A statistical hydrologic simulation model

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A STATISTICAL HYDROLOGIC SIMULATION MODEL

BY

RONALD LEE WYCOFF, 1945-

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CIVIL ENGINEERING ABSTRACT

A simulation model of a small watershed using probabilistic models derived from short term rainfall-runoff records is developed. The model is used to generate a synthetic flood series which is compared to the observed flood series.
PUBLICATION THESIS OPTION

This thesis has been prepared in the style utilized by The Journals of the American Society of Civil Engineers. The Vita and appendices A and B have been added for purposes normal to the thesis writing.
A STATISTICAL HYDROLOGIC SIMULATION MODEL

By Ronald L. Wycoff,¹ A.M. ASCE

KEY WORDS: hydrology; simulation; statistical analysis; probabilistic models; computers.

ABSTRACT: Simulation of a physical system requires knowledge of all components of the system and their interactions. A probabilistic simulation model of a hydrologic system is developed from short term continuous, synchronized rainfall-runoff records. The individual components of the system are: the time between and duration of storms, the depth of rain occurring in each time period of the storm, precipitation excess relations, and the watersheds unit hydrograph. The available data is used to define or develop the probabilistic models involved. The only nonprobabilistic model employed is the watersheds unit hydrograph. It is found that the time between and duration of storms may be represented by an exponential distribution model. In general the depth of rain may be modeled by a log-normal distribution; although a separate model was employed for rainfall extremes. A probabilistic precipitation excess model is developed which relates excess precipitation to total rainfall, season of the year and a random process. The total simulation model yielded synthetic flood frequency curves within the 90% confidence limits of the observed flood frequency curve.

¹ Graduate Student, University of Missouri at Rolla, Rolla, Missouri.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>v</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>WATERSHED SIMULATION</td>
<td>2</td>
</tr>
<tr>
<td>SIMULATION PROCESS</td>
<td>6</td>
</tr>
<tr>
<td>Description of Study Watershed</td>
<td>6</td>
</tr>
<tr>
<td>Available Rainfall Data</td>
<td>7</td>
</tr>
<tr>
<td>Time Between and Duration of Storms</td>
<td>8</td>
</tr>
<tr>
<td>Rainfall Depths</td>
<td>11</td>
</tr>
<tr>
<td>Extreme Rainfall Depths</td>
<td>17</td>
</tr>
<tr>
<td>Rainfall Depth Model</td>
<td>20</td>
</tr>
<tr>
<td>Precipitation Excess Model</td>
<td>20</td>
</tr>
<tr>
<td>Unit Hydrograph Model</td>
<td>24</td>
</tr>
<tr>
<td>COMPARISON OF SYNTHETIC TO OBSERVED FLOOD FREQUENCY CURVES</td>
<td>24</td>
</tr>
<tr>
<td>CONCLUSIONS</td>
<td>31</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>32</td>
</tr>
<tr>
<td>APPENDIX - I - REFERENCES</td>
<td>33</td>
</tr>
<tr>
<td>APPENDIX - II - NOTATION</td>
<td>34</td>
</tr>
<tr>
<td>VITA</td>
<td>35</td>
</tr>
<tr>
<td>APPENDIX - A - FORTRAN LISTING OF SIMULATION PROGRAM</td>
<td>36</td>
</tr>
<tr>
<td>APPENDIX - B - DATA AND DISTRIBUTION MODELS</td>
<td>45</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. General Simulation Flow Chart</td>
<td>5</td>
</tr>
<tr>
<td>2. Typical Mass Rainfall Curve</td>
<td>13</td>
</tr>
<tr>
<td>3. 15 Minute Unit Hydrographs</td>
<td>25</td>
</tr>
<tr>
<td>4. Generated and Observed Flood Frequency Curves</td>
<td>27</td>
</tr>
</tbody>
</table>


LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Sample Statistics for Time Between and Duration of Storms</td>
<td>10</td>
</tr>
<tr>
<td>2.</td>
<td>Sample Statistics for Rainfall Depths Occurring in a 15 Minute Period</td>
<td>16</td>
</tr>
<tr>
<td>3.</td>
<td>Flood Flow Estimates From Observed Flood Series and Synthetic Methods</td>
<td>29</td>
</tr>
</tbody>
</table>
INTRODUCTION

Simulation is a tool of that branch of applied mathematics known as Operations Research. Explanation of the various types of simulation and the actual techniques of simulation can be found in several texts which deal wholly or in part with the subject. Among them are Hiller and Lieberman 1967,\(^{(8)}\) and Evans, Wallace and Sutherland 1967.\(^{(3)}\) A discussion of simulation as a hydrologic tool is presented by Viessman, Harbaugh and Knapp 1972.\(^{(11)}\)

Simulation can be defined as the art and science of modeling a natural system. This process can be accomplished by use of various mathematical representations many of which are probabilistic or statistical in nature, or by use of a physical model of the system. The purpose of this study is to build a mathematical simulation model of a watershed system in order to generate synthetic stream flow records for a selected watershed. The general procedure used to construct a simulation model of a watershed is outlined as follows. First a watershed is selected which has continuous, synchronized rainfall-runoff data. These data are analyzed statistically to establish the various probability density functions used in defining the behavior of the systems components. The required statistical distributions are: the distribution of the time between storms, the distribution of the duration of storms, and the distribution of the depth of rain for each time period of the storm.

A crude Monte Carlo technique is used to obtain a random sampling of the various rainfall events. After the total
storm rainfall has been generated it becomes the input into a probabilistic precipitation excess model. The response or discharge of the watershed is then determined by use of the watershed's unit hydrograph. The total simulation model is programmed for operation on a digital computer and used to generate continuous stream flow records. A flood frequency curve derived from the synthetic stream flow records is established by sampling the annual maximum flood peaks.

WATERSHED SIMULATION

In general mathematical simulation models developed for the purpose of modeling a hydrologic system may be classed in two types, sequential, as used by Chow and Ramaseshan 1965(2) and event, as used by Hiemstra 1968,(5) and 1969(6) and Hiemstra and Creese 1970.(7)

In a sequential simulation model each time period is considered individually. For example if the object were to generate rainfall records, the rainfall depth occurring in any time period would be modeled in the form of a recursion formula. This recursion formula expresses the depth of rain in a given time period as a function of the depth of rain in the previous time period and a random process. Thus each time period in the simulation process is considered even if no rainfall occurred.

In an event simulation model only those time periods where an event occurs are considered. Considering a model for the generation of rainfall records the time interval
between storms is generated and all time periods in this interval are eliminated from individual computation. Thus only those time periods where rainfall occurs need be considered individually. The simulation model developed here is of the event type.

Five individual models are required in order to formulate a watershed simulation model. Four of the models are probabilistic and one is deterministic; they are: a model for the time interval between storms; a model for the duration of a storm; a model for the depth of rainfall for each time unit of the storm; a precipitation excess relation to determine the rainfall excess from the total rainfall; and a unit hydrograph to determine the time distribution of runoff resulting from the excess rainfall.

Given the five individual models, a logical step by step routine is required to develop a working simulation model. Since the overall objective is to attempt to generate synthetic flood flow data for a small watershed, the basic time unit employed in the model should be of short duration to insure adequate reproduction of the hydrologic response of the watershed. For this reason a time unit of 15 minutes was chosen thus providing a time line of 35,040 units for each year.

In general the operation of the watershed simulation model can be divided into three phases. First an array of rainfall depths is generated. This array contains 35,040 numbers, most of which are zero, and is known as the
precipitation total array. This array is represented by the symbol $P_t$. Second the $P_t$ array and the precipitation excess relations are used to determine the precipitation excess array for the year. This array is represented by the symbol $P_e$. Third, using the $P_e$ array and the unit hydrograph array, the stream flow record for the year is calculated. This process is repeated until a desired length of record is obtained.

More specifically the process can be viewed as a series of 10 steps, some of which are computational, and some of which are logical. Figure 1 is a general flow chart of the total simulation model. The procedure may be described as follows:

1. Generate the time between storms to the nearest whole time unit. Also set the $P_e$ array equal to zero from the end of the previous storm to the beginning of the present storm.
2. If the $P_e$ array for the year is generated go to step 8. If not go to step 3.
3. Generate the duration of the storm to the nearest whole time unit.
4. Generate the total depth of rain for each time unit of the storm. If the duration of the storm is short, arrange the rainfall depths into an advanced pattern. This step yields the $P_t$ array.
5. Sum the $P_t$ array for the storm and find the excess portion of the total rainfall using the precipitation excess relations.
6. Using the $P_t$ array, the excess portion of the rainfall from step 5, and the Phi-Index method calculate the $P_e$ array for the storm.
7. If the $P_e$ array for the year has been generated, go to step 8. If not, go to step 1.
START
Initilize and input UHG array
Generate time to storm

Is Pe array for year complete?

Is Pe array for storm complete?

Generate duration of storm.
Generate Pt array for storm.
Determine total runoff for storm.
Calculate Pe array for storm.

Is Pe array for year complete?

Calculate stream flow hydrograph for year

OUTPUT

Is req. data generated?

REINITILIZE
No
Yes STOP

FIG. 1. - GENERAL SIMULATION FLOW CHART
8. Calculate the runoff hydrograph for the year by multiplying the $P_e$ array by the unit hydrograph array.

9. Search the runoff hydrograph array and output the peak flow rate for the year.

10. If the required number of years of data have been generated stop. If not, set the year equal to the present year plus one, set the time unit equal to one, set the peak flow rate equal to zero and go to step 1.

SIMULATION PROCESS

Description of Study Watershed. - The study watershed has been gaged by the United States Geological Survey Water Resources Division since 1958, and is part of the Salt River Basin of northern Missouri. This watershed is listed by the U.S.G.S. as "Easdale Branch near Shelbyville, Mo.," downstream order number 5-5027, with a drainage area of .71 square miles. From a U.S.G.S. topographic map the main channel length, measured from the gage along the channel, projected to the watershed divided was determined to be 6,850 feet. Also the difference in elevation between the divide and the gage was found to be 80 feet. The soil type of the watershed could not be exactly determined from a large scale Missouri soils map because of the small size of the watershed. However, the soils of the region are either of group C or D according to the hydrologic soil classification system employed by the U.S.D.A. Soil Conservation Service. This indicates a higher than average runoff potential. The land use of the watershed may be described as mixed cover, rural, with no one land use predominating.
The data gathered from the above watershed is of two types. First, yearly maximum peak flow rate data has been obtained since 1958. Second, continuous rainfall-runoff data has been obtained since July 24, 1969. The period of record on which this study is based is from the above starting date until October 6, 1970, approximately 14 months.

Both the stream gage and the rain gage are located at the outlet of the watershed. The rain gage is a tipping bucket rain gage, which records each tenth of an inch of accumulated rainfall. The data output is in strip chart form, with the stream gage height and the rainfall record, recorded on the same chart.

Available Rainfall Data - In order to reduce the strip chart data several somewhat arbitrary definitions are necessary. Since it is the purpose of the study to simulate stream flow, those rainstorms which did not produce stream flow were not considered. Thus the basic event is defined to be a rainstorm which results in runoff.

Various events involved in the hydrologic process are subject to seasonal variations. Therefore several seasons of the year were defined. Winter is defined as the months of October thru March. Summer is defined as the months of April thru September. Early summer is defined as April, May and June and late summer is defined as July, August and September.

The time between storms is defined as that length of time between the end of one event and the beginning of another event. This length of time must be at least 12 hours. If
two events occur with less than a 12 hour dry period between them, they are not considered independent events, but are considered part of the same event.

Time Between and Duration of Storms. - The time between and duration of storms are considered according to the season of occurrence as either summer storms or winter storms. Of the 46 observed rainfall-runoff events 40 occurred in the summer and 6 occurred in the winter. An extremely small sample of winter events causes a large degree of uncertainty as to the probabilistic nature of these events. However, as reported by Sandhaus and Skelton, 1968, floods in Missouri are most likely to occur in June, March and April respectively and least likely to occur in November, December and January. Therefore the lack of winter data is not considered to be of importance in the generation of synthetic peak flood flows.

Probabilistic models in the form of statistical distributions are often used to mathematically describe or represent a random process. If a set of observations of the random process are available they may be used in the following manner to select a distribution model. The sample statistics of the observed data set are computed and the parameters of the assumed distribution models are estimated from these sample statistics. The overall fit of the assumed models may be tested by a group of statistical tests known as goodness-of-fit tests or by simultaneously plotting the model and observed histogram and visually comparing the fit. The model which yields the best fit should be accepted.
The exponential distribution is often used to model the time between events and may be used to model the duration of an event. As discussed by Benjamin and Cornell, 1970, (1) this distribution describes the time to the first occurrence of a poisson event. In addition if a random variable is exponentially distributed, the mean is equal to the standard deviation. As can be seen from Table 1 these sample statistics are numerically within 30% of each other. For the above reasons a exponential distribution fit was attempted for the time between and the duration of storms.

Considering the time between storms, the assumption was made that the random variable time is exponentially distributed with the mean equal to the computed mean of each of the two observed data sets. A chi-square test was performed on the assumed models. In each case the resulting significance level of the chi-square test is above 90%. Therefore the exponential distribution model was accepted and no other distribution model was investigated.

A tipping bucket gage begins to record only after a tenth of an inch of rain has fallen thus the exact time of the beginning of the storm cannot be determined. Occasionally the beginning of rise of the stage hydrograph occurred before the first tip of the bucket. In these cases the time at the start of rise of the stage hydrograph was considered to be the time of the beginning of the storm. When the beginning of rise of the stage hydrograph did not occur until after the first tip of the bucket, the time at the beginning of the storm was arbitrarily defined as the nearest whole time
TABLE 1. - SAMPLE STATISTICS FOR TIME BETWEEN AND DURATION OF STORMS

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Number of Observations</th>
<th>Mean, Number of 15 Minute Time Units</th>
<th>Standard Deviation, Number of 15 Minute Time Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time between storms</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>summer</td>
<td>39</td>
<td>597.59</td>
<td>610.41</td>
</tr>
<tr>
<td>winter</td>
<td>6</td>
<td>2565.33</td>
<td>3319.13</td>
</tr>
<tr>
<td>Duration of storms</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>summer</td>
<td>40</td>
<td>28.00</td>
<td>33.39</td>
</tr>
<tr>
<td>winter</td>
<td>6</td>
<td>46.83</td>
<td>41.87</td>
</tr>
</tbody>
</table>
unit before the first tip of the bucket. Therefore the duration of the storm is defined as that period of time from the beginning of rise of the stage hydrograph or the nearest whole time unit before the first tip of the bucket, which ever occurs first, to the last tip of the bucket.

The duration of storms was also assumed to be an exponentially distributed random variable with the mean equal to the computed mean of the two observed data sets. Chi-square tests performed on these two assumed models, resulted in significance levels above 80%. A log-normal distribution model was also considered; however the chi-square test results were lower than for the exponential model.

An exponential distribution is used to model the time between and the duration of both summer and winter storms. In order to generate a random observation of either the time between storms or the duration of a storm a uniformly distributed random number between 0 and 1, "RN," is used in conjunction with the exponential inverse transformation function. This function is expressed as follows:

\[ T = -\ln (RN) \bar{M} \]  

where \( T \) is a random observation of time; \( \bar{M} \) is the mean of the distribution of \( T \); and \( RN \) is a random number.

Rainfall Depths. - As noted previously the accumulated rainfall is recorded on one tenth of an inch intervals. Therefore depths of rainfall for each 15 minute interval could not be read directly from the strip chart. The following procedure was used to determine the depth of rain that fell during each time period of each of the 46 observed storms.
First the accumulated rainfall was plotted versus time on rectangular coordinate paper, for each storm. The plotted points are then connected with straight line segments. This procedure results in a mass rainfall curve, for each storm. By connecting the plotted points with straight line segments the rainfall intensity is assumed uniform between tips of the bucket. Dividing the time axis into 15 minute intervals and reading the depth of rain for each of these intervals, the depth of rain for each 15 minute interval of the storm can be obtained. Figure 2 is a typical mass rainfall curve, illustrating the procedure used to determine the rainfall depths. The final result is a tabulation, one for each of the observed storms, of depth of rainfall for each time unit of the storm.

The events are grouped as summer events and winter events. Summer events are further subdivided as either short duration events or long duration events. A short duration event is defined as one which lasted 4 hours or less. The reason for this subdivision of summer storm is that summer storms of short duration generally arise out of a thunderstorm mechanism whereas summer storms of long duration generally arise from more regional frontal storms. The selection of 4 hours is due to a natural grouping of the observed data on either side of this point. Inspection of the data reveals that 24 of the 40 summer events had durations of from 1 to 14 units (3½ hours) and the remaining 16 events had durations of from 18 units (4½ hours) to 132 units.
FIG. 2 - TYPICAL MASS RAINFALL CURVE
The reason for differentiating between long and short duration storms is that rainfall intensity and the time distribution of rainfall for these two groups of storms are significantly different. Therefore separate models will be developed to simulate both the depth of rainfall and the time distribution of rainfall occurring in short and long duration storms.

It is assumed that for long duration storms rainfall depths occurring in adjacent time periods can be modeled as independent events and therefore random observations of rainfall depths may be generated by Monte Carlo techniques. In general the time distribution of rainfall depths generated in this manner will be random and multi-peaked. This assumption is at best a simplification of the natural phenomenon. As reported by Heimstra and Creese,(7) Monte Carlo sampling of rainfall depths thru a univariate probability distribution does not account for the interdependence of the system and is therefore conceptually inaccurate. However, the long duration storms observed in this data set tend to be multi-peaked. Thus, long periods of low to moderate intensity rainfall were separated into distinct intervals by short bursts of moderate to high intensity rainfall. Therefore, it is concluded that the dependency between adjacent rainfall depths is at a minimum for long duration storms, and the assumption of independence is made in the interest of developing a simple working tool.

For short duration storms the dependency among rainfall depths is much more pronounced. The observed short duration
storms tend to have time distribution patterns of the advanced type. As can be seen from Table 2, short duration summer storms have a much higher mean depth of rain for a 15 minute time interval than do long duration storms. This means that short duration storms, i.e. thunderstorms, are high intensity events. Other investigators, as reported by Hiemstra, (5) have observed predominantly advanced patterns in high intensity storms. Therefore the short duration storms are assumed to have an advanced time distribution of rainfall depths.

In order to simulate these advanced storm patterns the following procedure is used. First a Monte Carlo sampling of the distribution of rainfall depths is used to generate the required number of rainfall depth observations. This procedure yields the sporadic or random rainfall pattern used in the simulation of long duration storms. The generated observations of rainfall depths are then sorted into descending order, according to their magnitude, resulting in the largest generated value of rainfall depth placed in the first time period of the storm, the second largest in the second time period, etcetera, until the complete array of rainfall depths is arranged in descending order.

It is assumed that the random variable, rainfall depth, is a univariate log-normal distribution with the mean and standard deviation of the logarithms equal to the computed mean and standard deviation of the logarithms of the three observed data sets as shown in Table 2. Chi-square tests were performed on the three assumed models. The resulting
TABLE 2. - SAMPLE STATISTICS FOR RAINFALL DEPTHS OCCURRING IN A 15 MINUTE PERIOD

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Number of Observations</th>
<th>Mean, Inches</th>
<th>Standard Deviation, Inches</th>
<th>Mean of Natural Logarithms</th>
<th>Standard Deviation of Natural Logarithms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer, short duration</td>
<td>170</td>
<td>0.0871</td>
<td>0.1193</td>
<td>-2.957</td>
<td>0.9235</td>
</tr>
<tr>
<td>storms</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Summer, long duration</td>
<td>950</td>
<td>0.0268</td>
<td>0.0560</td>
<td>-4.392</td>
<td>1.168</td>
</tr>
<tr>
<td>storms</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Winter storms</td>
<td>281</td>
<td>0.0448</td>
<td>0.0815</td>
<td>-3.822</td>
<td>1.0797</td>
</tr>
</tbody>
</table>
significance levels were 81%, 49% and less than 5%, for the depth of rain, summer short duration, summer long duration and winter respectively. The results obtained from the summer data indicates that the log-normal model yields a reasonable fit to the observed data. Because there are only six winter events and because no attempt was made to differentiate between short and long duration winter events the low chi-square test result for depth of rain winter storms was disregarded. Based on the chi-square test results for depth of rain summer storms all three log-normal distribution models were accepted for use in the total simulation model.

In order to generate a random observation of rainfall depth a normally distributed random variant having a mean of zero and a standard deviation of one is used in conjunction with the inverse log-normal transformation function. This function is as follows:

\[ D = \exp (NV \cdot SIG + \bar{d}) \]  

where \( D \) is a random observation of rainfall depth; \( NV \) is a random normal variate as defined above; \( SIG \) is the standard deviation of the logarithms of \( D \); and \( \bar{d} \) is the mean of the logarithms of \( D \). A normally distributed random variate is generated by application of the Central Limit Theorem to the sum of a series of uniformly distributed random numbers.

Extreme Rainfall Depths. - The annual maximum peak flow rate is influenced to a great extent by the extreme rainfall amounts. These extreme rainfall depths are represented by
the tails of the log-normal distribution models. Although the depth of rain occurring in a 15 minute duration is reasonably well defined by a log-normal model, in a overall sense, the representation of the extremes was found to be inadequate for two of the three models. For example, considering the distribution of the depth of rain for short duration summer storms the log-normal distribution model indicates that the probability of occurrence of a rainfall depth equal to or greater than 0.30 inches is about 3%. However the observed data indicates that this probability should be about 8%. Further, the log-normal model indicates that the probability of occurrence of a rainfall depth greater than or equal to 0.565 inches is 0.5%; whereas the data indicates that this probability should be 1.75%. In other words the chance of a rainfall depth exceeding 0.565 inches is observed to be $3\frac{1}{2}$ times greater than the probability indicated by the model. Obviously this model of extremes is inadequate for the generation of synthetic flood peaks.

This deviation in the tails of the model, although not as pronounced, was also observed in the distribution of the depth of rain for winter storms. The log-normal model for the depth of rain for long duration summer storms was found to give a usable fit over the whole range of depths. It is noted that the tail of the distribution was modeled best for the data set with the largest number of observations. From Table 2 it may be seen that the deviation of the observed to the modeled distributions is inversely related to the number
of observations. The fit was best for the distribution of long duration summer storms for which 950 observations were available and worst for the distribution of short duration summer storms for which only 170 observations were available. This indicates that the observed deviation in the distribution tails may be due to the limited data available, rather than an inability of the log-normal distribution to describe rainfall depths.

For the above reasons the log-normal model was not used to generate extreme rainfall depths for short duration summer storms or winter storms. A series of three straight line segments, derived directly from the cumulative histogram was employed to represent these extreme events. As noted by Evans, Wallace and Sutherland, (3) any set of observations may be converted to an empirical cumulative distribution function by a series of straight line segments. In most cases storing a curve in this manner would require a considerable amount of time and effort in addition to much computer storage. However in this case only the extreme tail of the distribution was modeled and just three line segments are necessary for each model.

In order to simulate the possibility of occurrence of a rainfall depth greater than the largest observed value the upper limit of possible rainfall depth was set as the 50 year point rainfall amount (1.55 inches) for short duration summer storms and the 5 year point rainfall amount (1.20 inches) for winter storms. These maximum rainfall depths were estimated
from rainfall-intensity-frequency-duration curves derived from Missouri data and based on a duration of 15 minutes.

Rainfall Depth Model - The procedure used to synthetically generate rainfall depths may be described as follows:

1. Determine which distribution is to be sampled. This may be either the distribution of short or long duration summer storms or the distribution of winter storms.

2. Generate a rainfall observation using the log-normal model for the appropriate distribution. If this observation is from the distribution of long duration summer storms use this observation and go to step 6.

3. If the rainfall depth generated from step 2 is from the distribution of short duration summer storms or winter storms, test to determine if the linear model applies. The linear model applies only if the generated rainfall observation is above 0.15 inches for short duration summer storms or above 0.08 inches for winter storms. The linear model will apply to about 12% of the depths generated from these two distribution models.

4. If the linear model does not apply use the log-normal observation and go to step 6.

5. If the linear model does apply, replace the original generated rainfall depth by a random observation of the appropriate linear model.

6. Repeat steps 2 thru 5 until a rainfall depth for each time unit of the storm has been generated.

7. If the storm is a short duration event arrange the rainfall depths in an advanced pattern. If the storm is a long duration event use the rainfall depths in the random pattern as generated. This yields the precipitation total array for the storm.

Precipitation Excess Model - A precipitation excess model is necessary in order to separate the excess portion of rainfall from the total rainfall. An analysis of the existing records was made to establish a relationship between total rainfall and precipitation excess.
In order to determine the total runoff for each storm the stage hydrograph as recorded on a strip chart was converted to a discharge hydrograph by means of the rating table provided for the gage site. This discharge hydrograph was integrated numerically to determine the total runoff. When the recorded rainfall was greater than the recorded runoff both observations were assumed to be valid values.

Three observed events recorded runoff greater than the recorded rainfall. In these three cases the observed value of runoff was assumed correct and the observed value of rainfall in error. In order to establish a rainfall depth value, it was further assumed that the watershed was in a wet antecedent state at the time of the storm and thus had high runoff potential. Using these assumptions and the known parameters of the watershed, the S.C.S. rainfall-runoff relation(9) was used to calculate a value of rainfall depth which will yield the observed value of runoff. These calculated values of rainfall are used in lieu of the observed values in the derivation of the model.

The 46 observed or adjusted rainfall-runoff data pairs were grouped according to season of occurrence as early summer, late summer and winter events. The data pairs for early summer were plotted rainfall versus direct runoff or precipitation excess on rectangular coordinate graph paper, in order to determine if a seasonal grouping occurs. These data exhibited a large degree of scatter and no definite correlation was evident.

The data was then subdivided into three sub-groups, each consisting of approximately one third of the total data set.
Sub-group one consists of those data pairs which yield a high direct runoff amount for a given rainfall amount. All data pairs in this group are assumed to have occurred when the watershed was in a wet antecedent condition. Sub-group two consists of all data pairs which yield an intermediate amount of direct runoff for a given amount of rainfall. These events are assumed to have occurred when the watershed was in a normal or mean antecedent condition. The last sub-group consists of the remaining events which are assumed to have occurred when the watershed was in a dry antecedent condition. The coordinates, zero, zero, are added to each of the three data sub-groups and a linear equation was established for each sub-group by least squares. The following three equations represent the precipitation excess relationship for the watershed in early summer for wet, normal and dry antecedent conditions respectively.

wet

\[
PE = 0.875512 \times PT - 0.168909, \quad Se = .385
\]

(3)

normal

\[
PE = 0.441014 \times PT - 0.052465, \quad Se = .080
\]

(4)

dry

\[
PE = 0.213372 \times PT - 0.035067, \quad Se = .033
\]

(5)

Where \( PE \) equals the precipitation excess in inches, \( PT \) equals the precipitation total in inches and \( Se \) equals the standard error in inches.

A similar analysis of the rainfall-runoff data for late summer storms yields the following three equations for wet, normal and dry antecedent conditions respectively.

wet

\[
PE = 0.9832794 \times PT - 0.177597, \quad Se = .217
\]

(6)

normal

\[
PE = 0.591734 \times PT - 0.197054, \quad Se = .256
\]

(7)

dry

\[
PE = 0.111443 \times PT - 0.016745, \quad Se = .062
\]

(8)
Because only six winter rainfall-runoff events are available for analysis no attempt was made to define different antecedent conditions. Instead all six data pairs plus the coordinates zero, zero were fit by a single linear least squares equation as follows:

$$PE = 0.813144 \times PT - 0.390253; \quad S_e = .714$$

For winter events the direct runoff for a given rainfall amount, is calculated directly from Eq. 9. However in the case of early or late summer events a random number is generated. The range of the random number, 0 to 1, is divided into three equal intervals and the precipitation excess equation is chosen dependent upon which one of the three intervals the generated random number occupies. Since each of the three precipitation excess equations is derived from one third of the observed data, it is assumed that the probability that any one equation applies is equal to one third.

At this point methods for generating or calculating the precipitation total array, the total precipitation, and the precipitation excess are known. A procedure to determine the precipitation excess array from the above known quantities is required. It is assumed that the rate of loss throughout the storm is constant and therefore a phi-index type model is employed. Knowing the precipitation total array and the precipitation excess a phi-index is found by a successive approximation technique. After the phi-index is determined it is subtracted from each rainfall depth in the precipitation total array. If the total precipitation in any time period is less
than or equal to the phi-index the precipitation excess for that time period is set equal to zero. This procedure yields the precipitation excess array.

**Unit Hydrograph Model** - The 15 minute unit hydrograph employed in the watershed simulation model was derived from the event of July 26, 1969 with an effective duration of 30 minutes and a direct runoff of .181 inches and the event of June 12, 1970, with an effective duration of 15 minutes and a direct runoff of .211 inches. The runoff hydrograph of the event of June 12, 1970, was converted directly into a 15 minute unit hydrograph by dividing the observed ordinates by the direct runoff. The event of July 26, 1969, was converted to a 30 minute unit hydrograph in a similar manner. This 30 minute unit hydrograph was then used to build a 30 minute S curve from which the 15 minute unit hydrograph was determined. Figure 3 shows the 15 minute unit hydrographs derived from each event. The unit hydrograph used in the total model is a mean curve lying between the two observed hydrographs and is also shown in Figure 3.

**COMPARISON OF SYNTHETIC TO OBSERVED FLOOD FREQUENCY CURVES**

The simulation program incorporating all of the previously described models was operated until 30 annual flood peaks had been generated. This was done three times in order to define three synthetic flood series. Each flood series was used to define a separate flood frequency curve.

An approximation of the observed flood frequency curve was obtained by calculating the return period of each observed
FIG. 3 - 15 MINUTE UNIT HYDROGRAPHS
annual maximum flood peak by the Weibull plotting position
formula; plotting the observed peaks versus the calculated
return periods on extreme value probability paper and estab-
lishing a line of best fit thru these plotted points. Figure
4 shows the observed flood peaks and the line of best fit.
Also shown in Figure 4 are the 90% confidence limits on the
observed curve, on the interval most used for design of small
drainage structures, i.e., 10 to 50 year return period. These
confidence limits were calculated by procedures reported by
Viessman, Harbaugh and Knapp.(11)

Approximations of the synthetic flood frequency curves
defined by the three generated flood series were obtained by
repeated application of the following formula.(11)

\[ Q_t = \bar{Q} + K(n,t) S_q \] (10)

where \( Q_t \) is an estimate of the, \( t, \) year flood; \( \bar{Q} \) is the calcu-
lated mean of the generated flood series; \( K(n,t) \) is a frequency
factor whose value depends on the sample size, \( n, \) and the flood
return period, \( t; \) and \( S_q \) is the calculated standard deviation
of the generated flood series. Equation 10 was used to cal-
culate flood peak estimates for several return periods, for
each of the three generated flood series. These flood peak
estimates were plotted on extreme value probability paper and
connected with a straight line. These generated flood frequency
curves are also shown on Figure 4. From Figure 4 it may be
seen that the three generated flood frequency curves plot con-
siderably below the observed curve but above the lower 90%
confidence limit. In addition, agreement between the three
generated flood series may be considered good.
FIG. 4.- GENERATED AND OBSERVED FLOOD FREQUENCY CURVES
Recently, a deterministic model for estimating synthetic flood frequency curves for small rural watersheds in Missouri was developed by Harbaugh and Thompson (1970)\(^4\). This model was developed from a multiple regression analysis of existing annual maximum peak flow rate data. The model takes the form of a set of regression equations relating the peak flow rate for a given return period to various physical parameters of the watershed. These equations were used to estimate the flood peaks for the 10, 25 and 50 year return periods for the study watershed.

Two additional synthetic flood frequency curves were estimated by deterministic methods. The first was calculated by methods used by the S.C.S.\(^9\) and the second was calculated by use of the rational method.

The writer has recently completed a study of the accuracy levels to be expected from six deterministic methods for estimating flood flows from small rural watersheds within the state of Missouri. This study was done for the Missouri State Highway Department and is as yet unpublished. Of the six methods investigated the above three were found to yield the best accuracy when applied to rural watersheds within the state of Missouri less than 1000 acres in size. Thus it is assumed that the synthetic flood flow estimates resulting from application of the above methods are among the best available.

Table 3 shows the 10, 25 and 50 year flood flows estimated from the observed flood series; the three generated flood series; and the three deterministic methods. From Table 3 it may be
TABLE 3. - FLOOD FLOW ESTIMATES FROM OBSERVED FLOOD SERIES AND SYNTHETIC METHODS

<table>
<thead>
<tr>
<th>Flood Frequency Curve</th>
<th>Mean, cubic feet per second</th>
<th>Standard Deviation, cubic feet per second</th>
<th>10 Year Flood cubic feet per second</th>
<th>25 Year Flood cubic feet per second</th>
<th>50 Year Flood cubic feet per second</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>449</td>
<td>211</td>
<td>725</td>
<td>880</td>
<td>995</td>
</tr>
<tr>
<td>Run No. 1</td>
<td>373</td>
<td>100</td>
<td>527</td>
<td>612</td>
<td>676</td>
</tr>
<tr>
<td>Run No. 2</td>
<td>374</td>
<td>115</td>
<td>551</td>
<td>649</td>
<td>722</td>
</tr>
<tr>
<td>Run No. 3</td>
<td>359</td>
<td>80</td>
<td>482</td>
<td>550</td>
<td>601</td>
</tr>
<tr>
<td>Regression Equations</td>
<td></td>
<td></td>
<td>391</td>
<td>500</td>
<td>588</td>
</tr>
<tr>
<td>S.C.S. Method</td>
<td></td>
<td></td>
<td>563</td>
<td>699</td>
<td>840</td>
</tr>
<tr>
<td>Rational Method</td>
<td></td>
<td></td>
<td>509</td>
<td>600</td>
<td>673</td>
</tr>
</tbody>
</table>
seen that all six synthetic flood estimates for all return periods considered are less than the observed values. Also the three generated flood frequency curves yield flood flow rate estimates which are quite comparable to the estimates obtained from the three deterministic methods. In general the simulation model did not yield flood frequency curves which are good representations of the observed curve. However the model did yield curves which agree well with other synthetic methods.

The model generates representative flood flows except for the more extreme events. For example, the largest flood flow generated in the total 90 years of simulation is 597 c.f.s. According to the observed flood frequency curve an event of this magnitude should have a return period equal to 5 years. This means that approximately 18% of the generated events should have had magnitudes equal to or greater than the maximum simulated event.

Extreme flood events arise out of a combination of extreme rainfall depths and wet antecedent moisture conditions. The model for rainfall depths was modified in order to incorporate a separate model for extreme events. In the case of the rainfall-runoff model a maximum of only three antecedent moisture conditions for each season were defined. Therefore, extreme precipitation excess events may not have been adequately simulated. The writer believes that the basic idea of a probabilistic precipitation excess model will work. However, more than three antecedent conditions may have to be defined. Special
attention should be given to the modeling of those rainfall-runoff events which are extreme. More research is needed in this area. Development of a relationship between total rainfall and rainfall excess in the form of a probabilistic model would be a valuable tool.

In general the statistical hydrologic simulation model developed here will yield an approximate estimate of the mean annual flood. However, due to its inability to reproduce higher order flood events the model yields low estimates of the standard deviation of floods.

CONCLUSIONS

Simulation modeling of a natural watershed system is a useful method for investigating the interactions of the systems components. In order to build a simulation model it is necessary to acquire an understanding of each component and its relative importance in the total system.

In this investigation a simulation model of a hydrologic system was developed. Several conclusions can be made based on this study as follows:

1. The time between and duration of storms may be represented by an exponential distribution model.

2. Monte Carlo sampling of these exponential models is adequate for generation of random observations of time between and duration of storms.

3. The depth of rain falling in a given time period may be represented by a log-normal distribution model. The fit of the log-normal model to the extreme rainfall depths is best for the distribution for which the most observations are available. Consideration should be given to providing a separate model for extreme rainfall events, if the log-normal fit is not adequate.
4. Monte Carlo sampling will not reproduce the interdependency of adjacent rainfall depths. Therefore, it is necessary to make some assumptions in regard to the time distribution of rainfall when using Monte Carlo techniques to generate observations of rainfall depths. Considering the interdependency of rainfall depths a sequential rather than a Monte Carlo simulation may be a better model for this component of the system.

5. The discrete, three state, probabilistic precipitation excess model developed in this study may not be adequate for use in a model whose primary purpose is to generate synthetic flood flows. However, this type model may be adequate if the purpose of the total model is not generation of extreme events. A conceptual rather than a probabilistic precipitation excess model may be the best approach to the simulation of this component of the system.

Although the simulation model did not yield a representation of the study watersheds flood frequency curve which could be considered adequate, the model did produce a curve which was as good as three other synthetic methods. The concept of using short term continuous rainfall-runoff data to build and calibrate a model for the generation of long term synthetic records is sound and deserves further investigation. At least some of the components of such a model can be well represented by probabilistic models. The optimum representation of a hydrologic system may be a combination of Monte Carlo, sequential and conceptual simulation models.

ACKNOWLEDGEMENTS

The writer would like to express his appreciation to Dr. T. E. Harbaugh for his advice during the study and his extensive review of the paper.
APPENDIX - I - REFERENCES


APPENDIX - II - NOTATION

The following symbols are used in this paper:

- \( D \) = Random observation of rainfall depth
- \( \exp \) = Exponential (natural antilogarithm)
- \( K(n,t) \) = Frequency factor
- \( \ln \) = Natural logarithm
- \( \bar{M} \) = Mean of the distribution of \( T \)
- \( \bar{M}_d \) = Mean of the distribution of the natural logarithms of \( D \)
- \( NV \) = Normal random variate i.e. a normally distributed random number with a mean of 0 and a standard deviation of 1
- \( PE \) = Precipitation excess for a given storm
- \( P_e \) = Precipitation excess array
- \( PT \) = Precipitation total for a given storm
- \( P_t \) = Precipitation total array
- \( \bar{Q} \) = Mean of a flood series
- \( Q_t \) = Estimate of the \( t \), year flood
- \( RN \) = Uniformly distributed random number between 0 and 1
- \( S_e \) = Standard error
- \( SIG \) = Standard deviation of the distribution of the natural logarithms of \( D \)
- \( S_q \) = Standard deviation of a flood series
- \( T \) = Random observation of time
VITA

Ronald Lee Wycoff was born on December 20, 1945, in St. Louis, Missouri. He received his primary and secondary education in Kirkwood, Missouri. He has received his college education from the University of Missouri at Rolla, in Rolla, Missouri, where he received a Bachelor of Science degree in Civil Engineering, in January, 1968.

Upon graduation he was employed by Brown and Root Inc. of Houston, Texas. In February, 1969, he joined the Missouri State Highway Department with whom he is presently employed. In January of 1970 he enrolled in the Graduate School of the University of Missouri at Rolla where he was a graduate Teaching Assistant. In December of 1970 he returned to full time employment with the Missouri State Highway Department in Springfield, Missouri, where he has been working on a part-time basis to complete his graduate education.
APPENDIX A

FORTRAN LISTING OF

SIMULATION PROGRAM
A STATISTICAL HYDROLOGIC SIMULATION MODEL

THE STATISTICAL DISTRIBUTIONS AND OTHER MATHEMATICAL MODELS APPEARING IN THIS PROGRAM ARE DERIVED FROM APPROXIMATELY ONE YEAR AND TWO MONTHS OF CONTINUOUS RAINFALL - RUNOFF RECORDS FOR A .71 SQUARE MILE WATERSHED, U.S.G.S. GAGE NO. 5-5027.0.

THE VARIABLES THAT APPEAR IN THIS PROGRAM ARE DEFINED AS FOLLOWS:

IT = TIME UNIT ( 1 UNIT = 15 MIN.)
IYEAR = YEAR
PT(IT) = PRECIPITATION TOTAL (RAIN) FOR EACH TIME UNIT
WSY = WATERSHED YIELD
RN = A UNIFORMLY DISTRIBUTED RANDOM NUMBER BETWEEN 0 AND 1.
RANN = THE RANDOM NUMBER GENERATOR SUBPROGRAM
ITS = TIME BETWEEN STORMS
ID = DURATION OF STORM
ITST = TIME AT START OF STORM
ITEND = TIME AT END OF STORM
PTOT = TOTAL PRECIPITATION FOR STORM
PEX = EXCESS PRECIPITATION (RUNOFF) FOR STORM
PI = PHI INDEX
SPEXX = SUM OF PRECIPITATION EXCESS FOR A TRIAL PHI INDEX
PEXX(J) = PRECIPITATION EXCESS FOR TIME PERIOD - J - AND A TRIAL PHI INDEX
DPEX = DIFFERENCE BETWEEN SPEXX AND PEX FOR A GIVEN TRIAL VALUE OF THE PHI INDEX
PE(IT) = PRECIPITATION EXCESS FOR EACH TIME UNIT
U(I) = UNIT HYDROGRAPH ARRAY
RUNO = RUNOFF, IE., PRECIPITATION EXCESS ARRAY MULTIPLIED BY THE UNIT HYDROGRAPH ARRAY IN C.F.S.
QP = PEAK RATE OF RUNOFF FOR YEAR

***** ***** ***** ***** ***** *****

DIMENSION AND INITIALIZE

DIMENSION PT(35050), PE(35050), U(24), PEXX(2000)
IT=1
IYEAR=1
PTOT=0.0
PEX=0.0
WSY=0.0
IRN1=2157873
M=2147483647
C1=1.0/SQRT(5.0/12.0)
C2=2.5*C1

READ IN AND WRITE OUT UNIT HYDROGRAPH ARRAY

READ(1,500)(U(J), J=1, 24)
500 FORMAT(F11.1)
DO 5 J=1, 24
WRITE(3,500)U(J)
5 CONTINUE

GENERATE TIME BETWEEN STORMS

10 CALL RANN(RN, IRN1, M)
IF(RN.EQ.0.0) RN=.00001
WRITE(3,2000)IYEAR, IT, RN, WSY
2000 FORMAT(JX,'YEAR=',I2,)X,'T=',I6,)X,'RN=',F11.8,)X,'WSY=',F11.8)
IF(IT.LE.17520) GO TO 20
ITS=ALOG(RN)/(-1.0/597.5895)+0.5
GO TO 30
20 ITS=ALOG(RN)/(-1.0/2565.333)+0.5
30 ITST=IT+ITS
   IF(ITST.GT.35040) ITST=35040
40 IF(IT.GE.ITST) GO TO 50
   PE(IT)=0.0
   IT=IT+1
   GO TO 40

C GENERATE DURATION OF STORM

C

50 CALL RANN(RN,IRN1,M)
   IF(RN.EQ.0.0) RN=.00001
   IF(IT.LE.17521) GO TO 60
   IF(IT.LT.35040) GO TO 70
   Go To 200
60 ID=ALOG(RN)/(-1.0/46.83332)+0.5
   Go To 80
70 ID=ALOG(RN)/(-1.0/28.0)+0.5

C GENERATE DEPTH OF RAIN FOR EACH TIME UNIT OF STORM

C

80 ITEND=IT+ID
   IF(ID.EQ.0) GO TO 10
   IF(ITEND.GT.35040) ITEND=35040
   PTOT=0.0
85 IF(IT.LT.17521) GO TO 90
   IF(ID.LE.16) GO TO 88
86 SUMRN=0.0
   DO 87 I=1,5
     CALL RANN(RN,IRN1,M)
     SUMRN=SUMRN+RN
87 CONTINUE
   RNORM=C1*SUMRN-C2
   PT(IT)=EXP(RNORM*1.168316-4.392096)
   GO TO 100
88 SUMRN=0.0
  DO 89 I=1,5
    CALL RANN(RN,IRN1,M)
    SUMRN=SUMRN+RN
  89 CONTINUE
  RNORM=C1*SUMRN-C2
  PT(IT)=EXP(RNORM*0.9234821-2.956989)
  IF(PT(IT).GT.0.150) GO TO 801
  GO TO 100

801 CALL RANN(RN,IRN1,M)
  IF(RN.LT.0.74) GO TO 805
  IF(RN.LT.0.96) GO TO 810
  CALL RANN(RN,IRN1,M)
  PT(IT)=0.80+RN*0.75
  GO TO 100

805 CALL RANN(RN,IRN1,M)
  PT(IT)=0.15+RN*0.20
  GO TO 100

810 CALL RANN(RN,IRN1,M)
  PT(IT)=0.35+RN*0.45
  GO TO 100

90 SUMRN=0.0
  DO 91 I=1,5
    CALL RANN(RN,IRN1,M)
    SUMRN=SUMRN+RN
  91 CONTINUE
  RNORM=C1*SUMRN-C2
  PT(IT)=EXP(RNORM*1.079724-3.822251)
  IF(PT(IT).GT.0.08) GO TO 901
  GO TO 100

901 CALL RANN(RN,IRN1,M)
  IF(RN.LT.0.79) GO TO 905
  IF(RN.LT.0.97) GO TO 910
  CALL RANN(RN,IRN1,M)
  PT(IT)=0.70+RN*0.50
  GO TO 100
CALL RANN(RN,IRN1,M)
PT(IT) = 0.08 + RN * 0.16
GO TO 100

CALL RANN(RN,IRN1,M)
PT(IT) = 0.24 + RN * 0.46

C

SUM RAINFALL DEPTHS
C

100 PTOT = PTOT + PT(IT)
   IT = IT + 1
   IF (IT .LE. ITEND) GO TO 85

C IF THE DURATION IS LESS THAN 4 HOURS SORT THE RAINFALL DEPTHS
C INTO DESCENDING ORDER
C

IF (ID .GT. 16) GO TO 9
IF (ID .LE. 1) GO TO 9
IT = ITST
   K = IT + ID - 2
   J = ITST
19 L = 0
   DO 2 IT = J, K
      IF (PT(IT) .GE. PT(IT + 1)) GO TO 2
   3 IF (L) 22, 21, 22
   21 J1 = IT - 1
   22 SAVE = PT(IT + 1)
      PT(IT) = PT(IT + 1)
      PT(IT + 1) = SAVE
      L = IT
   2 CONTINUE
      IF (L .EQ. 0) GO TO 9
   8 K = L
      IF (J1 .LE. 0) GO TO 6
   7 J = J1
   GO TO 19
   9 IT = ITEND + 1
SELECT RAINFALL RUNOFF RELATION AND CALCULATE TOTAL RUNOFF FOR STORM

WRITE(3,300)ID,PT(IT-1)
300 FORMAT(3X,'DUR=',16,3X,'PT=',F11.8)
IF(IT.LT.17521) GO TO 120
IF(IT.LT.26281) GO TO 130
GO TO 140
120 PEX=-0.390253+0.8131443*PTOT
GO TO 150
130 CALL RANN(RN,IRN1,M)
IF(RN.LE.3333) GO TO 132
IF(RN.LE.6667) GO TO 134
PEX=-0.168909+0.8755121*PTOT
GO TO 150
132 PEX=-0.05264467+0.441014*PTOT
GO TO 150
134 PEX=-0.0350666+0.213724*PTOT
GO TO 150
140 CALL RANN(RN,IRN1,M)
IF(RN.LE.3333) GO TO 142
IF(RN.LE.6667) GO TO 144
PEX=-0.1775965+0.9327939*PTOT
GO TO 150
142 PEX=-0.1970548+0.5917342*PTOT
GO TO 150
144 PEX=-0.01674533+1.11443*PTOT
150 IF(PEX.LT.0.0) PEX=0.0
IF(PEX.GT.PTOT) PEX=PTOT

SUM RUNOFF DEPTHS TO OBTAIN THE WATERSHED YIELD
WSY=WSY+PEX

SOLVE FOR PHI-INDEX
C
PI=(PTOT-PEX)/ID
165 J=1
SPEXX=0.0
170 PEXX(J)=PT(J+ITST-1)-PI
   IF(PEXX(J).LT.0.0) PEXX(J)=0.0
   SPEXX=SPEXX+PEXX(J)
   J=J+1
   IF(J.LE.ID) GO TO 170
   DPEX=SPEXX-PEX
   IF(DPEX.LT.0.0) GO TO 175
   PI=PI+(.01/ID)
   GO TO 165
C
SUBTRACT PHI-INDEX FROM PT ARRAY TO OBTAIN PE ARRAY FOR STORM
C
175 IT=ITST
180 PE(IT)=PT(IT)-PI
   IF(PE(IT).LT.0.0) PE(IT)=0.0
   IT=IT+1
   IF(IT.LE.ITEND) GO TO 180
   WRITE(3,4000)IT,PE(IT-1)
4000 FORMAT(3X,'T=',I6,3X,'PE=',F11.8)
C
IF PE ARRAY FOR YEAR HAS BEEN GENERATED THEN MULT. PE ARRAY BY THE
UHG ARRAY TO OBTAIN RUNOFF ARRAY, SEARCH THIS ARRAY FOR THE PEAK
FLOW RATE FOR THAT YEAR.
C
IF(IT.LT.35040) GO TO 10
200 QP=0.0
   DO 210 J=24,35040
      RUNO=PE(J)*U(1)+PE(J-1)*U(2)+PE(J-2)*U(3)+PE(J-3)*U(4)+PE(J-4)*U(5)
   1+PE(J-5)*U(6)+PE(J-6)*U(7)+PE(J-7)*U(8)+PE(J-8)*U(9)+PE(J-9)*U(10)
   2+PE(J-10)*U(11)+PE(J-11)*U(12)+PE(J-12)*U(13)+PE(J-13)*U(14)+PE(J)
   3-14)*U(15)+PE(J-15)*U(16)+PE(J-16)*U(17)+PE(J-17)*U(18)+PE(J-18)*U
4(19)+PE(J-19)*U(20)+PE(J-20)*U(21)+PE(J-21)*U(22)+PE(J-22)*U(23)+P
5E(J-23)*U(24)
IF(RUNO.GT.QP) QP=RUNO
210 CONTINUE

C OUTPUT REINITIALIZE AND CONTINUE
C
WRITE(3,1000)IYEAR,QP,WSY
1000 FORMAT('YEAR=',I2,'PEAK FLOW=',F10.1,'WATERShed YIEL
D=',F9.5)
IYEAR=IYEAR+1
IT=1
WSY=0.0
IF(IYEAR.LE.5) GO TO 10
STOP
END

SUBROUTINE RANN(RNN,IRNN1,MN)
IRNN1=IRNN1*131075
RNN1=IRNN1
DENOM=MN
RNN=ABS(RNN1/DENOM)
RETURN
END
APPENDIX B

DATA AND DISTRIBUTION MODELS

Figures A1 thru A7 illustrate the cumulative histogram of the observed data, designated "data" on the figure and the cumulative distribution model derived from this data, designated "model" on the figure, for each of the seven statistical distribution models used in the total model. The symbol x shown on the cumulative histograms indicates points of observed cumulative probability.

Table A1 is the rainfall-runoff data used to derive the seven linear least squares equations used in the rainfall-runoff model.

Table A2 is the observed and generated flood series.
FIG. A1. - TIME BETWEEN STORMS, SUMMER
FIG. A2 - TIME BETWEEN STORMS, WINTER
FIG. A3. - DURATION OF STORM, SUMMER
FIG. A4 - DURATION OF STORM, WINTER
FIG. A5.- DEPTH OF RAIN, SUMMER, SHORT DURATION
FIG. A6.- DEPTH OF RAIN, SUMMER, LONG DURATION
FIG. A7. - DEPTH OF RAIN, WINTER
TABLE A1 - OBSERVED RAINFALL-RUNOFF DATA

a) EARLY SUMMER

<table>
<thead>
<tr>
<th>Wet Antecedent Condition</th>
<th>Mean Antecedent Condition</th>
<th>Dry Antecedent Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rainfall Inches</td>
<td>Runoff Inches</td>
<td>Rainfall Inches</td>
</tr>
<tr>
<td>0.10</td>
<td>0.052</td>
<td>0.20</td>
</tr>
<tr>
<td>1.10</td>
<td>0.804</td>
<td>0.20</td>
</tr>
<tr>
<td>0.30</td>
<td>0.712</td>
<td>1.30</td>
</tr>
<tr>
<td>1.90</td>
<td>1.247</td>
<td>1.30</td>
</tr>
<tr>
<td>2.10</td>
<td>1.577</td>
<td>1.80</td>
</tr>
<tr>
<td>2.80</td>
<td>2.743</td>
<td></td>
</tr>
</tbody>
</table>

b) LATE SUMMER

<table>
<thead>
<tr>
<th>Wet Antecedent Condition</th>
<th>Mean Antecedent Condition</th>
<th>Dry Antecedent Condition</th>
</tr>
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<tbody>
<tr>
<td>Rainfall Inches</td>
<td>Runoff Inches</td>
<td>Rainfall Inches</td>
</tr>
<tr>
<td>0.10</td>
<td>0.069</td>
<td>0.30</td>
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<td>0.50</td>
</tr>
<tr>
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<td>0.60</td>
</tr>
<tr>
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<td>0.047</td>
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</tr>
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<td>0.121</td>
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</tr>
<tr>
<td>0.40</td>
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<td>3.30</td>
</tr>
<tr>
<td>2.50</td>
<td>2.367</td>
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c) WINTER

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<thead>
<tr>
<th>Rainfall Inches</th>
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<tr>
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<tr>
<td>0.90</td>
<td>0.161</td>
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<tr>
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<td>0.633</td>
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<tr>
<td>3.60</td>
<td>1.211</td>
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<tr>
<td>5.70</td>
<td>6.289</td>
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1. Adjusted to 1.50 inches
2. Adjusted to 1.20 inches
3. Adjusted to 7.40 inches
TABLE A2 - GENERATED AND OBSERVED FLOOD FLOWS

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<tr>
<th>Observed Floods</th>
<th>Generated Floods</th>
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<tbody>
<tr>
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