An accurate digital instrument to measure reactor period

Ernest Wayne Scott

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AN ACCURATE DIGITAL INSTRUMENT TO MEASURE REACTOR PERIOD

BY

ERNEST WAYNE SCOTT, 1943-

A THESIS

Presented to the Faculty of the Graduate School of the

UNIVERSITY OF MISSOURI-ROLLA

In Partial Fulfillment of the Requirements for the Degree

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

1972

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T2881
38 pages
C.1

226946
This thesis has been prepared in the style acceptable by the IEEE Transaction on Nuclear Science and pages iii, vi, and 1 thru 22 will be submitted to the Editorial Department for publication. Appendices A, B and C have been added to this thesis for additional information.
ABSTRACT

At facilities having a research-training reactor, such as the University of Missouri-Rolla Reactor (UMRR), one finds it necessary to perform a large number of rod calibrations during the course of the year. In practice rod worths are determined by measuring the reactor period created by an incremental withdrawal of the rod under calibration. Period is then related to reactivity thru the use of a publication such as the AEC publication, TID-4500.

This frequent measurement of period makes it desirable to have a simple, automatic and accurate method to make such measurements. At UMRR we have designed, constructed and installed such an instrument. The instrument measures doubling time rather than period but, thru the use of an internal time base conversion, displays a four bit decimal number that is the reactor period in seconds. The instrument is simple in concept and utilizes the 7400 Series integrated circuits in the largest portion of the unit. The instrument is easy to operate and once initiated, will automatically complete the measurement of the period displaying the results. Error in the instrument can be shown to be less than 1.5%. Thus the unit meets the three requirements of simplicity, accuracy and ease of operation and in addition is moderately inexpensive, less than $120.
ACKNOWLEDGEMENT

The author is grateful to Dr. Paul Stigall, Electrical Engineering Department of the University of Missouri-Rolla, for guidance and assistance in the preparation of the paper. A special word of thanks is also due Mr. Walt Henery, for invaluable assistance in the preparation of printed circuit boards for the project, and Mr. John Arvanis, who prepared the artwork for the paper.
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I. INTRODUCTION

The Atomic Energy Commission's licensing requirements make it necessary to determine rod worth curves at intervals not to exceed six months as well as recalibration following changes in core configuration. At research-training facilities like the University of Missouri-Rolla Reactor (UMRR), a 200 Kw thermal swimming-pool reactor, such rod calibrations occur quite frequently. At UMRR rod worth curves are performed by staff at six month intervals and following interim changes of core configuration. They are also performed as laboratory experiments by students in Nuclear Engineering.

Rod worth determinations are made by experimentally determining the reactor period generated by the incremental withdrawals of the rod under calibration, then relating the period to reactivity thru AEC publication, "TID 4500". Since period is defined as the amount of time required for reactor power to increase by a factor of the exponential constant "e", the measurement to be made is time as a function of power. The method of making this measurement is to withdraw the rod a specific distance and after the initial transients have died away a manual measurement is made of the time for reactor power to increase from 30 to 81% as observed on the Linear Power Recorder. The 30 to 81% change is approximately a factor of "e" increase. This method of measurement requires interpolation of the signal observed, since the minor divisions on the recorder in question are 2% increments. It is difficult to calculate the accuracy of measurements made in this manner since the operator is an integral part of the measurement. One may assume that, on the average, the operator is able to resolve reactor
power to within ± 1% with an occasional operator induced error. In addition to the resolving error the recorder has a quoted figure of ± 0.3% for linearity.

The purpose to this paper is to detail a digital system that is capable of measuring and displaying period. The system is automatic, except for an initiation signal from the operator. The instrument has an eight stage Analog to Digital Convertor with a resolving accuracy of plus or minus the least significant digit (± LSD). This is a resolving accuracy of ± 0.4%. The linearity figure for the convertor is ±¼ LSD or ± 0.2% if expressed as a percentage. The two sources of error create an uncertainty in the period measurement that is inversely proportional to the initial power level. (Refer to Topic IV) If one uses the optimum range of the instrument, initial power equal to one-half the full scale power, the maximum possible instrument error can be shown to be less than 1.5%. The device has been installed at the UMRR and fulfills its intended function during the rod calibration procedures.
II. THEORY OF OPERATION

Reactor period is defined as the time necessary for the reactor power to increase by the exponential constant "e" and is normally expressed in seconds. It may be measured by determining the elapsed time for such an increase or by measuring the time for power to increase by a factor of two, "Doubling Time", and then converting to period by dividing by the natural logarithm of two. This can be shown in the following manner:

\[ P(t) = P_0 \cdot e^{t/T} \]

where, \( t = \) time from \( P(0) = P_0 \)

\( T = \) Reactor Period

\( P_0 = \) Initial power level

When \( P(t) = 2 \cdot P_0 \) the power is twice the initial value and the variable, time, is defined to be \( t_2 \).

\[ 2 \cdot P_0 = P_0 \cdot e^{t_2/T} \]

\[ 2 = e^{t_2/T} \]

\[ \ln 2 = t_2/T \]

\[ T = t_2/\ln 2 = t_2/0.69 \]

The instrument installed at the UMRR measures the elapsed doubling time and internally converts real time seconds to equivalent period seconds. The instrument can be divided into three sections for discussion purposes, shown in Figure 1.

A. Analog to Digital Convertor

The section contains a tracking Analog to Digital Convertor whose
Figure 1. Block Diagram of the System
input is a zero to -5 volt analog signal that is proportional to the present reactor power and whose output is an unsigned 8 bit binary number which is sent to the Decision Logic board.

B. Decision Logic

This unit inputs the 8 bit number and upon initiation of a measurement locks the initial value of $P_0$ into a register. The board then begins to compare the present value received from the Analog to Digital Convertor against the initial value. This comparison continues until the present value is either twice or one-half the initial value. At this time the Decision Logic unit outputs a termination signal to the Display Logic board.

C. Display Logic

The unit is basically a four stage counter with an input gating circuit. The gate is opened when the START push button is pushed and is closed by the STOP signal from the Decision Logic board. Since the interval between initiation and termination is the doubling time, one must convert to reactor period by dividing by the natural logarithm of two. This is accomplished internally in the Display Unit by dividing the real time, time base by the natural logarithm of two and using the equivalent period time base as the input to the four digit display register.
III. CIRCUIT OPERATION

This section is intended to give a complete detailed description of the sub-sections of the device and to describe the interactions of the sub-sections in the complete system.

A. Analog to Digital Convertor

The heart of the Analog to Digital Convertor is a solid state Digital to Analog Convertor (DAC) module that converts an eight bit binary number to a proportional current. Referring to Figure 2 we examine the operation of the circuit. For an illustrative example we assume that the input to J1 is a -1 volt signal and the 8 bit counter is initially reset, zero current output, so the voltage at the test point (TP) will be negative. This negative signal is converted to a logic "1" by the μ710 comparator circuit. The direction Flip-Flop is set when the CLOCK1 timing pulse is generated. This will partially enable the upper NAND gate. The gate will be completely enabled when the CLOCK2 pulse is generated, however, the Up pulse doesn't enter the 74193 reversible counter until the trailing edge of CLOCK2, due to the triggering characteristics of the counter. With a count of 1 in the counter the current out of the DAC is about 7.8 microamperes which develops about +4.4 millivolts at pin 15 of the convertor, thus reducing the negative voltage at the TP by a small amount. The process is continued until the 8 bit counter reaches a count of \( 52 \) \(_{10}\) or \( 110100 \) \(_{2}\). The current out of the DAC will be about 406 micro-amps, while the current removed from the TP node by the -1 volt input signal is about 400 micro-amps. Thus the voltage at the TP is now a small positive voltage which is converted to a logic "0" by the μ710
Figure 2. Analog to Digital Converter
comparator. The direction Flip-Flop is reset by the CLOCK1 pulse and
the register is counted down one bit by the CLOCK2 pulse. At a count
of \(51_{10}\) or \(110011_2\) the voltage at the TP is negative which generates an
Up pulse during the next clock cycle. The convertor continues to
oscillate back and forth between a count of \(51_{10}\) and \(52_{10}\) until one
changes the input voltage. The Analog to Digital Convertor will con-
tinually track the input signal until one depresses the START push
button, at which time the timing circuit is momentarily frozen at
CLOCK4 time. This allows the transfer of the 8 bit word from the con-
vertor to the Decision Logic board to be free from switching transients.

The remaining circuit on the Analog to Digital board is the clock
and timing circuit which consists of an astable multivibrator and a
two stage binary counter. The two stage counter is decoded by four
NOR gates into four distinct timing pulses, CLOCK1 thru CLOCK4. The
astable runs at approximately 100 kHz and thus the timing pulse fre-
quency is about 25 kHz. This gives the convertor a resolution time
for a full scale step input of about 0.01 seconds, which would give a
tracking rate of 500 volts/sec. Comparing this to the maximum expected
input rate of change of 0.182 volts/sec. we see that the convertor's
tracking ability is well in excess of that required to follow input
variations.

B. Decision Logic Board

The Decision Logic board, Figure 3, takes as an input the 8 bit
binary number from the Analog to Digital Convertor. When the STROBE
signal from the START push button is received, the present digital
value of the reactor power is stored in the two 7475 latches. The inverted output of the latches is fed into two 8 bit adder chains. In the first adder chain the latch output is shifted one position left with the LSD and the input carry both hard-wired at a logic "1" level. This means that when the 8 bit number from the convertor has doubled from its' initial value the first adder chain will generate a carry out signal. As an illustrative example assume the initial digital value is a count of \(100_2\) or \(01100100_{10}\).

\[
P(0) = 01100100 \\
\overline{L} = 10011011
\]

Thus \(\overline{L}\) left shifted one position and with a hard-wired logic "1" LSD, the first input to the adder is:

\[
\overline{L^*} = 00110111
\]

The second input is the 8 bit digital input from the Analog to Digital Convertor, which at the region of interest will be twice \(P(0)\) or \(200_{10} = 11001000_2\). The third input is a hard-wired "1" carry-in. Thus the adder chain sums:

\[
2 \cdot P(0) = 11001000 \\
\overline{L^*} = 00110111 \\
\text{Carry-in} = \overline{1} \\
\overline{00000000} \text{ plus a "1" carry-out signal.}
\]

Thus the first adder chain generates a logic "1" carry-out signal when \(P(t)\) is greater than or equal to twice \(P(0)\).

The second adder chain works in a similar fashion, except the carry-out signal becomes a zero when \(P(t)\) is less than or equal to one-half \(P(0)\). For example:
Figure 3. Decision Logic Unit

From the Push Button

DECISION LOGIC UNIT

From the START Push Button

8 Bit Number From the ANALOG-DIGITAL CONVERTER

Carry In

7483 ADDER

L1

7475 LATCH

A1

A2

A3

A4

L2

L3

L4

8 Bit Number From the ANALOG-DIGITAL CONVERTER

A5

A6

A7

A8

L5

L6

L7

L8

STROBE

To DISPLAY UNIT

STOP

CLOCK 4

From A/D Conv.

1/4 7400

1/4 7400
\[ P(0) = 100_{10} = 01100100 \]

\[ L = 1001011 \]

Creating the modified signal, \( L^{**} \) by right shifting the binary number one position and making the MSD a "1".

\[ L^{**} = 11001101 \]

Plus \( \frac{1}{2} \cdot P(0) = 00110010 \)

\[ 11111111 \text{ with a } "0" \text{ carry-out.} \]

The logic "0" carry-out signal from the second adder chain is fed thru a NAND gate used as a simple invertor. This signal and the output of the first adder chain are gated with the CLOCK4 pulse from the Digital to Analog Convertor, creating a "0" \text{STOP} signal when the power level has either doubled or been reduced to one-half the original value frozen in the 7475 latches when the START push button was depressed.

One item in the circuit that is not self-explanatory is the purpose of the diode from the carry-out of the first adder chain to the \( L_8 \) output. The diode inhibits an erroneous \text{STOP} signal that would be generated when one would attempt to measure negative periods and the initial count in the register would contain a logic "1" in the MSD. Rather than belabor the point it is suggested that the reader examine the behavior of the first adder chain, as was done in a previous example, assuming an initial value of \( 128_{10} \). If this is done one will find that carry-out "1's" are generated immediately after the initiation of the measurement unless inhibited by the diode.

C. Display Logic Board

The Display Board, Figure 4, is simply a gated decade counter that
counts clock pulses from a crystal controlled oscillator. The gate is
opened in the following manner, by depressing the START push button
the NAND-NAND flip-flop is set, which partially enables the NOR gate.
When the push button is released the NOR gate is fully enabled and
clock pulses are then gated into the display counter via the NAND gate.

The oscillator frequency was determined as much by economics as
by reactor theory. The desired display was to be a four digit decimal
number including one decimal place. The time base counted into the
display unit is to be tenths of reactor period equivalent seconds
rather than real time tenths of seconds. Real time is converted to
equivalent period seconds by dividing by the natural logarithm of 2.0
which is 0.69315. This requires an input pulse rate of 14.427 pps
into the four stage decimal display register. For crystal stability
one is restricted to values above 100 kHz using divider circuits to
achieve the lower frequencies. Crystals of 144.270 kHz and 1.442 695 MHz
were found to be unrealistically priced. By multiplying 1.442 695 by 5
we find that 7.213 475 MHz lies in the middle of the 40 meter amateur
radio band where crystals can be ordered at a reasonable cost. The
additional cost of adding two more 7490 decade counters to the display
unit is minimal when compared to the cost of the special purpose
crystal. Thus the display unit consists of a 7.213 MHz oscillator, a
divide by five section of a 7490 decade counter, five divide by ten
decade counters (7490's) followed by four more decade counters (7490's)
those outputs are tied to BCD drivers (7441's) for the Nixie tube
display units. The counter is enabled as previously discussed and
counting is terminated by the STOP signal from the Decision Logic
board. The display is reset by depressing the START push button
To DECISION LOGIC

STROBE

START
To A/D CONV.

Crystal Oscillator 7.213MHz.

DISPLAY LOGIC

NIXIE Tubes

Decoder/Drivers

5 7490

Five Decade Counters 5 - 7490's

+ 5

RESET Line

Figure 4. Display Logic
The oscillator used in the system is an untuned Pierce with a common emitter follower stage for isolation. The output of the CE stage is clipped at a five volt level by a zener diode to make the oscillator output compatible with TTL logic circuits.
IV. CALIBRATION AND ACCURACY

The pico-ammeter at UMRR provides a 0 to -5 volt signal proportional to 0 to 120% of the instrument's range, with nine usable ranges from 2 watts to 200 kilowatts. The period measuring device should be calibrated so that a -5 volt input yields a count oscillating between 254\textsubscript{10} and 255\textsubscript{10}. This is accomplished by first shorting the input and observing that the 8 bit number from the A/D Convertor is oscillating between a count of zero and one, then one inputs a -5 volt signal and adjusts the potentiometer until the count from the convertor varies between 254\textsubscript{10} and 255\textsubscript{10}. The preceding item is the only adjustment required on the unit and following that, the unit may be installed and connected to the pico-ammeter.

Accuracy of the unit is determined by the four factors listed below:

1. Voltage resolution in the A/D Convertor.
2. Linearity of the A/D Convertor.
4. Time base inaccuracy.

The last two factors are disregarded as sources of error since the resolving time of the unit is much faster than the maximum rate of change of the input and the time base will have less than 0.01% deviation due to the use of the crystal oscillator. Thus the accuracy is determined by the two factors generated in the A/D Convertor.

The first source of error is related to the one bit variation in
the LSD of the A/D Convertor. This variation in count is an uncertainty in power since the signal we are performing the conversion on is proportional to power. We may relate uncertainty in power to uncertainty in time in the following manner:

1. Solve the power equation for time.
2. Differentiate with respect to time.
3. Replace \( dt \) and \( dP \) with \( \Delta t \) and \( \Delta P \), the respective uncertainties in time and power, then solve for \( \Delta t \).

At the beginning of a measurement there will not be any uncertainty in time because the counter is started by the START push button, however, there will be an uncertainty at the end of the measurement because of the power uncertainty created by the resolving error of the system. In the normal A/D Convertor the uncertainty is \( \pm \frac{1}{2} \) LSD. In tracking convertors the uncertainty is \( \pm \) LSD because the count varies above and below the true value, thus at any instant the true value could lie as much as one count above or below the indicated value. This \( \pm \) one count variation is doubled in the instrument creating a four count error band in the termination of the measurement. This error band has been reduced by half by gating the START signal with the CLOCK4 and the Up signals, refer to Figure 2. The process is to simply force the A/D Convertor to take on the upper value of the one bit variation so that the uncertainty at the beginning is \(-LSD\) rather than \(\pm LSD\), thus reducing the uncertainty at the termination value to \(\pm LSD\).

The steps involved in calculating the error are:
1. Define error created by the power uncertainty as:

\[
\text{Error}_1 = \left. \frac{\Delta t}{t_2} \right|_{P=2\cdot P_0} \times 100\% 
\]

2. Replace T in the expression for \( \Delta t \) by \( T = \frac{t_2}{0.69} \).

3. The power per bit resolution of the convertor, defined as \( \Delta P \), is expressed as the full scale power \( (P_{\text{F.S.}}) \) divided by 255.10.

4. Evaluate the error expression at the termination value of \( P = 2 \cdot P_0 \) replacing \( P_0 \) with \( K \cdot P_{\text{F.S.}} \). Where K is the fractional part of full scale power, i.e., \( K = \frac{P_0}{P_{\text{F.S.}}} \).

The culmination of these three steps is:

\[
\text{Error}_1 = \frac{1}{0.69 \cdot 2 \cdot 255 \cdot K} \times 100\% = \frac{0.28}{K} \%
\]

Thus, as one might expect, the accuracy of a measurement depends on the initial starting point of the measurement.

The second error term can be calculated in a similar manner using the figure of \( \pm \frac{1}{2} \) LSD for linearity quoted by the manufacturer of the DAC module used in the A/D unit. The only difference in the calculation is brought about by the fact that the nonlinearity does not have to have the same sign at the termination and initiation points, therefore, one will have to consider the effect of the uncertainty at both locations. The \( \pm \frac{1}{2} \) LSD uncertainty at the initiation point will be multiplied by two at the termination point and the termination nonlinearity will add another \( \pm \frac{1}{2} \) LSD so the effective uncertainty due to nonlinearity will be one and one-half counts.
Thus the error due to nonlinearity is expressed by:

\[
\text{Error}_2 = \frac{\Delta t_{\text{start}} + \Delta t_{\text{stop}}}{t_2} \times 100\% \\
= \pm \frac{\Delta t_{P=2P_0} + \Delta t_{P=2P_0}}{t_2} \times 100\% \\
\text{Error} = \pm \frac{0.43}{K} \times 100\%
\]

The maximum error possible would be the simple sum of the error sources so that:

\[
\text{Error}_{\text{Total}} = \text{Error}_1 + \text{Error}_2 = \pm \frac{0.71}{K} \%
\]

The possible error ranges from a low of \(\pm 1.5\%\) for \(K = 0.5\), initial power equal to one-half the full scale value, thru 7.1\% at \(K = 0.1\), initial power one-tenth the full scale value. As can be seen, one must utilize the upper half of the instrument's range to achieve maximum accuracy when making period measurements.
V. CONCLUSIONS

A basic digital instrument has been designed, constructed and installed on the UMRR. Although not particularly sophisticated the instrument has the advantages of modest cost, improved convenience and improved accuracy. The instrument automatically completes the measurement of reactor period, positive or negative, by physically measuring the doubling time and displaying the equivalent reactor period on a four digit decimal readout. The instrument can measure stable periods up to 999.9 sec. with a possible error of less than 1.5%, if one uses the entire measurement range. It is difficult to compare the 1.5% figure to the manual method of measurement because the operator was an integral part of the system, but one can show a two-to-one improvement in the power resolving ability of the digital device and an improvement of three-to-two in the linearity of the A/D Convertor when compared to the recorder. Thus we can simply say that there is an increase in the accuracy of the system.

There are three considerations to be observed when using the instrument to measure positive period:

1. The power must double from the initial value. Therefore, the pico-ammeter must be less than half scale prior to the initiation of a measurement.

2. Accuracy is inversely proportional to the initial power, therefore, one would like to start as near to half scale as possible.

3. Switching the pico-ammeter during a measurement will cause an erroneous reading.
When measuring negative periods one must observe two considerations to insure maximum accuracy.

1. The initial value should be near full scale.

2. Switching the pico-ammeter during a measurement invalidates the reading.

As long as the preceding considerations are followed the measured period will have an error of less than one and one-half percent.
BIBLIOGRAPHY


Ernest Wayne Scott was born in Kirksville, Missouri on September 12, 1943. He received all of his primary and secondary education in the Kirksville school system. His undergraduate education was accomplished at the Northeast Missouri State Teachers College, in Kirksville and the Missouri School of Mines in Rolla, Missouri, receiving a Bachelor of Science, Physics Major, and a Bachelor of Science in Electrical Engineering from the two schools in 1966. He was employed by the General Electric Ordnance Department from August 1966 to April 1970.

He has been enrolled in the Graduate School of the University of Missouri-Rolla since September 1970 working on a dual Master of Science program in the Electrical and Nuclear Engineering curriculums.
APPENDIX A

Accuracy Calculations

The basic equation we are working with is:

\[ P(t) = P_0 e^{t/T} \]  \hspace{1cm} (1)

However, we are interested in the behavior of time as a function of power so we solve Equation (1) for time.

\[ t(P) = T \ln \frac{P}{P_0} \]  \hspace{1cm} (2)

Taking the derivative of time with respect to power we have:

\[ \frac{dt}{dP} = \frac{T}{P} \]  \hspace{1cm} (3)

Thus the uncertainty in time (\( \Delta t \)) caused by an uncertainty in power (\( \Delta P \)) can be described as:

\[ \Delta t = \frac{T}{P} \Delta P \]  \hspace{1cm} (4)

evaluated at a particular power.

Errors in an Analog to Digital Converter that can create an uncertainty in the power measurement are, the quantization uncertainty and linearity errors. The quantization error (Q.E.) for a static A/D Converter is \( \pm \frac{1}{2} \) LSD, however, a tracking A/D Converter has a Q.E. of \( \pm \) LSD because of the continuous one count variation that denotes the tracking converter. The quantization error for the A/D Converter used is:

\[ \text{Q.E.} = \pm \frac{\text{Full Scale}}{2^n - 1} = \pm \frac{\text{Full Scale}}{255} \]  \hspace{1cm} (5)
where \( n \) is the number of stages in the convertor. Linearity error is determined by the DAC module used in the system and, as in most high accuracy convertors, the figure quoted by the manufacturer is \( \pm \) LSD.

The effect due to power uncertainty created by the quantization error is only felt at the termination value and is \( \pm \) LSD. Thus:

\[
\Delta t \bigg|_{P=2 \cdot P_0} = \pm \frac{T}{2 \cdot P_0} \frac{P_{F.S.}}{255}
\]  
(6)

Introducing the variable \( K \) that is defined in the following manner:

\[
K = \frac{P_0}{P_{F.S.}}
\]  
(7)

Making a substitution in Equation (6) for the initial power we have:

\[
\Delta t \bigg|_{P=2 \cdot P_0} = \pm \frac{T}{2 \cdot K \cdot P_{F.S.}} \frac{P_{F.S.}}{255} = \pm \frac{T}{510 \cdot K}
\]  
(8)

The error due to quantization uncertainty is then:

\[
\text{Error}_1 = \pm \frac{2P_0}{t_2} \times 100\%
\]  
(9)

Substitution of the relationship between period and doubling time we have:

\[
\text{Error}_1 = \pm \frac{1}{0.69 \cdot 510 \cdot K} \times 100\% = \pm \frac{0.28}{K}\%
\]  
(10)

Error due to non-linearity will be felt both at the starting point and the termination point, thus the error will be expressed by:
\[
\text{Error}_2 = \pm \frac{\Delta t (\Delta P@P_0) + \Delta t (\Delta P@2P_0)}{t_2} \times 100\% \tag{11}
\]

Because of the internal doubling in the instrument the error due to uncertainty of the initial value will be \(\pm\) LSD at termination. The uncertainty due to non-linearity at the termination value will be \(\pm\) LSD so that the total uncertainty due to combined non-linearity will be plus or minus one and one-half counts.

\[
\text{Error}_2 = \pm \frac{1.5 \cdot \Delta t |2P_0}{t_2} \times 100\% \tag{12}
\]

Making the same substitutions that were used in generating Equation (10) we have:

\[
\text{Error}_2 = \pm \frac{1.5}{0.69 \cdot 2 \cdot 255 \cdot K} \times 100\% = \pm \frac{0.43}{K} \% \tag{13}
\]

The worst possible error that one might expect is the simple sum of Error_1 and Error_2, and is:

\[
\text{Error}_{\text{Total}} = \pm \frac{0.71}{K} \% \tag{14}
\]

Since \(K\), for positive periods is limited to a maximum of one-half, because power must be allowed to double during a measurement, the minimum possible error that one may quote for the instrument is:

\[
\text{Error}_{\text{Total}} = \pm 1.42\% \tag{15}
\]

While this figure seems quite large one must remember that we have improved the power resolution by 50\% and have improved the linearity figure by 33\% over the manual method previously used to determine
the reactor period. Thus one may say that the instrument shows a
definite mathematic improvement in accuracy in addition to the
improvement created by the reduction of operator induced errors by
replacing the manual method by the semi-automated system.
APPENDIX B

Sample Data

The following is a rod worth calibration performed on rod number 4 at the University of Missouri-Rolla Reactor. This calibration was performed on November 16, 1972.

<table>
<thead>
<tr>
<th>Rod Position</th>
<th>Withdrawal Increment</th>
<th>Data* Point in see.</th>
<th>Period in secs.</th>
<th>Reactivity from TID-4500 x 10^-4</th>
<th>Total Reactivity x 10^-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>6&quot;</td>
<td>6&quot;</td>
<td>3&quot;</td>
<td>239.2</td>
<td>3.60</td>
<td>3.60</td>
</tr>
<tr>
<td>10&quot;</td>
<td>4&quot;</td>
<td>8&quot;</td>
<td>90.9</td>
<td>8.03</td>
<td>11.63</td>
</tr>
<tr>
<td>12&quot;</td>
<td>2&quot;</td>
<td>11&quot;</td>
<td>153.8</td>
<td>5.26</td>
<td>16.89</td>
</tr>
<tr>
<td>14&quot;</td>
<td>2&quot;</td>
<td>13&quot;</td>
<td>165.1</td>
<td>4.96</td>
<td>21.85</td>
</tr>
<tr>
<td>18&quot;</td>
<td>4&quot;</td>
<td>16&quot;</td>
<td>77.6</td>
<td>9.02</td>
<td>29.87</td>
</tr>
<tr>
<td>24&quot;</td>
<td>6&quot;</td>
<td>21&quot;</td>
<td>140.1</td>
<td>5.69</td>
<td>35.56</td>
</tr>
</tbody>
</table>

* The Data Point is determined by subtracting one-half the withdrawal increment from the current rod position.
Figure 5. Reactivity Vs. Rod Position
APPENDIX C

Printed Circuit Layout

Included in this section are photos of the three circuit boards used in the instrument. The Analog-to-Digital Convertor and the Display Logic boards are complete with the exception of a small number of power and ground jumpers. The location of the jumpers is reasonably straightforward and can be accomplished with the flat wires normally used for that purpose. The third board, Decision Logic, requires a large number of interconnections and it is suggested that one use the "wire wrap" technique to accomplish this.
Figure 6. Analog to Digital Convertor Printed Circuit
Figure 7. Decision Logic Printed Circuit
Figure 8. Display Logic Printed Circuit