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Analysis of Damping in Soil as Applied to Machine Foundations

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SYNOPSIS: The parameters needed for the design of machine foundations are usually evaluated from the model block resonance test conducted at a given site. Sometimes it may not be possible to attain complete resonance and the test results may be limited to the ascending part of the response. In this paper the possible effect of damping on the values of the design parameters evaluated from the ascending part of response is examined and the nature of damping which gives realistic values of design parameters is presented.

INTRODUCTION

One of the common methods of determining the in-situ dynamic properties of soil needed in the design of machine foundations is to conduct resonance test on model block constructed at a given site. Due to the limitation of instruments, the size of model block and the stiffness of soil at a given site, the test results may be limited to the ascending part of the response in the vertical mode of vibration. In such situation, IS : 5249 (1977) suggests a method of extrapolation based on the assumption that the response of the model block corresponds to that of viscously damped signle degree of freedom system. When this method of extrapolation is used on the test results reported by Prakash et al (1968) it gave very erratic results indicating the non-applicability of the assumptions made in the extrapolation and the need for re-examination.

EXTRAPOLATION OF THE TEST RESULTS

The response of soil foundation system in the vertical mode of vibration based on equivalent viscously damped single degree of freedom system is given by

$$A_z=(m_e e/m)\frac{r^2}{\sqrt{(1-r^2)^2+4D^2r^2}} \qquad (1)$$

where A_z = Amplitude in mm

m_e = Eccentric mass of the oscillator

e = Eccentricity of eccentric mass of oscillator in mm

r = f/f_n = Frequency ratio

D = Equivalent viscous damping ratio of the soil foundation system.

m = Equivalent mass of soil foundation system

f = Frequency of vibration in Hertz

f_n = Natural frequency of soil foundation system in Hertz

$m_e e/m$ = The amplitude of vibration of the system when the exciting frequency is infinity. Here afterwards this ratio is referred as amplitude at infinite frequency.

Rearrangement and simplification of equation (1) gives

$$Pf^4 + Qf^2A_z^2 - RA_z^2 = f^4A_z^2 \qquad (2)$$

where $P = (m_e e/m)^2$

$Q = 2f_n^2(1-2D^2)$

$R = f_n^4$

Equation (2) is a simultaneous equation in three unknowns P,Q and R which are constants and do not vary with frequency of oscillation and amplitude of oscillation for a given soil foundation system. To get the values of P,Q and R at least three pairs of amplitude frequency values obtained from test are necessary. When the test results are more than three pairs of amplitude frequency values, the least square fit method is employed to get unique values of P,Q and R.

Least Square Fit Method

Equation (2) can be written in the matrix form for the 'n' pairs of amplitude frequency values from the test as

$$[A]\{x\} = \{C\} \qquad (3)$$

Where [A] = Matrix with known elements with 'n' rows and 3 columns.

$\{x\}$ = Column vector containing unknown elements P,Q and R

$\{c\}$ = Column vector containing known elements with 'n' rows

For any given solution, equation (3) gives an error vector $\{e\}$ which can be written as

$$[A]\{x\} - \{c\} = \{e\} \quad (4)$$

In the least square fit method, a unique solution is obtained by minimizing the square of the error vector. Accordingly squaring both sides of equation (4) and minimizing error vector gives

$$[A]^T [A] \{x\} = [A]^T \{c\} \quad (5)$$

Solution of equation (5) gives unique values of the three unknowns P,Q and R.

TEST RESULTS ANALYSED

Prakash et al (1968) have reported the results of the vertical resonance test conducted on model block for forge hammer foundations of Hindustan Aeronautics Limited (H.A.L.), Koraput in which the test results were limited to the ascending part of the response curve as shown in figure 1. To assess the suitability of the extrapolation method suggested in IS: 5249 (1977) the test results reported by Prakash et al (1968) are analysed using equation (5) for each of the blocks 1,2,3,4 and 5. The values of P,Q and R obtained and the dynamic parameters of the system such as natural frequency, coefficient of elastic uniform compression, damping ratio and the amplitude at infinite frequency are tabulated in table 1. From this table it

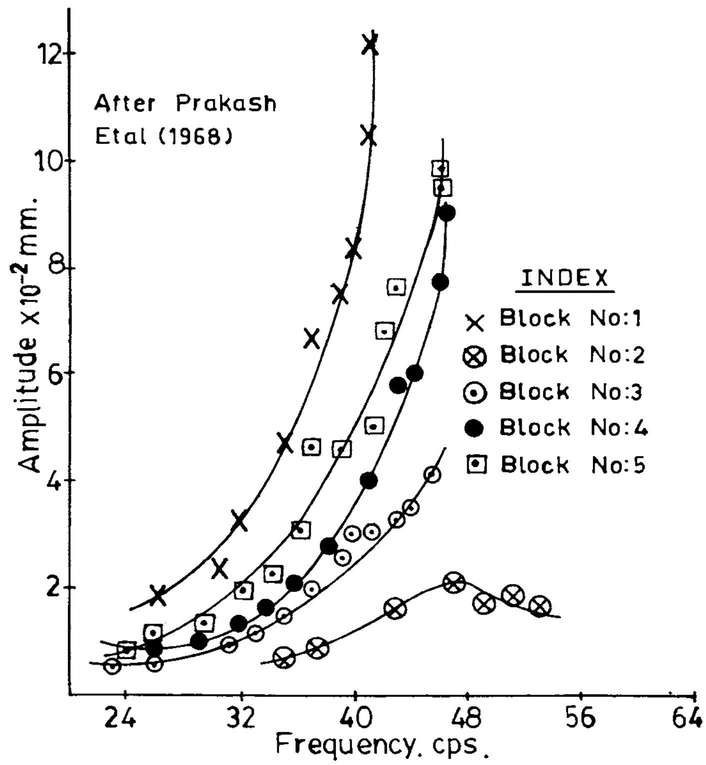

Fig.1 Amplitude Frequency Response - HAL, Koraput

TABLE 1: Results of analysis for experimental data of Prakash et al (1968)

Block No	P	Q	R	Natural frequency	Coefficient of elastic uniform compression	Damping Ratio	Amplitude at infinite frequency
	$(10^{-5}mm^2)$	(Hz^2)	(Hz^4)	(Hz)	(Kg/cm^3)	(%)	$(10^{-2}mm)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	-5.5800	2972.01	2176802	38.41	9.91	6.01	7.402*
2	2.4221	4073.02	4400143	45.80	14.18	12.07	4.921
3	-6.8820	3635.96	34466405	43.14	12.15	10.85	8.290*
4	-13.7261	4165.28	4425388	45.86	14.21	6.90	11.750*
5	25.5773	3914.15	3914137	44.55	13.42	8.45	16.000

*Imaginary

can be observed that the constant P is negative for blocks 1,3 and 4 as given in column 2. Since the amplitude at infinite frequency is the square root of P, the values of this amplitude becomes imaginary, thus indicating the inadequacy of the analysis. Treating the imaginary value of the amplitude at infinite frequency as real, the theoretical response obtained for block 1 from equation (1) is shown in figure 2. Similar results are obtained for other block and reported by Nagakumar (1988)

To study the problem in more detail, the tes results reported by Prakash and Basavanna (1968 were analysed. They have reported the results of vertical resonance test conducted on model block constructed at sites A, B and G of the proposed Aeroengine test beds at Aero Engine Factory (AEF), Chandigarh. At all these sites, test results are on both ascending and descend ing parts of the resonance as shown in figure 3. Using these test results the values of P,Q,R and the dynamic parameters of the system are

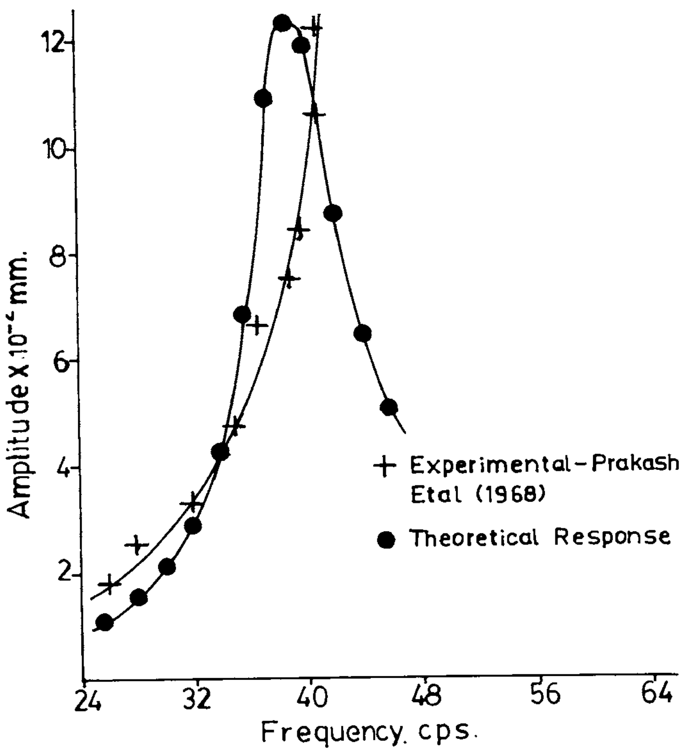

Fig.2 Amplitude Frequency Response - Block 1 HAL, Koraput

obtained and tabulated in Table 2 for all the sites. These results hereafterwards are referred as for full experimental data. The response obtained by substituting the values of the dynamic parameters in equation (1) for the block at Site A is shown in figure 4 and marked as theoretical response (full experimental data). Similar results are obtained for blocks at other sites and are reported by Nagakumar (1988).

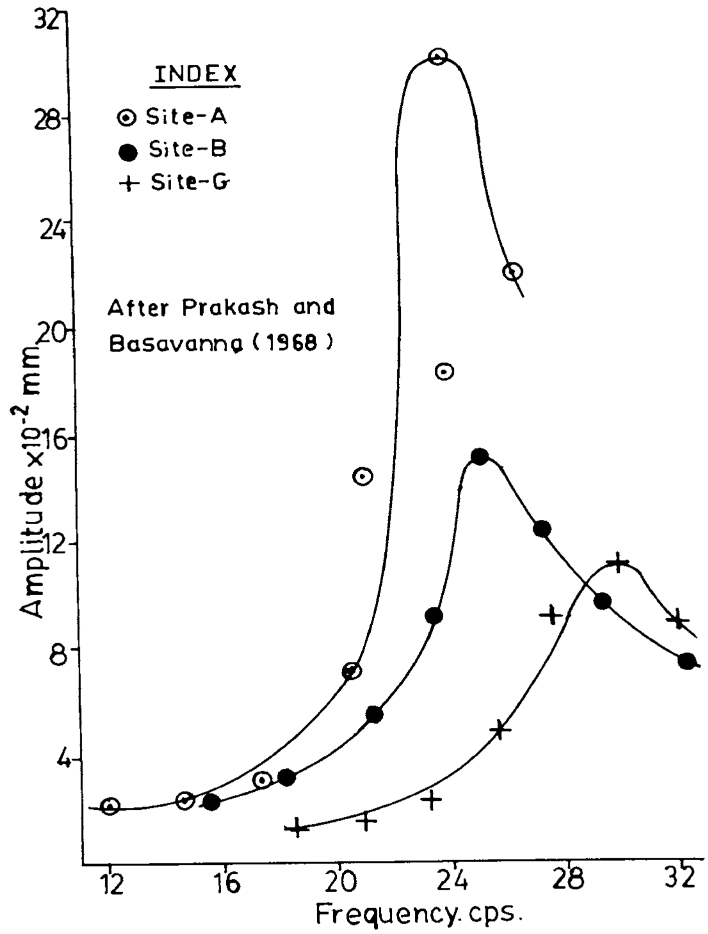

Fig.3 Amplitude Frequency Response - A.E.F. Chandigarh

obtained by substituting the dynamic parameters in equation (1) for block at Site A is shown

TABLE 2: Results of Analysis for Full Experimental Data of Prakash and Basavanna (1968)

Site	P	Q	R	Natural frequency	Coefficient of elastic uniform compression	Damping ratio	Amplitude at infinite frequency
	($10^{-5}mm^2$)	(Hz^2)	(Hz^4)	(Hz)	(Kg/cm^3)	(%)	($10^{-2}mm$)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A	14.0224	1190.143	358633	24.47	4.048	5.56	3.74
B	8.3249	1287.701	432015	25.63	4.440	10.10	2.88
G	3.2961	1713.523	751515	29.44	5.859	7.57	1.81

The constants P,Q and R and dynamic parameters of the system are also evaluated using the experimental data corresponding to ascending part only and are tabulated in table 3. These results hereafterwards are referred to as for ascending part of experimental data. Treating the imaginary value of the amplitude at infinite frequency as real, the theoretical response in figure 4 and marked as theoretical response (ascending part of experimental data only).

DISCUSSION ON TEST RESULTS ANALYSED

The values of constant P given in column 2

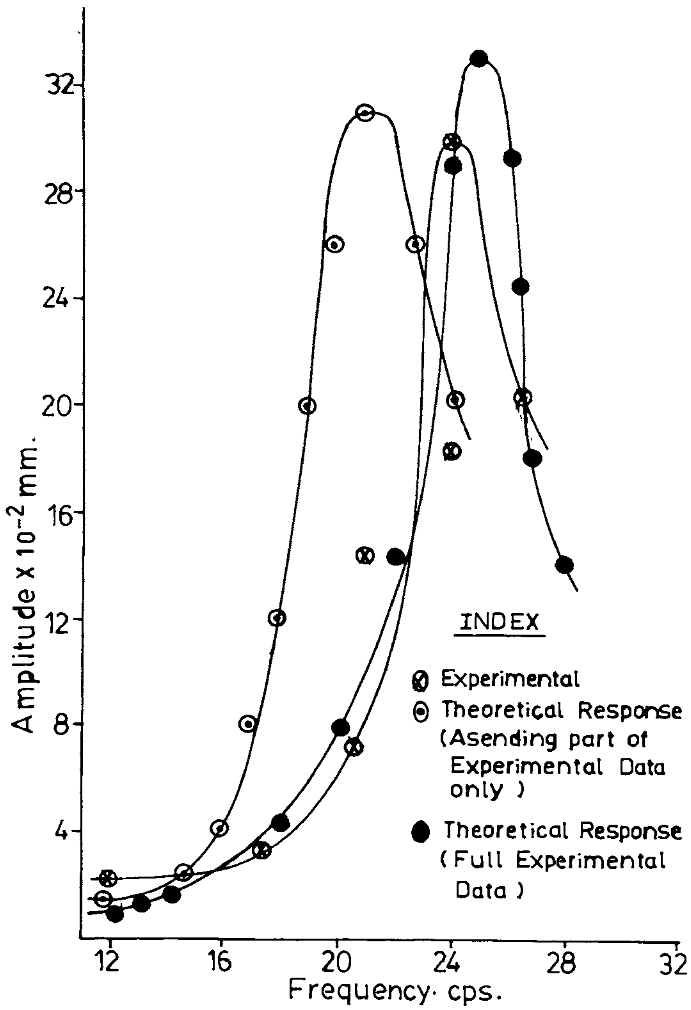

Fig.4 Amplitude Frequency Response - Site A A.E.F., Chandigarh

<u>Natural Frequency</u>

Comparison of the values of the natural frequency tabulated in tables 2 and 3 reveals tha these values obtained for full experimenta data are higher than those obtained for ascending part of experimental data. The percentag error between the values of the natural frequency for full experimental data and those obtained for ascending part of experimental data i. 13%, 6.5% and 25% respectively for sites A B and G.

<u>Coefficient of Elastic Uniform Compression</u>

From tables 2 and 3 it can be observed that the values of coefficient of elastic uniform compression for full experimental data are higher than those obtained for ascending part of experimental data. The percentage error in the determination of the coefficient of elastic uniform compression is 24.75, 12.61 and 43 respectively for sites A, B and G wher it is determined from the ascending part of experimental data.

<u>Damping Ratio</u>

The damping ratio obtained from full experimental data are higher than that obtained from ascending part of experimental data for sites A and B as revealed in column 7 of tables 2 and 3. However the damping ratio for site G is lower from full experimental response compared to that from ascending part of experimental response. Moreover for Site G the value of damping ratio from ascending part of experimental response is imaginary. The determination of damping ratio is more erratic compared to the determination of coefficient of elastic uniform compression and natural frequency.

<u>Theoretical Response</u>

Observation of figure 4 shows that the theoretical response obtained from ascending part of experimental data deviates too much from the experimental response, whereas the theoretical response obtained from full experimental data is very close to the experimental response.

From the above discussion it is clear that extrapolation of ascending part of experimental response based on the assumption that the response of the model block corresponds to that of viscously damped single degree of freedom

TABLE 3 : Results of Analysis for Experimental Data of Prakash and Basavanna (1968) when only the ascending part is used.

Site	P ($10^{-5}mm^2$)	Q (Hz^2)	R (Hz^4)	Natural frequency (Hz)	Coefficient of elastic uniform compression (Kg/cm^3)	Damping ratio (%)	Amplitude at infinite frequency ($10^{-2}mm$)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A	-2.5816	900.356	202416	21.27	3.04	4.969	1.606*
B	-6.2427	1139.552	329840	23.96	3.88	6.123	2.498*
G	-1.1709	1057.940	244578	22.23	3.34	10.763*	1.082*

*Imaginary

of table 3 for ascending part of experimental response shows that these values are negative and hence the amplitude at infinite frequency becomes imaginary, whereas for full experimental response, the values of constant P are positive and give real values of amplitude at infinite frequency.

ystem gives erroneous in-situ dynamic parameters.

'he parameters involved in the response of lock foundation resting on elastic halfspace ased on equivalent single degree of freedom ystem are mass 'm', siffness 'K' and damping C' of the foundation system. The nature of ariation of these parameter will influence he response of the system. Hence, to obtain .cceptable extrapolation of ascending part of experimental response, it is obvious to re-examine these parameters.

NATURE OF DESIGN PARAMETERS

According to Hsieh (1962) and Lysmer (1965) the equation of motion for the response of circular footing resting on elastic half-space, s

$$m\ddot{Z} + C_z\dot{Z} + K_z Z = m_e e\, w^2 e^{iwt} \qquad (6)$$

Where C_z = Equivalent damping constant

$$= \frac{4GR_oF_2}{w(1-\nu)\,(F_1^2 + F_2^2)} \qquad (7)$$

K_z = Equivalent spring stiffness

$$= \frac{4GR_oF_1}{(1-\nu)\,(F_1^2 + F_2^2)} \qquad (8)$$

m = Mass of the foundation and machine only

F_1, F_2 = Displacement functions which are functions of frequency ratio a_o

a_o = Frequency ratio

$= R_o w/V_s$

ν = Poisson's ratio of soil

R_o = Radius of circular footing

w = Circular frequency in radians per second

G = Modulus of rigidity of soil

V_s = Shear wave velocity of soil at site

$Z, \dot{Z}, \ddot{Z}$ = Vertical displacement, velocity and acceleration respectively of the footing.

Equation (6) is similar to equation of motion of viscously damped single degree of freedom system except that the stiffness K_z and damping C_z are functions of displacement functions which are functions of frequency of vibration. Therefore the nature of variation of spring stiffness K_z and damping C_z with frequency are discussed below.

Equivalent Spring Stiffness

After studying the variation of equivalent spring stiffness K_z with frequency, Lysmer (1965) suggested a constant value K which is equal to the spring stiffness of soil at zero frequency. Trial computation of K_z upto a_o less than or equal to 1.5 by the authors indicated that the above statement of Lysmer on K_z is approximately correct. Therefore a constant value of spring stiffness can be used in the analysis.

Equivalent Damping

By best fit of response, Lysmer (1965) has suggested a constant equivalent damping constant for the frequency ratio 'a_o' less than 1.0. Trial computation of the value of C_z given by equation (7) indicated that the value of C_z cannot be constant in the frequency ratio less than 1.5. Hence the use of constant equivalent viscous damping for response of soil foundation system is not justified and its variation with frequency is to be taken into account in any realistic analysis.

NATURE OF DAMPING

From equations (7) and (8) and from definition of damping ratio, the damping ratio D_z can be expressed as

$$D_z = w_n F_2/(2wF_1) \qquad (9)$$

Where w_n = Natural frequency in radians per sec.

Equation (9) shows that damping ratio is directly proportional to the ratio of displacement functions, as well as inversely proportional to frequency.

Variation of Ratio of Displacement Functions

The variation of displacement functions ratio with frequency ratio a_o is computed by the authors using the values of F_2 and F_1 given by Richart et al (1970) and shown in figure (5) by circles. Examination of these points indicates that the variation of (F_2/F_1) can be approximately represented by

$$(F_2/F_1) = Na_o^2 \qquad (10)$$

Where N is a constant and is approximately equal to 1.36. The variation of (F_2/F_1) given by the above equation (10) is shown by solid line in figure 5 and is very close to values of (F_2/F_1) computed from the values of F_1 and F_2 given by Richart et al (1970). Therfore equations (9) and (10) are used to study the variation of damping ratio with frequency.

Variation of Damping Ratio with Frequency

Substitution of equation (10) in equation (9) and simplification gives

$$D_z = D\, r \qquad (11)$$

Where D = a constant and is equal to damping ratio at frequency ratio equal to 1.0.

Equation (11) shows that the damping ratio

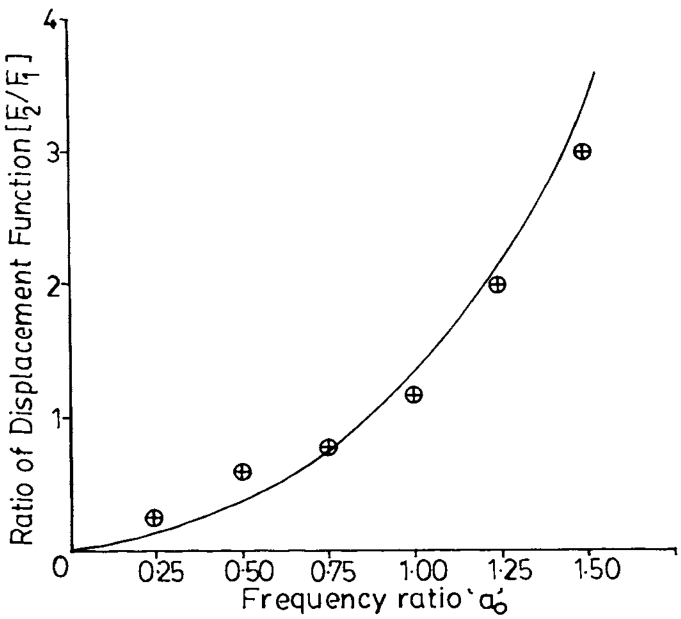

Fig. 5 Variation of Ratio of Displacement Functions

EFFECT OF NATURE OF DAMPING ON EXTRAPOLATIC OF ASCENDING PART OF EXPERIMENTAL RESPONSE

When damping ratio linearly increases wit frequency ratio the response of equivalen single degree freedom system with damping i given by

$$A_z = (m_e e/m)\frac{r^2}{\sqrt{(1-r^2)^2 + 4D^2r^4}} \quad (12$$

Rearrangement and simplification of equations (12) gives

$$Hw^4 - SA_z^2\, w^4 + TA_z^2\, w^2 = A_z^2 \quad (13$$

Where $H = (M_e e/mw_n^2)^2$

$T = (2/w_n^2)$

$S = (1 + 4D^2)/w_n^4$

The constants H,S, and T are obtained by solvin equation (13) by least square fit method fo the test results reported by Prakash an Basavanna (1968) and are given in columns 2, and 4 in table 4 and in table 5 when only th ascending part of experimental data are use in the analysis.

As can be seen from tables 4 and 5 the constant

TABLE 4: Results of Analysis For Full Experimental Data of Prakash and Basavanna (1968)
(Damping ratio varies linearly with frequency ratio)

Site	$H \times 10^{-13}$ $(mm^2\text{-}Sec^4)$	$S \times 10^{-10}$ (Sec^4)	$T \times 10^{-5}$ (Sec^2)	Natural frequency (Hz)	Coefficient of elastic uniform compression (Kg/cm^3)	Damping ratio (%)	Amplitude at infinite frequency $(10^{-2}mm)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A	23.960	17.62	8.347	24.63	4.101	5.475	3.709
B	12.655	14.72	7.520	25.95	4.552	10.154	2.919
G	2.806	8.52	5.770	29.63	5.935	7.717	1.836

TABLE 5: Results of Analysis For Ascending Part of Experimental Data of Prakash and Basavanna (1968) (Damping ratio varies linearly with frequency ratio)

Site	$H \times 10^{-13}$ $(mm^2\text{-}Sec^4)$	$S \times 10^{-10}$ (Sec^4)	$T \times 10^{-5}$ (Sec^2)	Natural frequency (Hz)	Coefficient of elastic uniform compression (Kg/cm^3)	Damping ratio (%)	Amplitude at infinite frequency $(10^{-2}mm)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A	18.596	19.44	8.773	24.03	3.903	5.099	3.108
B	9.451	16.77	8.056	25.07	4.249	8.974	2.142
G	1.781	10.27	6.234	28.50	5.491	11.816	1.353

is a linear function of frequency ratio 'r' and not a constant.

H,S and T are positive both when full experimental data and when only ascending part experimental data are used in the analysis. Using these

·onstants, the dynamic design parameters are btained and tabulated in columns 5,6,7 and of tables 4 and 5. The influence of variation ·f damping on the design parameters is discussed ·elow.

Natural Frequency

Comparison of the values of the natural frequency tabulated in column 5 of the tables 4 and 5 reveals that the natural frequency obtained using full experimental response are slightly higher than that obtained using only the ascending part of the experimental response. The percentage error between the natural frequency obtained from full experimental response and that obtained from ascending part is 2.43, 3.39 and 3.81 respectively for sites A, B and G.

Coefficient of Elastic Uniform Compression

From tables 4 and 5 it can be observed that the values of coefficient of elastic uniform compression obtained from full experimental response is slightly higher than that obtained from ascending part. The percentage error in determination of the coefficient of elastic uniform compression is 4.82, 6.65 and 7.48 percent respectively for sites A,B and G when ascending part of experimental response is used.

Damping Ratio

The damping ratio obtained using full experimental response are higher than that obtained using only the ascending part of the experimental response as revealed in column 7 of tables 4 and 5. However damping ratio for site G is lower when it is determined from ascending part of experimental response.

Theoretical Response

Figure 6 shows the experimental response as obtained by Prakash and Basavanna (1968) for Site A. In this figure, the theoretical response obtained from full experimental data as well as that obtained from the ascending part of the experimental data are shown for comparison. In the computation of theoretical response from the experimental data, the damping ratio is considered to vary linearly with frequency ratio. Similar results are obtained for other sites and are reported by Nagakumar (1988). Observation of these figures show that the theoretical response obtained using only the ascending part of experimental data and also that obtained using full experimental data are close to the experimental response, when the damping varies linearly with frequency.

From the above discussion on the natural frequency, coefficient of elastic uniform compression, damping ratio and theoretical response, it can be considered that at sites where the full experimental response is not obtained from the test and only the ascending part is obtained, the in-situ dynamic properties can be estimated with reasonable accuracy by least square fit method, when the damping linearly varies with frequency.

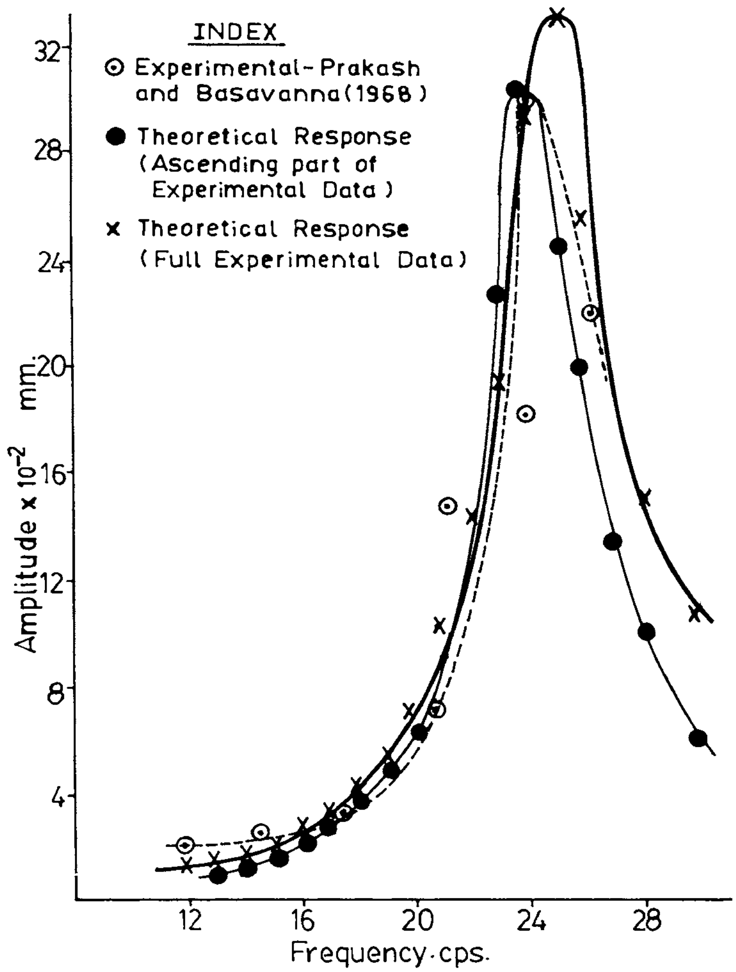

Fig.6 Amplitude Frequency Response When Damping Varies Linearly with Frequency (Site A - A.E.F., Chandigarh)

CONCLUSIONS

Based on the analysis presented here the following conclusions can be drawn:

1. The extrapolation method suggested in IS : 5249 (1977) for the determination of in-situe dynamic properties of soil when the model block test results are limited to the ascending part of the response sometimes give imaginary values of these properties.

2. The damping in soil is not constant but varies approximately linearly with frequency

3. In-situ dynamic properties of soil determined from the ascending part of the experimental response are close to actual values when damping is considered to be linearly varying with frequency.

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