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ABSTRACT

This paper describes a hybrid method that may be used to model the dynamic responses of multiple rectangular foundations resting on a homogeneous, viscoelastic stratum and excited by harmonic waves. The hybrid analysis described in this paper combines the dynamic stiffness and finite element methods. The method uses equivalent area disk loads for Green's functions at the soil surface. The dynamic stiffness method is used to obtain the stiffness matrix of the soil medium and the finite element method is used to obtain the stiffness matrix of surface foundations. The stiffness of the entire soil-structure system can be obtained using a substructure approach by assembling the stiffnesses of the soil medium and the foundations. The stiffness of the surface foundations affects the dynamic behavior of the soil-foundation system, especially with respect to its vertical motion. The finite element method described in this paper is used to construct the stiffness of the surface foundations. Equivalent area disks are placed at discretized points on the interface between the foundations and stratum surface. The stratum compliance matrix is constructed using the Green functions for vertical and lateral disk loads. Implementation of this model for analyzing soil-foundation system is current underway.

BACKGROUND

The through-soil interaction between adjacent identical foundations may magnify or lower the peak response at frequencies which are higher than the first resonant frequency of the stratum. Solutions for dynamic soil-structure interaction problem typically assume that the foundations are rigid or perfectly flexible. For small rectangular and circular foundations, the assumption that the foundations are rigid gives reasonable results. However, the assumption may be inappropriate for a mat foundation with a large length to width ratio. For accurate soil-foundation analysis, the stiffness of the foundation must be properly modeled.

The finite element, boundary element, hybrid, and experimental methods for soil-structure interaction analysis are frequently used by researchers. The finite element method (FEM) has gained considerable popularity from the availability of digital computers and sound theories. However, modeling wave radiation through a continuum that is unbounded in the horizontal direction causes numerical errors. To model an infinite horizontal continuum using the FEM, artificial boundaries must be introduced to reflect waves. (Vaish and Chopra 1973) suggested that artificial boundaries be placed far enough away from the region of interest to avoid undesirable wave reflections. Then the waves reflected from the boundaries are either dampened before waving back or arrive after the response time of interest. (Bettess and Zienkiewicz 1977) introduced the infinite element and (Kausel and Tassoulas 1981) recommended the transmitting boundaries which absorb wave effects emanating from the structure. While the finite element approach provides a method of modeling complex physical situations including system nonlinearities, the shortcomings of the finite element methods in soil-structure interaction analyses become apparent when models with numerous elements are needed. Computational costs rise and computational speed decreases.

The boundary element method (BEM) has been used by many researchers for dynamic soil-structure interaction analyses in the past decade. The advantage of the BEM over the FEM is that only the boundaries of the region being investigated have to be discretized. As a result, fewer discrete elements are needed to apply the BEM method than any scheme requiring internal subdivision of the whole body. The BEM can be applied when the governing differential equation is either linear or incrementally linear. (Emperador and Dominguez 1989) used the BEM to analyze the response of rigid cylindrical and hemispherical foundations embedded in uniform and layered viscoelastic half-spaces. (Israil and Ahmad 1989) used isoparametric quadratic boundary elements to study the dynamic response of rigid strip foundations in viscoelastic soils under vertical excitation. (Antes and Estorff 1989) used the BEM to determine the dynamic responses of rigid foundations and flexible elastic structures when placed on or embedded in an elastic stratum. (Dravinski 1983) used the BEM to investigate the amplification of harmonic waves by two alluvial valleys of arbitrary shape embedded in a half-space.

The BEM can be used in conjunction with other numerical techniques, such as finite element or finite difference methods and the combination of the two methods is called hybrid approach. Usually, the BEM is used for the soil stratum and the FEM is used for the foundations. It has very distinct advantage, saving computational effort, for problems of large physical dimensions. (Karabalis and Beskos 1985) obtained the dynamic responses of a three-dimension flexible foundation by using a time domain BEM for the soil medium and the FEM for the flexural foundation plate. (Spyrakos and Beskos 1986) employed a time domain boundary element model of an elastic, isotropic, and homogeneous soil medium and finite element model of a flexible foundation. The hybrid approach described in this paper is a combination of the dynamic stiffness method for the stratum and the FEM for the foundations.

In the experimental approach to studying soil-structure interaction, full or reduced models are tested under simulated earthquake loads. The experimental approach is then used to validate numerical models. However, it is difficult and costly to test those models in the nonlinear domain. Adequate predictions of the system behavior should be planned before the experimental approach is applied. (Weissman and Prevost 1989) presented a centrifuge model that is capable of representing soil-structure systems subjected to earthquake like excitation. The model was validated by performing free field soil tests, dynamic soil-structure interaction tests, and a numerical analysis of the experimental results. The centrifuge bucket's walls are lined with an clay-like material which can absorb wave energy and attenuate wave reflections. Without special treatment on walls, (Mita and Luco 1989) used a finite soil model, made of silicone rubber, to simulate the semi-infiniteness of the actual soil medium for soil-structure interaction problems and obtain the impedance functions and effective input motions for surface and embedded foundations.

OVERVIEW OF THE HYBRID SUBSTRUCTURE APPROACH

The hybrid analysis approach described in this paper combines the dynamic stiffness method and the FEM. The method involves a substructure approach and can be generalized for any number of surface foundations. The substructure approach was applied by (Vaish and Chopra 1974), Whittaker and (Whittaker and Christiano 1982), and (Adams and Christiano 1986). The dynamic force displacement equation for the soil-foundation system is expressed in Eqn. 1. F is force vector containing the external loads applied to the discrete points at the surface of soil stratum and Δ is complex displacement vector at the points. Eqn. 2 is used to find $Z_{system}(\omega)$ the impedance matrix of the entire system by combining impedances of masses and stratum. $Z_{fnd}(\omega)$ is found using the FEM component of the hybrid approach and $Z_{soil}(\omega)$ is found using the dynamic stiffness method.

Since there are no discrete nodes in the stratum, the vertically propagating seismic waves will transform into equivalent surface loads at the points of discretization. An equivalent force vector F_0 at the surface nodes may be expressed by Eqn. 3, where Δ_0 is the known free field displacement vector at the foundation-stratum interface. The dynamic response Δ of the entire system can be found by solving the system of equations in Eqn. 4.

$$Fe^{i\omega t} = Z_{system}(\omega)\Delta e^{i\omega t} \quad (1)$$

$$Z_{system}(\omega) = Z_{fnd}(\omega) + Z_{soil}(\omega) \quad (2)$$

$$F_0 e^{i\omega t} = Z_{soil}(\omega)\Delta_0 e^{i\omega t} \quad (3)$$

$$F_0 e^{i\omega t} = Z_{system}(\omega)\Delta e^{i\omega t} \quad (4)$$

A HYBRID MODEL FOR SINGLE FOUNDATION

In this section, a hybrid model that combines the dynamic stiffness and finite element methods with interface disks is described for analyzing the soil-structure interaction of multiple rectangular foundations founded on a viscoelastic stratum. Figure 1 illustrates the modeling technique for a single foundation. Models for multiple foundation systems can be built by assembling single foundation models. The dynamic stiffness method is used to find the stiffness of the soil stratum and the finite element method is used to find the stiffness of the foundations. In Figure 1, a single mat foundation is discretized into four finite elements. Figure 1 shows a view into the model between the foundation bottom surface and the stratum to surface. The soil surface is discretized at the points of contact with the finite element nodes of the mat foundation labeled 1-9. Assuming no slip between the soil surface and the foundation during excitation, the displacements at the discretized points on the soil surface and the nodes of the finite element at the bottom of foundations should be the same. The stiffness, damping, and mass matrices of the rectangular foundation are calculated for each finite element and then assembled for the entire foundation. Green functions are found for harmonic point loads at each discretized point on the soil surface. The resulting compliance matrix is inverted and combined with the stiffness of the foundation system to obtain the stiffness of the entire soil-foundation system.

Each foundations is discretized as solid finite elements with degrees of freedom in the x, y, and z directions at each node as shown for node 9. The Green's functions introduced by (Kausel 1981) for the harmonic vertical point and horizontal loads, the displacements at the on discretized points on the soil surface can be obtained, and the compliance matrix for the soil stratum can be formed. For accuracy, a mesh of small finite elements should be used, which will increase the computational efforts significantly because of a large and highly populated compliance matrix. To avoid high computational efforts without sacrificing accuracy, the Green's functions for harmonic disk loads rather than harmonic point loads should be used.

The structural responses to earthquake ground motion is for the most part influenced by the first few modes of vibration, so a mesh of large finite elements can be used. Because the element size is larger, the loads that are transferred down to the soil surface through the discretized points will be heavily concentrated points loads that would yield conservative compliance matrix. To minimize the computing efforts and possible errors, the use of interface disks at the discretized points between the foundation and soil surface is proposed. These disks denoted d_1 through d_9 in Figure 1 are weightless and thicknessless. Using the interface disks allows Green function for harmonic disk loads instead of point loads to be found for large-sized finite elements.

The hybrid method uses an assumption of equivalent area disks under the foundation. In Figure 1, the distributed areas of Rectangle $abcd$ at Node 5 and Disk d_5 are equal. Similarly, the distributed areas at the Node 1 of Rectangle $aefg$, and the area of the Disk d_1 are equal. The equivalent area disks under the foundation are used to construct the compliance matrix by finding the Green's functions for harmonic disk loads and using an averaging technique to get average displacement under disk. For example, the average displacements under Disk d_5 will be the displacements at Node 5 on the stratum surface.

FINITE ELEMENT METHOD FOR FOUNDATIONS

The impedance matrix of the foundations, $Z_{fnd}(\omega)$ in Eqn. 2, can be found using the FEM and Eqn. 5. K, C and M in Eqn. 5 are the structural stiffness, damping and mass matrices, respectively. For less discretized points, trilinear elements with eight nodes and twenty four degrees of freedom are used to form the stiffness and mass matrices. The K or M in Eqn. 5 can be obtained by assembling the stiffness matrix or mass matrix of each finite element, k in Eqn. 6 or m in Eqn. 7. Matrices N and B, in Eqns. 6 and 7 is the shape function and strain-displacement matrix respectively. ρ is the mass density of the foundation. The volume integrations are carried out over the element volume V_e. Four element matrices are assembled to form the stiffness and mass matrices of the surface foundation shown in Figure 1. Damping Matrix C is evaluated by Eqn. 8 using a popular scheme which combines a fraction α of stiffness matrix with a fraction β of the mass matrix.

$$Z_{fnd} = K + i\omega C - \omega^2 M \quad (5)$$

$$\underbrace{k}_{24\times 24} = \int_{V_e} B^{T} E B\, dV \quad (6)$$

$$\underbrace{m}_{24\times 24} = \int_{V_e} \rho N^{T} N\, dV \quad (7)$$

$$C = \alpha K + \beta M \quad (8)$$

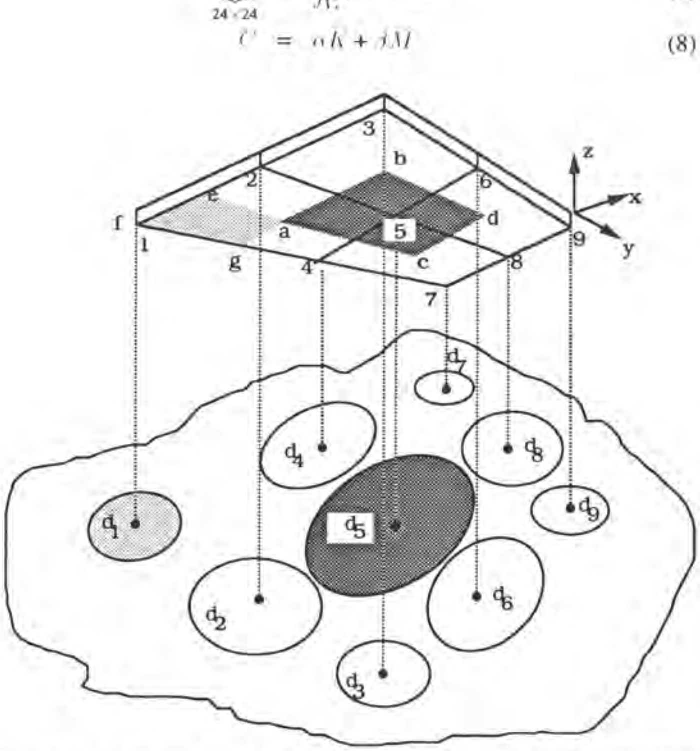

Figure 1: Boundary and finite element discretization of the foundation and the disk which transfer the loadings down to soil layer surface

DYNAMIC STIFFNESS METHOD FOR STRATUM

The stratum impedance matrix $Z_{soil}(\omega)$ in Eqn. 2 can be obtained by inverting the stratum compliance matrix. The coefficients of the stratum compliance matrix are obtained by evaluating the Green's functions (displacements due to unit disk loads including vertical and horizontal disk loads) and using an averaging technique, introduced by (Warburton et al 1971), to get the average displacements under one mass due to a unit load applied to another mass. The dynamic stiffness approach was applied by (Adams and Christiano 1986) for the analysis of interaction among clustered flexible masses.

To illustrate the method for constructing the compliance matrix for the stratum under the foundation in Figure 1, consider the arrangement shown in Figure 2. The surface disks at the FEM nodes are attached to the soil surface. Each disk area is equivalent to the contributing area of the mat foundation at that point on the stratum surface. The excitation applied at Disk d_5 produces displacements on the free surface at Disks d_1, d_2, d_3, d_4, d_6, d_7, d_8, and d_9. When the displacements distributed over Disk d_7 are averaged, the averaging technique is described by (Warburton et al 1971), the coefficient in the compliance matrix is obtained.

The Green's functions for disk loads can be obtained from a report written by (Kausel 1981). Each discretized node on the soil surface has three degrees of freedom, vertical and horizontal displacements, so that the Green's functions for vertical and horizontal disk loads are needed to construct the compliance matrix. The Green's functions are functions of wave number, wave shapes, the radius of the disk, Bessel functions of the first kind of order 0 and 1, and Hankel functions of the second kind of order 0 and 1.

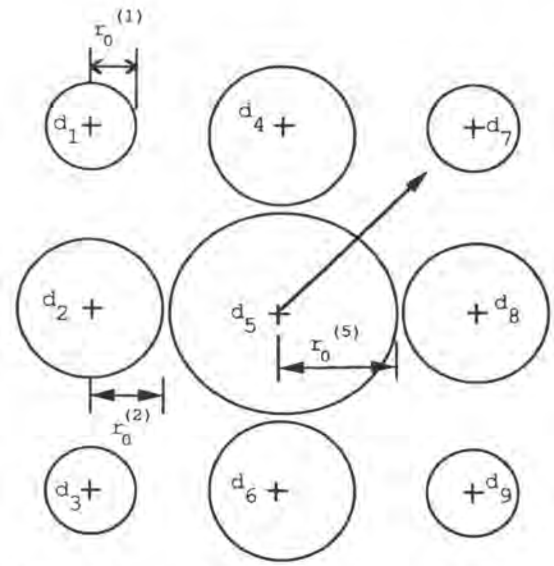

Figure 2: Disks for constructing compliance matrix.

The wave numbers and mode shapes in the Green functions are obtained by solving the eigenvalue problem, $P = K_s U$. Where K_s is the global stiffness matrix obtained by assembling all sublayer stiffness matrices, the stratum has to be divided into thin sublayers in this dynamic stiffness approach, and eliminating rigid body translation by imposing boundary conditions at the stratum's base, P is the loads and U is the displacements. The stiffness matrix of the i^{th} layer is:

$$K_i = A_i n^2 + B_i n + G_i - \omega^2 M_i \quad (9)$$

where n is the wavenumber, ω is frequency of excitation and A_i, B_i, G_i and M_i are matrices which involve material properties only. The eigenvalue problem yielding the natural modes of wave propagation in the stratum is obtained by setting the load vector equal to zero.

$$(A k_j^2 + B k_j + G - \omega^2 M)\phi_j = 0 \quad (10)$$

The subscript j refers to the various possible solutions which are $6l$ eigenvalues k_j and eigenvectors ϕ_j, with l being the total number of layers (and 6 being the total degrees of freedom at the top and bottom of each sublayer).

CONCLUSION

The paper describes a hybrid approach to analyze soil-foundation system. The approach combines dynamic stiffness method and the FEM. Equivalent area disks are presented for constructing a more accurate compliance matrix. The following observations indicate that this is a promising model for yielding an accurate approximate of the response of soil-foundation system while saving computing efforts.

1. Larger elements can be used to model the motion of the foundation because the structural responses to earthquake ground motion is mainly due to the first few modes of vibration of the structure.
2. Larger elements allow fewer elements to be used to model the foundation. With large elements, the discretized points contacting

the node, on the soil surface are farther apart on the stratum surface than using smaller mesh size that will yield a more conservative compliance matrix than if Green functions for the point loads are used for evaluating the soil surface displacements.

3. This proposed approach replaces the distributed area around a finite element node with an equivalent area disk and finds the Green functions for these equivalent area disks to construct the compliance matrix.

This approach is used to analyze the seismic interaction among numerous foundations and provide insight into the behavior of the soil-foundation system. The system is excited by vertically propagating waves and the influence of system parameters on the harmonic response of the system is studied. System parameters of interest include primarily the number, magnitude and spacing of surface foundation; the stiffnesses of the foundation and the stratum; and the frequency and motions associated with the seismic wave.

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