtained in [6]. A 95% confidence interval is used to generate the error bars.

Figure 4.1: BER of 2x2 System with Stationary Transmitters and Known CSI (BPSK)
4.1.2 BER of 2x2 System with No CSI at Receiver. Figure 4.2 shows the bit error rate of a 2x2 system which assumes stationary transmitters and no channel information at the receiver. It matches well with the results obtained in [10]. A 95% confidence interval is used to generate the error bars.

Figure 4.2: BER of 2x2 System with Stationary Transmitters and no CSI (BPSK)

Figure 4.3 illustrates the error in channel estimation in the linear least squares approach. It shows the error values obtained from simulation and the expected theoretical error. The error obtained from simulation matches the value obtained mathematically in Equation (18).
4.2 CHANNEL ESTIMATE (HALF-SINE MODEL)

Figure 4.4 shows the channel estimate of a half-sine model for a sine wave frequency of $f_b/300$ Hz where $f_b$ is the bit rate of the system which is assumed to be 1000 bits per second or 1 kbps.

It is clear from the figure that there is a periodic component in the channel. The spikes can be seen at values of 100, 400, 700 and so on. This periodic component rises due to rotation of the transmitters. As the frequency of rotation increases, the periodic component will repeat more often. It should be noted that if the frequency is increased to an extremely high value, then the time period for repetition becomes very small and it might again look like a random signal. It should be also noted that a minimum signal to noise ratio is required to see the effect of rotation. Figure 4.5 shows the channel estimate with the same sine wave frequency and with a very low SNR of 1dB.
Figure 4.4: Channel Estimate for Rotating Transmitters at 10dB (Half-Sine)

Figure 4.5: Channel Estimate for Rotating Transmitters at 1dB (Half-Sine)
4.3 CHANNEL ESTIMATE (PATCH ANTENNAS)

Figure 4.6 shows the estimate of the channel gain between the first transmitter and the first receiver for with patch antennas at the transmitters. The rotation speed is assumed to be 20 rot/1000 bits. A periodic component in the channel is clearly visible.

![Estimated channel co-efficient with speed of 20rot/1000 bits at 10 dB](image1)

![Actual channel co-efficient without spin at 10 dB](image2)

Figure 4.6: Channel Estimate for Rotating Transmitters (Patch Antennas)

4.4 BER OF 2X2 SYSTEM WITH ROTATING TRANSMITTERS

4.4.1 BER Comparison for Half-Sine Model. Figure 4.7 shows bit error rate comparison for systems with stationary transmitters with both transmitters radiating power towards the receiver, stationary transmitters with one antenna radiating power towards the receiver and rotating transmitters. The half-sine model is used to simulate spinning.
As depicted in the figure, it is found that spinning causes the performance of the system to degrade. The performance of the system will change depending on the orientation of the antenna, especially for slow rotations. These simulations assume that at least one transmitter is initially pointing in the right direction irrespective of the speed of rotation.

![Figure 4.7: BER Comparison for Half-Sine Model (BPSK)](image)

It can be seen that at extremely slow rotation speeds, the performance of the system is equivalent to a system with one stationary transmitter pointing towards the receiver. The results for a more practical spinning model based on the radiation pattern of patch antenna are shown below.
4.4.2 BER Comparison for Radiation Pattern Model. Figure 4.8 shows the bit error rate comparison when a sample patch antenna radiation pattern is used to model spinning. It is found that the bit error rate of the system degrades by about 4dB when one of the transmitters points directly to the receiver and the system is stationary. When the transmitters rotate at a speed of 20 rotations/1000 bits, there is a further degradation of about 2-3 dB.

![Figure 4.8: BER Comparison for Patch Antennas (BPSK)](image)

From the above figures, it can be inferred that the bit error rate of the system with rotating transmitters is higher for the case of half-sine model as compared to a sample radiation pattern of a patch antenna. From the simulations, it was found that the average antenna gain value for the half-sine model is 31.87%. This means that when the transmitters are rotating at 20 rot/1000 bits and the half-sine model is used for spinning,
the power received from each antenna on an average is 0.3187 times the power received if the both transmitters were stationary and pointing towards the receivers. The average channel gain value for rotating transmitters at a speed of 20 rot/1000 bits and when a sample radiation pattern is used for spinning is 42.32%. This could be the cause for the change in bit-error rates when different models for spinning are used.

**4.4.3 Results for QPSK.** Figures 4.9 through 4.12 illustrate the results for an Alamouti coded system with QPSK modulation. The bit-error rate for Alamouti coding with QPSK is higher than that for BPSK. Therefore, there is a tighter limit on the spin velocity if QPSK is used since the bit-error rate of the system will be higher even for slow rotations. The confidence interval used to generate the error bars is 95%.

![Figure 4.9: BER of 2x2 System with Stationary Transmitters and Known CSI (QPSK)](image-url)
Figure 4.10: BER of 2x2 System with Stationary Transmitters and No CSI (QPSK)

Figure 4.11: BER Comparison for Half-Sine Model (QPSK)
As expected the bit error rate of the system degrades with increasing rotation speed. The bit error rate of the system is higher in QPSK as compared to BPSK. Thus, there is a limitation on the speed of rotation. High rotation speeds might significantly degrade the overall performance of the system.
5. CONCLUSION

The performance of a Multiple-Input Multiple-Output (MIMO) communication system with the transmitters on a spinning vehicle is investigated. It is assumed that no Channel State Information is available at the transmitter or the receiver. The linear least squares channel estimation was used to estimate the channel. It is found that spinning causes a periodic component to occur in the channel which can be predicted based on the spin rate relative to the data rate. It is also found that spinning degrades the bit error rate of the system. Further, it is seen that Alamouti coding with QPSK modulation has a higher bit error rate than Alamouti coding with BPSK modulation.
BIBLIOGRAPHY


VITA

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