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EVALUATION OF THE FAILURE PROBABILITY OF SLOPES IN RESIDUAL SOIL FROM THE ABURRÁ VALLEY, MEDELLÍN COLOMBIA

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ABSTRACT

In recent years the evaluation of failure probability on slope stability analyses has become important because it has been presented as a representative method helping to complement the factor of safety. As part of this paper, it is proposed the evaluation of the failure probability in cut slopes on residual soils from the Aburrá valley. Statistical analyses were performed on the most important variables, considering continuous statistical distributions. Sensitivity analyses were performed to quantify their influence in the stability. As a result, some abacuses are developed in which the height and the slope angle are related to the failure probability and the factor of safety, proposed from the mean values of the chosen variables. To evaluate the failure probability, the Taylor series technique and the First Order Second Moment methodology were used. The obtained results allow the estimation of the probability that a slope of certain features may fail.

INTRODUCTION

An important aspect on the stability analysis of slopes is the determination of the factor of safety. For this purpose procedures are carried out, considering the equilibrium of forces and moments. Generally, the factors of safety used in geotechnical engineering for any design are based on experience and the study of the soil behavior. Nonetheless, it is common to give the same value of security to a given application like the stability analysis, without considering the degree of uncertainty associated to the procedure and the properties of the soils involved in the calculation (Griffiths, Fenton 2007).

The failure probability is important because it helps to complement the value of the factor of safety obtained for a particular case of study, which consider the statistical distribution of the most significant variables. To achieve that, is important to take into account the uncertainties in nature (Griffiths, Fenton 2007).

The probabilistic methods are not used frequently in the geotechnical engineering, in part because it is believed that analyses performed which consider the probability theory, require a computing time higher than usual and concern statistical concepts very complex (Duncan 2000).

Nowadays, due to the construction of real estate projects and

the high population living on the slopes of the Aburrá valley located in Medellín, Colombia, the stability analysis has become into an important issue. It is advocated that the studies of stability performed on the slopes, have a high degree of reliability enough to avoid unexpected behaviors on the slopes. These may lead to instabilities that cause damages and deaths, like those occurred in the 2008, where humans lives were lost in two large landslides presented in one of the main highways of the valley and in a disposal place of waste materials, or recently in the 2010 in Bello, a city which is part of the Aburrá valley, where a large landslide occurred, killing 82 persons.

The purpose of this article is to evaluate the failure probability in cut slopes on residual soils of the Aburrá valley. This is achieved by a detailed study of the Seismic Microzonification of the Aburrá valley and their soil properties database (Consorcio Microzonificación, 2006), where it were extracted the values of the mechanical properties of the variables mainly influencing the slope stability. Over these, statistical analyses were performed considering continuous statistical distributions (Griffiths, Fenton 2007) as sensitivity analyses, to watch the influence as well. Hence, some abacuses were obtained in which the results are shown as from the most likely values of the chosen variables.

This work is intended to present an evaluation method more complete for the slope stability analyses, by using rigorous

methodologies, based on probability theory and in the occurrence of the variables that influence the most the calculation of the stability, without abandoning the analysis of the factor of safety in favor of reliability.

METHODOLOGY

A geometry of study is defined, which refers to a cut slope frequently used in construction projects in the Aburrá valley. Four types of slope angles, 1V:1.5H, 1V:1H, 1V:0.50H and 1V:0.25H, are used. Also, five heights or elevations are defined, due to the frequency in the slopes in the Aburrá valley. The heights are in its order, 5 m, 10 m, 20 m, 35 m and 50 m. Fig. 1 shows a scheme of the geometry analyzed, where β y H are the inclination and the height of the slope, respectively.

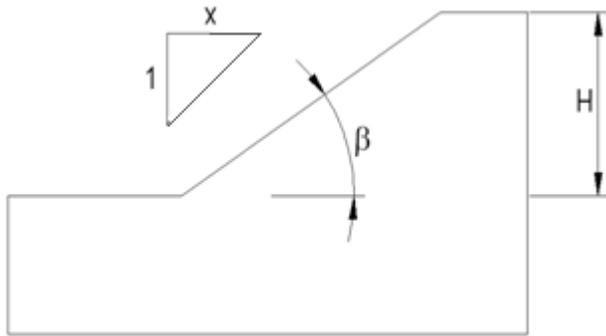


Fig. 1. Geometry of study

Based on the obtained information from the study Seismic Microzonification of the Aburrá valley (Consortio Microzonificación, 2006), mud flows and gneiss residual soils were used in this study. They were chosen due to their high frequency in the valley areas. The variables of more significance on slope stability analysis are the wet unit weight (γ_h), the effective angle of friction (ϕ') and the effective cohesion (c'). On these variables it was determined the parameters of the descriptive statistics like the mean value (μ), the standard deviation (σ) and the coefficient of variation (V), and they were assigned the probability density functions more adequate to observe and model their behavior. Sensitivity analyses were performed to study the influence of the variables on the calculation of the failure probability.

Failure probability

Analyses were performed to determine the reliability and the failure probability (P_f). For this, it was taking into account several procedures, the first order second moment (FOSM), the first order reliability method (FORM) and the Taylor series technique, which allow to obtain more information about an specific design condition.

In first place it is used the Taylor series technique (U.S. Army Corps. Of Engineers 1997, 1998; Duncan 2000) as an initial indicative of the value of failure probability and as a complement of the sensitivity analyses on the variables using the following equations:

$$\sigma_{FS} = \sqrt{\sum_{i=1}^n \left(\frac{\Delta FS_i}{2}\right)^2} \quad (1)$$

$$V_{FS} = \frac{\sigma_{FS}}{\overline{FS}} \quad (2)$$

Where $\Delta FS_i = (FS_i^+ - FS_i^-)$. FS_i^+ is the factor of safety calculated with the value of a variable n increased by one standard deviation from its mean value. FS_i^- is the factor of safety calculated with the value of a variable n decreased by one standard deviation from its mean value. At the moment of compute FS_i^+ y FS_i^- for a variable n , the rest of the variables must stay on their mean values.

\overline{FS} is the most likely factor of safety, σ_{FS} is the standard deviation of the factor of safety and V_{FS} is the coefficient of variation of the factor of safety.

First order second moment (FOSM)

The approximation to the first order second moment FOSM, provides an analytic approximation of the mean value and the standard deviation of the factor of safety, as a function of the mean value and standard deviation of several input factors and their correlations (Ang & Tang 1984).

Considering a function Y of random variables x_1, x_2, \dots, x_n ; so that $Y = f(x_1, x_2, \dots, x_n)$, the approximation of the mean value and variance of this function can be estimated by an expansion of the Taylor series of the function, around the mean values of the random variables and neglecting the terms of highest order. If the Taylor series is truncated in their lineal terms, it provides estimates of first order of the mean value and variance, as shown below:

$$\mu_Y = f(\mu_{x1}, \mu_{x2}, \dots, \mu_{xn}) \quad (3)$$

$$\sigma_Y^2 = \{b\}^T [C] \{b\} \quad (4)$$

In which the vector $\{b\}$ denotes $\partial Y / \partial x_i$, evaluated in the mean values of x_i , i.e.:

$$\{b\}^T = \{\partial Y / \partial x_1, \partial Y / \partial x_2, \dots, \partial Y / \partial x_n\} |_{\mu_x} \quad (5)$$

For the case of the slope stability Y is equal to the factor of safety.

The FOSM approximation provides estimates of the mean value and the standard deviation, which are not sufficient to evaluate the failure probability. To estimate the failure probability it must be assumed previously the probability density function of the factor of safety, defining a performance function $G(x)$, such that $G(x) \geq 0$ and $G(x) < 0$ means a satisfactory and unsatisfactory performance respectively, where x is vector of basic random variables that include resistance parameters, the geometry of the slope and the uncertainty of the model.

First order reliability method (FORM)

The FORM proposes a invariable definition of the reliability index (β) and it is referred to as a first order reliability method (Hasofer & Lind 1974). The reliability index is directly related to the failure probability and provides more information about the reliability than that given only by the factor of safety. It is defined as follows:

$$\beta = \frac{\mu_{G(x)}}{\sigma_{G(x)}} \quad (6)$$

The starting point is the definition of a performance function $G(x)$, in which x is the vector of basic random variables. For the case of study, it is defined as follows:

$$G(x) = FS - 1 \quad (7)$$

The performance function assumes a limit state in which, if $G(x)$ is less than zero it will a unsatisfactory performance on the slope, while if $G(x)$ is greater than zero, the slope will have a satisfactory performance. If the joint probability density function of all variables $f_x(x)$ is known, then the failure probability is given by:

$$P_f = \int_L f_x(x) dx \quad (8)$$

Where L is the domain of x in which the performance function $G(x)$ is less than zero. Generally, the above integral cannot be resolved analytically. In the FORM approximation, the vector of random variables x is transformed in the standard normal space U , where U is a vector of independent Gaussian variables, which mean value of zero and a standard deviation of one, and where $G(U)$ is a lineal function. Therefore the failure probability P_f is given by:

$$P_f = P[G(U) < 0] = \Phi(-\beta) = 1 - \Phi(\beta) \quad (9)$$

In which Φ is the standard cummulative normal distribution function.

When using the Taylor series technique the P_f is obtained considering a definition of the reliability index (β) for the

lognormal distribution, using the following formula:

$$\beta = \frac{\ln\left(\frac{\overline{FS}}{\sqrt{1 + V_{FS}^2}}\right)}{\sqrt{\ln(1 + V_{FS}^2)}} \quad (10)$$

where \overline{FS} is the most likely factor of safety and V_{FS} is the coefficient of variation of the factor of safety.

GEOLOGY OF THE VALLEY

The Aburrá valley is located on the north part of the central mountain range in Colombia. This corresponds to an elongated topographic depression. In terms of height there are variations in the mountains surrounding the valley which can reach up to 3000 m. The valley elevation is about 1500 m. There are also highland areas (Consortio Microzonificación, 2006).

The area of study includes the Aburrá valley and the highlands, where it has litological units with rocks of different age, origin and composition. In terms of age, they range from paleozoic rocks to quaternary deposits. In terms of origin and composition the valley has metamorphic rocks like schists, amphibolites and gneisses; igneous rocks like dunites, gabbros and basalts; volcanic-sedimentary rocks and alluvial deposits, besides of the anthropic fills (Consortio Microzonificación, 2006). Fig. 2 shows the geology of the Aburrá valley.

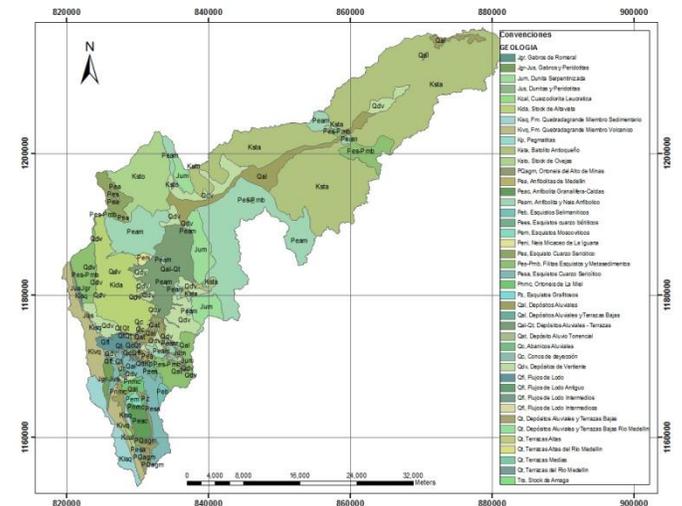


Fig. 2. Geology of the Aburrá valley

Mud flows and gneiss residual soils were used in this study. The first ones are formed by flows of several different ages whose soil particles showing different degrees of weathering. These are developed on the highest parts of the mountains, a saturation and a resistance loss, which make them vulnerable to phenomena like high precipitations or seismic movements. It has been found in the valley that deposits are quite thick with layers over 10 m with particles that not exceed the 35 % involved in a silt-clay matrix.

On the other hand, gneiss residual soils come from the weathering of the amphibole and feldspathic gneiss. These are characterized by producing soil layers of sandy texture to clay-loam texture with presence of rock fragments which can keep an inherited structure from bedrock. They are wet and they have plasticity that range medium to high, in part by the presence of feldspar which gives the soil a fine behavior. The samples found in the field usually have features of saprolitic soils. In the valley it has been found uniform layers about 20 m which classify as IC in the Deere & Paton classification. After 20 m the classification may vary to IIA having particle percentages less than 30 percent in some places.

Variables of study

The most important parameters of the descriptive statistics were determined on the variables of study, which are the wet unit weight (γ_h), the effective angle of friction (ϕ') and the effective cohesion (c'). In terms of the variables of study, it was only possible to obtain the values of the wet unit weight and a few values of cohesion and the friction angle. Hence, the SPT records of the Microzonification studies were used and it was determined the angle of friction following the methodology proposed by Gonzalez, 1999. It was used the formula by Kishida for the estimation of the angle of friction, considering a hammer efficiency of the 45° as:

$$\phi' = 15^\circ + (12,5 \cdot N_{45})^{0,5} \quad (11)$$

For the soils derived from mudslides it were taking into account 13 values of effective cohesion, 114 values of the wet unit weight and 423 values of the standard penetration resistance, N_{SPT} . For the gneiss residual soils, it were taking into account 9 values of effective cohesion, 61 values of the wet unit weight and 450 values of the N_{SPT} .

Given that the records of cohesion for both soils were scant, it was necessary to use a methodology proposed for the determination of the standard deviation of the data known as the Three-Sigma Rule (Dai & Wang 1992). This assumes that the 99.73 % of all values of a normally distributed variable fall within three standard deviation of the average. The rule is give by:

$$\sigma_{(3)} = \frac{HCV - LCV}{6} \quad (12)$$

In which HCV and LCV is the highest and lowest conceivable value of the variable respectively. This rule can only be applied when the number of data available is limited. For the use of equation (11) is not necessary the variable should be normally distributed. Table 1 shows the statistics parameters for the variables. These refer to the mean value (μ), the standard deviation (σ) and the coefficient of variation (V), besides the standard deviation according to equation (11)

($\sigma_{(3)}$) and the coefficient of variation for this standard deviation ($V \sigma_{(3)}$)

Table 1. Statistic parameters of the variables

Variable	γ_h (kN/m ³)		c' (kPa)		ϕ' (°)	
	Mud	Gneiss	Mud	Gneiss	Mud	Gneiss
μ	17.0	17.8	18.8	25.4	30.2	31.9
σ	1.5	2.9	9.0	11.8	5.2	5.6
V	9%	16%	48%	46%	17%	18%
$\sigma_{(3)}$	-	-	5.2	5.5	-	-
V $\sigma_{(3)}$	-	-	27%	22%	-	-
Max	20.9	27.1	36.0	13.6	49.2	47.6
Min	12.9	14.4	5.0	3.8	19.3	19.6

Accordingly to the obtained results it can be said the values of γ_h y ϕ' have the lower V. Harr 1984 y Kulhawy 1992, indicate that the expected ranges of V for the unit weight and the angle of friction go from 3 % to 7 % and from 2 % to 13 %, respectively. For the variable c' the values are quite high, above the 40 % for a standard deviation σ , and greater than 20 % for the three-sigma rule ($\sigma_{(3)}$).

The above is because the statistic sample of the cohesion values is small, hence it is not possible obtain a value with less uncertainty. By the use of the three-sigma rule it was obtained a standard deviation, even less of the range of the sample. The range is defined as the difference of the highest and lowest value of the sample.

SENSITIVITY ANALYSIS

It was carried out on the variables for two of the construction conditions given; the first one for a 1V:1.5H slope and for heights of 5 m and 50 m, and the second one for a 1V:0.25H slope for the same heights. A water table position (WT) was defined, which is inferred and conditioned by the geometry of the slope. In this way, it is possible to observe the behavior of the factor of safety (FS) against the variation of the variables.

In Fig. 3 to Fig. 6, the tendencies of variation of the FS for each variable of the gneiss residual soil are shown. Each value of FS it is calculated by performing a variation of one of the study variables, leaving the others constant, on their mean values. It was established for the x-axis of the charts a percentile range of the variables. This was done to compare the variables on the same scale of fluctuation of the FS. The percentile range it is defined as a percent which represents the occurrence of the value of a variable in respect to his mean value (μ). If the percentile range is equal to 0 % or 100 %, the variable takes his minimum and maximum value respectively. The mean value of the variables (μ) shown in Table 1 is the 50 %.

The obtained tendencies are intercepted in a percentile range of 50 %, which indicate that in this point of interception where the mean values of the variables are used, the mean factor of safety (FS) it is obtained. For the soils from mud flows, it was obtained a similar behavior to the gneiss residual soil

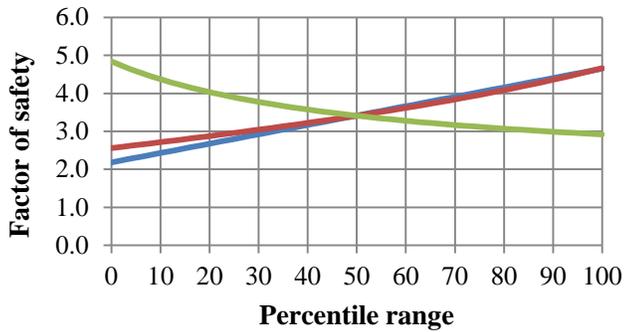


Fig. 3. Sensitivity analysis gneiss slope, $h=5\text{ m } 1V:1.5H$

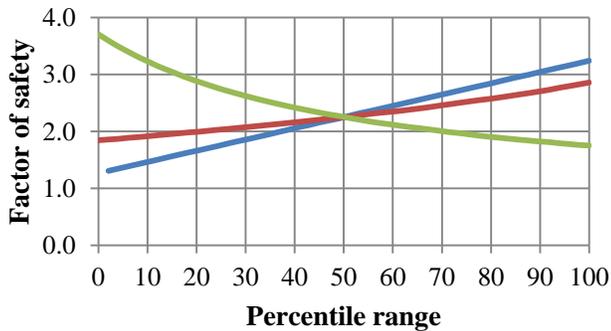


Fig. 4. Sensitivity analysis gneiss slope, $h=5\text{ m } 1V:0.25H$

For the case of Fig. 3 and Fig. 4, the inclination of the effective cohesion variable in color blue is higher than the inclination of the variable for the effective angle of friction in red. For the wet unit weight, the inclination is negative. The above means that the significant changes in the FS will be given by a change in the values of the effective cohesion.

From Fig. 5 and Fig. 6 the inclination in the tendency for the effective angle of friction is higher than the variable effective cohesion and wet unit weight. The influence of the variable wet unit weight is imperceptible, where it has a tendency almost flat when there is an increase in the percentile range. The above means that when increasing the height of the slope, the FS is mainly influenced by a change in the effective angle of friction.

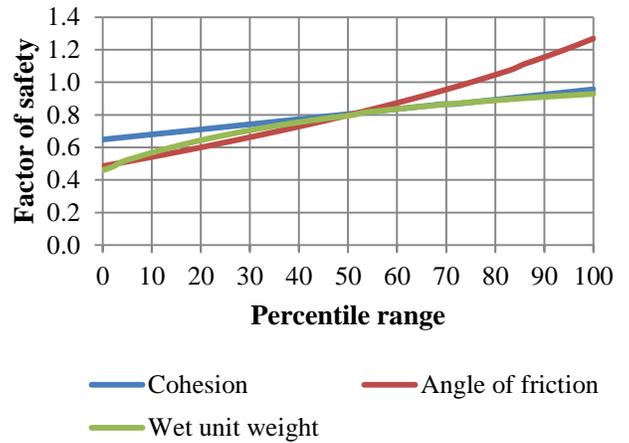


Fig. 5. Sensitivity analysis gneiss slope, $h=50\text{ m } 1V:1.5H$

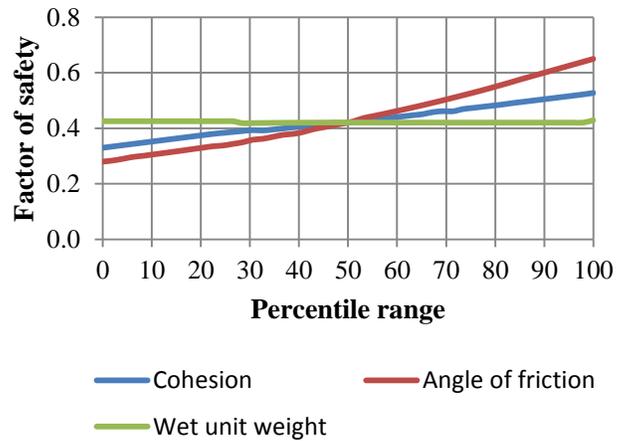


Fig. 6. Sensitivity analysis gneiss slope, $h=50\text{ m } 1V:0.25H$

Accordingly to the obtained results, it was decided to make special emphasis on the variables effective cohesion and effective angle of friction for the calculation of the failure probability.

Variation of the water table (WT)

One factor to consider in obtaining the FS is the variation of the water table (WT). Due to high rainfall and infiltration processes the value of the FS can change significantly for a slope. Taking into account that sometimes it is not possible to know the exact position of the water table (WT), it was performed a sensitivity analysis considering a normalized parameter of the pore pressure head (μ_n) and it is defined as follows:

$$\mu_n = \frac{y_w}{y} \quad (13)$$

In which y_w is the height between the minimum value of WT and the mean value of the WT, which as previously stated, was inferred and conditioned by the geometry of the slope. y

is the height between the minimum and maximum value of the WT. Fig. 7 shows a schematic of the heights.

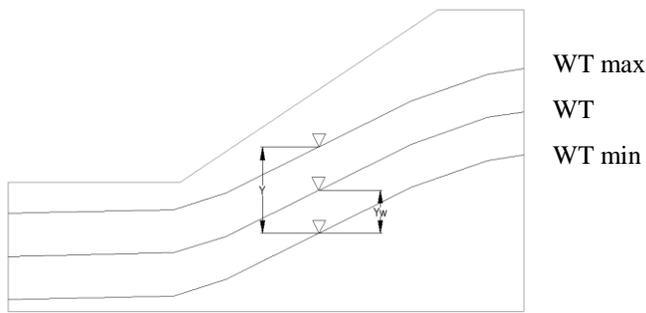


Fig. 7. Height scheme of water table (WT)

A μ_n value between 0 and 1 represents the minimum and maximum boundary of the WT, i.e. the lower and higher geometric position of the water table, respectively. For the case of study, the maximum boundary of the WT is defined as if the slope is saturated. In this way it is calculated the minimum boundary of the WT by assuming that the value of μ_n is 0.5, implying that the difference between a point on the maximum WT against other one located on the medium WT is similar to that which exists between the medium WT and the minimum WT.

Fig. 8 and Fig. 9 show the variation of the WT position against the FS for the gneiss residual soil for a 1V:1.5H slope with heights of 5 m and 50 m. The performed analysis is carried out only on these slopes, just to observe the behavior of the FS against a variation of the WT position.

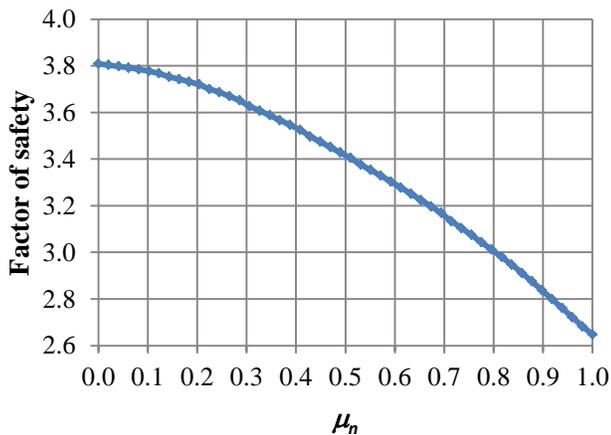


Fig. 8. Variation FS vs μ_n gneiss slope, $h=5$ m 1V:1.5H

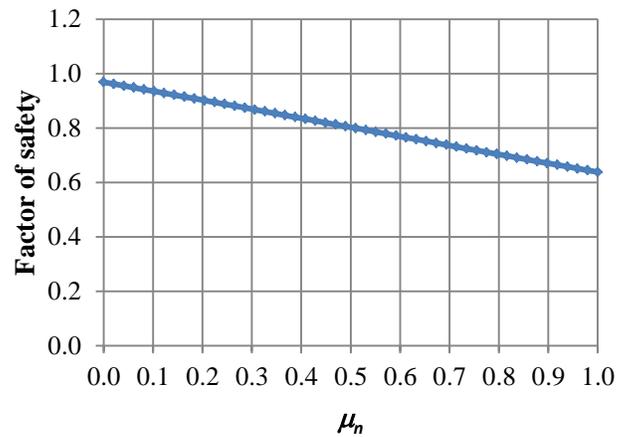


Fig. 9. Variation FS vs μ_n gneiss slope, $h=50$ m 1V:1.5H

In Fig. 8 and Fig. 9, it is observed that with an increase of the μ_n parameter there is decreasing of the FS. For the case when the slope has a height of 5 m, it can be seen that when μ_n range from 0.5 to 1, there is a significant variation around the FS, with a decreasing of 0.8 in its value. When the slope has a height of 50 m, the decreasing in the FS value is around 0.2 for the same change between a range from 0.5 to 1.

A variation on the WT can change the design condition of slopes and affect his behavior in a significant way. It should be noted that a generalization of medium WT, also deterministic, adds an important uncertainty to the stability analysis, which sometimes is not possible to quantify and eliminate. For the above case it is observed that a slope saturation in which μ_n is 1, is an undesired situation that produces low FS.

FAILURE PROBABILITY EVALUATION

An evaluation of the failure probability by using the FOSM and FORM methodologies and Taylor series technique it is performed, to observe the response of the slopes against the construction conditions.

Taylor series technique

An analysis it is performed using the Taylor series technique for the slopes of study of soils derived from mudslides and gneiss residual soils. The analysis was done for a 20 m height and for the inclines mentioned at the beginning. Table 2 and Table 3 shown the obtained results for the soil from mud flows and the gneiss residual soil, respectively, where it can be seen the results of the FS for the different slope angles and their P_f associated. From Table 2 and Table 3 it is also observed that for the both types of soils there is a gradual decreasing of the FS value with an increasing on the P_f , when the slope angles is increasing as well. This result indicates how sensible the FS against a variation in the slope angle is.

For the obtained values of ΔFS , it is observed that the highest magnitudes of these are for the effective angle of friction (ϕ'). This gives an indication of the contribution of the variables in the P_f calculation, where the angle of friction is the most significant variable in his calculation, following by the effective cohesion and the wet unit weight. In this sense, the Taylor series technique can be seen as a sensitivity analysis.

Table 2. Taylor series technique evaluation for the soil from mud flows

Slope Angle	FS	c' (kPa)			γ_h (kN/m ³)			ϕ' (°)			σFS	V	P_f
		FS+	FS-	ΔFS	FS+	FS-	ΔFS	FS+	FS-	ΔFS			
1V:1.5H	1.14	1.25	1.04	0.21	1.16	1.13	0.03	1.31	1.00	0.31	0.19	0.17	24.5%
1V:1H	0.96	1.06	0.87	0.20	0.96	0.95	0.01	1.09	0.84	0.25	0.16	0.17	62.8%
1V:0.5H	0.76	0.83	0.68	0.15	0.75	0.76	0.01	0.86	0.66	0.20	0.12	0.16	96.5%
1V:0.25H	0.67	0.73	0.61	0.12	0.67	0.67	0.00	0.77	0.58	0.19	0.11	0.17	99.3%

Table 3. Taylor series technique evaluation for the gneiss residual soil

Slope Angle	FS	c' (kPa)			γ_h (kN/m ³)			ϕ' (°)			σFS	V	P_f
		FS+	FS-	ΔFS	FS+	FS-	ΔFS	FS+	FS-	ΔFS			
1V:1.5H	1.34	1.45	1.23	0.23	1.35	1.31	0.04	1.53	1.17	0.36	0.21	0.16	3.9%
1V:1H	1.13	1.23	1.03	0.20	1.12	1.13	0.01	1.28	1.00	0.29	0.17	0.15	22.8%
1V:0.5H	0.89	0.97	0.81	0.16	0.87	0.90	0.04	1.00	0.78	0.22	0.14	0.16	79.2%
1V:0.25H	0.78	0.84	0.72	0.12	0.76	0.78	0.02	0.89	0.68	0.21	0.12	0.16	95.0%

The latter procedure allows obtaining the P_f and also setting a relation of this and the value of FS but it has the disadvantage that is made only for a slope of well known features, in this case, a 20 m height slope. Therefore, if is desired to set a new relation between the P_f and the FS for another slope height, new calculations must be made implying more time than expected. In this way and for the study purpose, the method used is not the best.

FOSM & FORM evaluation

It is performed a FOSM and FORM evaluation. The methodologies of the FOSM and FORM were used on the proposed slope, considering the heights and slope angles mentioned, besides the statistical values of the variables γ_h , c' y ϕ' . At the end it was possible to obtain some abacuses for each type of soil, which set a relation of the P_f against the height and angle of the slope. It was also generated abacuses that set a relation between the FS and the height and angle of the slope.

Accordingly to the FORM methodology, it is defined a parameter δx which represents the variation on the FS as a function of the variables of study around their mean values. Equation (5) is a definition of this. For the case of study it is set a value of 10 % for δx . If the range of values between the FS is going to vary can be predicted, a value of δx more accurate can be set.

Having statistical values more accurate of the variables of most influence in the calculation of FS, may causes the value of V decreases considerably and therefore, the P_f . This is because the calculation of the reliability index (β) it depends of the standard deviation of the FS, and this one in turn, it depends of the standard deviations of γ_h , c' and ϕ' .

It was determined the FS for the mean values of the variables, and then the values of FS against a change of δx on the variables of study. In this way, it is possible to set the difference between the FS for the mean values of the variables against the obtained FS for a δx variation. The difference is defined as follows:

$$\delta FS_i = FS_{\text{mean}} - |FS|_{\delta x_i} \quad (14)$$

In which FS_{mean} is the factor of safety when the variables are in their mean values. $|FS|_{\delta x_i}$ represents the factor of safety when there is a δx_i variation on the mean value of variable x_i . There will be as many differences as variables are used. Then it was determined the variation of these differences (δFS_i) against δx .

Based on these variations and considering the variances of the variables of study, it was obtained the variance of the FS_{mean} and finally, the reliability index for the performance function defined in equation (7). Based on the equation (9) the value of P_f was determined. Fig. 10 and Fig. 11 show the abacuses which relate the P_f and the FS with the height and angle of the slope for the soil from mudflows and the gneiss residual soil, respectively. In this way, depending of the type of soil, it is chosen a height and angle of inclination, and the P_f and FS is determined.

From Fig. 10 and Fig. 11 is possible to establish that combinations of small heights and angles produce values of P_f

low. As the height is increased and higher slope angles are used, values of high P_f are produced.

For the case of the gneiss residual soil there is a behavior more reliable around the heights of study than shown for the soil

derived from mudslides. The above is more appreciable for slopes with angles of inclination of 34° and 45° and heights over 10 m. In figures where the result is the obtention of the FS it is observed a decreasing of the factor of safety when the height of the slope is increasing.

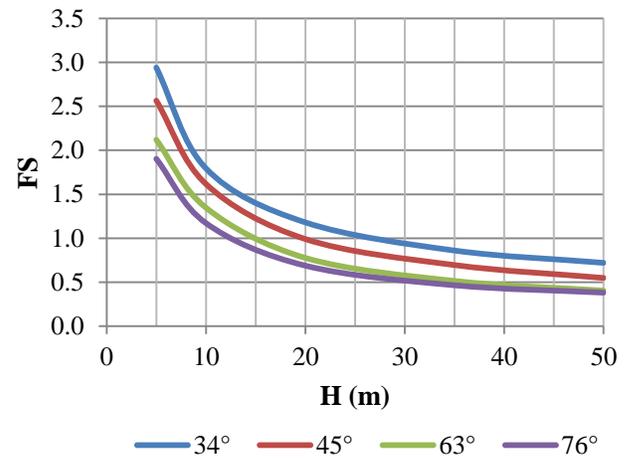
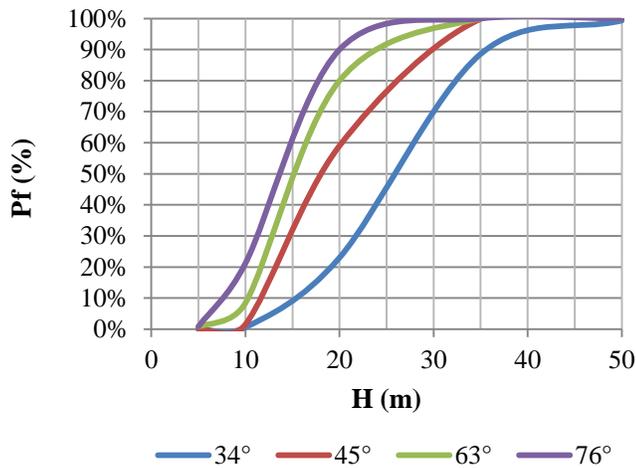


Fig. 10. Relationship between the slope height and its P_f and FS for soil from mud flows

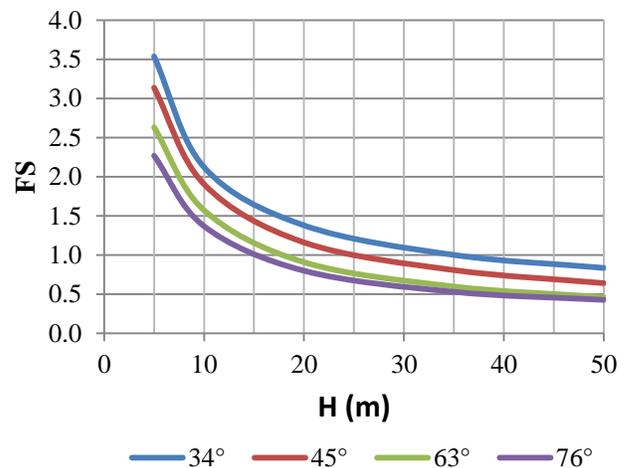
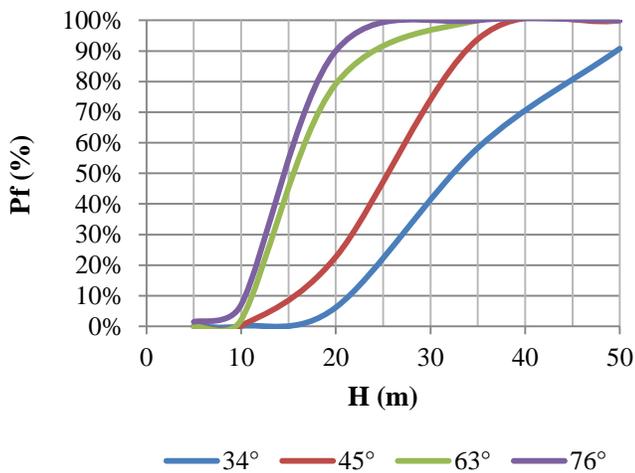


Fig. 11. Relationship between the slope height and its P_f and FS for gneiss residual soil

The studied soils give similar tendencies in their assessment for the P_f . For the gneiss residual soil there are values of FS higher than those obtained for the soils derived from mudslides. This allows to have heights higher for the gneiss soil than the mudslides soil, given a same value of P_f .

Given certain conditions of design is possible to determine in simultaneous way the value of P_f and FS based on the slope height. The performed analyses allow to corroborate the significant contribution of the effective cohesion when the slope height is small, and the effective angle of friction when the slope height is quite big.

Considering the implications that the increasing of height in the slope stability analysis has for the conditions of study, if a design is performed on a gneiss soil, there will be a less conservative solution than any given for a mudslide soil.

CONCLUSIONS

It was determined the probability of failure P_f , of the slopes for the soils studied, by the use of methodologies of approximation of the mean values of the most important variables in the FS calculation. The P_f is given as a function of the height and incline of the slope and the statistical

parameters of the most important variables.

According to the sensitivity analyses performed and the different methodologies of probabilistic estimation applied, it is observed that the variables which have more influence in the P_f are mainly the effective cohesion and the effective angle of friction, in complement with the height and the slope angle. For lower heights to 10 m, the cohesion has significant influence while for heights over 20 m the friction angle becomes important.

The use of the FOSM and FORM required the setting of the failure surface when the stability analyses were performed. In this way the results are significant despite that variation in the δx parameter can affect the FS and therefore, change the coordinates of the failure surface.

The performed work allows to estimate on often design conditions the P_f for a FS associated. The estimation is performed in reliable and simple way and do not require a significant effort compared to the done in the calculation of the FS. In this way the persons in charge of a design for slopes, would be able to perform approximate evaluations but useful, working with the same quantity of data and applying the same concepts used in the geotechnical engineering.

It is intended with this work to take the P_f as a measure of the reliability. In this way, any kind of answer which satisfying a condition of study is complemented.

LIMITATIONS

There are three limitations from the study; the first one it is related with the lack of data of the variables of study, particularly, the effective cohesion and the effective angle of friction. For the case of the effective cohesion, the absence of data impacted in a important way the calculation of P_f because of the high standard deviations. For the case of the effective angle of friction, using a correlation for the N_{SPT} values, provides uncertainty in the obtained results.

Although is not necessary to have an excessive quantity of data for the variables of study, it is important that on the obtained data exists certainty in his origin, and specially on the magnitudes of the values this variables can take. The engineer's judge plays an important role in choosing the most likely values of the parameters.

The second limitation refers to the fact of considering a water table WT, conditioned by the slope geometry. The WT geometry must respond to a variation of the piezometric values on the slope and it has to be estimated from instrumentation records. The stability analyses have to be performed considering these records, as well as a statistical analysis has to be done on the water heights measured.

The last limitation it is referred to the occurrence of uniform

thick layers. If there is any variation in the stratigraphy that may lead to different slope conditions, the results obtained, would be far from the estimate of the failure probability and will not be representative for the assumptions made in this study.

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