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Matching of a horn antenna to its dense surrounding media by the use of artificial dielectric methods

Paul Arthur Ray

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MATCHING OF A HORN ANTENNA TO ITS DENSE SURROUNDING MEDIA BY THE USE OF ARTIFICIAL DIELECTRIC METHODS

BY
PAUL A. RAY

A
THESIS
submitted to the faculty of the
UNIVERSITY OF MISSOURI AT ROLLA
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Rolla, Missouri
1965

Approved by

[Signatures]

[Signatures]
ABSTRACT

The purpose of this thesis is to design an artificial dielectric slug to match a horn in a media such as water and to discuss a method of determining experimentally its dielectric properties.

The design is based on known artificial dielectric methods. The method of determining the dielectric properties is by microwave waveguide reflection techniques.

This thesis deals with the theoretical aspects and suggests an experimental method of determining the practicability of such a matching method.
ACKNOWLEDGEMENTS

The author wishes to acknowledge the special assistance given to him by his advisor, Gabriel G. Sketek, Professor of Electrical Engineering at the University of Missouri at Rolla.

The author is also indebted to Frank Huskey of the Electrical Machine Shop for his help in constructing the experimental equipment.
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<td>$\bar{E}$, $\bar{H}$</td>
<td>General vector electric and magnetic field strength respectively.</td>
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<tr>
<td>$\bar{E}_p$</td>
<td>Dipole field produced by all obstacles in the array.</td>
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<td>$\bar{E}_e$</td>
<td>Effective field acting to polarize the obstacle at the origin.</td>
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<td>$\bar{E}_i$</td>
<td>Interaction field.</td>
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<td>$\bar{E}<em>{pa}$, $\bar{E}</em>{ea}$, $\bar{E}_{ia}$</td>
<td>Fields averaged over the volume of a unit cell (abc).</td>
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<td>$\bar{P}$</td>
<td>Average dipole polarization vector.</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>Dipole moment vector.</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Permittivity.</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Permeability.</td>
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<tr>
<td>$\varepsilon_0$, $\mu_0$</td>
<td>Permittivity and permeability of free space.</td>
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<tr>
<td>$\alpha$</td>
<td>Polarizability constant.</td>
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<tr>
<td>$\Gamma$</td>
<td>Reflection coefficient.</td>
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<tr>
<td>$t$</td>
<td>Transmission coefficient.</td>
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<td>$\gamma$</td>
<td>Propagation constant.</td>
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<td>Intrinsic impedance.</td>
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<td>$\sigma$</td>
<td>Conductivity.</td>
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<td>$\omega$</td>
<td>Radian frequency.</td>
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<td>$\lambda_g$, $\lambda_0$, $\lambda_1$</td>
<td>Waveguide, free space, and region 1 wavelength respectively.</td>
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<td>$Z_x$</td>
<td>Wave impedance in x direction.</td>
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I. INTRODUCTION

The problem of antenna matching has been investigated extensively by a number of people. The type of antenna matching with which these investigators dealt is that of matching the antenna to its feed system. The type of matching that this thesis investigates is that of matching the antenna to its dense surrounding media.

The need for matching of this type has arisen in the last few years. With the coming of the nuclear age, a need of concealing and protecting communication devices has developed. All of the devices, including the transmitting and receiving antennas, are immersed in a media that has dielectric properties other than those of free space.

In the author's investigation of this area, little material could be found which dealt with the antenna matching problem. Most available material deals with immersed long wire antennas at low frequencies (50kc to 400mc). The use of high frequencies will require waveguides and their corresponding radiating systems. The type of radiating devices used are slots or horns.

The laboratory equipment available to the author necessitated the selection of a horn antenna operating at 10kmc. Water was selected as the surrounding media because of the ease of immersing and removal of the horn antenna.
The purpose of this thesis is to design a device that will match a horn antenna to its surrounding media. The procedure for determining the properties of this device will also be discussed.
II. REVIEW OF THE LITERATURE

Literature Concerning Artificial Dielectrics

The subject of artificial dielectrics had its beginning in the mid 1940's. Kock(1), in the period 1944-1948, suggested the use of artificial dielectrics to replace the actual dielectric in order to overcome their major disadvantage. This disadvantage is the vast weight and mass of solid dielectrics.

In the following years, various authors contributed to this area; those who performed the major work are Carlson and Heins(2) in 1947, Brown(3) in 1950, Cohn(4) in 1949, Susskind(5) in 1952, and El-Kharadly and Jackson(6) in 1953. The reference, upon which most of the first part of this thesis is based, is by Collin(7).

Literature Concerning Measurement of Dielectric Properties

In the microwave range, ordinary dielectric measurement techniques become inferior. The macroscopic size of the wavelength proves of great advantage if standing-wave methods are applied, because a detector may travel directly through the profile of the wave pattern. Drude's(8) two classical methods utilize this possibility and have since been employed in many variations. These variations are too numerous to mention. Later, open transmission lines (Lecher systems(9)) were used but they had several handicaps. An empirical calibration of the condenser system was required, and
extreme care had to be taken to avoid perturbation of the waves by the detector system. These limitations have been overcome by enclosing the electromagnetic field in wave guides\(^{(10)}\). The type of wave-guide equipment still used today was developed for research during World War II\(^{(11)}\). The references, upon which most of this thesis is based, are A. von Hippel\(^{(12,13)}\).
III. DESIGN OF AN ARTIFICIAL DIELECTRIC SLUG

Introduction

The permittivity $\varepsilon$ of the slug to be designed will vary gradually with distance within the horn to produce a match between the feed and the media. This will necessitate the use of a variable dielectric material.

A major disadvantage of solid dielectric materials is their excessive bulk and weight. This disadvantage has been overcome by the use of artificial dielectrics. The properties of an artificial dielectric can also be more easily controlled.

An artificial dielectric is a large-scale model of an actual dielectric, obtained by arranging a large number of identical conducting obstacles in a regular three-dimensional pattern. The obstacles are supported by a lightweight binder or filler material.

A dielectric material can react to an electric field because it contains charge carriers that can be displaced. Under the action of an externally applied electric field, the charges on each conducting obstacle are displaced so as to set up an induced field which will oppose and thus cancel the applied field at the surface of the obstacle. This produces an electrically neutral obstacle which results in a dipole field. Each obstacle resembles a molecule in a regular dielectric in that it exhibits a dipole moment. The resultant effect of all the obstacles
is to produce a net average dipole polarization $\mathbf{P}$ per unit volume. The interpretation of $\mathbf{P}$ provides a transition from the macroscopic view to the molecular view. The dipole moment per unit volume can be thought of as resulting from the additive action of $n$ elementary dipole moments $\mathbf{p}$. Thus

$$\mathbf{P} = n\mathbf{p}$$

(1)

The average dipole moment $\mathbf{p}$ of the particle may be assumed to be proportional to the local electric field $\mathbf{E}'$ that acts on the particle. Therefore,

$$\mathbf{p} = \alpha \mathbf{E}'$$

(2)

The proportionality factor $\alpha$, called polarizability, measures the electric pliability of the particle, that is, the average dipole moment per unit field strength.

If $\mathbf{E}$ is the average net field in the medium, the dipole polarization $\mathbf{P}$ is related to $\mathbf{E}$ as follows:

$$\mathbf{P} = \varepsilon \mathbf{E} - \varepsilon_0 \mathbf{E} = (\varepsilon - \varepsilon_0) \mathbf{E}$$

(3)

Equations (1), (2), and (3) are three alternative expressions for the polarization, linking the macroscopically measured permittivity $\varepsilon$, to three molecular parameters: the number $n$ of contribution particles per unit volume; their polarizability $\alpha$; and the local acting electric field $\mathbf{E}'$. This field $\mathbf{E}'$ will normally be different from the applied field $\mathbf{E}$ due to the polarization of the surrounding dielectric medium.

In the next section a relationship will be formulated relating the dielectric constant as a function of the
polarizability, number of conducting obstacles and an interaction constant C which results from \( \bar{E} \). In this section there will be some discussion on the polarizability \( \alpha \) and the interaction constant C, but most of it will be concerned with the number of conducting obstacles N. The reason for this is that the design criteria will be based on the number of conducting obstacles placed in a given volume. The actual value of these parameters will not be needed because a method of determining the dielectric constant, as a function of the number of conducting obstacles N, will be discussed in the latter part of this thesis. All that is required for the design is that the dielectric constant varies with the number of conducting obstacles N.

In the preceding discussion, nothing was mentioned about the magnetic properties of the artificial dielectrics. Obstacles which have magnetic dipole moments, will have induced magnetic dipoles that oppose the inducing fields. Therefore, in general, an artificial dielectric exhibits magnetic dipole polarization as well as electric dipole polarization. In the following discussion, obstacles with magnetic dipole moments will be neglected. All discussion will be limited to symmetrical configurations (isotropic properties).
Theoretical Analysis

The term artificial dielectric is commonly used to incorporate structures such as parallel-plate arrays, stacks of square wave guides placed side by side, and structures made up of arrays of discrete obstacles. The following analysis will be limited to discrete obstacles.

Typical structures used for artificial dielectrics are illustrated in Figure 1.

(a) three-dimensional sphere  (b) three-dimensional disk

(c) two-dimensional strip  (d) two-dimensional rod

Fig. 1. Artificial Dielectric Structures.
When the electric field is perpendicular to the rods in (d) above, capacitive loading is produced, and $\varepsilon$ is increased. This method of increasing $\varepsilon$ will be used in the design criteria.

There are three basic approaches used in the analysis of artificial dielectrics. They are: the Lorentz theory, which is a classical theory; the Electrostatic Solution, which is not always easy to obtain; a Maxwell solution, which is based on the solution of Maxwell's equations. The approach to be used in this analysis will be the Lorentz theory.

According to Brown and Jackson (14), "This theory has a limited range of application, but reasonable agreement with experimental results is obtained if allowance is made for the short-range interaction forces between nearby conductors." The Lorentz theory considers only dipole interaction between obstacles and produces an accurate result when spacings are approximately $0.1 \lambda_0$. Since the wavelength involved in this analysis is about 4.6 cm, this theory can be used to yield a high degree of accuracy (less than 5% error).

Consider a three-dimensional regular array of identical conducting obstacles, as in Fig. 2. Apply an external electrostatic field $\bar{E}_0$ in the $y$ direction. Assume the obstacles to have their principal axes of polarization coincident with the coordinate axis. Then the induced
dipole moment \( \vec{p} \) of one obstacle is in the y direction for an applied field in the y direction.

\[ \vec{p} = \vec{a} \varepsilon_0 E_y \]

For nonsymmetrical obstacles, \( \varepsilon_e \) is different along different directions, and in general is a tensor quantity. The number \( N \) of obstacles per unit volume is equal to \( (abc)^{-1} \), therefore, the dipole polarization per unit volume is
given by

\[ \overline{P} = N \overline{P} = \overline{P}_{abc} = N a_0 \varepsilon_0 \overline{E}_{ey} \]  
(5)

The effective polarizing field \( \overline{E}_{ey} \) is

\[ \overline{E}_{e} = \overline{E}_0 + \overline{E}_p - \overline{E}_1 \]  
(6)

but \( \overline{E}_1 \) is

\[ \overline{E}_1 = \overline{E}_p - \overline{E}_1 \]  
(7)

therefore,

\[ \overline{E}_{ey} = \overline{E}_0 + \overline{E}_{iy} \]  
(8)

The interaction field \( \overline{E}_{iy} \) is proportional to \( \overline{P} \), consequently,

\[ \overline{E}_{iy} = \frac{\overline{CP}}{\varepsilon_0} \]  
(9)

where \( C \) is called the interaction constant. But substituting (9) into (4) and (5), we obtain

\[ \overline{P} = a_0 \varepsilon_0 (\overline{E}_0 + \overline{CP}) \]  
(10)

or

\[ \overline{P} = \frac{a_0 \varepsilon_0 \overline{E}_0}{1 - a_0 C} \]  
(11)

The dipole polarization per unit volume \( \overline{P} \) may be expressed as,

\[ \overline{P} = \frac{N a_0 \varepsilon_0 \overline{E}_0}{1 - a_0 C} \]  
(12)
The y component of the average displacement flux density is defined by

\[ \overline{D_{ay}} = \varepsilon_0 \overline{E_{ay}} + \overline{P} \]  

(13)

Also

\[ \overline{D_{ay}} = \varepsilon \overline{E_{ay}} = k \varepsilon_0 \overline{E_{ay}} \]  

(14)

From (13) and (14), the following expression is obtained.

\[ k = 1 + \frac{\overline{P}}{\varepsilon_0 \overline{E_{ay}}} = 1 + \frac{\overline{P}}{\varepsilon_0 (\overline{E_0} + \overline{E_{pay}})} \]  

(15)

For symmetrical obstacles about the coordinate planes passing through the center of the obstacle, the average field produced by all the induced dipoles is zero. With the applied field in the y direction, conducting plates may be inserted into the array at \( y = \pm b/2 + mb \), \( m = 0, \pm 1, \pm 2, \ldots \) without disturbing the field distribution.

If \( \alpha E_0 \) is the applied field, the potential at \( y = \pm b/2 \) may be chosen as \( -\varepsilon \alpha b \) and \( +\varepsilon \alpha b \) as in Fig. 3. Each unit volume will have a potential \( \phi \) that can be developed into a three-dimensional Fourier series. The applied potential for the unit volume at the origin is given by

\[ \phi_0 = -E_0 y. \]  

(16)

The induced potential \( \phi_i \) will be an even function of \( x \) and \( z \), and an odd function with \( y \). Thus \( \phi_i \) can be expressed as

\[ \phi_i = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} A_{nms} \cos \frac{2n\pi x}{a} \cos \frac{2s\pi z}{c} \sin \frac{2m\pi y}{b} \]  

(17)
The \( y \) component of the induced field is written as

\[
E_{py} = - \frac{\partial \phi_i}{\partial y} = - \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} A_{nms} \frac{2\pi n}{b} \cos \frac{2\pi s}{c} \cos \frac{2\pi b y}{b} \tag{18}
\]

This corresponds to the total field produced by all of the obstacles. The average induced field in a unit volume has a zero average because the cosine terms have zero average value.

Because of the above discussion, \( E_{pay} \) can be set equal to zero in (15). Thus,

\[
k = 1 + \frac{\overline{F}}{\varepsilon_0 \overline{E}_0} \tag{19}
\]

Substituting for \( \overline{F} \), we obtain

\[
k = 1 + \frac{N \alpha \varepsilon_0 \overline{E}_0}{\varepsilon_0 \overline{E}_0 (1 - \alpha e C)} \tag{20}
\]
which yields

\[
k = 1 + \frac{N \alpha_e}{1 - \frac{\alpha_e c}{e}}
\]  

(21)

This expression is known as the Clausius-Massotti relation. This expresses the dielectric constant as a function of the obstacle polarizability \( \alpha_e \), the interaction constant \( C \), and the number of obstacles \( N \). As stated in the introduction, the obstacles are immersed in a low-dielectric constant filler. Equation (21) is the dielectric constant relative to the dielectric constant of the filler material.

The main reason for the preceding analysis is to show that the dielectric constant varies with the number of conducting obstacles \( N \).

**Experimental Design of Slug to be Placed in the Horn Antenna.**

The design of the slug will be based on the fact that the dielectric constant can be made to vary from the feed end of the horn to the media. The techniques of designing a variable dielectric constant slug (as a function of distance) will make use of the expressions established in the theoretical analysis.

The type of horn antenna that is to be matched operates in the X band at approximately 10kmc. In Fig. 4 the dimensions are given on the top and side views of the antenna.
The dielectric constant of water is approximately 55 at 25°C. The dielectric constant of air is 1, therefore, \( \varepsilon \) will have to be increased (in a manner which will be discussed later) in order to match the antenna. As stated previously, the obstacles will have to be perpendicular to the electric fields, in order to increase \( \varepsilon \). Also, for the analysis in the previous section to coincide, the obstacle (rods in this case) will have to be as in Fig. 5. The cross-sectional view now coincides with that of Fig. 2.

![Fig. 4. Top and Side View of Horn Antenna](image)
The size of the holes that are drilled in the slug will have to be small compared with the spacings. The spacings are limited to $0.1 \lambda_0$ or about 0.46cm. This will necessitate the use of small drill bits. In order for small holes to be drilled, a material will have to be selected which is tightly packed (nonporous) such as polystyrene. The material chosen should have a dielectric constant close to one. Styrofoam, which has a $k$ of 1.03, was chosen by the author and tested. The type of conducting obstacles used in the drilled holes was water. The results will be discussed in the conclusion.

Fig. 5. Top View of Antenna.

Fig. 6. Cross-sectional View of Antenna.
A method of increasing $\varepsilon$ by increasing the number of obstacles will now be discussed. The cross-section of the slug should be divided into a number of equal sections as in Fig. 7.

![Cross-section diagram](image)

**Fig. 7. Cross-section**

Next a distribution (straight-line, logarithmic, etc.) should be chosen which describes the variation of $\varepsilon$ in the range $1 \leq \varepsilon \leq 55$ with distance. A sample of the material used for the slug should be prepared in order that it may be placed in the waveguide and its properties determined. The sample should consist of a 1" x $\frac{1}{2}$" x 1" block of material with holes drilled in it. This size was chosen because of the dimensions of the waveguide. Whatever conducting material is chosen the obstacle will be in the holes. The dielectric constant is found for each different hole distribution. A graph is plotted of the dielectric constant $\varepsilon$ versus the number of holes as in Fig. 8. Given the $\varepsilon$ needed in the chosen distribution, the number of holes that has to be drilled per unit volume can be found from this
graph. The actual procedure for determining the dielectric constant will be discussed in the next section.

![Graph of ε (the permittivity) versus N (the number of holes per unit volume)](image)

Fig. 8. Graph of ε (the permittivity) versus N (the number of holes per unit volume)

After the slug has been prepared, it should then be placed in the mouth of the antenna and tested. The types of experiments that might be used are: the field strength \(E\) some distance from the antenna, the effect in the input impedance, the effect on band-width. In Fig. 9, the block diagram of the equipment to be used is illustrated. Both the transmitting and receiving system are placed in a water filled tank which is 11.5 feet in diameter.

![Block diagram of transmit and receive systems](image)

Fig. 9. (a) Transmitting system. (b) Receiving system.
These experiments should be performed on each distribution chosen. A table might be made listing the effects opposite the given distribution. A comparison can be made between the different distributions and the better one chosen.
IV. DISCUSSION OF THE MEASUREMENT OF DIELECTRIC PROPERTIES

Introduction

In the laboratory, electromagnetic waves are confined by various types of boundaries. This might be looked upon as a disadvantage, but the reflection and refraction of fields by conductors or dielectrics provide the means for guiding and alternating the fields and for measuring the dielectric properties of media.

Theoretical Analysis

The incident, reflected, and refracted waves are "tied together" by the boundary condition which states that the tangential component of $\mathbf{E}$ and $\mathbf{H}$ must be continuous in traveling from one media to another. The amplitudes as well as the phases must be continuous. The first condition (amplitudes are continuous) leads to Fresnel's equation. The second condition (phases being continuous) introduces Snell's laws of reflection and refraction. Snell's laws and Fresnel's equations together determine the intensity, direction, and polarization of the reflected and refracted waves as a function of the properties of the two adjoining media.

When dealing with reflection and refraction characteristics, the relative amplitude and intensities of the waves are of importance, not the absolute values.
The ratio of the reflected amplitude to the incident amplitude at the boundary is defined as the reflection coefficient $r$.

\[
\begin{align*}
\Gamma_E & \triangleq \frac{E_1}{E_0}, & \Gamma_H & \triangleq \frac{H_1}{H_0}.
\end{align*}
\]

(22)

The subscript 1 indicates the reflected wave and the subscript 0 indicates the incident wave. By defining the ratio of the amplitudes of the transmitted to the incident as the transmission coefficient $t$, we have

\[
\begin{align*}
\tau_E & \triangleq \frac{E_2}{E_0}, & \tau_H & = \frac{H_2}{H_0}.
\end{align*}
\]

(23)

The subscript 2 indicates the transmitted wave. After establishing the above relationships, one of Fresnel's equations can be formulated as follows:

\[
\Gamma_E^p = \left(\frac{E_1}{E_0}\right)^p = \frac{n_2 \cos \theta_t - n_1 \cos \theta_i}{n_2 \cos \theta_t + n_1 \cos \theta_i}.
\]

(24)

This is for the case of the components parallel to the plane of incidence (indicated by subscript $p$). The angles are described in Fig. 10.

Fig. 10. Angle Conditions at Boundary.
In the above equation, \( \eta_1 \) and \( \eta_2 \) are the intrinsic impedances of media 1 and 2.

The angle of incidence \( \theta_i \) and the angle of refraction (or transmission) \( \theta_t \) are interrelated by Snell's refraction law:

\[
\sin \theta_i = \frac{\eta_2}{\eta_1} \sin \theta_t \tag{25}
\]

where \( \gamma_1 \) and \( \gamma_2 \) are the propagation constants of media 1 and 2 respectively, or in another form

\[
\frac{\sin \theta_t}{\sin \theta_i} = \sqrt{\frac{\varepsilon_1 \mu_1}{\varepsilon_2 \mu_2}} \tag{26}
\]

In order to simplify Fresnel's equation, perpendicular incidence is assumed \( (\theta_i = \theta_t = 0) \), therefore Fresnel's equation becomes

\[
\Gamma_E = \frac{n_2 - \eta_1}{n_2 + \eta_1} . \tag{27}
\]

The incident and reflected waves superpose and form by interference a standing-wave pattern. This resulting standing-wave pattern will help in determining the dielectric properties of the media.

Let \( \vec{E}_y \) and \( \vec{H}_z \) propagate in the positive x direction through medium 1 and strike the boundary of medium 2 at \( x = 0 \) as shown in Fig. 11. The reflected wave returning in the negative x direction combines with the forward wave to yield the resulting standing wave. The \( \vec{E} \) and \( \vec{H} \) fields of the standing wave pattern are expressed as follows:
\[
E_{y1} = E_0 e^{(j\omega t - \gamma_1 x)} + E_1 e^{(j\omega t + \gamma_1 x)} \\
H_{z1} = H_0 e^{(j\omega t - \gamma_1 x)} + H_1 e^{(j\omega t + \gamma_1 x)}
\]

(28)

Now let \( \Gamma_0 \) and \(-\Gamma_0\) be the reflection coefficients of the electric and magnetic waves at the boundary \( x = 0 \). Expressing the resulting fields we have
\[
E_{y1} = E_0 e^{(j\omega t - \gamma_1 x)} + \Gamma_0 E_0 e^{(j\omega t + \gamma_1 x)} \\
H_{z1} = H_0 e^{(j\omega t - \gamma_1 x)} - \Gamma_0 H_0 e^{(j\omega t + \gamma_1 x)}
\]

(29)

\[
E_{y1} = E_0 e^{j\omega t [e^{-\gamma_1 x} + \Gamma_0 e^{\gamma_1 x}]} \\
H_{z1} = \frac{E_0}{n_1} e^{j\omega t [e^{-\gamma_1 x} - \Gamma_0 e^{\gamma_1 x}]}
\]

(30)

Fig. 11. Formation of Standing Waves.
Assume, for the present, that the wave in media 2 continues without further interference. Thus,

\[
E_{y2} = E_2 e^{(j\omega t - \gamma_2 x)}
\]

\[
H_{z2} = H_2 e^{(j\omega t - \gamma_2 x)} \quad \text{or} \quad H_{z2} = \frac{E_2}{\eta_2} e^{(j\omega t - \gamma_2 x)}
\]

The reflection \( \Gamma_0 \) for normal incidence is given by

\[
\Gamma_0 = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}
\]

It can be seen from the above equation that if the intrinsic impedances of the two media are equal, there will be no reflection. Therefore,

\[
\eta_1 = \eta_2
\]

and

\[
\frac{\varepsilon_1^*}{\mu_1^*} = \frac{\varepsilon_2^*}{\mu_2^*} \quad \text{or} \quad \frac{\varepsilon_1^*}{\mu_1^*} = \frac{\varepsilon_2^*}{\mu_2^*}
\]

where * indicates a complex quantity. This condition requires that both the electric and magnetic flux densities must change by the same amounts in traveling across the boundary between the two media.

The opposite extreme, total reflection, requires that one intrinsic impedance must be much higher than the other impedance. This can be accomplished by using a metal for one of the media. The intrinsic impedance of a metal (magnetic loss neglected) may be written as
\[
\eta_2 = \sqrt{\frac{\mu_2^*}{\varepsilon_2^*}} = \frac{j\omega \mu_2^*}{\gamma_2} \quad \text{where} \quad \gamma_2 = j\omega \sqrt{\varepsilon_2^* \mu_2^*}
\]

(34)

\[
\eta_2 = \sqrt{\frac{\mu_2}{-j\varepsilon_2^*}} = \sqrt{\frac{j\mu_2}{\varepsilon_2^*}} \quad \text{but} \quad \varepsilon_2^* = \frac{\sigma_2}{\omega}
\]

(35)

Because of the high conductivity \(\sigma_2\), \(\eta_2 \ll \eta_1\), the reflection coefficient at the metal boundary becomes

\[
\Gamma_0 = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1
\]

(36)

Now assume that media 1 is loss-free. Then

\[
\gamma_1 = j\omega \sqrt{\varepsilon_0 \mu_0} = \frac{j\omega}{c} = j \frac{2\pi f}{\lambda_1} = j \frac{2\pi}{\lambda_1}
\]

(37)

Under the above conditions Eqs. (29) become

\[
E_{y_1} = -j2E_0 e^{j\omega t} \sin \frac{2\pi x}{\lambda_1}
\]

(38)

\[
H_{z_1} = \frac{2E_0}{\eta_1} e^{j\omega t} \cos \frac{2\pi x}{\lambda_1}
\]

\[
\sin \frac{2\pi x}{\lambda_1} = \frac{e^{j2\pi x} - e^{-j2\pi x}}{2j}
\]

(39)

\[
\cos \frac{2\pi x}{\lambda_1} = \frac{e^{j2\pi x} + e^{-j2\pi x}}{2}
\]

Now by changing over to the real field components we obtain

\[
\text{Re}(E_{y_1}) = 2E_0 \sin \omega t \sin \frac{2\pi x}{\lambda_1}
\]

(40)

\[
\text{Re}(H_{z_1}) = \frac{2E_0}{\eta_1} \cos \omega t \cos \frac{2\pi x}{\lambda_1}
\]
These equations represent a standing wave for which the electric field has its nodes at the metal boundary and at a distance which is given by

$$\sin \frac{2\pi x}{\lambda_1} = 0$$

$$\frac{2\pi x}{\lambda_1} = n\pi \quad x = \frac{n\lambda_1}{2} \quad (41)$$

In this case, $x$ is the distance from the metal surface as shown in Fig. 13.

The antinodes of $\vec{E}$ are located at the following points:

$$\sin \frac{2\pi x}{\lambda_1} = \frac{m\pi}{2} \quad m = (2k - 1)$$

$$x = \frac{m\lambda_1}{4} \quad k = 1, 2, 3, 4, \ldots \quad (42)$$

where $x$ is the distance explained above. The nodes and antinodes for the magnetic field are in reverse order.

The ratio of a component of $\vec{E}$ to a component of $\vec{H}$ is called the wave impedance in the direction defined by the cross-product rule as applied to the two components. Thus, continuity of tangential $\vec{E}$ and $\vec{H}$ requires that wave impedances normal to a material boundary must be continuous.

Using the symbol $Z_x$ to represent the wave impedance in the $x$ direction, we have

$$Z_x = \frac{E_y}{H_z}$$

The ratio of the two fields at the boundary ($x = 0$), is
given by

\[ Z_x(0) = \frac{E_y(0)}{E_z(0)} = \eta_1 \frac{E_0 e^{j\omega t} [e^{-\gamma_1(0)} + \Gamma_0 e^{\gamma_1(0)}]}{E_0 e^{j\omega t} [e^{-\gamma_1(0)} - \Gamma_0 e^{\gamma_1(0)}]} \] (43)

\[ Z_x(0) = \eta_1 \frac{1 + \Gamma_0}{1 - \Gamma_0} \] (44)

The wave impedance \(Z_x(0)\) can be found by measuring the amplitude \(|\Gamma_0|\) and the phase angle \(2\psi\) of the reflection coefficient \(\Gamma_0\).

By measuring the distance \(x_0\) of the first minimum from the boundary of the dielectric(12), the phase angle is given by

\[ 2\psi = 4\pi \left( \frac{1}{4} - \frac{x_0}{\lambda_1} \right) \] (45)

The ratio of the electric to the magnetic field strength at this minimum is expressed as

\[ \frac{E_{\text{MIN}}}{H_{\text{MAX}}} = \eta_1 \frac{1 - |\Gamma_0| e^{-2\alpha_1 x_0}}{1 + |\Gamma_0| e^{-2\alpha_1 x_0}} \] (46)

Since media 1 was assumed loss-free, \((\alpha_1 = 0)\) equation (46) reduces to

\[ \frac{E_{\text{MAX}}}{E_{\text{MIN}}} = \frac{1 + |\Gamma_0|}{1 - |\Gamma_0|} \] (47)

This ratio is called the voltage standing-wave ratio (VSWR).

By measuring the VSWR and the distance to the first minimum \((x_0)\), the magnitude and phase of the reflection coefficient may be determined. After the reflection
coefficient is known, the wave impedance $Z_x(0)$ is found providing the intrinsic impedance $\eta_1$ is known.

From the previous discussion on wave impedances, it is clear that the wave impedance for normal incidence is the same whether traveling toward the boundary or away from the boundary. Thus,

$$Z_x(0) = \frac{E_{y_1}(0)}{H_{z_1}(0)} = \frac{E_{y_2}(0)}{H_{z_1}(0)} \quad (48)$$

If no further reflection takes place in medium 2, the wave impedance measured by the VSWR in medium 1 is directly equal to the intrinsic impedance of medium 2. Therefore,

$$Z_x(0) = \eta_2$$

Medium 2 will, in general, not be of infinite length, but will consist of a layer of thickness $d$, followed by medium 3. Additional reflections will occur at the boundary 2-3 and a part of this reflected energy will return to medium 1 as described by Fig. 12.

![Fig. 12. Wave Reflection on Dielectric Layer.](image-url)
This complex situation will not effect the measurement of the standing-wave pattern in medium 1, but will change the requirements for the reflection coefficient $\Gamma_0$ and the wave impedance $Z_x(0)$. Before going on with this discussion, the wave impedance $Z_x(0)$ will be reformulated in terms of the directly measurable quantities, inverse VSWR, $x_0$, and $\lambda_1$, in order to arrive at a usable expression for actual calculations. Let

$$\Gamma_0 = e^{-2u}$$

where

$$u = p + j\psi$$

then the wave impedance is given by

$$Z_x(0) = n_1 \left( \frac{1 + \Gamma_0}{1 - \Gamma_0} \right) = n_1 \frac{1 + e^{-2u}}{1 - e^{-2u}}$$  \hspace{1cm} (49)$$

$$Z_x(0) = n_1 \frac{e^{-u}(e^u + e^{-u})}{e^{-u}(e^u - e^{-u})}$$ \hspace{1cm} (50)$$

$$Z_x(0) = n_1 \frac{e^u + e^{-u}}{2} = n_1 \frac{\cosh u}{\sinh u} = n_1 \coth u$$  \hspace{1cm} (51)$$

and the inverse VSWR becomes

$$\frac{E_{\text{MIN}}}{E_{\text{MAX}}} = \frac{1 - e^{-2p}}{1 + e^{-2p}} = \tanh p$$  \hspace{1cm} (52)$$

By expanding coth $u$ and using the relation for $\psi$ and $x_0$, the wave impedance can be expressed (This development may be found in the Appendix.) in terms of measurable quantities.
The wave impedance \( Z(0) \) is now determined experimentally by measurements of the VSWR pattern in medium 1.

In order to obtain from the previous equation the properties of medium 2, the situation at the boundary 2-3 at \( x = d \) has to be considered.

Due to the reflection at the boundary 2-3, there will be a standing wave set up in medium 2. The fields are given by

\[
E_{y_2} = E_2 e^{j\omega t} [e^{-\gamma_2 (x-d)} + \Gamma_{23} e^{\gamma_2 (x-d)}] \\
H_{z_2} = \frac{E_2}{\eta_2} e^{j\omega t} [e^{-\gamma_2 (x-d)} - \Gamma_{23} e^{\gamma_2 (x-d)}]
\]

(54)

The wave impedance in the negative \( x \) direction at \( x = 0 \) \( Z_{-x}(0) \) is expressed as

\[
Z_{-x}(0) = \frac{E_2(0)}{H_2(0)} = \eta_2 \frac{e^{\gamma_2 d} + \Gamma_{23} e^{-\gamma_2 d}}{e^{\gamma_2 d} - \Gamma_{23} e^{-\gamma_2 d}}
\]

(55)

If the transmitted wave continues in medium without interference, then the reflection coefficient at boundary 2-3 becomes

\[
\Gamma_{23} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2}
\]

(56)
Now, if the medium 3 is metal, total reflection will occur at $x = d$. Thus,

$$r_{23} = -1$$

Substituting the above result into Eq. (55), the wave impedance $Z_x(0)$ becomes

$$Z_x(0) = n_2 \frac{e^{\gamma_2 d} - e^{-\gamma_2 d}}{e^{\gamma_2 d} + e^{-\gamma_2 d}}$$

$$Z_x(0) = n_2 \tanh \gamma_2 d$$

This will be called the "short-circuit measurement" of medium 2.

This one measurement will not suffice for all media, therefore, another measurement must be performed. In this case, the shorting metal plate will be placed a distance $\Delta$ behind $d$ as shown in Fig. 13.

![Fig. 13. Arrangement for Measuring a Dielectric 2.](image)

Due to the reflection at boundary 2-3, a standing wave pattern will be formed in medium 3. These fields are expressed as
\[
E_{Y_3} = E_3 e^{j\omega t} [e^{-\gamma_3 (x-d-\Delta)} + e^{\gamma_3 (x-d-\Delta)}]
\]
\[
H_{z_3} = \frac{E_m}{n_3} e^{j\omega t} [e^{-\gamma_3 (x-d-\Delta)} - e^{\gamma_3 (x-d-\Delta)}]
\]

As stated before, the wave impedance at \( x = d \) is the same in either direction (-x or +x). Thus,

\[
Z_x(d) = Z_{-x}(d)
\]  
(59)

\[
\frac{E_{Y_2}}{H_{z_2}} = \frac{E_{Y_3}}{H_{z_3}}
\]  
(60)

\[
\eta_2 \frac{1 + \Gamma_{23}}{1 - \Gamma_{23}} = \eta_3 \frac{e^{\gamma_3 \Delta} - e^{-\gamma_3 \Delta}}{e^{\gamma_3 \Delta} + e^{-\gamma_3 \Delta}}
\]  
(61)

\[
\Gamma_{23} = \frac{\eta_3 (1 - e^{-2\gamma_3 \Delta}) - \eta_2 (1 + e^{-2\gamma_3 \Delta})}{\eta_3 (1 - e^{-2\gamma_3 \Delta}) + \eta_2 (1 + e^{-2\gamma_3 \Delta})}
\]

Now if \( \Delta \) is made equal to \( \frac{\lambda_3}{4} \) (quarter wave length), this measurement will be called an "open-circuit measurement." Assume that medium 3 is loss-free (\( \gamma_3 = j\frac{2\pi}{\lambda_3} \)).

From the above equation, \( \Gamma_{23} \) then becomes equal to 1.

The wave impedance \( Z_x(0) \) becomes

\[
Z(0) = \eta_2 \coth \gamma_2 d
\]  
(62)

The ratio of input impedance at \( x = 0 \) to the intrinsic impedance is given in Eq. (53). Thus,
\[
\frac{Z(0)}{\eta_1} = \frac{E_{\text{MIN}}}{E_{\text{MAX}}} - j \tan \frac{2\pi x_0}{\lambda_1}
\]

When medium 2 is terminated in a shorting plate \( (\Delta = 0) \) the impedance becomes

\[
Z(0) = \eta_2 \tanh \gamma_2 d.
\]  

(64)

Dividing by \( \eta_1 \) and transposing gives

\[
\frac{\eta_2}{\eta_1} \tanh \gamma_2 d = \frac{Z(0)}{\eta_1}
\]  

(65)

The relationship between the intrinsic impedance and the propagation factor is

\[
\eta = \frac{j \omega \mu_0}{\gamma} \quad (\mu^* = \mu_0)
\]

Now

\[
\frac{\eta_2}{\eta_1} = \frac{\gamma_1}{\gamma_2}
\]

Substituting this into Eq. (65) and dividing by \( d \) gives

\[
\frac{\tanh \gamma_2 d}{\gamma_2 d} = \frac{Z(0)}{\gamma_1 d \eta_1}
\]

(67)

If medium 1 is loss-free \((\gamma_1 = j \frac{2\pi}{\lambda_1})\), this reduces to

\[
\frac{\tanh \gamma_2 d}{\gamma_2 d} = -j \frac{\lambda_1}{2\pi d} \frac{Z(0)}{\eta_1}
\]  

(68)

When \( \Delta \) is equal to a quarter wavelength of medium 3, the open-circuit condition results. Therefore,

\[
Z(0) = \eta_2 \coth \gamma_2 d
\]

(69)
and again for a loss-free medium 1, we have

\[
\frac{\coth \gamma_2 d}{\gamma_2 d} = \frac{-j \lambda_1}{2\pi d} \frac{Z(0)}{n_1}
\] (70)

Equations (68) and (70) are expressions relating the unknown \( \gamma \) to the measured quantities \( d, \lambda_1, E_{\text{MIN}}/E_{\text{MAX}} \) and \( x_0 \). Equation (68) is for the sample mounted in front of a shorting plate and equation (70) is for the sample a quarter wavelength away. One stipulation on these expressions is that the bounding conductor walls are uniform throughout the three media. Equations (68) and (70) may be rewritten as follows:

\[
\frac{\tanh \gamma_2 d}{\gamma_2 d} = \frac{E_{\text{MIN}}}{E_{\text{MAX}}} - j \tan \frac{2\pi x_0}{\lambda_1} \frac{E_{\text{MIN}}}{E_{\text{MAX}}} \tan \frac{2\pi x_0}{\lambda_1} - j \frac{E_{\text{MIN}}}{E_{\text{MAX}}} \tan \frac{2\pi x_0}{\lambda_1}
\] (71)

\[
\frac{\coth \gamma_2 d}{\gamma_2 d} = \frac{-j \lambda_1}{2\pi d} \frac{E_{\text{MIN}}}{E_{\text{MAX}}} - j \tan \frac{2\pi x_0}{\lambda_1} \frac{E_{\text{MIN}}}{E_{\text{MAX}}} \tan \frac{2\pi x_0}{\lambda_1} - j \frac{E_{\text{MIN}}}{E_{\text{MAX}}} \tan \frac{2\pi x_0}{\lambda_1}
\]

These equations may be expressed in polar form:

\[
\frac{\tanh Te^{j\tau}}{Te^{j\tau}} = Ce^{j\xi}
\]

\[
\frac{\coth Te^{j\tau}}{Te^{j\tau}} = Ce^{j\xi}
\] (72)

A survey map has been drawn by S. Roberts(10) using the argument \( \tau \) as the ordinate, and the absolute value of \( \tau \) as
the abscissa, where C and \( \zeta \) are parameters of intersecting curves; and \( \tau \) and \( \zeta \) are expressed in degrees. Detailed charts may be found in work by von Hippel(12).

The relationship between \( \varepsilon_2^* \) and \( \gamma_2 \) depends on the cut-off wavelength \( \lambda_c \) of the wave guide. The cut-off wavelength \( \lambda_c \) is the lowest free-space wavelength for which propagation in a particular mode is possible. For the rectangular wave guide (TE\(_{01}\) mode), the cut-off wavelength \( \lambda_c \) is two times the width.

For TE modes in rectangular wave guides, the separation equation can be expressed as

\[
k_1^2 = k_x^2 + k_y^2 + k_z^2
\]

where

\[
k_x^2 = \left( \frac{m\pi}{a} \right)^2
\]

\[
k_y^2 = \left( \frac{n\pi}{b} \right)^2
\]

\[
k_z^2 = \omega^2 \mu \varepsilon
\]

\[
k_z^2 = \omega^2 \mu \varepsilon - \left( \frac{m\pi}{a} \right)^2 - \left( \frac{n\pi}{b} \right)^2
\]

where \( k_z = -j\gamma_z \)

\[
-\gamma_1^2 = +\omega^2 \mu \varepsilon - \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2
\]

\[
\gamma_1^2 = -\omega^2 \mu \varepsilon + \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2
\]

\[
\gamma_1 = j \sqrt{\omega^2 \mu \varepsilon - \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2}
\]

For propagation, the radical must be positive. Therefore:
The propagation constant can be written for region (2) as follows:

\[ k_2^2 = k_1^2 + \left( \frac{2\pi}{\lambda_c} \right)^2 \]

\[ k_{z_1}^2 = k_1^2 - \left( \frac{2\pi}{\lambda_c} \right)^2 \]

\[ \gamma_1^2 = -\omega^2 \varepsilon_0 \mu_0 + \left( \frac{2\pi}{\lambda_c} \right)^2 \]

But, from previous paragraphs, we recall that

\[ \gamma_1 = \frac{12\pi}{\lambda_1} \]

From the above equations, we obtain

\[ \varepsilon_2^* = \frac{-\gamma_2^2 + \left( \frac{2\pi}{\lambda_c} \right)^2}{\omega^2 \mu_0} \]

Multiplying by \( \varepsilon_0 \) gives

\[ \varepsilon_2^* = \varepsilon_0 \frac{-\gamma_2^2 + \left( \frac{2\pi}{\lambda_c} \right)^2}{\omega^2 \mu_0 \varepsilon_0} \]
From previous work, we have

\[ \omega^2 \mu_0 \varepsilon_0 = \left( \frac{2\pi}{\lambda_c} \right)^2 + \left( \frac{2\pi}{\lambda_1} \right)^2 \]  

\[ \varepsilon_2^* = \varepsilon_0 \frac{-\gamma_2^2 + \left( \frac{2\pi}{\lambda_c} \right)^2 \left( \frac{2\pi}{\lambda_1} \right)^2 \varepsilon_0^2}{\left( \frac{2\pi}{\lambda_c} \right)^2 + \left( \frac{2\pi}{\lambda_1} \right)^2} \quad (80) \]

\[ \varepsilon_2^* = \varepsilon_0 \frac{\left( \frac{1}{\lambda_c} \right)^2 - \left( \frac{\gamma_2}{2\pi} \right)^2}{\left( \frac{1}{\lambda_c} \right)^2 - \left( \frac{1}{\lambda_1} \right)^2} \quad (81) \]

Now by making a substitution we obtain

\[ u = \left( \frac{\lambda_1}{\lambda_c} \right)^2 \]

\[ \varepsilon_2^* = \varepsilon_0 \frac{\left( \frac{1}{\lambda_1} \right)^2 - \left( \frac{\lambda_2 d \lambda_1}{2\pi d} \right)^2}{\left( \frac{1}{\lambda_c} \right)^2 + 1} \quad (82) \]

\[ \varepsilon_2^* = \varepsilon_0 \frac{u - \left( \frac{\gamma_2 d \lambda_1}{2\pi d} \right)^2}{1 + u} \]

From the above equation, the complex permittivity \( \varepsilon_2^* \) can be found from the measurable quantities \( \lambda_1, \lambda_c, d \) and the calculated quantity \( \gamma_2 d \).

**Procedure for Measuring the Dielectric Properties**

The laboratory equipment available determines the procedure and method that should be used. A type of waveguide instrument has been developed for dielectric
measurements. A stabilized reflex klystron oscillator radiates monochromatic waves of a certain frequency into a hollow, rectangular wave guide; they are reflected by a shorting metallic boundary at the other end. Standing waves are set up and can be measured by a traveling probe detector along a narrow slot in the top of the guide parallel to its axis. The dielectric sample is placed in the closed end of the guide opposite the transmitter. For the open circuit measurement, the sample is located a quarter wavelength ahead of the short; for the short-circuit measurement, the sample is placed in direct contact with the short. The sample should fit as closely as possible to the short and walls of the guide, and its faces should be perpendicular to the guide axis. The block diagram of the laboratory set up is shown in Fig. 14.

![Block Diagram of Equipment used in the Laboratory.](image)

**Fig. 14.** Block Diagram of Equipment used in the Laboratory.

After the equipment has been tuned and is functioning properly, the location of the load should be established.
This is accomplished by placing a short at the end of the slotted section and recording the VSWR distribution along the guide as shown in Fig. 15.

![Standing Wave Pattern in Slotted Section](image)

Fig. 15. Standing Wave Pattern in Slotted Section.

After the location of the minima has been recorded, the short should be removed and replaced with the sample and the short. The choice of either the short-circuit or the open-circuit measurement will determine the short location.

The standing wave pattern is measured in air (medium 2). The wave impedance $Z(0)$ of medium 1 is found by determining the ratio of minimum to maximum electric field strength and the distance $x_0$ of the first minimum from the dielectric boundary. The distance $x_0$ is the shift of the minimum from the position recorded above. The VSWR readings along the line will be rather high (greater than 10). Therefore, the double minimum method should be used. For a detector of "square-law" response ($I \sim E^2$) as shown in Fig. 16, it can be shown that

$$\frac{E_{\text{MIN}}}{E_{\text{MAX}}} = \frac{\pi \Delta x}{\lambda_1}$$
The half wavelength \( \frac{\lambda_1}{2} \) in the air space of the guide may be obtained by measuring the distance between two minima of the standing wave pattern. The measurable quantities \( \frac{E_{\text{MIN}}}{E_{\text{MAX}}} \), \( x_0 \), \( d \), and \( \lambda_1 \) can be determined from this procedure.

![Graph of square law measurements](image)

**Fig. 16. Square Law Measurements.**

Summarizing, the procedure of determining \( \varepsilon_2^* \) begins with measurements of \( \frac{E_{\text{MIN}}}{E_{\text{MAX}}} \), \( x_0 \), \( d \), and \( \lambda_1 \). From these measured values, \( C \) and \( \xi \) are calculated from Eq. (71). From \( C \) and \( \xi \), the values of \( T \) and \( \tau \) are found by using the specified charts. By dividing \( T e^{j\tau} \) by \( d \), \( \gamma_2 \) is obtained. Finally \( \varepsilon_2^* \) is determined from \( \gamma_2 \) by Eq. (81) or Eq. (85).

For nonmagnetic materials, just one of the two measurement methods (short-circuit or open-circuit) has to be performed. Magnetic materials require a separate calculation for \( Z_2(0) \) and \( \gamma_2 \) because the impedance is of ratio form and the propagation factor is of product form, both forms involving \( \mu_2^* \) and \( \varepsilon_2^* \). In this case both measurement methods have to be performed.
V. CONCLUSIONS

The main problem in this matching technique is the selection of the material for the slug and also that of the conducting obstacles.

The material for the slug must be a nonporous, lightweight dielectric with a dielectric constant close to one. The only restriction placed on the obstacle, aside from being small, is that it have low losses. There is a wide variety of these materials from which to choose.

In the author's actual experiments on styrofoam dielectric, difficulties came up because of the choice of materials and obstacles. Styrofoam was chosen because of its light weight and low dielectric constant (1.03). The only problem with this material is that it is too porous. After the holes were drilled and water placed in these holes, the sample was then placed in the waveguide equipment. Before all of the measurements needed could be taken, most of the water either had leaked out of the sample or was absorbed by the styrofoam. From this problem, erratic data resulted. The problem may be bypassed by the use of a different obstacle (a solid material). If water is to be used as the obstacle, a different material should be used for the slug. Polystyrene is a non-porous, lightweight material with a dielectric constant of 2.56. Some compromise will have to be made with the materials to get the best results.
The method of determining the dielectric properties discussed in the thesis gave reasonably accurate results of solid samples tested. The value of the dielectric constant of styrofoam was found to be 1.035 which compares favorably with that given by von Hippel.

The author feels that a suitable matching slug can be designed using the procedure discussed under artificial dielectrics to match an antenna such as a horn to a dense media such as water. The dielectric properties of this matching slug may be determined by the proper use of the techniques discussed under the measurement of dielectric properties section.
APPENDIX

Wave Impedance Expressed in Terms of Measurable Quantities.

Starting with the wave impedance,

\[ Z(0) = \eta_1 \coth u \quad (A-1) \]

Make the substitution,

\[ u = p + j\psi \]

where

\[ \psi = \left( \frac{\pi}{2} - \frac{2\pi x_0}{\lambda_1} \right) \quad (A-2) \]

and expand \( \coth u \).

\[ Z(0) = \eta_1 \frac{\cosh (p + j\psi)}{\sinh (p + j\psi)} \quad (A-3) \]

\[ Z(0) = \eta_1 \frac{\cosh p \cos \psi + j \sinh p \sin \psi}{\sinh p \cos \psi + j \cosh p \sin \psi} \quad (A-4) \]

Divide (A-4) by \( \sinh p \).

\[ Z(0) = \eta_1 \frac{\coth p \cos \psi + j \sin \psi}{\cos \psi + j \coth p \sin \psi} \quad (A-5) \]

Divide (A-5) by \( \sin \psi \)

\[ Z(0) = \eta_1 \frac{\coth p \cot \psi + j}{\cot \psi + j \coth p} \quad (A-6) \]

\[ Z(0) = \eta_1 \frac{\cot \psi + j \tanh p}{\tanh p \cot \psi + j} \quad \frac{\tanh p}{\tanh p} \quad (A-7) \]

\[ Z(0) = \eta_1 \frac{\cot \psi + j \tanh p}{j + \tanh p \cot \psi} \quad (A-8) \]

Multiply (A-8) by \((-j)\).

\[ Z(0) = \eta_1 \frac{\tanh p - j \cot \psi}{1 - j \tanh p \cot \psi} \quad (A-9) \]
Substitute the values of \( \tanh p \) and \( \psi \) into (A-9).

\[
Z(0) = \eta_1 \left( \frac{E_{\text{MIN}}}{E_{\text{MAX}}} - j \tan \frac{2\pi x_0}{\lambda_1} \right) \frac{1 - j \frac{E_{\text{MIN}}}{E_{\text{MAX}}} \tan \frac{2\pi x_0}{\lambda_1}}{1 - j \tan \frac{2\pi x_0}{\lambda_1}}
\]  

(A-10)
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VITA

Paul Arthur Ray was born on December 21, 1941, in Rolla, Missouri. He received his primary and secondary education in the Rolla Public Schools.

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