A study of cathode drift compensation in D-C amplifiers

Joseph W. Rittenhouse

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A STUDY OF CATHODE DRIFT
COMPENSATION IN D-C AMPLIFIERS

BY

JOSEPH W. RITTENHOUSE

A

THESIS

submitted to the faculty of the

SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI

in partial fulfillment of the work required for the

Degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

Rolla, Missouri

1949

Approved by

Professor of Electrical Engineering
ACKNOWLEDGMENT

The author desires to express his sincere appreciation for the invaluable advice and recommendations given to him by Professor Carl Johnk during the course of this work.
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A Study of Cathode Drift Compensation in D-C Amplifiers.

I. Introduction

The expression, cathode drift, has to do with the random variations in vacuum tube plate current which are attributable to similar variations in emission velocities of electrons leaving hot cathodes. The fact that these random current changes appear to result from spurious drifts in cathode potential, since all other electrode potentials are unrelated to them, give rise to the expression, cathode drift.

The exact nature of the cause of cathode drift is not known in great detail. It is known, however, that factors contributing to the phenomenon are: variations in work function of the cathode (which are partially attributable to inability to produce absolutely pure cathode materials) and random fluctuations in cathode temperature (which may be caused by such things as unstable heater voltage, random changes in ambient temperature, and non-uniform flow of cooling gases over the cathode).

It is significant, in this respect, that the effects of cathode drift are exactly as though some random noise-voltage generator were located in the cathode lead of the vacuum tube in question. Such a generator would introduce random changes in cathode-to-grid potential difference which would exactly account for the variations in plate current which
are the only evidences of cathode drift.

The difficulties which arise from cathode drift are largely associated with the application of thermionic tubes to direct-coupled amplifier problems. For example: a triode operating as a voltage amplifier in the input stage of a direct-coupled amplifier is unable to distinguish between those changes in plate current which stem from changes in input signal and those which arise from the cathode-drift effect; consequently, both sources of plate current change will produce a change in output voltage.

Since, in a d-c amplifier, the coupling between stages is a conductive coupling, the relatively long-time-varying output voltage changes which were initiated in one stage by cathode drift appear to the next stage as an integral part of the amplified signal voltage.

The unique association of this problem with direct-coupled amplifiers, of course, is a result of the fact that the cathode-drift plate current changes invariably are composed of such low-frequency components that their passage through the coupling networks of any of the non-direct-coupled amplifiers is inhibited so that they hardly appear in the output signal at all.

The problem with which this paper is concerned is a study of some of the possibilities of compensating for cathode drift effects in a single amplifier stage. Specifically, this paper will:
(A) present a detailed mathematical and graphical analysis of the so-called Miller circuit and using this analysis, together with experimental evidence, will remove some of the limitations which the originator of this circuit initially associated with it; and

(B) propose a new dual triode circuit, having only one resistor at the low voltage level, which will compensate for cathode drift while operating from a single B voltage supply.

II. Review of Literature

It would appear that the simplest possible solution to the problem of cathode drift might be simply to eliminate its original cause. A review of the literature, however, will reveal that removal of the cause is hardly an economical approach to the solution, since such an approach would require one to develop sources of absolutely fixed voltages for filament supplies, and to produce cathode materials whose work functions are absolutely fixed in magnitude.

A more reasonable approach, then, would seem to be one which seeks to nullify the normal effect of cathode drift rather than to eliminate the drift itself. This is the approach taken by Miller(2) in his development of the circuit which bears his name.


circuit which bears his name.

This circuit, shown in Figure 1, consists of a dual triode, one section of which is connected as a conventional amplifier, while the other section behaves as a cathode follower with self bias which, through the effect of its plate current in the cathode resistors $R_1$ and $R_2$, produces the change in voltage at the amplifier cathode which is just necessary to prevent the change in amplifier plate current, $i_1$, which would normally accompany a given change in electron emission velocity.
The Miller Circuit, Showing the Equivalent Cathode Noise Voltage Generator, \( e_n \)

Figure 1
The voltage, $e_n$, shown in series with the cathodes of the tubes in Figure 1 is a fictitious voltage whose random variations are assumed to be responsible for the random changes in plate current that are caused by cathode drift.

The notation of Figure 1 will be employed throughout the discussion of the Miller circuit.

The mechanism by which compensation is accomplished in this circuit does not lend itself to simple qualitative explanation, yet it has been shown both mathematically and experimentally that compensation is effected.\(^{(3)(4)}\)

\(^{(3)}\) Ibid.


In his original paper, Mr. Miller did not include all the steps involved in his mathematical treatment of this circuit; however, on the basis of what he did present, the following analysis has been developed and it is believed to be very nearly identical to that which was prepared by Miller.

By referring to Figure 1, the following equations can be written if $R_L$ is assumed to be large enough to make $i_1$ large compared to $i_2$:

\[
i_2 = \frac{\mu_2(e_n - R_1i_2) + e_n}{R_1 + R_2 + r_2}
\]  \hspace{1cm} . . . (1)
\[ e = e_n - (R_1 + R_2)i_2 \] ... (2)

Solving for \( e_n \) from Equation (2) and substituting for \( e_n \) in Equation (1):
\[ i_2 = \frac{\mu_2[e + i_2(R_1 + R_2) - R_1i_2] + e + i_2(R_1 + R_2)}{R_1 + R_2 + r_2} \]

where:
\[ \mu_2 = \text{Amplification factor of } T_2 \]
\[ i_2 = \text{Plate current of } T_2 \]
\[ r_2 = \text{Plate resistance of } T_2 \]

Expanding and simplifying:
\[ i_2R_1 + i_2R_2 + i_2r_2 = \mu_2e + i_2\mu_2R_1 + i_2\mu_2R_2 - \mu_2i_2R_1 + e + i_2R_1 + i_2R_2 \]
\[ e(\mu_2 + 1) = i_2(r_2 - \mu_2R_2) \]
\[ e = \frac{i_2(r_2 - \mu_2R_2)}{\mu_2 + 1} \] ... (3)

From Equation (2)
\[ i_2 = \frac{e_n - e}{R_1 + R_2} \]

Substituting in (3)
\[ e = \frac{(e_n - e)(r_2 - \mu_2R_2)}{(\mu_2 + 1)(R_1 + R_2)} \]
\[ = \frac{e_n r_2 - \mu_2R_2 e_n - e r_2 + \mu_2 e R_2}{\mu_2 R_1 + \mu_2 R_2 + R_1 + R_2} \]

expanding and simplifying:
\[ e = \frac{e_n (r_2 - \mu_2 R_2)}{(\mu_2 + 1) R_1 + R_2 + r_2} \] ... (4)
Now, Miller reasons, if \( e \) becomes equal to zero, then the random variations in \( e_n \) will have been compensated; so if the right hand side of Equation (4) is equated to zero there is obtained the criterion:

\[
R_2 = \frac{r_2}{\mu_2} = \frac{1}{\varepsilon m_2}
\]

where \( \varepsilon m_2 \) = the transconductance of \( T_2 \).

From this work stems the conclusion that the circuit of Figure 1 will compensate for cathode drift effects provided that \( R_L \) is made very large and provided that \( R_2 = \frac{1}{\varepsilon m_2} \).

It will be observed that the assumption that \( R_L \) is large is a rather arbitrary one, the reason for which seems to be solely to simplify the writing of Equations (1) and (2) by permitting voltage drops to be written in terms of \( i_2 \) alone instead of in terms of the more exact expression, \((i_1 + i_2)\).

While it is true that such an assumption does facilitate the writing of the initial equations, it is equally true that it limits the applicability of the Miller circuit to problems in which frequency response is of secondary importance to gain requirements. This rather serious limitation, together with certain seeming errors in another author's discussion of the Miller circuit,

---

led the author into a preliminary investigation which seemed to corroborate his original suspicion that the foregoing limitation was an unnecessary one.

This same preliminary investigation revealed the additional possibility that there could be developed a new compensating circuit involving two triodes which would have a twofold advantage over the Miller circuit in that it could operate from a single regulated B+ supply, and it would have one less resistance at its low voltage level than does the Miller circuit.

It should be pointed out that the elimination of one resistance at a low voltage level becomes exceedingly important in those applications in which the noise voltages generated in the resistance are of the same approximate order of magnitude as are the signal voltages to be amplified.
III. A-C Equivalent Circuit Analysis of The Miller Circuit.

It will be recalled from Figure 1 and previous discussion that other workers (6)(7)(8) have considered it satisfactory to represent cathode drift as a random noise-voltage generator located in the cathode of the tube under consideration. In fact, Miller's analysis presumes that the noise voltages associated with the cathode drift of the two sections of a dual triode are identical if the two sections have a common cathode.

The fact that Miller's presumption is largely justifiable is evidenced by the high degree of compensation which his experiments revealed were possible; consequently, for the purposes of this portion of this paper it was assumed that the cathode drift effect can be represented in the Miller Circuit by a single random noise-voltage generator located in the common cathode lead of the dual triode.

Making this assumption the A-C equivalent circuit associated with the circuit of Figure 1, when the grid of $T_1$ is grounded, is shown in Figure 2 where:

\[ r_1 = \text{the plate resistance of } T_1 \]
\[ r_2 = \text{...} \]
\[ T_2 \]
A-C Equivalent Circuit for Miller Circuit

Figure 2
\(\mu_1\) = the amplification factor of \(T_1\)

\(\mu_2\) = " " " " " " \(T_2\)

From an analysis of Figure 1, taking into consideration proper phase relationships, it can be seen that with the grid of \(T_1\) grounded:

\[e_{g1} = (i_1 + i_2)(R_1 + R_2) - e_n \ldots (5)\]

and

\[e_{g2} = (i_1 + i_2)R_1 - e_n \ldots (6)\]

Using Equation (5) and applying Kirchhoff's law of summation of voltages around the top loop of Figure 2, the following expression was obtained:

\[\mu_1[(i_1 + i_2)(R_1 + R_2) - e_n] = e_n((i_1 + i_2)(R_1 + R_2) - i_1(R_3 + r_1)\]

Rearranging:

\[i_1[(\mu_1 + 1)(R_1 + R_2) + R_3 + r_1] + i_2(\mu_1 + 1)(R_1 + R_2) = (\mu_1 + 1)e_n \ldots (7)\]

Now, using Equation (6) in a similar summation around the bottom loop of Figure 2, the following results were achieved:

\[\mu_2[(i_1 + i_2)R_1 - e_n] = e_n((i_1 + i_2)(R_1 + R_2) - i_2r_2\]

Rearranging:

\[i_1[(\mu_2 + 1)R_1 + R_2] + i_2[(\mu_2 + 1)R_1 + R_2 + r_2] = (\mu_2 + 1)e_n \ldots (8)\]
Solving Equations (7) and (8) for \( i_1 \) by determinants:

\[
\frac{i_1}{\text{Denominator}} = \frac{\{(\mu_1 + 1)[(\mu_2 + 1) R_1 + R_2 + r_2] - (\mu_2 + 1)(\mu_1 + 1)(R_1 + R_2)\}}{\text{Denominator}}
\]

Since the denominator obviously consists of constants only:

\[
\frac{\delta i_1}{\delta e_n} = \frac{(\mu_1 + 1)[(\mu_2 + 1) R_1 + R_2 + r_2]}{X} - \frac{(\mu_2 + 1)(R_1 + R_2)(\mu_1 + 1)}{X}
\quad \cdots (9)
\]

Since \( \frac{\delta i_1}{\delta e_n} = 0 \) is the condition for which this circuit was originally developed, it was proposed to equate the right hand side of Equation (9) to zero and solve for the conditions under which this equality would exist.

Let

\[
\frac{\delta i_1}{\delta e_n} = 0
\]

Then from Equation (9)

\[
\frac{(\mu_1 + 1)[(\mu_2 + 1) R_1 + R_2 + r_2]}{X} - \frac{(\mu_2 + 1)(R_1 + R_2)(\mu_1 + 1)}{X} = 0
\]

But \((\mu_1 + 1)\) cannot equal zero and the denominator must be finite, and greater than zero, so:

\[
(\mu_2 + 1) R_1 + R_2 + r_2 - (\mu_2 + 1)(R_1 + R_2) = 0
\]
Simplifying:

\[ R_2 = \frac{r_2}{\mu_2} = \frac{1}{g_{m2}} \]  

\[ \ldots (10) \]

Where \( g_{m2} \) is the transconductance of \( T_2 \).

The significant aspect of this treatment is that it introduces no assumption whatsoever regarding the relative magnitudes of the various resistances in the circuit, yet the criterion for \( \frac{\partial I_1}{\partial \varepsilon_n} = 0 \) which it yields is identical to criterion which Miller obtained from a derivation based on the assumption that \( R_L \) is so large that the plate current of \( T_1 \) is negligible compared with the plate current of \( T_2 \).

This would seem to indicate that the requirement of a large \( R_L \) in this circuit is an unwarranted one. It was felt, however, that a mere mathematical treatment was inconclusive in firmly substantiating this contention; so a program of experimentation was initiated in an effort to confirm the foregoing indications that the Miller circuit will compensate for cathode drift even though \( R_L \) may be small.
IV. Graphical Analysis of the Miller Circuit Containing a Large Load Resistor

In order to compare the extent to which the Miller circuit would compensate for cathode drift effects when both large and small load resistances were used in the amplifier section, it was first necessary to determine the values of the various circuit elements to be employed in the circuit; consequently, the following graphical analysis was proposed as a basis for the selection of circuit components in the case in which a large load resistance is used. This analysis is for a circuit containing a 6SL7 dual triode.

On the characteristic curves of the 6SL7 in Figure 3 was drawn a load line, AB, to represent the 1-megohm load which was selected for the case in which $R_L$ is large. From this construction, it can be seen that a 255-volt supply was selected for the amplifier tube.

After having drawn this load line, a reasonable operating point was assumed as is indicated by point C in Figure 3. This operating point established the bias on $T_1$ as -2 volts and the static plate current of $T_1$ as 0.125 milliamperes.

Having established an operating point for $T_1$, values for $(R_1 + R_2)$ and $i_2$ were assumed such that the product of the assumed $(R_1 + R_2)$ and $(i_1 + i_2)$ would yield the desired bias of -2 volts on $T_1$. 
Since the assumed value of \((R_1 + R_2)\) is the only load on \(T_2\), a load line could be drawn on Figure 3 for \(T_2\). In this case \((R_1 + R_2)\) was found to be 1778 ohms when \(i_2\) was assumed to be 1 milliampere; so line DE in Figure 3 represents a 1778-ohm load line when a plate supply voltage of 105 volts is used with \(T_2\).

On this latter load line, DE, the assumed plate current of 1 milliampere was found to exist at a grid bias of -0.61 volts as indicated by point F in Figure 3; so at this operating point the reciprocal of the transconductance of \(T_2\) was calculated by graphically taking the reciprocal of the slope of the plate current vs. plate voltage curve and dividing this reciprocal slope by the amplification factor, \(\mu_2\), of the tube.

Reference to Figure 3 will reveal that the reciprocal slope of the curve in question is equal to \(\frac{\Delta E_p}{\Delta i_2}\) which in this instance is \(47.7 \times 10^3\) ohms, and since \(\mu\) for a 6SL7 equals 70, the reciprocal of \(g_m2\) in this case will equal \(\frac{47.7 \times 10^3}{70}\) or 680 ohms.

It will be recalled from Equation (10) that this is the value which must be used for \(R_2\) in order that \(\frac{\Delta i_1}{\Delta e_n}\) shall equal zero; so immediately, since \(R_1 + R_2 = 1778\) ohms, \(R_1\) can be found to be 1098 ohms.

At this point it was necessary to determine whether this value for \(R_1\) multiplied by the sum \((i_1 + i_2)\),
Graphical Construction For Miller Circuit
Having A Large Load Resistor

Figure 3
1.125 \times 10^3 \text{ amperes}, would yield the previously determined value of -0.61 volts as a bias on $T_2$. Actually the product of $(i_1 + i_2) R_1 = 1.235$ volts; consequently, some of the previously used values had to be adjusted by trial and error processes until a set of values was found which would produce the assumed operating conditions for both tubes.

Examination of Figure 3 will reveal that a relatively large change in $R_1$ will produce very little effect on the slope of the load line of $T_2$. Furthermore, no change in $R_1$ could produce a change in $i_1$ of more than about $\pm 0.130$ milliamperes; so it seemed logical to try changing $R_1$ in an effort to achieve more nearly the originally assumed operating conditions.

If a value of $R_1 = 542$ ohms were used, the sum $(R_1 + R_2)$ would equal $(680 + 542) = 1222$ ohms. As has been observed, a 1222-ohm load line on Figure 3 in place of the existing 1778-ohm line would hardly introduce a perceptible difference; so the 1778-ohm line was retained.

Still assuming $i_2 = 1$ milliampere, and considering the change in $i_1$ to be negligible so far as its effect on bias values is concerned, the new bias on $T_1$ was found to equal approximately $1.125 \times 1222 \times 10^{-3} = 1.375$ volts and the bias on $T_2$ was calculated as: $1.125 \times 542 \times 10^{-3} = .61$ volts.

These values suitably agreed with the originally assumed values, and since the operating point of $T_2$ was
still the same as it was when \( \frac{1}{C \text{m}^2} \) was originally calculated, it could be expected that when the following values, as determined above, were employed in the Miller circuit (Figure 1), compensation for cathode drift could be anticipated:

\[
\begin{align*}
R_1 &= 542 \text{ ohms} \\
R_2 &= 680 \text{ ohms} \\
R_3 &= 10^6 \text{ ohms} \\
E_{bb1} &= 255 \text{ volts} \\
E_{bb2} &= 105 \text{ volts}
\end{align*}
\]

It is significant in this analysis that the smallness of \( i_1 \) as compared to \( i_2 \) did permit some simplification in the graphical procedures just as it did in Miller's mathematical analysis; the work of the following section, however, will reveal that this smallness of \( i_1 \) is not a necessary condition to successful operation of the circuit.
The procedure for selecting circuit parameters when $R_L$ is small was similar to the foregoing graphical analysis in that it resolved into a series of solutions based on assumed values, each of which solutions contained certain corrections for errors which became apparent in the next preceding solution.

In this specific case, a 6SL7 tube was used with a value of $R_L = 40,000$-ohms; so, as in the preceding case, a load line for a 40,000-ohm resistance was drawn on the tube characteristic curves, as shown by line AB, in Figure 4. A B+ supply of 255-volts was used.

A value of bias voltage on $T_1$ equal to $-2.25$ volts was assumed, thereby establishing the operating point as shown at C in Figure 4. The static value of $i_1$ was established as 1.1 milliamperes.

For $T_2$ a value of plate-supply voltage equal to 105 volts and a plate current of 1 milliampere were assumed.

As was true in the previous analysis, the sum of this plate current plus the plate current of $T_1$ was used to determine the required value of $(R_1 + R_2)$ by observing that

$$R_1 + R_2 = \frac{E_{c1}}{i_1 + i_2}$$

where $E_{c1}$ is the bias on $T_1$. 
Graphical Construction for the Miller Circuit
Having a Small Load Resistor

Figure 4
In this case,

\[
R_1 + R_2 = \frac{2.25}{2.1 \times 10^{-3}} = 1070 \text{ ohms}
\]

Using the value of 1070 ohms as the load on T_2 a load line, DE, was drawn through E_{bb2} = 105 V on the tube characteristic curves as shown in Figure 4. On this load line at point F it was found that the assumed plate current of 1 milliampere occurred at a grid bias, E_{c2}, of about -0.61 volts again. In other words, the operating point of T_2 in this case was very little different from the operating point in the case involving a large load resistance. This of course meant that \( \frac{1}{g_{m2}} \) and consequently R_2 would still be 680 ohms, and since R_1 + R_2 = 1070 ohms, R_1 became 1070 - 680 = 390 ohms.

Having found this value for R_1 it was necessary to calculate a value for E_{c2} to determine whether the actual value would equal the previously assumed value of -0.61 volts. Therefore:

\[
E_{c2} = (i_1 + i_2) R_1 = (1.1 + 1) \times 10^{-3} \times 390 = .82 \text{ volts}
\]

This value is obviously larger than the assumed value; so it was decided to try a new value of bias, E_{c1}, on T_1.

The value of -2.1 volts was used and as is indicated at point G in Figure 4 this new operating point yielded a new i_1 of 1.2 milliamperes.
This change in $i_1$ required a new calculation of $(R_1 + R_2)$:

$$R_1 + R_2 = \frac{E_{c2}}{I_1}$$

$$= \frac{2.1}{2.2 \times 10^{-3}} = 955 \text{ ohms}$$

Theoretically this new value of $(R_1 + R_2)$ would have necessitated the construction of a new load line for $T_2$, but practically there would have been a negligible difference between the 1070 ohm line previously constructed and the new 955 ohm line; so the former line was retained.

Since the operating point, $F$ in Figure 4, of $T_2$ was not changed, $\frac{1}{5m2}$ and consequently $R_2$ remained at 680 ohms; so the new value of $R_1$ was calculated as follows:

$$R_1 = 955 - 680$$

$$= 275 \text{ ohms}$$

Using this value of $R_1$ the actual value of bias, $E_{c2}$, on $T_2$ was again calculated:

$$E_{c2} = (i_1 + i_2) R_1$$

$$= (1.2 + 1) \times 10^{-3} \times 275$$

$$= 0.605 \text{ volts}$$

It was concluded that this value of $E_{c2}$ was sufficiently close to the originally assumed value of 0.61 volts to justify the use of the following circuit constants taken from the foregoing analysis:

$$R_1 = 275 \text{ ohms}$$

$$R_2 = 680 \text{ ohms}$$
\[ R_L = 40,000 \text{ ohms} \]
\[ E_{bbl} = 255 \text{ volts} \]
\[ E_{bb2} = 105 \text{ volts} \]

It will be observed that in this treatment it was not necessary to assume any predetermined relationship between the relative magnitudes of \( i_1 \) and \( i_2 \).
VI. Experimental Work on the Miller Circuit

The preceding mathematical analysis demonstrates the existence of a criterion for cathode-drift compensation by the Miller circuit without regard for the relative magnitude of the load resistance in the amplifier section of the circuit.

The foregoing graphical analyses demonstrate that it is possible to select circuit constants such that reasonable operating points can be achieved for both tubes of the circuit and at the same time the required condition for control, \( R_2 = \frac{1}{2m_2} \), can be established regardless of whether the load resistance for \( T_1 \) is large or small.

It would seem reasonable, then, to test the effect upon compensation of varying \( R_L \) from the large value considered, \( 10^6 \) ohms, to the small value considered, 40,000 ohms. To this end the following program of experimentation was pursued:

The circuit of Figure 5 was constructed using the following equipment:

- **Galvanometer** -- Leeds & Northrup, Type 2500
- **Standard Cell** -- Eppely Cat. #100
- **Potentiometer** -- Leeds & Northrup, Type K-2
- **\( R_3 \)** -- 20.1 megohm, .1 megohm Voltage Divider consisting of Shallcross #441 & 442 Decade Units.
- **B Battery** -- Burgess #2308 Batteries
- **Heater Battery** -- Edison Cells
The Test Circuit Used with the Miller Circuit

Figure 3
In Run #1, with values of $R_1$, $R_2$, and $R_L$ corresponding to those found in Section IV (which treats the case involving a large load resistance), heater voltage was varied, to simulate cathode drift, over a range of 10% below and 10% above some reference value near rated voltage. For each value of heater voltage the voltage at the plate of the amplifier tube was measured by means of the potentiometer and voltage divider. This data is contained in Appendix A.

Curve #1 in Figure 6 represents the manner in which output voltage varied with respect to heater voltage; consequently it is indicative of the extent to which the simulated cathode drift effects were reflected in the output of the stage.

In Run #2, with values of $R_1$, $R_2$, and $R_L$ corresponding to those found in Section V (which treats the case involving a small load resistance), a set of data similar to that described above was taken and is included in Appendix A.

Curve #2 in Figure 6 represents the manner in which output voltage varied with heater voltage in this case.

In Run #3, with the plate, grid, cathode, and heater of $T_2$ disconnected from the rest of the circuit and grounded, $R_L$ was again adjusted to $10^6$ ohms while $(R_1 + R_2)$
Drift Curves for the Miller Circuit

Figure 6
was adjusted to produce the same plate current in $T_1$ as had existed in Run #1. With these adjustments the circuit simply became a single triode amplifier stage with no compensation for cathode drift.

After the above adjustments had been made a third set of heater voltage versus output voltage data (see Appendix A) was taken and plotted as Curve #3 in Figure 6. This curve represents the extent to which the simulated cathode drift effect is reflected in the output of a completely uncompensated triode amplifier circuit. It is significant also that this curve agrees closely with the accepted general values\(^{(9)}\) of output voltage drift over a 20% change in heater voltage in an uncompensated triode amplifier stage.


It is believed that a visual comparison of the slopes of Curves 1 and 2 with the slope of Curve 3 is sufficient to indicate that the Miller circuit is indeed quite effective in compensating for cathode drift effects.

It is further believed that the curves of Figure 6 leave little question as to the ability of this circuit to affect such compensation whether the load resistance be large or small.
VII. Additional Mathematical Analyses of The Miller Circuit

In studying the Miller circuit the author came to regard its two required B+ voltages as a distinct disadvantage, in that a regulated power supply to provide these two voltages is inherently more bulky and more expensive than would be a similarly regulated supply to provide only one B+ voltage. (A bit of reflection will reveal that B+ supplies intended for use with highly stable D-C amplifiers must always be very well regulated lest a random-varying B voltage introduce the same sort of spurious plate current changes as are caused by cathode drift.)

Furthermore, it was believed that the use of two resistors in the common cathode lead of the two tubes was objectionable in that such use introduces two possible sources of resistance-noise-voltage at a low voltage level in the circuit; consequently, it was proposed to explore at greater length the possibilities of additional mathematical treatment of the circuit of Figure 1 with the hope that such exploration might produce a clue to an improved control circuit.

It will be recalled that throughout all the previous discussion it has been assumed that a single noise voltage generator, indicated as $e_n$ in Figure 1, could adequately represent the cathode drift effects of both $T_1$ and $T_2$ in the Miller circuit. Actually, however, it is highly improbable
that the random effective changes in potential of the
cathode of $T_1$ are exactly equal to such changes in $T_2$; in
fact, it is quite probable that the differences in cathode
drifts of $T_1$ and $T_2$ are at least partly responsible for the
deviation of Curves 1 and 2 of Figure 6 from straight hori­
zontal lines.

Recognizing that the elimination of this inconsistency
might yield a clue to a new cathode-control circuit, the
revision shown in Figure 7 was made in the A-C equivalent
of the Miller circuit. It will be observed that the inser­
tion of noise voltages, $e_{n1}$ and $e_{n2}$, at the two different
locations is equivalent to removing $e_n$ from Figure 1 and
placing $e_{n1}$ and $e_{n2}$ in the individual cathodes of $T_1$ and
$T_2$ respectively.

By using the circuit of Figure 7, the following expres­sions were obtained:

\[
e_{g1} = (i_1 + i_2)(R_1 + R_2) - e_{n1} \quad \ldots \quad (11)
\]

\[
e_{g2} = (i_1 + i_2)R_1 - e_{n2} \quad \ldots \quad (12)
\]

Using Equation (11) and summing voltages around the top
loop of Figure 7 yields:

\[
\mu_1[(i_1 + i_2)(R_1 + R_2) - e_{n1}] = e_{n1} - (i_1 + i_2)(R_1 + R_2)
- i_1(R_L + r_1)
\]

Expanding and collecting terms:

\[
i_1[(\mu_1 + 1)(R_1 + R_2) + R_L + r_1] + i_2(\mu_1 + 1)(R_1 + R_2)
= (\mu_1 + 1)e_{n1} \quad \ldots \quad (13)
\]
A-C Equivalent Circuit for the Miller Circuit
Assuming Different Noise Voltages

Figure 7
Using Equation (12) and summing voltages around the bottom loop of Figure 7 yielded:

\[ \mu_2 [(i_1 + i_2) R_1 - e_{n2}] = e_{n2} - (i_1 + i_2)(R_1 + R_2) - i_2 r_2 \]

Expanding and collecting terms:

\[ i_1 [(\mu_2 + 1)R_1 + R_2] + i_2 [(\mu_2 + 1)R_1 + R_2 + r_2] = (\mu_2 + 1)e_{n2} \quad \ldots (14) \]

Solving Equations (13) and (14) for \( i_1 \) by determinants:

\[
\begin{vmatrix}
(\mu_1 + 1)e_{n1} & (\mu_1 + 1)(R_1 + R_2) \\
(\mu_2 + 1)e_{n2} & [(\mu_2 + 1)R_1 + R_2 + r_2]
\end{vmatrix}
\]

\[
i_1 = \frac{(\mu_1 + 1)e_{n1}[(\mu_2 + 1)R_1 + R_2 + r_2]}{Denominator} \]

\[
i_1 = \frac{-(\mu_2 + 1)e_{n2}(\mu_1 + 1)(R_1 + R_2)}{Denominator}
\]

Since the denominator will obviously consist of constants only,

\[
i_1 = \frac{(\mu_1 + 1)e_{n1}[(\mu_2 + 1)R_1 + R_2 + r_2]}{Denominator}
\]

\[
i_1 = -\frac{(\mu_2 + 1)e_{n2}(\mu_1 + 1)(R_1 + R_2)}{K} \quad \ldots (15)
\]

It was proposed to differentiate Equation (15) first with respect to \( e_{n1} \) and then with respect to \( e_{n2} \), and then equate each of the resulting derivatives to zero in order to determine whether there existed a reasonable criterion for either \( \frac{\partial i_1}{\partial e_{n1}} = 0 \) or \( \frac{\partial i_1}{\partial e_{n2}} = 0 \). It was hoped that such criteria might provide a suggestion for an approach to a new cathode control circuit.
\[
\frac{d \lambda_1}{d e_{n1}} = \frac{(\mu_1 + 1)(\mu_2 + 1)R_1 + R_2 + r_2}{K}
\]

Let \[ \frac{d \lambda_1}{d e_{n1}} = 0 \]

\[ 0 = \frac{(\mu_1 + 1)(\mu_2 + 1)R_1 + R_2 + r_2}{K} \]

But since \( \mu_1 + 1 \neq 0 \) and \( K \) is finite:

\[ 0 = (\mu_2 + 1)R_1 + R_2 + r_2 \]

or

\[ R_1 = -\frac{R_2 + r_2}{\mu_2 + 1} \]

and if \( \mu_2 \gg 1 \)

\[ R_1 = -\frac{R_2 + r_2}{\mu_2} \quad \ldots (16) \]

Also,

\[
\frac{d \lambda_1}{d e_{n2}} = -\frac{(\mu_2 + 1)(\mu_1 + 1)(R_1 + R_2)}{K}
\]

Let \[ \frac{d \lambda_1}{d e_{n2}} = 0 \]

\[ 0 = -\frac{(\mu_2 + 1)(\mu_1 + 1)(R_1 + R_2)}{K} \]

Since \( \mu_2 + 1 \neq 0 \), \( \mu_1 + 1 \neq 0 \) and \( K \) is finite:

\[ 0 = R_1 + R_2 \]

\[ R_1 = -R_2 \quad \ldots (17) \]

An examination of the two criteria, shown in Equations (16) and (17) respectively, for \( \frac{d \lambda_1}{d e_{n1}} = 0 \) and \( \frac{d \lambda_1}{d e_{n2}} = 0 \) failed to provide any suggestion for a new control circuit; so it was proposed to consider the possibility in which \( e_{n1} \) is proportional to \( e_{n2} \) in another effort to find some suggestion which might lead to a new control circuit.
Substituting the condition, \( e_{n_2} = K'e_{n_1} \), into Equation (15) provided the following expression for \( i_1 \):

\[
i_1 = \frac{(\mu_1 + 1)e_{n_1}[(\mu_2 + 1)R_1 + R_2 + r_2]}{K}
\]

\[
- \frac{K'(\mu_2 + 1)e_{n_1}(\mu_1 + 1)(R_1 + R_2)}{K}
\]

\[
\frac{\partial i_1}{\partial e_{n_1}} = \frac{(\mu_1 + 1)[(\mu_2 + 1)R_1 + R_2 + r_2]}{K}
\]

\[
- \frac{K'(\mu_2 + 1)(\mu_1 + 1)(R_1 + R_2)}{K}
\]

Let \( \frac{\partial i_1}{\partial e_{n_1}} = 0 \)

\[
o = \frac{(\mu_1 + 1)[(\mu_2 + 1)R_1 + R_2 + r_2]}{K}
\]

\[
- \frac{K'(\mu_2 + 1)(\mu_1 + 1)(R_1 + R_2)}{K}
\]

and since \((\mu_1 + 1) \neq 0\) and \(K\) is finite:

\[
o = (\mu_2 + 1)\frac{R_1 + R_2 + r_2 - K'(\mu_2 + 1)(R_1 + R_2)}{K}
\]

Solve for \(R_2\):

\[
(\mu_2 + 1)R_1 + R_2 + r_2 = K'(\mu_2 + 1)R_1 + K'(\mu_2 + 1)R_2
\]

\[
[1 - K'(\mu_2 + 1)] R_2 = K'(\mu_2 + 1)R_1 - (\mu_2 + 1)R_1 - r_2
\]

\[
R_2 = \frac{K'(\mu_2 + 1)R_1 - (\mu_2 + 1)R_1 - r_2}{1 - K'(\mu_2 + 1)}
\]

or:

\[
R_2 = \frac{(K' - 1)(\mu_2 + 1)R_1 - r_2}{1 - K'(\mu_2 + 1)} \ldots (18)
\]

As would be expected, Equation (18) reduces to Equation (10) if \(K'\) is equated to unity; though unfortunately there
is nothing in Equation (18) which suggested a new approach to a cathode control circuit. It is believed significant, however, that Equation (18) does not agree with the conclusion drawn by Miller which stipulated that if $e_{n1} = K'e_{n2}$, the criterion for $\frac{dI_1}{de_{n1}} = 0$ would be $R_2 = \frac{K'}{E_{m2}}$.

Even though the author obtained a microfilm copy of Miller's original paper (10) it was not possible to determine the exact cause of this seeming contradiction since this paper did not contain a proof of the criterion, $R_2 = \frac{K'}{E_{m2}}$.

It is believed, however, that the general approach taken by Miller (the assumptions regarding relative magnitudes of plate currents and the actual circuit analysis rather than equivalent circuit analysis) was such as might easily have led to prejudiced conclusions.
VIII. Mathematical Analysis of a Single Triode Circuit

When both of the analyses of Section VII failed to suggest an approach to a new circuit it was concluded that a study of the equation for $\frac{di}{de_n}$ in a single uncompensated triode stage, as shown in Figure 8, might suggest a desirable approach; so the equivalent A-C circuit for such a stage was drawn, as shown in Figure 9, with the equivalent cathode drift noise voltage generator located in the cathode lead. In Figures 8 and 9, the following symbols were used:

- $r =$ tube plate resistance
- $R_L =$ load resistor
- $R_K =$ cathode resistor
- $e_n =$ equivalent cathode drift noise voltage
- $i =$ tube plate current
- $\mu =$ amplification factor of the tube
- $e_g =$ input signal voltage

With the grid of the tube in Figure 8 grounded, the equation for $e_g$ became:

$$e_g = iR_K - e_n \quad \ldots \quad (19)$$

Using Equation (19) and summing voltages around the loop of Figure 9 yielded:

$$\mu(iR_K - e_n) = e_n - i(R_K + R_L + r)$$

Expanding and solving for $i$:

$$i \left[ (\mu + 1)R_K + R_L + r \right] = (\mu + 1)e_n$$
Conventional Triode Amplifier

Figure 8

A-C Equivalent Circuit for a Conventional Triode Amplifier

Figure 9
or:

\[ i = \frac{(\mu + 1)e_n}{(\mu + 1)R_K + R_L + r} \]

and:

\[ \frac{\delta i}{\delta e_n} = \frac{\mu + 1}{(\mu + 1)R_K + R_L + r} \]

Let \( \frac{\delta i}{\delta e_n} = 0 \) and:

\[ 0 = \frac{\delta i}{\delta e_n} = \frac{\mu + 1}{(\mu + 1)R_K + R_L + r} \]

But since the denominator must be finite:

\[ 0 = \mu + 1 \]

or:

\[ \mu = -1 \]

became the criterion for \( \frac{\delta i}{\delta e_n} = 0 \) in the circuit of Figure 8. This unfortunately was of no assistance in the solution of the problem at hand; so it was determined that a new approach must be taken.
IX. A Proposed New Circuit

A detailed examination of the functioning of the circuit in Figure 1 will reveal that the major function of $R_1$ in that circuit is to provide bias for $T_2$ in order to limit the current, $i_2$, to reasonable values. The fact that $R_1$ actually contributes nothing to the actual compensation for cathode drift is evidenced by the non-existence of $R_1$ in Equation (10), the criterion for minimization of cathode drift.

Realizing that the rather secondary function of holding $i_2$ within reasonable limits could as well be performed by a plate-loading resistor in the circuit of $T_2$ as it could by a biasing resistor, the author proposed the circuit of Figure 10 as a possible cathode control circuit which eliminates both of the objectionable features of the Miller circuit mentioned in Section VII.

It can be observed that the circuit of Figure 10 is essentially that of two separate triode amplifier stages, one of which, $T_1$, is biased by the action of both plate currents flowing through a common cathode resistor, $R_K$, while the other, $T_2$, operates at zero bias.

A brief investigation will also reveal that the only signal impressed upon $T_2$ is the equivalent cathode drift noise voltage. This means then that any change in $e_n$ will immediately produce a proportional change in $i_2$ which through a degenerative feedback effect on the bias
The Proposed Circuit Showing the Equivalent Cathode Drift Noise Voltage Generator, $e_n$

Figure 10
of $T_1$, will tend to nullify the cathode drift effect on $T_1$.

It can be observed that this circuit, unlike the Miller circuit, does not produce a negative feedback effect on $T_2$ since no amount of change in $i_2$ or $i_1$ can affect the zero bias on $T_2$. It can also be observed that the use of $R_{L2}$ provides the possibility of using a common plate-supply voltage, thereby appreciably simplifying the power supply requirements.
X. Mathematical Analysis of The Proposed Circuit

Having proposed the circuit of Figure 10, it seemed in order to analyze it in the manner of Section III in order to determine whether criteria existed for $\frac{dI_1}{dE_n} = 0$; so the A-C equivalent circuit of Figure 11 was drawn to be used as a basis for the analysis. The symbols used in Figure 11 are defined below and they will be used throughout the ensuing discussion of the proposed circuit:

- $r_1$ = Plate resistance of $T_1$
- $r_2$ = Plate resistance of $T_2$
- $R_K$ = Cathode Resistor
- $R_{L1}$ = Load resistance on $T_1$
- $R_{L2}$ = Load resistance on $T_2$
- $\mu_1$ = Amplification factor of $T_1$
- $\mu_2$ = Amplification factor of $T_2$
- $e_{g1}$ = Input signal for $T_1$
- $e_{g2}$ = Input signal for $T_2$
- $e_n$ = Equivalent cathode drift noise voltage
- $i_1$ = Plate current of $T_1$
- $i_2$ = Plate current of $T_2$

Assuming that the grid of $T_1$ was grounded, the following observations were made from Figure 10:

\[ e_{g1} = (i_1 + i_2)R_K - e_n \] \hspace{1cm} (20)

\[ e_{g2} = -e_n \] \hspace{1cm} (21)
A-C Equivalent Circuit for the Proposed Circuit

Figure 11
Using Equation (20) and summing voltages around the top loop of Figure 11 yielded the following expression:

\[ \mu_1 [(i_1 + i_2)R_K - e_n] = e_n - (i_1 + i_2)R_K - i_1(R_{LL} + r_1) \]

Expanding and collecting terms:

\[ (\mu_1 + 1)e_n = i_1[(\mu_1 + 1)R_K + R_{LL} + r_1] + i_2(\mu_1 + 1)R_K \]

... (22)

Using Equation (21) and summing voltages around the bottom loop of Figure 11 yielded the following expression:

\[-\mu_2 e_n = e_n - (i_1 + i_2)R_K - i_2(R_{L2} + r_2)\]

Expanding and collecting terms:

\[ (\mu_2 + 1)e_n = i_1R_K + i_2(R_{L2} + R_K + r_2) \]

... (23)

Solving Equations (22) and (23) for \( i_1 \) by determinants:

\[ i_1 = \frac{(\mu_1 + 1)e_n - (\mu_1 + 1)R_K}{\text{Denominator}} \]

Or, since the denominator obviously is a constant:

\[ i_1 = \frac{(\mu_1 + 1)(R_{L2} + R_K + r_2)e_n}{K} - \frac{(\mu_2 + 1)(\mu_1 + 1)R_K e_n}{K} \]

\[ \frac{\partial i_1}{\partial e_n} = \frac{(\mu_1 + 1)(R_{L2} + R_K + r_2) - (\mu_2 + 1)(\mu_1 + 1)R_K}{K} \]

... (24)

In order to determine a criterion for \( \frac{\partial i_1}{\partial e_n} = 0 \), the right hand side of Equation (24) was equated to zero, and since \( (\mu_1 + 1) \neq 0 \) and \( K \) must be finite:

\[ 0 = R_{L2} + R_K + r_2 - (\mu_2 + 1)R_K \]

or;

\[ R_K = \frac{R_{L2} + r_2}{\mu_2} = \frac{R_{L2}}{\mu_2} + \frac{1}{\epsilon_n} \]

... (25)
where $g_{m2}$ equals the transconductance of $T_2$. 
XI. Graphical Analysis of The Proposed Circuit

With the derivation of Equation (25) the ability of the proposed circuit to compensate for cathode drift effects seemed assured; consequently there remained only the problem of selecting circuit values which would permit the establishment of reasonable operating points for the two tubes, T1 and T2, and which at the same time would satisfy the necessary condition for cathode control, \( R_K = \frac{R_{L2}}{\mu^2} + \frac{1}{\beta m C^2} \).

The selection of such values again resolved into a trial-and-error type of graphical solution similar to that of Sections IV and V. It is with the exact nature of this solution that this section will be concerned.

A 6SL7 tube was again used with a value for \( R_{L1} \) of 1 megohm and an assumed grid bias for T1 of -3 volts; consequently on the characteristic curves for this tube a load line representing \( 10^6 \) ohms and passing through the assumed plate supply voltage, 255 volts, was drawn as AB in Figure 12. Point C in Figure 12 represents the assumed operating point for T1.

Assuming a plate current, \( i_2 \), of 4 milliamperes for the control tube, an operating point for this tube was automatically established as Point D in Figure 12 since it was known that this point must lie on the zero bias curve; consequently the load line for this tube was drawn by connecting the operating point, D, with the common plate supply voltage, 255 volts, at Point A.
Graphical Constructions for the Proposed Circuit Using a 6SL7 Tube

Figure 12
Taking the reciprocal of the slope of line AD provided the value:

\[ R_{L2} = 18700 \text{ ohms} \]  \hspace{1cm} \ldots \text{(26)}

Now, realizing that the 3 volts of grid bias on T1 must be provided by the sum of the two plate currents flowing through \( R_K \), it can be seen that

\[ R_K = \frac{E_{C1}}{i_1 + i_2} \]

Where \( E_{C1} \) = the grid bias on T1.

But:

\[ E_{C1} = 3 \text{ volts by assumption} \]
\[ i_2 = 4 \times 10^{-3} \text{ amperes by assumption} \]

and:

\[ i_1 \approx 0.125 \times 10^{-3} \text{ amperes from Figure 12} \]

So:

\[ R_K = \frac{3 \times 10^3}{4 + 0.125} \]
\[ = 728 \text{ ohms} \] \hspace{1cm} \ldots \text{(27)}

From Equation (25), however,

\[ R_K = \frac{R_{L2}}{\mu_2} + \frac{1}{\beta m_2} \] \hspace{1cm} \ldots \text{(28)}

So substituting from Equations (26) and (27) into Equation (28), and since \( \mu \) for a 6SL7 equals 70:

\[ 728 = \frac{18700}{70} + \frac{1}{\beta m_2} \]

or:

\[ \frac{1}{\beta m_2} = 461 \text{ ohms} \] \hspace{1cm} \ldots \text{(29)}
In other words, if the foregoing choice of an operating point for $T_2$ was such that the reciprocal of the transconductance of the tube at that point is 461 ohms, then the circuit values selected in this solution when employed in the circuit of Figure 10 should produce a cathode control circuit in which $\frac{dI_1}{de_n}$ approaches zero.

By taking the reciprocal of the slope of the zero bias curve at Point D and dividing it by $\mu$ it was found that at Point D:

$$\frac{1}{Em_2} = 492$$

...(30)

This value was believed to be sufficiently close to the value found in Equation (29) to justify using the circuit constants which had been obtained by this analysis.

It of course is apparent that the choice of an operating point for $T_2$ in this procedure will not always lead to the fortunate agreement between Equations (29) and (30) that was found in this case. In fact it was not the author's good fortune to find such immediate agreement with his first choice of this operating point; consequently, as has undoubtedly been surmised, it was necessary to duplicate the foregoing work a number of times, each time using a different set of assumed starting values, before the happy coincidence of agreement between Equations (29) and (30) was reached. The solution given here, then, is only the last one of a large number of previous attempts.
The circuit constants obtained from this analysis are summarized below:

\[ R_K = 728 \text{ ohms} \]
\[ R_{L1} = 10^6 \text{ ohms} \]
\[ R_{L2} = 18700 \text{ ohms} \]
\[ E_{bb1} = E_{bb2} = 255 \text{ volts} \]
XII. Some Seeming Limitations of The Proposed Circuit

An examination of Figure 12 will immediately reveal that, inherent in the final solution in Section XI, is an operating point for T1 which allows only a very small grid signal swing (about ± 0.75 volts), if relatively distortion-free operation is to be accomplished; it is significant in this respect that even though the author tried a large number of different initial assumptions he was not able to arrive at a set of circuit constants which would avoid this near-cutoff operating point for T1 so long as a 6SL7 tube was employed.

A critical analysis of the manner in which Equation (25) fits into the graphical solution of Section XI to determine the operating point of T1 revealed the possibility that the above limitation might be avoided if some tube having a large transconductance were employed.

By examining several sets of tube characteristics it was discovered that a 6J6 has a transconductance which will range from three to four times as large as the transconductance of a 6SL7; so it was proposed to repeat the work of Section XI for a 6J6 employed in the circuit of Figure 10. The circuit constants obtained from this work (the complete analysis is contained in Appendix B) were as follows:

\[ R_K = 585 \text{ ohms} \]
\[ R_{L1} = 255000 \text{ ohms} \]
$R_{L2} = 16350 \text{ ohms}$

$E_{bb1} = E_{bb2} = 255 \text{ volts}$

These circuit values gave a bias of -6.43 volts on T₁ which, as can be seen in Appendix B, permits a grid swing of 2 or perhaps 3 volts before the region of high distortion is invaded.
XIII. Experimental Work on The Proposed Circuit

In order to determine whether and to what extent the proposed circuit would compensate for cathode drift effects, a program of experimentation essentially the same as that of Section VI was initiated. The circuit employed was similar to that of Figure 5, the difference being, as shown in Figure 13, that the actual circuit undergoing test was the proposed circuit of Figure 10 rather than the Miller circuit of Figure 1.

All test apparatus was the same as is listed in Section VII.

In Run Number 1, using a 6SL7 as the tube in Figure 13 and with the circuit constants essentially as found in Section XI, data for heater voltage versus output voltage was taken as the heater voltage was varied from about 10% below to about 10% above some reference value near to rated voltage. These data are contained in Appendix C and are plotted as Curve Number 1 in Figure 14.

In order to get a comparison of compensated versus uncompensated performance the data for Curve Number 3 of Figure 6 (which it will be recalled is for an uncompensated 6SL7) was replotted as Curve Number 2 on Figure 14. It is believed that a comparison of these two curves will amply attest the fact that the proposed circuit provides excellent compensation over a wide range of heater voltage limits.
The Test Circuit Used With the Proposed Circuit

Figure 13
Drift Curves for the Proposed Circuit Using a 6SL7 Tube

Figure 14
In Run Number 2 the 6SL7 of Run Number 1 was replaced by an RCA 6J6 tube, and the circuit constants were changed to correspond essentially to those found in Section XII; then, data of heater voltage versus output voltage were again taken over a range of heater voltage from about 10% below to about 10% above a reference value near rated voltage. These data are plotted as Curve Number 1 in Figure 15.

With the grid, plate, cathode, and heater of T2 disconnected from the circuit and grounded, and with Rk adjusted to produce the same bias on T1 as existed in Run Number 2, data for a 6J6 single unit triode stage was taken in the same manner as was employed in Run Number 2. These data are plotted in Figure 15 as Curve Number 2.

It is believed that Figure 15 stands as further evidence of the ability of the proposed circuit to compensate for cathode drift effects.

It is also believed that the compensation achieved with a 6J6, having an allowable low distortion grid voltage swing of 2 or 3 volts, fully demonstrates that the limitations of the proposed circuit, as they were discussed in Section XII, are actually limitations on the applicability of a specific tube rather than inherent characteristics of the circuit itself.
Drift Curves for the Proposed Circuit Using a 6J6 Tube

Figure 15
XIV. Summary and Conclusions

It is believed that the conclusions to be drawn from this work can be quite simply stated as follows:

A. The former belief that the amplifier load resistance must be large in order for the Miller circuit to affect cathode control has been shown to be without justification.

B. There has been developed a new circuit for cathode control which not only eliminates a source of noise voltage (one cathode resistor) at a low voltage level in the circuit but also eliminates the double plate supply voltage requirement which is associated with many existing cathode control circuits. This circuit is original with this paper to the best of the author's knowledge.

In connection with subparagraph A above it is important to point out that the earlier belief, that $R_L$ in Figure 1 must be large, represented a serious limitation on the applicability of an excellent circuit to problems requiring wide frequency response.

When it is observed that many of the applications of D-C amplifiers, as in high quality cathode ray oscilloscopes, combine the difficult requirements of high stability and wide frequency response it can be readily realized that many valuable applications of the low cathode drift effect of the Miller circuit would never be made if the circuit
were one with which only relatively narrow frequency response were possible.

While it is not intended to demonstrate that the new circuit of subparagraph B above is generally superior to either the Miller circuit or to any other cathode-control circuit, it is believed that its single plate supply voltage requirement will constitute a distinct advantage in many applications.

The importance of the removal of one cathode resistor can best be emphasized by observing that in numerous applications of D-C amplifiers, such as in the investigation of voltages generated by animal nervous systems, the order of magnitude of the voltages being studied might easily be the same as or even less than that of the noise voltages generated in a cathode resistor.
# Appendix A

## Table I

Drift Data for Miller Circuit Having a 10^6 ohm Load Resistor

<table>
<thead>
<tr>
<th>Change in Heater Voltage (% of Rated)</th>
<th>Change in Output Voltage (% of Reference*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 9.53</td>
<td>+ .542</td>
</tr>
<tr>
<td>- 6.35</td>
<td>+ .407</td>
</tr>
<tr>
<td>- 3.18</td>
<td>+ .127</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>+ 3.18</td>
<td>- .095</td>
</tr>
<tr>
<td>+ 6.35</td>
<td>- .218</td>
</tr>
<tr>
<td>+ 9.53</td>
<td>- .437</td>
</tr>
</tbody>
</table>

*The reference value of output voltage was taken as that value which existed at 0\% change in heater voltage.*
### Table II

Drift Data for Miller Circuit Having 40K ohm Load Resistor

<table>
<thead>
<tr>
<th>Change in Heater Voltage (% of Rated)</th>
<th>Change in Output Voltage (% of Reference*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 9.53</td>
<td>+ .157</td>
</tr>
<tr>
<td>- 6.35</td>
<td>+ .099</td>
</tr>
<tr>
<td>- 3.18</td>
<td>+ .054</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>+ 3.18</td>
<td>- .095</td>
</tr>
<tr>
<td>+ 6.35</td>
<td>- .197</td>
</tr>
<tr>
<td>+ 9.53</td>
<td>- .285</td>
</tr>
</tbody>
</table>

*The reference value of output voltage was taken as that value which existed at 0% change in heater voltage.
Table III

Drift Data For Conventional Triode Amplifier Using a 6SL7 Tube

<table>
<thead>
<tr>
<th>Change in Heater Voltage (% of Rated)</th>
<th>Change in Output Voltage (% of Reference$^*$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 9.53</td>
<td>+ 4.32</td>
</tr>
<tr>
<td>- 6.35</td>
<td>+ 2.93</td>
</tr>
<tr>
<td>- 3.18</td>
<td>+ 1.41</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>+ 3.18</td>
<td>- 1.39</td>
</tr>
<tr>
<td>+ 6.35</td>
<td>- 2.62</td>
</tr>
<tr>
<td>+ 9.53</td>
<td>- 3.93</td>
</tr>
</tbody>
</table>

$^*$The reference value of output voltage was taken as that value which existed at 0% change in heater voltage.
Appendix B

Graphical Analysis For A 6J6 Tube Used In The Proposed Cathode Control Circuit

Using the characteristic curves for a 6J6 dual triode a load line for a 255,000-ohm load resistance was drawn as AB in Figure 16.

A plate current, $i_1$, for $T_1$ was assumed to be 0.4 milliamperes, thereby establishing an assumed operating point on the -6 volt bias line at C in Figure 16.

Assuming a plate current, $i_2$, of 10.6 milliamperes for $T_2$ established the load line AD in Figure 16. This line represents a load resistor, $R_{L2}$, of 16,350 ohms.

Taking the reciprocal of the slope of the zero bias curve at Point D provided a value of 5880 ohms for the plate resistance, $r_2$, of $T_2$.

Substituting the foregoing values for $R_{L2}$ and $r_2$ into Equation (25) and observing that $\mu$ for a 6J6 equals 38, a value for $R_K$ was obtained as follows:

$$R_K = \frac{R_{L2} + r_2}{\mu_2}$$
$$= \frac{16350 + 5880}{38}$$
$$= 585 \text{ ohms}$$

In order to determine whether the foregoing values would produce proper operating conditions it was only necessary to see whether the above value of $R_K$ would produce the
Graphical Constructions
for the Proposed Circuit Using a 6J6 Tube

Figure 16
originally assumed value of bias on $T_1$; so, observing that the bias on $T_1$, $E_{C1}$, is developed by the sum of the two plate currents flowing through $R_K$:

$$E_{C1} = (i_1 + i_2)R_K$$

Substituting from above:

$$E_{C1} = (0.4 + 10.6) \times 585 \times 10^3$$

$$= 6.43 \text{ volts}$$

This value deviates slightly from the originally assumed value of 6 volts, but it can be observed that this deviation serves solely to introduce a negligible change in the plate current of $T_1$, and even if $i_1$ were changed by 100%, no appreciable difference would be observed in the calculated bias of 6.43 volts; therefore the foregoing solution was deemed to be acceptable.
## Appendix C

### Table IV

Drift Data For The Proposed Circuit Using a 6SL7 Tube

<table>
<thead>
<tr>
<th>Change in Heater Voltage (% of Rated)</th>
<th>Change in Output Voltage (% of Reference*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 9.53</td>
<td>- .176</td>
</tr>
<tr>
<td>- 6.35</td>
<td>- .120</td>
</tr>
<tr>
<td>- 3.18</td>
<td>- .051</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>+ 3.18</td>
<td>+ .038</td>
</tr>
<tr>
<td>+ 6.35</td>
<td>+ .087</td>
</tr>
<tr>
<td>+ 9.53</td>
<td>0</td>
</tr>
</tbody>
</table>

*The reference value of output voltage was taken as that value which existed at 0% change in heater voltage.*
**Table V**

Drift Data For The Proposed Circuit Using a 6J6 Tube

<table>
<thead>
<tr>
<th>Change in Heater Voltage (% of Reference*)</th>
<th>Change in Output Voltage (% of Reference**)</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 13.5</td>
<td>- .355</td>
</tr>
<tr>
<td>- 6.79</td>
<td>- .055</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>+ 6.79</td>
<td>- .163</td>
</tr>
<tr>
<td>+ 13.5</td>
<td>- .598</td>
</tr>
<tr>
<td>+ 20.4</td>
<td>-1.35</td>
</tr>
</tbody>
</table>

*Reference heater voltage was taken as 5.9 volts since that was the apparent point of zero slope of the curve.*

**Reference output voltage was taken as the value that existed at Reference heater voltage.*
Table VI

Drift Data For A Conventional Triode Amplifier Using a 6J6 Tube

<table>
<thead>
<tr>
<th>Change in Heater Voltage (% of Reference*)</th>
<th>Change in Output Voltage (% of Reference*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-13.5</td>
<td>+1.46</td>
</tr>
<tr>
<td>-6.79</td>
<td>+.62</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>+6.79</td>
<td>-.81</td>
</tr>
<tr>
<td>+13.5</td>
<td>-1.67</td>
</tr>
<tr>
<td>+20.4</td>
<td>-2.43</td>
</tr>
</tbody>
</table>

*The reference values used here were the same as those used for Table V.


Vita

The author was born on a farm about 8 miles west of Neosho, Missouri in Newton County on January 22, 1917.

His pre-college education was obtained in the public schools of Springfield, Missouri between the years 1922 and 1934.

In 1934 he entered Drury College in Springfield where he pursued a pre-engineering type of training until 1936 when he transferred to Purdue University. He graduated from Purdue with the degree of Bachelor of Science in Electrical Engineering in 1939.

From 1939 until 1943 he was employed by James R. Kearney Corporation of St. Louis, Missouri in various capacities as an engineer.

In 1943 he was commissioned a Second Lieutenant in the Signal Corps of the U. S. Army. He became a Captain in the Active Reserve of the Officers' Reserve Corps upon his separation from active duty in 1946.

In 1947 he became an Instructor at the School of Mines and Metallurgy of the University of Missouri in which capacity he still serves.