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A New Method for Determining the Anisotropic Parameters of Materials Under True Triaxial Cyclic Loading

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ABSTRACT In this paper a new approach to solving the basic equations of elastic cross-anisotropic stress and strain relationships is presented. The uniqueness of the equations is demonstrated and the conditions for their use are easily controlled in the true triaxial test equipment available in the laboratory. By this approximate solution the four anisotropic parameters of cross-anisotropic materials can be obtained. In addition, so-called fuzzy loagic weighting factors are introduced to assess the differences among the observed principal deformations of samples tested. A computer program ANISOLV has been written to calculate the formulated parameters. Some typical examples are given to demonstrate the reliability and validity of the method and the program.

INTRODUCTION

The constitutive laws of idealised cross-anisotropic elastic materials can be easily found in the text books such as those by Love (1927), Lekhnitsikii (1977) and Hearman (1961). However when they are adopted to solve engineering problems such as the calculation of the stresses and strains of pavement materials there are limitations in the characterisation of the material parameters. Moreover, to date, it seems impossible to determine all the parameters of cross-anisotropic elastic materials by any single device (Shackel, 1991). That is because the available tests cannot completely simulate in the laboratory the stresses generated by the movement of a wheel along a pavement. Even though several new devices reported by Sousa (1987) and Arthur (1988) have been developed to evaluate the elastic constants necessary to characterise fully a cross-anisotropic elastic material in which the elastic parameters are independent of one another, they all have limitations in the interpretation of the data and their assumptions are not always completely rational. Therefore, it is desirable to find some way to simulate anisotropic problems. In this respect the cyclic loading cubic triaxial apparatus available at the University of New South Wales, Australia, may be used to estimate four of the five parameters needed to characterise a cross-anisotropic elastic material.

In this paper, a new method is presented to solve the equations describing true cubical triaxial cyclic stress-strain relationships. The reliability and validity of the approach is assessed. So-called fuzzy logic weighting factors are introduced to evaluate the differences among the measured principal strains of the tested samples. A computer program for analysing the test results is described. This is followed by examples to demonstrate the application and limitations of this new methodology of testing.

BASIC THEORY

Pavement materials under traffic loading may demonstrate cross-anisotropic elastic properties in the small strain domain. The model reflecting the elastic properties to obtain the relations between the strain and stress is taken to be a continuous medium following the generalised Hooke's law. With the co-ordinate axes 1, 2 and 3 coinciding with the principal axes, and the vertical axis being axis-3, as shown in Fig. 1, the generalised Hooke's law can be expressed as follows:

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{13} \\ \gamma_{23} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} A & B & C & & & \\ B & A & C & & & \\ C & C & D & & & \\ & & & F & & \\ & & & & F & \\ & & & & & A-B \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{13} \\ \tau_{23} \\ \tau_{12} \end{Bmatrix} \qquad \text{(1a-f)}$$

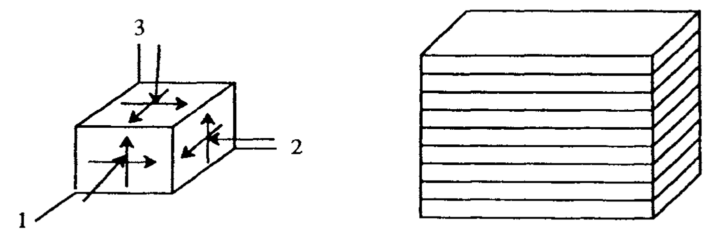

Fig. 1 Orientation of cross-anisotropic specimen relative to a Cartesian co-ordinate system

In eq. (1a-f) there are five independent parameters A, B, C, D and F to be determined in the elastic compliance matrix. No single apparatus currently available can be used to provide the five parameters. Theoretically, based on the assumption that the elasticity of the materials is kept unchanged during the loading and unloading process, the principle of superposition may be used to analyse cross-anisotropic problems. That is to say, the element stress state shown in Fig. 1 may be divided into the four simple stress states illustrated in Fig. 2.

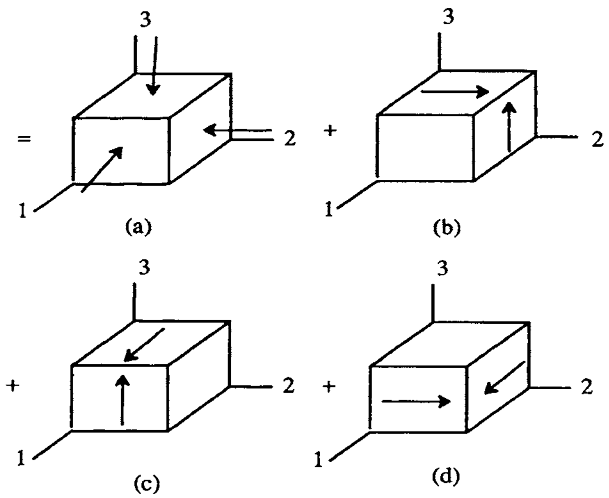

Fig. 2 Superposition of element stress states

The principal stress state in Fig. 2(a) may be simulated by cubical true triaxial testing equipment and the pure shearing stress states in Fig. 2 (b), (c) and (d) may be simulated by pure shearing testing apparatus. It is easily shown that the parameters describing stress states in Fig. 2 (b) and (c) are equivalent and either of them can provide the elastic constant F. The parameters A, B, C and D in eqs (1a-c) cannot be obtained through solving the three normal simultaneous equations unless parameter, A-B, in eq. (1f) can be solved by pure shearing stress testing facility, because there are only three independent equations for four unknown variables. Normally, the cross-anisotropic elastic constants are determined through convention triaxial testing apparatus by performing tests with specimens having vertical and horizontal orientations and monitoring the relevant deformations. Ideally, the three-dimensional principal stress and strain states are simulated by true triaxial testing equipment. In this specific apparatus the three mutually perpendicular principal stresses can be applied on a cubical sample and the corresponding stresses and strains can be monitored simultaneously by the relevant computer system. On the other hand, the parameter F in eq. (1d) or (1e) requires a test in which pure shear stresses are applied externally to horizontal and vertical surfaces of the specimen. Thus the determination of the five elastic constants requires additional equipment.

To date, it seems too expensive to perform such complicated experiments to obtain the five parameters. Therefore, what concerns us most is first to find approximate solutions to eqs. (1a-f) to meet the requirements of engineering practice.

SOLUTION TO TRUE TRIAXIAL TESTS

As mentioned above, the four parameters in eqs. (1a-c) cannot be directly determined. Graham and Houlsby (1983) proposed that there exist simple interrelationships among the four elastic constants such that just three independent parameters remain. These may be determined by static triaxial testing. Diverging from the assumptions made by Graham and Houlsby (1983), an approach based on least squares method is now presented to evaluate the four independent parameters of the pavement materials which are undergoing repeated loadings by means of a set of results of true triaxial cyclic testing. In addition, so-called fuzzy logic weighting factors (FLWF) are introduced in the following section to assess the differences among the observed principal deformations of samples tested. In the true triaxial tests the incremental stresses and strains are recorded simultaneously by a computer system. Thus the data to be used to calculate the elastic parameters in eqs. (1a-c) may be expressed in the incremental form.

$$\begin{Bmatrix}\delta\epsilon_1\\ \delta\epsilon_2\\ \delta\epsilon_3\end{Bmatrix}=\begin{bmatrix}A & B & C\\ B & A & C\\ C & C & D\end{bmatrix}\begin{Bmatrix}\delta\sigma_1\\ \delta\sigma_2\\ \delta\sigma_3\end{Bmatrix} \qquad \text{(2a-c)}$$

From the data represented in eqs. (2a-c), it seems impossible to directly solve the three equations of four unknowns A, B, C and D. Rather at least two sets of tests have to be carried out to obtain the four parameters. However, if two sets of tests are available, there are then six equations with four variables. Thus they produce a considerable amount of mathematically redundant information about the cross-anisotropic elasticity of compacted soils. Furthermore, the intrinsic differences which may arise from in presumably similar samples and because of the presence of small experimental errors resulting from the measuring system, the two groups of equations derived from the test results will be mutually inconsistent to some extent. Such a problem may be solved by the least squares method to minimise the random errors.

By using stress and strain increments as independent variables in eqs. (2a-c), a least squares solution of the four parameters can be readily obtained. If the measured values of three principal stress increments $\delta\sigma_i$, $i=1,2,3$ are obtained and input to the three relevant equations, then the principal incremental strains $(\delta\epsilon_i)_c$, $i=1,2,3$ can be calculated from

$$\begin{Bmatrix}(\delta\epsilon_1)_c\\ (\delta\epsilon_2)_c\\ (\delta\epsilon_3)_c\end{Bmatrix}=\begin{bmatrix}A & B & C\\ B & A & C\\ C & C & D\end{bmatrix}\begin{Bmatrix}\delta\sigma_1\\ \delta\sigma_2\\ \delta\sigma_3\end{Bmatrix} \qquad \text{(3a-c)}$$

The errors of the principal strain increments $(\delta\epsilon_i)_e$, $i=1,2,3$ are then obtained by the difference between the calculated and measured values.

$$\begin{Bmatrix}(\delta\epsilon_1)_e\\ (\delta\epsilon_2)_e\\ (\delta\epsilon_3)_e\end{Bmatrix}=\begin{bmatrix}A & B & C\\ B & A & C\\ C & C & D\end{bmatrix}\begin{Bmatrix}\delta\sigma_1\\ \delta\sigma_2\\ \delta\sigma_3\end{Bmatrix}-\begin{Bmatrix}\delta\epsilon_1\\ \delta\epsilon_2\\ \delta\epsilon_3\end{Bmatrix} \qquad \text{(4a-c)}$$

where $\delta\epsilon_i$, i= 1, 2, 3 are the corresponding measured principal strain increments in the true triaxial tests.

The sum of the square of errors in the strain increments for all available tests is given by

$$\Delta = \sum^{\text{Tests}} (A\delta\sigma_1 + B\delta\sigma_2 + C\delta\sigma_3 - \delta\epsilon_1)^2 + (B\delta\sigma_1 + A\delta\sigma_2 + C\delta\sigma_3 - \delta\epsilon_2)^2 + (C\delta\sigma_1 + C\delta\sigma_2 + D\delta\sigma_3 - \delta\epsilon_3)^2 \quad (5)$$

The least squares solution for parameters A, B, C and D from the redundant equations is found by setting the differentials of the error measurement with respect to each of the parameters A, B, C and D in turn to zero.

$$\frac{\partial\Delta}{\partial A} = \Sigma 2(A\delta\sigma_1 + B\delta\sigma_2 + C\delta\sigma_3 - \delta\epsilon_1)\delta\sigma_1 + 2(B\delta\sigma_1 + A\delta\sigma_2 + C\delta\sigma_3 - \delta\epsilon_2)\delta\sigma_2 = 0 \quad (6)$$

$$\frac{\partial\Delta}{\partial B} = \Sigma 2(A\delta\sigma_1 + B\delta\sigma_2 + C\delta\sigma_3 - \delta\epsilon_1)\delta\sigma_2 + 2(B\delta\sigma_1 + A\delta\sigma_2 + C\delta\sigma_3 - \delta\epsilon_2)\delta\sigma_1 = 0 \quad (7)$$

$$\frac{\partial\Delta}{\partial C} = \Sigma 2(A\delta\sigma_1 + B\delta\sigma_2 + C\delta\sigma_3 - \delta\epsilon_1)\delta\sigma_3 + 2(B\delta\sigma_1 + A\delta\sigma_2 + C\delta\sigma_3 - \delta\epsilon_2)\delta\sigma_3 + 2(C\delta\sigma_1 + C\delta\sigma_2 + D\delta\sigma_3 - \delta\epsilon_3)(\delta\sigma_1 + \delta\sigma_2) = 0 \quad (8)$$

$$\frac{\partial\Delta}{\partial D} = \Sigma 2(C\delta\sigma_1 + C\delta\sigma_2 + D\delta\sigma_3 - \delta\epsilon_3)\delta\sigma_3 = 0 \quad (9)$$

Equations (6)-(9) may be expressed in matrix form

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & 0 \\ m_{21} & m_{22} & m_{23} & 0 \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & m_{43} & m_{44} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix} = \begin{Bmatrix} d_{11} \\ d_{12} \\ d_{13} \\ d_{14} \end{Bmatrix} \quad (10a\text{-}d)$$

where

$$\left.\begin{aligned} m_{11} &= \Sigma(\delta\sigma_1^2 + \delta\sigma_2^2) \\ m_{12} &= m_{21} = \Sigma 2\delta\sigma_1\delta\sigma_2 \\ m_{13} &= m_{31} = \Sigma(\delta\sigma_1 + \delta\sigma_2)\delta\sigma_3 \\ m_{23} &= m_{32} = \Sigma(\delta\sigma_1 + \delta\sigma_2)\delta\sigma_3 \\ m_{33} &= \Sigma[(\delta\sigma_1 + \delta\sigma_2)^2 + 2\delta\sigma_3^2] \\ m_{34} &= m_{43} = \Sigma(\delta\sigma_1 + \delta\sigma_2)\delta\sigma_3 \\ m_{44} &= \Sigma\delta\sigma_3^2 \\ d_{11} &= \Sigma(\delta\epsilon_1\delta\sigma_1 + \delta\epsilon_2\delta\sigma_2) \\ d_{12} &= \Sigma(\delta\epsilon_1\delta\sigma_2 + \delta\epsilon_2\delta\sigma_1) \\ d_{13} &= \Sigma[(\delta\epsilon_1 + \delta\epsilon_2)\delta\sigma_3 + (\delta\sigma_1 + \delta\sigma_2)\epsilon_3] \\ d_{14} &= \Sigma\delta\sigma_3\delta\epsilon_3 \end{aligned}\right\} \quad (10e\text{-}o)$$

It should be pointed out that in any two sets of true triaxial tests the applied principal stress increments must meet the following condition

$$\frac{(\delta\sigma_i)_1}{(\delta\sigma_j)_2} \neq k \qquad (i=j=1,2,3) \quad (11)$$

in which k is a constant.

As is well-known, the necessary and sufficient condition that eqs. (10a-d) have unique solution is that the determinant of the coefficient matrix is not equal to zero. That is,

$$D_4 = \begin{vmatrix} m_{11} & m_{12} & m_{13} & 0 \\ m_{21} & m_{22} & m_{23} & 0 \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & m_{43} & m_{44} \end{vmatrix} \neq 0 \quad (12)$$

From eq.(10e-o) and (12) we have

$$D_4 = \Sigma(\delta\sigma_1 - \delta\sigma_2)^2\left[\Sigma(\delta\sigma_1 + \delta\sigma_2)^2 + 2\Sigma\delta\sigma_3^2\right] * \left[\Sigma(\delta\sigma_1 + \delta\sigma_2)^2\Sigma\delta\sigma_3^2 - [\Sigma(\delta\sigma_1 + \delta\sigma_2)\delta\sigma_3]^2\right] \quad (13)$$

It can be seen that D_4 are all dependent on the increments of applied principal stresses.

Only if

$$\delta\sigma_1 \neq 0,\ \delta\sigma_2 \neq 0,\ \delta\sigma_1 \neq \delta\sigma_2 \text{ and } \Sigma(\delta\sigma_1 + \delta\sigma_2) \neq \Sigma\delta\sigma_3 \quad (14)$$

then

$$\Sigma(\delta\sigma_1 - \delta\sigma_2)^2 > 0$$
$$\{\Sigma(\delta\sigma_1 + \delta\sigma_2)^2\Sigma\delta\sigma_3^2 - [\Sigma(\delta\sigma_1 + \delta\sigma_2)\delta\sigma_3]^2\} > 0 \quad (15)$$

thus $D_4 > 0$

It can be concluded, therefore, that once the conditions in eq. (14) are satisfied, the solution of eqs. (10a-d) will be unique. All these stress incremental conditions are readily controlled in true triaxial cyclic testing. Therefore the approximate method described in detail above can be used to estimate the parameters A, B, C and D for elastic cross-anisotropic materials under cyclic loading.

A FORTRAN computer program can be used to solve the linear simultaneous equations (10a-d). Because of the symmetry of the coefficient matrix the Gauss method is used in the arrangement of the computer code.

SOLUTION TO CONVENTIONAL TRIAXIAL TESTS

For the conventional triaxial test where $\delta\sigma_2=\delta\sigma_3$ and $\delta\epsilon_2=\delta\epsilon_3$, thus eqs. (2a-c) further simplifies to

$$\begin{Bmatrix}\delta\epsilon_1\\ \delta\epsilon_3\end{Bmatrix}=\begin{bmatrix}D & 2C\\ C & A+B\end{bmatrix}\begin{Bmatrix}\delta\sigma_1\\ \delta\sigma_3\end{Bmatrix} \tag{16a-b}$$

Where axis-1 is denoted as the vertical axis for engineering convention. This is made to keep the notation consistent with the conventional designation.

Let H=A+B, then we get

$$\begin{Bmatrix}\delta\epsilon_1\\ \delta\epsilon_3\end{Bmatrix}=\begin{bmatrix}D & 2C\\ C & H\end{bmatrix}\begin{Bmatrix}\delta\sigma_1\\ \delta\sigma_3\end{Bmatrix} \tag{17a-b}$$

Similarly, the errors of the strain increments may be expressed by

$$(\delta\epsilon_1)_e=D\delta\sigma_1+2C\delta\sigma_3-\delta\epsilon_1 \tag{18}$$

$$(\delta\epsilon_3)_e=C\delta\sigma_1+H\delta\sigma_3-\delta\epsilon_3 \tag{19}$$

The sum of the square of errors in the strain increments for all available tests is given by

$$\Delta=\sum^{\text{Tests}}\left[(D\delta\sigma_1+2C\delta\sigma_3-\delta\epsilon_1)^2+(C\delta\sigma_1+H\delta\sigma_3-\delta\epsilon_3)^2\right] \tag{20}$$

Then let

$$\frac{\partial\Delta}{\partial D}=\Sigma 2(D\delta\sigma_1+2C\delta\sigma_3-\delta\epsilon_1)\delta\sigma_1=0 \tag{21}$$

$$\frac{\partial\Delta}{\partial C}=\Sigma 2(D\delta\sigma_1+2C\delta\sigma_3-\delta\epsilon_1)(2\delta\sigma_3)+2(C\delta\sigma_1+H\delta\sigma_3-\delta\epsilon_3)\delta\sigma_1=0 \tag{22}$$

$$\frac{\partial\Delta}{\partial H}=\Sigma 2(C\delta\sigma_1+H\delta\sigma_3-\delta\epsilon_3)\delta\sigma_3=0 \tag{23}$$

Thus the following equations may be obtained

$$\begin{bmatrix}n_{11} & n_{12} & 0\\ n_{21} & n_{22} & n_{23}\\ 0 & n_{32} & n_{33}\end{bmatrix}\begin{Bmatrix}D\\ C\\ H\end{Bmatrix}=\begin{Bmatrix}\Sigma\delta\epsilon_1\delta\sigma_1\\ \Sigma(\delta\epsilon_1\delta\sigma_3+\delta\epsilon_3\delta\sigma_1)\\ \Sigma\delta\epsilon_3\delta\sigma_3\end{Bmatrix} \tag{24a-c}$$

where

$$\left.\begin{aligned} n_{11}&=\Sigma\delta\sigma_1^2\\ n_{12}&=n_{21}=\Sigma 2\delta\sigma_1\delta\sigma_3\\ n_{22}&=\Sigma(\delta\sigma_1^2+4\delta\sigma_3^2)\\ n_{13}&=n_{31}=\Sigma\delta\sigma_1\delta\sigma_3\\ n_{33}&=\Sigma\delta\sigma_3^2\end{aligned}\right\} \tag{24d-h}$$

The determinant of the third order matrix coefficient can be calculated from

$$D_3=\left[\Sigma\delta\sigma_1^2+4\Sigma\delta\sigma_3^2\right]\left[\Sigma\delta\sigma_1^2\Sigma\delta\sigma_3^2-\Sigma(\delta\sigma_1\delta\sigma_3)^2\right] \tag{25}$$

Provided

$$\delta\sigma_1\neq 0 \text{ and } \delta\sigma_3\neq 0\ ,\ \Sigma\delta\sigma_1\neq\Sigma\delta\sigma_3 \tag{26}$$

Then $D_3 > 0$

Thus it can be concluded that eqs. (24a-c) will have unique solution once the conditions in eq. (26) are met. These are easily controlled in the conventional triaxial test.

The next step is to solve the set of simultaneous eqs. (24a-c). Parameters D, C and H can be readily evaluated by Cramer's Rule. Thus the three elastic constants of cross-anisotropic materials can be obtained from conventional triaxial testing.

DETERMINATION OF ENGINEERING PARAMETERS

By considering the symmetry of the cross-anisotropic compliance matrix and the condition of isotropy in the horizontal plane, the generalised Hooke' law can be expressed as follows:

$$\begin{Bmatrix}\delta\epsilon_1\\ \delta\epsilon_2\\ \delta\epsilon_3\\ \delta\gamma_{13}\\ \delta\gamma_{23}\\ \delta\gamma_{12}\end{Bmatrix}=\begin{bmatrix}\frac{1}{E_h} & \frac{-\nu_{hh}}{E_h} & \frac{-\nu_{hv}}{E_h} & & & \\ \frac{-\nu_{hh}}{E_h} & \frac{1}{E_h} & \frac{-\nu_{hv}}{E_h} & & & \\ \frac{-\nu_{hv}}{E_h} & \frac{-\nu_{hv}}{E_h} & \frac{1}{E_v} & & & \\ & & & \frac{1}{G_{hv}} & & \\ & & & & \frac{1}{G_{hv}} & \\ & & & & & \frac{1}{G_{hh}}\end{bmatrix}\begin{Bmatrix}\delta\sigma_1\\ \delta\sigma_2\\ \delta\sigma_3\\ \delta\tau_{13}\\ \delta\tau_{23}\\ \delta\tau_{12}\end{Bmatrix} \tag{27a-f}$$

$$\frac{\nu_{hv}}{E_h}=\frac{\nu_{vh}}{E_v};\quad G_{hh}=\frac{E_h}{2(1+\nu_{hh})} \tag{28}$$

where

E_v = Young's modulus in vertical direction;
E_h = Young's modulus in horizontal direction;
ν_{hv} = Poisson's ratio of strain in vertical direction to applied strain in horizontal direction;
ν_{vh} = Poisson's ratio of strain in horizontal direction to applied strain in vertical direction;
ν_{hh} = Poisson's ratio of strain in horizontal direction to applied strain in orthogonal horizontal direction;
G_{hv} = Shear modulus in any vertical plane, and
G_{hh} = Shear modulus in any horizontal plane.

By comparing the (1a-f) and (27a-f), thus we get

$$E_v=\frac{1}{D};\quad E_h=\frac{1}{A};\quad \nu_{hv}=\frac{-C}{A};\quad \nu_{hh}=\frac{-B}{A};\quad G_{hh}=\frac{E_h}{2(1+\nu_{hh}} \tag{29}$$

It is worth pointing out that all the above mentioned calculations can be implemented by the computer program ANISOLV.

FUZZY LOGIC WEIGHTING FACTORS

It is difficult to achieve two specimen with 'same' physical, geological and micro-fabric properties in the laboratory preparation of samples because of the presence of intrinsic variability. Also the experimental system itself may introduce errors. Because of this, supposedly identical samples and tests produce different test results. It is, therefore, prudent to introduce fuzzy logic weighting factors (FLWF), f_i, i=1,2,3 to reflect the differences among the measured principal strains $\delta\epsilon_i$, i=1,2,3 of samples tested, which enable the user to properly assess the sample quality and similarity. However it should be emphasised that great effort has to be made to obtain the reasonable FLWFs. By introducing the FLWFs, f_i, i=1,2,3, eqs.(10a-d) and (10e-o) yield the more general forms

$$\begin{bmatrix} m'_{11} & m'_{12} & m'_{13} & 0 \\ m'_{21} & m'_{22} & m'_{23} & 0 \\ m'_{31} & m'_{32} & m'_{33} & m'_{34} \\ 0 & 0 & m'_{43} & m'_{44} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix} = \begin{Bmatrix} d'_{11} \\ d'_{12} \\ d'_{13} \\ d'_{14} \end{Bmatrix} \tag{30a-d}$$

$$\left.\begin{aligned} m'_{11} &= \Sigma(f_1\delta\sigma_1^2+f_2\delta\sigma_2^2) \\ m'_{12} &= m'_{21}=\Sigma 2f_1f_2\delta\sigma_1\delta\sigma_2 \\ m'_{13} &= m'_{31}=\Sigma(f_1\delta\sigma_1+f_2\delta\sigma_2)\delta\sigma_3 \\ m'_{23} &= m'_{32}=\Sigma(f_1\delta\sigma_1+f_2\delta\sigma_2)\delta\sigma_3 \\ m'_{33} &= \Sigma[f_3(\delta\sigma_1+\delta\sigma_2)^2+2f_1f_2\delta\sigma_3^2] \\ m'_{34} &= m'_{43}=\Sigma f_3(\delta\sigma_1+\delta\sigma_2)\delta\sigma_3 \\ m'_{44} &= \Sigma f_3\delta\sigma_3^2 \\ d'_{11} &= \Sigma(f_1\delta\epsilon_1\delta\sigma_1+f_2\delta\epsilon_2\delta\sigma_2) \\ d'_{12} &= \Sigma(f_1\delta\epsilon_1\delta\sigma_2+f_2\delta\epsilon_2\delta\sigma_1) \\ d'_{13} &= \Sigma[(f_1\delta\epsilon_1+f_2\delta\epsilon_2)\delta\sigma_3+f_3(\delta\sigma_1+\delta\sigma_2)\delta\epsilon_3] \\ d'_{14} &= \Sigma f_3\delta\sigma_3\delta\epsilon_3 \end{aligned}\right\} \tag{30e-o}$$

Similarly we can applied the FLWFs to measured strains of conventional tests and obtain expressions similar to those described above. It is interesting to note that eqs. (30a-d) and (30e-o) are reduced to eqs.(10a-d) and (10e-o) respectively if the fuzzy logic weighting factors are unity. All these factors are incorporated in the program, ANISOLV.

ILLUSTRATIVE EXAMPLES

The ANISOLV program needs an input file which is edited according to the true triaxial cyclic testing conditions. The input parameters are the number of simultaneous equations, the number of the collected data groups, the three principal stress increments and the corresponding three principal strain increments. The fuzzy logic weighting factors are optional, and largely depend on the sample assessment before and after the samples are tested. The ANISOLV program permits the engineering parameters E_v, E_h, ν_{hv}, ν_{hh} to be readily obtained. Ten calculation examples are given in Appendix A. These results show that the approximate approach to determining the four elastic parameters is practical and valid and that the program ANISOLV is effective and reliable.

CONCLUSIONS

A new approximate approach to solving the basic linear simultaneous equations of the elastic stress and strain relationships of cross-anisotropic materials has been presented. This is shown to be practical and valid. The uniqueness of the solution is conditional upon assumptions which are readily satisfied by true triaxial cyclic testing in the laboratory. Fuzzy logic weighting factors may be introduced to make it possible to assess the differences among the principal deformations of samples tested.

The program ANISOLV implementing the formulation of the approximate method appears to be reliable and useful in dealing with the testing data. By this method the elastic constants required for the calculation of the stresses in the elastic media and their surface settlements under cyclic loading are readily obtained.

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Appendix A: Some examples for demonstration of the ANISOLV program for determining the four elastic parameters of cross-anisotropic materials under true triaxial cyclic loading.

No.	$\delta\sigma_1$	$\delta\sigma_2$	$\delta\sigma_3$	$\delta\epsilon_1$	$\delta\epsilon_2$	$\delta\epsilon_3$	E_v	E_h	ν_{hv}	ν_{hh}
1-a	5.000	8.000	14.00	-.108	0.054	0.0980	61.28	28.05	.283	.509
1-b	5.010	8.020	14.03	-.1082	0.053	0.0970				
2-a	5.000	8.000	14.00	-.108	0.054	0.0980	50.80	25.28	.347	.360
2-b	5.010	8.020	14.03	-.1082	0.053	0.0968				
3-a	3.000	4.000	10.00	-.0760	-.022	0.1020	54.52	27.04	.314	.481
3-b	3.010	4.010	10.00	-.0765	-.021	0.1019				
4-a	10.00	14.00	20.00	-.0760	0.140	0.0640	52.22	25.36	.338	.337
4-b	10.00	14.01	20.00	-.0765	0.140	0.0633				
5-a	6.000	8.000	16.00	-.0960	0.012	0.1240	48.80	24.68	.359	.327
5-b	6.000	8.010	16.00	-.9550	.01218	0.1243				
6-a	6.000	9.000	18.00	-.1240	-.0160	0.1640	59.11	40.75	.382	.464
6-b	6.000	9.010	18.00	-.1239	-.0160	0.1638				
7-a	2.000	2.500	3.000	-.00365	.01116	-.002813	117.5	50.45	.318	.492
7-b	2.000	2.500	3.000	-.003645	.011158	-.002813				
8-a	18.00	25.00	28.00	-.0350	.11457	.00495	144.1	69.08	.304	.476
8-b	18.00	25.00	28.00	-.0350	.11457	.00510				
9-a	18.00	25.00	28.00	-.0350	.11457	.00494	94.48	62.42	.423	.334
9-b	18.00	25.00	28.00	-.0350	.11457	.00508				
10-a	18.00	25.00	28.00	-.0350	.1145	.00494	149.1	69.56	.296	.486
10-b	18.00	25.00	28.00	-.03498	.1145	.00508				