Seismic Response Analysis of Pile-Supported Structure: Assessment of Commonly Used Approximations

Toyoaki Nogami
University of California at San Diego, La Jolla, California

Reed L. Mosher
U.S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi

H. Wayne Jones
U.S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi

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SYNOPSIS: The seismic response of a pile-supported structure is formulated by the approach developed by the first author. Using this formulation, some of the crude approximations frequently used in the seismic response analysis of a soil-pile-structure system are examined. Those involved in the analysis procedure are assessed under the linear elastic condition. A commonly used nonlinear soil model for the dynamic pile response analysis is also assessed. It is found that those approximations routinely used in the analysis procedure and numerical modelling can cause significant errors in the computed response of a pile-supported structure.

INTRODUCTION

A structure is frequently supported by a pile foundation. When this structure is analyzed for its seismic responses, the pile foundation must be properly taken into account in the analysis. Seismic responses of pile foundations are complex and their analyses generally require a large amount of computations, particularly when pile groups and nonlinear soil behavior are considered. Thus, various crude approximations are used in the analysis. This paper investigates errors caused by some of those approximations frequently used in the seismic response analysis of pile-supported structures.

Novak (1975) has developed an approach to analyze the dynamic response of linear elastic single pile foundations within the frame of the Winkler's hypothesis. Nogami and his colleagues have extended this approach for nonlinear pile foundations and pile groups, including 1) linear elastic pile foundations in the frequency-domain analysis (Nogami, 1980; Nogami, 1983; Nogami, 1985), 2) nonlinear pile foundations in the frequency-domain analysis (Nogami and Chen, 1987a), 3) linear elastic pile foundations in the time-domain analysis (Konagai and Nogami, 1987; Nogami and Konagai, 1986; Nogami and Konagai, 1988a) and 4) nonlinear pile foundations in the time-domain analysis (Nogami and Konagai, 1987b; Nogami et al., 1988b; Nogami et al., 1991). This approach has been verified by various people (e.g. Sanchez-Salinero, 1983; Nogami, 1983; Roesset, 1984) and is used herein for the assessment of the approximations frequently used in the seismic response analysis of pile-supported structures.

SOIL MODEL AND FORMULATION OF SEISMIC RESPONSE OF PILE-SUPPORTED STRUCTURES

Vertical pile groups are considered herein as a general case. A horizontal slice of a soil-pile system with a unit thickness is idealized as shown in Fig. 1, in which each one of pile shafts is enclosed by a rigid ring and mechanical systems are located outside and inside of the ring. The soil-pile interaction produces the nonlinearity in the inside mechanical system only, whereas the free-field nonlinearity affects both the outside and inside mechanical systems. When a linear elastic soil medium is considered, the radius of the rigid ring is set equal to the radius of the pile; i.e. no inside mechanical soil model. Details of those mechanical systems developed in both the frequency-domain and time-domain, can be found in the papers published by the first author. According to the model shown, each of the vertical and horizontal displacements are expressed at the pile shafts as

$$\mathbf{u} - u_0\mathbf{1} = \mathbf{fp} \tag{1}$$

where $\mathbf{u}$ = vector containing displacements at the piles; u_0 = free-field displacement; $\mathbf{1}$ = unit vector; $\mathbf{f}$ = flexibility matrix of the system (soil); and $\mathbf{p}$ = vector containing soil-pile interaction forces. The nonlinearity caused by the soil-pile interaction affects only the diagonal terms of the matrix $\mathbf{f}$.

Eq. 1 is coupled with the equations of motion of pile shafts to formulate the dynamic response of pile foundations. Those equations are typically formulated with a lumped mass pile model or a continuous beam pile model. When a continuous

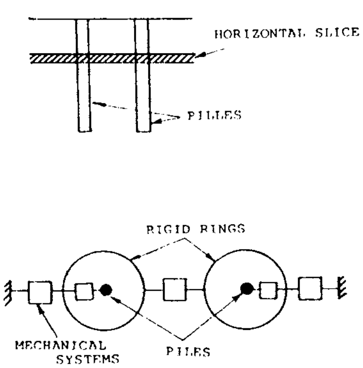

Fig. 1 Horizontal Slice in Soil-Pile System and Plan View of Soil Model

beam model is used, the flexural and axial responses of pile shafts are described by, respectively,

$$\mathbf{EI}\ \frac{\partial^4}{\partial z^4}\mathbf{u} + \mathbf{m}\ \frac{\partial^2}{\partial t^2}\mathbf{u} = -\mathbf{p}$$

$$-\mathbf{EA}\ \frac{\partial^2}{\partial z^2}\mathbf{u} + \mathbf{m}\ \frac{\partial^2}{\partial t^2}\mathbf{u} = -\mathbf{p} \tag{2}$$

where: **EI** = diagonal matrix containing flexural rigidities of piles; **EA** = diagonal matrix containing axial rigidities of piles; and **m** = diagonal matrix containing masses of piles. Writing Eq. 1 in the either time-domain or frequency-domain, Eq. 2 can be solved in one of those domains.

With the steady state response to the harmonic horizontal bedrock motion, the lateral soil response is expressed as.

$$u_0(z) = (a\ e^{i\gamma z} + b\ e^{-i\gamma z})\ u_0(H) \tag{3}$$

where a and b = constants determined by the boundary conditions of the free-field soil; $u_0(H)$ = bedrock displacement; and $\gamma = \omega/v_s$ with ω = circular frequency and v_s = shear wave velocity of soil

Substituting Eq. 3 into Eq. 1, the expressions of the lateral and axial pile responses in the frequency-domain can be obtained from Eqs. 1 and 2 as, respectively,

$$\mathbf{u}(z) = \sum_{n=1}^{N}\left(A_n e^{\lambda_n z} + B_n e^{-\lambda_n z} + C_n e^{i\lambda_n z} + D_n e^{-i\lambda_n z}\right)\eta_n + u_0(H)\left(\alpha e^{i\gamma z} + \beta e^{-i\gamma z}\right)\mathbf{1}$$

$$\mathbf{u}(z) = \sum_{n=1}^{N}\left(A_n e^{i\lambda_n z} + B_n e^{-i\lambda_n z}\right)\eta_n \tag{4}$$

where; N = number of piles; A_n, B_n, C_n and D_n = constants determined by the boundary conditions of the piles; λ_n and η_n = n-the eigenvalue and eigenvector obtained from $(\lambda^4\mathbf{EI}+\mathbf{k}-\omega^2\mathbf{m})\eta = \mathbf{0}$ for the lateral response and $(\lambda^2\mathbf{EA}+\mathbf{k}-\omega^2\mathbf{m})\eta = \mathbf{0}$ for axial response, with $\mathbf{k} = \mathbf{f}^{-1}$; $\alpha = a(\gamma^4\mathbf{EI}+\mathbf{k}-\omega^2\mathbf{m})^{-1}\mathbf{k}$; and $\beta = b(\gamma^4\mathbf{EI}+\mathbf{k}-\omega^2\mathbf{m})^{-1}\mathbf{k}$. Then, all other responses associated with the flexural and axial pile response are expressed using Eq. 4 as, respectively,

$$\left(\phi(z), \mathbf{EI}^{-1}\mathbf{P}(z), \mathbf{EI}^{-1}\mathbf{M}(z)\right) = \left(\frac{d}{dz}\mathbf{u}(z), \frac{d^3}{dz^3}\mathbf{u}(z), \frac{-d^2}{dz^2}\mathbf{u}(z)\right)$$

$$\mathbf{EA}^{-1}\mathbf{P}(z) = \frac{d}{dz}\ \mathbf{u}(z) \tag{5}$$

After determining the unknown constants in Eq. 4 for the boundary conditions of the piles, the force and displacement responses of the piles are completely described by Eqs. 4 and 5. With those expressions, the force-displacement relationship of a pile group attached to a rigid cap can be written in the form of

$$\mathbf{P} = \mathbf{K_f U} + \Delta\mathbf{P} \tag{6}$$

where **U** = vector containing the lateral displacement (U) and rotational displacement (Φ) of the rigid cap; **P** = vector containing the lateral force (P) and moment (M) applied at the rigid cap by the super-structure motions; $\mathbf{K_f}$ = stiffness matrix of the pile group attached to a rigid cap; and $\Delta\mathbf{P}$ = force produced at the cap by the free-field soil motion, containing α and β.

For simplicity, the super-structure is considered to be a mass attached to the top of the pile cap. Combining the structure and Eq.4, the equation of motion of the pile-supported super-structure subjected to the seismic excitation is written as

$$\mathbf{M_s}\ddot{\mathbf{U}} + \mathbf{K_f U} + \Delta\mathbf{P} = \mathbf{0} \tag{7}$$

where $\mathbf{M_s}$ = mass of super-structure. Eq.7 can be split into

$$\mathbf{K_f U_1} + \Delta\mathbf{P} = \mathbf{0}$$

$$\mathbf{M_s}\ddot{\mathbf{U}}_2 + \mathbf{K_f U_2} = -\mathbf{M_s}\ddot{\mathbf{U}}_1 \tag{8}$$

where $\mathbf{U} = \mathbf{U}_1 + \mathbf{U}_2$. Eq. 8 is interpreted such that the motions, $\mathbf{U}_1$, are transmitted to excite the super-structure and generate the feedback motions, $\mathbf{U}_2$, (second equation) and that the transmitted motions, $\mathbf{U}_1$, are the seismic responses of pile foundation at the cap without any super-structure according to the first equation. This is illustrated in Fig. 2.

APPROXIMATE SEISMIC RESPONSE ANALYSIS OF PILE-SUPPORTED STRUCTURE

Approximate Methods Various approximations are adopted in the seismic response analysis of pile-supported structures. Those often used are 1) no pile-soil-pile interaction for a pile group, 2) frequency independent spring and dashpot for the Winkler subgrade model and 3) analysis by applying the free-field ground

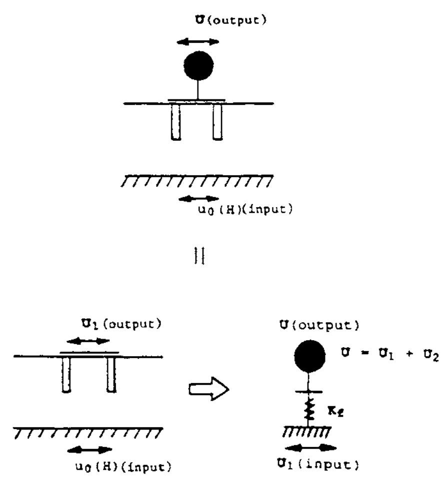

Fig. 2 Transmitted and Feedback Motions in Seismic Response of Pile-Supported Structure

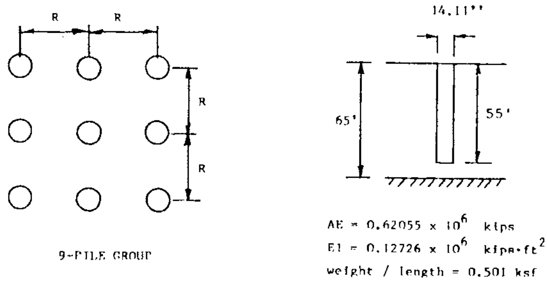

Fig. 3 Pile Foundations Considered

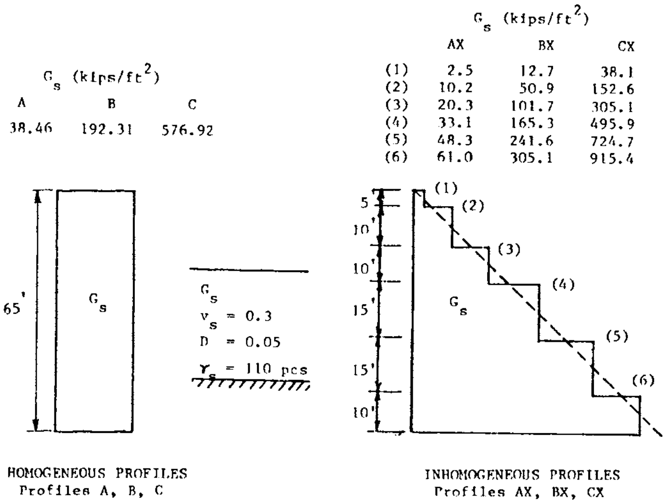

Fig. 4 Soil Profiles Considered

surface motion at the base of the pile supported structure, in which the third approximation corresponds to the use of the free-field ground surface motion for U_1 in the second equation in Eq. 8.

The soil model presented herein can reproduce the dynamic responses very well, if the mechanical systems in the soil model are defined from the dynamic response behavior of multiple infinitely long rigid massless cylinders, vertically inserted in an infinite medium (Konagai and Nogami, 1987; Nogami, 1983: Nogami et al., 1986, 1987a, 1987b, 1988a, 1988b, 1991). Thus, defining the soil model by this approach, the afore-given formulation is used to examine the effects of the above listed approximations and is referred herein as "relatively rigorous method".

The above listed first approximation is introduced by defining the model parameters for a single cylinder in the medium rather than a group of cylinders. The second is introduced by using the soil model parameters defined at $\omega = 0.02\ v_s/r_0$ for all frequencies, in addition to the conditions used in the first approximation: r_0 = radius of pile and v_s = shear wave velocity of the medium.

<u>Conditions Considered</u> Pile foundations considered include those made of a single pile and 3x3 piles attached to a rigid cap (Fig. 3). Homogeneous soil profiles (A,B and C) and inhomogeneous soil profiles (AX, BX, and CX) as shown in Fig. 4 are considered. Assuming the linearly increasing shear modulus with depth, inhomogeneous profiles are defined such that their fundamental natural frequencies are identical to those of the homogeneous profiles. Fig. 5 shows the normalized acceleration response of the free-field ground surface to harmonic horizontal bedrock motions.

A single mass, effective only to the lateral translational motion, is considered for the super-structure. The masses supported by single piles are M_s = 5 kips.sec^2/ft for piles in homogeneous profiles and M_s = 1.292 kips.sec^2/ft^2 for piles in inhomogeneous profiles. Those M_s result in an identical $K_{xx}(\omega = 0.02 v_s/r_0)/M_s$ ratio between profile B and profile BX. The masses supported by 9-pile groups are nine times of those of the structures supported by single piles, and thus are respectively M_s = 45 kips.sec^2/ft and 11.628 kips.sec^2/ft for profile B and profile BX.

Both harmonic and random motions are considered for input horizontal bedrock motions. The time history of the random bedrock motion is shown in Fig. 6.

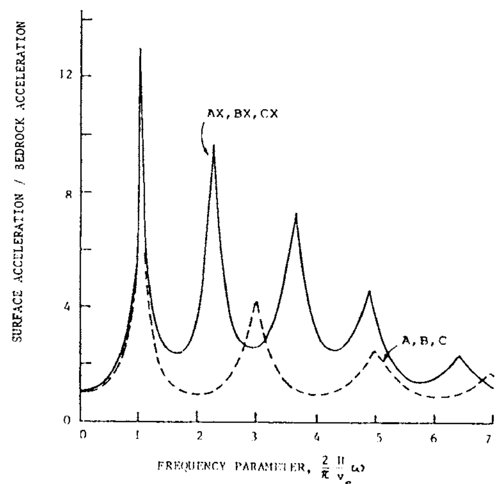

Fig. 5 Free-Field Ground Surface Response to Harmonic Bedrock Motions

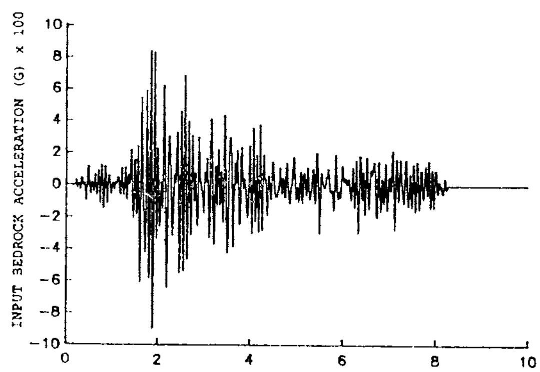

Fig. 6 Acceleration Bedrock Motion Time History

Computed Results As the second equation in Eq. 8 indicates, seismic responses of pile-supported structures are governed by the transmitted motions ($\mathbf{U}_1$) and impedance of the pile foundation ($\mathbf{K_f}$), all which are affected by the pile foundation in general. $\mathbf{K_f}$ and $\mathbf{U}_1$ are computed for the single piles by the relatively rigorous method and the computed results are shown in Figs. 7 and 8. Transmitted motions in the figure are normalized by multiplying the lateral motion by $1/u_0(0)$ and the rocking motion by $L/u_0(0)$, in which $u_0(0)$ = free-field lateral motion at the ground surface and L = pile length. Nondimensional stiffness parameters shown in Fig. 8 are obtained by multiplying the real part of the impedances K_j by $1/(E_sL)$, $1/(E_sL^2)$, and $1/(E_sL^3)$ respectively for j = xx, xϕ and $\phi\phi$, in which E_s = Young's modulus of soil and

$$\mathbf{K_f} = \begin{bmatrix} K_{xx} & K_{x\phi} \\ K_{x\phi} & K_{\phi\phi} \end{bmatrix} \tag{9}$$

Nondimensional damping parameters are obtained by multiplying the imaginary parts of K_j by $1/(E_sL)/a_0$, $1/(E_sL^2)/a_0$ and $1/(E_sL^3)/a_0$ respectively for j = xx, xϕ and $\phi\phi$, in which $a_0 = \omega r_0/v_s$. The transmitted motions contain not only the lateral motion but also rotational motion, as seen in the figure. The lateral transmitted motions tend to increase a little bit first and then decrease with frequency to become smaller than the free-field ground surface motion. The curves shift to the left with increasing the soil softness, particularly for inhomogeneous than homogeneous profiles. The stiffness parameters of single-pile foundations vary very little with frequency but the damping parameters rapidly decrease with frequency at low frequencies. Transmitted motions and impedance functions for 9-pile groups in profiles B and BX are shown in Figs. 9 and 10. Similar trends as those observed for single-pile foundations can be seen in the transmitted motions but the rotational ones are much smaller than those for the single-pile foundations. The pile-soil-pile interaction effects in the transmitted motions are negligibly small at very low frequencies and become more pronounced at higher frequencies. The pile-soil-pile interaction affects far more significantly the stiffness and damping parameters than the transmitted motions.

Seismic responses of the structures supported by single piles are computed for harmonic bedrock motions by the relatively rigorous method and the approximate method. The approximate method adopts crude approximations including frequency independent soil model and use of the free-field ground surface motion as input $\mathbf{U}_1$ motion. The computed results for profiles B and BX are shown in Fig. 11. Those approximations appear to be acceptable for the piles in homogeneous soil but overestimate the response for the piles in inhomogeneous soil. Similarly, seismic responses of the structures supported by 9-pile groups are computed for harmonic bedrock motions. Two different approximate methods are used in this case. In the first approximate method, the frequency independent soil model is used and also no pile-soil-pile interaction effects are taken into account. In the second approximate method, the free-field ground surface motion is used for $\mathbf{U}_1$ in addition to

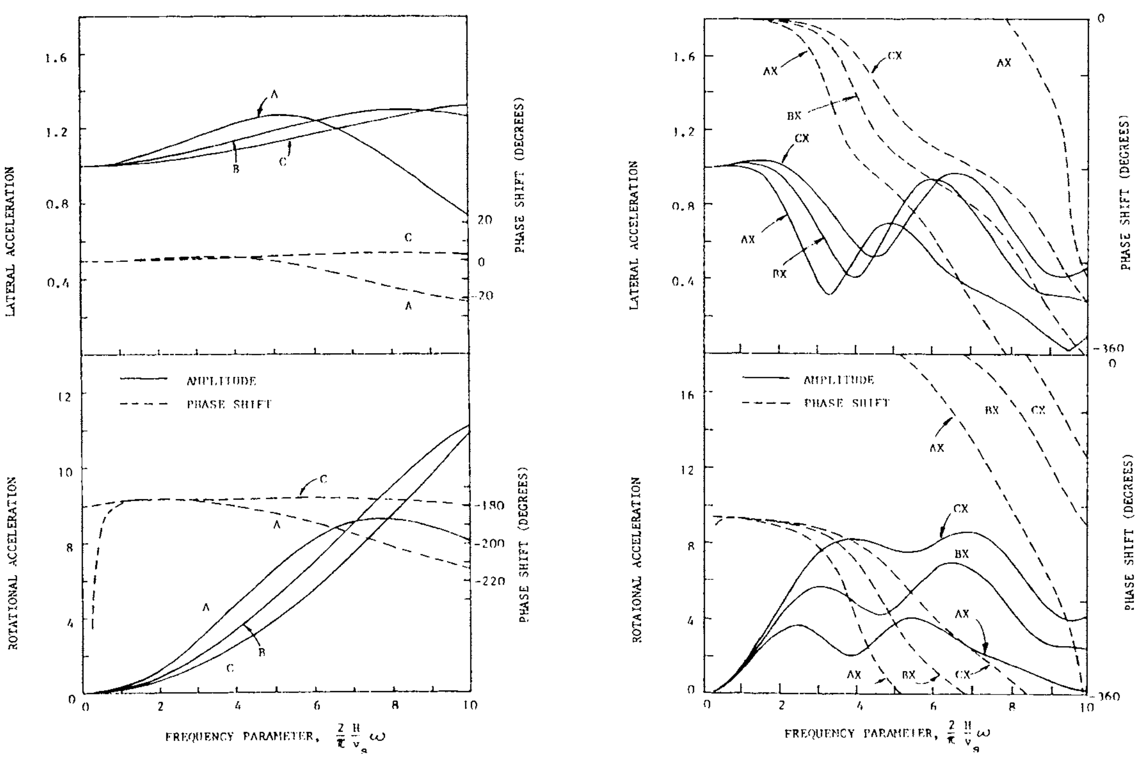

Fig. 7 Transmitted Motions for Single Piles

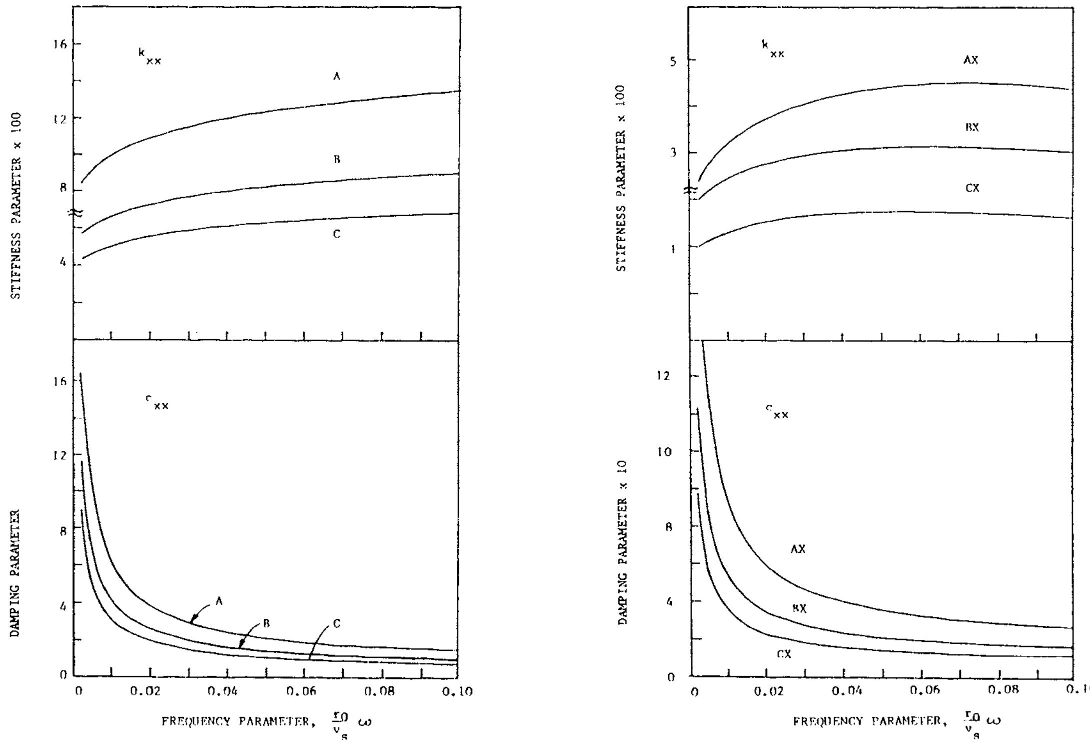

Fig. 8 Stiffness and Damping Parameters of Single Piles

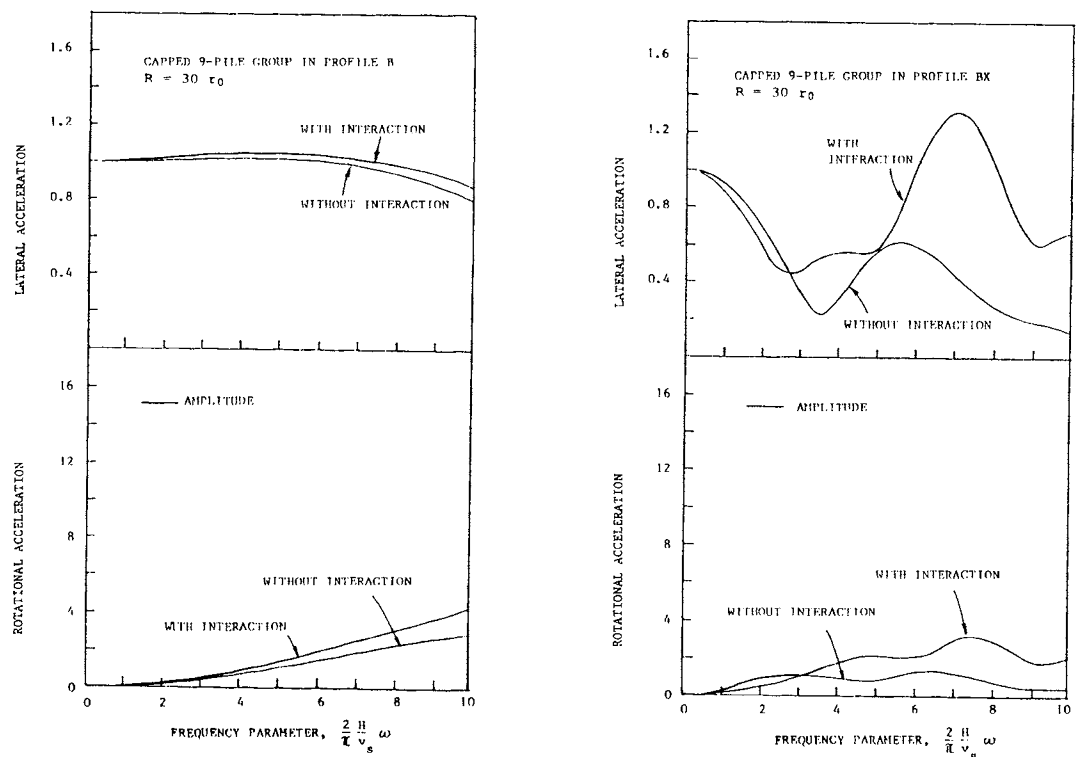

Fig. 9 Transmitted Motions for 9-Pile Groups

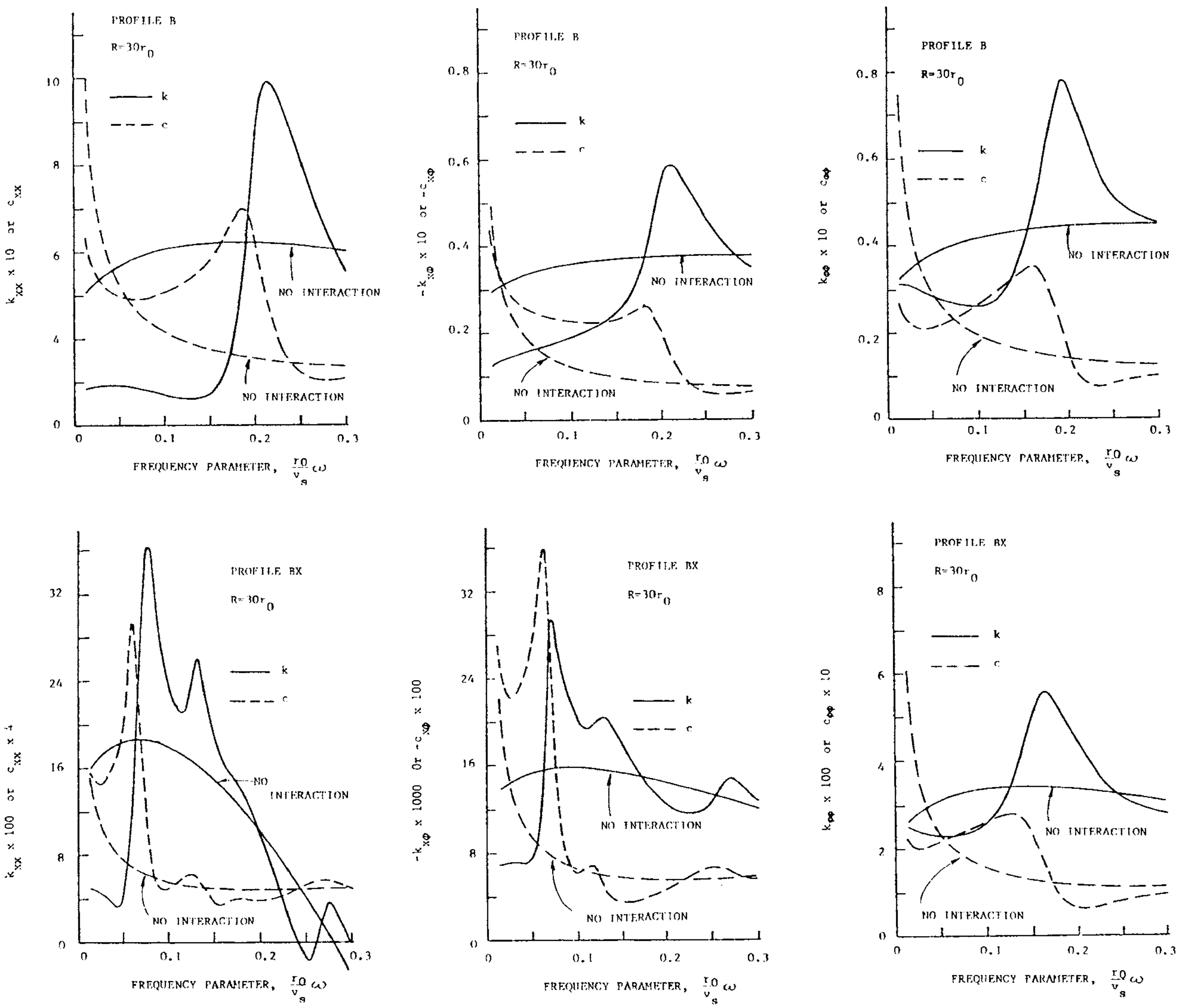

Fig. 10 Stiffness and Damping Parameters of 9-Pile Groups

approximations used in the first approximate method. The results computed for profiles B and BX are shown in Fig. 12. In the results obtained for profile B, the difference between the peak values computed by the relatively rigorous method and the first approximate method appears to mostly result from the differences in the stiffness and damping parameters between the two cases: the first approximate method yields higher stiffnesses and lower dampings. The use of the free-field ground surface motion for $\mathbf{U}_1$ does not produce significant errors in this particular case. In the results obtained for profile BX, the response curves computed by the relatively rigorous method and the first approximate method are very similar to the transmitted motions presented in Fig. 9 and are relatively close to each other. On the other hand, the response curve computed by the second approximate method is significantly different from the other two. Therefore, the use of the free-field ground surface motion causes significant errors in the computed seismic response of the structure supported by 9-pile group in profile BX.

The real seismic motions are random and contain various frequency components. Thus, the above observed errors caused by approximations at various frequencies are all included in the response time history. One random earthquake bedrock motion time history is used to see how those errors are reflected in the time-domain responses. Fig. 13 shows the seismic responses of the structures supported by single-piles, computed by the relatively rigorous method and the approximate method. As expected from the previous observation for harmonic bedrock motions, the approximate method produces the responses very close to those computed by the relatively rigorous method for profile B but amplifies the predominant frequency component responses excessively for profile BX. Fig. 14 shows the seismic responses of the structures supported by 9-pile groups, computed by the relatively rigorous method and the approximate method. The approximate method herein is the second approximate method explained previously. This approximate method amplifies the high frequency component responses excessively for both profiles. This is more pronounced for the

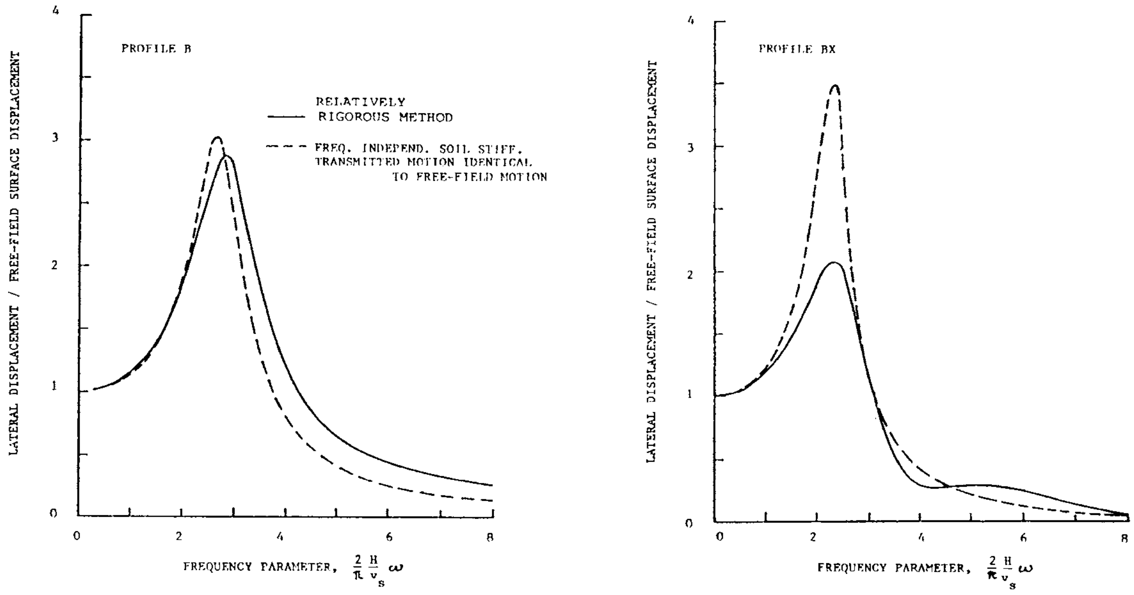

Fig. 11 Responses of Structures Supported by Single Piles to Harmonic Bedrock Motions

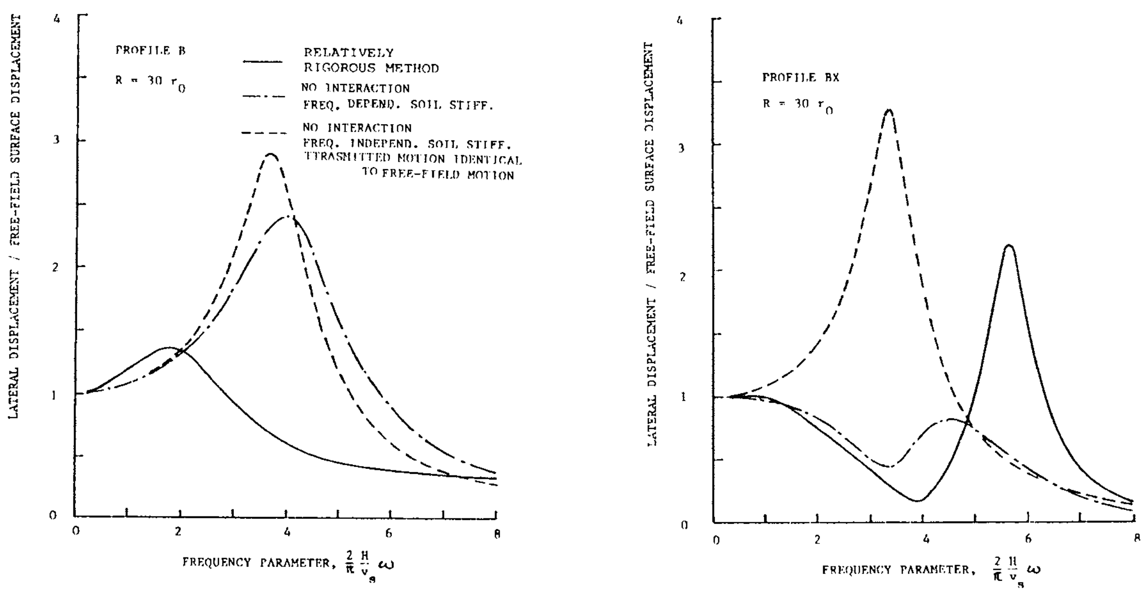

Fig. 12 Response of Structures Supported by 9-Pile-Groups to Harmonic Bedrock Motions

inhomogeneous profile than the homogeneous profile.

CONVENTIONAL WINKLER MODEL FOR NONLINEAR SOIL-PILE INTERACTION

A conventional Winkler model is made of the nonlinear spring and dashpot placed in a mutually parallel position, in which the latter is to reproduce the radiation damping and its property is independent of the displacement (Matlock et al., 1978). The soil model presented herein can account for the nonlinear soil-pile interaction by using the nonlinear mechanical system inside the rigid ring and the frequency independent mechanical system outside the ring. Details of those systems can be found in the papers by the first author contained in the references.

Under the steady state harmonic response, the complex force-displacement relationship of the conventional model is schematically illustrated in Fig. 15: the real and imaginary parts are respectively the backbone curve and the curve related to the damping (area of the hysteresis loop). The damping in this case is simply the summation of the nonlinear damping and radiation damping, and thus the nonlinearity induced in the vicinity of the pile always increases the damping since the dashpot is not affected by the nonlinear behavior. Fig. 16 shows the complex

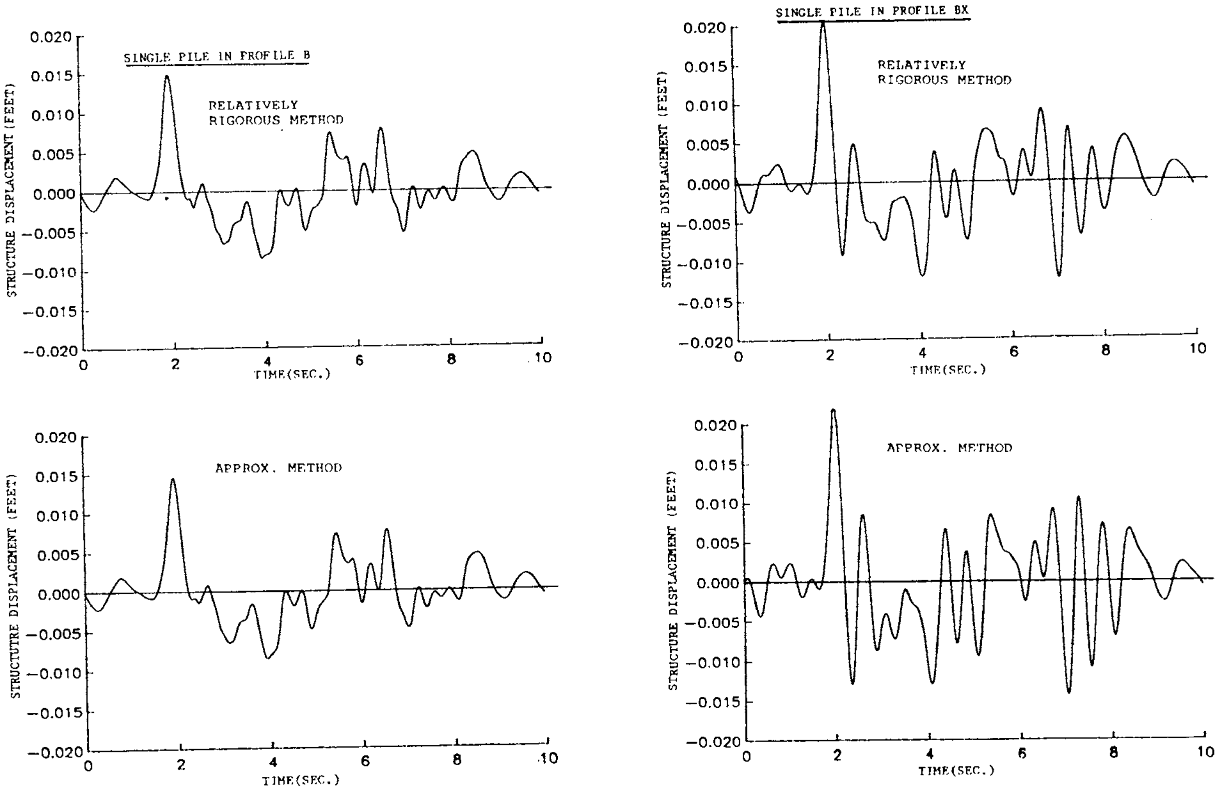

Fig. 13 Responses of Structures Supported by Single Piles to Random Bedrock Motions

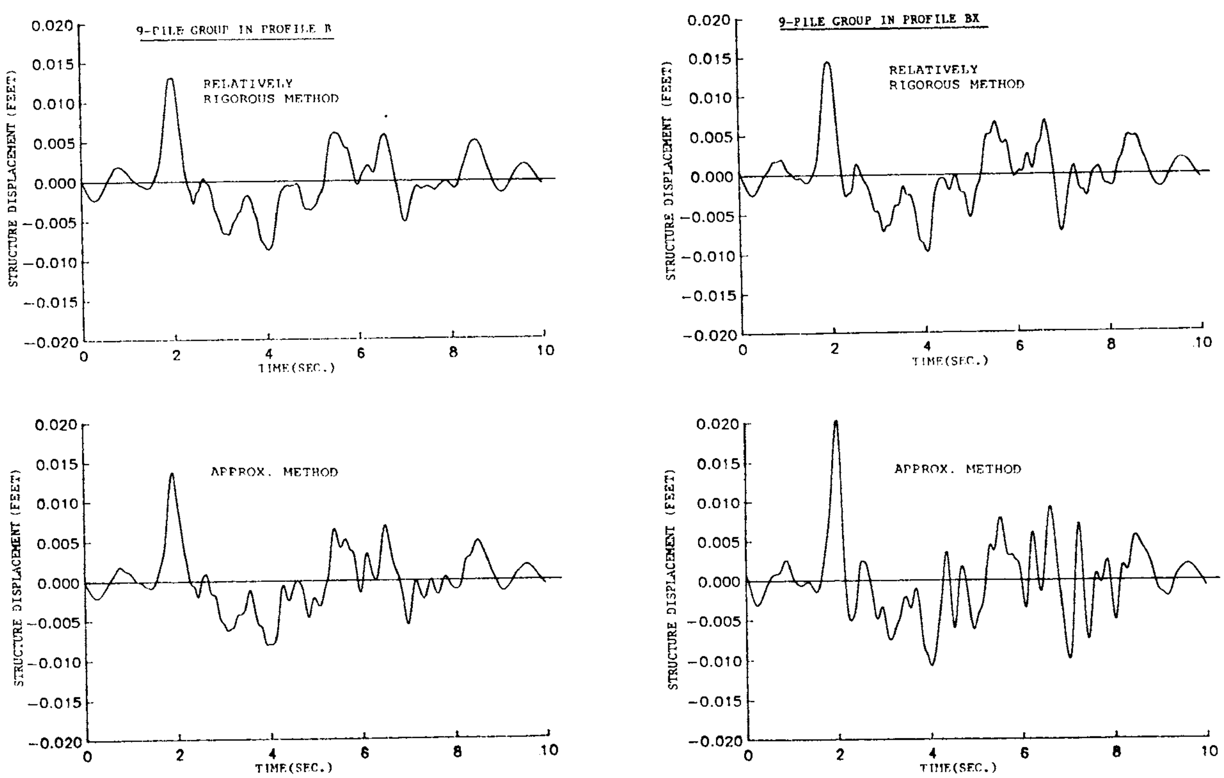

Fig. 14 Responses of Structures Supported by 9-Pile Groups to Random Bedrock Motions

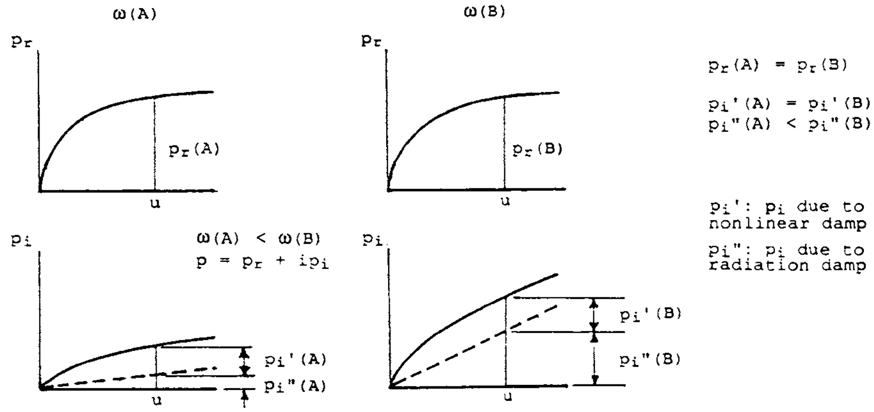

Fig. 15 Complex Force-Displacement Relationship of Conventional Soil Model

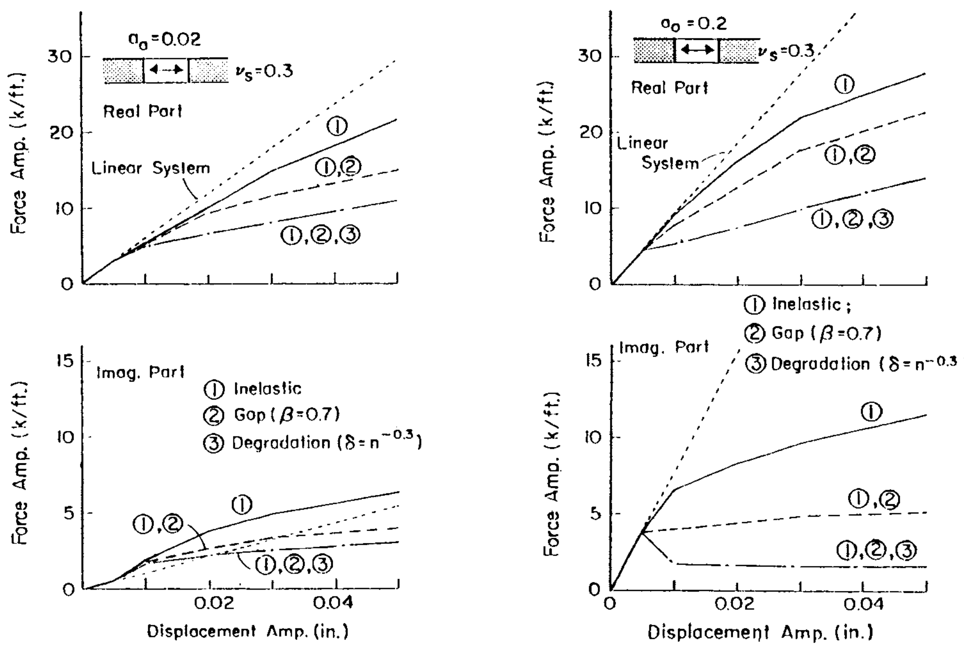

Fig. 16 Complex Force-Displacement Relationship of Nogami Soil Model

force-displacement relationship of the presented soil model. In this model, the nonlinear behavior in the vicinity of the pile foundation (mechanical system inside the ring) generates the damping but at the same time reduces the energy transmitted to the infinity (radiation damping). The net effect is to increase the damping at the frequencies where the radiation damping is small (low frequencies), but to reduce the damping at the frequencies where the radiation damping is large (high frequencies).

CONCLUSIONS

Seismic responses of pile-supported structures are formulated with the approach developed by the first author. Approximations frequently used in the seismic response analysis of pile-supported structures are assessed by using this formulation. Some of the routinely used approximations can cause significant errors in the computed responses. This is generally pronounced for pile groups and soil profiles containing soft soils at shallow depth. A commonly used nonlinear Winkler soil mode fails to reproduce the coupling between the nonlinear and radiation dampings, and thus overestimates the damping in the high frequency component responses. All those warrant us that the seismic response of pile-supported structures must be analyzed by using the methods and numerical models based on a careful and rational consideration.

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