Air cooled engine cooling fins redesigned by use of the membrane analogy for heat conduction

Eberhard Hammel Miller

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AIR COOLED ENGINE COOLING FINS REDESIGNED BY USE OF THE MEMBRANE ANALOGY FOR HEAT CONDUCTION

BY

EBERHARD H. MILLER

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A

THESIS

submitted to the faculty of the

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Degree of

MASTER OF SCIENCE, MECHANICAL ENGINEERING MAJOR

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1948

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The author is indebted to Professor A. J. Miles for his timely suggestions, assistance, and guidance while the investigation was being conducted.
The purpose of this investigation is to determine the feasibility of applying the two-dimensional heat conduction membrane analogy to the redesign of cooling fins of air-cooled engines.
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PART I

HISTORY OF AIR-COOLED ENGINE COOLING
Since the advent of the air-cooled engine its design has largely been a cut-and-try method. The heat transfer from a cylinder is considered as that which is left after the heat dissipation to other sources has been subtracted from the original heat in the cylinder due to combustion and other sources. (1) To transfer this heat through the cylinder wall to the air, the cylinder designers have found from the general heat-transfer equation, (2) (1)

\[ H = UA(T_1 - T_a) \]  

where \( H \) is the heat transferred in Btu/hr.

\( U \) is the overall heat transfer coefficient for the part of the cylinder being studied, Btu/hr per sq ft of base area per degree F difference between the average temperature (barrel or head) and the cooling air.

\( T_1 \) is the average temperature of the head or barrel being considered, degree F.

\( T_a \) is the temperature of the air, degree F.

\( A \) is the base area of the cylinder head or barrel exposed to the air, sq ft.


that the area required to dissipate the heat left over for steady state conditions is directly proportional to this heat to be transferred. The area required for dissipation of the heat was found to be much more than the area of a finless cylinder such as is used in the liquid cooled engine. Also the amount of cooling medium could not be controlled as in the case of the liquid cooled engine. The amount of cooling medium


(in this case air) flowing over heated area and the type of flow of the medium (laminar or turbulent) as it passes the heated area determine the amount of heat dissipated to the cooling medium. Tests (4) showed that


the cooling was very effective in the front part of the cylinder, but as the air neared the rear of the cylinder the heat dissipation became less, due to the approach of the air temperature to that of the cylinder temperature.

About 1933, to aid in the transfer of heat, designers added a deflector to the path of the air (5) (called a baffle) to direct more air

(5) Ibid.

around to the rear of the cylinder and partially eliminated the stagnant point at the rear of the cylinder. This baffle also aided in giving high
turbulence to the air flowing around the cylinder, giving a higher heat
transfer coefficient and, therefore, a higher rate of heat transfer.

The cooling efficiency of the air passing between the cooling fins
of the cylinder is controlled in part by the turbulence. Greater turb-
ulence or mixing gives better cooling. The flow may be turbulent or
laminar depending on the value of a dimensionless ratio known as Reynolds
Number (6) (2)

\[
\text{Re} = \frac{\nu \cdot \rho}{\mu}
\]

(2)

where \( V \) is the velocity of the fluid, ft/sec.

\( \rho \) is the density of the air, lb/cu ft.

\( \mu \) is the viscosity of the air, lb-sec/ft.

\( r_h \) is \( S/b \) (hydraulic radius)

\( S \) is the cross sectional area perpendicular to the fluid flow,
sq ft.

\( b \) is the wetted perimeter, ft.

It is general practice to consider that a fluid has laminar flow when
the Reynolds Number is below 2100, and a fluid has turbulent flow when
the Reynolds Number is 2100 or above. (7)

(7) Ibid.
The effects of the type of the inside surface of the heated cylinder barrel and the type of flow of the hot gases within the cylinder are combined into a value called the surface-heat-transfer coefficient "q" in the following equation.

\[
q = C_p \rho V x f \left( \frac{\nu V_0}{\mu}, \frac{\mu C_p}{K}, r_1, r_2, r_3, \ldots \right)
\]

Where \( C_p \) is the specific heat of the hot gas at constant pressure, Btu/lb/deg F.

\( \rho \) is the gas density, lb/cu ft.

\( V \) is the gas velocity, ft/sec.

\( \mu \) is the gas viscosity, lb-sec/ft.

\( r_h \) is a characteristic dimension called the hydraulic radius.

\( k \) is the thermal conductivity of the metal, Btu/sq ft/deg F through one ft/hr.

\( \frac{\nu V_0}{\mu} \) is the Reynolds Number, dimensionless.

\( \frac{\mu C_p}{K} \) is the Prandtl Number, dimensionless.

\( r_1, r_2, r_3, \ldots \) are dimensionless ratios of other important dimensions of the body to "r_h".

and "f" is read "function of." This coefficient "q" is then combined with other cylinder dimensions not considered in "q" and called the overall-heat-transfer coefficient "U" between the hot gases inside of
the cylinder and the cooling medium.

Biermann and Pinkel (9) combined "q" and the cylinder dimensions in an empirical formula for the overall-heat-transfer coefficient;

$$U = \frac{2q}{\pi c} \left[ \frac{2}{\alpha} \left( 1 + \frac{W}{2R_b} \right) \tanh(\alpha W) + \frac{S_b}{a} \right]$$

(4)

where q is the surface heat transfer coefficient, Btu/sq in. total surface area/hr/deg F temperature difference between the hot gases inside the cylinder and the inside cylinder wall as found in equation (3).

\( t \) is the average fin thickness, inches.

\( S \) is the average space between fins, inches.

\( W \) is the fin width, inches.

\( W' \) is the effective fin width, \( W' \neq t_f/2 \), inches.

\( t_t \) is the fin-tip thickness, inches.

\( R_b \) is the radius from the center of the cylinder to the fin root, inches.

\( a \) is \( 2q/kt \).

\( k \) is the thermal conductivity of the metal, Btu/sq in./deg F through one inch/hr.

\( S_b \) is the distance between adjacent fin surfaces at the fin root, ft.

In order to obtain (4) the temperature inside the cylinder was
maintained constant and the temperatures at the tip and root of a few fins were measured with a thermocouple under simulated operating conditions. The empirical formula (4) was then fitted to these data.

The amount of air passing between the fins was another problem which was solved, after much experimenting, by use of a covering or cowling (10) around the engine with openings in the front and rear to allow for the passage of air. The rear openings were so constructed that the open area could be controlled, thereby controlling the amount of air flowing past the cylinder fins.

Fin development has since become a problem of production simplifications (11) in the arrangement, shape, and size of fins rather than a heat-transfer problem. This thesis, by the use of the membrane analogy for conduction of heat through a homogenous body, will study the temperature distribution and heat transfer in the rectangular fin.


(11) Ibid.
PART II

PREVIOUS INVESTIGATIONS APPLIED

TO THE MEMBRANE ANALOGY
Application of the membrane analogy has been made in several fields since its first use in 1917 by Taylor and Griffith (12) to solve problems in torsion in various shaped members.

This came from a variation in the identity, shown by Prandtl, (13)

between the equation for force acting on a soap film supported at its edge and the equation for torsion in an elastic bar.

\[
\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{p}{2T}
\]

(5)

where \( T \) is the tension in the membrane.

\( p \) is the pressure per unit area.

\( x \) is a direction.

\( y \) is a direction perpendicular to \( x \).

\( z \) is a direction perpendicular to both \( x \) and \( y \).

In 1937, Miles and Stephenson (14) saw that Laplace's equation was similar to that of the distribution about gas or oil wells, and from this found the pressure distribution in reservoirs of irregular shapes when drained by several wells.


(13) Ibid.

The actual use of the membrane analogy to solve heat-conduction problems was introduced by Emmons (15) in 1943, and Wilson (16) in 1947,


further developed the membrane analogy for use in steady-state heat conduction.

Using these developments as a background, a study will be made of the temperature variation inside a cylinder fin.
PART III

PROCEDURE AND EXPERIMENTAL DATA
Before the membrane may actually be used, some preliminary calculations must be made to find the boundary conditions of the typical cylinder barrel fin shown in figure 1.

Since these fins are bounded by baffles, as shown by the dotted line, each space between fins may be considered as an enclosed pipe for the calculation of the turbulence by the use of Reynolds Number (equation 2). The air velocity is assumed to be 134 miles per hour or 196.5 feet per second through the space between the fins. From tables for air, the density was found to be 0.0745 pounds per cubic foot and viscosity to be $4 \times 10^{-7}$ pound-seconds per foot. From the dimensions shown in figure (1) other values needed in equation (2) may be found.

\[ S = \frac{196 + 196}{12 \times 2} \times \frac{473}{12} = 0.00646 \text{ sq ft} \]

and

\[ b = \frac{2 \times 473 + 2 \times 186}{12} = 0.1115 \text{ ft} \]

dividing \( S \) by \( b \) to find \( r_h \)

\[ r_h = \frac{S}{b} = \frac{0.00646}{0.1115} = 0.0578 \text{ ft} \]

and substituting in equation (2) for the dimensionless Reynolds Number

\[ RN = \frac{4 \times 0.0578 \times 196.5 \times 0.745}{4 \times 10^{-5}} \]

\[ RN = 846,000 \]

Since, as mentioned before, a Reynolds Number above 2100 is considered as indicating turbulence, it is obvious that the flow between the fins of an air-cooled cylinder is turbulent.

The Reynolds Number found is used in finding the heat transfer coefficient "h" between the air and the outside of the cylinder fin (18)


which is important in calculating the heat transferred from the cylinder fins to the cooling air.

The equation from McAdams (19) used to find the heat transfer coefficient "h" is

\[ 2h = f \cdot c_p \cdot G \]  

(6)

where \( f \) is the dimensionless-friction factor as found from a value of 
Reynolds Number (20)


\( c_p \) is the specific heat at constant pressure, Btu/(lb of fluid) 
(deg F).

\( G \) is the mass velocity, equal to the product of the velocity in
feet/hr and density in lb/cu ft.

Since the constituents of the equation are not dependent on the

The fin is rectangular in cross-section. (Fin being studied)
The fin is long enough, such that heat loss from the end is the same as if the fin had no end and were longer by one-half its end thickness.

Using these assumptions, the following equation for heat loss may be set up from the general heat transfer equations with dimensions taken from figure (2).

\[
2k \frac{\partial^2 \theta}{\partial x^2} = 2k \frac{\partial^2 \theta}{\partial y^2} + \frac{1}{\alpha_f} \int 2k \frac{\partial^2 \theta}{\partial x^2} \, dx + 2h \Theta \, dx
\]  

(7)

where \( k \) is the heat-transfer coefficient of the metal, \( \text{Btu/ (hr) (sq ft) (deg F/ft)} \),

\( \Theta \) is the temperature above atmosphere temperature, \( \text{deg F} \),

\( h \) is the heat-transfer coefficient between the cooling air and the metal, \( \text{Btu/(hr)(sq ft)(deg F)} \),

\( x \) and \( y \) are dimensions as shown in figure (2).
This equation is not readily solvable. This, however, is a case of two-dimensional heat flow of the type used by Wilson (22) to link heat-transfer by conduction to the membrane analogy.

From the assumption that the fin is long compared to its thickness, "t", it is obvious that heat will flow out of the fin only in the "y" direction or from the top and bottom only. This statement when considered at the edge of the fin, where "y" is equal to t/2, may be put into equation form for any point along the edge.

\[ k \frac{\partial \Theta}{\partial y} = -h \Theta \]  

(8)

Where \( k \) is the heat-transfer coefficient of the metal, Btu/(hr)

(sq ft)(deg F/ft).

\( \frac{\partial \Theta}{\partial y} \) is the temperature gradient in the "y" direction only, (at the edge of the fin),

\( h \) is the heat-transfer coefficient between the cooling air and the metal, Btu/(hr)(sq ft)(deg F),

\( \Theta \) is the temperature at the point where the temperature gradient is measured above atmosphere temperature, deg F.

This indicates that at the edge of the fin the temperature gradient in a direction perpendicular to the fin is directly proportional to the fin temperature, at the edge, above air temperature. For simplification, the air temperature is taken as base temperature with
all other temperatures measured above this base temperature.

"h" and "k" being constant the second derivative of equation (8) gives;

\[ \frac{\partial^2 \theta}{\partial y^2} = - \frac{h}{k} \frac{\partial \theta}{\partial y} = \frac{h}{k^2} \theta \]  \hspace{1cm} (9)

Using the LaPlacian equation

\[ \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0 \]  \hspace{1cm} (10)

and substituting for the second derivative of "\( \theta \)" with respect to "\( y \)" from equation (9) in equation (10)

\[ \frac{\partial \theta}{\partial x^2} + \frac{h}{k^2} \theta = 0 \]  \hspace{1cm} (11)

This is an ordinary differential equation of the second order with constant coefficients. The general solution of which is

\[ \theta = A \sin \left( \frac{hx}{k} \right) + B \cos \left( \frac{hx}{k} \right) \]  \hspace{1cm} (12)

A boundary condition is \( \theta = \Theta_0 \), when \( x = 0 \)

\[ \Theta_0 = A \sin (\omega) + B \cos (\omega) = B \]

giving

\[ \theta = A \sin \left( \frac{hx}{k} \right) + \Theta_0 \cos \left( \frac{hx}{k} \right) \]

and \( \theta = 0 \), when \( x = W' \)

\[ 0 = A \sin \left( \frac{hW'}{k} \right) + \Theta_0 \cos \left( \frac{hW'}{k} \right) \]

or

\[ A = \Theta_0 \cot \left( \frac{hW'}{k} \right) \]
The general equation for "q" in terms of the variable "x" is now

\[ \theta = \Theta_0 \cos \frac{h x}{k} - \cot \left( \frac{h w}{k} \right) \sin \frac{h x}{k} \] (13)

Equation (13) when solved, using the following values gives the temperature "q" along the edge of the fin as "x" is changed from zero to \( W' \), and "y" is held constant at \( t/2 \),

\[ h = 22.2 \text{ Btu/(hr)(sq ft)(deg F)} \] as found by equation (6),

\[ W' = W/2 \text{ inches, as defined by equation (4),} \]

\[ k = 26 \text{ Btu/(hr)(sq ft)(deg F/ft) for steel,} \]

\[ 30 \text{ Btu/(hr)(sq ft)(deg F/ft) for cast iron,} \]

\[ 119 \text{ Btu/(hr)(sq ft)(deg F/ft) for aluminum,} \]


gives a general curve as shown in figure (3).

This sine curve is known to be incorrect since the temperature gradient is higher, algebraically, nearer the source of heat than at any other point.

If the boundary conditions are such that the temperature is zero at "x" equals zero, and at "x" equals "W" is "-\( \Theta_0 \)", then equation (12) reduces to

\[ \theta = -\Theta_0 \sin \frac{h x}{k} \] (14)
Fig. 3
CURVE TAKEN FROM EQUATION 12
The curve formed by this equation is shown in figures (4) and (5). These are views of the edge curve that will be used as the first test in finding the final curve. This is shown in full scale.

Before trying a test run on the curve found by equation (14), a practice curve of a straight line was tried in order to see at what stage of "cure" the membrane must attain to give accurate results. "Cure" is the length of time that the membrane is allowed to drain off the excess water. Since the straight line gives a curve of constant slope, there should be no variation in the elevation perpendicular to the known straight edge. It will be noticed that all curves taken, as shown in figure (6) are straight lines with the exception of curve number five (points not connected since not accurate) which has a slight sag in the middle. This curve was taken after just a one-minute "cure" to test the possibility of working with less curing time. The other curves were taken after the membrane was allowed to "cure" for at least two minutes. The use of a shorter curing time than was suggested by Wilson (23) is due to the membrane draining from three of the four sides instead of its draining only to one or two sides, as was the case during Wilson's investigations in heat conduction.

The edge of the plate must be sharp so that the membrane will attach itself at the desired points. This was accomplished by undercutting the inside of the curved support with a hand-grinder until a knife edge was had along the curve. A hand-grinder was used instead
Fig. 5. Plates and equipment used for fin data.
Fig. 6

CONSTANT ELEVATION CONTOURS USING CONSTANT SLOPE FIN EDGE

Micrometer Reading in Inches

Curve Number
of a milling machine, because it was known that curves other than a straight line or circular arc was to be used, thereby making the milling machine impractical. The hand-grinder was held at an angle of approximately forty-five degrees and grinding slowly on a line about one tenth of an inch inside the curve until a sharp knife edge was achieved along the curve support.

Using the data obtained from equation (14) a sine curve as indicated in figure (4) was constructed, (figure (5)) for the fin used for membrane calculations. A constant elevation run, as shown in figure (7), was taken using this sine curve edge to see if this curve satisfied the condition that the temperature gradient at the edge of the fin was proportional to the temperature at the edge of the fin. It is obvious from figure (7) that this condition is true in the case of the sine curve. No calculations will be made, since the constant of proportionality will not be needed.

From elementary heat-transfer it is known that under steady-state conditions all the heat (24) going into an area must come out of that area, so that the net heat-transfer around any closed curve must equal zero. This condition may be stated mathematically in the following equation

\[ \oint_{\text{area}} d\mathbf{h} \cdot d\mathbf{s} = 0. \]  

(15)
Fig. 7
CONSTANT ELEVATION CONTOURS USING SINE CURVE FIN EDGE
where "dh" is the heat-transfer in the direction perpendicular to "ds" and parallel to "dn" over the curve "S" as indicated in figure (8).

The above integration may be accomplished graphically by replacing this curve with a series of straight lines which form a closed area as shown in figure (9), which represents the curves that are to be used in this investigation. By use of the center line of the fin as one of these lines, the task is simplified. This line is a line of symmetry; hence there is no heat flow across it. (25) Since the direction of the heat-transfer is known, we may find the total heat-transfer across the upper curve and set this equal to the heat-transfer across the other two curves, the flow across the fourth side is zero, (the center line).

Figure (9) shows a top view of the area bounded by the curve described above, figure (10) shows an end view of the fin indicating the change in elevation along the upper and lower lines of the curve, and figure (11) shows the elevation changes along the edge of the fin and directly inside of the fin. From these three charts, the data in table (1) were tabulated to determine the possibility of accurately finding the heat-transfer through a certain area.

Using the data in table (1) in conjunction with equation (15) modified as follows

\[ \frac{\Delta h_x}{\Delta n_x} \Delta S_x = 0. \]  

(16)
Fig. 8
A CURVE TO FOLLOW FOR HEAT DETERMINATION
Fig. 9
MODIFIED du CURVE FOR SIMPLIFICATION
(Enlarged 4 times)

Curves 1, 2, and 3 represent different sets of data or measurements, with lines indicating constant elevation. The diagram shows a series of points marked with circles, connected by lines, indicating a pattern or trend in the data. The vertical axis is labeled 'Wall', and the horizontal axis is labeled 'Fixed Edge', with various lines indicating 'Lines of Constant Elevation' and 'Micrometer Reading'.
Fig. 10
END VIEW OF FIN SHOWING ELEVATION CHANGES
OF CURVES (1) and (2)
Fig. 11

EDGE VIEW OF FIN SHOWING ELEVATION CHANGES
OF CURVE 3

Distance Along Edges  Inches

Elongation Inches
where $\Delta h$, $\Delta n$, $\Delta s$ are the same as $dh$, $dn$, and $ds$ in equation (15) but are of a measurable distance instead of being infinitesimals.

$x$ indicates the curve measured as shown in figure (9), curves 1, 2 and 3.

The total heat transfer across curve number one was found to be proportional to

$$\frac{\Delta h_1}{\Delta n_1} \Delta s_1 \approx \frac{7.54}{0.282} (30 \times 0.2) = 164.0$$

across curve number two

$$\frac{\Delta h_2}{\Delta n_2} \Delta s_2 \approx \frac{-44.1}{0.282} (30 \times 0.2) = -95.4$$

and across curve number three

$$\frac{\Delta h_3}{\Delta n_3} \Delta s_3 \approx \frac{-486}{0.3} (21 \times 0.2) = -68.8$$

Adding the above three results algebraically we get

$$164.0 - 95.4 - 68.8 = -0.2$$

which is very near to the theoretical results, or zero.
Fig. 12. Membrane apparatus with fin model in place.
TABLE 1
Tabulated Data Taken from Figures 9, 10 and 11

<table>
<thead>
<tr>
<th>Curve No. 1</th>
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<td>0.3680</td>
<td>0.1950</td>
<td></td>
</tr>
</tbody>
</table>

ds = 0.20   
dn = 0.282  
ds = 0.20   
dn = 0.282  
PART IV

CONCLUSIONS
It is shown by the above measurements in the membrane representing the rectangular fin that it is possible to determine exactly what fraction or amount of the heat is given off to the cooling air and what portion is conducted toward the cool end of the fin. Fortunately the rectangular fin lent itself to an analytical solution and the membrane solution checked with this to a high degree of accuracy.

Heat transfer problems dealing with the steady-state where the boundary conditions are known offer little difficulty. However, if the boundary conditions are not known, as is the general case along the surface of the fin exposed to the cooling air, the accuracy of this membrane method will suffice to determine the effectiveness of each increment of fin length regardless of whether the fin is rectangular, or otherwise all that is needed is the temperature of the fin at its base and temperature of the cooling air. The temperature of the fin surface is less at points farthest from the cylinder barrel. Therefore, in the case of a fin of known but varying cross-sections, it is only necessary to begin with the section of the fin at the base and divide it into a number of equal lengths, say five to ten, and determine the temperature at each division by the above method, one point at a time. Thus, the effectiveness of a fin design and the effectiveness of the parts of the fin, such as the tips, can be determined. It was hoped that a few designs other than the rectangular fin could be checked, unfortunately the summer term was too short for this.
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VITA

The author was born February 11, 1922, at Denver, Colorado. His high school education was completed in 1940, at Southwest High School in St. Louis, Missouri, at which time he entered the Missouri School of Mines and Metallurgy at Rolla, Missouri. In June, 1943, his education was interrupted when he was called to active duty with the United States Army. After serving three years, he returned to the Missouri School of Mines in June, 1946, and was graduated August, 1946, with a B. S. degree in Mechanical Engineering, and June, 1947, with a B. S. degree in Electrical Engineering.