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A study of plane wave filter analysis with flow

Thomas Valle Huber

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A STUDY OF PLANE WAVE FILTER ANALYSIS WITH FLOW

BY

THOMAS VALLE HUBER, 1944-

A THESIS

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ABSTRACT

The purpose of this investigation is to determine the effect of slow steady flow on the reflection and transmission characteristics of acoustic filters.

A standing wave tube and transmission tube apparatus, designed and built by a previous investigator, was modified to accept slow steady flow. Reflection characteristics were determined from measurements made on the inlet side of the filters investigated. These data along with measurements taken with the transmission tube apparatus were used to determine transmission characteristics. The calculations were simplified by terminating the filters anechoically. The filters used in this investigation were an expansion chamber, an automobile type muffler with adjustable elements, and a replacement automobile muffler. Experimentally determined characteristics of the expansion chamber were in good agreement with theoretically determined characteristics. A major conclusion of this investigation is that standing wave measurements can be made in the presence of slow steady flow. Recommendations for future investigations are made.
ACKNOWLEDGEMENTS

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I INTRODUCTION

Noise is a major problem in today's society. As a result, new and improved methods for controlling noise are needed. The basic rule of noise control is to design or modify the construction of the machine or device so that it is less noisy. If, after modification of the design, the device is still too noisy, then one must resort to one or more of the three standard techniques for noise control as stated by Beranek [2]. They are relocating, muffling, or enclosing the device. This report is concerned with the noise control technique of muffling.

Presently, intake and exhaust silencers are designed by "trial and error". This is done by building a prototype and testing it on the vehicle or apparatus to be silenced. After being evaluated, the prototype design is either accepted or modified. If modified, it is again installed on the vehicle or apparatus to be silenced and re-evaluated. This process is continued until the desired results are obtained.

This process can be improved by designing silencers from individual elements whose acoustic characteristics have been determined under operating flow and
temperature conditions. These characteristics could then be combined according to plane wave theory to predict the performance of the silencer.

Previous investigators [3, 8, 28] have determined static (no flow) values of the acoustic characteristics of individual elements commonly used in silencers which operate under steady flow conditions. The objective of this investigation was to modify the existing standing wave tube (Fig. 1) so that it would be capable of analyzing the reflection characteristics of silencer elements and complete silencers under operating conditions of steady flow.

This research is a step towards attaining the level of knowledge to design or modify a silencer without the need for building prototypes and evaluating them on a vehicle.
Figure 1  Side View of 2 Inch I.D. Standing Wave Tube Apparatus
II REVIEW OF LITERATURE

A. Effect of Tube Attenuation

When working with sound waves traveling in acoustical wave guides, energy loss within the tube must be considered. This loss, due to the combination of viscous and thermal conduction effects at the tube boundaries, causes attenuation of the sound pressure of a wave propagating through the tube and a decrease in propagation velocity.

The attenuation of the sound pressure is exponential and increase linearly with distance. It is of the form [13]:

\[ P_2 = P_1 e^{-\alpha (x_2 - x_1)} \]  

(1)

where

- \( P_1 \) = sound pressure at \( x_1 \)
- \( P_2 \) = sound pressure at \( x_2 \)
- \( \alpha \) = attenuation constant

Helmholtz and Kirchhoff [4] were the first to show the theoretical value of the attenuation constant to be

\[ \alpha = \frac{1}{2Cr} \left[ \omega L_\lambda + \omega (\gamma - 1)L_\nu \right] \]  

(2)

where

- \( c \) = propagation velocity of sound
- \( r \) = tube radius
- \( \omega \) = frequency of sound signal, rad./sec.
\[ \gamma = \text{ratio of the specific heats, } \frac{C_p}{C_v} \]

\[ L_u = \left[ \frac{2\mu}{\omega} \right]^\frac{1}{2}, \quad L_v = \left[ \frac{2K}{\rho C_p \omega} \right]^\frac{1}{2} \], where

- \( \mu \) = kinematic viscosity
- \( K \) = thermal conductivity
- \( \rho \) = mass density
- \( C_p \) = specific heat at constant pressure
- \( C_v \) = specific heat at constant volume

After taking the results of numerous investigators into consideration, Beranek [1] concluded that the attenuation constant for the case of normal undried air in tubes with thin walls exposed to the surroundings is best described by Kirchhoff's equation increased by 15 percent. The attenuation constant then becomes

\[ \alpha = 3.18 \times 10^{-5} f^\frac{1}{2}/r, \text{ cm}^{-1} \]  

(3)

This is the expression for the theoretical attenuation constants used by Simon [28] and in this thesis.

The equation [27] for the speed of sound in a tube of radius \( r \), according to Kirchhoff, is

\[ c' = c \left[ 1 - \frac{L_v h}{2r(\pi f)^{1/2}} \right] \text{ cm/sec} \]  

(4)

*This corresponds closely to the conditions under which experimental measurements were made by Simon [28] and in this investigation.
where \( L_{vh} = \left[ v \right]^{1/2} + (\gamma - 1) \left[ \frac{K}{\rho C_p} \right]^{1/2} \)

- \( c \) = adiabatic speed of sound in free space
- \( v \) = kinematic viscosity, cm²/sec
- \( \gamma \) = ratio of specific heats
- \( \frac{K}{\rho C_p} \) = thermal diffusivity, cm²/sec

Experimental verification of this formulation in the frequency range of 400 to 4,000 cps is given by Scott [26]. He found that the theoretical and experimental values are in close agreement.

B. Three-Dimensional Effects

Higher order transverse modes of vibration can exist if boundary conditions are not satisfied by the plane wave assumption. Transverse modes are excited in a tube if the wavelength of sound propagating through it is less than a critical multiplier of the tube radius. Transverse modes are excited at a discontinuity but do not propagate unless the wavelength is less than a critical multiplier of the maximum transverse dimension.

Rayleigh [23] predicted that higher modes could be neglected for \( \lambda \geq 3.1413 r \).

Hartig and Swanson [11], and Miles [19,20] found that the first transverse mode is excited and propagated when \( \lambda = 1.64 r \).
Sabine [25] stated that transverse modes of vibration can be expected to occur at $\lambda = 4\pi$. However, with the apparatus he used, interference was not encountered until the frequency approached the second transverse mode or $\lambda = 2\pi$.

Davis, Stokes, Moore, and Stevens [6] found that experimental data agreed closely with plane wave theory for cylindrical mufflers until the first transverse mode was excited within the mufflers. This occurred whenever $\lambda \leq 0.82d$, where $d$ is the internal diameter of the muffler.

Gatley [8] and Buckley [3] found discrepancies in experimental phase angle measurements as compared to values predicted from plane wave theory. A probable cause appeared to be excitation of higher-order transverse modes of vibration near discontinuities.

Simon [28] investigated the effects of higher-order transverse modes of vibration. The experimental apparatus consisted of standing wave and transmission tube equipment plus an expansion chamber. The expansion chamber was 12.46 inches long, had an internal diameter of 9.96 inches, and inlet and outlet diameters of 0.430 inches. The objective of his examination was to determine if correction factors could be obtained that would permit the use of one-dimensional plane wave theory in a region where three-dimensional effects could not be neglected. The results were not conclusive.
C. **Steady Flow and Temperature Effects**

Several authors have investigated the effects of steady flow on the propagation of plane waves. From Morse and Ingard [21], the time of travel for a plane wave pulse back and forth in a tube of length \( L \) with no flow is

\[
T_i = \frac{2L}{c} \quad (5)
\]

Therefore, the fundamental plane wave resonance frequency of the tube is

\[
f_i = \frac{c}{2L} \quad (6)
\]

The round trip time for a pulse with flow is

\[
T'_i = \left[ \frac{L}{(c+v)} \right] + \left[ \frac{L}{(c-v)} \right] \quad (7)
\]

where \( V \) = velocity of flow.

Equation (7) can be reduced to

\[
T'_i = T_i \left( \frac{1 - M^2}{1 + M^2} \right) \quad (8)
\]

where

\[
M = \frac{V}{c}
\]

The resonance frequency becomes

\[
f'_i = (1 - M^2) f_i \quad (9)
\]

From (9) it can be seen that the presence of flow lowers the resonance frequency of the tube.

Trimmer [30] made a theoretical approach to the behavior of sound waves in a pipe through which the medium is moving at a steady velocity \( V \). Theory indicates that
the resonances of pipes are affected by steady flow so that the resonance peaks are flattened, while the separation between resonance points is reduced by the factor \(1 - (v/c)^2\).

In two companion publications, Lambert [15, 16] investigated the effect of a moving medium on the insertion loss of a side-branch filter. In the first publication calculations of insertion loss were carried out for a constant-velocity source and a side-branch resonator. It was found that the positions of the cut-off frequencies and the level of the characteristic impedance of the filter were functions of the flow Mach number, as well as geometry. The scale at which Mach effects become important was reported to be dependent upon the geometry of the filter. In a second paper, Lambert stated that the insertion loss of a side-branch filter was particularly sensitive to flow velocities for frequencies in the vicinity of the fundamental resonance point. Experiment showed that the insertion loss property of the filter can be destroyed unless the flow velocities were associated with very low Mach number (less than 0.01). For sufficiently high flows, independent measurements of sound excitation of the side branch by flow indicated that
appreciable "self-noise" caused by flow-induced forced vibrations was in evidence. Vortices shed over the mouth of the resonator were thought to be the cause of these vibrations. It appeared as if the vortex system was stabilized somewhat by the action of the resonant system.

Meyer, Mechel, and Kurtze [18] experimentally investigated the influence of a turbulent air stream on the sound attenuation in ducts lined with sound absorbing materials or structures of different kinds. A decrease of absorption with increasing flow velocity was observed for linings consisting either of porous materials or sufficiently damped Helmholtz resonators. Helmholtz resonators were damped by placing silk tissue over the holes, causing increased flow resistance. An increase of the frequency of maximum absorption for the resonators was observed. The authors reported that negative attenuation as well as self-excitation can occur when undamped Helmholtz resonators are used for the lining of silencer ducts.

Powell [22] calculates the reflection and transmission characteristics at cross section variations in a duct with flow. The analytical results apply to "almost conical" ducts, either convergent or divergent with the incident wave propagating with or against the flow direction. Based
upon the analytical results, an approximate method is demonstrated for ducts of other form.

An analytical approach was made by Smith [29] to determine the effect of a slow steady flow* of ideal fluid upon a superimposed pure-tone sound field. For one dimensional fields, it was shown that the steady flow has no direct effect on the acoustic admittance or the magnitudes of acoustic pressure and particle velocity.

Ronneberger and Schilz [24] investigated the propagation of sound in air flow through tubes with variations in cross section at flow velocities of 100 - 200 m/s. Measurements were made in a circular tube (85 mm I.D.) at frequencies from 0.3 KHz to 1.5 KHz. The reflection coefficient at the tube end was reduced to less than 2% by an adjustable closing. Pure tones of 130 - 140 db were used to obtain sound levels well above the flow noise. Nonlinearities were disregarded in the range of measuring accuracy. It was demonstrated that the acoustic characteristics of a smooth variation in cross section agreed well with theory. The theory states that the reflection of

*He considered a flow slow enough to justify approximating it as incompressible, without any terms of second order in the steady flow variables.
acoustic energy from such transitions shows little change during flow, while the sound pressure reflection coefficient is increased considerably, and that the sum of the transmitted and reflected sound energies is equal to the incident energy.

Harris [10] investigated the effect of temperature on attenuation at a relative humidity of 50%, for constant values of frequency. He found that attenuation decreased with an increase of temperature in the temperature range of 10 to 30°C. In this temperature range the attenuation coefficient decreased from .06 to .03 for a frequency of 12.5 KHz, and from .02 to .01 for a frequency of 6.3 KHz.

D. Acoustic Filter Analysis

The following is a review of literature from 1950 through 1969. An extensive review of literature back to 1922 can be found in a thesis prepared by Simon [28].

In an analysis and investigation of the performance of multiple Helmholtz resonators by McGinnis [17], it was found that in most cases the uncoupled natural frequencies of separate cavities are very different from the frequencies of a coupled system. The large number of parameters involved in the calculations made it possible to have more than one configuration of the network for a
given frequency spectrum or, as stated by the author, if a frequency spectrum is analyzed, the resonator system is not thereby determined. As resonators were added, the number of modes of vibrations was seen to increase. An increase in the conductivity of the throat or neck, as in the case of a single resonator, was said to be attended by an increase in the natural frequencies of vibration, and an increase in the volume of any cavity resulted in a decrease in the natural frequencies. Also, increasing the length of the aperture was reported to bring about a decrease in the resonant frequency.

In a study by Lambert [14] the factors influencing the damping of resonators containing air indicated that mechanical losses due to the vibrations of the walls were experimentally significant. During experimental studies on sound waves propagated in a thin-walled cylindrical tube, the author found that the measured attenuation was dependent upon the thickness of the shell. It was also found that mechanical loss due to the damped vibration of the side walls was in evidence. The author noted that the role played by this factor should not be over-looked in precise studies employing thin-walled tubes.

Davis, Stokes, Moore, and Stevens [6] made an
investigation to determine the limitations of the equations for the attenuation of various types of muffler configurations. Included were single-chamber and multiple-chamber mufflers of both the expansion chamber and resonator types, tuned side-branch tubes, and the combination of an expansion chamber with a resonator. Measurements were made with an impedance tube at room temperature without flow.

Igarashi and Toyama [12] have theoretically and experimentally investigated the attenuation characteristics of expansion chambers, resonators and combinations of the two. The effect of sound absorbing material placed within the filters was also considered. Four terminal network theory was used to calculate the attenuation characteristics of the filters. Experimental measurements were made with the filters terminated anechoically at both ends. Tee fittings connected a sound source and fixed microphone to the filter's inlet and outlet. The anechoic terminations, sound source, and microphone were packed in sand to prevent any system vibrations. Experimentally determined attenuation characteristics were in good agreement with the theoretically predicted values except at higher frequencies, where one dimensional theory was no longer applicable due
to the presence of higher order transverse modes of
vibration.

In a study by Danner [5] the importance of muffler
location was investigated. Three locations were investi-
gated: 1) near the front of the vehicle or where the
acoustical length of the exhaust pipe equals one half the
tail-pipe length, 2) in the center of the exhaust system
or where the acoustical length of the exhaust pipe and
the tail pipe are approximately equal, and 3) with the
muffler located at the extreme rear of the exhaust system
where the entire system is made up of an exhaust pipe and
a muffler with a short stub tail pipe. Sound pressure
level (dB) data were obtained with a microphone located
approximately 18 inches from the end of the tail pipe.
Curves of firing frequency versus sound pressure level
(dB) were plotted for the different muffler locations.
Danner found that a muffler must be designed for a given
location to obtain maximum silencing.

Gatley [8] proposed a design procedure for mufflers
used in the suction and discharge lines of small refriger-
ation systems. The procedure requires that the reflection
and transmission characteristics of the muffler's component
elements be determined. The overall muffler characteristics
are then determined using plane wave theory. This is
accomplished by combining the elements in a sequential manner. Gatley also predicted the performance of a muffler in an operating refrigeration system by combining the reflection and transmission characteristics for the muffler with the acoustical characteristics of the system at the inlet and outlet of the muffler.

Gatley analyzed and compared three different methods for measuring reflection characteristics of small acoustical elements. A standing wave tube specifically designed for small diameter (0.5 in.) tubing is recommended. An anechoic termination for the measuring system was designed to simplify the calculation of both transmission and reflection factors from measured data.

Comparison of theoretical and measured reflection and transmission characteristics for plane discontinuities and expansion chambers showed relatively good agreement at the lower frequencies but not at the higher frequencies where the effects of higher-order transverse modes of vibration become pronounced.

Buckley [3] discussed the design, construction, and evaluation of standing wave tubes of 0.43 inch O.D. and 1.87 inch O.D. These sizes were selected because of their potential use in refrigeration and automotive exhaust applications. The experimental analysis of a
variable length expansion chamber is compared to its theoretically determined characteristics to demonstrate the usefulness of the equipment.

Simon [28] determined the reflection and transmission characteristics of seven reactive filter elements in the frequency range from 500 to 2000 Hz. The elements consisted of bends, coils, and tees in 0.430 inch I.D. tubes.

Simon describes the design, construction and evaluation of a 2-inch I.D. standing wave tube apparatus. The apparatus was evaluated over a frequency range from 50 to 3500 Hz. and was found to produce very accurate and consistent results. This apparatus with modifications for flow was used in the investigation to analyze the reflection characteristics of acoustic filters under operating conditions of steady flow.

Simon also describes a method for measuring the attenuation effects produced in 0.430 inch I.D. hardened copper refrigeration tubing.

Simon's thesis is particularly valuable for its extensive list of references.

Fukuda [7] developed and experimentally evaluated attenuation formulas for the design of mufflers as opposed to transmission loss formulas. He defines attenuation as the difference between the sound pressure level (SPL)
with no silencer attached and sound pressure level (SPL) with the silencer attached, at a fixed distance from the end of the pipe.

Findings of other investigators presented in the literature review were considered when making this investigation. This investigation develops a method for analyzing filters in the presence of slow steady flow. The effects of flow are determined for several filters.
III ACOUSTIC PLANE WAVE THEORY

A. Solution of the Plane Wave Equation

The derivation of the acoustic wave equation can be found in any basic text in acoustics. The three dimensional wave equation in Cartesian coordinates is

$$\nabla^2 P = \frac{1}{c^2} \frac{\partial^2 P}{\partial t^2}$$

(10)

where $P = \text{sound pressure}$

$c = \text{propagation velocity of sound}$

$t = \text{time}$

Sound pressure is a function of one dimension and time for plane waves. Choosing $x$ as the independent dimension and removing $y$ and $z$ dependency, (10) becomes the plane wave equation

$$\frac{\partial^2 P}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 P}{\partial t^2}$$

(11)

The general solution to the plane wave equation (11), according to Beranek [2], can be expressed as the sum of two terms

$$P(x, t) = f(t - \frac{x}{c}) + g(t + \frac{x}{c})$$

(12)

where $f$ and $g$ are arbitrary functions. It is necessary, however, that $f$ and $g$ both have continuous first and second derivatives.

Equation (12) states that sound pressure at location $x$ for any time $t$ is composed of two pressure
waves traveling in opposite directions. The wave \( f(t - x/c) \) is traveling in the positive \( x \) direction (away from the source), while the wave \( g(t + x/c) \) is traveling in the negative \( x \) direction.

Equation (11) can also be expressed in exponential notation for a harmonic boundary condition of the form \( e^{j\omega t} \):

\[
P(x,t) = (A_e^{+} e^{j \frac{\omega}{c} x} + B_e^{+} e^{-j \frac{\omega}{c} x}) e^{j\omega t} \quad (13)
\]

or

\[
P(x,t) = (A_m e^{j \frac{\omega}{c} x} + B_m e^{j (\phi - \frac{\omega}{c} x)}) e^{j\omega t} \quad (14)
\]

where (Fig. 2) \( p(x,t) \) = The complex value of the sound pressure at location \( x \) for any time \( t \).

\( A_e^{+} = \) The complex value of a sound pressure wave traveling in the negative \( x \) direction and having magnitude \( A_m = |A_e^{+}| \) and phase angle of zero degrees.

\( B_e^{+} = \) The complex value of a sound pressure wave traveling in the positive \( x \) direction having magnitude \( B_m = |B_e^{+}| \) and a phase angle with respect to \( A_e^{+} \).
Figure 2. Sound Pressure Wave Relationship
Recalling equation (12) it is seen that for frequency $\omega$

$$f(t-\frac{x}{c}) = B_m e^{j(\theta - \frac{\omega x}{c})} e^{j\omega t}$$  \hspace{1cm} (15)$$

and

$$g(t+\frac{x}{c}) = A_m e^{j\frac{\omega x}{c}} e^{j\omega t}$$  \hspace{1cm} (16)$$

B. Reflection Factor

Consider the situation as shown in Fig. (3). $A_1$ is the wave incident on the filter, $A_2$ is the transmitted wave, and $B_1$ is the reflected wave. Since the filter is terminated anechoically, $B_2$ is zero. We may thus define a reflection factor $R$ as

$$R = \frac{B_1}{A_1} = \frac{B_m e^{j\theta}}{A_m}$$  \hspace{1cm} (17)$$

where $e^{j\theta}$ expresses the phase relationship between the reflected and the incident waves.

The transmission factor $T$ is defined as

$$T = \frac{A_2}{A_1}$$  \hspace{1cm} (18)$$

where $T$ and $A_2$ are complex values. Terminating the filter anechoically means that only the free progressive wave $A_2$ exists behind the filter.

The wave equation (14) can be modified to account for tube attenuation by including an exponential change in the magnitudes of $A$ and $B$. The resulting sound pressure is

$$p(x,t) = (A_1 e^{\alpha x} e^{j\frac{\omega x}{c}} + B_1 e^{-\alpha x} e^{j(\theta - \frac{\omega x}{c})}) e^{j\omega t}$$  \hspace{1cm} (19)$$
Figure 3  Sound Pressure Wave Relationship for an Arbitrary Filter
It can be shown [3, 8, 28] that the resulting expression for reflection factor magnitude, knowing a sound pressure minimum and a sound pressure maximum, is

$$|R| = \frac{B_m}{A_m} = \frac{P_{\text{max}} e^{\alpha X_{\text{min}}} - P_{\text{min}} e^{\alpha X_{\text{max}}}}{P_{\text{max}} e^{\alpha X_{\text{min}}} + P_{\text{min}} e^{\alpha X_{\text{max}}}}$$  \hspace{1cm} (20)

where

- $P_{\text{max}}$ = maximum sound pressure
- $P_{\text{min}}$ = minimum sound pressure
- $X_{\text{max}}$ = location of maximum sound pressure relative to the filter
- $X_{\text{min}}$ = location of minimum sound pressure relative to the filter
- $\alpha$ = attenuation constant
- $e$ = base of natural logarithms

Using a sound pressure minimum, the expression for the reflection factor phase angle is

$$\theta = 2 \frac{\omega}{c t} X_{\text{min}} - n \pi, \quad n = \text{smallest odd integer yielding a negative value for } \theta$$  \hspace{1cm} (21)

where

- $c'$ = speed of sound in a tube
- $\omega$ = circular frequency, radians per second

To determine the reflection factor magnitude at low frequencies, the measurement and location of one
sound pressure minimum and another arbitrary sound pressure are used.* The expression for the reflection factor magnitude using one sound pressure minimum and another arbitrary sound pressure is \([3, 8, 28]\):

\[
|R| = \frac{A_m e^{2\alpha x_{\min}} - P_{\min} e^{\alpha x_{\min}}}{A_m}
\]

(22)

where

\[
A_m = \frac{1}{2} \left[ P_{\min} + \left( P_{\min} - 2 \left( \frac{P_{\min}^2 - P_{\max}^2}{1 - \cos \phi} \right) \right)^{1/2} \right]
\]

(23)

The phase angle associated with \(R\) is determined as before from (21).

C. Transmission Factor

Transmission factor is defined as the ratio of the amplitudes of the transmitted wave to the incident wave and the relative phase angle between the transmitted wave and the incident wave. It is determined by combining the complex value of reflection factor with values of and relative phase angle between, the sound pressure obtained at known locations on either side of the filter \([3, 8, 28]\).

The graphical calculation of the transmission factor, as shown in Fig. (4), begins by assuming that the value of the incident wave at the filter inlet, denoted by \(A_1\), has a value of one at an angle of zero degrees. The reflected wave at the filter inlet, \(B_1 \Delta \theta\), is then obtained

*In the standing wave tube used in this investigation two sound pressure minima can not be detected at or below 200 Hz.
\[ A_1 = \text{Incident Wave Amplitude at Filter Inlet} \]
\[ B_1 = \text{Reflected Wave Amplitude at Filter Inlet at Angle } \theta_R \]
\[ A'_1 = \text{Incident Wave at } x_1 \]
\[ B'_1 = \text{Reflected Wave at } x_1 \]
\[ C_1 = \text{Total Sound Pressure at } x_1 \]
\[ C_2 = \text{Total Sound Pressure at } x_2 \]
\[ A_2 = \text{Transmitted Wave at Filter Outlet} \]
\[ T = \frac{A_2}{A_1} \text{ at Angle } \theta_T \]

Figure 4 Graphical Determination of the Transmission Factor For an Arbitrary Filter
from the complex product of the reflection factor and $A_1$. At location $X_1$ the incident wave $A_1$ has a positive phase angle of $\frac{\omega}{c} X_1$ and due to attenuation has a magnitude greater than $A_1$ of $|A_1'| = |A_1| e^{\omega X_1}$. Similarly, the reflected wave at $X_1$, $B_1'$, has a negative phase angle $\frac{\omega}{c} X_1$, and has a magnitude less than $B_1$ of $|B_1'| = |B_1| e^{-\omega X_1}$. Rotating $A_1'$ and $B_1'$ through the proper phase angles and adding them vectorially yields the total sound pressure vector, $C_1$, at location $X_1$. The measured values of total sound pressure at locations $X_1$ and $X_2$ are $P_1$ and $P_2$ respectively. Multiplying $C_1$ by the ratio and rotating the results through the relative phase angle between $P_2$ and $P_1$ yields the total sound pressure vector, $C_2$, of the transmitted wave at location $X_2$. Since the filter is terminated anechoically the total sound pressure at $X_2$ is due solely to the transmitted wave. To find $A_2$, the value of the transmitted wave at the filter outlet, $C_2$ is rotated through the positive phase angle $\frac{\omega}{c} X_2$, and corrected for attenuation by increasing its magnitude by $e^{\omega X_2}$, i.e., $|A_2'| = |C_2| e^{\omega X_2}$. The complex value of the transmission factor, $T$ is then

$$T = \frac{A_2}{A_1} \Delta \theta_T \tag{24}$$

D. **Expansion Chamber Characteristics**

An expansion chamber consists of two plane discontinuities with a connecting tube in between. The
expansion chamber in Fig. (5) consists of a simple expanding element having a cross-sectional area increase of \( S_1 \) to \( S_2 \), a connecting tube of cross-sectional area \( S_2 \), and a simple contracting element having a cross-sectional area decrease from \( S_2 \) to \( S_1 \). The complex value of the reflection factor for the expansion chamber is

\[
R = R_1 + \frac{R_3 T_1 T_3 e^{-j\omega L}}{1 - R_2 R_3 e^{-j\omega L}}
\]  
(25)

The complex value of the transmission factor for the expansion chamber is

\[
T = \frac{T_1 T_3 e^{-j\omega L}}{1 - R_2 R_3 e^{-j\omega L}}
\]  
(26)

A complete derivation of these equations can be found in G Britain [8], Buckley [3], Simon [28], and Gegesky [9].
\[ S_1 = \text{Internal Cross-Sectional Area of Inlet and Outlet} \]

\[ S_2 = \text{Internal Cross-Sectional Area of Chamber} \]

\[ R_1 = \frac{S_1 - S_2}{S_2 + S_1} = R_4 \]

\[ R_2 = \frac{S_2 - S_1}{S_2 + S_1} = R_4 \]

\[ T_1 = \frac{2S_1}{S_2 + S_1} = T_4 \]

\[ T_2 = \frac{2S_2}{S_2 + S_1} = T_3 \]

**Figure 5** A Simple Expansion Chamber
IV. DESIGN, DEVELOPMENT AND EVALUATION OF STEADY FLOW APPARATUS FOR TWO-INCH I.D. STANDING WAVE TUBE

A. Introduction

In most applications of reactive type filters, flow through the filter is present. This investigation deals with automotive type filters used in both intake and exhaust silencing. Flow information for this application was needed to design the test equipment for simulating actual conditions.

Typical flow information was obtained from a 330 cu.in. V-8 engine with single exhaust.* A pitot tube was installed in the exhaust pipe eight inches upstream from the open end to measure velocity pressure. Engine rpm was held constant until the slope gauge reading stabilized.** Exhaust velocity pressure was then read in inches of water. Keeping these values in mind, a steady flow system was then designed for the two-inch I.D. standing wave tube constructed by Simon [28]. This standing wave tube is shown in Fig. 6. Detailed information on the design and evaluation of this standing wave tube can be found in a thesis by Simon [28].

*Information from this vehicle was also used in ref. (9).
**Flow rates ranged from 31 to 79 cfm for engine rpm between 500 and 2500.
Figure 6 Schematic of Standing Wave Tube, Flow System, and Instrumentation
B. Flow Induction System

Several methods of introducing flow were tried, leading to the design used in this investigation. Flow was introduced into the standing wave tube at the driver end with a small centrifugal blower. The sound signal was then introduced by an acoustic driver, as illustrated in Fig. (7).*

Approximate flow rates were measured by the use of a pitot static tube located at the blower outlet and monitored with a slope gauge (Fig. 8), calibrated in inches of water velocity pressure. By locating the pitot tube at the driver end of the standing wave tube, it had no effect on the standing wave pattern developed by the filter being investigated.

The blower used was capable of simulating typical idle flow conditions of a 330 cubic inch V-8 with single exhaust. The noise in the standing wave tube due to the blower and/or air flow passing the microphone probe was found to be at least 30 db below the standing wave minimums encountered in this investigation.** (See Appendix 2).

*A list of the equipment used in all phases of this investigation can be found in Appendix 7.

**Sound pressure levels of 110-120 db were used in this investigation. The microphone output was filtered by a 1% frequency analyzer tuned to the frequency of interest.
Figure 7  Blower, Driver, and Enclosure
Figure 8  2-Inch I.D. Standing Wave Tube with Blower and Driver Enclosure
Due to the intensity of the sound propagating from the blower inlet, it was necessary to enclose the blower and driver in a fiberglass lined enclosure. (Figs. 7 and 8)

C. Anechoic Termination

Reflection and transmission factor calculations were simplified by terminating the filter or filter element being investigated anechoically. An anechoic termination which allows flow was required for this investigation.

Various designs were evaluated by observing reflections with the standing wave tube. A tuneable side arm tee and tube dividers were investigated and proved unsatisfactory. A successful design (Fig. 9) was constructed by inserting a steel wool cone, tapering from ½" to 3" in eight feet, into a sheet metal cone tapering from 2" ID to 4" ID in 8 feet 6 inches. This configuration allowed a one inch space around the steel wool cone for air flow.

The anechoic termination was evaluated by measuring its reflection characteristics. The termination was attached to the standing wave tube and evaluated with and

*A true anechoic termination produces no reflections and therefore no standing waves are present.**The reflection data obtained from the standing wave measurements is analyzed by a digital computer program developed by Simon [28] during his investigation. This program is presented in Appendix 4.
Figure 9  Anechoic Terminations

Static Anechoic Termination (No Flow)

Anechoic Termination for Flow
without flow over a frequency range from 50 to 1500 Hz. The results of the reflection factor measurements are shown graphically in Fig. 10, along with the reflection factor characteristics of the anechoic termination* used by previous investigators [3,28]. As can be seen from Fig. 10, the values of the reflection factor were about 10% or less for the frequency range of 200 Hz to 1500 Hz. This was true for the flow termination both with and without flow. Because of significant reflections from the anechoic termination below 200 Hz, acoustic filter performance at low frequencies is subject to interpretation, depending on how the magnitude and phase of reflection from the anechoic termination affect the standing wave. For example, reflection from the anechoic termination would produce a standing wave between the test filter and the anechoic termination. The magnitude of the transmission factor obtained under such conditions would be smaller than the anechoic value if the outlet sound pressure was measured near a sound pressure minimum and greater than the anechoic value if it was measured near a sound pressure maximum. The effect on phase angle is unknown.

*This termination consisted of a steel wool cone backed by a cylinder of steel wool and an adjustable piston, inserted in a 7 foot length of 1.87 inch I.D. aluminum tubing.
Figure 10  Reflection Characteristics of Anechoic Terminations
D. Conclusions and Recommendations

It was concluded that slow steady flow can be introduced with a minimum of noise, therefore making standing wave measurements possible. It was also concluded that an anechoic termination can be developed to handle slow steady flow.

It is believed that higher flow rates are possible with the use of a larger blower and the addition of an inlet silencer to reduce the blower noise transmitted into the standing wave tube. With higher flow rates it may become necessary to provide the microphone cavity with a pressure-relief channel or a means of pressure equalization.

It is believed that this apparatus, with a minimum number of modifications, will be capable of handling steady flow at elevated temperatures. This could be accomplished by ducting the air from the anechoic termination outlet to the blower inlet and adding a heating element and thermostat to this air return to maintain a constant air temperature.

It is recommended that the effort required to traverse the microphone be reduced. This may be accomplished by modifying the sealing method between the steel band and aluminum tube or by the addition of a power traversing system.
The addition of a micrometer type adjustment mechanism would increase the accuracy of the measuring system.

It is also recommended that an anechoic termination which produces smaller reflections be developed. It is believed that this may be accomplished by increasing the length of the anechoic termination. Smaller reflection factors are definitely needed for accurate low frequency analysis. If the reflection factor of the element being tested is low, any reflection from the anechoic termination will affect the total measured reflection. However, if the reflection from the element is high, reflection from the anechoic termination is less important.
V. EXPERIMENTAL DETERMINATION OF THE ACOUSTIC CHARACTERISTICS OF REACTIVE FILTERS

A. Introduction

Three reactive type filters were used in this part of the experimental investigation. They were an expansion chamber, an automotive muffler with adjustable elements, and a typical replacement automobile muffler. They are shown in Fig. (11). The expansion chamber is 20\(\frac{1}{2}\) inches long and has an inside diameter of 5-1/8 inches. It was constructed from a double wrapped muffler casing. The inlet and outlet tubes were constructed from 1.87 inch I.D. aluminum tubing and were centrally located in .25 inch thick aluminum end caps.

The adjustable muffler was of the tri-flow design. It is shown schematically in Fig. (12). Its internal construction was equivalent to the replacement muffler* also used in this investigation. Three internal adjustments were possible: one chamber divider could be adjusted so as to change the length of the outlet expansion chamber; and at both the inlet and outlet a section of louvered pipe could be covered or uncovered. More detailed information on construction and static (no flow) evaluation

---
*The automobile muffler was a replacement muffler for a 1965 Chevelle with a 283 cubic inch V-8 engine.
Figure 11  Reactive Filters Investigated

- Expansion Chamber
- Adjustable Muffler
- Automobile Muffler
Figure 12 Schematic of Adjustable Muffler
of this muffler can be found in ref. [9] by Gegesky.

All three filters tested were evaluated by determining their reflection factors and transmission factors both with and without flow at frequencies of 50, 100, 150, 200, 300, 400, 600, 800, 1000, 1200, 1350, and 1500 Hz. All measurements were made with the filters terminated anechoically.

B. Reflection Factor Measurements

The reflection factor magnitude was determined by the measurement of a sound pressure minimum, a sound pressure maximum, and the distance between the adjacent sound pressure minima on either side of the sound pressure maximum. Since the location of a sound pressure minimum can be measured with greater accuracy than a sound pressure maximum, the location of the sound pressure maximum was determined from the relationship

$$x_{\text{max}} = \frac{x_{2\text{min}} - x_{1\text{min}}}{2} + x_{1\text{min}}$$

(27)

where $x_{1\text{min}}$ and $x_{2\text{min}}$ are the locations of two adjacent pressure minima. Accurate measurement of the location of a sound pressure minimum entailed measuring the locations of equal sound pressures occurring on either side of the minimum and using the mean value as its location. From Section III the expression for reflection factor magnitude,
knowing a sound pressure minimum and a sound pressure
maximum, is

\[ |R| = \frac{P_{\text{max}} e^{-\alpha X_{\text{min}}} - P_{\text{min}} e^{-\alpha X_{\text{max}}}}{P_{\text{max}} e^{-\alpha X_{\text{min}}} + P_{\text{min}} e^{-\alpha X_{\text{max}}}} \]  

(28)

where

- \( P_{\text{max}} \) = maximum sound pressure
- \( P_{\text{min}} \) = minimum sound pressure
- \( X_{\text{max}} \) = location of maximum sound pressure relative to the filter
- \( X_{\text{min}} \) = location of minimum sound pressure relative to the filter
- \( \alpha \) = attenuation constant
- \( e \) = base of natural logarithms

Using a sound pressure minimum, the expression for the reflection factor phase angle is

\[ \theta = 2 \frac{\omega}{c'} X_{1\text{min}} - n\pi \]  

(29)

Since \( \omega = 2\pi f \), \( c' = f\lambda \) and the distance between two adjacent sound pressure minimum is one half of a wavelength, the phase angle can be expressed as

\[ \theta = \frac{2\pi X_{1\text{min}}}{X_{2\text{min}} - X_{1\text{min}}} - n\pi \]  

(30), where \( n = \text{largest positive integer yielding a positive value for } \theta \)

The measurement of \( \lambda/2 \) must be accurate to within approximately 1/3% for accurate phase angle calculations \([3,8]\). To minimize the error in phase angle calculations, the sound pressure minimum nearest the filter was used.
To determine the reflection factor magnitude at low frequencies, the measurement and location of one sound pressure minimum and another arbitrary sound pressure are used. The expression for the reflection factor magnitude using one sound pressure minimum and another arbitrary sound pressure is, from Section III-B,

$$|R| = \frac{A_m e^{2\alpha x_{min}} - P_{min} e^{\alpha x_{min}}}{A_m}$$  \hspace{1cm} (22)

where

$$A_m = \frac{1}{2} \left[ P_{min} + \left( P_{min}^2 - 2 \left( \frac{P_{min}^2 - P^2(x)}{1 - \cos \phi} \right) \right)^{1/2} \right]$$  \hspace{1cm} (23)

The phase angle associated with R is determined as before from (21). This technique for determining reflection factor magnitude is required due to the fact that at low frequencies two sound pressure minima do not always exist in the distance traversed by the microphone. In the standing wave tube used in this investigation two sound pressure minima cannot be detected at or below 200 Hz. Therefore, this technique was used for all reflection factor calculations below 200 Hz. The computer programs for these methods can be found in Appendix 3.

A sensitive vacuum tube voltmeter was used to read sound pressure magnitudes. The microphone output was passed through a sound level meter and then through a sound analyzer before it was measured by the voltmeter.
A sound analyzer with a continuously tuneable 1/10-octave filter was used when taking data at frequencies below 200 Hz and a sound analyzer with a 1% bandwidth filter was used for frequencies of 200 Hz and above. All measurements were made as quickly as possible to minimize error due to drift. Standing wave drift may be caused by variations in frequency, temperature, or driver output. The location of the sound pressures were measured with the cursor and meter stick measuring system.

C. Transmission Factor Measurements

From Section III, the transmission factor was determined by combining the reflection factor data with data obtained from transmission measurements. The transmission measurements consisted of determining the magnitudes of relative phase angle between the sound pressures that exist at known distances on either side of the filter or filter element being investigated. Measurements were made with the use of the transmission tube apparatus constructed by Buckley [3], Figure 13. The apparatus consisted of two 1.87 inch I.D. brass tubes with microphone stations. The microphone stations are fixed at 18 inches from the acoustic filter. These locations are identical to simplify the calculations. The transmission tubes
Figure 13 Apparatus For Measuring Transmission Factors
were coupled to either side of the filter or filter element as shown in Fig. (13).

The same instrumentation as used for reflection factor measurements was used to measure the sound pressure magnitudes at each microphone station. Matched microphones were used at the two stations; however, data taken with these microphones were modified to account for slight differences in microphone response. A precision electronic phase meter was used to determine the relative phase angle between the two sound pressures. The equipment used in all phases of this investigation is shown in Fig. (14).

D. Evaluation of Expansion Chamber

The expansion chamber (See Section V) was evaluated at frequencies of 50, 100, 150, 200, 300, 400, 600, 800, 1000, 1200, 1350, and 1500 Hz and at flow rates of 0 and 37 CFM. All sound pressure and distance measurements were each measured twice before they were recorded. In practically all instances, no deviation between the two measurements was observed. If a deviation did occur, the average of the two measurements was recorded. All measurements were made in an air conditioned laboratory to minimize the effects of temperature and humidity variations. Therefore, differences in the expansion
Figure 14  Instrumentation Used For Acoustic Measurements
chamber's acoustic characteristics with and without flow were assumed to be due to flow effects. Figures 15 and 16 compare the expansion chamber's experimental characteristics (both with and without flow) to theory.* As can be seen from Fig. (15), correlation between experimental and theoretical reflection factor and reflection angle is good. The reflection factor of this expansion chamber with the flow termination and no flow was on the average 3% greater than the reflection factor of the expansion chamber with the static termination. This increase in reflection factor is probably due to the slightly greater reflection factor of the flow termination. There was also an average of 2 degrees increase in reflection phase angle for the flow termination with no flow as compared to the static termination. Therefore, the flow termination design is comparable to the static termination design. The flow rate of 37 CFM caused a 0-11% increase in reflection factor over the frequency range tested and an average of 3 degrees increase in reflection phase angle.

Fig. (16) shows that at certain frequencies the experimental transmission factor deviates from theoretical. It is believed that the deviation below 200 Hz

*Theoretical characteristics were obtained from derivation by Gatley [8], Buckley [3], and Simon [28].
Figure 15 Reflection Characteristics of the Expansion Chamber
Figure 16 Transmission Characteristics of the Expansion Chamber
is due to the anechoic termination not being completely reflection free, (See Section IV) and the low transmission of the expansion chamber in this range. The magnitude of the transmission factor obtained under such conditions would be smaller than the anechoic value if the outlet sound pressure was measured near a sound pressure minimum and greater than the anechoic value if it was measured near a sound pressure maximum. The effect on phase angle is unknown. Transmission factor deviation from theoretical at 600, 1000, and 1350 Hz may be due to the rapid rate of change of the reflection factor at these frequencies. Since reflection factor is used in the calculation of transmission factor, a discrepancy in the reflection factor will cause an error in transmission factor. Sound level variations and differences in microphone response may also introduce error. At frequencies where correlation is good between experimental and theoretical transmission factor, a flow rate of 37 CFM caused a 8-19% increase in transmission factor. The effect of flow on transmission phase angle appears to be unpredictable.

E. Evaluation of Adjustable Muffler

The adjustable muffler was evaluated at frequencies
of 50, 100, 150, 200, 300, 400, 600, 800, 1000, 1200, 1350, and 1500 Hz and with and without flow. A flow rate of 30 CFM was used. Reflection and transmission characteristics with an outlet expansion chamber length of 3\(\frac{1}{4}\) in., inlet and outlet louvers opened and closed, and with and without flow are shown in Figures 17 and 18. Characteristics with an outlet expansion chamber length of 5\(\frac{1}{4}\) in., inlet and outlet louvers opened and closed, and with and without flow are shown in Figures 19 and 20.

The varying effect of flow on reflection factor at 200 Hz or less in Figures 17 and 19 is believed to be due to the fact that the anechoic termination is not reflection free at these frequencies. Above 200 Hz a flow rate of 30 CFM caused a 3-6% increase in reflection factor and -10 to a 2 degree change in reflection phase angle. In Fig. (17) the effect of the 3\(\frac{1}{4}\) in. long outlet expansion chamber and louver combination can be seen at 1000 Hz. Closing the louvers causes a decrease in reflection factor and corresponding increase in transmission factor. This change suggests that one of the resonant modes of the expansion chamber closed-louver combination is approximately 1000 Hz, and therefore a decrease in performance occurs at this
Figure 17  Reflection Characteristics of the Adjustable Muffler with an Outlet Expansion Chamber Length of 3\(\frac{1}{4}\) Inches
Figure 18  Transmission Characteristics of the Adjustable Muffler with an Outlet Expansion Chamber Length of 3\(\frac{1}{4}\) Inches
Figure 19 Reflection Characteristics of the Adjustable Muffler with an Outlet Expansion Chamber Length of 5\(\frac{1}{4}\) Inches
Figure 20 Transmission Characteristics of the Adjustable Muffler with an Outlet Expansion Chamber Length of 5 1/4 inches
frequency (Figure 18). A similar effect can be seen with 5\(\frac{1}{4}\) inch long outlet expansion chamber. Increasing the length shifts the resonance to a lower frequency, and therefore the effect is not as noticeable at 1000 Hz.

The large deviation in transmission factor at 1350 Hz, shown in Figures 18 and 20, may be due to a rapid rate of change of the reflection factor around 1350 Hz or a resonant frequency of the system at approximately 1350 Hz. The effect of the 30 CFM flow rate and louvers on transmission phase angle appears to be unpredictable.

F. Evaluation of Automobile Muffler

The automobile replacement muffler was evaluated at frequencies of 50, 100, 150, 200, 300, 400, 600, 800, 1000, 1200, 1350, and 1500 Hz, and at flow rates of 0, 23.5, and 30 CFM.* Reflection factor, reflection phase angle, transmission factor and transmission phase angle are shown graphically in Figures (21 and 22). From Fig. (21) it can be seen that below 600 Hz a flow rate of 23.5 CFM caused a 5-10% decrease in reflection factor magnitude and a 5-10 degree increase in reflection phase angle, while a flow rate of 30 CFM caused a 10-15% decrease in reflection factor magnitude and a 5-10 degree

*30 CFM was maximum flow with the automobile muffler.
23.5 CFM was obtained by restricting the blower inlet.
Figure 21 Reflection Characteristics of Automobile Muffler
Figure 22 Transmission Characteristics of Automobile Muffler
increase in reflection phase angle. At 600 Hz and above, a flow rate of 23.5 CFM caused an average of 5% increase in reflection factor and an average increase in reflection phase angle of 6 degrees. A flow rate of 30 CFM caused a 7.5% decrease to a 3.5% increase in reflection factor and -25 to 10 degree change in reflection phase angle.

From Fig. (22) it can be seen that the transmission factor decreased with an increase in flow rate. In general, transmission phase angle also decreased with an increase in flow rate. The large deviation in transmission factor at 1000 Hz and 1350 Hz may be due to the same factors that produced the variations observed with the experimental analysis of the adjustable muffler, i.e. a rapid rate of change of the reflection factor near these frequencies or a muffler resonance at or near 1000 and 1350 Hz.
VI. CONCLUSIONS AND RECOMMENDATIONS

Major conclusions of this investigation can be summarized as follows:

1. Standing wave measurements can be made in the presence of slow steady flow.
2. A flow rate of 37 CFM increased reflection factor from 0 - 11% and increased transmission factor from 8 - 19% for a simple expansion chamber. Flow effects should be considered when analyzing expansion chamber type filters commonly used with flow.
3. For a combination of filter elements evaluated and for a typical automotive muffler, a flow rate of 30 CFM* does not have a significant effect on transmission and reflection characteristics.
4. It is recommended that the characteristics of louvered tubes with flow should be investigated to determine their effect when used in combination with other filter elements.
5. It is recommended that in future investigations of this type, higher flow rates be used to simulate an engine under load at higher speeds. These higher flow rates may cause significant changes in filter characteristics.

*For a 330 cu. in. engine, 30 CFM represents a no load idle condition.
6. Since high temperature, as well as flow, is common in many filter applications, it is recommended that the combined effects of temperature and flow be investigated.

7. It is recommended that filter characteristics be investigated for sound pressure level dependency. This may be a cause for deviations in the results of this investigation, since sound pressure maximums of 110 - 130 dB in the standing wave tube were used.
VII. APPENDICES
Appendix I

Attenuation Constants and Speed of Sound in Two Inch I.D. Tube

The attenuation constants and speed of sound used for calculating were obtained from the modified Kirchhoff equation and Kirchhoff's equation for the speed of sound in a tube respectively [28]:

$$\alpha = 3.18 \times 10^{-5} \frac{f^{1/2}}{r}$$

where $f$ = frequency in Hz

$r$ = tube radius in centimeters

and

where $c$ = speed of sound in free space

$r$ = radius of tube centimeters

$f$ = frequency in Hz

The speed of sound in free space was evaluated for dry air at 74°F and 760 mmHg and has a value of 34,529 cm/sec.
The data are presented below:

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Attenuation Constant for 2 inch ID Tube (x 10 cm)</th>
<th>Speed of Sound in 2 inch ID Tube c' (cm/sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.88</td>
<td>34,117</td>
</tr>
<tr>
<td>100</td>
<td>1.25</td>
<td>34,238</td>
</tr>
<tr>
<td>150</td>
<td>1.53</td>
<td>34,291</td>
</tr>
<tr>
<td>200</td>
<td>1.77</td>
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<td>300</td>
<td>2.17</td>
<td>34,361</td>
</tr>
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After design and construction of the flow system, it was necessary to determine the sound level in the standing wave tube due to the blower and noise caused by air flow around the microphone probe. Spectrums (Fig. A) in the frequency range of interest were taken with the mufflers used in this investigation attached to the standing wave tube and terminated with the anechoic termination. The microphone was located in the center of its travel in the standing wave tube.

From Gatley [8] and automobile engine exhaust sound level measurements, sound pressure maximums of 110 to 130 db in the tube were selected for this investigation. As can be seen from the spectrum in Fig. (A) the noise due to the air flow or the air flow system was found to be between 30 and 50 db below the sound level chosen for this investigation. With a reflection factor magnitude of .9 and a standing wave maximum sound pressure of 130 db, the minimum would be about 104 db. This is a sufficient distance from the air flow noise so that the noise has virtually no effect on measurements taken.
Figure A  Air Flow Noise in the Standing Wave Tube at 23.5 CFM Using a 7% (1/10 Octave) Bandwidth Filter
Appendix 3

Computer Programs for the Calculation of Experimental Reflection Factors

Two sound pressure minima cannot be detected at or below 200 Hz in the 2 inch I.D. standing wave tube used in this investigation. Therefore, two separate programs are required, one for the calculation of reflection factor at or below 200 Hz and one for the calculation of reflection factor above 200 Hz.

The notation and programs follow:

1) Notation for computer program using two sound pressure minima;

\[ \text{ATT} = \text{attenuation constant} \]

\[ \text{DELTA} = \text{correction factor for the differences between the measured and actual distances from the discontinuity to the sound pressure minima} \]

\[ \text{DELTAX} = \text{distance between first and second sound pressure minima} \]

\[ \text{DELTB} = \text{DELTAX} \]

\[ F = \text{frequency in Hz} \]

\[ M = \text{number of input data cards} \]

\[ \text{PHASE} = \text{phase angle associated with reflection factor} \]

\[ \text{PMAX} = \text{maximum sound pressure} \]

\[ \text{PMIN} = \text{minimum sound pressure} \]
\( R_1 \) = calculated magnitude of the reflection factor

neglecting attenuation

\( R_2 \) = calculated magnitude of the reflection factor

considering the effect of attenuation

\( R_{FATT} = R_2 \)

\( R_{FO} = R_1 \)

\( X_1, X_2 \) = measured location (distance from discontinuity)

of equal sound pressure magnitudes on either
side of the first sound pressure minimum

\( X_3, X_4 \) = measured location (distance from discontinuity)

of equal sound pressure magnitudes on either
side of the second sound pressure minimum

\( X_{1MIN} \) = actual distance from the discontinuity to the

first sound pressure minimum

\( X_{2MIN} \) = actual distance from the discontinuity to the

second sound pressure minimum

\( X_{MAX} \) = actual distance from the discontinuity to the

sound pressure maximum

Computer program using two sound pressure minima
along with a sample of output;
C GENERAL PROGRAM FOR REFLECTION FACTORS AND PHASE ANGLES
C FREQUENCY IN CYCLES PER SECOND
C SOUND PRESSURE IN VOLTS
C X IN CENTIMETERS
C PHASE ANGLE IN DEGREES
C ATTENUATION FACTOR PER CENTIMETER
C OUTPUT IS IN THE FOLLOWING ORDER

WRITE (3,200)
DO10 J 1, 32
READ (1,100)F, PMA, PMIN, X1, X2, X3, X4, ATT, DELTA
X1MIN ((X2-X1)/2.0) DELTA
X2MIN ((X4-X3)/2.0) DELTA
DELTB X2MIN-X1MIN
XMAX (DELTB/2.0) X1MIN
R1 (PMA-PMIN)/(PMA PMIN)
A ATT*X1MIN
B ATT*XMAX
C EXP(A)
D EXP(B)
R2 ((PMA*C)-(PMIN*D))/((PMA/C) (PMIN/D))
Z X1MIN/DELTB
DO 2 N 1,99,2
PI 3.1416
PH (2.0*PI*Z)-N*PI
PHASE PH*(180.0/PI)
IF(ABS(PHASE)-360.0)1,1,2
CONTINUE
WRITE (3,201)F,PMA, PMIN,X1MIN, X2MIN, DELTB, XMAX, R1, ATT, R2, PHASE
10 CONTINUE
CALL EXIT
100 FORMAT (9F8.0)
200 FORMAT (3X, 'FREQUENCY', 5X, 'PMAX', 5X, 'PMIN', 5X, 'X1MIN', 6X, 'X2MIN', 6X, 
      6 ANGLE')
      7F5.3, 5X, E10.3, 5X, F5.3, 5X, F7.2)
END

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<th>PMIN</th>
<th>X1MIN</th>
<th>X2MIN</th>
<th>DELTA X</th>
<th>XMAX</th>
<th>RFO</th>
<th>ATTENUATION</th>
<th>RFATT</th>
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OBJECT CODE 1792 BYTES, ARRAY AREA 0 BYTES, UNUSED 48208 BYTES

HASP-II JOB STATISTICS-- 65 CARDS READ-- 71 LINES PRINTED--
0 CARDS PUNCHES-- 1.73 SEC CPU TIME
2) Notation for computer program using one sound pressure minimum and another arbitrary sound pressure:

\[ AO = \text{calculated magnitude of the incident sound pressure wave} \]

\[ ATT = \text{attenuation constant} \]

\[ C = \text{speed of sound in the standing wave tube} \]

\[ F = \text{frequency in Hz} \]

\[ M = \text{number of input data cards} \]

\[ \text{PHASE} = \text{phase angle associated with the reflection factor} \]

\[ \text{PMIN} = \text{minimum pressure} \]

\[ \text{PREF} = \text{arbitrary pressure} \]

\[ RA = \text{calculated magnitude of the reflection factor considering the effect of attenuation} \]

\[ \text{RFATT} = RA \]

\[ \text{RFQ} = \text{calculated magnitude of the reflection factor neglecting attenuation} \]

\[ RO = RFO \]

\[ \text{XMIN} = \text{distance of sound pressure minimum from discontinuity} \]

\[ \text{XREF} = \text{arbitrary pressure location relative to the minimum pressure location} \]

Computer program using one sound pressure minimum and another arbitrary sound pressure along with a sample of output;
C PROGRAM TO SOLVE FOR LOW FREQUENCY REFLECTION FACTORS
AND REFLECTION ANGLES 50-200 CPS

1  PI 3.1416
2  WRITE (3,200)
3  DO 10 J 1, 16
4  READ (1,100)F,ATT,C,PREF,PMIN,XREF,XMIN
5  B (4.0*PI*F)/C
6  Z B*XMIN
7  DO 2 N 1,99,2
8  PH Z-N*PI
9  PHASE PH*(180.0/PI)
10  IF(ABS(PHASE)-360.0)1,1,2
11  2 CONTINUE
12  ETA B*ABS(XMIN-XREF)
13  DEM 1.0-COS(ETA)
14  AO (PMIN SQRT(PMIN**2-(2.0*(PMIN**2-PREF**2)/DEM)))/2.0
15  RO (AO-PMIN)/AO
16  RA ((AO*EXP(2.0*ATT*XMIN))-(PMIN*EXP(ATT*XMIN)))/AO
17  WRITE (3,201)F,PREF,XREF,PMIN,XMIN,RO,RA,PHASE
18  10 CONTINUE
19  CALL EXIT
20  100 FORMAT(7F10.0)
21  200 FORMAT(3X,'FREQUENCY',2X,'PREF',8X,'XREF',8X,'PMIN',8X,'XMIN',8X,'
1RF0',9X,'RFATT',6X,'PHASE')
22  201 FORMAT(3X,F6.1,5X,F7.3,5X,F7.2,5X,F7.3,5X,F7.2,5X,F7.3,5X,F7.3,5X,
2F8.2)
23  END

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OBJECT CODE 1624 BYTES, ARRAY AREA 0 BYTES, UNUSED 48376 BYTES

HASP-11 JOB STATISTICS-- 43 CARDS READ-- 49 LINES PRINTED

CARDS PUNCHED-- 1.06 SEC. CPU TIME
Appendix 4

Computer Program for the Calculation of Experimental Transmission Factors

The graphical calculation of the experimental transmission factor, presented in Section III, is adapted to digital computer solution by the use of complex numbers and phasor notation.

The notation and the program follow:

\( A \) = phase angle between \( B_0 \) and \( B_1 - Z \)

\( \text{ANGLE} \) = calculated reflection factor phase angle expressed in radians

\( A_0 \) = incident sound pressure wave at discontinuity inlet with assumed value of one and phase angle zero

\( A_1 \) = calculated incident sound pressure wave at \( x_1 \)

\( A_2 \) = calculated transmitted wave at outlet of discontinuity

\( \text{ATT} \) = attenuation constant

\( B_0 \) = calculated reflected sound pressure wave at discontinuity inlet

\( B_1 \) = calculated reflected sound pressure wave at \( x_1 \)

\( C_1 \) = calculated total sound pressure at \( x_1 \)

\( C_2 \) = calculated total sound pressure at \( x_2 \)

\( \text{EX} \) = measured relative phase angle between \( P_1 \) and \( P_2 \) expressed in radians
DELTA = distance between first and second sound pressure minima or 0.5 times the wave length

F = frequency in Hz

M = number of input data cards

P = calculated transmission phase angle expressed in degrees

$P_1$ = measured value of the sound pressure at the microphone station located at $x_1$ on the inlet side of the discontinuity

$P_2$ = measured value of the sound pressure at the microphone station located at $x_2$ on the outlet side of the discontinuity

R = magnitude of calculated reflection factor considering the effects of attenuation

RANG = calculated reflection factor phase angle expressed in degrees

$RO$ = complex value of the calculated reflection factor

RP = pressure ratio $P_2/P_1$

TANG = measured relative phase angle between $P_1$ and $P_2$ expressed in degrees

VAL = calculated magnitude of the transmission factor

$W_1$ = distance from the discontinuity inlet to the microphone station located at $x_1$ on the inlet side of the discontinuity
W2 = distance from the discontinuity outlet to the microphone station located at x2 on the outlet side of the discontinuity

Z = phase angle between AO and A1 due to path distance W1

Z2 = phase angle between C2 and A2 due to path distance W2

Note: Anechoic conditions are assumed to exist at the muffler outlet.
C GENERAL PROGRAM TO SOLVE FOR TRANSMISSION FACTORS AND
C TRANSMISSION ANGLES
COMPLEX RO, BO, Y, A1, V, B1, C1, Z1, Y1, A2, COMPLEX, CEXP
WRITE(3,200)
3  PI 3.1416
DO 10 J 1, 72
READ(1,100)F, ATT, R, RANG, DELTA, W1, W2, P1, P2, TANG
AO 1.0
ANGLE RANG/57.3
F1 R*COS(ANGLE)
F2 R*SIN(ANGLE)
RO CMPLX(F1, F2)
BO RO*AO
Z (P1*W1)/DELTA
Y CMPLX(0.0, Z)
D1 ATT*W1
D1P EXP(D1)
A1 D1P*AO*CEXP(Y)
A -1.0*Z
V CMPLX(0.0, A)
D2 -D1
D2P EXP(D2)
B1 D2P*BO*CEXP(V)
C1 A1 B1
RP P2/P1
CX TANG/57.3
Z1 CMPLX(0.0, CX)
C2 C1*RP*CEXP(Z1)
D3 ATT*W2
D3P EXP(D3)
29  Z2 (PI*W2)/DELTA
30  Y1 CMPLX(0.0,Z2)
31  A2 D3P*C2*CEXP(Y1)
32  G REAL(A2)
33  H AIMAG(A2)
34  VAL SQRT(G**2 H**2)
35  IF(G)12,13,12
36  13 IF(H)14,14,15
37  14 P -90.0
38  GO TO 20
39  15 P 90.0
40  GO TO 20
41  12 P (ATAN2(ABS(H),ABS(G)))*57.3
42  IF(G)91,17,17
43  91 IF(H)18,18,19
44  17 IF(H)21,21,20
45  18 P P-180.0
46  GO TO 20
47  19 P 180.0-P
48  GO TO 20
49  21 P -1.0*P
50  20 WRITE(3,201)F,R,RANG,VAL,P
51  10 CONTINUE
52  CALL EXIT
53  100 FORMAT(1OF7.0)
54  200 FORMAT(3X,'FREQUENCY',3X,'REFLECTION FACTOR',3X,'REFLECTION ANGLE'
      1,3X,'TRANSMISSION FACTOR',3X,'TRANSMISSION ANGLE')
55  201 FORMAT(5X,F6.1,8X,F7.3,13X,F8.2,14X,F7.3,13X,F8.2)
56  END
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OBJECT CODE 2944 BYTES, ARRAY AREA 0 BYTES, UNUSED 47056 BYTES

HASP-II JOB STATISTICS-- 99 CARDS READ-- 105 LINES PRINTED-- 0 CARDS PUNCHED-- 15.04 SEC.CPU TIME

/END
Appendix 5

List of Equipment

5. General Radio Co. Sound Level Calibrator, Type 1562-A, Serial No. 322.
12. General Radio Data Recorder, Type 1525-A, Serial No. 119.
14. Ling Electronics Electronic Voltmeter, Type 2416, Model VTVM-2, Serial No. 186526.
15. AD YU Electronics, Inc. Precision Phase Meter, Type 406L, Serial No. 3399 L.


18. Dwyer Flex-tube Manometer, Series 25.


## VIII. BIBLIOGRAPHY


IX. VITA

Thomas Valle Huber was born on March 25, 1944 in St. Louis, Missouri. He received his primary and secondary education in Affton, Missouri. In September, 1962 he enrolled at the University of Missouri - Rolla and received a Bachelor of Science Degree in Mechanical Engineering in January 1968.

While completing his undergraduate work, Mr. Huber was a co-op student with McDonnell Douglas Corporation. As a graduate student, he had a graduate assistantship at the University of Missouri - Rolla and was a Research Engineer for Ford Motor Company during the summer of 1968. Since September 1969, he has been a Research Engineer for Ford Motor Company.

Mr. Huber has received a Curators Award and been on the Honor Roll. He is a member of Delta Sigma Phi Social Fraternity, Society of Automotive Engineers, and Phi Eta Sigma. He is married and is a U. S. citizen.