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Fluid flow in pipes of rectangular cross sections

Bruce Harold Bradford

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FLUID FLOW IN PIPES OF
RECTANGULAR CROSS SECTION

BY

BRUCE HAROLD BRADFORD – 1963

A

THESIS

submitted to the faculty of the
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Approved by

[Signatures]
ABSTRACT

Head loss due to fluid friction was measured in smooth conduits of rectangular cross section. The data covers the laminar and turbulent range. Results were compared with previous theoretical and empirical data.

The customary practice of calculating head loss in non-circular conduits by use of the friction factor obtained on an "equivalent" circular pipe was discussed. It was found that this practice is successful in square cross sections undergoing turbulent flow. In laminar flow, however, this method is not accurate.
ACKNOWLEDGMENTS

The author wishes to express his deepest appreciation to Professors Jerry R. Bayless and Paul R. Munger for their constant guidance and counsel in the preparation of this thesis.

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NOMENCLATURE

The symbols are defined as they appear in the thesis and appear here in alphabetical order.

A  Cross sectional area, feet squared
a  Half width of cross section
b  Half depth of cross section
D  Circular pipe diameter, feet
De Equivalent non-circular pipe diameter, feet
d  Distance from a friction surface, feet
dp Potential depth of viscous shear, feet
e  Mean height of roughness projections, feet
f  Friction factor, dimensionless
Fi  Inertial force
Fv  Viscous force
g  Acceleration of gravity, feet per second squared
hf  Head loss, feet of water
L  Length of conduit, feet
l  Characteristic length dimension
P  Wetted perimeter, feet
Q  Volume flowrate, cubic feet per second
R  Reynolds number, dimensionless
Rh  Hydraulic radius, feet
s.g. Specific gravity, dimensionless
t  Characteristic time dimension
V  Average velocity, feet per second
v  Characteristic velocity dimension
\[ \frac{v^2}{2g} \] Velocity head, feet of water

\[ \rho \] Density of water, slugs per cubic foot

\[ \gamma \] Viscosity of water, slugs per foot second
I. INTRODUCTION

The effects of fluid friction in conduits of circular cross section are well known through many experimental studies. These data are sufficient to establish the Reynolds number effect on friction in both the laminar and turbulent regions and show that head loss, $h_f$, can be expressed as a function of length, $L$, pipe diameter, $D$, and velocity head, $V^2/2g$, by use of the "Darcy equation," $h_f = f\frac{L V^2}{D 2g}$.

For circular pipes the friction factor is a function of Reynolds number and the relative roughness where relative roughness is the dimensionless ratio $e/D$, $e$ being the mean height of the roughness projections and $D$ the diameter of the pipe. When the surface roughness is very small the pipe is said to be hydraulically "smooth" and friction factor is a function of Reynolds number, $R = \frac{V D e}{\nu}$ alone.

Friction factors for turbulent flow in smooth pipes can be approximated empirically by the "Blasius equation," $f = .316/R^{0.25}$. In the laminar region theory and experiment validate the relationship $f = 64/R$ for both smooth and rough pipes.

In dealing with conduits of noncircular cross section it is necessary to replace the pipe diameter by an equivalent diameter, $D_e$, which is usually taken as four times the hydraulic radius where hydraulic radius is defined as cross sectional area divided by wetted perimeter. For laminar flow the equation $f = 64/R$ can be applied to non-circular cross sections by multiplying it by a constant which varies with the shape of the cross section. In turbulent flow, however, no such adjustment is necessary; the "Blasius equation" is used for non-circular cross sections simply by replacing diameter with equivalent diameter.
The purposes of this investigation are as follows: to present friction data on smooth conduits of rectangular cross section, to discuss this data and to compare it with previous work of a similar nature, to discuss the concept of equivalent diameter as applied to non-circular cross sections, to suggest a new criteria for finding equivalent diameter, and to discuss its effect on the experimental data.

The data will be presented graphically as a logarithmic plot of friction factor versus Reynolds number. The data in the laminar range will be compared with the theoretical relationships for various cross sectional shapes. A least squares curve fit of the data in the turbulent region will be determined and compared to the "Blasius equation."

The reasons for using four times the hydraulic radius as the equivalent diameter will be discussed. It will be shown that hydraulic radius is the proper length dimension in Reynolds number subject to certain assumptions. By altering these assumptions slightly a new length dimension, potential depth, can be determined. This will be used in place of hydraulic radius and its effect on the experimental data will be discussed.
II. LITERATURE REVIEW

Head loss due to friction in flowing pipes has been the subject of experiment and analysis for many years. As early as 1850 experiments on the flow of water in long, straight, uniform pipes indicated that head loss, $h_f$, was directly proportional to length, $L$, and velocity head, $V^2/2g$, and inversely proportional to pipe diameter, $D$. Darcy, Weisbach, and others proposed the equation

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

This equation, known as the "Darcy Equation," is still the basic equation for head loss caused by pipe friction in long, straight, uniform pipes. In the following investigation the friction factor, $f$, will be defined by the Darcy equation, i.e., $f = \frac{h_f D}{L} \frac{2g}{V^2}$.

Dimensional analysis shows that friction factor is a function of Reynolds number alone in pipes of negligible roughness. Reynolds number is defined as the ratio of the inertial force, $F_i$, to the viscous force, $F_v$. This can be determined as follows:

$$F_i = ma \quad F_v = \mathcal{N} \frac{(dy)}{dy} A$$

written dimensionally $F_i = \left( \frac{\rho}{L^3} \right) \frac{(v)}{(t)} \quad F_v = \mathcal{N} \frac{(v)}{(l)} 1^2$

and the ratio $\frac{F_i}{F_v} = \frac{\rho L^3 v/\mathcal{N}}{t} = \frac{(1)}{(t)} \rho 1 = \frac{V D \rho}{\mathcal{N}}$

For convenience the characteristic velocity dimension is taken as the average velocity, $V$, and the characteristic length dimension is taken as the pipe diameter, $D$, defining Reynolds number as $\frac{V D \rho}{\mathcal{N}}$. Characteristic lengths and velocities other than pipe diameter and average velocity are occasionally used.
In dealing with non-circular cross sections it is customary to use an equivalent diameter, $D_e$, in place of the pipe diameter, $D$. By analogy with circular cross sections $D_e$ is taken as four times the hydraulic radius where hydraulic radius, $R_h$, equals the cross sectional area occupied by the fluid divided by the perimeter on which the fluid exerts skin friction.

Stanton and Pannell\(^1\) as well as many others have done extensive work on flow of water in pipes of circular cross section. Their work is presented graphically as a logarithmic plot of friction factor against Reynolds number for various values of pipe roughness.

Little data is available for pipes of non-circular cross section. Walker, Whan and Rothfus\(^2\) as well as Owen\(^3\) have collected data on annular pipes. Fromm\(^4\) worked with pipes in which the ratio of sides was never less than six to one and dealt only with turbulent flow. Davies and White\(^5\) worked with sections whose side ratios were never less than forty to one, so that laminar flow could be calculated from the formula for flow between infinitely wide parallel plates; and Cornish\(^6\) worked with a pipe 1.178 centimeters wide by .404 centimeters deep (ratio of sides = 2.92).

This investigation will deal with four cross sections, three square sections of widths 1.0, 2.0 and 3.0 inches and one rectangular section 0.5 inches by 1.5 inches (ratio of sides = 3.0).

\(^1\)Superscripts refer to references in the bibliography.
III. TESTING EQUIPMENT

Tests were run on four pipes all of which were similar in construction and use, Figures 1, 2 and 3.

All sections were constructed from strips of one-fourth inch plexiglass which were eight feet long and were glued to their adjacent sides with chloroform. Two eight foot sections were constructed for each section and were spliced together to make their total length sixteen feet, Figure 2.

Manometer connections were placed nine and fifteen feet from the inlet providing an upstream length greater than fifty equivalent diameters in each case for establishment of flow and a length of six feet over which head loss could be measured.

Pressure drop was measured with one of three vertical u-tube manometers. For very small differences, up to one inch of water, monochlorobenzine was used as the manometer fluid. Larger differences were measured with carbon tetrachloride, and for still larger differences mercury was used. The specific gravities of these fluids under testing conditions were calculated as shown by the figure and calculations on Page 9.

At the inlet and outlet of each section a piece of three quarter inch plexiglass was threaded and a one inch diameter nipple was screwed into it, Figure 2. These were glued to the ends of the pipe and coupled to one inch inlet and outlet pipes.

The pipe was fed by a pressure tank which could operate at pressures from zero to fifty p.s.i. Flow was controlled by means of a valve at the outlet of the pipes. The water flowed into a tank mounted on scales for collection and weighing to determine flowrate.
SIDE VIEW OF TESTING APPARATUS

FIGURE 1
CONNECTION PLATES

SPlice

MANOMETER

Connecting Plates, Splice, Manometer

Figure 2
TESTING APPARATUS

FIGURE 3
Determination of specific gravity of manometer fluids:

(Pressure in inches of H₂O)

\[ P_A = P_0 + X + (s.g.) \cdot h \]

\[ P_B = P_0 + \ell + X + h \]

\[ s.g. - 1 = \ell / h \]

\[ s.g. = 1 + \ell / h \]

Monochlorobenzine:

\[ s.g. = 1 + \ell / h = 1 + \frac{7.20 - 6.05}{5.10 + 5.45} = 1.11 \]

Carbon Tetrachloride:

\[ s.g. = 1 + \ell / h = 1 + \frac{8.50 - 6.70}{1.10 + 1.90} = 1.60 \]

Mercury:

\[ s.g. = 1 + \ell / h = 1 + \frac{101.0 + 2.5}{19.2 - 11.0} = 13.6 \]
IV. TESTING PROCEDURE

The purpose of the testing procedure was to determine friction factor, \( f = h_f \frac{D}{L} \frac{2g}{V^2} \), over a range of Reynolds number, \( R = \frac{VD\rho}{\mu} \), for each cross section. In order to make these calculations head loss, length, equivalent diameter, velocity head, velocity, and the density, \( \rho \), and viscosity, \( \mu \), of the water must be known. For a particular cross section all these values remain constant except velocity and head loss.

In order to determine head loss and velocity of a particular run three values were needed: the weight of water collected, the time interval over which it was collected, and the manometer reading at this particular flow. The weight flowrate is the weight of water collected divided by the time interval. This is converted to volume flowrate, \( Q \), by dividing by the specific weight of water. The velocity is then found by dividing flowrate by the cross sectional area of the pipe. The head loss is converted to feet of water by taking the differential manometer reading in inches times the quantity specific gravity of the fluid minus one and dividing by twelve. Friction factor and Reynolds number are then found. A typical sample calculation follows:

Test on 1 in. by 1 in. cross section

Manometer fluid monochlorobenzine, s.g. = 1.11

\( A = \text{Area} = 1 \text{ in.}^2 \) \( P = \text{Wetted Perimeter} = 4 \text{ in.} \)

\( D_e = 4 \frac{A}{P} = 4.4 = 1 \text{ in.} \) \( L = 6 \text{ ft.} \)

\( g = 32.2 \text{ ft./sec.}^2 \) \( \rho = 1.94 \text{ slug/ft.}^3 \) \( \mu = 20.98 \times 10^{-6} \text{ slug/ft.-sec.} \)

Manometer Reading = 7.4 in. Time = 53.6 sec. Weight = 100 lb.

\( Q = \frac{100 \text{ lb.}}{62.4 \text{ lb./ft.}^3 /53.6 \text{ sec.}} = .0299 \text{ ft.}^3/\text{sec.} \)
\[
V = \frac{0.0299 \text{ ft.}^3/\text{sec.}}{1/144 \text{ ft.}^2} = 4.31 \text{ ft./sec.}
\]

\[
R = \frac{\frac{\text{VD}^6}{4}}{\frac{f}{20.98 \times 10^{-6} \text{ slug/ft. sec.}}} = 4.31 \frac{\text{ft.}}{\text{sec.}} \times \frac{1}{12 \text{ ft.}} \times 1.94 \frac{\text{slug}}{3 \text{ ft.}} = 33100
\]

\[
h_f = \frac{7.4 \text{ in.}}{12 \text{ in./ft.}} (1.11 - 1.00) = 0.0677 \text{ ft. of H}_2\text{O}
\]

\[
f = \frac{h_f D}{V^2} \frac{2g}{L} = \frac{0.0677 \text{ ft.}}{1 \text{ in.}} \frac{1 \text{ in.}}{12 \text{ in./ft.}} \frac{6 \text{ ft.}}{\text{ft.}} \frac{2.32.2}{(4.31)^2} \frac{1}{\text{ft.}} = 0.0326
\]
V. DISCUSSION OF RESULTS

For the purpose of discussion, the data will be divided into three groups: laminar flow, turbulent flow and potential depth. The transition zone is not discussed as the equipment used was not sensitive enough to record a significant number of points in this region.

A. Turbulent Flow

The onset of fully turbulent flow is characterized by an abrupt change in slope on the logarithmic plot of friction factor versus Reynolds number. As is shown in Figure 4 the Reynolds number at which turbulent flow begins varies with each section. The 0.5 by 1.5 inch pipe and the 1.0 by 1.0 inch cross section becomes fully turbulent at a Reynolds number of approximately 6,500. The 1.5 by 1.5 inch section becomes fully turbulent at \( R = 10,000 \) and the 2.0 by 2.0 inch section doesn't become fully turbulent until \( R = 20,000 \). These values are rather high but plexiglass is smoother than pipe materials which are ordinarily assumed to be hydraulically smooth, such as brass. This would tend to make the onset of turbulent flow occur at a larger Reynolds number.

As was pointed out by Vennard the use of four times the hydraulic radius as the equivalent diameter can be expected to give satisfactory results for turbulent flow. The least squares curve fit of the form \( f = C R^p \) for turbulent flow of all square sections tested was \( f = .316/R^{.26} \). This compares closely to the generally accepted Blasius equation, \( f = .316/R^{.25} \). The least squares curve fit is plotted on Figure 4; the dotted lines above and below this line represent the standard deviation of the data. When the data for the 0.5 by 1.5 inch
cross section are included the least squares curve fit becomes \( f = 0.221/R^{0.23} \). As shown in Figure 8 the points with higher Reynolds number for this section correspond closely to the Blasius equation. It is the points in the lower turbulent region which introduce the error. Also, the difference in values computed by \( f = 0.316/R^{0.26} \) and \( f = 0.221/R^{0.23} \) is less than would be expected. This can be seen in Figure 9 which is a logarithmic plot of these two equations and the "Blasius equation" between \( R = 10^4 \) and \( R = 10^5 \).

B. Laminar Flow

The theoretical relationship between friction factor and Reynolds number in a square pipe undergoing laminar flow is \( f = 0.89(64)/R \). For a pipe of infinite width to depth ratio the relationship is \( f = 1.5(64)/R \). These relationships are plotted on Figure 4.

The 0.5 by 1.5 inch section, \( a/b = 3 \), should fall between these two lines. The calculated values for this section are approximately correct but tend to be greater than the theoretical values.

As can be seen by Figure 4 the friction factor in the laminar range appears to increase as the size of the section increases. Theoretically, however, the friction factor in the laminar range is dependent upon the shape of the section but not upon its size.

This discrepancy could be due to a number of reasons. As it was pointed out earlier turbulent flow begins at exceptionally high Reynolds numbers in the larger sections. It is then possible that these curves do not represent laminar flow.

It is also possible that these errors are due to imperfections in the testing apparatus. Laminar flow, for these sections, deals with very low flow rates, approximately 0.002 to 0.006 cubic feet per second, and
FIGURE 4. FRICTION FACTOR VERSUS REYNOLDS NUMBER
SQUARE CROSS SECTIONS
FIGURE 5. FRICTION FACTOR VERSUS REYNOLDS NUMBER

.1 IN. BY 1 IN. CROSS SECTION
FIGURE 6. FRICTION FACTOR VERSUS REYNOLDS NUMBER

$1\frac{1}{2}$ BY $1\frac{1}{2}$ IN. CROSS SECTION
Figure 7. Friction Factor Versus Reynolds Number

2 in. by 2 in. Cross Section

Reynolds Number = \( \frac{V D \rho}{\mu} \)
FIGURE 8. FRICTION FACTOR VERSUS REYNOLDS NUMBER

\[ \frac{1}{h} \text{ IN.} \times \frac{3}{4} \text{ IN. BY } 1 \frac{1}{2} \text{ IN. CROSS SECTION} \]
FIGURE 9. $f = \frac{.316}{R^{.26}}, f = \frac{.221}{R^{.23}}$ AND BLASIUS EQUATION

REYNOLDS NUMBER $= \frac{VD\rho}{\nu}$
FIGURE 10. FRICTION FACTOR VERSUS REYNOLDS NUMBER
1 IN. BY 1 IN. AND 1/2 IN. BY 1 1/2 IN. CROSS SECTIONS
POTENTIAL DEPTH USED IN PLACE OF HYDRAULIC RADIUS
leaks in the apparatus which would be insignificant for turbulent flow could cause significant error in laminar flow. Also large errors could be introduced by air in the manometer line or a small error in reading the manometer.

However, these errors would only tend to make the data erratic. It does not explain why the friction factor becomes consistently greater as the size of the section increases. It is also interesting to note that the smaller cross sections are in closer agreement with theory and that previous work in this area dealt with cross sections which were smaller than those tested here.

C. Potential Depth

As it was mentioned in the introduction to this investigation the concept of equivalent diameter as applied to pipes of non-circular cross section was to be discussed and a new criteria for finding equivalent diameter would be suggested. It was shown that Reynolds number could be represented by the dimensionless number \( \frac{VDf}{Af} \). As is shown by Muir\(^ 8 \), the length involved in this number represents the average distance from a friction surface. Consider the rectangular section shown in Figure 11a, with half width \( a \) and half depth \( b \). Assume that the bisectors of the
corner angles separate the flow into segments each of which is influenced only by the nearest wall in determining viscous resistance. The viscous shear depth for area ABEF is equal to b and acts over a length a-b. The average depth of BCE is b/2 and acts over a length equal to b. Similarly area CDE has an average depth of b/2 over a length of b. A weighted mean with respect to length yields

\[
\frac{b(a-b) + \frac{b}{2} b + \frac{b}{2} b}{a + b} = \frac{ab}{a + b}
\]

Note that due to symmetry it was necessary to consider only one fourth of the cross section's perimeter. The above quantity, \(\frac{ab}{a + b}\), is equivalent to the hydraulic radius which tends to validate the use of the hydraulic radius in determining Reynolds number.

According to the original assumption the point G of Figure 11a would have a viscous shear depth equal to d. However, it is the author's opinion that this point would have a viscous shear depth less than d since it would be influenced by wall CD as well as wall AC. It would seem logical that the depth distribution would be as shown in Figure 11b which is a cross section with contours of constant viscous shear depth. A distribution such as this follows the differential equation

\[
\frac{d^2(d)}{dx^2} + \frac{d^2(d)}{dy^2} = 0
\]

which is known as the Laplace equation.

![Figure 11b](image-url)
The Laplace equation appears in the theoretical derivation of many physical phenomena such as torsion on non-circular cross sections, ideal fluid flow, and steady state heat transfer. The finite difference operator for this equation is given by

\[ d_{i+1,j} + d_{i-1,j} + d_{i,j+1} + d_{i,j-1} - 4d_{i,j} = 0. \]

Therefore, if the section is broken into a grid system, a linear equation may be written for each nodal point and the simultaneous solution of these equations will give the effective depth of each nodal point. The accuracy of this solution depends upon the distance between nodal points. Figure 11c shows a rectangular section broken up into square grids of width \( \delta \). Due to symmetry it is necessary to use only one fourth of the entire section. The depths along the boundary of the grid system are assumed as shown in Figure 11c. This particular section would require the solution of twelve linear equations with twelve unknowns, i.e., an equation and unknown for each particular point. For instance the equation for point (1,1) would be \( d_{1,2} + 1/4 + d_{2,1} + 0 - 4d_{1,1} = 0 \) and the equation for point (2,3) would be \( d_{2,4} + d_{2,2} + d_{3,3} + d_{1,3} - 4d_{2,3} = 0 \). After these depths have been computed the mean depth can be determined by finding the weighted mean with respect to perimeter length as follows:
\[ d = \frac{1}{2} \phi(1) + \phi(1) + \phi(d_{3,2}) + \phi(d_{2,3}) + \phi(d_{1,4}) + \phi(0) + \phi(d_{1,4}) + \phi(d_{2,3}) + \phi(d_{3,2}) + \frac{1}{2} \phi(1) / \phi. \]

This gives a method for finding hydraulic mean depth for rectangular cross sections by a slightly different criteria. It is referred to here as potential depth, \( d_p \), because Laplacian distributions are sometimes known as "potential" functions. The following table compares \( d_p \) to \( R_h \) for various \( a/b \) ratios.

<table>
<thead>
<tr>
<th>a/b</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>( \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_h )</td>
<td>.50b</td>
<td>.60b</td>
<td>.67b</td>
<td>.75b</td>
<td>b</td>
</tr>
<tr>
<td>( d_p )</td>
<td>.38b</td>
<td>.50b</td>
<td>.58b</td>
<td>.69b</td>
<td>b</td>
</tr>
</tbody>
</table>

Figure 10 is similar to Figure 4 with the exception that \( 4 \, d_p \) is used in place of \( 4 \, R_h \) as the equivalent diameter. Note that this brings the 1 x 1 inch and the 0.5 x 1.5 inch cross sections much closer to their expected theoretical values. It does not, however, keep the friction factor from increasing with the size of the cross section.
VI. CONCLUSIONS

Results of this investigation lead to the following conclusions:

1. Size has no effect on the relationship between friction factor and Reynolds number for rectangular conduits undergoing turbulent flow.

2. The use of four times the area divided by the wetted perimeter as the diameter of an equivalent circular pipe is in error for laminar flow and a different length dimension is necessary to uniquely correlate pipes of non-circular cross section with those of circular cross section.

3. Because of this a new length dimension, potential depth, was used. This dimension was to represent the average viscous shear depth. The use of potential depth in place of hydraulic radius makes the data correspond more closely with theory but does not explain why the data indicates that friction factor increases with the size of the cross section.

4. The use of equivalent diameter gives satisfactory results for all sizes and shapes of smooth rectangular pipes undergoing turbulent flow.
VII. RECOMMENDATIONS

The author feels that the following related areas of study should be investigated:

1. An analytical investigation should be made to determine a length dimension to be used in Reynolds number which would uniquely correlate the relationship of friction factor to Reynolds number for all cross sectional shapes.

2. Tests should be conducted to determine if size has any effect on the relationship between friction factor and Reynolds number in the laminar region.

3. Tests should be conducted to determine if size has any effect on the upper limit of laminar flow and the lower limit of turbulent flow.

Tests of this nature should be made on an apparatus which has a number of improvements over the one used in this investigation. Longer upstream calming lengths and a longer distance over which head loss could be measured would be of benefit. Also a better manometer set up which could measure very small pressure differences over a wide range of Reynolds number and eliminate the possibility of leaks and air in the lines would be necessary. The apparatus should be designed so that dye could be injected in order to determine the upper limit of laminar flow and the lower limit of fully turbulent flow.

It is the author's opinion that only square sections would be necessary since the problem is one of size rather than shape. One very small section should be tested along with a number of larger sections. Descriptions of testing equipment such as that of Cornish would be helpful in designing such an apparatus.
BIBLIOGRAPHY


VITA

Bruce Harold Bradford was born September 1, 1943 in Kansas City, Missouri. He is the son of Harold E. and Elizabeth L. (Esry) Bradford.

He attended Blenheim Elementary School and Southeast High School in Kansas City, graduating in 1961.

In September, 1961, he enrolled at the Missouri School of Mines and Metallurgy (now University of Missouri at Rolla) and received a Bachelor of Science Degree in Civil Engineering.

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