A heuristic algorithm for determining a constructive suboptimal solution to the combinatorial problem of facility allocation

Harry Kerry Edwards

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A HEURISTIC ALGORITHM FOR DETERMINING A CONSTRUCTIVE
SUBOPTIMAL SOLUTION TO THE COMBINATORIAL
PROBLEM OF FACILITY ALLOCATION

BY

HARRY KERRY EDWARDS, 1940

A DISSERTATION
Presented to the Faculty of the Graduate School of the
UNIVERSITY OF MISSOURI - ROLLA

In Partial Fulfillment of the Requirements for the Degree
DOCTOR OF PHILOSOPHY
in
MATHEMATICS
1970

T2367
c.1
61 pages

193937
ABSTRACT

The major problem in plant layout is to determine the most economical relative location of facilities. There are two distinct types of suboptimal solutions to this combinatorial problem: construction and improvement. The writer has developed Modular Allocation Technique (MAT) which is the first useful construction suboptimal technique.

The MAT general algorithm, a theorem relating the MAT solution to the optimal solution and an example problem are given.

A computer program has been written that will apply MAT to the allocation problem for a maximum of 40 facilities. Results are given to demonstrate how MAT solutions may be used as initial assignments for the improvement techniques. MAT solutions are compared with other allocation techniques with respect to solution quality and computer time. The various options of the MAT computer program are given to illustrate the flexibility of the technique.
ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation to Dr. Billy E. Gillett for his help and guidance in the writing of this dissertation and for serving as an advisor throughout the majority of the author's years of graduate study.

The author also extends his appreciation to Professor Ralph E. Lee for everything he has done to make this dissertation possible and for consenting to serve as second reader. The time and efforts of the other members of the author's committee, Dr. Lee J. Bain, Dr. Anthony J. Penico, and Dr. Hughes M. Zenor, in the author's behalf are greatly appreciated.

A very special thanks to my wife Cathie, her parents, and my own parents for their understanding, encouragement, and sacrifices during my years as a graduate student and to Linda Giem for an excellent job of typing this dissertation.
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I. INTRODUCTION

The allocation of facilities problem consists of assigning $n$ available facilities to $n$ fixed locations with each facility assigned to one and only one location. Hence, there are $n!$ possible assignments. Each pair of locations has an associated non-negative cost factor and between each pair of facilities is a non-negative intensity or weighting coefficient. The value of assigning a pair of facilities to a pair of locations is the product of the cost factor for the pair of locations and the intensity or weighting coefficient for the pair of facilities that have been assigned to the locations. The total value of the assignment is the sum of the above products over all facility-location pairs. The assignment with the minimum non-negative value is called the optimal assignment or arrangement.

In an industrial concern, for example, a facility may consist of one machine tool, a group of machines which must operate as a unit, a department within a plant, or the plant itself if a group of plants is to be allocated to a given set of locations. The intensity coefficient would be the number of loads transported per unit time between the pairs of machine tools, groups of machines operating as a unit, departments or plants. The cost factor for a pair of locations could be the linear distance between the locations but not necessarily since obstacles between the
locations could create computational problems. The gauges on the instrument panel of a space capsule could also be considered as facilities with the intensity coefficient between each pair of gauges being the desirability of having two gauges close to each other to facilitate the smooth operation of the capsule. The cost factor for a location pair could be the distance the eyes of the operator must travel to read the gauges placed in the two locations.

In order to produce an optimal arrangement it is necessary to consider an objective function and its attributes. To create an objective function a measure of effectiveness is needed. The measure of effectiveness used in this dissertation will be the sum over all pairs of facilities of the product of the linear distance between the locations where the two facilities are located and the loads per unit time transported between the two facilities. E. S. Buffa (1)*, indicates that "This measure of effectiveness closely approximates material-handling cost. The variable costs associated with a material-handling operation (mainly labor plus power) are related to distance." According to R. W. Mallick (2) "the question of whether plant layout determines material-handling or material-handling determines plant layout is debatable. The fact is that an improvement

* All numbers (a) refer to the bibliography while the numbers (a,b) refer to equations.
made in the layout invariably influences material-handling methods. It seems to matter but little which change comes first, so long as the end result is the same." Thus this measure of effectiveness is related to an actual cost possibly up to a constant of proportionality.

To formalize the objective function using the above measure of effectiveness, E, we have

\[
\text{Minimize } [E = \sum_{i<j}^{n} \sum_{j}^{n} f_{ij}g_{ij}],
\]

where:

- \(f_{ij}\) = loads per unit time to be transported between facilities at locations \(i\) and \(j\), and
- \(g_{ij}\) = linear distance between locations \(i\) and \(j\).

The only constraint imposed on the problem is that certain facilities may need to be assigned to certain specified locations.

It has been concluded by Nugent, Vollman, and Rumml (3) that there do not exist any computationally feasible optimal-producing procedures and that suboptimal procedures seem to be the only compromise between time and a reasonable solution to the combinatorial problem.

There are two types of suboptimal producing procedures; construction and improvement. A construction technique produces a suboptimal solution to (1.1) based on the loads between all pairs of facilities per unit time and the linear distance between all pairs of locations where the
facilities are to be located. An improvement technique is one which requires, in addition to the data utilized in a construction technique, an initial assignment so that the technique may improve upon the initial assignment to reach a suboptimal solution to (1.1).

Presently there are a number of useful improvement techniques available but the initial assignments for these procedures must be chosen randomly. A construction technique is needed to eliminate the guesswork involved in generating an initial assignment.

The purpose of this dissertation was to develop a useful construction technique. Using the experimental data in (3), results are given to show how the construction technique may be used to generate initial assignments which are then used by an improvement technique. The construction technique was compared with other allocation techniques in (3) with respect to solution quality and computer time. Various options of the construction technique are given to illustrate the flexibility of the technique. This construction technique, developed by the author, was called Modular Allocation Technique or equivalently, MAT.
II. REVIEW OF LITERATURE

Since there are only a finite number of ways, namely n!, of assigning n facilities to n locations, enumeration of all possible assignments must be considered as a method of solution. Enumeration can quickly be eliminated for all problems of interesting size. If 20 facilities are to be allocated to 20 locations arranged in four rows and five columns, there are 20! or approximately $2.4 \times 10^{18}$ possible arrangements. This number can be reduced to approximately $6.0 \times 10^{17}$ by eliminating symmetric solutions but enumeration is still not feasible. Just as enumeration can be eliminated there is no known algorithm that can feasibly be used to produce the optimal arrangement according to Buffa and Armour (4).

There are two semi-enumerative techniques that produce the optimal assignment. The first, developed by Gilmore (5) and Lawler (6), independently, is severely limited by computation. Gilmore (5) states that his algorithm is "Probably not computationally feasible for n much larger than 15".

The second semi-enumerative technique given by Gavett and Plyter (7) was developed by adapting a procedure given by Little, Murty, Sweeney, and Karel (8) for the traveling salesman problem. The largest value of n which could be computationally handled was n=8 for this method and 42 minutes of IBM 7074 computer time was consumed to yield the
solution. Again, this technique is computationally infeasible for all but the smallest problems. Koopman and Beckmann (9) reveal some results which tend to confirm the hypothesis that there is no formal mathematical technique which can be applied to equation (1.1) to yield the optimal assignment.

According to Nugent, Vollman and Ruml (3) "One is forced to conclude that no computationally feasible optimal-producing procedures exist at present. Interest must focus on suboptimal procedures." Only improvement techniques were considered for further testing in (3).

In contrast to this point of view, this author feels that there are two construction techniques that are worthy of further testing. The first is MAT, created by the author, which is completely covered in Chapter III.

The second method is Systematic Plant Layout given by Muther (10). This is probably the most widely used and practical method of facility allocation in existence. Computerization of this method was attempted by Lee and Moore (11) but with little success. Since it has not been computerized and does rely upon a different type of input data from the other allocation techniques, this method of allocation of facilities will not be considered to any greater extent in this dissertation.

The improvement procedures selected for testing in (3) were Hillier's 1963 Procedure (12), Hillier's and Connor's 1966 Procedure (13), CRAFT (4), and Biased Sampling
The above procedures were tested on eight problems of different sizes: 5, 6, 7, 8, 12, 15, 20, and 30 facilities. Five starting solutions were generated at random for each of the eight test problems and each of the above procedures was run on all eight problems with all five starting solutions. In Chapter III MAT solutions are compared to the solutions yielded by the above techniques based on the common data given by Nugent, Vollman, and Rumml (3). MAT solutions are also used as input to the CRAFT procedure. Results of this technique are also given in Chapter III.

There are two other improvement techniques which have not been fully developed for release. Automated Layout Design Program (ALDEP) which was written by Seehof and Evans (14) of IBM and Systemized Hospital And Plan Evaluation (SHAPE) which was developed by Gathers and Gathers (15). Since complete documentation and computer source decks are not available for these techniques, results for comparison of these techniques with the other techniques in this thesis were not available.

Fraley (16) gives some results concerning the practical aspects of utilizing computerized algorithms for plant layout. Suboptimal solutions to (1.1) are of interest to industry and Fraley (16) has classified a number of techniques with respect to applicability under specified conditions.
III. DISCUSSION

While (1.1) is the general form of the objective function for the allocation of facilities problem, the objective function most frequently and practically used in the assignment of facilities to locations problem is as follows:

Minimize \[ E = \sum_{i<j}^{n} \sum_{i}^{n} a_{ij} m_{ij} x_{ij} \]  

where:

- \( a_{ij} \) = loads per unit time to be transported between facilities at locations \( i \) and \( j \), \((a_{ij} = 0 \text{ for } i=j)\) and
- \( m_{ij} \) = cost per load per unit distance to transport material between facilities at locations \( i \) and \( j \), \((m_{ij} = 0 \text{ for } i=j)\) and
- \( x_{ij} \) = distance between locations \( i \) and \( j \), \((x_{ij} = 0 \text{ for } i=j)\).

This form of the objective function satisfied the requirements which were set forth in Chapter I and to show the relationship between (1.1) and (3.1) let \( f_{ij} = a_{ij} m_{ij} \) and \( g_{ij} = x_{ij} \).

Definition: The most economical location of facilities is the arrangement that places the facilities in locations relative to each other, such that (3.1) is satisfied. This will be called the optimum arrangement or assignment.
The restriction on $E$ where $i < j$ implies that the total flow between the facilities at locations $i$ and $j$ is expressible as a single quantity. This is referred to as a symmetric problem and only symmetric problems are considered in this dissertation.

Another constraint usually imposed on $E$ is $m_{ij} = 1$ (for all $i \neq j$), i.e., the cost per load per unit distance to transport material between facilities at locations $i$ and $j$ is constant. Hence, the minimization is with respect to distance. When $m_{ij} \neq$ constant (for all $i \neq j$) the objective function becomes responsive to the use of different material-handling systems for different materials. This form of the objective function is actually the one used by Armour and Buffa (19). The option of a non-constant $m_{ij}$ is also available in the Modular Allocation Technique (MAT) and the above constraint of $m_{ij} = 1$ is imposed only for the purpose of comparing MAT results with the results given by Nugent, Vollman, and Ruml (3).

The Modular Allocation Technique (MAT) is based upon the following theorem:

Theorem I: Suppose $u_i, v_i, i=1,\ldots,m$ are given.

Then $\sum_{i=1}^{m} u_i v_i$ will not decrease under any interchange of the order of two elements in $(u_1,\ldots,u_m)$ if and only if $(u_k - u_l)(v_k - v_l) \leq 0$ for all $k, l=1,\ldots,m$. 
Proof: The proof of the above theorem is an obvious consequence of the identity

\[(u_k v_k + u \cdot v \cdot) - (u_k v_k + u \cdot v \cdot) = (u_k - u \cdot)(v_k - v \cdot)\]

The above theorem states that the sum of pairwise products of two sequences of real numbers is minimized if one sequence is arranged in nondecreasing order and the other in nonincreasing order. [(7), (18)]

Before demonstrating how this theorem and MAT are related consider the following general MAT procedure.

A. General MAT Procedure

Assume that the distance between any two locations and the number of loads between all pairs of facilities for a given time period are known. Given the matrix

\[
A = \begin{bmatrix}
0 & a_{12} & \ldots & \ldots & a_{1n} \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
a_{n1} & \ldots & a_{n,n-1} & 0 \\
\end{bmatrix}_{n \times n}
\]

where \(a_{ij}\) represents the loads transported between facilities \(i\) and \(j\) in both directions in unit time (\(a_{ij} = a_{ji}\)) and the distance matrix
where \( x_{ij} \) represents the distance from location \( i \) to location \( j \) (\( x_{ij}=x_{ji} \)) consider the assignment of the \( n \) facilities to the \( n \) locations such that the total distance traveled will be a minimum.

**Step 1**

If certain facilities must necessarily be assigned to specific locations, make these assignments and go to Step 2.

**Step 2**

Order the \( a_{ij} \) elements from largest to smallest \((i<j)\) and order the \( x_{ij} \) elements from smallest to largest \((i<j)\). Go to Step 3.

**Step 3**

Let \( m = n(n-1)/2 \) and let \( S \) be an \( m x 2 \) matrix consisting of the row and column subscripts of the ordered (largest to smallest) \( a_{ij} \) elements. Likewise, let \( T \) be an \( m x 2 \) matrix consisting of the row and column subscripts of the ordered (smallest to largest) \( x_{ij} \) elements.
**Step 4**

Beginning with the first row of $S$ examine each row successively for one of the four following conditions:

(a) Neither of the facilities corresponding to a given row has been assigned and more than two facilities remain to be assigned. Select these facilities for assignment and go to Step 5.

(b) Neither of the facilities corresponding to a given row has been assigned and they are the only two facilities that remain to be assigned. Go to Step 9.

(c) Only one of the facilities corresponding to a given row has been assigned. Go to Step 10.

(d) Both of the facilities corresponding to a given row have been assigned. If all of the $n$ facilities have not been assigned, examine the next row of $S$ and go to Step 4a, 4b, 4c, or 4d as dictated by the outcome of the examination; otherwise, the MAT assignment is complete.

**Step 5**

Continue to examine the remaining rows of $S$, in order, until a row is found that contains one of the facilities selected in Step 4a. Select the facilities of this row for assignment and go to Step 6.
Step 6

Beginning with the first row of T examine each row successively for a pair of locations such that neither has a facility assigned to it. Select this pair of locations to make assignment to. Go to Step 7.

Step 7

Continue to examine the rows of T for a pair of locations containing one of the locations from the pair selected in Step 6. Select this pair of locations to make assignments to. Go to Step 8.

Step 8

Suppose facilities i and j are selected in Step 4a and facilities m and j in Step 5. Further, suppose locations p and q are selected in Step 6 and locations q and r in Step 7. Make the assignment:

<table>
<thead>
<tr>
<th>Facility</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>p</td>
</tr>
<tr>
<td>j</td>
<td>q</td>
</tr>
<tr>
<td>m</td>
<td>r</td>
</tr>
</tbody>
</table>

and go to Step 11.

Step 9

Arbitrarily assign the two remaining facilities to the two available locations. Compute the total distance traveled for this assignment. Interchange the assignment
of the facilities to the locations. Compute the total
distance traveled for this assignment. Retain the assign-
ment with the minimum total distance traveled. Go to
Step 11.

**Step 10**

Beginning with the first row of T examine each row
successively for a pair of locations such that one of the
locations has the assigned facility assigned to it and
the other location is available for an assignment. Assign
the unassigned facility to the available location. Go to
Step 11.

**Step 11**

Determine if all n facilities have been assigned to
the n locations. If not, go to Step 4. If so, the MAT
assignment is complete.

The relationship between the MAT procedure and the
previously stated theorem can be explained in the following
manner.

Arrange the distances $x_{ij}$ between all pairs of the n
locations as a nondecreasing sequence of real numbers and
arrange the loads transported between all pairs of n
facilities per unit time as a nonincreasing sequence of real
numbers. Each sequence contains $m = \frac{n(n-1)}{2}$ elements. The
$m \times 2$ matrices $S$ and $T$ contain the row and column sub-
scripts of the ordered loads and distances respectively.
The MAT assignment is created from the S and T matrices as described above. If the assignment could be made such that no internal contradictions would exist between the facilities and their assigned locations, then the optimal assignment would be reached using MAT by the above theorem. However, these internal contradictions between the facilities and their assigned locations almost inevitably occur and hence the MAT assignment is only an approximation to the optimal assignment. That is, MAT is a suboptimal procedure.

B. Example:

Given the matrix $A$ of loads between all pairs of facilities,

$$
\begin{array}{c|ccccc}
\text{FACILITIES} & A & B & C & D & E \\
\hline
F & 0 & 10 & 19 & 6 & 20 \\
A & 10 & 0 & 27 & 7 & 19 \\
C & 19 & 27 & 0 & 5 & 11 \\
C & 6 & 7 & 5 & 0 & 3 \\
E & 20 & 19 & 11 & 3 & 0 \\
\end{array}
$$

and the matrix $X$ of distances between all possible pairs of locations,
consider only the elements above the main diagonal in both matrices.

Order the elements of the distance matrix, \( X \), from smallest to largest and the elements of the load matrix, \( A \), from largest to smallest. During the ordering processes, the facilities corresponding to the loads and the locations corresponding to the distances will be listed according to the respective loads and distances.

Both the \( A \) and \( X \) matrices are symmetric about the principal diagonal. The distance between locations \( i \) and \( j \) is taken to be the same as the distance between locations \( j \) and \( i \), while the total loads between facilities \( i \) and \( j \) is entered in the symmetric locations about the principal diagonal.

The ordering of the distances and loads with respective locations and facilities is as follows:

\[
X = \begin{bmatrix}
L & O & C & A & T & I \\
1 & 0 & 10 & 5 & 20 & 4 \\
2 & 10 & 0 & 3 & 10 & 7 \\
3 & 5 & 3 & 0 & 12 & 30 \\
4 & 20 & 10 & 12 & 0 & 9 \\
5 & 4 & 7 & 30 & 9 & 0 \\
\end{bmatrix}
\]
<table>
<thead>
<tr>
<th>LOCATIONS(T)</th>
<th>DISTANCE(X)</th>
<th>FACILITIES(S)</th>
<th>LOADS(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-3</td>
<td>3</td>
<td>B-C</td>
<td>27</td>
</tr>
<tr>
<td>1-5</td>
<td>4</td>
<td>A-E</td>
<td>20</td>
</tr>
<tr>
<td>1-3</td>
<td>5</td>
<td>A-C</td>
<td>19</td>
</tr>
<tr>
<td>2-5</td>
<td>7</td>
<td>B-E</td>
<td>19</td>
</tr>
<tr>
<td>4-5</td>
<td>9</td>
<td>C-E</td>
<td>11</td>
</tr>
<tr>
<td>1-2</td>
<td>10</td>
<td>A-B</td>
<td>10</td>
</tr>
<tr>
<td>2-4</td>
<td>10</td>
<td>B-D</td>
<td>7</td>
</tr>
<tr>
<td>3-4</td>
<td>12</td>
<td>A-D</td>
<td>6</td>
</tr>
<tr>
<td>1-4</td>
<td>20</td>
<td>C-D</td>
<td>5</td>
</tr>
<tr>
<td>3-5</td>
<td>30</td>
<td>D-E</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure 1. Ordering of Distances and Loads

The first two facilities, B-C, are to be assigned to the first two locations, 2-3, in some manner. By looking for the first repeat of one of the first two facilities, C is found repeated first with A. Doing the same for the locations, 3 is found repeated first with 1. The following has been selected.

Assign Facilities to Locations

B - C to 2 - 3
A - C to 1 - 3

If both of these relations are to hold, then the following one-to-one assignments must be made:
Thus, three of the five assignments have been made. The next assignment(s) is (are) made by searching the facility column for the first pair of facilities where only one or neither of the facilities have been assigned. The second pair of facilities is A-E. A has been assigned to location 1 but E has not been assigned. By searching the location column for the first pair of locations containing a 1 and another location not assigned, the second pair 1-5 is found to satisfy these conditions. The following relation is then established.

<table>
<thead>
<tr>
<th>Assign Facility</th>
<th>to</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Assign Facilities to Locations
A - E           1 - 5

This implies that facility E must be assigned to location 5 since A has been assigned to location 1.

Only facility D and location 4 remain to be assigned. Therefore, the entire one-to-one assignment is:

<table>
<thead>
<tr>
<th>Assign Facility</th>
<th>to</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>
By enumerating all $5! = 120$ assignments, the assignment with minimum travel distance, 1047 units, is

<table>
<thead>
<tr>
<th>Assign Facility</th>
<th>to</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

The total distance traveled for the MAT assignment is 1096 units. There are two other assignments, excluding the optimal assignment, that are better than the assignment given by MAT. The travel distances for these two assignments are 1077 and 1071 units.

In order to attempt an improvement of the MAT assignment, an interchanging routine developed by L. G. Clark\textsuperscript{(17)} was adapted to manipulate the MAT assignment.

Beginning with the first pair of assignments and continuing to the last pair, adjacent facilities in the assignment are interchanged and if any improvement is found, the interchange is kept and the next two facilities are interchanged. After the $n$ and $n-1$ facilities are interchanged, the $n^{th}$ facility at that time is assigned to its present location. The routine then goes back and does the same for the $n-1$ remaining facilities and assigns the
(n-1)st facility to its present location. This interchanging continues until all n facilities are assigned. Only an improvement of the MAT assignment can be accomplished. No improvement was found for the example problem using this interchanging method.

C. Experiments

Using the experimental data in (3), MAT was used to generate initial assignments which were then used as input to CRAFT (19). The assignments given by the MAT interchange routine were also used as input to CRAFT. The cost results are given in Table I. Actual assignments are given in Appendix I.

The original computer program which implemented MAT was written in FORTRAN IV for the IBM 7094 Model II computer. In order to get a time comparison with the techniques in (3), the 7094 times were converted to equivalent G.E. 265 times. The conversion equation is \[ \text{G.E. 265 time} = 5 \text{[7094 time]} \]. The time results are given in Table II, with all times given in seconds. In Table II, the total times for the CRAFT Assignment with Interchange Input do not include the times for the MAT assignment which is improved by the interchange procedure.

For purposes of comparison, the average final costs for CRAFT with random initial assignments are shown in Table I and the average seconds of G.E. 265 time consumed by CRAFT with random initial assignments are shown in Table II, as given in (3).
<table>
<thead>
<tr>
<th>Number of Facilities</th>
<th>MAT Cost</th>
<th>CRAFT Cost with MAT Input</th>
<th>Interchange Cost</th>
<th>CRAFT Cost with Interchange Input</th>
<th>CRAFT Average Costs with Random Initial Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>28.2</td>
</tr>
<tr>
<td>6</td>
<td>55</td>
<td>46</td>
<td>47</td>
<td>46</td>
<td>44.2</td>
</tr>
<tr>
<td>7</td>
<td>80</td>
<td>78</td>
<td>80</td>
<td>78</td>
<td>79.6</td>
</tr>
<tr>
<td>8</td>
<td>128</td>
<td>119</td>
<td>124</td>
<td>119</td>
<td>113.4</td>
</tr>
<tr>
<td>12</td>
<td>337</td>
<td>303</td>
<td>309</td>
<td>309</td>
<td>296.2</td>
</tr>
<tr>
<td>15</td>
<td>741</td>
<td>604</td>
<td>691</td>
<td>614</td>
<td>606.0</td>
</tr>
<tr>
<td>20</td>
<td>1450</td>
<td>1349</td>
<td>1437</td>
<td>1318</td>
<td>1339.0</td>
</tr>
<tr>
<td>30</td>
<td>3711</td>
<td>3216</td>
<td>3542</td>
<td>3137</td>
<td>3189.6</td>
</tr>
</tbody>
</table>
### TABLE II

**SECONDS OF G.E. 265 TIME CONSUMED**

<table>
<thead>
<tr>
<th>Number of Facilities</th>
<th>MAT Assignment with MAT Input</th>
<th>CRAFT Assignment with Interchange Assignment Input</th>
<th>Total Input</th>
<th>Interchange Assignment</th>
<th>Total</th>
<th>CRAFT Average with Random Initial Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.02</td>
<td>1.65</td>
<td>2.67</td>
<td>.15</td>
<td>1.65</td>
<td>11.80</td>
</tr>
<tr>
<td>6</td>
<td>1.26</td>
<td>1.95</td>
<td>3.21</td>
<td>.21</td>
<td>1.83</td>
<td>2.04</td>
</tr>
<tr>
<td>7</td>
<td>1.62</td>
<td>2.79</td>
<td>4.41</td>
<td>.30</td>
<td>2.76</td>
<td>3.06</td>
</tr>
<tr>
<td>8</td>
<td>2.10</td>
<td>2.61</td>
<td>4.71</td>
<td>.39</td>
<td>2.43</td>
<td>2.82</td>
</tr>
<tr>
<td>12</td>
<td>6.12</td>
<td>5.43</td>
<td>11.55</td>
<td>1.32</td>
<td>4.08</td>
<td>5.40</td>
</tr>
<tr>
<td>15</td>
<td>13.20</td>
<td>9.87</td>
<td>23.07</td>
<td>3.00</td>
<td>9.06</td>
<td>12.06</td>
</tr>
<tr>
<td>20</td>
<td>47.49</td>
<td>22.68</td>
<td>70.17</td>
<td>9.15</td>
<td>22.86</td>
<td>32.01</td>
</tr>
<tr>
<td>30</td>
<td>346.71</td>
<td>86.73</td>
<td>433.44</td>
<td>45.63</td>
<td>95.04</td>
<td>140.67</td>
</tr>
</tbody>
</table>
D. Calculation of Distances ($x_{ij}$)

The MAT program will accept the distances between locations in two ways. First, as pre-established distances, i.e., distances computed by the user as in the example problem. Second, in terms of standard x - y Cartesian coordinates of the centers of the locations. The distance, $x_{ij}$, between the centers of locations $i$, $(x_i, y_i)$, and $j$, $(x_j, y_j)$, is calculated as $x_{ij} = |x_j - x_i| + |y_j - y_i|$. This "taxicab geometry" is applicable to the allocation of facilities problem since it simulates aisle travel in an industrial situation. An example of the second method of specifying distances between locations is given in Figure 2. As a result of the second method, MAT lends itself to the allocation of facilities in multi-story buildings as shown in Figure 2. Group I could represent first floor locations and Group II second floor locations. Any number of floor groupings can be used with the current maximum number of locations being 40.
Figure 2. Second Method of Specifying Distances Between Locations.

E. Investigation of Ties

If the number of loads is equal for a group of facility pairs, this is referred to as a tie category. In Figure 1, facility pairs A-C and B-E form a tie category since there are 19 loads transported between both of these facility pairs in unit time. By permuting the facility pairs in a tie category it is possible to generate alternate assignments using MAT. If there are $n_1$ facility pairs in a tie category, then $n_1!$ possible assignments result. Thus, we are once
again faced with a combinatorial problem which becomes computationally restrictive if \( n_1 \) becomes much larger than four or five.

In order to examine a portion of the alternate assignments resulting from the tie category concept, the following policy was adopted:

Let 1, 2, ..., \( n_1 \) represent the order of the facility pairs in a tie category under the original ordering of facility pairs. Then \((n_1-1)\) alternate assignments are generated for this tie category where the following arrangements of facility pairs are considered: \( \phi_i \) for \( i = 1, 2, ..., n_1-1 \) and \( 1 \rightarrow n_1 \), where \( \phi \) is the mapping carrying the \((i+1)\)st facility pair into the \(i\)th facility pair and the first facility pair to the \(n_1\)th facility pair.

The MAT program contains the option of specifying whether or not any tie categories are to be investigated and how many categories beginning with category one are to be considered if tie categories are to be investigated. A tie category with only one element must be counted but no alternate assignment is generated for this category.

The results of investigating tie categories for the data given by Nugent, Vollman, and Rumml (3), are given in Table III. All tie categories corresponding to non-zero loads were investigated for the problems containing 5, 6, 7, 8, 12, and 15 facilities. Only five of the eight tie categories for the 20 facilities problem and only two of the eight tie categories for the 30 facilities problem were
investigated. An examination of the time required to investigate the alternates for the 20 facilities and 30 facilities problems in Table III clearly shows why all tie categories corresponding to non-zero loads were not investigated for these two problems. Table III also gives the final cost associated with CRAFT (19) utilizing as the initial assignment the best solution obtained from the investigation of tie categories from the MAT program. The actual assignments are given in Appendix I and the complete MAT program, with a description of input data and output results, is given in Appendix II. The MAT program is written in FORTRAN IV and was run on the IBM 360/50. All times in Table III are given in minutes and seconds (min., sec.) of IBM 360/50 time.

F. Quality of Solutions

The relationships of the costs of solutions obtained by MAT and CRAFT, with MAT program input, to the optimal costs are of interest to anyone involved in the allocation of facilities problem. Lower bound costs were computed for all problems using Theorem I applied to the ordered load and distance vectors. When Theorem I is applied to the data in Figure 1, the lower bound cost for the example problem is 920 units of distance. It is worthwhile to note that the actual optimal cost of 1096 units of distance is 19.13 percent overage from the lower bound cost.
TABLE III

RESULTS OF THE INVESTIGATION OF TIE CATEGORIES

<table>
<thead>
<tr>
<th>Number of Facilities</th>
<th>Number of Alternates Investigated</th>
<th>Time (min., sec.) Consumed to Investigate Alternates</th>
<th>Final Cost of the Best Solution from the MAT Program</th>
<th>CRAFT Final Cost with MAT Best Solution Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
<td>00,01</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>00,01</td>
<td>47</td>
<td>46</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>00,03</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>00,04</td>
<td>107</td>
<td>107</td>
</tr>
<tr>
<td>12</td>
<td>38</td>
<td>00,47</td>
<td>309</td>
<td>309</td>
</tr>
<tr>
<td>15</td>
<td>69</td>
<td>02,52</td>
<td>629</td>
<td>624</td>
</tr>
<tr>
<td>20</td>
<td>75</td>
<td>09,43</td>
<td>1384</td>
<td>1354</td>
</tr>
<tr>
<td>30</td>
<td>39</td>
<td>27,50</td>
<td>3430</td>
<td>3229</td>
</tr>
</tbody>
</table>
For the 5 and 8 facility problems, the optimal solutions were attained by the MAT program as shown in Table IV. However, the percent overage from the lower bound in the 8 facility problem is 17.58, which supports the hypothesis that the best solutions obtained may well be close to the optimal solutions. The results of Table IV, for the problems where the optimal costs are known, also support this conjecture. The general unattainability of the lower bound costs is clear when observing that the percent overage of the optimal cost rose to 17.58 above the lower bound cost for the 8 facility problem. Above 8 facilities, the increase in percent overage is not as large as the increase below the 12 facility problem. In fact, the 27.16 to 44.28 percent overage is really less than would be expected from extrapolation of the results from the smaller problems.

The increase in per cent overage from 12 to 15 and from 15 to 20 is small enough to suppose that the solutions given by MAT and CRAFT may well have costs which are very close to the optimal costs. Since the optimal solutions are not known for the larger problems, nothing can be said with certainty about the goodness of the best solutions obtained.
TABLE IV
PERCENTAGE BY WHICH COSTS OBTAINED BY HEURISTIC PROCEDURES EXCEED THE LOWER BOUND COSTS

<table>
<thead>
<tr>
<th>Number of Facilities</th>
<th>Lower Bound</th>
<th>Best Solution From MAT Program</th>
<th>Percent Overage</th>
<th>CRAFT with MAT Best Solution Input</th>
<th>Percent Overage</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>25 (25)*</td>
<td>25</td>
<td>0.00</td>
<td>25</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>41 (43)</td>
<td>47</td>
<td>14.63</td>
<td>46</td>
<td>12.20</td>
</tr>
<tr>
<td>7</td>
<td>67 (74)</td>
<td>75</td>
<td>11.94</td>
<td>75</td>
<td>11.94</td>
</tr>
<tr>
<td>8</td>
<td>91 (107)</td>
<td>107</td>
<td>17.58</td>
<td>107</td>
<td>17.58</td>
</tr>
<tr>
<td>12</td>
<td>243 (?)</td>
<td>309</td>
<td>27.16</td>
<td>309</td>
<td>27.16</td>
</tr>
<tr>
<td>15</td>
<td>479 (?)</td>
<td>629</td>
<td>31.32</td>
<td>624</td>
<td>30.27</td>
</tr>
<tr>
<td>20</td>
<td>1014 (?)</td>
<td>1384</td>
<td>36.49</td>
<td>1354</td>
<td>33.53</td>
</tr>
<tr>
<td>30</td>
<td>2238 (?)</td>
<td>3430</td>
<td>53.26</td>
<td>3229</td>
<td>44.28</td>
</tr>
</tbody>
</table>

*Optimal solution costs given in parentheses where the optimal solutions could be obtained.
IV. RESULTS AND CONCLUSIONS

MAT is a construction technique which produces a suboptimal assignment of facilities to locations based on the loads between all pairs of facilities per unit time and the distance between all pairs of locations where the facilities are to be assigned. It is also possible to force the objective function (3.1) to become responsive to the use of different material-handling systems for different materials by adjusting $f_{ij}$ in equation (1.1). A similar statement holds for the responsiveness of equation (3.1) to weighted distances by adjusting $g_{ij}$ in equation (1.1).

The significant advantages of MAT and the MAT program are the following:

1. MAT is a construction technique which attempts to minimize the same objective function, equation (3.1), as the improvement techniques.
2. MAT eliminates the randomness associated with selecting an initial assignment to be used as input to the improvement techniques.
3. Table II shows that the solution times for MAT and Interchange assignments are considerably better than the times given for the improvement procedures by Nugent, Vollmann, and Ruml (3). The solution times for the CRAFT procedures were
significantly reduced from the times given in (3) when MAT and Interchange input were used by CRAFT. This is due to the quality of the solutions yielded by the MAT and Interchange procedures.

4. The results of Table I and Table III indicate that the solution quality of the MAT program assignments are comparable to the quality of the solutions given by the improvement techniques in (3). Comparing the final cost values for CRAFT, with MAT program input, from Table III to the final cost values obtained by the procedures given in (3) definitely shows the superiority of combining the output of the MAT program with an improvement procedure such as CRAFT.

5. Figure 2 demonstrates how MAT is applicable to the allocation of facilities in multi-story buildings due to the flexibility of the location input. MAT can also be adapted to problems in which several locations are disjoint from the majority of the locations where the facilities are to be assigned. It is also possible to specify more locations than facilities by proper coding of the MAT program input data given in Appendix II.

6. The MAT program contains the option of investigating tie categories for a possible improvement in the MAT
program assignment. Any number of tie categories to be investigated may be specified. Obviously, an increase in computer time is encountered as shown in Table III.

7. The simplicity of MAT allows assignments to be generated manually for a reasonable number of facilities as shown in the example problem.

8. A lower bound cost is generated for every problem run with the MAT program given in Appendix II. This allows a percent overage to be calculated for the MAT solution as shown for the test problems in Table IV. Some measure of the goodness of the MAT solution can be obtained by the use of the percent overage.

When a layout for a new building is to be constructed or a major rearrangement of existing facilities is being planned in an industrial setting, the results are usually permanent for at least the life of a new product or the revision of an established line. Thus, every effort should be made to provide the "best" allocation of facilities to locations. A small amount of computer time invested in layout planning can yield a large return in cost avoidance. Of course, all of the techniques available for layout planning pre-suppose that a good method of data collection is available to the layout planner.
As a result of the investigations conducted, it is recommended that when allocating facilities to locations, a procedure combining the results of the MAT program, utilizing the tie category option, with an improvement technique be used. Although in this dissertation only CRAFT was utilized, it is reasonable to assume that meaningful results could be expected by combining the results of the MAT program with other improvement procedures, such as those given by Nugent, Vollman, and Ruml (3).
BIBLIOGRAPHY


APPENDIX I

Actual Assignments for Principal Procedures

Facility numbers given below indicate the assignments of facilities to locations. The numbers are given in order of locations 1, 2, ..., n. Location numbers follow the conventional facility sequence of left to right, top to bottom, for the patterns given in (3).

<table>
<thead>
<tr>
<th>No. of Facilities</th>
<th>MAT Assignment</th>
<th>CRAFT with MAT Assignment Interchange</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4,5,1,2,3</td>
<td>4,5,1,2,3</td>
</tr>
<tr>
<td>6</td>
<td>5,6,1,4,3,2</td>
<td>5,6,2,4,1,3</td>
</tr>
<tr>
<td>7</td>
<td>5,6,7,3,4,2</td>
<td>5,6,7,2,4,1,3</td>
</tr>
<tr>
<td>8</td>
<td>8,7,6,5,4,1,2,3</td>
<td>7,4,6,5,8,1,2,3</td>
</tr>
<tr>
<td>12</td>
<td>8,7,6,5,4,12,10,3,11,9,1,2</td>
<td>8,7,6,5,4,11,10,12,9,3,2</td>
</tr>
<tr>
<td>15</td>
<td>13,2,15,6,12,4,9,10,11,8,5,1,3,4,7</td>
<td>1,2,13,9,12,10,6,14,8,11,4,15,3,5,7</td>
</tr>
</tbody>
</table>

Continued
## APPENDIX I (Continued)

<table>
<thead>
<tr>
<th>No. of Facilities</th>
<th>MAT Assignment</th>
<th>CRAFT with MAT Assignment Input</th>
<th>Interchange Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>11,4,2,16,13</td>
<td>17,1,5,6,13,</td>
<td>4,11,14,16,13,</td>
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<td>20,7,1,5,3,</td>
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<td>20,7,1,5,3,</td>
</tr>
<tr>
<td></td>
<td>15,19,17,18,9</td>
<td>15,19,2,14,16</td>
<td>19,15,17,18,9</td>
</tr>
<tr>
<td>30</td>
<td>29,19,6,22,30,</td>
<td>29,16,20,14,30,</td>
<td>2,29,19,22,30</td>
</tr>
<tr>
<td></td>
<td>11,2,9,13,20,</td>
<td>4,21,13,19,3,</td>
<td>27,6,9,13,4,</td>
</tr>
<tr>
<td></td>
<td>27,10,8,7,4,</td>
<td>27,9,2,7,8,</td>
<td>11,10,20,7,8,</td>
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<td></td>
<td>14,16,26,25,28</td>
<td>22,10,11,28,6,</td>
<td>14,16,26,28,25,</td>
</tr>
<tr>
<td></td>
<td>1,18,23,21,24,</td>
<td>25,18,23,26,5,</td>
<td>1,18,23,17,12,</td>
</tr>
<tr>
<td></td>
<td>12,5,3,17,15</td>
<td>12,24,1,17,15</td>
<td>24,21,3,5,15</td>
</tr>
</tbody>
</table>

Continued
<table>
<thead>
<tr>
<th>No. of Facilities</th>
<th>CRAFT with</th>
<th>Best Solution from the MAT Program</th>
<th>CRAFT with MAT Best Solution Input</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Interchange Assignment Input</td>
<td></td>
<td></td>
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<td>4,5,1,2,3</td>
<td>4,5,1,2,3</td>
</tr>
<tr>
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<td>5,4,1,6,2,3</td>
<td>5,4,1,6,3,2</td>
<td>5,4,1,6,2,3</td>
</tr>
<tr>
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<td>5,6,7,2,4,3,1</td>
<td>7,6,5,4,3,2</td>
<td>7,6,5,4,3,2</td>
</tr>
<tr>
<td>8</td>
<td>7,4,6,5,8,1,2,3</td>
<td>6,7,8,3,5,4,1,2</td>
<td>6,7,8,3,5,4,1,2</td>
</tr>
<tr>
<td>12</td>
<td>8,7,6,10,4,11,5,1,12,9,3,2</td>
<td>8,7,6,10,4,11,5,1,12,9,3,2</td>
<td>8,7,6,10,4,11,5,1,12,9,3,2</td>
</tr>
<tr>
<td>15</td>
<td>6,15,5,10,12,2,14,7,8,11,1,13,4,3,9</td>
<td>2,13,9,11,7,1,14,8,3,4,6,15,5,10,12</td>
<td>2,13,8,11,7,1,14,9,3,4,6,15,5,10,12</td>
</tr>
</tbody>
</table>

Continued
<table>
<thead>
<tr>
<th>No. of Facilities</th>
<th>CRAFT with Interchange Assignment Input</th>
<th>Best Solution From the MAT Program</th>
<th>CRAFT with MAT Best Solution Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>4,11,19,16,9, 20,12,2,14,13, 7,8,15,5,6, 17,1,18,10,3</td>
<td>7,12,20,19,4, 1,8,15,11,2, 6,10,3,14,18, 13,5,9,16,17</td>
<td>7,20,8,11,4, 1,12,15,2,19, 6,10,3,14,18, 13,5,9,16,17</td>
</tr>
<tr>
<td>30</td>
<td>2,29,16,19,30, 4,21,9,7,3,11, 27,25,13,10, 22,8,20,28,6, 1,18,23,14,5, 12,24,26,17,15</td>
<td>12,6,18,23,2, 5,7, 1,22,19, 29,14,16,11,30, 8,4,20,9,10, 27,25,28,3,13, 26,15,24,17,21</td>
<td>12,6,23,18,2, 5,1,22,7,19, 3,21,25,11,8, 30,4,28,13,10, 9,16,29,14,24, 26,17,27,15,20</td>
</tr>
</tbody>
</table>
APPENDIX II

MAT Program Specifications

INPUT DATA SEQUENCE FOR THE MAT PROGRAM

The following data is required for each data set.

1. Specification data - 5I3
   Number of Facilities
   Number of Locations
   Previous Assignment Code
   Distance specification code
   Tie interchange code.

2. Load Data - 10F7.2
   Load data is read a row at a time with each row beginning on a new card. Only the upper half of the load matrix is utilized.

3. Distance Data - 10F7.2
   If distances are to be read as pre-calculated quantities, then the data is read a row at a time with each row beginning on a card. Only the upper half of the distance matrix is utilized.
   If distances are to be calculated from the x-y Cartesian coordinates of the location points then the coordinates of each point are read in the following manner:
   Five coordinate pairs per card with the X coordinate of each location point preceding the y coordinate of that location point.
4. Number of tie categories to be investigated - I3
   If a number of tie categories are to be investigated
   then the number of tie categories to be investigated
   is read at this time. A category with only one
   entry must be counted as a tie category.

5. Number of previous assignments - I3
   If previous assignments are to be specified then
   the number of assignments to be made is read at
   this time.

6. Previous assignment data - 20I3
   If previous assignments are to be made then the
   assignments are specified in the following manner:
   The numbers of the facilities to be assigned
   are read. The corresponding location numbers
   for the above facilities are read with the
   location numbers beginning on a new card
   following the facility numbers.

If more than one data set is to be run then each new data
set begins with Step 1., the specification data, and is
placed behind the first data set.

An explanation of the specification card data is given below
in the comment section of the MAT program.
MAT PROGRAM

C INPUT OF NUMBER OF FACILITIES, NF MAX. OF 40
C INPUT OF NUMBER OF LOCATIONS, NL MAX. OF 40
C INPUT OF CODE FOR PREVIOUS ASSIGNMENTS, IPA
C IF IPA=1, NO PREVIOUS ASSIGNMENTS
C IF IPA=2, PREVIOUS ASSIGNMENTS TO BE MADE
C INPUT OF CODE FOR DISTANCE SPECIFICATIONS, IDS
C IF IDS=1, DISTANCES TO BE PRE-CALCULATED
C D(I,J) IN UPPER TRIANGULAR FORM
C IF IDS=2, DISTANCES TO BE CALCULATED
C X,Y LOCATIONS SPECIFIED
C D(I,J)=ABS(X(J)-X(I))+ABS(Y(J)
C -Y(I))
C INPUT OF CODE FOR TIES IN NUMBER OF LOADS, ITS
C IF ITS=1, NO TIE INTERCHANGES EXAMINED
C IF ITS=2, ALL SPECIFIED TIES INVESTIGATED
D
DIMENSION T(40,40),IM(780),JM(780),Y(780),D(40,40),
1IS(780),JS(780),X(780),XX(40),YY(40),Z(780),IJM(780),
2JJM(780),NTIC(250),MAC(40),IXSPAC(40),MACT(40),
3ISPACT(40),L(780)
REAL*8 TIME
C INPUT OF I/O LOGICAL DEVICE NUMBERS
10 IR=1
IW=3
420 CALL TIMER(TIME,IFIX,1)
WRITE(IW,2130) TIME
2130 FORMAT(1H0,3X,'TIME FOR THIS DATA SET IS',3X,A8)
READ(IR,1000,END=999) NF,NL,IPA,IDS,ITS
1000 FORMAT(513)
C INITIALIZATION
KOUNT=1
JFK=0
NNA=0
KOUNT1=1
K=1
C LOAD INPUT
NN=NF-1
DO 20 I=1,NN
M=I+1
READ(IR,1010) (T(I,J),J=M,NF)
1010 FORMAT(10F7.2)
DO 20 J=M,NF
T(J,I)=T(I,J)
IM(K)=I
JM(K)=J
Y(K)=T(I,J)
20 K=K+1
C DETERMINE DISTANCE INPUT
GO TO(30,40),IDS

30 NN=NL-1
K1=1
DO 50 I-1,NN
M=I+1
READ(IR,1010) (D(I,J),J=M,NL)
DO 50 J=M,NL
D(J,I)=D(I,J)
IS(K1)=I
JS(K1)=J
X(K1)=D(I,J)
50 K1=K1+1
GO TO 60

40 NN=NL-1
K1=1
READ(IR,1010) (XX(I),YY(I),I=1,NL)
DO 70 I=1,NN
M=I+1
DO 70 J=M,NL
D(I,J)=ABS(XX(J)-XX(I))+ABS(YY(J)-YY(I))
D(J,I)=D(I,J)
IS(K1)=I
JS(K1)=J
X(K1)=D(I,J)
70 K1=K1+1
60 K=K-1
KK=K
K1=K1-1
KK1=K1
C ORDERING OF LOAD DATA
CALL SORTN(Y,K,L,IM,JM)
C ORDERING OF DISTANCE DATA
CALL SORTN(X,K1,L,IS,JS)
C SORTN ORDERS FROM MIN TO MAX, SO Y MUST BE INVERTED
DO 80 I=1,K
Z(I)=Y(I)
IJM(I)=IM(I)
80 JJM(I)=JM(I)
DO 90 I=1,K
Z(I)=Z(KK)
IM(I)=IJM(KK)
JM(I)=JJM(KK)
90 KK=KK-1
KK=K
C STORE DISTANCES INVERTED FOR UPPER BOUND CALCULATIONS
DO 100 I=1,K1
Z(I)=X(KK1)
IJM(I)=IS(KK1)
JJM(I)=JS(KK1)
100 KK1=KK1-1
KK1=K1
C OUTPUT OF DIST FROM MIN TO MAX WITH LOCATION PAIRS
C OUTPUT OF LOADS FROM MAX TO MIN WITH FACILITY PAIRS
WRITE(IW,2000) NF,NL
2000 FORMAT(1H1,42X,I3,'FACILITIES AND ',I3,'LOCATIONS'
1///9X,'THE DISTANCES FROM MIN. TO MAX. AND LOADS'
2FROM MAX. TO MIN. FOLLOW'///9X,'IS JS DIST.'
3,14X,'IM JM LOADS')
DO 110 I=1,K
110 WRITE(IW,2010) IS(I),JS(I),X(I),IM(I),JM(I),Y(I)
2010 FORMAT(/9X,I2,3X,F10.1,14X,I2,3X,F9.1)
C TEST TO SEE IF NF=NL AND IF NOT, WRITE REMAINING
C DISTANCES
IF(NF.EQ.NL) GO TO 120
DO 130 I=K,K1
130 WRITE(IW,2010) IS(I),JS(I),X(I)
C COMPUTE UPPER AND LOWER BOUNDS ON ASSIGNMENTS AND
C OUTPUT THEM
120 SUM1=0.0
SUM2=0.0
DO 140 I=1,K
SUM1=SUM1+X(I)*Y(I)
140 SUM2=SUM2+Z(I)*Y(I)
WRITE(IW,2020) SUM1,SUM2
2020 FORMAT(1H1,8X,'A LOWER BOUND FOR THIS PROBLEM IS'
1F18.8//8X,'AN UPPER BOUND FOR THIS PROBLEM IS'F18.8)
C BEGIN THE ROUTINE TO DETERMINE NUMBER OF TIE CATEGOR
C TIES AND THE NUMBER OF TIES IN EACH CATEGORY IF ANY
C TIES ARE TO BE INVESTIGATED
IF(ITS.EQ.1) GO TO 150
NTC=1
DO 160 I=1,250
160 NTIC(I)=1
IDUM=K-1
DO 170 I=1,IDUM
IF(ABS(Y(I+1)-Y(I)).LE..001) GO TO 180
NTC=NTC+1
GO TO 170
180 NTIC(NTC)=NTIC(NTC)+1
170 CONTINUE
WRITE(IW,2030) NTC
2030 FORMAT(1H0,8X,'THERE ARE',I3,' TIE CATEGORIES FOR'
1THIS PROBLEM')
C INPUT OF NUMBER OF TIE CATEGORIES TO BE INVESTIGATED
READ(IR,1000) NTCTBI
WRITE(IW,2040) NTCTBI
2040 FORMAT(1H0,8X,'OF THESE CATEGORIES ARE TO BE'
1INVESTIGATED,BEGINNING WITH CATEGORY 1'///9X,'THE'
2CATEGORY AND NUMBER OF TIES IN EACH CATEGORY FOLLOW'
3///9X,'CATEGORY NUMBER IN THIS CATEGORY'///)
DO 190 I=1,NTC
190 WRITE(IW,2050) I,NTIC(I)
2050 FORMAT(14X,I3,13X,I3)
C TEST TO SEE IF ANY PREVIOUS ASSIGNMENTS ARE TO BE MADE
150 GO TO (200,210),IPA
C INPUT PREVIOUS ASSIGNMENTS
210 READ(IR,1000) NA
   READ(IR,1020) (MAC(I),I=1,NA)
   READ(IR,1020) (ISPAC(I),I=1,NA)
1020 FORMAT(20I3)
   DO 3070 I=1,NA
      MACT(I)=MAC(I)
      ISPACT(I)=ISPAC(I)
   NNA=NA
4020 I=0
   GO TO 220
C INITIAL ASSIGNMENTS IF NO PREVIOUS ASSIGNMENTS HAVE BEEN MADE
200 NA=1
   I=1
   IK=2
   JK=2
280 IF(IM(I).NE.IM(IK)) GO TO 230
   MAC(NA)=IM(I)
   MAC(NA+1)=JM(I)
   MAC(NA+2)=JM(IK)
   GO TO 240
230 IF(IM(I).NE.JM(IK)) GO TO 250
   MAC(NA)=IM(I)
   MAC(NA+1)=JM(I)
   MAC(NA+2)=IM(IK)
   GO TO 240
250 IF(JM(I).NE.IM(IK)) GO TO 260
   MAC(NA)=JM(I)
   MAC(NA+1)=IM(I)
   MAC(NA+2)=JM(IK)
   GO TO 240
260 IF(JM(I).NE.JM(IK)) GO TO 270
   MAC(NA)=JM(I)
   MAC(NA+1)=IM(I)
   MAC(NA+2)=IM(IK)
   GO TO 240
270 IK=IK+1
   KOUNT=KOUNT+1
   GO TO 280
240 IF(IS(I).NE.IS(JK)) GO TO 290
   ISPAC(NA)=IS(I)
   ISPAC(NA+1)=JS(I)
   ISPAC(NA+2)=JS(JK)
   GO TO 300
290 IF(IS(I).NE.JS(JK)) GO TO 310
   ISPAC(NA)=IS(I)
   ISPAC(NA+1)=JS(I)
   ISPAC(NA+2)=IS(JK)
   GO TO 300
310 IF(JS(I).NE.IS(JK)) GO TO 320
ISPAC(NA)=JS(I)
ISPAC(NA+1)=IS(I)
ISPAC(NA+2)=JS(JK)
GO TO 300
320 IF(JS(I).NE.JS(JK)) GO TO 330
ISPAC(NA)=JS(I)
ISPAC(NA+1)=IS(I)
ISPAC(NA+2)=JS(JK)
GO TO 300
330 JK=JK+1
KOUNT1=KOUNT1+1
GO TO 240
300 NA=NA+2
C DETERMINE IF ALL ASSIGNMENTS HAVE BEEN MADE
IF(NA+1.EQ.NF) GO TO 340
IF(NA+1.GT.NF) GO TO 350
220 J=1
950 I=I+1
C DETERMINE WHICH FACILITIES HAVE BEEN ASSIGNED
DO 360 KA=1,NA
IF(IM(I).EQ.MAC(KA)) GO TO 370
360 CONTINUE
DO 380 KB=1,NA
IF(JM(I).EQ.MAC(KB)) GO TO 390
380 CONTINUE
IF(NA+2.LT.NF) GO TO 400
IF(NA+2.EQ.NF) GO TO 410
WRITE(IW.2060)
2060 FORMAT(1X,'ERROR, CHECK LOGIC, GOING TO NEW DATA')
GO TO 420
C IF NEITHER FACILITY HAS BEEN ASSIGNED AND ONLY TWO
C REMAIN TO BE ASSIGNED
410 DO 430 IL=1,K
DO 440 MI=1,NA
IF(IS(J).EQ.ISPAC(MI)) GO TO 430
440 CONTINUE
DO 450 IIM=1,NA
IF(JS(J).EQ.ISPAC(IIM)) GO TO 430
450 CONTINUE
GO TO 460
430 J=J+1
460 DO 470 LK=1,2
IF(LK.EQ.2) GO TO 480
NA=NA+1
MAC(NA)=IM(I)
MAC(NA+1)=JM(I)
ISPAC(NA)=IS(J)
ISPAC(NA+1)=JS(J)
GO TO 490
480 ISPAC(NA)=JS(J)
ISPAC(NA+1)=IS(J)
490 SUM=0.0
   DO 500 I=1,NN
   M=I+1
   IMM=MAC(I)
   ISS=ISPAC(I)
   DO 500 J1=M,NF
   JMM=MAC(J1)
   JSS=ISPAC(J1)
   500 SUM=SUM+D(ISS,JSS)*T(IMM,JMM)
   IF(LK.GT.1) GO TO 510
   SUM1=SUM
   GO TO 470
   510 IF(SUM.LT.SUM1) GO TO 470
   ISPAC(NA)=IS(J)
   ISPAC(NA+1)=JS(J)
   470 CONTINUE
   GO TO 350
C IF NEITHER FACILITY HAS BEEN ASSIGNED AND MORE THAN
C TWO FACILITIES REMAIN TO BE ASSIGNED

400 IJ=1
   610 IF(I.EQ.IJ) GO TO 520
   IF(IM(I).NE.IM(IJ)) GO TO 530
   DO 540 IC=1,NA
   IF(JM(IJ).EQ.MAC(IC)) GO TO 520
   540 CONTINUE
   NA=NA+1
   MAC(NA)=IM(I)
   MAC(NA+1)=JM(I)
   MAC(NA+2)=JM(IJ)
   GO TO 550
   530 IF(IM(I).NE.JM(IJ)) GO TO 560
   DO 570 IC=1,NA
   IF(IM(IJ).EQ.MAC(IC)) GO TO 520
   570 CONTINUE
   NA=NA+1
   MAC(NA)=IM(I)
   MAC(NA+1)=JM(I)
   MAC(NA+2)=IM(IJ)
   GO TO 550
   560 IF(JM(I).NE.IM(IJ)) GO TO 580
   DO 590 IC=1,NA
   IF(JM(IJ).EQ.MAC(IC)) GO TO 520
   590 CONTINUE
   NA=NA+1
   MAC(NA)=JM(I)
   MAC(NA+1)=IM(I)
   MAC(NA+2)=JM(IJ)
   GO TO 550
   580 IF(JM(I).EQ.JM(IJ)) GO TO 600
   520 IJ=IJ+1
   GO TO 610
600 DO 620 IC=1,NA
   IF(IM(IJ).EQ.MAC(IC)) GO TO 520
620 CONTINUE
   NA=NA+1
   MAC(NA)=JM(I)
   MAC(NA+1)=IM(I)
   MAC(NA+2)=IM(IJ)
550 NA=NA-1
   DO 630 IE=1,K
   DO 640 IF=1,NA
   IF(IS(J).EQ.ISPAC(IF)) GO TO 630
640 CONTINUE
   DO 650 IG=1,NA
   IF(JS(J).EQ.ISPAC(IG)) GO TO 630
650 CONTINUE
   GO TO 660
630 J=J+1
660 IH=1
760 IF(J.EQ.IH) GO TO 670
   IF(IS(J).NE.IS(IH)) GO TO 680
   DO 690 JJI=1,NA
   IF(JS(IH).EQ.ISPAC(JJI)) GO TO 670
690 CONTINUE
   NA=NA+1
   ISPAC(NA)=IS(J)
   ISPAC(NA+1)=JS(J)
   ISPAC(NA+2)=JS(IH)
   NA=NA+2
   GO TO 700
680 IF(IS(J).NE.JS(IH)) GO TO 710
   DO 720 JJI=1,NA
   IF(JS(IH).EQ.ISPAC(JJI)) GO TO 670
720 CONTINUE
   NA=NA+1
   ISPAC(NA)=IS(J)
   ISPAC(NA+1)=JS(J)
   ISPAC(NA+2)=IS(IH)
   NA=NA+2
   GO TO 700
710 IF(JS(J).NE.IS(IH)) GO TO 730
   DO 740 JJI=1,NA
   IF(JS(IH).EQ.ISPAC(JJI)) GO TO 670
740 CONTINUE
   NA=NA+1
   ISPAC(NA)=JS(J)
   ISPAC(NA+1)=IS(J)
   ISPAC(NA+2)=JS(IH)
   NA=NA+2
   GO TO 700
730 IF(JS(J).NE.JS(IH)) GO TO 670
DO 750 JJI=1,NA
IF(IS(IH).EQ.ISPAC(JJI)) GO TO 670
750 CONTINUE
NA=NA+1
ISPAC(NA)=JS(J)
ISPAC(NA+1)=IS(J)
ISPAC(NA+2)=IS(IH)
NA=NA+2
GO TO 700
670 IH=IH+1
GO TO 760
C IF THE FIRST FACILITY HAS BEEN ASSIGNED
770 DO 780 IB=1,K
    IF(IS(J).NE.ISPAC(KA)) GO TO 790
    DO 800 IA=1,NA
    IF(JS(J).EQ.ISPAC(IA)) GO TO 810
800 CONTINUE
NA=NA+1
MAC(NA)=JM(I)
ISPAC(NA)=JS(J)
GO TO 700
790 IF(JS(J).NE.ISPAC(KA)) GO TO 810
    DO 820 IC=1,NA
    IF(IS(J).EQ.ISPAC(IC)) GO TO 810
820 CONTINUE
NA=NA+1
MAC(NA)=JM(I)
ISPAC(NA)=JS(J)
GO TO 700
810 IF(IB.GE.K) GO TO 220
780 CONTINUE
C IF THE SECOND FACILITY HAS BEEN ASSIGNED
390 DO 830 IB=1,K
    IF(IS(J).NE.ISPAC(KB)) GO TO 840
    DO 850 IA=1,NA
    IF(JS(J).EQ.ISPAC(IA)) GO TO 860
850 CONTINUE
NA=NA+1
MAC(NA)=IM(I)
ISPAC(NA)=JS(J)
GO TO 700
840 IF(JS(J).NE.ISPAC(KB)) GO TO 860
    DO 870 IC=1,NA
    IF(IS(J).EQ.ISPAC(IC)) GO TO 860
870 CONTINUE
NA=NA+1
MAC(NA)=IM(I)
ISPAC(NA)=IS(J)
GO TO 700
860 J=J+1
IF(IB.GE.K) GO TO 220
830 CONTINUE
C TEST TO SEE HOW MANY HAVE BEEN ASSIGNED TO THIS POINT
700 IF(NA.EQ.NF) GO TO 350
 IF(NA+1.EQ.NF) GO TO 340
 GO TO 220
C ONE FACILITY REMAINS TO BE ASSIGNED
340 DO 880 IA=1,NF
 DO 890 IB=1,NA
 IF(IA.EQ.MAC(IB)) GO TO 880
890 CONTINUE
 GO TO 900
880 CONTINUE
900 MAC(NF)=IA
 DO 910 IA=1,NF
 DO 920 IB=1,NA
 IF(IA.EQ.ISPAC(IB)) GO TO 910
920 CONTINUE
 GO TO 930
910 CONTINUE
930 ISPAC(NF)=IA
 GO TO 350
C IF BOTH FACILITIES HAVE BEEN ASSIGNED
370 DO 940 JA=1,NA
 IF(JM(I) .EQ.MAC(JA)) GO TO 950
940 CONTINUE
 GO TO 770
C BEGIN OUTPUT
350 WRITE(IW,2070) NF
 2070 FORMAT(1H1,2X,'FOR ',I3,' FACILITIES THE ASSIGNMENT
 1FOLLOWS')
 WRITE(IW,2080)
 2080 FORMAT(1H0,2X,'ASSIGN FACILITY TO LOCATION'/)
 DO 960 I=1,NF
960 WRITE(IW,2090) MAC(I),ISPAC(I)
 2090 FORMAT(13X,I3,11X,I3)
C SUMMING TOTAL MEASURE OF EFFECTIVENESS FOR THIS
C ASSIGNMENT
 SUM=0.0
 DO 970 I=1,NN
 M=I+1
 ISS=ISPAC(I)
 IMM=MAC(I)
 DO 970 J=M,NF
 IIS=ISPAC(J)
 IIM=MAC(J)
970 SUM=SUM+D(ISS,IIS)*T(IMM,IIM)
 WRITE(IW,2100) SUM
 2100 FORMAT(1H0,3X,'MEASURE OF EFFECTIVENESS FOR THIS
 1ASSIGNMENT IS ',2X,F18.8)
C BEGINNING INTERCHANGE ASSIGNMENT
 KKK=1
 IJK=0
 INN=NF
DO 990 I=1,NN
M=I+1
ISS=ISPAC(I)
IMM=MAC(I)
DO 990 J=M,NF
JSS=ISPAC(J)
JMM=MAC(J)
SUM=SUM+D(ISS,JSS)*T(IMM,JMM)
IF(IJK.GT.0) GO TO 3000

K1=KKK+1
IA=MAC(KKK)
IB=MAC(K1)
MAC(KKK)=IB
MAC(K1)=IA
IF(IJK.EQ.1) GO TO 980
SUM1=SUM
IJK=1
GO TO 980

IF(SUM1.GT.SUM) GO TO 3020
MAC(KKK)=IA
MAC(K1)=IB
GO TO 3030

3020 SUM1=SUM
3030 IF(K1.EQ.INN) GO TO 3040
KKK=KKK+1
GO TO 3010

3040 INN=INN-1
IF(INN.EQ.1) GO TO 3050
KKK=1
GO TO 3010

WRITE(IW,2110)
2110 FORMAT(1H0,2X,'ASSIGNMENT FOR INTERCHANGE ROUTINE
1FOLLOWS I)
WRITE(IW,2080)
DO 3060 I=1,NF
3060 WRITE(IW,2090) MAC(I),ISPAC(I)
WRITE(IW,2100) SUM1

BEGINNING TIED ASSIGNMENTS, IF ANY ARE TO BE
INVESTIGATED

IF(ITS.EQ.1) GO TO 420
4040 IF(JFK.NE.0) GO TO 3080
NTI=0
DO 3090 I=1,NTCTBI
3090 NTI=NTI+NTIC(I)
WRITE(IW,2120)NTI

FORMAT(///3X,I5,' TIES ARE INVESTIGATED AND THE
1ASSIGNMENTS FOR THESE TIES FOLLOW')
IC1=1
IC2=NTIC(1)
IC3=1
3080 IF(JFK.EQ.NTI) GO TO 420
IF(JFK.NE.IC2) GO TO 4000
IC1=IC1+NTIC(IC3)
IC2=IC2+NTIC(IC3+1)
IC3=IC3+1

4000 TEMP1=IM(IC2)
TEMP2=JM(IC2)
JFK=JFK+1
IF(NTIC(IC3).EQ.1) GO TO 4040
IC4=IC2-IC1
DO 4010 I=1,IC4
IM(IC2+1-I)=IM(IC2-I)
4010 JM(IC2+1-I)=JM(IC2-I)
IM(IC1)=TEMP1
JM(IC1)=TEMP2
IF(JFK.EQ.IC2) GO TO 3080
IF(NNA.EQ.0) GO TO 200
DO 4030 I=1,NNA
MAC(I)=MACT(I)
4030 ISPAC(I)=ISPACT(I)
NA=NNA
GO TO 4020

999 STOP
END

SUBROUTINE SORTN(X,N,L,IS,JS)
DIMENSION X(1) ,L(1) ,XA(780),IS(1) ,JS(1)
IF(N.EQ.1) GO TO 100
DO 10 I=1,N
10 XA(I)=X(I)
I=1
K=1
40 J=I+1
30 IF(X(I).LE.X(J)) GO TO 20
A=X(I)
B=X(J)
X(I)=B
X(J)=A
IA=IS(I)
IB=IS(J)
IS(I)=IB
IS(J)=IA
IA=JS(I)
IB=JS(J)
JS(I)=IB
JS(J)=IA
20 J=J+1
IF(J.LE.N) GO TO 30
I=I+1
IF(I.LT.N) GO TO 40
DO 50 I=1,N
J=I-1
50 IF(ABS(X(I)-XA(K)).LT..001) GO TO 60
K=K+1
GO TO 70
60 L(I)=K
   IF(J.LE.0) GO TO 80
   DO 90 M=1,J
   IF(K.NE.L(M)) GO TO 90
   K=K+1
   GO TO 70
90 CONTINUE
80 K=1
50 CONTINUE
100 RETURN
END
OUTPUT DATA FOR THE MAT PROGRAM

6 FACILITIES AND 6 LOCATIONS

THE DISTANCES FROM MIN. TO MAX. AND LOADS FROM MAX. TO MIN. FOLLOW.

<table>
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<th>IS</th>
<th>JS</th>
<th>DIST.</th>
<th>IM</th>
<th>JM</th>
<th>LOADS</th>
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<td>6</td>
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<td>1</td>
<td>6</td>
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</table>

A LOWER BOUND FOR THIS PROBLEM IS 41.0000000
AN UPPER BOUND FOR THIS PROBLEM IS 84.0000000
THERE ARE 7 TIE CATEGORIES FOR THIS PROBLEM
6 OF THESE CATEGORIES ARE TO BE INVESTIGATED, BEGINNING WITH CATEGORY 1

THE CATEGORY AND NUMBER OF TIES IN EACH CATEGORY FOLLOW

CATEGORY   NUMBER IN THIS CATEGORY
          1
1          1
2          2
3          1
4          1
5          4
6          1
7          5
FOR 6 FACILITIES THE ASSIGNMENT Follows

ASSIGN FACILITY TO LOCATION

| 5 | 1 |
| 6 | 2 |
| 4 | 4 |
| 2 | 6 |
| 1 | 3 |
| 3 | 5 |

MEASURE OF EFFECTIVENESS FOR THIS ASSIGNMENT IS 55.0000000

ASSIGNMENT FOR INTERCHANGE ROUTINE Follows

ASSIGN FACILITY TO LOCATION

| 5 | 1 |
| 4 | 2 |
| 6 | 4 |
| 2 | 6 |
| 1 | 3 |
| 3 | 5 |

MEASURE OF EFFECTIVENESS FOR THIS ASSIGNMENT IS 47,0000000

10 TIES ARE INVESTIGATED AND THE ASSIGNMENTS FOR THESE TIES FOLLOW

NOTE: The assignments for the ties are identical to the above assignment and are not included since they appear in exactly the same format as the above assignment. It should also be noted that there would only be 4 additional assignments generated since there will always be one less assignment for each tie category than the number of ties in that category.
VITA

Harry Kerry Edwards was born on August 16, 1940, in Sharon, Pennsylvania. He received his primary and secondary education in Sharpsville, Pennsylvania, his college education at Youngstown University in Youngstown, Ohio, Brown University in Providence, Rhode Island, and the University of Missouri-Rolla, Rolla, Missouri. In August 1962, he received a Bachelor of Science Degree in Education, Major in Mathematics, from Youngstown University and was employed as a mathematics and English teacher from 1961 to 1963 in Brookfield, Ohio. He attended Brown University as a graduate student during the summer of 1963 and became a graduate assistant in the Computer Science Center at the University of Missouri-Rolla in September 1963. In August 1964, he received a Master of Science in Mathematics from the University of Missouri-Rolla and was an instructor of Mathematics at the University of Missouri-Rolla until June 1965. He was employed as a Mathematician and Statistician at the Allison Division of General Motors in Indianapolis, Indiana and was a part time Mathematics Instructor at Purdue University from 1965 to 1966.

In September 1966, he returned to the University of Missouri-Rolla as an Instructor of Mathematics and as a graduate student enrolled in the Graduate School of the University of Missouri-Rolla. He became a member of
Kappa Mu Epsilon, the honorary Mathematics Fraternity in 1966 and was initiated as an honorary associate member of Triangle Fraternity in April 1967. He became faculty advisor to Triangle Fraternity in September 1968 and remained in that capacity until June 1970.