An investigation of the limitations of the antenna range at the University of Missouri at Rolla

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AN INVESTIGATION OF THE LIMITATIONS

OF THE

ANTENNA RANGE AT THE UNIVERSITY OF MISSOURI AT ROLLA

BY

RONALD EARL NELSON

A

THESIS

submitted to the faculty of

THE UNIVERSITY OF MISSOURI AT ROLLA

in partial fulfillment of the requirements for the

Degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

Rolla, Missouri

1966

Approved by

(advisor)

Richard C. Hanson
ABSTRACT

The University of Missouri Antenna Range is described and criteria on range components for acceptable operation are discussed. Discussion of the results of an experimental investigation of component systems of the antenna range is presented. Experimental results are compared to criteria for acceptable operation and recommendations for the improvement of the facility is made. Suggestions are made as to its usage at higher frequencies.
ACKNOWLEDGEMENT

The author wishes to express his thanks to R. Schroeder and H. A. Noble for their capable technical assistance. Thanks are also due to G. Brunner for his help in running some of the experiments.

The author is also indebted to Prof. G. G. Skitek for his guidance and encouragement.

Credit is also due to my wife, Joan, whose moral support and assistance in preparation of the first draft were greatly appreciated.
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A. THE PROBLEM

The objective of this thesis was to investigate the limitations of the Antenna Range at the University of Missouri at Rolla. Since a particular system was under study, special attention was paid to the specific properties of this system, rather than to the properties of antenna ranges in general. The system was divided into four sub-systems and the properties of each were evaluated experimentally. From this information conclusions regarding the operation of the system as a whole were drawn.

B. IMPORTANCE OF THE PROBLEM

One of the goals in the operation of the Antenna Range at the University of Missouri at Rolla is use of the range as a research instrument. The range provides a polar plot of relative radiated power density as a function of angular displacement about the antenna in the plane of rotation. By rotating the antenna in various planes, data concerning the three dimensional radiation pattern may be obtained. In this way calculated antenna patterns may be checked or the radiation characteristics of unknown structures may be obtained experimentally.

In order that the antenna range may be used in this way its characteristics must be determined. Without knowledge of these characteristics experimental results obtained with the range would have little meaning.
This problem was selected because of the author's interest in antenna pattern measurement and his desire to see the antenna range developed as a useful research instrument.
CHAPTER II
REVIEW OF THE LITERATURE

In order to investigate the limitations of a given system it was first necessary to define the objectives of the system and limits within which the performance of these objectives was considered satisfactory. In this connection, a reference work on antenna pattern measurement by Hollis\textsuperscript{2} provided considerable information. Work by Kraus\textsuperscript{1} was also valuable. Certain criteria were also taken from the work of Ousely\textsuperscript{4} in a thesis on antenna range design. General information on antenna theory used in explaining these criteria was provided by Silver\textsuperscript{3} and Kraus\textsuperscript{1}.

Information on equipment used in experimental measurements on the system was found in work by Montgomery\textsuperscript{5}.

Information on the different types of servomechanisms used in the antenna range system is given by Kuo\textsuperscript{6}. Schwartz\textsuperscript{7} develops expressions involving bandwidth and response time of certain networks which are of use in evaluating certain components of the receiving system.

Work by Cohn and Maltese\textsuperscript{8} on antenna range design was used in development of recommendations.
CHAPTER III
A SURVEY OF THE ANTENNA RANGE

A. GENERAL

The Antenna Range at the University of Missouri at Rolla is located on the roof of the Electrical Engineering Building. Essentially it consists of a generator of radio frequency power, a transmitting antenna, a rotating tower to support the antenna under test and receiving equipment which detects the signal from the antenna under test. An automatic plotter then plots the amplitude of this signal as a function of angular position of the antenna with respect to some reference. A block diagram of the range is shown in Fig. 1.

B. MICROWAVE GENERATING EQUIPMENT

The source of microwave power presently used on the Antenna Range is the Type 124 A Power Oscillator manufactured by the Airborne Instruments Laboratory of Mineola, New York. It has continuous frequency coverage from 300 megacycles per second to 2500 megacycles per second and is rated to produce a minimum of five watts of radio frequency power on all frequencies covered. The output impedance of the oscillator is 50 ohms. An internal modulator provides for the modulation of the output signal with sinusoidal signals at frequencies of 400, 1000, and 577 cycles per second. The 577 cycle per second section is a modification of the original design which has been made in order to make the oscillator
Fig. 1. BLOCK DIAGRAM OF ANTENNA RANGE
compatible with the range receiving system, which requires 577 cycle per second modulation of the received signal. A trimming potentiometer allows adjustment of this frequency.

The Type 124 A Oscillator is a triode oscillator which uses co-axial resonant cavities for the resonant grid and plate circuits. The frequency is varied by changing the physical length of the cavities. The triode oscillator tube is mounted co-axially with the two cavities.

C. TRANSMITTING ANTENNAS AND TRANSMISSION PATH

Four transmitting antennas are available: two parabolas and two horns. The large parabola covers a frequency range from 500 to 1800 megacycles per second using two separate exciter antennas. The small parabola covers a range from 1750 to 3150 megacycles per second. The large horn covers the range from 3100 to 5700 megacycles per second and the small horn from 5700 to 10,000 megacycles per second.

The transmission path is approximately 12.9 meters long over the roof of the Electrical Engineering Building. The roof is not a perfectly conducting ground plane and several large objects are present in the radiation field of the transmitting antenna. Four of the most prominent are the tower mount for the old antenna range, the small building housing the antenna range transmitting and receiving equipment, and two large ventilators. The old range tower mount is to the left of the equipment building looking from
the radiating antenna and the two ventilators are beyond it and slightly to the right. The carriage which supports the tower to which the antenna under test is attached is mounted on the roof of the equipment building directly in front of the transmitting antenna. This carriage has a large metal covering which also may be presumed to have an effect on the field. An elevation view of the antenna range is shown in Fig. 2 and a plan view is shown in Fig. 3.

Two support structures for the antenna under test are available. One is a fiber tower which has provision for rotation of the antenna in a vertical, as well as a horizontal, plane. The other is a wooden tower consisting of a plywood base and an upright consisting of a single one-half inch dowel. The wooden tower has provision for rotation in the horizontal plane only. The wooden tower was constructed especially to have minimum effect on the radiation pattern of antennas under test.

D. RECEIVING EQUIPMENT

The signal from the antenna under test is detected by use of a bolometer. A bolometer is used rather than a crystal because its detection characteristic is very close to square law over a large range of power levels. Current bias for the bolometer is obtained from an adjustable supply associated with the high gain audio amplifier which amplifies the detected signal.

The detected signal from the bolometer is applied to the input of a narrow band, high gain audio amplifier tuned to
Fig. 2. ELEVATION VIEW OF THE ANTENNA RANGE
Fig. 3. PLAN VIEW OF THE ANTENNA RANGE
approximately 577 cycles per second. Following the last stage of amplification the 577 cycle per second signal is applied to an infinite impedance detector, the output of which is filtered by a resistor and capacitor combination. The output of the high gain amplifier is then a direct current voltage proportional to the power level received by the antenna under test. A square root section is also available in the high gain amplifier. When this section is in use the output of the high gain amplifier is proportional to the voltage induced in the antenna under test. Both power and electric field patterns can be taken by use of the linear and square root sections of the amplifier, respectively.

The direct current voltage output of the high gain amplifier is applied to a pen servo amplifier which drives a recording pen. In the pen servo amplifier a 400 cycle per second signal is generated and fed to the cathodes of two identical triodes with a common plate connection. This is done in such a way that the signals on the two cathodes are equal in magnitude but 180 degrees out of phase. As long as the grids of the two triodes are held at the same potential, zero net signal current flows in the common plate circuit of the two tubes. However, if the two grids are at different potentials a net signal is seen in the plate circuit. The direct current voltage from the high gain audio amplifier is applied to the grid of one of the triodes and a potential controlled by a potentiometer rotated by the pen servomotor is applied to the grid of the other. The first of these may be called the signal grid and the second the reference grid. When the signal
level from the output of the high gain audio amplifier changes, an unbalanced condition exists and a 400 cycle per second signal flows in the common plate circuit of the two triodes. This signal is amplified and applied to one winding of the two phase pen servomotor. The motor rotates until the potentiometer connected to it brings the voltage level of the reference grid to that of the signal grid. At this point a balanced condition is achieved and the rotation of the motor stops. Voltage for the second winding of the two phase servomotor is the original 400 cycle per second signal amplified and shifted ninety degrees in phase. The way in which the system becomes unbalanced determines which tube conducts a greater amount of signal and hence the phase of the output signal. It is this phase which determines the direction of motor rotation so that the system is driven towards a balanced condition.

E. TOWER DRIVE AND SERVO EQUIPMENT

The system which provides for the rotation of the antenna under test is called the tower drive system. It consists of a motor control unit and a set of direct current motors which are driven one at a time. The torque of each motor is varied by changing the current in its field winding. A permanent magnet generator rotated by each motor produces a direct current voltage proportional to the speed of that motor. This voltage is fed back to the motor control unit as negative feedback and controls the speed of the motor by changing the torque as the load changes.
An increase in load causes the motor to slow. This reduces the negative feedback and a greater field is applied to the direct current motor. This increases the torque and the speed remains nearly constant. In a similar manner reduction of the load on the motor causes a reduction of torque and the speed is held relatively constant.

The recorder turntable is made to follow the motion of the tower rotation system by means of a servo system. The stator windings of a synchro transmitter on the tower and a synchro control transformer coupled to the turntable are fed with the same signal. Under these conditions a signal applied to the rotor winding of the synchro transmitter produces a signal at the rotor terminals of the synchro control transformer which is proportional to the difference in angular displacement between the two rotors. This error signal is amplified and applied to one winding of the two phase servomotor which drives the recorder turntable. The other winding is driven with the same signal applied to the synchro transmitter rotor windings. The motor is connected in such a way that the synchro transformer is driven in the direction of minimum output. This is the position corresponding to equivalence of position of the tower rotation device and the recorder turntable. An increase in accuracy is effected by use of a synchro control transformer geared to turn thirty-six times for each total rotation of the tower. It is switched in automatically after a one to one synchro control transformer brings
the recorder turntable to within ten degrees of the correct position.

A position indicator is also provided which, by means of circular scales marked from zero to three hundred and sixty degrees, shows the angular position of the antenna in both the horizontal and the vertical plane.
CHAPTER IV
CRITERIA FOR ACCEPTABLE OPERATION

A. GENERAL

In order to investigate the limitations of a system a standard to which the system may be compared is necessary. Certain criteria for antenna ranges have been developed by investigators of the problem of antenna pattern measurement. In this chapter these criteria are summarized. Since certain terms used in the description of these criteria should be defined and explained, a brief summary of antenna theory is in order.

An antenna is a device which guides electromagnetic energy during the transition of the wave from a guided wave to a free space wave. Thus an antenna is a radiator of electromagnetic energy. An antenna which radiates equal amounts of power in all directions is called an isotropic radiator. Physically such an antenna does not exist as all radiating structures exhibit directional properties, that is, they radiate more power in some directions than in others.

One measure of how much power a certain structure radiates in a given direction is the amount of power radiated per unit solid angle in that direction. This quantity has been called radiation intensity, and has units of watts per steradian. In this discussion radiation intensity will be represented by the symbol \( \mathcal{U} \).
Consider an antenna placed at the center of a spherical coordinate system as shown in Fig. 4. Radiation intensity is a function only of $\theta$ and $\phi$ and may be expressed as

$$U = Kf(\theta, \phi)$$  \hspace{1cm} (4-1)

where $K$ is some constant. An average radiation intensity may be defined as the ratio of total radiated power to $4\pi$, the total number of steradians in a sphere.

$$U_{\text{avg}} = \frac{\text{Total Radiated Power}}{4\pi}$$  \hspace{1cm} (4-2)

At this point, directivity may be defined as the ratio of maximum radiation intensity to average radiation intensity. Directivity will be denoted by the symbol $D$.

$$D = \frac{U_{\text{max.}}}{U_{\text{avg.}}}$$  \hspace{1cm} (4-3)

The gain of an antenna with respect to an isotropic radiator is defined as the ratio of maximum radiation intensity to the radiation intensity from a lossless isotropic source with the same power input.\(^1\) Hence, gain is also an expression of antenna efficiency. It can be shown\(^1\) that the gain of an antenna is equal to the product of the directivity times the antenna efficiency.
Fig. 4. SPHERICAL CO-ORDINATE SYSTEM
\[ G = \eta D \]  
\((4-4)\)

where \( \eta \) represents antenna efficiency.

Radiated power density, expressed in watts per square meter, can be expressed in terms of antenna directivity and total transmitted power. Let power density be assigned the symbol \( S \). Then, in the direction of maximum radiation intensity\(^2\),

\[ S = \frac{P_d D}{4\pi R^2} \]  
\((4-5)\)

where \( P_t \) is the total transmitted power and \( R \) is the distance in meters from the antenna.

The effective aperture of an antenna is defined as the ratio of power received in the terminating impedance to the power density of the wave incident upon the antenna.\(^1\)

\[ A_e = \frac{\text{Power in Terminating Load}}{S} \]  
\((4-6)\)

This definition is made under the assumption that the antenna is oriented for maximum reception of electromagnetic energy from the incident wave.

In general both directivity and gain may be expressed as functions of direction from the antenna. The definition of
directivity then becomes the ratio of radiation intensity of the antenna *in a given direction* to the average radiation intensity. Similarly the gain of the antenna with respect to an isotropic radiator is defined as the ratio of the radiation intensity of the antenna *in a given direction* to the radiation intensity of a lossless isotropic source with the same power input. Unless specifically stated otherwise, directivities and gains used in this thesis will be understood to be maximum directivities and gains, that is, directivities and gains taken in the direction of maximum radiation intensity.

The quantities discussed above all are related to the far field or Fraunhofer region of the antenna. The field immediately surrounding an antenna is called the Fresnel region or near field. In the far field, only propagating electromagnetic waves are measureable and the shape of the polar plot of electric field intensity as a function of angular displacement about the antenna is independent of distance from the antenna. Such a plot is commonly known as the field pattern of the antenna. In the near field nonpropagating fields are present and the shape of the field pattern may, in general, be a function of radius. The boundary between the near and far fields may be taken as

$$\gamma_b = \frac{2L}{\lambda^3},$$

where $L$ is the maximum physical aperture dimension of the antenna, $\lambda$ denotes the wavelength at the frequency at which the antenna is being operated and $\gamma_b$ is the distance of the boundary from the antenna in any direction.

A reciprocity theorem for antennas, first stated by Carson, is very important in the measurement of antenna patterns. It
states that, in a linear homogeneous and isotropic medium, if a given voltage difference is applied to the terminals of an antenna A and a certain current is measured at the terminals of an antenna B, an equal current will be measured at the terminals of antenna A if the same voltage difference is applied to the terminals of antenna B. As a direct result of this theorem the transmitting and receiving field patterns of an antenna are the same. This fact allows measurement of the field pattern of an antenna by using it as either a transmitting or receiving antenna.

B. TRANSMITTING EQUIPMENT

The device used to generate radio frequency energy must be capable of producing enough power to produce a signal at the receiver large enough to cause full scale deflection of the recording pen when the antenna under test is oriented for maximum received signal. Given the power density in an incident wave, this power level depends upon the receiver and upon the effective aperture of the antenna under test. Thus it is usually necessary to specify a minimum effective aperture for antennas to be tested.

An expression of this level of power may be developed by assigning losses to each portion of the transmission path from the terminals of the transmitter to the terminals of the receiver. A similar procedure is used by Ousley.4

Let the power produced by the radio frequency generator be expressed in decibels with respect to one milliwatt as \( W_\text{dBm} \).
A certain amount of attenuation will be present in the transmission line and some power will be lost due to reflections at the antenna and due to ohmic losses in the antenna. If these two losses are combined and called generator losses, denoted by $L_{\text{gt}}$, the transmitted power in dbm is given by

$$W_T = W - L_{\text{gt}} \quad (4-6)$$

Conversion of $W_T$ from dbm to milliwatts yields,

$$W_{\text{tm}} = 10^{\frac{W_T}{10}} \quad (4-7)$$

The power density at the receiving antenna may then be expressed as

$$S_r = \frac{W_{\text{tm}} D_c}{4\pi R^2} \quad (4-8)$$

Where $D_c$ is the directivity of the transmitting antenna and $R$ is the distance between the transmitting and receiving antennas. If the effective aperture of the receiving antenna is $A_{ev}$, the power dissipated in the total impedance connected to the receiving antenna terminals may be expressed as

$$P = \frac{W_{\text{tm}} D_c A_{ev}}{4\pi R^2} \quad (4-9)$$

Conversion of this quantity to decibels with respect to one milliwatt yields
Substituting for $V_J$ yields

$$P_{db} = 10 \log_{10} \frac{W_{tm} D_t A_{er}}{4\pi R^2} \quad (4-10)$$

$$P_{db} = 10 \left[ \log_{10} \frac{A_{er} D_t}{4\pi} + \log_{10} W_{tm} - 2 \log_{10} R \right] \quad (4-11)$$

$$P_{db} = 10 \log_{10} \frac{A_{er} D_t}{4\pi} + W_t - 20 \log_{10} R \quad (4-12)$$

Substituting for $W_t$ yields

$$P_{db} = W_g - L_{gt} + 10 \log_{10} \frac{A_{er} D_t}{4\pi} - 20 \log_{10} R \quad (4-13)$$

Not all of this power is usable, however, since losses will occur due to mismatches and ohmic losses in the receiving antenna and due to attenuation in the receiving antenna transmission line. These losses may be combined and called receiver transmission losses denoted by $L_{rt}$. Then, usable power may be expressed as

$$P_{ddb} = W_g - L_{gt} - L_{rt} + 10 \log_{10} \frac{A_{er} D_t}{4\pi} - 20 \log_{10} R. \quad (4-14)$$
If \( L'_{\text{t}} \) and \( L_{\text{t}} \) are combined into a single transmission loss \( K_{\text{t}} \), the expression becomes

\[
P_{\text{ddb}} = W_3 - K_{\text{t}} + 10 \log_{10} \frac{A e r}{4\pi} D_{\text{t}} - 20 \log_{10} R.
\]  \hspace{1cm} (4-15)

Solving for \( W_3 \)

\[
W_3 = P_{\text{ddb}} + K_{\text{t}} - 10 \log_{10} \frac{A e r}{4\pi} D_{\text{t}} + 20 \log_{10} R. \]  \hspace{1cm} (4-16)

If the minimum power necessary to cause full scale deflection of the pen is known, and the other quantities are either known or can be estimated, a minimum necessary generated power \( W_3 \) can be determined from this equation.

Other criteria for the source of radio frequency power are frequency stability and amplitude stability. In many cases antennas and the networks with which they are fed are frequency sensitive. If the frequency of the radio frequency source changes during measurement of a field pattern the received power level will vary due to frequency changes as well as changes in the angular position of the antenna. Thus a true picture of the antenna radiation pattern will not be obtained. In the same manner variations of power output of the source during measurement of a field pattern would introduce variations in received power not due to variation in antenna position.
Definition of an acceptable level of variation of power and frequency depends upon the anticipated accuracy of the measurements. In general the best approach is to use the best equipment available in these respects and to measure the extent of amplitude and frequency drift. Experimental results must then be interpreted on the basis that they contain error due to these effects. The extent of error may be estimated from knowledge gained by measurement of the extent of drift.

C. ANTENNAS AND TRANSMISSION PATH

The main requirement on the transmitting antennas and the transmission path is that together they must produce a wave front at the antenna under test which closely approximates a plane wave, that is, a propagating wave in which electric and magnetic field intensity vectors lie in a plane perpendicular to the direction of propagation and are uniform in phase and have the same magnitude everywhere in that plane. The reason for this is that it is desirable to test the antenna under conditions which approach actual operating conditions. When a receiving antenna is widely separated from the transmitting antenna, as is the case in practice, the shape of the wave front incident on the receiving antenna is essentially plane.

The transmitting antennas used in measurement of antenna patterns should be directional in order to minimize reflections from the transmission path and other obstacles. At the same time,
the beam width of these antennas should be wide enough to provide essentially constant power density over the aperture of the antenna under test.

The transmission path should either have negligible effect on the radiated field from the transmitting antenna or it should interfere with that field in a way which presents a good approximation of a plane wave to the receiving antenna.

A set of requirements on the plane wave which must be produced by the transmitting antennas and transmission path is an acceptable criterion for this combination. Such requirements have been developed by workers in the field and the set presented by Kraus ¹ is typical. A maximum phase variation of $\lambda/16$, where $\lambda$ is the wavelength at the frequency being used, is considered acceptable. In free space this corresponds to a variation of $22.5^\circ$. A maximum amplitude variation of less than 0.25 db is considered acceptable for variations in field amplitude.

D. RECEIVING EQUIPMENT

A first requirement on receiving equipment of the type used in the University of Missouri Antenna Range is that the detector used must have a uniform law of detection over a large range of powers. If square law detection is specified, the signal from the detector should be proportional to the square of the voltage applied to it over a large range of powers. Since the square of the voltage applied to the detector is proportional to the
received power, the signal from the detector is directly proportional to the power.

The signal from the detector is applied to a high gain audio amplifier which in turn drives a servo amplifier. The response of this system must be linear over the range of signals anticipated in order that the deflection of the pen be directly proportional to the input signal. Thus the deflection of the pen is directly proportional to received power.

A square root section is available in the high gain amplifier. Obviously, the use of this section should cause an output signal proportional to the square root of the received power.

In order for the system to determine the position of nulls in the antenna pattern the receiving system must be able to distinguish a signal at some lower level than that necessary for full scale deflection. A criterion used by Ousley for accurate determination of nulls is that the receiver must be able to detect a signal 20 decibels below that required for maximum pen deflection in the presence of noise.

Finally, the servo system which drives the recording pen should be able to follow the maximum anticipated variation of input signal with respect to time. This depends upon the shape of the field pattern and upon the speed of rotation of the antenna under test. Given a slowest speed of rotation possible, a limit to the derivative of the antenna pattern with respect to angular variation is reached beyond which the pen can no longer
follow the variation.

E. TOWER DRIVE AND SERVO EQUIPMENT

The motor control must supply an even speed of rotation for the tower. Sharp variations in speed due to changes in load on the tower due to wind or other causes could cause the signal input to change so rapidly that the recording pen could not follow it. An absolutely constant speed is not necessary but variations in speed should be kept below a level that would cause pen response problems.

The servo system must provide for good tracking accuracy between the tower and the recorder turntable. There can be no set level of acceptable tracking accuracy. The tracking should be made as good as possible and the residual error measured. Then experimental results may be interpreted in the light of this known error.

Any mechanical backlash in the servo system should be either so small that it may be neglected or correctable in some way. This is because it is desirable to run patterns in both directions as a time saving measure.
CHAPTER V
MEASURED CHARACTERISTICS
OF
MICROWAVE GENERATING EQUIPMENT

A. GENERAL

The microwave oscillator in use at the University of Missouri at Rolla Antenna Range was tested experimentally to determine the magnitude of shift in frequency and amplitude with respect to time. Power output was not checked experimentally, since equipment to measure microwave power levels of one watt and higher was not available. The oscillator was assumed to deliver its rated output.

B. FREQUENCY DRIFT OF MICROWAVE OSCILLATOR

The frequency drift of the microwave oscillator was measured by taking periodic frequency measurements during the warmup period immediately after the microwave oscillator had been turned on. The same method of frequency control of the oscillator is used at all frequencies and so the shift of frequency versus time during warmup near the frequency at which this test was run may be considered typical. The circuit used to measure warmup frequency drift is shown in Fig. 5. A coaxial wave meter was used to measure the wavelength which was subsequently converted to frequency. The 40 decibels of attenuation between the oscillator and the detector reduce the power to a level which the detector can safely handle.
Fig. 5. EXPERIMENTAL CIRCUIT FOR MEASUREMENT OF OSCILLATOR FREQUENCY SHIFT DURING WARMUP
Because of the isolating effects of the 40 decibels of attenuation the oscillator is presented with a load very close to its rated 50 ohms. A crystal detector was usable since the detector and meter were used only to detect power peaks indicating resonant points.

The coaxial wavemeter consists of a cavity formed by two coaxial cylinders. A moveable plunger is used to vary the length of the cavity and microwave energy is transmitted through the cavity when it is at a resonant length. Since these resonant points are one half wavelength apart, measuring the distance between two of them yields information from which the frequency can be calculated. The wavemeter was read by means of a micrometer dial which easily yielded accuracy to three places to the right of the decimal point. Any error due to wear of the wavemeter was neglected and maximum points were approached from the same direction for all of the readings in order to eliminate backlash error. The wavemeter was the main source of error in this experiment. The measurements were taken to be as read with negligible error, since with well constructed wavemeters of this type wavelengths can be measured to within an absolute accuracy of 0.05% in the 3000 megacycle per second range. Thus the accuracy would be considerably better in the range where these frequencies were measured, near 1750 megacycles per second.

The experimental data is presented in graphical form in Fig. 6. The horizontal axis of the graph represents time. The
Fig. 6. VARIATION IN OSCILLATOR FREQUENCY WITH RESPECT TO TIME DURING OSCILLATOR WARMUP
oscillator was turned on at time equal to zero. The vertical axis represents \( \Delta f \), the change in frequency from the frequency measured at time equal zero. The data is presented as a broken line graph since no trend was observed which could be described as a smooth curve until the frequency stabilized at 65 minutes. After 65 minutes there was no measurable change in frequency. The total drift during the 90 minutes after the oscillator power was turned on was less than two megacycles.

C. AMPLITUDE DRIFT OF MICROWAVE OSCILLATOR

The experimental circuit for measurement of power variation of the microwave oscillator during its warmup period is shown in Fig. 7. The attenuator reduced the power to a level which was safe to apply to the bolometer detector. The HP415B standing wave ratio meter was set at a level of one decibel as a reference when the oscillator was turned on. Then power variations in decibels were recorded once every minute for the first five minutes and once every five minutes thereafter. The expanded decibel scale which was used for the readings was divided into 0.1 decibel divisions. Readings could be estimated to about one tenth division or about 0.01 decibel.

The experimental data is shown in graphical form in Fig. 8. The data points were connected by straight line segments since after the first five minutes readings were taken at five minute intervals and how the amplitude varied during these intervals is not accurately known. The meter was watched for several minutes
at various times during the test, however, and shifts in amplitude were observed to occur slowly. Large or erratic variations in power level did not occur. After a large initial change of 0.29 decibels in the first minute, the power level stabilized and varied no more than 0.03 decibels during the remaining 59 minutes of the test.
Fig. 7. EXPERIMENTAL CIRCUIT FOR MEASUREMENT OF OSCILLATOR POWER VARIATION DURING WARMUP
Fig. 8. VARIATION IN POWER OUTPUT OF OSCILLATOR WITH RESPECT TO TIME DURING WARMUP
A. GENERAL

The incident field at the receiving location consists of the direct radiation from the transmitting antenna and the radiation reflected from the transmission path. Except in situations involving simple geometry it is very difficult to predict this field from knowledge of the properties of the antenna and transmission path. Ordinarily the best approach is the measurement of the field at the receiving location. This is the method used in this investigation.

Although the tower used to support the antenna under test is a part of the transmission path it is unique in the fact that it rotates with the antenna. Thus the way in which the supporting structure affects the measurement of the pattern of the antenna under test differs fundamentally from the way in which this pattern is affected by the rest of the transmission path. For this reason the effects of the transmitting antennas and the stationary portion of the transmission path are measured with the supporting structure absent. Then the effects of the supporting structure are investigated separately.

Measurements are taken at two frequencies, 1750 megacycles per second and 2400 megacycles per second. These frequencies were decided upon because they are typical of the frequency range of
the antenna presently installed. While these measurements accurately represent the operation of the antenna range only at these frequencies, they are a good indication of how the system will behave at other frequencies.

B. TRANSMITTING ANTENNAS AND STATIONARY TRANSMISSION PATH

In order to measure the field at the receiving location a wooden track was constructed and a half wave dipole antenna was attached to a runner which was moved along the track. Measurements were taken along a vertical line through the position of the antenna under test. Readings were taken every two centimeters from one meter below to one meter above the normal position of the antenna under test. Measurements were also taken every two centimeters along a horizontal line from 0.5 meters to the left to 0.5 meters to the right of the position of the antenna under test. The position of the antenna under test was taken to be 2.4 meters above the base of the receiving tower turntable.

The experimental circuit is shown in block diagram form in Fig. 9. A bolometer was used to detect the signal and an HP415B voltage standing wave ratio meter was used to read variations in received power. A reference level was set on the meter and changes in received power were read in decibels as the position of the antenna was changed.

The first source of error to be considered in this experiment was the effect of unbalanced currents on the transmission line.
Fig. 9. EXPERIMENTAL CIRCUIT FOR MEASUREMENT OF FIELD VARIATION WITH RESPECT TO POSITION
to the test antenna. The half wave dipole which was used to make the measurements is inherently a balanced device while the coaxial transmission line with which it was fed is an unbalanced device. Because of the unbalance of the transmission line radio frequency currents flow on its outer conductor and it becomes a radiator of radio frequency energy. In effect, the line becomes a part of the antenna. Changing the position of the transmission line with respect to nearby objects or moving it into a different part of the incident field changes the condition of the currents on the transmission line. This changes the impedance seen by the antenna and the power absorbed by the antenna varies. Since raising or lowering the antenna necessarily involves movement of the transmission line, steps were taken to reduce the unbalanced currents.

A balun is a device used to match a balanced system to an unbalanced system. One type of balun which is effective at the frequencies at which these tests were run is the metal sleeve balun described by Kraus.\(^1\) It consists of a metal sleeve one quarter wavelength long fitted around the coaxial transmission line just below the antenna. This sleeve was shorted to the outer conductor of the transmission line at the end furthest from the antenna. Together with the outer conductor of the transmission line the metal sleeve forms a short circuited section of transmission line one quarter wavelength long. Since the impedance of this section is very high at the frequency of operation radio frequency currents
were prevented from flowing on the outer conductor of the feed line to the antenna.

A balun of this type was constructed but it did not completely eliminate radio frequency current on the outer conductor of the transmission line. Touching the transmission line caused variations in received power of 0.5 db. This indicated the presence of radio frequency current on the transmission line. As an additional step in reducing these currents a radio frequency choke was constructed by winding the coaxial feed line on a coil form. The choke consisted of 42 turns of RG71-U coaxial cable. The coil was 18 inches long and had a mean diameter of 1.5 inches. Insertion of this coil in the line below the antenna reduced unbalanced current effects to about 0.2 decibels of power variation at both frequencies.

Another source of error is the interception and reradiation of radio frequency power by the test antenna feed line. This effect can be reduced by orienting the feed lines in a direction normal to the direction of polarization of the incident field. The test antenna is polarized in the same direction as the incident field, so this orientation also provides for minimum coupling between the feed line and the test antenna. The field from the transmitting antenna is polarized in a horizontal direction. Ideally, therefore, the test antenna feed line should be oriented in a vertical direction. This is not possible in practice, however, since raising and lowering of the test antenna changes the amount of feed line accumulated at the base of the receiving antenna.
In addition, because the tower was moved from side to side, it was necessary that part of the feed line deviate from an orientation exactly orthogonal to the incident field. The first of these effects was reduced by positioning the accumulated feed line beneath the conducting receiving antenna tower base. In this way the accumulated feed line was effectively shielded from the test antenna. Reradiation from the feed line was found not to be serious. With the antenna at the test position, movement of the maximum necessary length of horizontally oriented feed line showed no noticeable power variation. Movement of the feed line back and forth caused power indications to vary about 0.2 decibels. This was assumed to have been caused by reradiation effects since the variations observed were in addition to those caused by touching the transmission line.

The last source of error was reflections from the experimenter who had to be present in the electromagnetic field in order to raise and lower the test antenna. In order to reduce this effect the experimenter moved away from the antenna while each reading was taken. As long as the experimenter avoided a position directly in line with the receiving and transmitting antennas and remained about three meters away from the test antenna no change in received power was noted as the experimenter moved.

The above errors could conceivably add. Adding these two errors as a conservative estimate, an error of ± 0.4 db is
considered possible on any reading.

Experimental data was plotted for 1750 megacycles in Fig. 10 and Fig. 11. Fig. 10 is a plot of power variation in the vertical direction and Fig. 11 is a plot of power variation in the horizontal position. Data for 2400 megacycles was plotted in Fig. 12 and Fig. 13. These two figures represent power variations in the vertical and horizontal directions respectively. The data points in all of these graphs are connected by straight line segments. Since measurements were taken fairly close together in space this method gives a good idea of the magnitude of power variations and the general shape of the curve.

Neither of the two curves representing variations in power with respect to vertical position appears to be a simple combination of a direct and a reflected field. Both appear to be the result of multiple reflections from surrounding objects in the incident field. The greater variations in power appear to occur below the antenna. These variations are probably due mainly to the large metal carriage cover which is positional below the antenna under test. It is noted that these variations are reduced in magnitude as distance away from the cover increases. Also, the major variations appear to be from 1.5 to 2.0 wavelengths long which would indicate reflection from an object below the antenna at an angle of about 60°. This was the position of the carriage when these measurements were taken.

The variations in power amplitude in the horizontal direction were found to be less than 0.6 db for both frequencies. This level
is only slightly above the estimated accuracy of the experiment but results are included for completeness. These results also indicate that accuracy of the experiment may be somewhat better than expected.
Fig. 10. VARIATION IN POWER AS A FUNCTION VERTICAL DISTANCE AT 1750 MC.
Fig. 11. VARIATION IN POWER AS A FUNCTION OF HORIZONTAL DISTANCE AT 1750 MEGACYCLES
Fig. 12. VARIATION IN POWER AS A FUNCTION OF VERTICAL HEIGHT AT 2400 MC
Fig. 13. VARIATION IN POWER AS A FUNCTION HORIZONTAL DISTANCE AT 2400 MC
C. COMPARISON OF SUPPORTING STRUCTURES

The two towers used to support the antenna under test were compared by taking actual power patterns of the same antenna using the two different towers. The two measured patterns of the antenna were then compared. Since the wooden tower contained no metal and had little material of any kind near the antenna it was assumed to have very little effect on the antenna patterns. The antennas used were one half wavelength and three halves wavelength dipoles with sleeve type baluns. These antenna lengths were chosen because the shape of their power patterns is well known. These antennas were not constructed for use as a perfect standard in calibrating the antenna range. They are meant only for use in comparing the effects of the antenna towers.

Fig. 14 is a sketch of the antenna position on the fiber tower.

Fig. 15 is a reproduction of a power pattern of a dipole antenna 1.5 wavelengths long at 1750 megacycles per second taken with the wooden antenna tower. The position of the antenna with respect to the pattern is shown by the two lines representing the two halves of a dipole. The small lobes at zero and 180 degrees show that the antenna is a little short and was beginning to radiate some energy like a one-half wavelength dipole.

Fig. 16 shows the pattern of the same antenna using the fiber tower. The fiber tower has a large gear box or tower head made of dielectric material which provides for rotation of the
Fig. 14. SKETCH OF ANTENNA POSITION ON FIBER TOWER
antenna in the vertical plane. The antenna is supported almost on the same level as this gear box by a horizontal standard. The position of the dielectric gear box is shown by a rectangular box. In subsequent figures the position of the antenna and gear box will be shown in a similar manner.

The pattern shown in Fig. 16 is clearly affected by the presence of the fiber tower as can be seen by comparison with Fig. 15. Only the two lobes at 135 and 225 degrees and the minor lobe at 180 degrees exhibit significant distortion, however. As can be seen from Fig. 16, these are the two lobes which are directed toward the dielectric gear box. Fig. 17 shows the effect of the dielectric gear box on the pattern when the antenna is rotated in such a way that one of the major lobes is directed toward the tower head. The lobe so directed is highly distorted. This is because the tower head was interposed between the transmitting antenna and the antenna under test at the position of this lobe and thus has a large effect on the incident wave.

Fig. 18 is the power pattern of a half wavelength dipole antenna at 1750 megacycles using the wooden tower. A pattern of the same antenna is shown in Fig. 19, taken with the fiber tower. A special effort was made to reduce the effects of the tower by directing the major lobes of the antenna away from the tower head. Distortion of the two lobes is clearly evident. The pattern shown in Fig. 20 was taken with one of the major lobes directed toward the tower head. Considerable distortion of this lobe resulted.
Fig. 15. PATTERN OF A 1.5 WAVELENGTH DIPOLE AT 1750 MC.
WOODEN TOWER
Fig. 16. PATTERN OF A 1.5 WAVELENGTH DIPOLE AT 1750 MC, FIBER TOWER, MINIMUM INTERACTION
Fig. 17. PATTERN OF A 1.5 WAVELENGTH DIPOLE AT 1750 MC, FIBER TOWER, MAXIMUM INTERACTION
Fig. 18. PATTERN OF A 0.5 WAVELENGTH DIPOLE AT 1750 MC, WOODEN TOWER
Fig. 19. PATTERN OF A 0.5 WAVELENGTH DIPOLE AT 1750 MC, FIBER TOWER, MINIMUM INTERACTION
Fig. 20. PATTERN OF A 0.5 WAVELENGTH DIPOLE AT 1750 MC,
FIBER TOWER, MAXIMUM INTERACTION
A similar procedure was followed using a frequency of 2400 megacycles per second. A pattern taken by use of the wooden tower for the 1.5 wavelength is shown in Fig. 21 and the same pattern taken with the fiber tower is shown in Fig. 22. More distortion is seen here than was noted in the case of the 1750 megacycle antenna of the same type. Also when one of the lobes was directed toward the tower head the power of that lobe was increased, not reduced as was the case at 1750 megacycles per second. The pattern obtained for this situation is shown in Fig. 23. The power pattern for a half wavelength dipole taken at 2400 megacycles per second with the wooden tower is shown in Fig. 24. The same pattern taken with the fiber tower is shown in Fig. 25. For this pattern the antenna was oriented for minimum interaction with the fiber tower. The pattern taken with the antenna oriented for maximum interaction is shown in Fig. 26. Rather than a radical reduction of power in the lobe directed toward the tower head, as was noticed in the case of the half wave dipole at 1750 megacycles, a division of the major lobes is evident. The greatest distortion is still seen in the lobe oriented toward the tower head, however.

The difference in the way the antenna patterns were distorted by the tower at the two different frequencies indicates that the manner in which the fiber tower interacts with the antennas is frequency dependent.
Fig. 21. PATTERN OF A 1.5 WAVELENGTH DIPOLE AT 2400 MC, WOODEN TOWER
Fig. 22. PATTERN OF A 1.5 WAVELENGTH DIPOLE AT 2400 MC.
FIBER TOWER, MINIMUM INTERACTION
Fig. 23. PATTERN OF A 1.5 WAVELENGTH DIPOLE AT 2400 MC,
FIBER TOWER, MAXIMUM INTERACTION
Fig. 24. PATTERN OF A 0.5 WAVELENGTH DIPOLE AT 2400 MC, WOODEN TOWER
Fig. 25. PATTERN OF A 0.5 WAVELENGTH DIPOLE AT 2400 MC, FIBER TOWER MINIMUM INTERACTION
Fig. 26. PATTERN OF A 0.5 WAVELENGTH DIPOLE AT 2400 MC, FIBER TOWER, MAXIMUM INTERACTION
CHAPTER VII
RECEIVING EQUIPMENT

The characteristics of the receiving equipment were investigated by treating the high gain amplifier and recording pen as a unit. Linearity of system response was determined by plotting the pen deflection in inches against RMS voltage at the input of the high gain amplifier. Bandwidth characteristics of the high gain amplifier were determined by varying the frequency of the input signal and measuring the output of the high gain amplifier. Response of the pen recorder was determined by examining its response to a step input.

The experimental circuit for investigation of linearity characteristics of the high gain amplifier and pen combination is shown in Fig. 27. The voltage $V_o$ and the resistance $R_o$ used for the various gain positions are shown in Fig. 28. The voltage at the input of the high gain amplifier was varied by means of the calibrated 1000 ohm precision potentiometer and values of pen deflection were recorded. From knowledge of the resistance across the input, the voltage to the input of the high gain amplifier was calculated. The input impedance to the high gain amplifier was taken as its rated value of 200 ohms.

The main sources of error in this experiment are the device used to measure the voltage $V_o$ and the resistors used in the voltage dividing circuit. The voltmeter used to measure $V_o$ was an RCA Voltomyst which has an accuracy of $\pm 3\%$ of full scale so...
Fig. 27. EXPERIMENTAL CIRCUIT FOR MEASUREMENT OF LINEARITY OF AMPLIFIER AND RECORDER
<table>
<thead>
<tr>
<th>GAIN POSITION</th>
<th>SECTION</th>
<th>RMS VOLTS</th>
<th>MEGOHMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>linear</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>square root</td>
<td>20</td>
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<td>10</td>
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<tr>
<td>20</td>
<td>square root</td>
<td>0.5</td>
<td>3</td>
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Fig. 28. TABLE OF RESISTANCE AND VOLTAGES USED IN LINEARITY EXPERIMENT
the voltage $V_o$ is accurate to within at least $\pm$ 3%.

The voltage at the terminals of the high gain amplifier was computed by assuming that the current in the circuit is controlled by the large resistance $R_o$ and the 1000 ohm potentiometer. The addition of the 200 ohm input impedance of the high gain amplifier in parallel with the portion of the potentiometer resistance denoted as $Y$ in Fig. 27 has the greatest effect on current when $Y$ is largest. The greatest value $Y$ ever has is slightly less than 100 ohms. Using 100 ohms as the value of $Y$, the addition of the 200 ohm resistance is seen to change the value of the current by less than 0.004%, an entirely negligible value. This value was computed using $R_o$ as 1 megohm. For higher values of $R_o$ the change is even less.

The current source representation of the circuit is shown in Fig. 29. Since the voltage $V_o$ is accurate to within only $\pm$ 3% and the value of the resistors used in the voltage divider are accurate to within $\pm$ 1% the current $I$ is accurate to within 4.04% of its nominal value. The manner in which this is determined is shown in Appendix A.

The resistance $Y$, shown in Fig. 27 is known to be accurate to within $\pm$ 5% of its nominal value. If the 200 ohm input impedance to the high gain amplifier is assumed also to be accurate to within $\pm$ 5% of its nominal value, the resistance of the parallel combination of the two will be accurate at least to within $\pm$ 16.05 of its nominal value. The manner in which this is
Fig. 29. CURRENT SOURCE REPRESENTATION OF EXPERIMENTAL CIRCUIT
determined is presented in Appendix B.

The voltage to the input terminals of the high gain amplifier is given by the product of the current \( I \) and the value of resistance of the parallel combination of the resistor \( Y \) and the 200 ohm input impedance of the high gain amplifier. Equation B-3 of Appendix B shows that if two positive numbers \( R_a \) and \( R_b \) are accurate to within fractions \( f_a \) and \( f_b \) of their nominal values respectively, that their product will at least be accurate to within a fraction \( (f_a + f_b + f_a f_b) \) of its nominal value. This fraction has its greatest value when \( f_a \) and \( f_b \) are both positive.

Since \( I \) is accurate to within 4.04% of its nominal value and the value of the parallel combination of \( Y \) and the 200 ohm input impedance is accurate to within 16.05% of its nominal value, it follows from the statements above that the value of the input voltage to the high gain amplifier is accurate to within 21% of its nominal value.

For measurements of linearity characteristics, however, the accuracy with which the change in voltage to the input of the high gain amplifier is known is important. The precision potentiometer which varies the resistance \( Y \) is linear to within 0.1% and the amount of shaft rotation of the potentiometer can be read to an estimated accuracy of \( \pm 1\% \). Thus the value of the resistance \( Y \) can be changed by an amount which is known to be accurate to within \( \pm 1.1\% \). Then the change in resistance of the parallel combination of \( Y \) and the 200 ohm input impedance can be determined to within 1.1% of its nominal value.
The manner in which these values are determined is shown in Appendix C.

Since the current \( I \) is assumed to be a constant, the change in voltage is the product of \( I \) times the change in the resistance of the parallel combination of \( Y \) and the 200 ohm input impedance. As was mentioned above, from considerations developed in Appendix B, the product of two numbers is at least accurate to within a fraction of error equal to the sum of the absolute values of the fractions of error of the two numbers plus the product of the absolute value of these two fractions. Thus, since the current is known to be accurate to within \( \pm 4.04\% \) and the change in resistance of the parallel combination is known to be accurate to within \( \pm 1.1\% \), the value of the change in voltage to the high gain amplifier is known to within 5.2\% of its nominal value. It should be noted that this figure assumes the worst case conditions throughout the system.

The experimental data on the linearity characteristics of the high gain amplifier and pen recorder is shown in graphical form in Fig. 30 thru Fig. 35. Fig. 30, 32, and 34, show pen deflection plotted versus voltage at the input of the high gain amplifier using the linear amplification section. The gain of the amplifier and pen recorder is very close to a constant. The deviation from a straight line of the graph representing linearity characteristics for gain position number one is within 0.1 inch of a straight line. This represents a maximum error of 2\% of full scale. The accuracy on gain position 10 is also within 2\% of full scale as is the accuracy on gain position 20.
Fig. 30. PEN DEFLECTION VERSUS VOLTAGE TO HIGH GAIN AMPLIFIER, LINEAR SECTION, POSITION 1
Fig. 31. PEN DEFLECTION VERSUS VOLTAGE TO HIGH GAIN AMPLIFIER, SQUARE ROOT SECTION, POSITION 1
Fig. 32. PEN DEFLECTION VERSUS VOLTAGE TO HIGH GAIN AMPLIFIER, LINEAR SECTION, POSITION 10
Fig. 33. Pen deflection versus voltage to high gain amplifier, square root section, position 10.
Fig. 34. PEN DEFLECTION VERSUS VOLTAGE TO HIGH GAIN AMPLIFIER,
LINEAR SECTION, POSITION 20
Fig. 35. PEN DEFLECTION VERSUS VOLTAGE TO HIGH GAIN AMPLIFIER, SQUARE ROOT SECTION, POSITION 20
Since a numerical value for the gain of the system is not known, the error is measured from a straight line passing as an average through the experimental data points. All of the errors above are below the estimated accuracy of the experiment.

The characteristics of the square root section of the amplifier are shown in Fig. 31, Fig. 33, and Fig. 35. The pen deflection in inches is plotted versus the voltage at the input of the high gain amplifier on log-log co-ordinates. This procedure would yield a straight line graph if the square root section worked perfectly. The curves for the square root section do not approximate closely the desired straight line, especially at small pen deflections.

The bandwidth characteristics of the high gain amplifier are shown in Fig. 36. Relative gain is plotted against variations in frequency. The experimental circuit for the determination of these characteristics is shown in Fig. 37. The voltage output of the high gain amplifier was read with an RCA Voltohmyst which has an accuracy of ± 3% of full scale. The frequency was read to the nearest cycle using a digital frequency counter. The curve was run with the amplifier in the linear mode of operation on gain position 1.

The half power bandwidth of the high gain amplifier is about 8.5 cycles.

Noise is not a problem, since the noise generated in the system was not sufficient to cause deflection of the pen on gain settings below position 20. This was determined by observing the
Fig. 36. BANDWIDTH CHARACTERISTICS OF HIGH GAIN AMPLIFIER
Fig. 37. EXPERIMENTAL CIRCUIT FOR MEASUREMENT OF HIGH GAIN AMPLIFIER BANDWIDTH
recording pen with zero signal applied to the input of the high gain amplifier.

The characteristics of the pen recorder were determined by switching a signal voltage to the input of the high gain amplifier and pen recorder combination at time equal to zero. This action applied a step input to the pen recorder. Any transients associated with the high gain amplifier were assumed to have a negligibly short duration with respect to those occurring in the pen recorder. Steady state error was assumed to be negligibly small.

The response of the pen recorder to the step input thus applied is shown in Fig. 38. The response has been normalized to a unit step.

The transfer function of the pen recording system is of the form

\[ \frac{P(s)}{V(s)} = \frac{K}{s^2 + Bs + C} \]  

(7-1)

where \( K \), \( B \) and \( C \) are positive numbers. This response is developed in Appendix D. This equation may be written

\[ \frac{P(s)}{V(s)} = \frac{(K/c)C}{s^2 + Bs + C} \]  

(7-2)

The transient response of this system to a unit step input is expressed as
Fig. 38. RESPONSE OF PEN RECORDER TO A STEP INPUT
This equation may be written as

\[ P(s) = \frac{K}{\omega^n} \frac{\omega^2}{s + 2\delta \omega \rho + \omega^2} \]  \hspace{1cm} (7-4)

where \( C = \omega^2 \) and \( B = 2\delta \omega \). Then \( \omega \) is the undamped natural frequency of the system and \( \delta \) is the damping ratio of the system.\(^6\) A table of Laplace transforms given by Kuo\(^6\) gives the inverse transform of this expression as

\[ P(t) = \frac{K}{\omega^n} \left[ 1 - \frac{e^{-\delta \omega t}}{\sqrt{1 - \delta^2}} \sin \left( \omega \sqrt{1 - \delta^2} t + \tan^{-1} \frac{\delta}{\sqrt{1 - \delta^2}} \right) \right] \]  \hspace{1cm} (7-5)

This response has its first maximum value at time

\[ T_m = \frac{\pi}{\omega \sqrt{1 - \delta^2}} \]  \hspace{1cm} (7-6)

The first term in the brackets represents the steady state response of the system while the second represents the transient response. The rate at which the transient response dies out is controlled by the coefficient of \( t \) in the exponent of \( e \) in the expression above. The time at which the transient has decayed to 36.8% of
its original value is given by \( \tau_d = \frac{1}{3 \omega_n} \). From the graph of
the response of the pen to a unit step input, the values \( \tau_m \) and \( \tau_d \)
can be approximately determined.

Examination of equation 7-5 shows that the transient
response of the system is the difference between the steady
state response and the total response. From Fig. 38 the absolute
value of the normalized response is very close to unity at time
equal to zero. Inspection of Fig. 38 shows that the transient
response is equal to 36.8% of this value at time \( t = 0.118 \)
seconds. This quantity is a good approximation to the value of
\( \tau_d \). From Fig. 38 \( \tau_m \) is very close to 0.188 seconds.

From this information the set of simultaneous equations

\[
\tau_d = \frac{1}{3 \omega_n} \quad (7-7)
\]

and

\[
\tau_m = \frac{\pi}{\omega_n \sqrt{1 - \delta^2}} \quad (7-8)
\]

may be solved for \( \delta \) and \( \omega_n \).

Solution of equations 7-6 and 7-7 yields values of 0.453
and 18.69 for \( \delta \) and \( \omega_n \) respectively. From knowledge of these
two quantities, the transfer function of the pen recorder may be
determined as
From this equation a Bode plot of system gain in decibels versus frequency can be developed. Since the term $K/349.3$ would add a constant to the value of gain at each frequency, only the term in brackets was plotted. Thus the curve represents relative gain.

The plot of the relative gain of this system versus $\omega$ is shown in Fig. 39. The pen recorder is noted to have a half power bandwidth of about 4.15 cycles per second.
Fig. 39. RELATIVE GAIN VERSUS ANGULAR FREQUENCY FOR PEN RECORDER
CHAPTER VIII

EXPERIMENTAL MEASUREMENTS ON TOWER DRIVE AND SERVO EQUIPMENT

The tower drive equipment was observed to be free of erratic movement. There were no changes in speed of rotation rapid enough to be noticeable.

The accuracy of tracking of the tower by the recorder turntable was measured by manually applying a momentary signal to the high gain amplifier for each fifteen degrees of tower rotation. This caused the recording pen to deflect and mark on polar co-ordinate paper the position of the recorder turntable. The experimental circuit used in this measurement is shown in Fig. 40.

Tracking accuracy was found to be accurate within human error in applying the marking signal. Mechanical backlash was found to be constant at about $4^\circ$. This was determined by running the measurement first in the clockwise and then in the counter-clockwise direction.
Fig. 40. EXPERIMENTAL CIRCUIT FOR MEASUREMENT OF TRACKING ACCURACY
OF TOWER SERVO EQUIPMENT
A. MICROWAVE GENERATING EQUIPMENT

The oscillator used by the University of Missouri at Rolla Antenna Range is capable of delivering at least one watt of microwave power on all frequencies. Using equation 4-13 the power required from the generator may be calculated. Equation 4-13 is reproduced below.

\[ W_0 = P_{ddb} + K_T - 10 \log_{10} \frac{A_{mr} D_r}{4\pi} + 20 \log_{10} R \]  \hspace{1cm} (8-1)

where \( P_{ddb} \) is the power necessary at the detector to cause full scale deflection of the recording pen, \( K_T \) represents losses in feeding the transmitting and receiving antennas and \( R \) is the distance between the two antennas. \( A_{mr} \) and \( D_r \) are the effective aperture of the receiving antenna and the directivity of the transmitting antenna respectively.

The value of \( K_T \) was estimated as 20 decibels. This value assumes a loss of 10 db due to losses in the antenna transmission lines.

The effective aperture of the receiving antenna, \( A_{ar} \), is taken as the effective aperture of an isotropic radiator. The effective aperture of an isotropic radiator is given by Kraus\(^1\) as

\[ A_{em} = \frac{A_{em2}}{D_z} \]  \hspace{1cm} (8-2)

where \( A_{em2} \) and \( D_z \) are the effective aperture and directivity,
respectively, of any antenna. By definition of directivity, the maximum effective aperture of any antenna with directive properties must be greater than the effective aperture of an isotropic radiator. Thus using the value of effective aperture of an isotropic radiator in computing the necessary generated power places a more stringent requirement on the system than will ever be encountered in practice. Substitution of $A_{e\text{ms}}$ and $D_e$ for any antenna in equation 8-2 yields

$$A_{e\text{ms}} = 0.079 \lambda^2.$$  

(8-3)

At the highest frequency of interest, 2400 megacycles per second, $A_{e\text{ms}} = 0.0012$ square meters. At 1750 megacycles per second $A_{e\text{ms}} = 0.023$ square meters.

The directivity of the parabola used may be calculated by assuming that the parabola is equivalent to a uniformly illuminated circular aperture several wavelengths in diameter. Kraus\textsuperscript{1} gives the expression for this directivity as

$$D = 9.87 D_{\lambda}^2$$  

(8-4)

where $D_{\lambda}$ is the diameter of the aperture in wavelengths. The parabola is 0.59 meters in diameter. At 2400 megacycles $D_e = 219.8$ and at 1750 megacycles $D_e = 117.45$.

The distance $R$, between the transmitting and receiving antennas, is 12.9 meters.

Inspection of the curves presented in Fig. 34 shows that the minimum voltage required at the input of the high gain amplifier is 2.2 microvolts RMS. The high gain amplifier is operated
at gain position 20 and in the linear mode. Using the rated 200 ohms as the input impedance of the high gain amplifier, the power necessary at the input to the high gain amplifier for full scale recording pen deflection is $2.42 \times 10^{-8}$ microwatts.

The sensitivity of the bolometer used was taken as 5 ohms per milliwatt. The bolometer was biased with a current of $8.75 \times 10^{-3}$ amps. Because of the bolometer resistance only one of half the voltage caused by the change in resistance of the bolometer appears across the input terminals of the high gain amplifier. Thus the resistance must change periodically by an amount which will cause a voltage change of 4.4 microvolts RMS or 6.23 microvolts peak. At 8.75 ma the required change in resistance is $0.712 \times 10^{-3}$ ohms. The power necessary to produce this resistance change is $0.142 \times 10^{-3}$ milliwatts. Assuming 50% modulation, this value represents one half of the required value of the instantaneous peak microwave power dissipated in the bolometer. The required peak power of the modulated signal is then $0.284 \times 10^{-3}$ milliwatts at the input to the bolometer detector. Thus, the power level required at the input to the detector to cause full scale deflection of the recording pen is given in decibels with respect to one milliwatt as

$$P_{db} = 10 \log_{10} \frac{1.89 \times 10^{-4}}{I} \quad (8-5)$$

or
\[ P_{db} = -37.24 \] \hspace{1cm} (8-6)

Expression of the values in equation 8-1 in decibels with respect to a level of one milliwatt yields

\[ W_3 = -37.24 + 20 \log_{10} \frac{A_{cr} D_c}{4\pi} + 20 \log_{10} R \] \hspace{1cm} (8-7)

Solution of equation 8-7 at a frequency of 2400 megacycles per second yields

\[ W_3 = -37.24 + 20 - 10 \log_{10} \frac{0.264}{4\pi} + 20 \log_{10} 12.9 \] \hspace{1cm} (8-8)

\[ W_3 = -37.24 + 20 + 16.8 + 22.2 \] \hspace{1cm} (8-9)

\[ W_3 = 21.74 \text{ dBm} \] \hspace{1cm} (8-10)

Conversion of this quantity to milliwatts gives the necessary generator power at 2400 megacycles as

\[ P_3 = 149.4 \text{ mw} \] \hspace{1cm} (8-11)

Similarly, the generated power necessary at a frequency of 1750 megacycles per second may be computed as
Both of these power levels are within the capability of the oscillator.

Frequency variation of the oscillator was not serious. At the frequency at which the test was run, near 1750 megacycles, the total variation in frequency caused a variation in wavelength of less than 0.2 millimeter. This is about 0.001 \( \lambda \), very adequate for most applications.

Amplitude variation of the oscillator was very stable after warmup. The power drift was less than 0.03 decibels. This corresponds to a variation in power level of less than 0.7 percent. This corresponds to a pen deflection of less than 0.04 inches at full scale.
B. TRANSMITTING ANTENNAS AND TRANSMISSION PATH

The transmitting antennas and transmission path did not provide an approximation to a plane wave at the receiving antenna which satisfied the generally accepted criteria. The power in the incident wave varied at least 1.5 db along a vertical axis through the position of the antenna under test. The maximum error in making power level measurements was $\pm$ 0.4 db. Let $V_m$ be the measured value of the power variation. Then $V_m$ is equal to the actual power variation, $V_a$, plus the error in measurement of the power variation $V_e$. Then

$$V_a = V_m - V_e$$  \hfill (8-15)

If $V_m$ is greater than $V_e$ and $V_a$ and $V_m$ are positive, then $V_a$ has its smallest possible value when $V_e$ has its largest possible positive value.

If the error in reading the power level is $+$ 0.4 db where the power is at a maximum and $-$ 0.4 db where the power is at a minimum, the error in the change in power from a maximum to a minimum will be 0.8 db. This is the largest positive value of error in power variation, that is, it is the largest positive value of $V_e$. Then, since $V_m$ is equal to 1.5 db, the smallest possible value of $V_a$ is given by

$$V_{ao} = 0.7 \text{ db}$$  \hfill (8-16)
The smallest possible value of actual power variation is still greater than the accepted maximum value of power variation, 0.25 db. Measured variations along a horizontal line through the normal position of the antenna under test were less than about 0.6 db. Since these variations were smaller than the accuracy of measurement no conclusions may be drawn.

The supporting structures for the antenna under test had a significant effect upon the patterns of the antenna. This effect appeared to change with frequency but no trends could be observed since the tests were run at only two frequencies.

At 1750 megacycles the patterns indicate that the tower head interfered with the antenna mainly by shielding the antenna under test from the incident wave. This is evident from the drastic reduction in received power when the tower head is positioned between the antenna under test and the transmitting antenna. At 2400 megacycles, however, there is apparent interaction between the antenna under test and the supporting structure. This interaction is evidenced by the formation of distinct new lobes and evident changes in the structure of those parts of the pattern where the tower head is not positioned between the transmitting and receiving antennas. From these considerations it seems reasonable to assume that the interaction between the supporting structure and antenna increases with frequency.

C. RECEIVING EQUIPMENT

The receiving equipment satisfies the criteria set for it.
Equipment was not available to obtain experimental results on the bolometer. Typically, however, bolometers have very good square law detection over a large range of powers. The high gain amplifier and pen recorder are linear within experimental error in the linear mode of operation. Considerable error in the square root section is evident, however, especially at lower values of pen deflection. A correction curve could be developed experimentally if very accurate results were desired. The receiving equipment meets the criterion requiring response to a signal level 20 decibels below the signal level required for maximum pen deflection. This corresponds to response to a voltage level of one tenth or less of that required for maximum deflection. That this condition is satisfied can be determined by inspection of the curves of pen deflection versus input voltage to the high gain amplifier.

The steady state error in the pen recorder was considered negligible. Since the plot of pen deflection versus voltage at the input to the high gain amplifier was within 0.1 inch of a straight line through the origin of the graph, the steady state error is less than ±0.1 inch or less than 2% of full scale. The frequency response of the recording pen was investigated by examining the response of the system to a step input. The half power bandwidth was found to be about 4.15 cycles per second. Because of the reciprocal relationship between bandwidth and rise time7 the shortest rise time which this system can follow is about 0.241 seconds. For a full scale rise this would correspond to a slope of 22.8 inches per second. If this rise time is doubles as a factor of safety a slope of 11.4 inches per second is the
steepest allowable. Any steeper slopes should be viewed with suspicion. The slowest speed in the counterclockwise direction is 3.2 degrees per second. Thus patterns which change in amplitude at a rate of less than 3.56 inch per degree can be accurately measured at this speed.

D. TOWER DRIVE AND SERVO EQUIPMENT

The tower drive equipment provided for smooth rotation of the antenna and there was negligible error in the tracking of the tower by the turntable.

Mechanical backlash is not a serious problem. As long as the test antenna is oriented with its reference plane directed toward the transmitting antenna when the turntable passes through zero in one direction, say counterclockwise, the patterns run in this direction will give a true picture of the actual pattern. Patterns run in the opposite direction will be the same shape but will be rotated four degrees in the counterclockwise direction on the polar co-ordinate paper. Because of the desirability of running patterns in both directions in order to save time the turntable may be rotated four degrees in such a direction that the backlash is cancelled out. Because of the difficulty of properly positioning the turntable it is usually best to run a given set of patterns in the same direction.
CHAPTER X
RECOMMENDATIONS

A. IMPROVEMENT OF EXISTING FACILITIES

All of the antennas range components meet the criteria necessary for acceptable operation of the range except the transmitting antennas and transmission path. In addition certain improvements are possible in the range facilities which affect the convenience and efficiency of operation. Recommendations for improvement of these conditions will now be discussed.

The incident wave at the position of the test antenna differs from a plane wave because of multiple reflections from objects in the transmission path. Because of the relatively complicated geometry of these objects the manner in which these reflected waves add to produce the incident wave is not easily determined. Under these conditions one approach is to change the geometry of the situation to a form which is more easily to deal with mathematically. This approach has been used at the Lincoln Laboratory Antenna Test Range as reported by A. Cohen and A. W. Maltese.

The geometry of the University of Missouri at Rolla Antenna Range may be simplified by making the transmission path between the transmitting antenna and the antenna under test a good approximation to a ground plane. This could be accomplished by covering the surface of the roof between the receiving and transmitting antennas with a conducting sheet made of copper screen. This conducting sheet should be at least as wide as the width of the
broadest major lobe of the transmitting antenna anticipated. A large baffle placed in front of the equipment building and well grounded to the conducting sheet along its lower edge will minimize reflections from the transmitting and receiving equipment.

The three possible paths by which radiation may be reflected back to the transmitting antenna are:

1. From the baffle to the transmitting antenna
2. From the baffle to the conducting plane to the transmitting antenna
3. From the conducting plane to the baffle to the transmitting antenna.

Reflection along the first of these three paths is more serious because the first path is the shortest. Reflection along this path will be eliminated completely if the reflecting baffle is either vertical or inclined toward the transmitting antenna by some angle $\theta_b$.

Use of a vertical baffle will be considered. By use of ray paths and the laws of reflection of electromagnetic waves the correct position of the baffle may be determined geometrically as shown in Fig. 41. The transmitting and receiving antennas are represented by $\mathbf{A}_t$ and $\mathbf{A}_r$, respectively. $h$ is the height above the ground plane of the transmitting and receiving antennas and $R_a$ is the distance between these two antennas. $D_d$ is the horizontal distance between the transmitting antenna and the position of the baffle. Image antenna $\mathbf{A}_{i_u}$ is an image antenna of the transmitting antenna and is the apparent source of reflection from the infinite ground plane shown in Fig. 41. $\mathbf{A}_{i_2}$ is also an
Fig. 41. DETERMINATION OF BAFFLE STRUCTURE BY GEOMETRY
image antenna of the transmitting antenna and is the apparent source of reflections from the infinite vertical ground plane represented by the line PP'. Paths OABC and ODAE represent the actual ray paths of the radiation represented in Fig. 41 as originating from antenna A_{i1} and radiating along paths P_1 and P_2, respectively.

Reflection back to the transmitting antenna from the vertical baffle and the horizontal ground plane with which it intersects may be considered to be eliminated if radiation from the transmitting antenna reflected from the infinite ground plane PP' to the image antenna, A_{i1}, is eliminated. This follows directly from the concept of an image antenna. If image antenna number one is considered the receiver, a second image antenna, A_{i2}, may be used to determine the signal reflected to image antenna A_{i1}. This is shown in Fig. 41. If it is desirable to remove incident radiation on image antenna number one to a distance d, above and below that antenna all radiation from antenna A_{i2} between paths P_1 and P_2 must be eliminated. This radiation can be removed by covering the ground plane with microwave absorbing material to a distance from the baffle equal to the larger of the two distances l_1 and l_2. From the geometry of the situation l_1 will always be the larger of the two. Then

\[ l_1 = D - \frac{h - d}{\tan \left( \tan^{-1} \frac{2h - d}{2D} \right)} \]  

(9-1)
Microwave absorbing material can also be used to cover the upper edge of the baffle in order to reduce reflections from the edge of the baffle. The tower carriage which is a major source of reflection and which cannot be removed from the incident field should also be covered with microwave absorbing material.

The above precautions should reduce all reflections except those from the ground plane to a negligible value. Using the expression for the pattern of a parabolic antenna given by Kraus\(^1\), the expression for the incident wave at the position of the antenna under test is developed in Appendix E. Using this expression the power variation in decibels along a vertical line through the normal position of the antenna was computed using a digital computer. These variations are plotted versus vertical height in Fig. 42 and Fig. 43. As can be seen from these figures the power variations is less than 0.25 db over a vertical distance for at least 0.4 meters above and below the position of the antenna under test. This corresponds to a reflected electric field of 0.029 times the direct field. This field will cause a maximum phase variation when it is ninety degrees out of phase. This maximum phase variation will be about 1.67 degrees, well within acceptable limits.

Other desirable improvements are an easy method of changing transmitting antennas and provision of a boresight telescope to position the test antenna with respect to the transmitting antenna. The first of these improvements is now under
consideration. Careful attention should also be given to development of suitable supporting structures which will have a minimum effect on the antenna under test.

B. EXTENSION OF USEFULNESS OF THE ANTENNA RANGE

The University of Missouri at Rolla Antenna Range could be used at higher frequencies than those examined in this thesis. At higher frequencies, however, transmission line attenuation becomes very great. To overcome this difficulty different methods of feeding the transmitting antenna must be devised. It is also probable that suitable supporting structures for the antenna under test will become a greater problem.

Investigation of these problems and others introduced by the use of higher frequencies is a first step toward improving the future usefulness of the antenna range.
Fig. 42. COMPUTED POWER VARIATION VERSUS HEIGHT, 2400 MEGACYCLES
Fig. 43. COMPUTED POWER VARIATION VERSUS HEIGHT, 1750 MEGACYCLES
BIBLIOGRAPHY


APPENDIX A

PER CENT OF ERROR
IN DIVISION OF TWO NUMBERS

Consider the quotient of two real numbers \( X \) and \( Y \).
Let \( X \) be accurate to within a positive fraction \( A \) of its nominal value and let \( Y \) be accurate to within a positive fraction \( B \) of its nominal value. The maximum variation of the quotient of \( X \) and \( Y \) from its nominal value will occur when the values of the numerator and denominator change in opposite directions, one increasing, the other decreasing. If the values of \( X \) and \( Y \) are assumed to vary from their nominal values by the maximum amount allowed by the conditions of accuracy imposed, and if \( X \) is assumed to increase and \( Y \) is assumed to decrease, the quotient of the two is given by

\[
Q = \frac{X + AX}{Y - BY} \quad \text{(A-1)}
\]

The nominal quotient, \( Q_0 \), is given by

\[
Q_0 = \frac{X}{Y} \quad \text{(A-2)}
\]

The difference between the two quotients \( \Delta Q \) is given by

\[
\Delta Q = \frac{X + AX}{Y - BY} - \frac{X}{Y} \quad \text{(A-3)}
\]
$$\Delta Q = \frac{XY + AXY - XY + BXY}{Y(Y - BY)} \quad (A-4)$$

$$\Delta Q = \frac{X}{Y} \frac{A + B}{1 - B} \quad (A-5)$$

Thus the quotient is seen to be accurate to within a fraction $\frac{A + B}{1 - B}$ of its nominal value. A similar operation may be performed allowing $Y$ to increase and $X$ to decrease. This situation results in a quotient which is accurate to within a fraction $\frac{A + B}{1 + B}$ of its nominal value. This value is a smaller fraction than the one developed above. Therefore, the fraction $\frac{A + B}{1 - B}$ above is the worst case and is the fraction which must be used to indicate the accuracy of the quotient.
APPENDIX B
PER CENT OF ERROR IN PARALLEL COMBINATION OF TWO RESISTORS

Consider the parallel combination of two resistors $R_a$ and $R_b$. Let the value of $R_a$ be known to be accurate to within a fraction $f_a$ of its nominal value and let the value of $R_b$ be known to be accurate to within a fraction $f_b$ of its nominal value. $f_a$ and $f_b$ may be either positive or negative but are restricted to have absolute value less than one. Also let $R_{ao}$ and $R_{bo}$ be the nominal values of $R_a$ and $R_b$ respectively. Then, assuming worst case conditions, the resistance of the parallel combination of $R_a$ and $R_b$ is given by

$$R_p = \frac{(R_{ao} + f_a R_{ao})(R_{bo} + f_b R_{bo})}{R_{ao} + f_a R_{ao} + R_{bo} + f_b R_{bo}} \quad (B\text{-}1)$$

The nominal value of $R_p$ is given by

$$R_{po} = \frac{R_{ao} R_{bo}}{R_{ao} + R_{bo}} \quad (B\text{-}2)$$

Let the numerator of the right side of equation B-1 be denoted by $N_p$. Then

$$N_p = R_{ao} R_{bo} + R_{ao} R_{bo} (f_a + f_b + f_a f_b) \quad (B\text{-}3)$$

Thus the numerator of the right side of equation B-1 is equal
to the numerator of the right side of equation B-1 is equal to the numerator of the nominal value of \( R_p \) within the fraction \( \left( f_a + f_b + f_c \right) \) of that nominal value. This fraction is largest when \( f_a \) and \( f_b \) are both positive values. Let this largest value be represented by the symbol \( A \).

Let the denominator of the right hand side of equation B-1 be represented by the symbol \( D_p \) and let the denominator of the nominal value of \( R_p \) given in equation B-2 be denoted by \( D_{p_0} \). Then

\[
D_p = R_{ao} + R_{bo} + f_a R_{ao} + f_b R_{bo}.
\]

(B-4)

The equation above shows that \( D_p \) differs from \( D_{p_0} \) by the amount \( f_a R_{ao} + f_b R_{bo} \). The absolute value of the fraction of error of \( D_p \) is given by dividing the absolute value of \( f_a R_{ao} + f_b R_{bo} \) by the absolute value \( |D_{p_0}| \). Let this fraction be denoted by \( \tilde{J} \). Then

\[
|\tilde{J}| = \frac{|f_a R_{ao} + f_b R_{bo}|}{|R_{ao} + R_{bo}|}.
\]

(B-5)

\( |\tilde{J}| \) will be greatest when \( f_a \) and \( f_b \) are of the same sign since \( R_{ao} \) and \( R_{bo} \) are both positive quantities. Let the largest possible value of \( |\tilde{J}| \) be represented by the symbol \( |\tilde{J}_m| \).

Under the assumption that \( |\tilde{J}| \) is made to have its maximum value by forcing \( f_a \) and \( f_b \) to have the same sign, this maximum
value may be written

\[ |J_m| = \left| \frac{f_a R_{ao} + f_b R_{bo}}{R_{ao} + R_{bo}} \right| \]  \hspace{1cm} (B-6)

Let the absolute value \( |f_a| \) be greater than the absolute value of \( f_b \). Then, since \( f_a \) and \( f_b \) are assumed to be the same sign

\[ \left| \frac{f_a R_{ao} + f_b R_{bo}}{R_{ao} + R_{bo}} \right| < \left| \frac{f_a R_{ao} + f_a R_{bo}}{R_{ao} + R_{bo}} \right| \]  \hspace{1cm} (B-7)

or

\[ |J_m| < |f_a| \]  \hspace{1cm} (B-8)

Similarly if the absolute value of \( f_b \) is assumed to be larger than the absolute value of \( f_a \), it can be shown that

\[ |J_m| < |f_b| \]  \hspace{1cm} (B-9)

Therefore, it is seen that the absolute value of the maximum fraction of error in \( D_p \) is less than the greater of the two absolute values \( |f_a| \) and \( |f_b| \). Thus the greater value of the two must be taken as the absolute value of the maximum fraction of error for the denominator. Let this greater value be denoted by the symbol \( |F| \).
From the previous development it is seen that the numerator of the expression for \( R_p \) is at least within some fraction \( A \) of the numerator of the expression for the nominal value of \( R_p \). Also, it is seen that the denominator of the expression for \( R_p \) is at least within some fraction \( F \) of the value of the denominator of the expression for the nominal value of \( R_p \). If the errors in the numerator and denominator are assumed to have these maximum values, the value of \( R_p \) may be expressed as

\[
R_p = \frac{R_{10}R_{b0} + \bar{\rho}R_{10}R_{b0}}{R_{10} + R_{b0} + F(R_{10} + R_{b0})} \quad \text{(B-10)}
\]

In Appendix A it is shown that the quotient \( \frac{X}{Y} \) is accurate to within a fraction of \( \frac{A + B}{1 - B} \) of its nominal value if \( X \) is accurate to within a positive fraction \( A \) of its nominal value and \( Y \) is accurate to within a positive fraction \( B \) of its nominal value. Using this result, it is seen that \( R_p \) is accurate at least to within a fraction \( \frac{A + F}{1 - F} \) of its nominal value.
APPENDIX C

ACCURACY OF THE CHANGE IN RESISTANCE OF A PARALLEL COMBINATION OF TWO RESISTORS

Let two resistors, \( R \), and \( R_2 \), be connected in parallel. Assume that it is possible to change the value of the resistance \( R \), by a value \( \Delta R \), by changing some parameter \( \alpha \). Let the change in resistance of the parallel combination of \( R \), and \( R_2 \) be called \( \Delta R_p \).

Let \( \Delta R \), be a linear function of the change in \( \alpha \) so that the nominal value of \( \Delta R \), is given by

\[
\Delta R = K_0 \Delta \alpha
\]

where \( K_0 \) is some constant and \( \Delta \alpha \) is the change in \( \alpha \).

Now let \( \Delta \alpha \) be known to within a fraction \( p \) of its nominal value and let the constant \( K_0 \) be known to within a fraction \( q_0 \) of its nominal value. Let the value of the change in resistance of \( R \), with this error introduced be denoted by \( R_{1\varepsilon} \). Then

\[
\Delta R_{1\varepsilon} = (\Delta \alpha + p\Delta \alpha)(K_0 + q_k K_0)
\]

or

\[
\Delta R_{1\varepsilon} = \Delta \alpha K_0 + (p + q_k + p_2 q_k) \Delta \alpha K_0
\]

or

\[
\Delta R_{1\varepsilon} = \Delta R_1 + (p + q_k + p_2 q_k) \Delta R_1
\]
From equation C-4 above the change in $R_i$ is seen to be within a fraction $(P + q_b + P_b)$ of its nominal value if $\Delta \alpha$ is accurate to within a fraction $p$ of its nominal value and $K_i$ is accurate to within a fraction $q$ of its nominal value. If both $P$ and $q$ are assumed to have absolute values less than one, the fraction $(P + q + P_b)$ will have its maximum value when $P$ and $q$ are both positive. Let this maximum value of the fraction $(P + q + P_b)$ be denoted by $h_0$. Then the value of the change in the resistance $R_i$ is always within the fraction $h_0$ of its nominal value. In the following discussion the absolute value of $h_0$ is restricted to values less than one.

The nominal value of the resistance of the parallel combination of the two resistors $R_i$ and $R_z$ is given by

$$R_{po} = \frac{R_i R_z}{R_i + R_z}. \quad (C-5)$$

If the resistance $R_i$ is changed by a value $\Delta R_i$ the absolute value of the nominal change in resistance of this parallel combination is given by

$$\Delta R_{po} = \left| \frac{R_i R_z}{R_i + R_z} - \frac{(R_i + \Delta R_i) R_z}{(R_i + \Delta R_i + R_z)} \right| \quad (C-6)$$

$$\Delta R_{po} = \left| \frac{-\Delta R_i R_z^2}{(R_i + R_z)(R_i + \Delta R_i + R_z)} \right|. \quad (C-7)$$
If the error in $\Delta R_i$ is assumed to have its maximum value $h_0 \Delta R_i$, the absolute value of the actual change in the resistance of the parallel combination is given by

$$\Delta R_{PE} = \left| \frac{R_i R_z}{R_i + R_z} - \frac{(R_i + \Delta R_i + h_0 \Delta R_i)(R_z)}{(R_i + \Delta R_i + h_0 \Delta R_i + R_z)} \right|. \quad (C-8)$$

The difference between $|\Delta R_{P0}|$ and $|\Delta R_{PE}|$ is the error in $\Delta R_P$ caused by the error in $\Delta R_i$. Let this difference be denoted by $E_{\Delta R_P}$.

An increase in the resistance of one of the branches of a parallel connection of two resistors increases the total resistance of the combination. A reduction in resistance of one of the branches reduces the resistance of the parallel combination. It follows that if the absolute value of $h_0$ is restricted to values less than one, $\Delta R_{P0}$ and $\Delta R_{PE}$ have the same sign. Therefore, the difference in the absolute values of these two quantities is the same as the absolute value of their difference. Then

$$|E_{\Delta R_P}| = |\Delta R_{P0} - \Delta R_{PE}| \quad (C-9)$$

$$|E_{\Delta R_P}| = \left| \left( \frac{R_i}{R_i + R_z} - \frac{(R_i + \Delta R_i) R_z}{(R_i + \Delta R_i + R_z)} \right) - \left( \frac{R_i}{R_i + R_z} - \frac{(R_i + \Delta R_i + h_0 \Delta R_i) R_z}{(R_i + \Delta R_i + h_0 \Delta R_i + R_z)} \right) \right|. \quad (C-10)$$
\[ |E_{\Delta R_P}| = \frac{(R + \Delta R + h_0 \Delta R_j)R_z}{(R + \Delta R + h_0 \Delta R_j + R_z)} - \frac{(R + \Delta R)R_z}{(R + \Delta R + R_z)} \]  \hspace{1cm} (C-11)

or

\[ |E_{\Delta R_P}| = \left| \frac{h_0 \Delta R_i R_z^2}{(R + \Delta R + R_z) (R + \Delta R + h_0 \Delta R_j + R_z)} \right| \]  \hspace{1cm} (C-12)

The absolute value of the fraction of error in \( \Delta R_P \) is given by the absolute value of the error in \( \Delta R_P \) divided by the absolute value of the nominal value of \( \Delta R_P \). If this fraction of error is called \( m \) then

\[ |m| = \frac{|E_{\Delta R_P}|}{|\Delta R_{P_0}|} \]  \hspace{1cm} (C-13)

\[ = \left| \frac{h_0 \Delta R_i R_z^2}{(R + \Delta R + R_z) (R + \Delta R + h_0 \Delta R_j + R_z)} \right| \frac{- \Delta R_i R_z^2}{(R + R_z) (R + \Delta R + R_z)} \]  \hspace{1cm} (C-14)

or

\[ |m| = \left| \frac{-h_0 (R + R_z)}{R + \Delta R + h_0 \Delta R_j + R_z} \right| \]  \hspace{1cm} (C-15)
This expression may be written

\[ |M| = \left| -h_0 \right| \left| \frac{R_1 + R_2}{R_1 + \Delta R_1 + h_0 \Delta R_1 + R_2} \right| \]  \hspace{1cm} (C-16)

As long as \( \Delta R_1 \) is positive, the fraction \( \frac{R_1 + R_2}{R_1 + \Delta R_1 + h_0 \Delta R_1 + R_2} \) in equation C-16 above is always a positive number less than one. From this fact it follows that

\[ |m| < \left| -h \right| \]  \hspace{1cm} (C-17)

or

\[ |m| < \left| h \right| \]  \hspace{1cm} (C-18)

Therefore, if the value of resistance of a parallel combination of two resistors is changed an amount \( \Delta R_p \) by changing the value of one of them by a positive amount \( \Delta R_1 \), and if \( \Delta R_1 \) is known within \( \mu \) per cent of its nominal value, then the change \( \Delta R_p \) is accurate to within \( \mu \) per cent of its nominal value and \( \mu \) is less than \( \mu \).
APPENDIX D

GENERAL FORM OF PEN RECORDER RESPONSE

A block diagram of the pen recorder system is shown in Fig. 44. \( P(t) \) represents the position of the recording pen as a function of time. The balanced bridge represents the two triodes at the input of the pen servo amplifier as described in Chapter III. Its output is proportional to the difference of the voltages \( V_o(t) \) and \( V(t) \). The transfer function \( M(t) \) associated with the motor represents the angular position of the motor shaft divided by the voltage \( V_m(t) \) from the amplifier. The differentiator provides a signal proportional to the speed of the recording pen, which is applied as negative feedback. The two transfer function \( C_o \) and \( C_r \) represent constants of proportionality by which \( V(t) \) and \( P(t) \), respectively, are related to the position of the motor shaft.

The voltage applied to the motor is given by

\[
V_m(t) = A C_e (V_o(t) - V(t)) . \tag{D-1}
\]

\( V(t) \) is directly proportional to \( \theta_m(t) \) and may be expressed as

\[
V(t) = C_o \theta_m(t) . \tag{D-2}
\]

then

\[
V_m(t) = A C_e (V_o(t) - C_o \theta_m(t)) . \tag{D-3}
\]
Fig. 44. BLOCK DIAGRAM OF PEN RECORDER SYSTEM
Taking the Laplace Transform of both sides of this equation under the assumption of zero initial condition yields

\[ V_m(s) = AC_z \left( V_o(s) - C_o \theta_m(s) \right) \quad (D-4) \]

The transfer function of a two phase induction motor operating under linear conditions has been developed by Kuo\(^6\) as

\[ \frac{\theta_m(s)}{V_m(s)} = \frac{K_m}{s(1 + s\gamma_m)} \quad (D-5) \]

where \(K_m\) and \(\gamma_m\) are constants of the motor. Since only the form of the results of concern is the numerical value of these constants is unimportant.

With knowledge of the motor transfer function, equation D-4 may be written as

\[ \frac{\theta_m(s)}{AC_z} \frac{s(1 + \gamma_m) + K_m AC_o C_z}{K_m} = V_o(s) \quad (D-6) \]

The transfer function from the input of the balanced bridge to the motor shaft is

\[ \frac{\theta_m(s)}{V_o(s)} = \frac{AC_z K_m}{s(1 + s\gamma_m) + K_m AC_o C_z} \quad (D-7) \]

The pen deflection is directly proportional to the amount of shaft rotation of the motor. Therefore,

\[ \frac{P(s)}{V_o(s)} = \frac{AC_o C_z K_m}{s(1 + s\gamma_m) + K_m AC_o C_z} \quad (D-8) \]
where $P(s)$ is the Laplace Transform of $P(t)$ assuming zero initial conditions. Under these conditions the Laplace Transform of the derivative of $P(t)$ with respect to time is $SP(s)$. The signal $V_o(t)$ is the difference between $V_s(t)$ and the time derivative of $P(t)$. Therefore the Laplace Transform of $V_o(t)$ is

$$V_o(s) = V_s(s) - SP(s) \quad \text{(D-9)}$$

Equation D-8 may be written

$$\frac{P(s)}{V_s(s) - SP(s)} = \frac{AC_1C_zK_m}{S(1 + S\gamma_m) + K_mAC_oC_z} \quad \text{(D-10)}$$

or

$$P(s) = \frac{(V_s(s) - SP(s))AC_1C_zK_m}{S(1 + S\gamma_m) + K_mAC_oC_z} \quad \text{(D-11)}$$

The total transfer function is given as

$$\frac{P(s)}{V_s(s)} = \frac{AC_1C_zK_m}{\gamma_m S^2 + (1 + AC_1C_zK_m)S + K_mAC_oC_z} \quad \text{(D-12)}$$

or

$$\frac{P(s)}{V_s(s)} = \frac{AC_1C_zK_m/\gamma_m}{S^2 + S(1 + AC_1C_zK_m)/\gamma_m + (K_mAC_oC_z)/\gamma_m} \quad \text{(D-13)}$$
Let

\[(AC_c z K) / \gamma_m = K\]

\[(1 + AC_c z K_m) / \gamma_m = B\]

and

\[(K_m AC_c C_z) / \gamma_m = C\]

Then the transfer function is of the form

\[
\frac{P(s)}{V(s)} = \frac{K}{s^2 + Bs + C} \quad \text{(D-14)}
\]
APPENDIX E

VARIATION IN FIELD INTENSITY
OF A PARABOLIC ANTENNA ABOVE A GROUND PLANE
AS A FUNCTION OF HEIGHT

Consider a transmitting antenna \( A_0 \) suspended above a ground plane as shown in Fig. 45. The variation in incident power at the receiving position as a function of the vertical distance \( x \) is desired. The transmitting antenna will be assumed to be a parabolic reflector excited by a horizontally polarized dipole. The pattern of this type antenna is given by Kraus\(^1\) as

\[
E(\theta) = \frac{2\lambda}{\pi D} \frac{J_1\left(\frac{\pi D \sin \theta}{\lambda}\right)}{\sin \theta}
\]

(E-1)

where \( D \) is the diameter of the parabola, \( \lambda \) is wavelength at the frequency of operation and \( J_1\left(\frac{\pi D \sin \theta}{\lambda}\right) \) is the Bessel function of first order of \( \frac{\pi D \sin \theta}{\lambda} \). \( \theta \) is measured as shown in Fig. 45.

By use of image theory the field along the vertical line \( PP' \) may be considered the sum of the fields of the antenna and the image antenna \( A_i \), which has a field 180 degrees out of phase with antenna \( A_0 \).

Let the electric field intensity be normalized to a value of one unit at a distance \( L \) from each of the antennas. The power reduction factor due to path length may then be expressed as the square of the reciprocal of one
plus the increase in path length as a fraction of $L$. For the real antenna this factor is given as

$$P_r = \frac{1}{(1 + \frac{a_0 - L}{L})^2} \tag{E-2}$$

and for the image antenna as

$$P_i = \frac{1}{(1 + \frac{a_i - L}{L})^2} \tag{E-3}$$

From geometry

$$\theta = \tan^{-1} \frac{x}{L} \tag{E-4}$$

and

$$\theta' = \tan^{-1} \frac{2h + x}{L} \tag{E-5}$$

Then

$$Q_o = \frac{L}{\cos \theta} \tag{E-6}$$

and

$$Q_i = \frac{L}{\cos \theta'} \tag{E-7}$$

Then

$$E_o(x) = \left(\frac{1}{1 + \frac{a_i - L}{L}}\right)^2 \frac{2\lambda}{D\pi} \frac{J_1(\pi R \sin \theta)}{\sin \theta} \tag{E-8}$$
and

\[ E_l(\chi) = \left( \frac{1}{1 + \frac{Q_l - L}{L}} \right)^2 \frac{2\lambda}{\pi D} \frac{J_i(\frac{\pi D}{\lambda} \sin \theta)}{\sin \theta}. \]  

(E-9)

since \( \theta \), \( Q_l \) and \( Q_o \) are functions of \( \chi \). The phase angle between the two waves is given as

\[ \phi = (\pi - \beta (Q_l - Q_o)) \]  

(E-10)

where \( \beta \) is the wave propagation constant in free space given by

\[ \beta = \frac{2\pi}{\lambda}. \]  

(E-11)

Thus at some distance \( \chi \) along the plane \( PP' \)

\[ E(\chi) = (E_0(\chi) + E_i(\chi) \cos \phi) + (E_i(\chi) \sin \phi)^2. \]  

(E-12)

Let the electric field intensity at \( \chi \) equal to zero be represented by \( E_0 \). Then the power variation in decibels with respect to the power at \( \chi = 0 \) is given by

\[ P = 20 \log_{10} \frac{E(\chi)}{E_0}. \]  

(E-13)
Fig. 45. ANTENNA ABOVE GROUND PLANE
VITA

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