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Structural Collapse Behavior of Sands in Undrained Shear

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SYNOPSIS: Results from undrained torsional shear tests are presented and used to further examine the concepts of steady state and structural collapse in sands, with emphasis on the influence of the collapse mechanism on soil behavior at steady state. It is shown that the degree to which the material collapses is dependent on the loading conditions and the initial fabric of the sand. Consequently, the steady state position in the state diagram is also function of these parameters.

INTRODUCTION

Significant advances have been made over the past 25 years in improving the understanding of the behavior of saturated cohesionless soils subjected to monotonic and cyclic loading under undrained conditions. Included in these developments are a number of procedures which use the steady state line as a reference state (e.g. Bean & Jefferies, 1985; Poulos et al., 1985). The key to the use of these methods is the condition that the steady state line in undrained shear is only a function of void ratio and conflicting evidence on this matter exists in the literature (Poulos et al., 1985; Alarcon and Leonards, 1988; Dennis, 1988a, 1988b; Poulos et al., 1988; Konrad, 1990a, 1990b; Vaid et al., 1990).

The concept of structural collapse was formulated to explain the behavior of sands in undrained shear (Alarcon et al., 1988). Strain softening behavior during undrained loading is associated with the fact that the structure of contractive sands can be metastable. As a result, in a collapsive skeleton, small shear strains are considered to be sufficient to produce a sudden rearrangement of grains and loss of contact points between neighboring grains. For the case of undrained shearing, collapse of the structure results in the load being suddenly transferred from the sand skeleton to the water and produces a sharp increase in pore-water pressure. The substantial reduction in shear strength leads to large deformations of the sand in a short period of time.

Results from undrained torsional shear tests (Frost, 1989) are presented and used in this paper to further examine the concepts of steady state and structural collapse.

STEADY STATE CONCEPT

The principle of steady state deformation was first formally presented in 1971 (Poulos, 1971) as an extension of the concepts of critical void ratio (Casagrande, 1940) and residual strength (Skempton, 1964). The steady state of deformation for an assembly of particles is that state in which the body is deforming at constant volume, constant normal effective stress,

constant shear stress and constant velocity. It is achieved only after all particle re-orientation has reached a statistically steady state condition and after all particle breakage, if any, is complete (Poulos, 1981). If straining of the specimen is stopped, the specimen is no longer in the steady state of deformation.

There are conflicting opinions on the relative influence of the factors (composition, fabric, initial state and method of loading) affecting the stress strain behavior of sands at steady state. These differences in interpretation are based on conceptual issues as well as conflicting test results. It has been suggested (Poulos et al., 1985; Poulos, Robinsky et al., 1985; Castro, 1987; and Poulos et al., 1988) that the steady state line (SSL) is unique for a given soil and is only a function of composition and (a) the normal effective stress for drained tests or (b) the void ratio for undrained tests. However, re-interpretation of data obtained by Castro (Alarcon and Leonards, 1988) shows that the SSL in drained shear is significantly different than that obtained in undrained shear.

It has been argued (Alarcon and Leonards, 1988; Alarcon et al., 1988 and 1989) that the pore pressure response of sand in undrained shear depends not only on the potential volume changes, as determined in drained tests, but also on the tendency of the sand structure to collapse as displayed by the very rapid change in compressibility with respect to shear strain at small strain amplitudes. This collapse potential is a function of composition, fabric, and initial state of stress of the sand and thus these factors all influence the position of the steady state line.

Results presented by Dennis (1988a and 1988b) show significant variations in the slope and position of the SSL resulting from what were considered minor changes in the preparation procedure. Other test results (Chen et al, 1988; Castro, 1988) show, in general, a unique steady state line even though a range of specimen preparation, initial stress and load methods were

used. The effect of the rate and method of loading as well as the applied stress path were also examined by Torrey (1981). The results of that study indicated that the method of loading (incremental versus ramp loading using stress controlled tests) and the total stress path applied influenced the position of the SSL as much, if not more, than the rate of loading. The suggested dependence of the SSL location on the total stress path is of particular interest in view of the current state of practice whereby its position is usually determined from the results of axial compression triaxial tests, while cyclic tests are frequently performed using different devices (such as torsional shear devices which simulate earthquake loading more closely) and hence different stress paths. More recent results (Vaid et al., 1990) also suggest that the steady state line of a given sand is not unique. These tests show that for a given void ratio, the steady state strength is smaller in extension than in compression. Similarly, Konrad (1990a and 1990b) has shown differences in steady state strengths depending on the initial states of the specimens.

STRUCTURAL COLLAPSE CONCEPT

The concept of structural collapse was formulated to explain the behavior of sands in undrained shear (Alarcon, 1986). Strain softening behavior during undrained loading is associated with the fact that the structure of contractive sands can be metastable. In a collapsive skeleton, small shear strains may be sufficient to produce a sudden rearrangement of grains and loss of contact points between neighboring grains. In undrained shearing, collapse of the structure results in the load being suddenly transferred from the sand skeleton to the water, resulting in a sharp increase in pore water pressure. Consequently, the shear strength is reduced substantially and the sand undergoes large deformations. In the process of deformation, the sand grains reach a statistically steady-state orientation after which the shear stress needed to continue deformation eventually reaches a very low, constant (steady state) value (as discussed by Poulos, 1981). In discussing the collapse hypothesis, Alarcon et al. (1988) presented interpretations of the results of torsional shear tests as evidence of the proposed mechanism.

Typical results from an undrained cyclic torsional shear test at constant shear stress amplitude were presented by Alarcon et al. (1988) and are reproduced in Figure 1. First of all, it was noted that strain softening was initiated at a stress ratio (ϕ'_{mob}) that was higher than the stress ratio at the peak monotonic strength ($\phi'_{mob} = 16.6^\circ$ vs 13° obtained in a monotonic test on a replicate specimen). However, more significantly, collapse occurred when the stress state during cyclic loading reached the effective stress path from the monotonic test; thereafter, the two stress paths were essentially the same. The initiation of strain softening behavior was clearly characterized by a sharp increase in pore water pressure and the development of large shear strains (point A in Figure 1). The abrupt increase in the rate of pore pressure generation and the corresponding development of large shear strains was considered to be strong evidence in support of the concept of structural collapse.

Tests performed at different shear stress amplitudes yielded similar results (Alarcon, 1986). At a given void ratio, the monotonic stress path seemed to constitute a collapse boundary that determined the initiation of strain softening behavior under cyclic loading, provided that the stress paths were similar and the cyclic shear stress amplitude was larger than the steady-state strength. Consequently, ϕ'_{mob} at the instant of collapse was not a constant value but a variable one depending upon the cyclic shear stress amplitude.

In torsional shear tests on isotropically consolidated specimens the axis of the major principal stress changes its orientation from -45° to $+45^\circ$ from the vertical in the course of reversing the shear stress from clockwise to counterclockwise, hence the collapse boundary shown in Figure 1 corresponded to this specific rotation of the principal stresses. For different amounts of rotation of the major principal stresses it was considered likely that this boundary would also change. Evidence for this can also be found in the work by Symes et al. (1984). Accordingly, triaxial tests and torsional shear tests would likely yield different undrained cyclic shear strengths, even in the absence of initial inherent anisotropy of the sand. For the same reasons, the collapse boundary would not be independent of the initial state of stress, since it would be influenced by the magnitude of the rotation of the planes of principal stress and their orientation with respect to the principal axes of anisotropy.

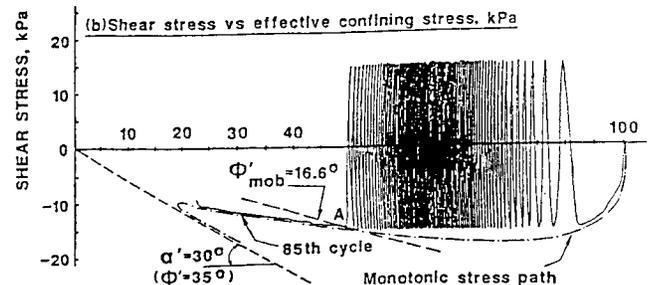
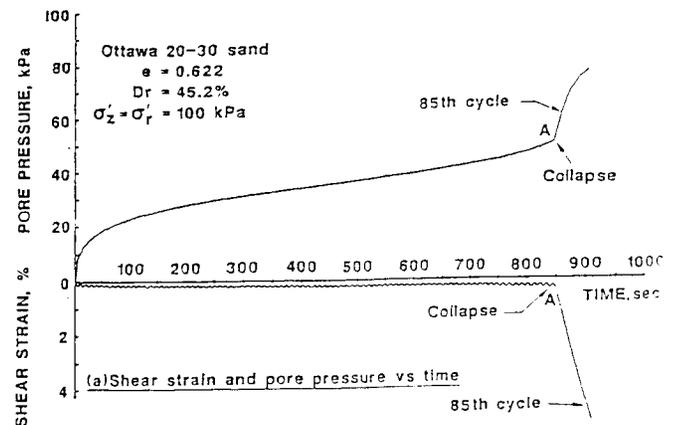


Figure 1 Undrained Cyclic Torsional Shear Test Results

On the basis of the foregoing evidence Alarcon et al. (1988) explained why the undrained steady state strength (F line) might be considerably smaller than the strength obtained from constant volume drained tests (S line) as was shown elsewhere using Castro's data (Alarcon and Leonards, 1988). It was postulated that the pore water pressure response of sand specimens in undrained shear not only depended on the magnitude of the potential volume changes but also on the tendency for very rapid changes in compressibility close to the mobilization of the peak monotonic strength. Thus structural collapse or instability would be of little consequence in drained shear however would result in a sharp increase in pore water pressure in undrained shear and consequently a large decrease in the undrained shear strength.

The remainder of this paper presents the results of additional undrained torsional shear tests and discusses the concept of structural collapse in light of these results.

EXPERIMENTAL APPARATUS

The apparatus used for the experimental work described in this paper is a combined resonant column - torsional shear device (Alarcon, 1986; Alarcon et al., 1988; Frost, 1989). The apparatus combines conventional triaxial loading features with resonant column and torsional shear capabilities. It can be used to test both solid and hollow cylindrical specimens, which have been consolidated either isotropically or anisotropically, over the full range of shear strains of engineering interest, from $10^{-4}\%$ up to about 20%. A schematic of the test device is shown in Figure 2. Both solid and hollow cylinder specimens with outside diameters of about 7 cm and lengths of 19.8 cm can be prepared and tested. The hollow cylinder specimens have an inside diameter of approximately 3.8 cm.

Confining pressures of up to 700 kPa (design limit of chamber) are applied by air pressure. The effective stress on the specimen is measured with a Sensotec P30P differential pressure transducer. One port of the transducer opens to the base of the confining chamber and the other is connected to the bottom of the specimen. Anisotropic stress conditions are applied by means of a Bellofram double acting air piston. Axial load is measured with a Lebow 3169 load cell located at the base of the cell. Axial deformation is monitored with a Schaevitz 1002XS-D linear variable differential transformer (LVDT).

Small amplitude shear strain loading ($10^{-4}\%$ to about $10^{-2}\%$) is applied to the soil specimen by a Hardin electromagnetic oscillator. The torsional response of the soil specimen to the applied vibrations is monitored by a piezoelectric accelerometer mounted on the top platen. Larger amplitude shear strain loading ($10^{-2}\%$ to 20%) is applied by a Compumotor AX83-135 stepper motor and indexer. Further details on the operation of the stepper motor are given below. The torque applied to the specimen is measured with a Lebow 2102-500 torque transducer located at the base of the cell. Rotation of the top of the specimen with respect to the fixed base during quasi-static torsional loading is monitored by a pair of Kaman KD2310-45 non-

contacting displacement transducers which use eccentric cams connected to the top platen of the specimen as metal targets.

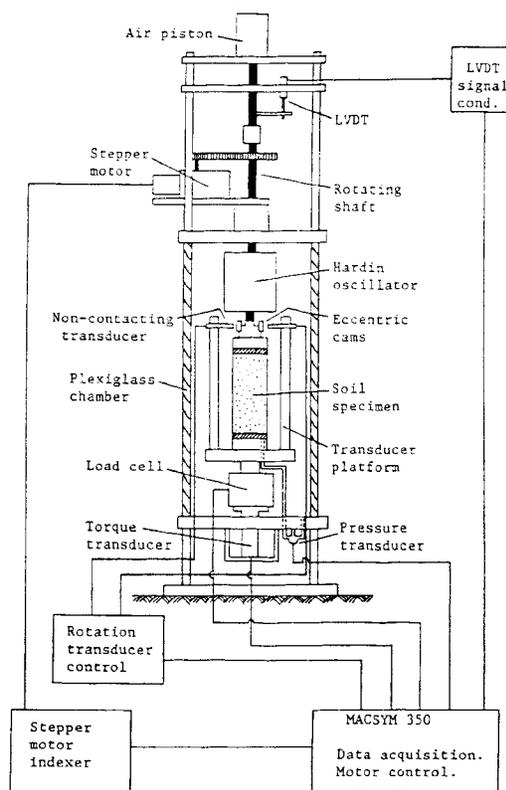


Figure 2 Schematic of Resonant Column - Torsional Shear Device

Transducer excitation, stepper motor control and transducer output monitoring and/or recording is achieved with the use of an Analog Devices Macsym 350 computer system which consists of a standard Macsym 150 microcomputer connected to a Macsym 200 intelligent front-end. All operations of the Macsym 350 during testing are controlled by a single computer program written in the Macbasic language and allow for user selection of various tasks from a menu.

The stepper motor and indexer permit control to be implemented directly through a serial port of the Macsym 350. In this manner, the motor can be programmed to apply a wide range of loading histories including non-uniform cyclic loads. Selection of parameters such as acceleration, velocity and direction of rotation of the motor is made through the keyboard of the computer. These parameters are transmitted in ASCII code to the indexer where they are interpreted and transmitted to the motor. An indication of the versatility of this system is that one revolution of the stepper motor (2 degree rotation of top of specimen) consists of 12,800 steps and the motor can be instructed to move continuously or as finite a distance as one step.

TEST PROCEDURES

All tests reported in this paper were performed on reconstituted specimens of Ottawa

20-30 sand. The characteristics of this sand and the grain size distribution curve are shown in Figure 3. All material was washed before testing to remove any surficial impurities and the sand used was discarded after the test to avoid any effects of grain breakage due to prior loading/testing. Most specimens were reconstituted using a modified air pluviation specimen preparation apparatus (Frost, 1989). The method allows for preparation of specimens with relative densities ranging between 20 and 100% by varying the fall height and/or intensity of flow.

A number of tests were performed on specimens reconstituted using a funnel attached to a plexiglass tube which extended to the bottom of the mold. Loose specimens were obtained by keeping the tube full of sand and slowly retracting it from the mold. This was the same procedure used by Alarcon (1986), and thus was also used in some of the tests summarized in this paper for comparison purposes.

Ottawa 20-30	
Predominant Mineral	Quartz
Grain Shape	bulky
Angularity	
Specific Gravity	2.65
D_{10} (mm)	0.60
D_{30} (mm)	0.72
D_{50} (mm)	0.75
D_{60} (mm)	0.83
$C_u = D_{60}/D_{10}$	1.25
C_{cu}	0.738
C_{uc}	0.501
C_{uc}^{max}	0.237

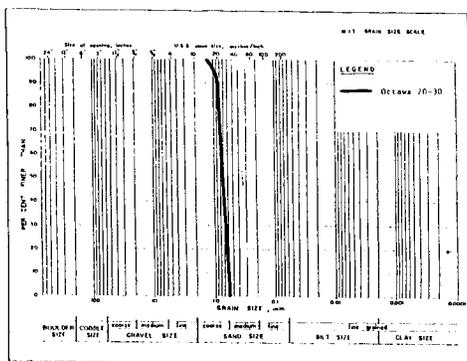


Figure 3 Characteristics of Test Materials

FURTHER EVIDENCE OF STRUCTURAL COLLAPSE

The results of undrained torsional shear tests are presented and discussed in this section within the framework of structural collapse and its influence on steady state.

Structural Collapse in Cyclic Tests

As discussed earlier, evidence of structural collapse is found in cyclic tests (Figure 1) where the initiation of strain softening behavior is characterized by an increase in the rate of pore pressure generation. This was confirmed by the results of a large number of cyclic torsional shear tests performed at different initial effective confining stresses and void ratios using different specimen preparation techniques. The manner in which the pore pressure generation rate increases for a given void ratio is related to the actual stress state at which the stress path during cyclic loading reaches the effective stress path from the monotonic test (Point A in Figure 1b). This is illustrated by the results in Figure 4, where slopes of pore pressure versus time curves immediately before and immediately after the initiation of collapse are compared. As the stiffness of the sample decreases (larger values of pore pressure ratio), the degree to which the sample collapses is reduced, as evidenced by the change in the rate of pore pressure generation at the initiation of collapse.

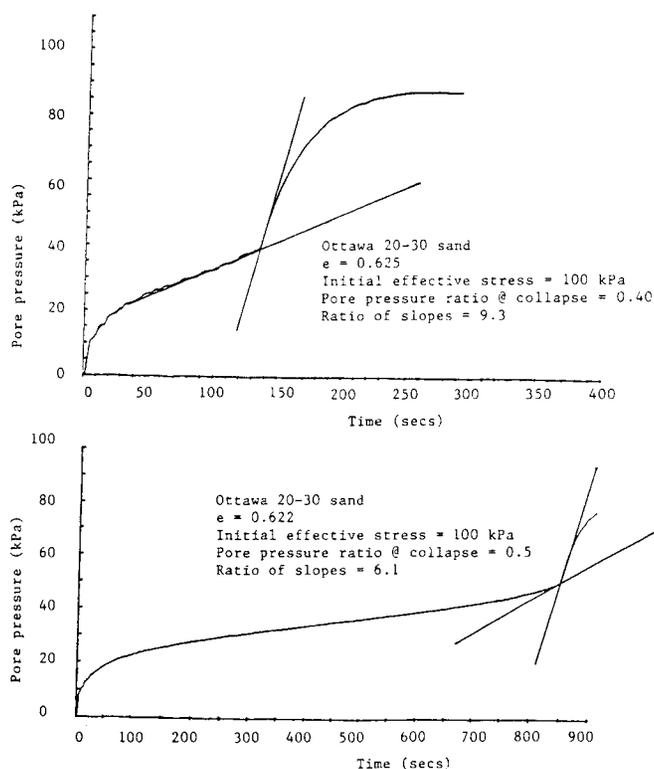


Figure 4 Collapse of Specimens as Function of Pore Pressure Ratio

Effect of Initial Fabric on Steady State

As noted earlier, there is conflicting evidence in the literature as to the effect of specimen preparation and stress path on steady state. The results of undrained monotonic torsional shear tests performed to determine the steady state line for Ottawa 20-30 sand are summarized in Table 1 and Figures 5 and 6. In particular, Figure 5 compares results from tests performed on specimens at approximately the same void ratio but prepared using different techniques. The specimens exhibit significantly different behavior in terms of both peak strength and post-peak strain softening. The axial strains during isotropic compression were comparable regardless of the method of preparation. It is postulated that differences in non-uniformity of local fabric play a role in the observed behavior, however the small differences in void ratio (0.646 versus 0.653) certainly contributed to these differences as well. It is also noted that the stress-strain curve for the specimen prepared using the dry tubing method exhibits larger local fluctuations when compared to the specimen prepared with the modified air pluviation method and this is believed to reflect the importance of fabric at a more local level even at shear strain levels approaching steady state.

When the results from these tests are plotted, along with results of other tests on the same sand (Figure 6), the steady state points for specimens prepared with the tubing technique (Tests 64 and 66) are outside the steady state line representative of tests prepared using the modified air pluviation technique, and thus indicate a separate steady state line is

appropriate for specimens prepared in this manner. These results agree with data reported by Dennis (1988a and 1988b) and suggest that the steady state during undrained shear is not only a function of void ratio but is also related to the initial structure of the sand. This finding deviates from the usual assumption regarding uniqueness of the steady state line.

Table 1 Summary of Steady State Data

Test number	Initial consol. void ratio	Initial mean effective stress (kPa)	S.S. shear stress (kPa)	S.S. horiz. stress (kPa)	S.S. vert. effective stress (kPa)	Approx. strain rate (%/min)	Remarks
14	0.621	109.5	31	70.1	72.5	7.5	
27	0.634	103.2	24.7	51.2	59.9	7.5	
28	0.667	101.0	13.5	26.7	33.1	7.5	
62	0.671	99.6	14.3	27.7	36.4	7.5	
63	0.646	98.3	20	44.7	51.8	7.5	
64	0.653	99.9	11.4	21.9	29.3	7.5	Tube preparation
66	0.646	100.4	16.1	34.6	38.2	7.5	Tube preparation
B9	0.673	100.9	10.4	25.4	26.8	2.5	
B10	0.671	99.8	6.3	19.3	23.8	25.0	
B16	0.676	99.4	11.2	20.9	24.6	150.0	
B36	0.659	51.5	11.8	26.4	31.5	2.5	
B36a	0.637	50.2	6.9	12.3	16.8	2.5	Reconsolidated
B11	0.682	101.3	5.1	15.1	18.9	2.5	
B11a	0.640	100.9	2.4	12.5	15.1	2.5	Reconsolidated

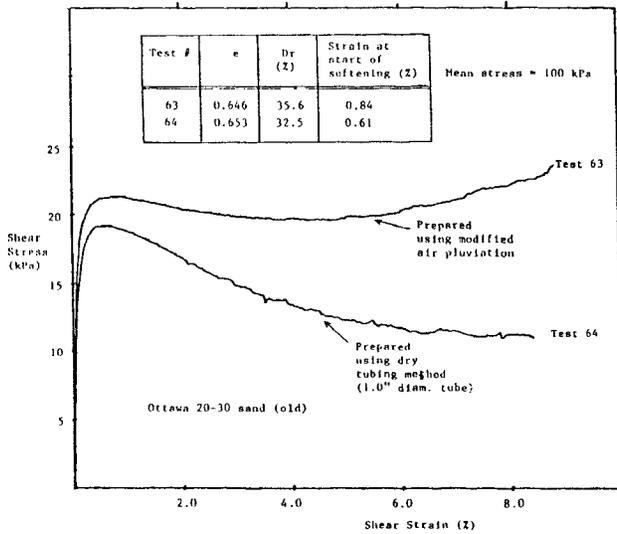


Figure 5 Effect of Method of Preparation Undrained Torsional Shear Tests.

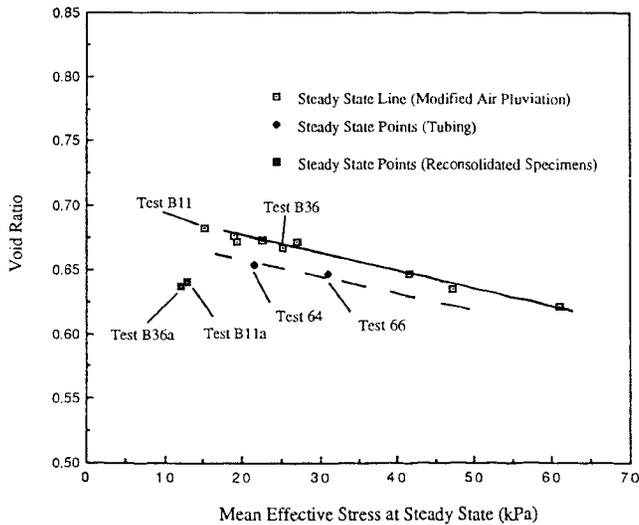


Figure 6 Steady State Line - Ottawa 20-30 Sand

The importance of initial structure is further evidenced by the result of tests B36 and B36a shown in Figure 6. A specimen was consolidated initially under an isotropic confining stress of about 50 kPa with a void ratio of 0.659 ($D_r = 33\%$). During loading the specimen reached a quasi-steady state of deformation and was able to sustain a shear stress of about 11.8 kPa at a mean normal stress of 28.1 kPa. The shear stress was then removed from the specimen and it was reconsolidated under the same isotropic confining stress of about 50 kPa to a void ratio of 0.637 ($D_r = 43\%$). On reloading, the denser specimen again reached a quasi-steady state of deformation but was able to sustain a shear stress of only 6.9 kPa at a mean normal stress of 12.1 kPa. On a state diagram (point B36a in Figure 6), the steady state point for the reload test plots far below the steady state line determined for virgin specimens including the initial test. This suggests that the structure of the reloaded specimen, while denser, was more conducive to pore pressure development during undrained loading.

A similar sequence of cyclic undrained torsional shear tests was also performed. A specimen was isotropically consolidated to a mean confining stress of 100 kPa and a void ratio of 0.682 ($D_r = 24\%$). The maximum shear modulus was determined to be 82.4 MPa. The specimen was then subjected to a cyclic shear stress of about 14 kPa. Strain softening behavior was initiated at a shear strain of about 0.3% after the application of 9 cycles as shown in Figure 7, at which stage the rate of pore pressure development increased sharply. A steady state condition was reached at a shear strain of about 5% and the test was terminated at a shear strain of 13%. The shear stress was then removed from the specimen and it was reconsolidated isotropically to a mean effective confining stress of 99.5 kPa. The void ratio at this stage had decreased to 0.641 ($D_r = 41\%$) and the maximum shear modulus was determined to be 77.5 MPa. It is noted that for this combination of confining stress and void ratio for Ottawa 20-30 sand, a G_{max} value of about 90 MPa would have been expected (Frost, 1989).

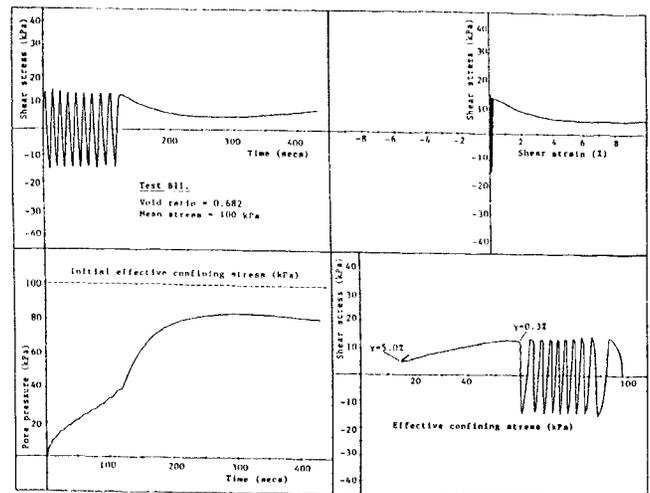


Figure 7 Undrained Cyclic Torsional Shear Test - Initial Loading

This reduction in G_{max} is attributed to a change in the grain structure during the initial loading, thus while the specimen is denser following reconsolidation, it has a structure which is less stiff in the direction of the applied torsional vibration loading. It was then intended to subject the specimen to a cyclic shear stress of about 14 kPa as in the previous test, however, during the first load cycle, strain softening accompanied by the development of large pore pressure was observed (Figure 8) with the specimen reaching steady state at a shear strain of about 3.5%.

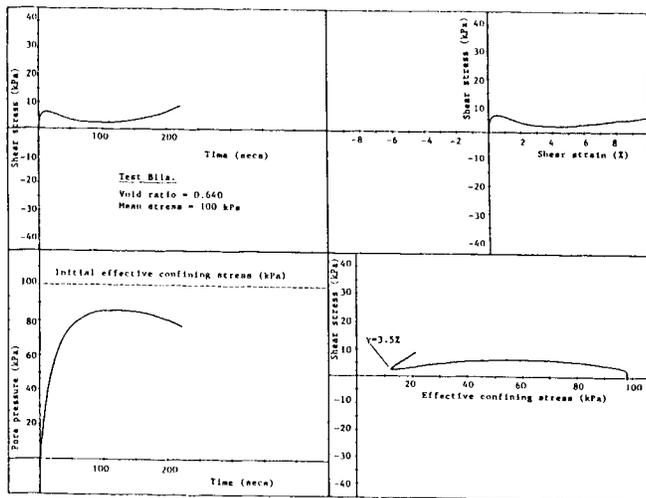


Figure 8 Undrained Cyclic Torsional Shear Test - Reloading

Several aspects of the results in Figures 7 and 8 are of interest. They confirm that the application of large shear strains during the initial loading resulted in a particle arrangement which while denser was more conducive to pore pressure build-up during subsequent undrained loading. During the initial loading, the specimen reached the steady state of deformation at which stage it was able to sustain a shear stress of about 5.2 kPa at a mean normal stress of 15 kPa. Following reconsolidation, the denser specimen was able to sustain a shear stress of only 2.6 kPa at a mean normal stress of 12 kPa. Again, it can be seen on the state diagram that the steady state test point (point B11a in Figure 6) for the reload test plots far below the steady state line determined for virgin specimens (Point B11). This difference is again of interest in view of the fact that it contradicts the assertion that the steady state condition is only a function of initial void ratio for undrained tests. The results of these tests further substantiate that the fabric may also be reflected in the behavior at steady state.

CONCLUSION

The results of undrained monotonic and cyclic torsional shear tests have been presented with a view to further evaluating the concept of structural collapse (Alarcon et al., 1988) and in particular the influence of such a collapse mechanism on the steady state line. The main conclusion based on evaluation of a large number

of undrained torsional shear tests is: The degree to which a specimen collapses, and hence its steady state position in the state diagram, is dependent on the loading conditions and the initial fabric of the sand. This finding is consistent with other published test results and suggests that further research is needed in order to obtain reliable values of insitu steady state strengths.

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