Experimental and analytical evaluation of FRP-confined large size reinforced concrete columns

Silvia Rocca

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EXPERIMENTAL AND ANALYTICAL EVALUATION OF FRP-CONFINED
LARGE SIZE REINFORCED CONCRETE COLUMNS

by

SILVIA ROCCA

A DISSERTATION
Presented to the Faculty of the Graduate School of the
UNIVERSITY OF MISSOURI-ROLLA
In Partial Fulfillment of the Requirements for the Degree

DOCTOR OF PHILOSOPHY

in

CIVIL ENGINEERING

2007

__________________________                              __________________________
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Abdeldjelil Belarbi                                                    Nicholas Carino
PUBLICATION DISSERTATION OPTION

This dissertation consists of the following three articles that have been submitted for publication as follows:

The first paper consisting of pages 10 through 60 has been prepared in the style used by the American Concrete Institute (ACI) – Special Publication and accepted for publication. The second paper consisting of pages 61 to 107 has been prepared in the style utilized by the American Society of Civil Engineers (ASCE) – Journal of Composite for Construction and accepted for publication in that journal. The third paper consisting of pages 108 to 154 has been prepared in the style used by the Journal of Construction and Building Materials and submitted for publication to that journal.
Fiber Reinforced Polymer (FRP) jacketing of Reinforced Concrete (RC) columns, as a rapidly growing strengthening technique, requires appropriate design methods. However, the current available design guides are based on models, in their majority semi-empirical, and calibrated with small-scale plain concrete specimens. These models might not be reliable in predicting the strength and ductility enhancement sought through FRP-confinement in RC columns with geometric and material properties as found in practice.

This dissertation presents the research conducted in three technical papers. In the first paper, the performance of 22 FRP-wrapped RC columns of medium- and large-scale is studied. The results of the experimental campaign were used to develop and evaluate an analytical method to determine the confining pressure and compressive capacity in a non-circular column. The model performed well in predicting the capacity of relevant size FRP-confined RC columns available from the literature.

A second paper is devoted to the review of state-of-the-art design methodologies available for the case of FRP-confined RC columns, and their ability to predict the increment of compressive strength and ductility. The observed outcomes were used to identify and remark upon the limits beyond the ones specifically stated by each of the guides and that reflect the absence of effects not considered in current models.

Finally, a third paper addresses the subject of FRP-confined RC columns subjected to combined axial load and bending moment. A design-oriented methodology is presented for the construction of a simplified interaction diagram in the compression-controlled region. Its performance was evaluated on the base of available data from the literature.
ACKNOWLEDGMENTS

I cannot present this work without first expressing my appreciation and gratitude to Dr. Antonio Nanni, Dr. Abdeldjelil Belarbi, Dr. Ashraf Ayoub, Dr. Victor Birman, and Dr. Lokeswarappa Dharani. I am honored to have you as my committee members and professors. Furthermore, I would like to acknowledge the funding and support received from: National Science Foundation (supplement grant number 0453808), MAPEI S.p.A. in Milan (Italy) for donating the FRP material, NSF Industry/University Cooperative Research Center on Repair of Buildings and Bridges with Composites (RB²C). The University Transportation Center on Advanced Materials and NDT Technologies based at the University of Missouri – Rolla is also recognized for its financial support.

A special recognition is given to the National Institute of Standards and Technology (NIST) and University of California - San Diego (UCSD), in the persons of Dr. Nicholas Carino and Mr. Frank Davis at NIST, and Dr. Gianmario Benzoni and Mr. Donato Innamorato at UCSD, for their cooperation conducting the tests. Thank you for attention, availability and willingness to help at any time.

Last but not least, all my gratitude to Nestore, my brothers, my parents and my friends, thank you for your love, support, and understanding.
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1. INTRODUCTION

Reinforced Concrete (RC) columns as vertical structural members that transmit axial compressive loads with or without moments are of critical importance for the performance and the safety of structures. Nowadays, it is commonly seen the need of strengthening and/or rehabilitating these members due to different reasons, namely: higher load capacity demands because of design/construction errors, change in the facility use, or revisions of code requirements. Ductility enhancement is typically required in existing columns that are subjected to a combination of axial load and bending moment because of reasons similar to those listed for strengthening. Among these reasons, seismic upgrade and correction of detailing defects (i.e., improper splicing of the longitudinal reinforcement or lack of transverse ties) are the most common.

Confinement of concrete is an efficient technique used to increase the load carrying capacity and/or ductility of a column. It is precisely the lateral pressure that induces in the concrete a tri-axial state of stresses and consequently an increment of compressive strength and ultimate axial strain.

Strengthening of RC columns was first conducted by means of steel jackets grouted to the concrete core, but at the beginning of the 1990s they were replaced by Fiber Reinforced Polymer (FRP) jackets. The application of FRP materials for column strengthening gained popularity in the last years, not only for its ease of installation, flexibility, and aesthetics, but mainly for its intrinsic properties as a material, such as: high strength to weight ratio, and good corrosion behavior. The disadvantages that come
along with the advantages in FRP applications are namely: lower quality control, and environmental stability (long term performance of certain components of the FRP jacket might not be optimum under different effects like ultraviolet radiation, thermal cycles, and humidity).

FRP confinement is accomplished by placing the fibers mainly transverse to the longitudinal axis of the column providing passive confinement, which is activated once the concrete core starts dilating as a result of Poisson’s effect and internal cracking.

The confinement of non-circular columns is widely accepted to be less efficient than the confinement of circular columns, since in the latter case, the jacket provides circumferentially uniform confining pressure to the radial expansion of the concrete. In non-circular columns, the confinement is concentrated at the corners rather than over the entire perimeter. Based on studies on steel-confined concrete, the generally accepted theoretical approach is to develop an effectively confined area defined by four second-degree parabolas that intersect the edges at 45°. The shape of the parabolas and the resulting effectively confined area are function of the dimensions of the cross-section and the chamfered corner radius.

Extensive work on both the experimental and analytical areas has been conducted on small-scale plain concrete specimens of circular and non-circular cross-sections confined with FRP and subjected to pure axial compressive loading (De Lorenzis and Tepfers 2001; Lam and Teng 2003a,b; Masia et al. 2004). Studies focused on RC columns of both circular and non-circular cross-sections of considerable size (one minimum dimension of the cross-section of about 300 mm [12 in.]) have also been conducted (Demers and Neale 1994; Kestner et al. 1997; Wang and Restrepo 2001;
Youssef 2003; Carey and Harries 2003; Matthys et al. 2005); however, this experimental research has been limited due to high cost and lack of high-capacity testing equipment. This situation has been the main reason for overlooking the following important effects on element performance not accounted for in most of the available models: (a) the size of the cross-sectional area; (b) the dimensional aspect ratio of the cross-sectional area (ratio of long side to short side); (c) the presence and possible detrimental effect of longitudinal steel reinforcement instability; (d) the concrete dilation; and (e) the contribution of the internal transverse steel reinforcement to the confining pressure.

In spite of these obstacles, design methods have been proposed based on the performance of small-scale plain concrete specimens (De Lorenzis and Tepfers, 2001; Lam and Teng, 2003a,b; Harajli et al., 2006). However, they might not be reliable in predicting the strength and ductility enhancement that might be achieved for larger columns found in practice. At this point, it was required additional experimental evidence on size-relevant specimens that complement the limited existing data, and that would allow addressing the above listed effects on the behavior of RC columns.

The work presented in this dissertation provides complementary experimental evidence on relevant size FRP-wrapped RC columns subject to pure axial compressive loading. These specimens were internally and externally extensively instrumented with strain gages and linear transducers. However, not all the readings were found to be useful. For example, in the non-circular specimens, the strain gauges located on the FRP jacket at the mid-distance on each face provided higher strain readings that the ones close to the corners which was the opposite of what expected. This difference is the result of two effects: a tensile strain due to confinement effect and a flexural strain due to the non-
uniform concrete dilation inducing a lateral pressure on the jacket (bulging). The latter is of a higher magnitude since the out-of-plane (flexural) stiffness of the FRP jacket is much lower than its axial (tensile) stiffness. This difference in strain also reflects the lack of confinement along the sides of the non-circular cross-sections. Future research should consider these effects in designing the instrumentation of non-circular specimens.

The difficulty to correlate the measured transverse strains at the corners to the confining pressure in the non-circular specimens, led to the development of a new analytical model to determine the lateral pressure, which on the contrary to the ones available in the literature, it is not based on an equivalent circular section, but on the actual shape of the effectively confined concrete area. The model was calibrated with the data from the experimental program and its performance was validated with data collected from the literature. Further research is recommended to refine and verify the basic assumption of this approach.

Parallel to this experimental work, a constructive critical review of the state-of-the-art design guidelines for FRP-confinement of RC columns was conducted. The purpose of this study was to use pertinent experimental evidence to identify and remark on the differences in the design methodologies used by the existing available design guides on the FRP confinement of RC columns of different cross-sections and subject to pure axial loading. The following fundamental reasons further clarify the purpose of this study:

- Understanding of pure axial performance addresses both the load capacity increase and ductility enhancement. The latter, in particular, is the result of strain enhancement in terms of both ultimate value and shape of the stress-strain curve;
- Increase in strength is an immediate and evident outcome typically expressed as the peak load resistance (or peak strength obtained as maximum load minus the contribution of steel and divided by the gross concrete area). On the contrary, increase in ductility is a more complex performance indicator that needs to be translated into the ability of a member to sustain rotation and drift without a substantial loss in strength. Ductility enhancement under pure compression conditions is not of great use in itself, but it becomes of great importance in the case of combined axial compressive load and bending moment;

- Depending on the level of confinement, the stress-strain behavior of a RC column can be modified to the point that peak capacity corresponds to the ultimate attainable axial strain;

- Understanding the effect of all the parameters contributing to the behavior of a FRP-confined RC column (contrast of the ideal cross-section behavior with that of a column where phenomena such as the instability of longitudinal reinforcement that is member-dependant and not simply cross-section dependant, needs to be considered) will help represent and predict such behavior. Thus, the performance of a column subjected to combined axial force and bending moment would be greatly simplified, and addressing structural ductility in more rigorous terms would become easier.

This study addressed both, the increment of concrete compressive strength and ductility. The observed outcomes were used to identify and remark upon the limits
beyond the ones specifically stated by each of the guides and that reflect the absence of effects not considered in current models. The purpose of this study is to present a constructive critical review of current design methodologies available for the case of FRP-confined concrete RC columns and to indicate a path for future research.

As already mentioned, the purpose of column strengthening is twofold: increment of axial compressive capacity and increment of ductility in terms of axial deformation. Ductility enhancement is typically required in existing columns that are subjected to a combination of axial load and bending moment, which is the most common situation in practice.

A design-oriented methodology for the construction of a simplified interaction (P-M) diagram limited to the compression-controlled region is presented. In the proposed method, the analysis of FRP-confined columns is carried out based on principles of equilibrium and strain compatibility equivalent to that of conventional RC columns. Thus far the available procedures for interaction diagrams are complex, iterative, and not readily use for design, therefore the importance of this proposed method relies on the simplicity of its application, in particular for practitioners. The proposed analytical procedure has been adopted by the American Concrete Institute (ACI) committee 440 for the design of members subjected to axial force and bending moment. This procedure was validated on a relatively small set of experimental data. Additional research is needed to confirm the level of safety of this approach.
2. OUTLINE OF THE DISSERTATION

Three papers are presented in this dissertation, and the general conclusions are listed in a section following the papers. Additional information is also presented in three appendices.

The first paper in this dissertation, which is titled “Axial Load Behavior of Large-Size Reinforced Concrete Columns Strengthened with Carbon FRP,” presents the results of an experimental investigation on the axial behavior of 22 medium- and large-scale RC columns of circular and non-circular cross-sections, and strengthened with unidirectional Carbon FRP (CFRP) wraps. Factors influencing the strength and ductility enhancement such as: the effectiveness of the confinement depending on the cross-section geometry, the side-aspect ratio, and the area-aspect ratio; and the actual rupture strain of the FRP at ultimate are addressed. Additionally, a new analytical method that allowed estimating the confining pressure in non-circular cross-sections from the transverse strains at the corners is proposed. The obtained confining pressures and experimental results from this study allowed calibrating a strength model, which was validated with the available experimental data in the literature. Finally, the predictions of this strength model were comparable to the ones by the model of Lam and Teng.

In a second paper titled “Review of Design Guidelines for FRP Confinement of Reinforced Concrete Columns of Non-Circular Cross-Sections,” four design guidelines are introduced, and a comparative study is presented. This study is based on the increment of concrete compressive strength and ductility, and includes the experimental results from six RC columns of different cross-section shapes presented in the first paper. The purpose of this study is to present a constructive critical review of the state-of-the-art
design methodologies available for the case of FRP-confined concrete RC columns and to indicate a direction for future developments.

Before the need for a design-oriented methodology for the construction of an interaction diagram, a third paper titled “Interaction Diagram Methodology for Design of FRP-Confined Reinforced Concrete Columns” is presented. Based on previous studies indicating that significant enhancement due to FRP-confinement is expected in compression-controlled RC members (Nanni and Norris, 1995; Teng et al., 2002), a simplified P-M diagram limited to the compression-controlled region, is introduced. The experimental results are compared to the theoretical simplified P-M diagrams obtained following the proposed methodology. Data points appear to be consistent with the analytical predictions.

The three papers composing this dissertation are complemented with material presented in Appendices A, B and C. The material included in appendix A is electronic and it is organized as follows:

1. Specimens’ specifications: two files in “PDF” format are included, one for the specimens tested at the laboratory at the University of California – San Diego (UCSD), and one related to the specimens tested at the National Institute of Standards and Technology (NIST). Information regarding the fabrication, strengthening, and instrumentation is included in these files. These two documents are accessible when opening the DVD as a unit system;

2. Videos of specimens’ failures: for the specimens at UCSD two views are available (overview and platen view). For NIST specimens only an overview is shown.
Appendix B contains in an electronic format the report submitted to the National Science Foundation (NSF) on the conducted research program. This document reports the entire set of experimental data and it has been cited in the three technical papers composing the dissertation.

Appendix C presents the chapter concerning FRP strengthening of members subjected to axial force and bending moment from ACI Committee 440 design guideline, where the methodology for the construction of a P-M diagram proposed in the third paper has been adopted.
I. AXIAL LOAD BEHAVIOR OF LARGE-SIZE REINFORCED CONCRETE COLUMNS STRENGTHENED WITH CARBON FRP

Silvia Rocca and Antonio Nanni

Synopsis: This paper presents the results of an experimental investigation on the axial behavior of medium and large scale Reinforced Concrete (RC) columns of circular and non-circular cross-sections strengthened with unidirectional Carbon Fiber Reinforced Polymer (CFRP) wraps. A test matrix was designed to investigate the effect of different variables, such as the geometry of the specimen cross-section (circular, square, and rectangular), the side-aspect ratio, and the area-aspect ratio. A total of 22 specimens were divided into six series of three specimens each and two series of two specimens each. The largest and smallest columns featured cross-sectional areas of 0.8 m² (9 ft²) and 0.1 m² (1 ft²), respectively. All the specimens were subjected to pure axial compressive loading. Factors influencing the strength and ductility enhancement such as: the effectiveness of the confinement depending on the cross-section geometry, and the actual rupture strain of the FRP at ultimate, are addressed. Additionally, a new analytical method that allowed estimating the confining pressure in non-circular cross-sections (due to steel reinforcement and FRP) from the transverse strains at the corners is proposed. The obtained confining pressures and experimental results from this study allowed calibrating a strength model, which was validated with the available experimental data in
the literature. Finally, the predictions of this strength model were compared to the ones by the model of Lam and Teng yielding close agreement.

**Keywords:** Confinement, Ductility, FRP-Strengthening, Non-Circular Columns, Reinforced Concrete.

ACI member Silvia Rocca is a Ph.D candidate in the Dept. of Civil, Arch., and Environ. Engineering at the University of Missouri-Rolla. She obtained her B.Sc. in Civil Engineering at the University of Piura – Peru. She received her M.Sc. degree in Civil Engineering at the University of Missouri – Rolla. Her research interests are in analysis, design, and retrofitting of RC structures.

FACI member Antonio Nanni is a professor and chair of the Dept. of Civil, Arch., and Environ. Engineering at the University of Miami and professor at the University of Naples – Federico II. He was the founding Chair of ACI Committee 440 (Fiber Reinforced Polymer Reinforcement) and is the Chair of ACI Committee 437 (Strength Evaluation of Existing Concrete Structures.)
INTRODUCTION

Confinement of Reinforced Concrete (RC) columns by means of Fiber Reinforced Polymer (FRP) jackets is a technique being used with growing frequency to seek the increment of load carrying capacity and/or ductility of such compression members.

The confinement of non-circular columns is generally acknowledged to be less efficient than the confinement of circular columns, since in the latter case, the wrapping provides circumferentially uniform confining pressure to the radial expansion of the compression member. In non-circular columns, the confinement pressure is concentrated at the corners rather than over the entire perimeter. Based on studies on steel-confined concrete, it is commonly accepted that for non-circular columns, the effectively confined area is defined by four second-degree parabolas that intersect the edges at 45° (Figure 1). In fact, this notion has been applied to FRP-confinement models for non-circular cross-sections by the following authors: Wang and Restrepo (2001), Lam and Teng (2003b), Harajli et al. (2006).

Extensive work on both the experimental and analytical areas has been conducted on small-scale plain concrete specimens of circular and non-circular cross-sections confined with FRP and subjected to pure axial compressive loading (De Lorenzis and Tepfers 2001; Lam and Teng 2003a,b; Masia et al. 2004). Studies focused on RC columns of both circular and non-circular cross-sections of considerable size (one minimum dimension of the cross-section of about 300mm [12 in.]) have also been conducted (Demers and Neale 1994; Kestner et al. 1997; Wang and Restrepo 2001; Youssef 2003; Carey and Harries 2003; Matthys et al. 2005); however, this experimental
research has been limited due to high cost and lack of high-capacity testing equipment. This situation has been the main reason for overlooking the following important effects on element performance not accounted for in most of the available models: (a) the size of the cross-sectional area; (b) the dimensional aspect ratio of the cross-sectional area (ratio of long side to short side); (c) the presence and possible detrimental effect of longitudinal steel reinforcement instability; (d) the concrete dilation; and (e) the contribution of the internal transverse steel reinforcement to the confining pressure.

In spite of these obstacles, design methods have been proposed based on the performance of small-scale plain concrete specimens. However, they might not be reliable in predicting the strength and ductility enhancement that might be achieved for larger columns found in practice for the reasons above stated. It is therefore necessary additional experimental evidence on size-relevant specimens that complement the limited existing data that would allow the evaluation and/or update of the current models.

**RESEARCH SIGNIFICANCE**

The behavior of FRP-confined RC columns of circular cross-section has been studied but there is a limited knowledge on the performance of RC columns of non-circular cross-section, in particular of large-size.

This research is of practical relevance in that there are thousands of RC structures (bridges and buildings) having non-circular columns that due to increases in load demands, changes in use or additions, or code updates require rapid and efficient strengthening with minimum disruption to users. Wrapping non-circular columns with
FRP has the potential to achieve increments in strength and ductility with ease of installation, provided that fundamental behavior is understood.

A systematic experimental investigation of the effect of column cross-section size and geometry is presented herein. This research study is now limited to specimens under pure axial loading condition, which is considered the first step to understand the confinement process to be later considered within the effects of combined axial force and bending moment to develop practical design interaction diagrams.

**EXPERIMENTAL PROGRAM**

**Test Matrix**

The test matrix (Table 1) was designed considering different variables, namely: side-aspect ratio \((h/b)\), area-aspect ratio (based on an area of 457×457 mm \([18\times18\text{ in.}]\)), and height-to-side aspect ratio \((H/h)\). It was composed of a total of 22 RC specimens divided into two groups based on the laboratories where the experiments were conducted: CALTRANS Seismic Response Modification Device Testing Laboratory (SRMD) at the University of California San Diego (UCSD) with 18 specimens (six series of three specimens each, that is: A, B, C, D, E, and F), and the Building and Fire Research Laboratory at the National Institute of Standards and Technology (NIST) with four specimens (two series of two specimens each, namely: G and H).

In Table 1 the first column denotes the specimens’ codes grouped by series. The geometrical and material properties are presented in the following order: diameter of the...
cross-section \( (D) \) of circular specimens or side dimensions of the non-circular specimens \((b \times h)\), side-aspect ratio \((h/b)\), height of the specimen \((H)\), height-to-side aspect ratio \((H/h)\), cross-section gross area \((A_g)\), area-aspect ratio \((A_g/A_{g(C)})\) based on gross area section of specimens in series C \((A_{g(C)})\), ratio of the area of longitudinal steel reinforcement to the cross-sectional area of the specimen \((\rho_s)\), volumetric ratio of transverse steel reinforcement to concrete core \((\rho_t)\), volumetric ratio of FRP reinforcement \((\rho_f)\), unconfined concrete compressive strength \((f'_{\text{c}})\), yield strength of longitudinal steel reinforcement \((f_y)\), and yield strength of transverse steel reinforcement \((f_{yt})\).

The dimensions of the specimens were selected as follows: the testing machine at UCSD dictated a specimen height limitation of 1.5 m (5 ft), therefore a height-to-side ratio \((H/h)\) of 2 was selected; otherwise a higher ratio would have compromised the dimension of the smaller cross-section specimens necessary to the study of the size effect. This ratio was considered appropriate based on experimental studies on length effect by Mirmiran et al. (1998). With the ratio of \(H/h\) of 2, the largest of the column specimens tested at UCSD featured a 648\(\times\)648 mm (25.5\(\times\)25.5 in.) cross-sectional area and 1.4 m (54 in.) of height (series D). Specimens of circular, square, and rectangular cross-section shapes, and of gross area sections half of the one corresponding to series D, were included in the matrix (series A, B, and C, respectively); note that the same height-to-side ratio \((H/h)\) was kept constant. To complement the variation on the size of the gross area section, two series of specimens of 324\(\times\)324 mm (12.75\(\times\)12.75 in.) were introduced, which are series E and F, respectively; the height-to-side ratio for series F was twice the original value. Finally, the largest specimens of square and rectangular geometry were
defined with cross-sectional areas of four times the ones from series B and C, respectively; the height-to-side ratio remained constant ($H/h = 2$). Very slight variations in the dimensions of the specimens were considered necessary for constructability purposes, for such reason, among the series of specimens the height-to-side ratio varies in between 2.2 and 2.1.

Each of series A, B, C, D, E, and F consists of three specimens each: one control unit (A1, B1, C1, D1, E1, and F1), one unit strengthened according to fib guideline (fib Bulletin 14 2001) to achieve an increment of 30% of load carrying capacity featuring a full wrapping scheme (A2, B2, C2, D2, E2, and F2), and a third unit whose thickness of FRP jacket matched the same number of plies used in specimen A2 (specimens B3, C3, and D3). Specimens A3, E3, and F3 were partially wrapped for a 30% increment of carrying capacity as well. Series G and H were composed of two test units each: one control (G1 and H1), and one strengthened to gain the same level of increase in axial capacity (G2 and H2) as the previous groups.

Regarding the wrapping scheme of all the strengthened specimens, a gap of about 7-13 mm (0.25-0.5 in.) was left at the top and bottom ends of the columns to avoid direct axial compressive loading of the FRP jacket. The partially wrapped specimens featured strips of 133 mm (5.25 in.) wide and a pitch of 210 mm (8.25 in.). Further information on the construction and strengthening of the specimens can be found in Rocca et al. (2006a).

The detailing of all the specimens was designed according to conventional RC practice (ACI 318-02). They featured a clear concrete cover of 38 mm (1.5 in.), and the non-circular specimens were designed with the radius of the chamfered corner of 30 mm
(1.2 in.). To prevent premature failure of the specimens at the top and bottom ends, closely spaced steel transverse reinforcement was placed at these locations (Figure 2).

**Materials Properties**

**Concrete** -- A nominal concrete compressive strength of 28 MPa (4 ksi) for the entire test program was considered appropriate for representing a common strength in current building structures. Since the specimens were cast at two different locations, the concrete constituents and properties are presented separately.

All of the specimens at UCSD were built up from one single batch of ready-mix concrete having constituents and mix proportions as follows: Portland cement 284 kg/m³ (478 lb/yd³), fly ash 53 kg/m³ (90 lb/yd³), 12.5 mm (0.5 in.) coarse gravel 682 kg/m³ (1150 lb/yd³), 9.5 mm (3/8 in) coarse gravel 309 kg/m³ (521 lb/yd³), sand 737 kg/m³ (1242 lb/yd³), water 208 kg/m³ (350 lb/yd³), Water Reducing Admixture (WRDA-64) 10 kg/m³ (17 lb/yd³), and 2 percent air-entraining agent. Standard concrete cylinders 152×305 mm (6×12 in.) were prepared and cured under the same conditions of the specimens. These cylinders were tested according to ASTM C39 (2004) at 7, 14, 21, 28 days, and at the corresponding age at which the related specimens were tested. The average compressive strength for the characteristic ages were 20.1 MPa (2.92 ksi), 23.7 MPa (3.44 ksi), 26.3 MPa (3.81 ksi), and 30.5 MPa (4.43 ksi), respectively.

Regarding the concrete for the specimens at NIST, its constituents and mix proportions were as follows: Portland Cement Type I-II 307 kg/m³ (517 lb/yd³), fine aggregate 987 kg/m³ (1664 lb/yd³), coarse aggregate (#8 gravel) 934 kg/m³ (1575 lb/yd³),
water 148 kg/m$^3$ (250 lb/yd$^3$), and High-Range Water Reducer (HRWR) 0.77 kg/m$^3$ (1.29 lb/yd$^3$). Standard concrete cylinders were cast and tested at 7, 28 days and at the time of the actual testing of the specimens. Due to the high congestion of the steel reinforcement at the top and bottom ends of the larger specimens (series G and H), a minimum slump of 20 cm (8 in.) was considered appropriate for the concrete to flow through the steel grids.

**Reinforcing Steel** -- Both UCSD and NIST specimens were designed with a Grade 60 (420 MPa) longitudinal steel reinforcement at a ratio ($\rho_g$) of approximately 1.5%. The transverse steel reinforcement ratio ($\rho_t$) in each of the specimens was based on the building code requirements ACI 318-02 (2002). The values of yielding strength shown in Table 1 correspond to the average values obtained from tensile tests on coupons performed according to ASTM A370 (2003).

**Carbon FRP (CFRP)** -- Unidirectional CFRP of one-ply nominal thickness ($t_f$) of 0.167 mm (0.0067 in.) was the wrapping material used for the entire research project. The mechanical properties provided by the manufacturer were used in the preliminary design. One and two-plies tensile coupons test was performed to determine the mechanical properties of the CFRP material used in the evaluation of the test results (ASTM D3039 2000). This characterization yielded an ultimate tensile strain $\varepsilon_{fu}$ of 0.93%, an ultimate tensile strength $f_{fu}$ of 2668 MPa (387 ksi), and a modulus of elasticity $E_f$ of 291 GPa (42,200 ksi).
Specimens Preparation, Instrumentation, and Test Setup

Specimens of series A to F were constructed and instrumented at the UCSD laboratory. The steel reinforcement assembling and corresponding instrumentation installation of NIST specimens (series G and H) were performed at the laboratory of the University of Missouri-Rolla. The steel cages were transported to the laboratory at NIST for their casting and testing. Due to the steel layout (fairly congested at the top and bottom ends) and the high probability of having large voids in the concrete, NIST specimens were cast horizontally.

The wrapping or FRP jacket of all the specimens was characterized by fiber orientation perpendicular to the longitudinal axis of the column. The method used for the application of the CFRP is known as manual dry lay-up. Preparation of concrete surface including grinding, leveling of imperfections, and removal of loosen particles, was followed by the application of layers of primer, putty, and saturant (in that order). The dry fabric was placed on the prepared surface and another layer of saturant was applied with the use of a ribbed roller for complete impregnation of the fibers and elimination of possible air voids.

The instrumentation in all the specimens consisted of electrical strain gauges located on the longitudinal and transverse steel reinforcement, and on the FRP jacket at critical locations (corner areas and mid-distance on each face of the non-circular specimens) along the perimeter of the cross-section on the central region of the strengthened specimens. Additionally, external sensors to measure axial deformation
such as potentiometers or Linear Variable Transducer Transformers (LVDTs) were affixed to the faces of the columns at about mid-height.

The equipment at both UCSD and NIST laboratories is capable of applying an axial compressive force of 53 MN (12,000 kip). However, due to the height limitation of the former (1.5 m [5 ft]), the largest specimens (series G and H) were tested at NIST. At both locations, the specimens were centered on the platen and capped with a 6-13 mm (0.25-0.5 in.) layer of hydro stone plaster. The loading was conducted in five cycles in increments of one fifth of the expected capacity of each specimen; the minimum load level (unloading) corresponded to approximately 5% of the total expected capacity or to a level that allowed the machine to remain engaged.

**EXPERIMENTAL RESULTS**

Table 2 reports the test results of the specimens in terms of the following parameters: maximum load for the unconfined case \(P_{co}\) or the increase in axial compressive loading \(\frac{P_{cc}}{P_{co}}\), concrete compressive strength corresponding to the maximum load for the unconfined case \(f'_{co}\) or the strengthening ratio \(\frac{f'_{cc}}{f'_{co}}\), concrete compressive strength at ultimate \(f_{cu}\), axial compressive strain at maximum load (\(\varepsilon'_{c}\) and \(\varepsilon'_{cc}\) for the case of unconfined and confined, respectively), axial compressive strain at ultimate for the unconfined case \(\varepsilon_{cu}\) or the ratio of ultimate axial strain of confined member to unconfined \(\frac{\varepsilon_{ccu}}{\varepsilon_{cu}}\), the average transverse strain at maximum load \(\varepsilon'_{tc}\), the average transverse strain on the FRP jacket at ultimate \(\varepsilon_{tu}\), the average transverse strain on the FRP at jacket rupture \(\varepsilon_{tr}\), the transverse strain on FRP at jacket rupture measured...
as close as possible to failure location ($\varepsilon_{fr}$), and the strain efficiency factors ($\kappa_{e1}$ and $\kappa_{e2}$). These factors are computed as the ratios of $\varepsilon_{tr}$ to $\varepsilon_{fus}$, and $\varepsilon_{fr}$ to $\varepsilon_{fus}$, respectively.

The axial strains reported in Table 2 are the average values obtained from linear potentiometers or LVDTs fixed to the sides of the specimens. The experimental values of $f_{cu}$ and $\varepsilon_{ccu}$ (or $\varepsilon_{cu}$ for the unconfined case) were reported according to the following definition: $\varepsilon_{cu}$ is the ultimate strain of the unconfined RC column corresponding to $0.85f'_c$ (Figure 3-curve a). For the confined RC column, $\varepsilon_{ccu}$ may correspond to one of the following values: a) $0.85f'_c$, in the case of a lightly confined member (Figure 3-curve b); b) the failure strain in the heavily confined, softening case when the failure stress is larger than $0.85f'_c$ (Figure 3-curve c); or the heavily confined, hardening case, where ultimate strength corresponds to ultimate strain (Figure 3-curve d). The definition of $\varepsilon_{ccu}$ at 85% of $f'_c$ (or less) is arbitrary, although consistent with modeling of conventional concrete (MacGregor 1997), and such that the descending branch of the stress-strain curve at that level of stress ($0.85f'_c$ or higher) is not as sensitive to the test procedure in terms of rate of loading and stiffness of the equipment used (Rocca et al. 2006b). Axial stress-strain and axial stress-transverse strain curves corresponding to each of the specimens are shown in Figure 4.

Regarding the transverse strains $\varepsilon_{tc}'$ and $\varepsilon_{tu}'$, in the case of the circular specimens they correspond to the average value of the measurements obtained by the strain gauges on the jacket, and in the case of the non-circular specimens, they correspond to the average of value given by the gauges located at the mid-distance on each of the faces. In the rectangular specimens (series B and H) these values correspond to the average of the longer sides. The strain values of $\varepsilon_{tr}$ are the average of the values given by the strain...
gauges located nearest to the corners. The values of $\varepsilon_{tr}$ and $\varepsilon_{fr}$ (and consequently $\kappa_{e1}$ and $\kappa_{e2}$) are not reported for specimens B3 and F2 because the FRP jacket did not rupture at instrumentation level, therefore the values of strain are believed not representative for these parameters.

In Table 2, the strain values corresponding to specimen H1 are not reported due to an inconvenience with the Data Acquisition System (DAS) in the last load cycle applied to this specimen. Additionally, the values of transverse strain on the FRP jacket of specimen H2 seemed to be inaccurate, and therefore are not reported either.

Regarding the failure mode of the FRP-wrapped specimens, it was characterized by rupture of the jacket mainly in the central region and it was generally followed by buckling of the longitudinal steel reinforcement. In some instances the jacket rupture extended partially along the height of the specimen. In the case of the non-circular specimens, the FRP rupture occurred at the corners of the section. In the cases of specimens A2, A3, C2, D2, D3, F2, F3, G2, and H2, the definitive breakage of the jacket was preceded by a slight rupture of a narrow FRP strip at the last loading cycle. In cases of specimens A2 and A3, this rupture originated at the location of one of the potentiometer brackets. Disengagement of steel transverse reinforcement was observed only in specimen D2. Figure 5 shows each of the specimens after testing.

Series A, which corresponded to specimens of circular cross-section and area-aspect ratio of 1.0 ($A_g/A_{g(C)} = 1$; recall that $A_{g(C)}$ is the gross cross-sectional area of series C specimens and it is equal to 2090 cm$^2$ [324 in.$^2$]), showed the highest increments of concrete compressive strength: 44% and 49% for the case of specimens A2 (fully wrapped with two plies) and A3 (partially wrapped with four plies), respectively.
Increments of axial deformation at ultimate attained by these specimens were of 360% and 465%, in the same order. Regarding the stress-strain performance, specimen A2 is the lone specimen that yielded a bilinear curve with an ascending second portion. The behavior of specimen A3 was represented by a bilinear curve with a second portion almost flat possibly due to the progressive failure of plies.

The behavior of strengthened specimens from series B to H was unfortunately characterized by stress-strain curves with either flat or descending second portions beyond the peak load. In the cases of specimens C3, specimens in series D, and specimen E2, an abrupt decay of the curve was followed after reaching the maximum compressive strength. In the remaining cases the second portion of the stress-strain curves extended up to a certain level of strain indicating the increment of ductility in terms of axial deformation in spite of the limited increase of compressive strength.

Series B was composed of specimens of rectangular cross-section with side-aspect ratio \((h/b)\) of 2 and area-aspect ratio of 1. Specimen B2 was wrapped with seven plies and specimen B3 with two plies. Increments of 24% and 1% of maximum axial compressive strength were attained by specimens B2 and B3, respectively. The enhancements of axial deformation were of 905% and 254%, in the same order.

Benchmark for the gross-area section were square specimens of series C \((A_g(C) = 2090 \text{ cm}^2 [324 \text{ in.}^2])\). The number of plies used in specimens C2 and C3 were four and two, respectively. Specimen C2 reached an increment of compressive strength of 12%, and C3 of 6%. Increments of axial deformation were of 352% and 67%, respectively.

Specimens of series D featured square cross-sections with area-aspect ratio of 2.0. Test unit D2 was wrapped with five plies and D3 with two plies. These specimens
showed increase of compressive strength (D2 of 20% and D3 of 7%), but marginal increase of axial deformation, that is: 35% in the case of specimen D2 and 44% in the case of D3.

Series E and F were composed of specimens with equal characteristics (material and geometrical) with the sole difference being the overall height of the specimens, that is: 0.7 m (27 in.) and 1.4 m (54 in.), for series E and F, respectively. These specimens featured square cross-section shapes and area-aspect ratio of 0.5. Specimens E2 and F2 were fully wrapped with two plies, and E3 and F3 were partially wrapped with four plies. In Table 2 it is noted the higher ratios of increment of axial loading capacity \( \frac{P_{cc}}{P_{co}} \) and therefore concrete compressive strength \( \frac{f'_{cc}}{f'_{co}} \) from series E specimens when compared to series F. This difference is due to the unexpected premature failure of control unit E1 (about 25% below the expected maximum load carrying capacity). The possible cause maybe the limited number of ties that was able to be placed along the height of the specimen (the spacing of the ties as per code requirements was 254 mm [10 in.]). Additionally, stress concentrations induced at the top and bottom ends might have affected the overall strength as well. For these reasons, the experimental results of specimens in series E are believed to be not truly representative and therefore are not included in the analysis. Regarding series F, both strengthened specimens reached an increment of compressive strength of 14%, and an increase of axial deformation of 68% and 245% for the case of specimen F2 and F3, respectively.

Series G was composed of two specimens of square cross-section and area-aspect ratio of 4.0. The strengthened unit (G2) was wrapped with eight plies. It achieved
increases of 8% and 86% of concrete compressive strength and axial deformation, respectively.

The cross-section geometry of specimens from series H was rectangular with side-aspect ratio of 2 and area-aspect ratio of 4. The strengthened specimen (H2) was wrapped with 19 plies. The increment of compressive strength attained by this specimen was of 19%. The maximum axial strain deformation was 0.54%. There is no record of the maximum axial strain from the control unit H1 due to an inconvenience with the DAS during the last loading cycle.

**ANALYSIS AND DISCUSSION OF EXPERIMENTAL RESULTS**

**Performance of Strengthened Specimens**

This section addresses the results obtained in the experimental program and the behavior of the specimens with respect to current available data of size-relevant RC columns subjected to axial compressive loading as well.

Figure 6(a), 6(b), and 6(c) present the performance of the specimens in terms of the strengthening ratio $f'_{cc}/f'_{co}$, which is a measurement of the effectiveness of the confinement, and the variables: side-aspect ratio ($h/b$) and area-aspect ratio ($A_g/A_{g(C)}$). Recall that $A_{g(C)}$ is the gross area section of specimens in series C (2090 cm$^2$ [324 in.$^2$]). In these figures, the label numbers within brackets represent the FRP volumetric ratio in percentages. Figure 6(a) shows the influence of the cross-sectional shape in the strengthening performance of specimens of a constant cross-sectional area ($A_g/A_{g(C)} = 1$).
In this figure, note that among specimens featuring similar FRP volumetric ratio, and taking the circular specimens as a “benchmark” with a ratio $h/b$ equal to zero, the level of confinement effectiveness decreases as the side-aspect ratio increases. That is: the values of $f'_{cc}/f'_{co}$ are 1.44, 1.06, and 1.01 for specimens A2, C3 ($h/b = 1$), and B3 ($h/b = 2$), respectively. This is consistent with the accepted distribution of the confinement pressure in the cross-section: in a circular cross-section, the confinement pressure is uniform yielding the entire gross area to be effectively confined; while in a non-circular cross-section, the confinement pressure is concentrated at the corners and the effectively confined area is contained within four second-degree parabolas, which define larger unconfined concrete areas as the side length increases (Teng et al. 2002).

Figure 6(b) and 6(c) show the effect of the area-aspect ratio for specimens of square and rectangular cross-sections, respectively. In Figure 6(b), the performance of specimens in series D seems to be not in accordance with the observed trend given by the other series. Specimens wrapped with two plies (F2, C3, and D3) were expected to show a continuous decrease of the strengthening ratio as the area-aspect ratio increased. This is observed for F2 and C3, however, specimen D3 attained a ratio $f'_{cc}/f'_{co}$ of only 1% difference with respect to the one given by specimen C3 in spite of having a ratio $A_g/A_g(C)$ of 2. Among specimens C2, D2, and G2, the unexpected performance of D2 is more notorious. Note that specimen G2, which features the same FRP volumetric ratio as specimen C2 (0.58%), showed a decrease in strengthening ratio of 3.6% with respect to C2, reflecting the possible size-effect. Additionally, the reduction in strengthening ratios obtained by specimens F2 and D3 compared to the circular specimen A2 were approximately 20.8% and 25.7%, respectively.
In Figure 6(c), even though specimen H2 was strengthened with a higher FRP volumetric ratio (1.5% compared to 1.12% from specimen B2), the ratio $f'_{ce}/f'_{co}$ decreased, and this could also be due to the size-effect. In Figure 6(b) and 6(c), there seems to be a decreasing confinement effectiveness trend as the area-aspect ratio increases, however, at this point with the limited data available, it is difficult to draw a definite conclusion with respect to the effect of the cross-sectional size in the confinement effectiveness in non-circular specimens.

With the exception of series A, the test units strengthened to increase their axial loading capacity by 30%, did not achieve this level. In this experimental program, at the time of the design of the test matrix, the approach presented by fib seemed to be the most appropriate when compared to other international guidelines (ACI Committee 440.2R 2002, S806 Canadian Standard Association 2002, Concrete Society Technical Report 55 2004). However, based on the results from this project and a recent study conducted by the authors focused on the review, identification of limitations, and comparison of the state-of-the-art design methods, the approach presented by fib for non-circular columns overestimated the expected capacities. Further details on the methodologies presented by the current international guidelines can be found in Rocca et al. (2006b).

The results from this experimental program are also presented along with collected available data on RC specimens of circular and non-circular cross-sections with one minimum dimension of the cross-section of 300 mm (12 in.), side-aspect ratios not greater than 2, and FRP jackets with the fibers oriented perpendicular to the longitudinal axis of the column. The collected data on circular and non-circular RC specimens is shown in Table 3 and Table 4, respectively. Table 3 is composed of a total of 20
specimens divided in five sets of experiments, and Table 4 presents 13 specimens divided in six experimental sets. The specimens’ codes correspond to the studies conducted by the following authors: “DN” to Demers and Neale (1994), “KE” to Kestner et al. (1997), “YO” to Youssef (2003), “CH” to Carey and Harries (2003), “MA” to Matthys et al. (2005), and “WR” to Wang and Restrepo (2001). From these databases, only specimens “DN” were pre-loaded up to the corresponding peak load and/or to the point where the cracks became visible, before strengthening and re-testing to failure. Additionally, circular specimens “YO” were divided in two groups depending on the type of transverse steel reinforcement, that is: “-s” for spiral and “-h” for hoops. In Table 3 and Table 4, the data is presented in terms of the following parameters: type of FRP used, diameter of the circular cross-sections ($D$); side dimensions ($b$, $h$) and the chamfered corner radius of the non-circular specimens ($r$); overall height ($H$); longitudinal and transverse steel reinforcement ratio ($\rho_g$ and $\rho_t$); FRP volumetric ratio ($\rho_f$); unconfined concrete compressive strength ($f'_c$); yield strength of the longitudinal and transverse steel reinforcement ($f_y$ and $f_{yt}$); FRP mechanical properties ($E_f$, $f_{fu}$, and $\varepsilon_{fu}$); nominal ply thickness of FRP ($t_f$); maximum loads for the unconfined cases ($P_{co}$) or the increase in axial compressive loading ($P_{cc}/P_{co}$); and concrete compressive strengths corresponding to the maximum load for the unconfined cases ($f'_{co}$) or the strengthening ratio ($f'_{cc}/f'_{co}$).

All the experimental data is presented in Figure 7 in terms of trends of the strengthening ratio $f'_{cc}/f'_{co}$ versus the parameter $\rho_f E_f / E_c$, which represents the stiffness of the FRP jacket to the axial stiffness of the concrete. The product of the parameters $\rho_f$ and $E_f$ resembles the theoretical stiffness of the FRP jacket ($E_j$), also known as confinement modulus or lateral modulus (Xiao and Wu 2000; De Lorenzis and Tepfers 2001). This
parameter represents the capacity of the FRP jacket of restraining the lateral dilation of the concrete. In the case of circular specimens, $E_j$ is directly related to the maximum confining pressure as follows:

$$f_{i,f} = \left( \frac{2\pi f_{c}E_{j}}{D} \right) \varepsilon_{f} = \left( \frac{1}{2} \rho_{f} E_{j} \right) \varepsilon_{f} = \left( E_{j} \right) \varepsilon_{f}$$

However, in the case of non-circular specimens, $E_j$ depends on the definition given to determine the equivalent confining pressure $f_{i,f}$, which in most of the cases is multiplied by a shape factor. Different authors have suggested varied expressions for this shape factor along with their applicability limitations being mainly the side-aspect ratio $(h/b)$ and maximum side dimension. These expressions depend basically on the geometry of the cross-section, chamfered corner radius, and longitudinal steel reinforcement ratio (Lam and Teng 2003b; Rocca et al. 2006b). Since for non-circular cross-sections it has not been established a definite expression for the equivalent confining pressure, and consequently for the lateral modulus $E_j$, it was considered appropriate to present the collected data and experimental results using the parameters $\rho_f$ and $E_j$. Additionally, the use of the concrete modulus $E_c$ was included to reflect the variation of concrete compressive strengths ($f'_{c}$) among the different specimens’ experiments.

Figure 7(a), 7(b), and 7(c) show the cases of specimens of circular, square and rectangular cross-sections, respectively. In the legends, each acronym is followed by a number(s) that indicate the dimensions of the cross-section $(D, b, h)$. The labels “RO” represent the specimens from the presented experimental study. Fig. 7(d) presents the linear trends of the types of cross-sections and their reliability indexes obtained by
regression analysis corresponding to each data-set. In the legend of this figure, the sub-indexes of the ratios $f'_{cc}/f'_{co}$ indicate the corresponding cross-section shapes, that is: “C” for circular, “S” for square, and “R” for rectangular.

In Fig. 7(a), the circular cross-section data-set, note the uniformity of the trend and minor scattering. No pattern reflecting the effect of cross-sectional area size is identified leading to believe on the lack of such effect on this type of cross-section. Regarding Fig. 7(b), the square cross-section data-set, the scatter of data is more pronounced. With respect to the specimens of rectangular cross-sections (Figure 7(c)), no definite observation can be concluded due to the high level of data scattering and the limited number of data points. The linear trends of the three data-sets presented in Figure 7(d) reflect the level of effectiveness of the FRP confinement in the axial strengthening. The slopes of the trends corresponding to the non-circular specimens reflect the reduced confinement effectiveness when compared to specimens of circular cross-sections.

**Efficiency of FRP Jacket**

The efficiency of the FRP jacket is directly related to the strain efficiency factor ($\kappa_e$). This factor accounts for the difference between the actual tensile strain at rupture of FRP jacket and the ultimate strain reported from flat coupon tests. The factor “$\kappa_e$” has been recognized by different authors (Kestner et al. 1997; Spoelstra and Monti 1999; Xiao and Wu 2000; De Lorenzis and Tepfers 2001; Carey and Harries 2003; Lam and Teng 2004; Matthys et al. 2005); however, the causes for this phenomenon and their level of effect are still under investigation. Lam and Teng (2004) conducted an experimental
study focused on the parameter “κe” and its possible causes. They carried out three tests: FRP-confined plain concrete cylinders subject to pure axial compression, flat coupon tensile test, and ring splitting test. Among the concluding causes of the premature failure of the FRP they included: (a) the possible stress-concentrations at concrete crack locations; (b) the effect of the chamfered corners in a non-circular column; (c) the multi-axial state of stress that the FRP jacket undergoes, that is, tension in the hoop direction, possible axial compression (either by direct loading or by transfer of axial loading from the concrete to the FRP through bond), and the lateral pressure due to concrete dilation; (d) air voids or misalignment of fibers from the FRP lay up process; (e) residual stress resulting from temperature, creep, and shrinkage incompatibility between the concrete and FRP jacket; and (f) the lower FRP transverse strains that could be observed in overlapping zones of the jacket and that reduce the average transverse strain, but do not result in a lower confining pressure in such zone due to the thickness of the jacket (Lam and Teng 2004, Matthys et al. 2005).

In the available literature, the value of FRP strain efficiency factor has been mostly determined in small-scale cylinders as the ratio of the average ultimate strain transverse on the FRP to the ultimate tensile strain obtained from flat coupon testing. Based on an experimental calibration using CFRP-confined concrete cylinders, an average value of 0.586 was computed by Lam and Teng (2003a). Similarly, based on an extensive database of concrete cylinders strengthened with different types of FRP, Carey and Harries (2003) computed value of 0.55 for specimens wrapped only with CFRP. Demers and Neale (1994) and Kestner et al. (1997) reported average values of 0.31 and 0.46 for the case of large-scale RC circular specimens, respectively. Carey and Harries
obtained values of 0.55 and 0.59 from medium- and large-scale RC circular specimens. Matthys et al. (2005) reported an average value of 0.61 based on experiments of CFRP-wrapped large scale RC circular columns. For the case of non-circular RC specimens, Kestner et al. (1997) obtained 0.39 from a square specimen wrapped with CFRP. Additionally, Carey and Harries (2003) reported values of 0.13 and 0.16 for one CFRP-wrapped medium- and one large-scale square RC specimen. In summary, the experimentally observed FRP strain efficiency factor in RC specimens varies approximately from 0.31 to 0.61 for RC circular specimens. In the case of non-circular specimens, factors of 0.13, 0.16, and 0.39 were found.

In the non-circular specimens of this experimental program, the strain gauges located on the FRP jacket at the mid-distance on each face provided higher strain readings that the ones close to the corners. This difference is the result of two effects: a tensile strain due to confinement effect and a flexural strain due to the non-uniform concrete dilation inducing a lateral pressure on the jacket (bulging). The latter is of a higher magnitude since the out-of-plane (flexural) stiffness of the FRP jacket is much lower than its axial (tensile) stiffness. This difference in strain also reflects the lack of confinement along the sides of the non-circular cross-sections. Recall that effective confinement is obtained once the FRP is in full tension product of uniform concrete dilation as in the case of circular cross-sections. The readings at the locations closer to the corners are minimally affected by the concrete bulging, therefore they are considered acceptable for an approximation to compute a strain efficiency factor in this section.

The last two columns in Table 2 show the strain efficiency factors ($\kappa_{e1}$ and $\kappa_{e2}$) for each of the specimens in the presented experimental program. Recall that “$\kappa_{e1}$” is
related to the average transverse strain from the sensors close to the corners at FRP rupture \( (\varepsilon_{tr}/\varepsilon_{fu}) \) and \( \kappa_{e2} \) is to the local strain at failure location \( (\varepsilon_{fr}/\varepsilon_{fu}) \). It is apparent that the values corresponding to specimen A3 are inaccurate, therefore, they should not be considered. In the case of non-circular specimens, the ranges for \( \kappa_{e1} \) and \( \kappa_{e2} \) obtained from ten of the strengthened specimens are as follows: \( \kappa_{e1} \) varies from 0.51 to 0.80, and \( \kappa_{e2} \) varies from 0.50 to 0.98. Both value ranges are considerably higher than the values of strain reduction factors reported in the literature for non-circular specimens. This difference might be due to the locations of the sensors with respect to the breakage of the FRP jacket in the different specimens.

**Proposed Analytical Model**

It is widely acknowledged that in a FRP wrapped column of non-circular cross-section, the effectively confined concrete area is defined by four second-degree parabolas (Figure 1). The current approach to determine the confining pressure for this type of cross-section consists in defining an equivalent circular section having a reduced efficiency identified by the shape factor that accounts for the geometry of the section. The diameter of this equivalent circular section varies according to different authors, such as: the smallest side dimension (Mirmiran et al. 1998), the diagonal of the non-circular cross-section (Lam and Teng 2003b).

Transverse or hoop strains measured in a circular cross-section can be directly used to compute the total acting confining pressure \( f_l \), however, in the case of a non-circular section, measured strains along the perimeter cannot be directly correlated to \( f_l \)
A proposed idealization of this problem is presented in what follows.

The relation between “$f_l$” and the transverse strain in the FRP and steel reinforcement may be obtained by idealizing a portion of the concrete confined area as a two-hinged parabolic symmetrical arch restrained by an horizontal tie representing FRP and transverse steel reinforcement, and subjected to a uniformly distributed load representing the confining pressure (Figure 8). For the analysis, the following assumptions/simplifications were considered: (a) the intersection of the parabola with the edges of the section is 45° (Lam and Teng 2003b); (b) the span “$L$” of the arch coincides with the side of the cross-section; (c) the thickness of the arch is constant and equal to twice the chamfered corner radius; (d) the loading span is equal to the side of the cross-section. The total internal force in the horizontal tie of the arch is equal to $T_f + T_s$, where “$T_f$” is the FRP tensile force and “$T_s$” is the steel tensile force. In a longitudinal portion of the column of height equal to the transverse steel reinforcement pitch “$s$”, “$T_f$” and “$T_s$” can be computed with Eqs. (2) and (3) (Fertis 1996):

$$T_f = E_f \varepsilon_f n t_s s$$  \hspace{1cm} (2)

$$T_s = A_s E_s \varepsilon_s$$  \hspace{1cm} (3)

The confining pressures due to the steel reinforcement and FRP can be obtained by solving the structure defined in Figure 8 based on equilibrium and the theorem of Castigliano (Fertis 1996), leading to the following expressions:
Where, “$h_a$” is the height of the parabolic arch at its centerline. The rest of the parameters in the equations above have been previously defined in the text. The derivation of Equations (4) and (5) is shown in the appendix of this paper.

In the presented experimental program, transverse strains on the steel and FRP were recorded at locations close to the corners of the specimens (approximately at the change of curvature). These strains were used to estimate the values of “$T_f$”, “$T_s$”, and the corresponding confining pressures ($f_{l,f}$ and $f_{l,s}$). With the values of “$f_{l,f}$” and “$f_{l,s}$”, and the concrete compressive strength of the confined and unconfined concrete ($f'_{cc}$ and $f'_{co}$), two efficiency factors “$k$” were calibrated following the empirical formula proposed by Richart et al. (1928):

$$\frac{f'_{cc}}{f'_{co}} = 1 + k \frac{f_l}{f'_{co}}$$

For the first case, the total acting confining pressure “$f_l$” is composed by two terms reflecting both the contribution of the FRP jacket and the contribution of the steel transverse reinforcement, that is: $f_l = f_{l,f} + f_{l,s} A_{cc} / A_g$. For the second case, “$f_l$” only accounts for the pressure induced by the FRP jacket, that is: $f_l = f_{l,f}$. These two situations
were considered to observe the possible contribution of the internal transverse steel reinforcement to the confinement. Values of $f'_{cc}/f'_{co}$ versus the ratio $f/l/f'_{co}$ corresponding to the non-circular specimens of the presented experimental program are plotted in Figure 9. Both sets of data-points show very similar ratios $f/l/f'_{co}$ which indicates that the contribution of the transverse steel reinforcement to the confinement pressure is minimal and therefore may be neglected. Then, the estimated value of $k = 0.61$ with a reliability index of 0.77 resulting from a regression analysis and corresponding to the second case is used to evaluate this strength model.

The performance of this proposed model to estimate the confining pressure in a non-circular section and therefore the increment of concrete compressive strength ($f'_{cc}/f'_{co}$) is evaluated using the available collected data presented in Table 4 and the experimental results from this study. Additionally, these predictions are compared to those given by the model of Lam and Teng (2003b), since this experimental model was calibrated with an extensive database of small plain concrete prisms and it has shown to yield acceptable predictions when estimating the capacity of RC confined columns (Rocca et al. 2006b).

Figure 10 shows the theoretical versus experimental ratios of $f'_{cc}/f'_{co}$. The 45° line corresponds to a perfect agreement between predictions and experiments. The points falling above this line represent overestimations of the experimental values. An average absolute error, standard deviation, and coefficient of variation of 4.79%, 2.46%, and 51% showed that the proposed model performed well in predicting the experimental results. The close agreement between both models constitutes a verification of the proposed
analytical method to correlate the measured transverse strains (on steel reinforcement and FRP) to the confining pressure and compute the increment of compressive strength.

**SUMMARY AND CONCLUSIONS**

The paper presents the experimental results of FRP-confined RC columns of circular and non-circular cross-sections with minimum and maximum cross-sectional areas of 0.1 m² (1 ft²) and 0.8 m² (9 ft²), respectively. A test matrix composed of a total of 22 RC columns divided into six series of three specimens each, and two series of two specimens each, was composed to study the effects of variable cross-sectional area, shape (circular, square, and rectangular), and side-aspect ratio. These specimens were tested under pure axial compressive loading condition as the first step to understand the confinement process to be later considered within the effects of combined axial force and bending moment to develop practical design interaction diagrams.

The results obtained in this experimental program were compared to RC columns of relevant size available in the literature. The performance of the specimens was compared based on the strengthening ratio \( f'_{cc}/f'_{co} \), and the variables of side-aspect ratio and area-aspect ratio for square and rectangular specimens. Additionally, FRP strain efficiency factors, and a proposed new analytical method to correlate the transverse strains on steel and FRP to the confining pressures were evaluated. The following conclusions can be made from this study:

- Even though the increments of compressive strength in the non-circular specimens was not significant, test units B2, C2, E3, F2, F3, and G2, did exhibit
ductility in terms of axial deformation. The highest level of increment of compressive strength and ductility were observed in specimen B2 (wrapped with seven plies) with 24% and 905%, respectively;

- The level of confinement effectiveness for specimens of different cross-sectional shape featuring the same cross-sectional area size and similar FRP volumetric ratio, decreases as the side-aspect ratio increases;

- Specimens of circular cross-section showed the highest level of confinement effectiveness in axial strengthening. No pattern reflecting the effect of cross-sectional area size is identified leading to believe on the lack of such effect on this type of cross-section;

- For the case of non-circular specimens, few indicatives of the possible negative effect of cross-sectional area size in the axial strengthening were noted, however, the scattering and limitation of data-points do not allow at the present time to draw a definite conclusion;

- For CFRP wrapped non-circular specimens, the FRP strain efficiency factor related to the average transverse strains at corners varied from 0.51 to 0.80, and the factor related to the “in-situ” strain at rupture location varied from 0.5 to 0.98. These factors are function of strains at locations close to the corners of the non-circular specimens rather than strains at the middle of the faces of the cross-section, as they have been reported in the literature. The former strains are more appropriate for the determination of a strain efficiency factor since it is at the corners of the section that the confining pressure is concentrated and consequently where the rupture generally occurs. The lowest observed factor (0.5) is
comparable to the one recommended in the literature by Lam and Teng (2003a,b), Matthys et al. (2005), and Carey and Harries (2003);

- Since in the non-circular specimens, the transverse strains measured along the perimeter cannot be directly related to the confinement pressure “$f_l$”, a new analytical approach was proposed to determine “$f_l$” using the strains close to the corners of the section. This method consists of idealizing a portion of the concrete confined area as a two-hinged parabolic arch restrained by a horizontal tie representing FRP and transverse steel reinforcement, and subjected to a uniformly distributed load. With the obtained values of confining pressures it was possible to calibrate a strength model and evaluate it with the collected experimental data. Additionally, the performance of this strength model was compared to the one by Lam and Teng showing close agreement in the predictions;

- The contribution of the transverse steel reinforcement to the confining pressure in the specimens of this experimental program computed using the proposed analytical model was found to be negligible;

- Since the proposed analytical method and strength model were calibrated with the experimental data from the present study, and was validated with limited experimental data available in the literature, further experimental evidence is needed to confirm the validity of the model.
ACKNOWLEDGMENTS

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NOTATION

The following symbols are used in this paper:

\begin{align*}
A_c & \quad \text{Concrete cross-sectional area} \\
A_{cc} & \quad \text{Concrete core area} \\
A_g & \quad \text{Total cross-sectional area} \\
A_g(C) & \quad \text{Total cross-sectional area corresponding to specimens in series C = 457×457mm (18×18in.)} \\
A_s & \quad \text{Area of longitudinal steel reinforcement} = A_g \cdot \rho_g \\
b & \quad \text{Short side dimension of a non-circular cross-section} \\
b_f & \quad \text{Width of FRP strip in partial wrapping} \\
D & \quad \text{Diameter of circular cross-section}
\end{align*}
\( E_f \)  Tensile modulus of elasticity of FRP

\( E_j \)  Stiffness of the FRP jacket or confinement modulus = \((1/2)\rho_f E_f\)

\( f'_c \)  Characteristic concrete compressive strength determined from standard cylinder

\( f'_{cc} \)  Compressive strength of confined concrete (For experiments: peak load minus the steel contribution and divided by the cross-sectional concrete area)

\( f'_{co} \)  Compressive strength of unconfined concrete (For experiments: peak load minus the steel contribution and divided by the cross-sectional concrete area)

\( f_{cu} \)  Compressive strength of concrete at ultimate

\( f_{fu} \)  Ultimate tensile strength of FRP

\( f_l \)  Total confining pressure

\( f_{lf} \)  FRP confining pressure

\( f_{ls} \)  Steel confining pressure

\( f_y \)  Yield strength of longitudinal steel reinforcement

\( f_{yt} \)  Yield strength of transverse steel reinforcement

\( H \)  Height of specimen

\( h \)  Long side dimension of a non-circular cross-section

\( h_a \)  Height of the parabolic arch at centerline = \(L_i/4+r\)

\( k \)  Confinement effectiveness factor

\( L \)  Clear span in concrete confined two-hinged arch model

\( n \)  Number of FRP plies composing the jacket

\( P_{co} \)  Maximum axial compressive load of unconfined column

\( P_{cc} \)  Maximum axial compressive load of confined column

\( r \)  Chamfered corner radius of the strengthened non-circular specimens
\( s_t \) Pitch in partial wrapping
\( s \) Pitch in transverse steel reinforcement
\( s'_t \) Clear spacing between FRP strips
\( t_f \) FRP nominal ply thickness
\( T_f \) FRP tensile force
\( T_s \) Steel tensile force
\( U \) Total strain energy of concrete arch
\( \varepsilon'_c \) Axial compressive strain corresponding to \( f'_c \)
\( \varepsilon'_{cc} \) Axial compressive strain corresponding to \( f'_{cc} \)
\( \varepsilon_{ccu} \) Ultimate axial compressive strain of confined concrete
\( \varepsilon_{cu} \) Ultimate axial compressive strain of unconfined concrete
\( \varepsilon_{fr} \) Transverse strain of the FRP at jacket rupture measured as close as possible to breakage location
\( \varepsilon_{fu} \) Ultimate tensile strain of the FRP
\( \varepsilon'_{ic} \) Average transverse strain corresponding to \( f'_{cc} \)
\( \varepsilon_{ir} \) Average transverse strain of the FRP at jacket rupture (for non-circular specimens is the average value of strain gages located adjacent to the corners)
\( \varepsilon_{fu} \) Average transverse strain of the FRP at ultimate
\( \kappa_s \) Shape factor for steel confining pressure = \( \kappa_{cs} \cdot \kappa_{ys} \)
\( \kappa_{\varepsilon} \) FRP strain efficiency factor
\( \kappa_{e1} \) FRP strain efficiency factor = ratio of average transverse strain at corners to the ultimate tensile FRP strain \( \varepsilon_{ir} / \varepsilon_{fu} \)
κ_{e2} FRP strain efficiency factor = ratio of “in-situ” FRP strain measured as close as possible to rupture location to the ultimate tensile FRP strain = \varepsilon_{fr} / \varepsilon_{fu}

κ_e FRP strain efficiency factor

ρ_{cc} Ratio of the area of longitudinal steel reinforcement to the concrete core area of a compression member

\[ \rho_f = \begin{cases} \frac{4nt_f}{D} \left( \frac{b_j}{s_f} \right) & \text{Circular} \\ \frac{2nt_f (b + h)}{bh} \left( \frac{b_f}{s_f} \right) & \text{Non-Circular} \end{cases} \]

ρ_s Ratio of the area of longitudinal steel reinforcement to the cross-sectional area of a compression member = \( A_s / A_g \)

ρ_t Volumetric ratio of transverse steel reinforcement to concrete core

**APPENDIX: DERIVATION OF EQUATIONS**

This appendix presents the derivation of Eqs. (4) and (5) used in section 5.3. Given the free-body diagram of a section of the parabolic arch at a distance “x” from the support A (Figure A1), the moment “\( M_x \)” may be obtained by equilibrium at point “O” as follows:

\[ \sum M_0 = M_x + f_i \frac{x^2}{2} + Ty - f_t \frac{L}{2} x = 0 \]
Where, \( y = \frac{4h_a x}{L^2} (L - x) = \frac{x}{L^2} (L + 2r)(L - x) \)

Then, \( M_x = f_i \frac{L}{2} x - f_i \frac{x^2}{2} - T \left( \frac{x}{L^2} (L + 2r)(L - x) \right) \)

The total strain energy “\( U \)” of the parabolic arch is composed by the strain energy due to axial force, bending moment, and shear force. However, since the thickness of the arch is considerably lower than the height of the arch, then strain energy due to axial and shear forces maybe neglected. Then, the strain energy is given by:

\[
U = \int_0^L \left( \frac{M_x^2}{2EI_c} \right) dx = \int_0^L \frac{1}{2EI_c} \left( f_i \frac{L}{2} x - f_i \frac{x^2}{2} - T \left( \frac{x}{L^2} (L + 2r)(L - x) \right) \right)^2 dx
\]

By assuming that the two hinges cannot translate and that the entire horizontal reaction is absorbed by the steel and FRP, a minimizing condition is applied to obtain the value of the tensile forces in the transverse tie of arch:

\[
\frac{\partial U}{\partial T} = 0 = \int_0^L \left( f_i \frac{L}{2} x - f_i \frac{x^2}{2} - T \left( \frac{x}{L^2} (L + 2r)(L - x) \right) \right) \left( \frac{x}{L^2} (L + 2r)(L - x) \right) dx
\]

After integrating and operating the equation above, the tensile force is given by:

\[
\frac{8}{15} h_a T = \frac{1}{15} f_i L^2 \Rightarrow T = \frac{f_i L^2}{8h_a}
\]
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Note: 1 mm = 0.04 in.; 1 cm² = 0.155 in.²; 1 MPa = 0.145 ksi; NA = Not Applicable
Table 2 -- Test Results

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Note: 1 kN = 0.225 kip; 1 MPa = 0.145 ksi; NA = Not Applicable; NR = Not Reported; a Strain corresponding to $\varepsilon'_{cu}$
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Note: 1 mm = 0.04 in.; 1 MPa = 0.145 ksi; 1 kN = 0.225 kip; NA = Not Applicable
### Table 4 -- Experimental Data – Non-Circular RC Specimens

| Test Unit | Type | b (mm) | h (mm) | h/b | H (m) | r (mm) | ρt (%) | ρr (%) | f′<sub>c</sub> (MPa) | fy (MPa) | fty (MPa) | Er (MPa) | fu (MPa) | εfu (%) | tr (mm) | P<sub>co</sub> (kN) | P′<sub>co</sub> (MPa) | \[P<sub>co</sub> \] or \[P′<sub>co</sub> \] | \[f′<sub>c</sub> \] or \[f′<sub>c</sub> \] |
|-----------|------|--------|--------|-----|-------|--------|--------|--------|--------------------|---------|-----------|---------|---------|--------|--------|-----------|-----------------|-----------------|-----------------|------------------|
| KE1       | NA   | 457    | 457    | 1.0 | 1.8   | 38     | 1.48   | 0.11   | 0.00              | 31.5    | 457       | 502     | NA      | NA      | NA      | 7340      | 28.81           | \[P<sub>co</sub> \] | \[f′<sub>c</sub> \] |
| KE3       | GFRP | 457    | 457    | 1.0 | 1.8   | 38     | 1.48   | 0.11   | 2.27              | 31.5    | 457       | 502     | 25000   | 443     | 1.90    | 0.86      | [1.13]          | [1.16]          | \[P<sub>co</sub> \] | \[f′<sub>c</sub> \] |
| KE4       | CFRP | 457    | 457    | 1.0 | 1.8   | 38     | 1.48   | 0.11   | 0.43              | 31.5    | 457       | 502     | 230909  | 3515    | 1.50    | 0.17      | 2127            | 17.31           | \[P<sub>co</sub> \] | \[f′<sub>c</sub> \] |
| WR1       | NA   | 300    | 300    | 1.0 | 0.9   | 30     | 1.50   | 0.62   | 0.00              | 18.9    | 439       | 365     | NA      | NA      | NA      | 2127      | 17.31           | \[P<sub>co</sub> \] | \[f′<sub>c</sub> \] |
| WR2       | GFRP | 300    | 300    | 1.0 | 0.9   | 30     | 1.50   | 0.62   | 3.39              | 18.9    | 439       | 365     | 20500   | 375     | 2.00    | 1.27      | 3268            | 17.89           | \[P<sub>co</sub> \] | \[f′<sub>c</sub> \] |
| WR3       | NA   | 300    | 450    | 1.5 | 0.9   | 30     | 1.50   | 0.62   | 0.00              | 18.9    | 439       | 365     | NA      | NA      | NA      | 3268      | 17.89           | \[P<sub>co</sub> \] | \[f′<sub>c</sub> \] |
| WR4       | GFRP | 300    | 450    | 1.5 | 0.9   | 30     | 1.50   | 0.62   | 2.82              | 18.9    | 439       | 365     | 20500   | 375     | 2.00    | 1.27      | 3966            | 34.92           | \[P<sub>co</sub> \] | \[f′<sub>c</sub> \] |
| YO1       | NA   | 381    | 381    | 1.0 | 0.8   | 38     | 1.60   | 0.55   | 0.00              | 41.2    | 414       | 276     | NA      | NA      | NA      | 5967      | 35.05           | \[P<sub>co</sub> \] | \[f′<sub>c</sub> \] |
| YO2       | CFRP | 381    | 381    | 1.0 | 0.8   | 38     | 1.60   | 0.55   | 2.45              | 41.2    | 414       | 276     | 103839  | 1246    | 1.25    | 0.58      | 3966            | 34.92           | \[P<sub>co</sub> \] | \[f′<sub>c</sub> \] |
| YO3       | NA   | 254    | 381    | 1.5 | 0.8   | 38     | 1.60   | 0.74   | 0.00              | 41.1    | 414       | 276     | NA      | NA      | NA      | 3966      | 34.92           | \[P<sub>co</sub> \] | \[f′<sub>c</sub> \] |
| YO4       | CFRP | 254    | 381    | 1.5 | 0.8   | 38     | 1.60   | 0.74   | 3.07              | 41.1    | 414       | 276     | 103839  | 1246    | 1.25    | 0.58      | 14196           | 43.46           | \[P<sub>co</sub> \] | \[f′<sub>c</sub> \] |
| CH1       | NA   | 540    | 540    | 1.0 | 1.6   | 51     | 1.10   | 0.41   | 0.00              | 33.5    | 414       | 441     | NA      | NA      | NA      | 14196     | 43.46           | \[P<sub>co</sub> \] | \[f′<sub>c</sub> \] |
| CH2       | CFRP | 540    | 540    | 1.0 | 1.6   | 51     | 1.10   | 0.41   | 2.22              | 33.5    | 414       | 441     | 72500   | 875     | 1.21    | 1.00      | 14196           | 43.46           | \[P<sub>co</sub> \] | \[f′<sub>c</sub> \] |

**Note:** 1 mm = 0.04 in.; 1 MPa = 0.145 ksi; 1 kN = 0.225 kip; NA = Not Applicable
Figure 1 -- Effectively Confined Concrete in a Non-Circular Cross-Section

Figure 2 -- Schematic of Reinforcement Layout – Specimens Series H
Figure 3 -- Schematic Stress-Strain Behavior of Unconfined and Confined Concrete
Figure 4 -- Stress-Strain Behavior of Specimens: (a) Series A; (b) Series B; (c) Series C; (d) Series D; (e) Series E; (f) Series F; (g) Series G; (h) Series H
Figure 5 -- FRP-Wrapped Specimens after Testing: (a) A2; (b) A3; (c) B2; (d) B3; (e) C2; (f) C3; (g) D2; (h) D3; (i) E2; (j) E3; (k) F2; (l) F3; (m) G2; (n) H2
Figure 6 -- Performance of Strengthened Specimens: (a) Strengthening Ratio vs. Side-Aspect Ratio of Specimens of Different Cross-Section Shapes; (b) Strengthening Ratio vs. Area-Aspect Ratio of Square Specimens; and (c) Strengthening Ratio vs. Area-Aspect Ratio of Rectangular Specimens
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II. REVIEW OF DESIGN GUIDELINES FOR FRP CONFINEMENT OF REINFORCED CONCRETE COLUMNS OF NON-CIRCULAR CROSS-SECTIONS

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Abstract: Current international design guidelines provide predictive design equations for the strengthening of Reinforced Concrete (RC) columns of both circular and prismatic cross-sections by means of FRP confinement and subjected to pure axial loading. Extensive studies (experimental and analytical) have been conducted on columns with circular cross-sections, and limited studies have been conducted on members with non-circular cross-sections. In fact, the majority of available research work has been on small-scale, plain concrete specimens. In this review paper, four design guidelines are introduced, and a comparative study is presented. This study is based on the increment of concrete compressive strength and ductility and includes the experimental results from six RC columns of different cross-section shapes. The observed outcomes are used to identify and remark upon the limits beyond the ones specifically stated by each of the guides and that reflect the absence of effects not considered in current models. The purpose of this study is to present a constructive critical review of the state-of-the-art design methodologies available for the case of FRP-confined concrete RC columns and to indicate a direction for future developments.

CE Database subject headings: Axial Compression, Circular Columns, Confinement, Confining Pressure, Design, Ductility, FRP, Non-circular Columns, Reinforced Concrete, Strengthening.
1. INTRODUCTION

Confinement of Reinforced Concrete (RC) columns by means of Fiber Reinforced Polymers (FRP) jackets is a technique being frequently used to seek the increment of load carrying capacity and/or ductility of such compression members. The need for improved strength results from higher load capacity demands because of change in the structure’s use or more stringent code requirements. Improving ductility stems from the need for energy dissipation, which allows the plastic behavior of the element and, ultimately, of the structure. Ductility enhancement is typically required in existing columns that are subjected to a combination of axial load and bending moment because of a change in code (e.g., to account for seismic provisions) or a correction for design or construction errors (e.g., improper splicing of the longitudinal reinforcement or lack of transverse ties).

Extensive work in both the experimental and analytical areas has been conducted on small plain concrete specimens of circular and non-circular cross-sections confined with FRP and subjected to pure axial compressive loading. This work has led to the development of several stress-strain models (the majority being empirical) of two types (Lam and Teng 2003a): design-oriented, where the axial compressive strength, the ultimate axial strain, and the stress-strain behavior are determined using closed-form expressions mainly obtained by best-fitting the results.

Studies focused on RC columns of both circular and non-circular cross-sections of considerable size (minimum side dimension of 300mm [12 in]) have also been conducted (Demers and Neale 1994, Kestner et al. 1997, Chaallal and Shahawy 2000, Wang and Restrepo 2001, Youssef 2003, Carey and Harries 2003, Matthys et al. 2005); however, this experimental research has been limited due to high cost and lack of high-capacity testing equipment. This situation has been the main reason for overlooking the following important effects on element performance not accounted for in most of the available models:

- The size of the cross-sectional area
- The dimensional aspect ratio of the cross-sectional area
- The presence and possible detrimental effect of longitudinal steel reinforcement instability
- The concrete dilation dependant on a pseudo-Poisson ratio
- The contribution of the internal transverse steel reinforcement

In spite of these obstacles, several models have been proposed for the case of non-circular columns (Harries et al. 1997, Wang and Restrepo 2001, Lam and Teng 2003b, Maalej et al. 2003) and have become the basis for design provisions. In particular, the predictive equations found in the current design guides (ACI Committee 440.2R 2002, S806 Canadian Standard Association 2002, Concrete
Society Technical Report 55 2004, *fib* Bulletin 14 2001) are mostly based on approaches created for columns with circular cross-section and then modified by a “shape factor” or “efficiency factor.” This factor is intended to account for the geometry of the section and its effect on the confining pressure, which is no longer uniformly applied by the FRP jacket as for the case of circular cross-sections.

2. RESPONSE TO AXIAL LOAD

The purpose of this study is to use pertinent experimental evidence to identify and remark on the differences in the design methodologies used by the existing available design guides on the FRP confinement of RC columns of different cross-sections and subject to pure axial loading. The following fundamental reasons further clarify the purpose of this study:

- Understanding of pure axial performance addresses both the load capacity increase (strengthening) and ductility enhancement. The latter, in particular, is the result of strain enhancement in terms of both ultimate value and shape of the stress-strain curve (See Fig. 1).

- Increase in strength is an immediate and evident outcome typically expressed as the peak load resistance (or peak strength obtained as maximum load minus the contribution of steel and divided by the gross concrete area). Conversely, increase in ductility is a more complex performance indicator that needs to be translated into the ability of a member to sustain rotation and drift without a substantial loss in strength. Ductility enhancement under pure compression conditions is not of great use in itself, but it becomes of great importance in the case of combined axial compressive load and bending moment.
• Depending on the level of confinement, the stress-strain behavior of a RC column can be modified to the point that peak capacity corresponds to the ultimate attainable axial strain. This modification would be ideally represented by a practically bilinear stress-strain diagram with an ascending (rather than descending) second branch (See curve d - Fig. 1).

• Understanding the effect of all the parameters contributing to the behavior of a FRP-confined RC column (contrast of the ideal cross-section behavior with that of a column where phenomena such as the instability of longitudinal reinforcement that is member-dependant and not simply cross-section dependant, needs to be considered) will help represent and predict such behavior (See Fig. 1). Thus, the performance of a column subjected to combined axial force and bending moment would be greatly simplified, and addressing structural ductility in more rigorous terms would become easier.

For the purpose of this paper and for the interpretation of experimental results, clear and unequivocal definitions of strength and ductility parameters are necessary:

• \( f'_{co} \) and \( f'_{cc} \) represent the peak concrete strengths corresponding to the maximum load carried by the RC column for unconfined and confined cases, respectively.

• \( \varepsilon_{cu} \) is the ultimate strain of the unconfined RC column corresponding to \( 0.85f'_{co} \) (See curve a - Fig. 1). For the confined RC column, \( \varepsilon_{ccu} \) may correspond to one of the following values: a) \( 0.85f'_{cc} \) in the case of a lightly confined member (See curve b - Fig. 1); b) the failure strain in the heavily confined, softening case when the failure stress is larger than \( 0.85f'_{cc} \) (See
curve c - Fig. 1); or the heavily confined, hardening case, where ultimate strength corresponds to ultimate strain (See curve d - Fig. 1).

The definition of $\varepsilon_{ceu}$ at 85 percent of $f'_{cc}$ (or less) is arbitrary, although consistent with modeling of conventional concrete (Hognestad 1951), and such that the descending branch of the stress-strain curve at that level of stress ($0.85f'_{cc}$ or higher) is not as sensitive to the test procedure in terms of rate of loading and stiffness of the equipment utilized.

3. REVIEW OF DESIGN GUIDELINES


In the presentation and discussion of the different design methods provided by the guidelines, a uniform set of parameters, which may be different from the original ones, has been adopted for consistency. They are referenced in a notation list at the end of the document.

No design guideline or recommendations from the Japan Concrete Institute (JCI) or the Japan Society of Civil Engineers (JSCE) are included in this discussion.
because the case of pure axial strengthening of columns is not specifically addressed. In fact, the available documents only refer to enhancement of ductility in terms of drift under seismic loads.

Regarding the design philosophies adopted by each of these codes, the recommendations for the design of RC members strengthened with FRP are based on limit states design principles, which provide acceptable levels of safety against ultimate (collapse) and serviceability limit states. The combinations of loads to be considered when determining the design capacity of a structural member are affected by amplifications factors (greater than 1.0), which account for the probability of the actual loads being larger than the expected ones. The design capacity is also affected by reduction factors that take into consideration the possibility of the resistances being less than calculated (MacGregor 1997).

While all guidelines have a consistent approach to the load amplification factors (even if the coefficients may be different), strength reduction factors are addressed in two different ways. For ACI, the strength reduction factors (less than the value of 1.0) multiply the computed overall nominal capacity, and they are internal force dependant: normal (flexure and axial compression) and shear. For CSA, the Concrete Society, and fib, material safety factors are applied individually to each of the material components of the member (concrete, steel reinforcement, and FRP when applicable) during the computation of the resistance. These material safety factors are indicated as $\gamma$ factors larger than the value of 1.0 and used as dividers, with the exception of CSA (where the factors are less than 1.0 and used as multipliers).

For the case of FRP materials, ACI and the Concrete Society consider additional material safety factors, which depend upon the type of composite material, manufacturing process, method of application, and the exposure condition
Table 1 shows the reduction factors and material safety factors used by the different guidelines. Note that the subscripts “c,” “s,” and “f” refer to concrete, reinforcing steel, and FRP, respectively. Since the guideline provided by ACI Committee 440 is based on the requirements of the building code ACI 318-1999 edition, the reduction factors presented in Table 1 correspond to such edition for the case of axial loading.

Table 2 presents the limits of the design guidelines and type of models adopted. The first column in the table features the guide acronym. The second column shows the type of cross-section. The third column presents the restrictions, which are related to the type of compressive load application (concentric), maximum side dimensions, maximum side-aspect-ratio ($h/b$), and minimum corner radius of the non-circular cross-section ($r$). All the guidelines, with the exception of fib, set a maximum side-aspect-ratio equal to 1.5 and state that confining effects for cases beyond this limit should be neglected, unless demonstrated by experimental evidence. Neither CSA nor fib point out any limiting value regarding a maximum dimension of the sides of the cross-section. An upper limit is given by the Concrete Society with a value of 200 mm (8 in) followed by ACI with 900 mm (36 in). The fourth column in the table shows the design approaches and the models adopted by each guideline. Note that the models adopted by both ACI and fib may be considered as steel-based models, because of the common “root” on the Mander model (Mander et al. 1988), originally developed for steel-confined concrete. The rest may be classified as empirical or analytical models, directly developed for FRP-confined concrete (De Lorenzis and Tepfers 2001).

Tables 3 and 4 present a synopsis of the expressions provided by each guideline for the calculation of the effective confinement pressure ($f_l$), maximum
compressive strength \(f'_{cc}\), and ultimate axial strain \(\varepsilon_{ccu}\) for the cases of FRP-confined RC columns of circular and non-circular cross-sections, respectively.

### 3.1. American Concrete Institute (ACI Committee 440.2R-02, 2002)

The approach presented by the current ACI Committee 440 for compressive strength enhancement is based on the formula originally developed by Mander et al. (1988) for steel-confined concrete, which was later shown by Spoelstra and Monti (1999) to be applicable for the case of FRP-confined concrete. More details on the work conducted by the latter authors on this model are presented when addressing the *fib* document.

The formula by Mander (1988) was adapted for the determination of the maximum strength enhancement that FRP confinement is able to provide. This was based on the fact that up to the yielding point of the steel no difference exists in the mechanics of confinement in between steel and FRP because they both behave linearly elastic.

The formula for \(f'_{cc}\) in ACI provides the peak axial stress of the Mander curve corresponding to a confinement pressure limited by the effective transverse or hoop strain attained at failure by the FRP \(\varepsilon_{fe}\). This expression is explicitly for members under both compression and bending effects, and since no recommendation is noted for pure compression, it is inferred that it is applicable also to this case. The definition of this effective strain \(\varepsilon_{fe}\) was based on experimental evidence of completely FRP-wrapped columns and beams (Priestley et al. 1996), where loss of aggregate interlock in the concrete had been observed to occur at fiber strain levels less than the ultimate fiber strain. Hence, to avoid this type of failure, the maximum strain used for shear strengthening applications was set to the lesser of 0.004 or 75
percent of the FRP rupture strain $e_{fu}$. Although this guideline introduces a limiting level of FRP strain, such restriction is not based on the generally acknowledged fact that the value of the FRP failure strain is less than the one observed in pure tensile test because the FRP is subjected to combined tensile stress and laterally applied pressure resulting from concrete dilation. The confinement pressure is also affected by a factor $k_s$, or efficiency factor (Restrepo and De Vino 1996) introduced to account for the geometry of the non-circular cross-section.

ACI provides an expression to determine the axial deformation corresponding to the peak strength for columns of circular and prismatic cross-sections ($e_{cc}$) (Mander et al. 1988). This strain corresponds to the ultimate axial strain $e_{ccu}$ in the cases where a bilinear stress-strain curve with an ascending second branch is observed (See curve d - Fig. 1). However, for cases corresponding to lightly confined columns (See curve b - Fig. 1) or the heavily confined, softening case (See curve c - Fig. 1), the expression provided by this guide yields the strain corresponding to the peak stress and not the ultimate strain.

**3.2. Canadian Standard Association S806-02, 2002**

Regarding CSA S806-02 guideline, the maximum confined concrete compressive strength (for which no model of reference is provided in the guide) is given by Eq. (1).

\[
f'_{cc} = 0.85f'^c_k + k_s k_c f'_l
\]  

(1)

The definition of the parameter $k_l$ resembles the expression empirically derived by Saatcioglu and Razvi (1992) for the confinement coefficient in the well-
known equation provided by Richart et al. (1928): 
\[ f'_{cc} = f'_{c} + k_{1} f_l \], where \( k_{1} = 6.7(f_l)^{0.17} \).

Since this equation was obtained from experiments on cylindrical concrete specimens confined under hydrostatic pressure, the introduction of the shape factor \( k_s \) in this guideline is intended to account for the different shape of the cross-section. In fact, \( k_s \) is equal to 1.0 and 0.25 for circular and non-circular cross-sections, respectively. The confinement pressure \( f_l \), in the case of non-circular cross-sections, is computed based on the formula derived from the equilibrium of forces developed in a circular cross-section under confinement action, where the diameter \( D \) corresponds to the minimum side dimension of the non-circular cross-section (CSA-A.23.3-94 1994). The maximum stress that the FRP jacket can attain at failure (\( f_{fe} \)) is based on the same strain limitation given by ACI. No expression for the calculation of the ultimate axial strain for confined concrete is provided in the guide.


For columns of circular cross-sections, the Concrete Society proposes a design-oriented model, developed by Lam and Teng (2003a). This model was calibrated against all the experimental data available at the time. As shown in Fig. 2, the confined concrete model is basically composed of an initial parabolic branch followed by an ascending linear branch with a smooth transition at the strain value \( \varepsilon_t \). The model is only applicable for monotonically increasing values of confined compressive strength (no softening or descending second branch); therefore, a criterion of minimum confinement (Xiao and Wu 2000) is established and is given as follows:
The value of the confined concrete compressive strength $f'_{cc}$ is given by Eq. (3), which was shown to yield good agreement with tests conducted on both CFRP-wrapped specimens and Concrete-Filled FRP Tubes (CFFT), although experimentally based on the latter (Lillistone 2000). Note that in Eq. (3) $f'_{cc}$ is based on the characteristic unconfined cube strength $f'_{cu}$, and it differs from the one actually recommended by the model of Lam and Teng (2003a).

$$f'_{cc} = f'_{cu} + 0.05 \left( \frac{2nt_{f}}{D} \right) E_{f} \quad (3)$$

With regards to the ultimate axial strain of the confined concrete $\varepsilon_{ccu}$, the expression provided in this guideline was the one proposed by Lam and Teng (2003a) in their original model (See Table 5). This expression implies the dependence of the ultimate axial deformation on the stiffness provided by the FRP jacket and contemplates non-linearity determined through trends from test data (same database used for calibration of model). The non-linearity coefficients reflect the fact that the secant Poisson ratio of FRP-confined concrete at ultimate depends strongly on the confinement stiffness ratio: $2E_{j}nt_{f}/E_{sec}D$. Additionally, the suggested equation for the ultimate strain includes a strain efficiency factor (0.6) proposed by the authors of the model, and again, calibrated experimentally. The guideline recommends that if the ultimate axial strain $\varepsilon_{ccu}$ were to be greater than 0.01, then the design failure stress
should be taken as the value corresponding to \( \varepsilon_{ecu} \) equal to 0.01 from the stress-strain curve (See Table 4).

The expression to predict the compressive strength of FRP-confined concrete in members of non-circular cross-sections was based on an equation originally developed for circular cross-sections: 
\[
f'_{cc} = f'_c + k_l k_s f_l
\]
where \( k_l \) was derived empirically and conservatively taken as 2.0. Additionally, \( k_s \) represents the effective confinement area ratio \( \left( \frac{A_e}{A_c} \right) \) divided by the side-aspect-ratio \( (h/b) \). The confinement pressure \( f_l \) is given in terms of the equivalent diameter \( D \), which is defined by Teng et al. (2002) as the diagonal distance of the cross-section (See Fig. 3). The model was originally proposed by Teng et al. (2002) and calibrated against a database composed of plain concrete specimens of minimum and maximum cross-sectional dimensions of 150 x 150 mm (6 x 6 in) and 150 x 225 mm (6 x 9 in), respectively, and side-aspect-ratios of 1.0, 1.3, and 1.5. While this model considers the generally accepted approach of an effectively confined area defined by four second-degree parabolas with initial slopes of diagonal lines between the column corners, the Concrete Society recommends a simpler assumption of the initial slopes starting at 45 degrees to the face of the column. The concept of an ineffectively confined area when the parabolas overlap (for side-aspect-ratios \( h/b \) greater than the value of 2.0) is introduced in the calculation of the effectively confined area of concrete \( (A_e) \) (See Fig. 3). This feature was adopted from the model proposed by Maalej et al. (2003).

For the case of FRP-confined concrete members of non-circular cross-sections, no provision is given for the calculation of the ultimate axial strain \( \varepsilon_{ecu} \).

The design recommendations provided by fib for columns of circular and non-circular cross-sections are based on the model proposed by Spoelstra and Monti (1999). These authors developed an iterative analysis-oriented model for circular columns from which two sets of closed-form equations for maximum confined concrete compressive strength $f'_{cc}$ and ultimate axial strain $\varepsilon_{ccu}$ were derived: “exact” and “approximate” formulas. The former requires the prior calculation of the parameters $f'_{cc}$ and $\varepsilon'_{cc}$ of the Mander stress-strain curve, and the secant modulus of elasticity at ultimate $E_{sec,u}$ (Eq. (4)). The latter are alternative expressions obtained by Spoelstra and Monti (1999) based on regression analysis of the proposed model results, and they only require the prior calculation of the confinement pressure $f_l$. These formulas are more readily used for design purposes. This analysis was based on the assumptions of $\varepsilon'_c$ of 0.2 percent and a variation of $E_c$ of 20 percent with respect to the reference value of $5700\sqrt{f'_c}$ for a range of $f'_c = 30 – 50$ MPa (4.4 – 7.3 ksi). Note that in both sets of formulas, the value of $\varepsilon_{fu}$ (Table 3 and Table 4) should be taken as $f_{fu}/E_f$.

$$E_{sec,a} = \frac{E_c}{1 + 2\beta_f} \Rightarrow f'_c = E_{sec,a} \varepsilon_{ccu} = \frac{E_c \varepsilon_{ccu}}{1 + 2\beta_f}$$

(A4)

A particular feature of the model presented by Spoelstra and Monti (1999) is the inclusion of a parameter $\beta$ to account for the physical degradation of concrete when subjected to loading (Eq. (5)). This parameter was originally developed by Pantazopoulou and Mills (1995) in a constitutive model for unconfined concrete
under uni-axial compressive loading and was first obtained in terms of physical properties (e.g., the volumetric fraction of paste per unit volume of concrete and the water-cement ratio). However, the parameter was adapted by Spoelstra and Monti to depend on more commonly available mechanical properties, such as \( f'_{c}, \varepsilon'_{c}, \) and \( E_{c} \):

\[
\beta = \frac{5700}{\sqrt{f'_c}} - 500 \quad (f'_c \text{ in MPa}) \tag{5}
\]

In addition, fib highlights that the hoop failure strain of the FRP jacket, based on experimental evidence, is lower than the ultimate strain obtained by tensile testing of the material. The guideline points out that this reduction is due to several reasons, such as the quality of execution (fibers not perfectly aligned or surface preparation not appropriate), the size effect when applying several layers, the effect of wrapping the material on the corners of low radius, and the combined state of stress of the FRP wrapping. Because of the lack of data on these effects, no appropriate reduction factors are suggested at the present time.

In the case of columns of circular cross-sections, for the calculation of the effective confinement pressure exerted by the FRP jacket \( f_i \), fib provides a confinement effectiveness coefficient \( k_e \) less than 1.0 for a confinement by partial wrapping and equal to 1.0 for a confinement by full wrapping (See Fig. 4 and Eq. (6)) (Mander et al. 1988). In the case of non-circular columns, a parameter \( k_s \) still introduces the confinement effectiveness but in a geometrical way (Eq. (7)). The guideline does not include provisions for the consideration of an additional factor that accounts for the effect of partial wrapping in non-circular specimens.
\[ k_e = \left(1 - \frac{s'}{2D}\right)^2 \]  

(6)

\[ k_s = 1 - \frac{(b-2r)^2 + (h-2r)^2}{3A_g (1 - \rho_l)} \]  

(7)

4. COMPARATIVE STUDY OF GUIDELINES PREDICTIVE EQUATIONS

To evaluate the performance and contrast the different approaches taken by the guidelines for the determination of the compressive strength \( f'_{cc} \) and the ultimate axial compressive strain for confined concrete \( \varepsilon_{ccu} \), a total of six RC column specimens (three strengthened specimens with their corresponding control units) of different cross-section shapes (circular, square, and rectangular) and equal gross areas \( A_g \) were selected, designed, constructed, and tested. These specimens were part of a research study on the size-effect of FRP-confined RC columns recently conducted (Rocca et al. 2006). This assessment is not intended to be comprehensive, but the three relevant cases presented here indicate the trends of the guidelines under study.

Table 5 shows the characteristics of each of the specimens selected. The first column shows the specimen acronym, where the letter in each label indicates the shape of the cross-section: C-circular, S-square, and R-rectangular. The following parameters are presented in the table in the same order: the cross-section dimensions (diameter of the circular cross-section \( D \) and sides \( b \) and \( h \) of the non-circular cross-sections), side-aspect-ratio \( h/b \), total column height \( H \), gross cross-section area \( A_g \), longitudinal steel reinforcement ratio \( \rho_l \), true longitudinal steel yield strength \( f_y \), characteristic concrete compressive strength \( f'_{cc} \) based on standard cylinders, and the FRP volumetric ratio \( \rho_f \) (all the strengthened specimens featured two plies of Carbon
FRP with fiber orientation perpendicular to the longitudinal axis of the column). All the specimens featured a clear concrete cover of 38 mm (1.5 in) and hoops or ties as internal transverse steel reinforcement. The specimens of non-circular cross-section were designed with a corner radius of 30 mm (1.2 in). Additionally, the experimental values of $f'_{co}$ or the strengthening ratios $f'_cc / f'_co$ and ultimate axial strain $\varepsilon_{cu}$ or strain ratios $\varepsilon_{ccu} / \varepsilon_{cu}$, are shown in the last two columns of the table. The experimental values of $\varepsilon_{cu}$ and $\varepsilon_{ccu}$ are reported accordingly to the definitions presented in Section 2 (See Fig. 1).

The material properties of the CFRP used in this study, as experimentally determined in pure tension tests using one and two-ply laminates (Rocca et al. 2006), are as follows:

- Nominal thickness of lamina: $t_f = 0.167$ mm (0.0066 in)
- Ultimate tensile strain: $\varepsilon_{fu} = 0.93\%$
- Modulus of elasticity: $E_f = 291$ GPa (42,200 ksi)

In all the specimens, besides the strain gages on longitudinal steel bars and ties, and the ones on the FRP jacket (at mid-height), two linear potentiometers were fixed to two opposite sides of each specimen in order to measure the axial shortening.

The non-circular CFRP-wrapped specimens failed by FRP rupture at approximately mid-height and at the corners. In the case of the circular specimen, the FRP rupture originated at mid-height and by the conclusion of the test practically the entire jacket debonded (See Fig. 5).

Table 6 presents the theoretical values of maximum axial compressive strength $f'_cc$ and ultimate axial strain $\varepsilon_{ccu}$ for confined concrete. This table is divided in three main horizontal sections, each of which corresponding to the selected cross-sections:
circular, square, and rectangular. Each section presents the values of $f'_{cc}$, $\varepsilon_{ccu}$, and their corresponding enhancement ratios $f'_{cc}/f'_c$ and $\varepsilon_{ccu}/\varepsilon_{cu}$, obtained in accordance to each of the guidelines. In the analytical calculations of $f'_{cc}$ and $\varepsilon_{ccu}$ all the safety and material factors were set equal to 1.0. Regarding the predictive equations for $\varepsilon_{ccu}$, not all the guidelines provide expressions for its determination; for this reason such cases are indicated as Not Applicable (NA). The theoretical values of limiting axial strain of unconfined concrete $\varepsilon_{cu}$ assumed by each guideline are as follows: 0.003 in the case of ACI and 0.0035 in the case of the Concrete Society and fib.

Table 8 presents the theoretical to experimental ratios of maximum compressive strength and axial deformation enhancement: $\left(\frac{f'_{cc}/f'_c}{f'_{cc}/f'_c}\right)_{\text{theo}}/\left(\frac{f'_{cc}/f'_c}{f'_{cc}/f'_c}\right)_{\text{exp}}$ and $\left(\frac{\varepsilon_{ccu}/\varepsilon_{cu}}{\varepsilon_{ccu}/\varepsilon_{cu}}\right)_{\text{theo}}/\left(\frac{\varepsilon_{ccu}/\varepsilon_{cu}}{\varepsilon_{ccu}/\varepsilon_{cu}}\right)_{\text{exp}}$, respectively. The table is divided in three main columns of results corresponding to each selected cross-section shape: circular, square, and rectangular. The experimental values of $\varepsilon_{ccu}$ depend on each of the stress-strain behaviors observed for each specimen, i.e.: the circular column featured a diagram corresponding to curve d in Fig. 1, and both the square and rectangular columns featured stress-strain behaviors similar to curve c in Fig. 1 (where the ultimate axial strain $\varepsilon_{ccu}$ corresponds to the 0.85 percent of the maximum confined compressive strength $f'_{cc}$). The stress-strain curves for the CFRP-wrapped specimens are shown in Fig. 6. The axial and transverse strain values correspond to the average value provided by the linear potentiometers, and the average value provided by the strain gages located at mid-height, respectively. The transverse strain for the case of the prismatic specimens corresponds to the average values of four or two strain gages located on opposite sides of the cross-section.

Fig. 7 shows the strengthening ratios $f'_{cc}/f'_c$ obtained from the experiments and according to the guidelines. The overall trends of the lines corresponding to ACI,
the Concrete Society, and fib predictions are in agreement with the notion that for approximately the same FRP volumetric ratio, the increment of confined compressive strength for non-circular cross-sections, in particular rectangular, is less than for the case of circular cross-sections. Note that within the prismatic specimens, CSA provides a higher strengthening ratio for specimens with rectangular cross-sections when compared to specimens with square cross-sections. The reason for this may be the fact that in this guideline, the computation of the confining pressure is dictated by the equivalent circular cross-section whose diameter is the minimum dimension of the non-circular cross-section in analysis (CSA-A.23.3-94). In other words, it is similar to confining smaller circular cross-sectional columns with the same amount of FRP reinforcement, yielding an increasing trend of the strengthening ratio with the side-aspect-ratio. Note, however, that for both cases of non-circular cross-sections, the theoretical level of strengthening provided by CSA can be considered as negligible for being less than or approximately equal to 1.0.

Fig. 8 shows the accuracy of the different codes with respect to the experimental results in terms of strength enhancement by plotting the ratios given in Table 7 \( \left( \frac{f'_{cc}}{f'_{co}} \right)_{\text{theo}} / \left( \frac{f'_{cc}}{f'_{co}} \right)_{\text{exp}} \). For the case of circular cross-sections, only the Concrete Society and the “exact” equations by fib slightly overestimate the strength enhancement (approximately three percent). Regarding the non-circular cross-sections, only ACI and the Concrete Society overestimate the strength increase for both square and rectangular cross-sections. The “exact” formulas by fib overestimate the strength enhancement for only the square type of cross-section.

Fig. 9 shows the accuracy of the guidelines in predicting the ultimate axial strain enhancement \( \varepsilon_{ccu} / \varepsilon_{cu} \). A theoretical value of \( \varepsilon_{cu} \) equal to 0.003 was used in the case of ACI, and a value of 0.0035 was used in the cases of the Concrete Society and
fib. Recall that CSA does not provide expressions for the calculation of $\varepsilon_{ccu}$. The estimations vary within a range of approximately ±50 percent of the experimental ratios, with the exception of the value corresponding to the “exact” equations from fib for the case of square columns (about 250 percent). As the ultimate axial strain of concrete is a function of different parameters (size and type of aggregates; mix proportions; water/cement ratio; and in the case of confined concrete, greatly influenced by the stiffness of the jacket material (De Lorenzis and Tepfers, 2001), the accurate analytical representation of $\varepsilon_{ccu}$ for design purposes is a challenge. This particular challenge is based on the difficulty posed by the inclusion of the influencing parameters, and in particular, the interaction of concrete dilation with confining FRP.

Table 8 presents the theoretical to experimental ratios of load-carrying capacity of the FRP-strengthened RC columns ($P_{theo}/P_{exp}$). The theoretical or design values of axial resistance were computed considering the material safety factors and/or the strength reduction factors as required by each guideline (See Table 1). Additionally, a value of 0.95 was assigned for the parameter $C_E$ (CFRP applications and interior exposure) in ACI, and a value of 1.2 was assigned for $\gamma_f$ (good quality control on application conditions and application process) in fib. The results presented in Table 8 are plotted in Fig. 10. All the predictions appear to be conservative. The results mainly vary in a range from about 60 to 95 percent of the experimentally obtained load-carrying capacity, with the exception of the ratios corresponding to CSA that show a minimum percentage of about 40, which can be considered as too conservative.
5. DISCUSSION

The limits presented in Table 2, which primarily dealt with the dimensions of the specimens’ cross-sections, side-aspect-ratios \((h/b)\), and loading types (concentric), are the results of the limited experimental evidence on the area of FRP-confinement of real-size RC columns. Additionally, these limits have not allowed the appropriate implementation of key effects in the current models. These effects have been identified as follows:

- The instability of longitudinal steel reinforcement
- The concrete dilation dependant on the pseudo-Poisson ratio
- The contribution of the internal transverse steel reinforcement to the confinement
- An appropriate reduction factor to account for the premature failure of the FRP jacket. Only the model presented by the Concrete Society introduces this parameter in the predicting equations. Two reasons may be the cause of this phenomenon: namely, the triaxial state of stress to which the FRP wrap is subjected as opposed to the pure axial state of coupons under material characterization (hoop stress in addition to the pressure laterally applied as a result of the concrete dilation) and the creation of stress concentration regions along the wrap product of the cracking of the concrete as it dilates.

Regarding the maximum confined concrete compressive strength \(f'_{cc}\), the models presented by ACI and fib, which are both based on the Mander formula for steel-confined concrete, yield different results because of the selection of different expressions for key parameters. In the case of ACI, the maximum confined compressive strength \(f'_{cc}\) is directly computed from the peak strength given by the
Mander formula. However, the predictive equations from *fib* are developed based on two other relationships “merged” in the model by Spoelstra and Monti (1999): Popovics, a concrete stress-strain curve under a constant confining action and the one developed by Pantazopoulou and Mills (1995) for the dilation characteristics of concrete, allowing establishing a relationship between the axial and the transverse strain (Monti 2001). The design approaches presented by CSA and the Concrete Society belong to the empirical or analytical type, and the values given to their parameters were based on plain concrete specimens.

Regarding the determination of the confining pressure in a column of non-circular cross-section, all the guidelines except CSA consider the effect of the cross-section geometry by the inclusion of a confinement effectiveness coefficient (\(k_s\)), which is expressed as a function of the side dimensions (\(b, h\)), corner radius (\(r\)), and the ratio of the area of longitudinal steel reinforcement to the cross-sectional area of the column (\(\rho_l\)). CSA dictates a constant value of 0.25 in addition to an equivalent diameter taken as the minimum side dimension of a non-circular cross-section.

With respect to the consideration of a “strain efficiency factor” accounting for the premature failure of the FRP jacket, ACI and CSA limit the level of hoop strain in the FRP to be attained at failure (\(\varepsilon_{fe}\)) by taking the minimum value of either 0.004 or 0.75\(\varepsilon_{fu}\). However, this limitation is not based on FRP performance considerations (see Section 3.1). The Concrete Society includes a limiting value of strain (\(\varepsilon_{fe}\)) corresponding to 60 percent of \(\varepsilon_{fu}\) (based on experimental evidence). *fib* does not recommend a reduction factor at the present time.

The internal damage (cracking) of concrete under loading is only introduced in the models adopted by the Concrete Society and *fib*. In the former, the concrete deterioration is represented by experimentally fitted coefficients in the expression for
the calculation of $\varepsilon_{ccu}$ (Lam and Teng 2003a). In the latter, the damage is based on a constitutive model for unconfined concrete under uni-axial compression proposed by Pantazopoulou and Mills (1995), whose equations are rearranged to yield a relationship between lateral and axial strain. None of the presented guidelines introduces the effect of longitudinal steel reinforcement instability.

With respect to the accuracy of the predictive equations, CSA was the only guide providing a strengthening ratio for a column of rectangular cross-section higher than the ratio corresponding to a square column, which is contrary to the intuitive notion that for increments of the side-aspect-ratio ($h/b$), the resulting $f'_{cc}$ decreases at an equal FRP volumetric ratio. However, the predictive values for both non-circular columns tested in this program were not larger than 1.0, value associated with no strengthening.

The theoretical to experimental strength enhancement ratios for circular and non-circular cross-sections ($[(f'_{cc}/f'_{co})_{theo}]/[(f'_{cc}/f'_{co})_{exp}]$) were best approximated by the predictive equations provided by the Concrete Society.

Regarding the prediction of the axial strain enhancement, only ACI and fib provide equations for circular and non-circular columns. The Concrete Society provides an equation solely for the case of circular columns. In general, the scatter of the predictions for strain enhancement was much larger than for strength enhancement. In fact, the “exact” equations by fib appear to overestimate the strain enhancement for the case of square columns by about 250 percent. This overestimation may be partly because of the difficulty in accurately representing the effects of parameters such as size and type of aggregates; mix proportions; water/cement ratio; and, in the case of confined concrete, the stiffness of the FRP jacket.
The design axial capacities of the strengthened RC columns were compared to the values experimentally obtained. Accounting for the guide-specific reduction and material factors, the design predictions from all guidelines were conservative, with the highest level provided by CSA and the lowest by the “exact” formulation from fib.

6. CONCLUSIONS

Design approaches for FRP-confined RC columns from four international design guidelines were presented, reviewed, and compared. Limits and design equations for the calculation of the maximum axial compressive strength $f'_{cc}$ and ultimate axial strain $\varepsilon_{ccu}$ of FRP-confined RC members for circular and non-circular cross-section shapes were outlined. The experimental results from six RC columns of different cross-section shapes (circular, square, and rectangular) were contrasted to theoretical predictions obtained in accordance to each guideline.

For the purpose of an appropriate comparison of axial deformation enhancement, it is necessary to set a stress value at which the measurement is made, particularly for the case of stress-strain curves with a descending second branch (See curves b and c in Fig. 1). For this, an arbitrary value of $0.85 f'_{cc}$, was adopted (Hognestad 1951).

Given the present knowledge and experimental evidence, the research community should consider further experimental and analytical work allowing confirming the basic assumptions, and providing relevant and substantial data information to feed and correctly calibrate numerical and analytical models. Although a vast experimental campaign on real-size RC columns following the conventional testing methodology is a choice, the current available sensing technology used in a few dimensionally-relevant specimens represents an innovative
alternative testing protocol, allowing obtaining accurate information, and most importantly allowing the understanding of the physical phenomena. The measurements should be targeted to the strain distribution along the perimeter of the FRP jacket, the strain distribution of the longitudinal and transverse steel reinforcement, the lateral (outward) deformation of the longitudinal steel bars product of the concrete lateral dilation (bar instability), the concrete dilation, and crack propagation detection. A more meaningful interpretation of the experimental data currently available in the literature would become possible once performance phenomena and controlling parameters are fully understood.

7. ACKNOWLEDGEMENTS

The authors would like to acknowledge the funding and support received from: National Science Foundation (supplement grant number 0453808), MAPEI S.p.A. in Milan (Italy), NSF Industry/University Cooperative Research Center on Repair of Buildings and Bridges with Composites (RB^2C), and the University Transportation Center on Advanced Materials and NDT Technologies based at the University of Missouri-Rolla (UMR).

8. NOTATION

The following symbols are used in this paper:

\[ A_c \text{ Cross-sectional area of concrete in column (Concrete Society) = } A_g \left(1 - \rho_t\right) \]

\[ A_e \text{ Effectively confined area = } A_g \left(\frac{(h-2r)^2 + (b-2r)^2}{3} - \rho_t A_g\right) \]

\[ A_g \text{ Total cross-sectional area = bh} \]
Only for Concrete Society = \( bh - (4 - \pi)r^2 \)

\( A_{ol} \) Area of overlap of the parabolas in a prismatic cross-section with side-aspect-ratio greater than 2.0 (Concrete Society) =

\[
\begin{cases}
0 & \text{if } 2b \geq (h - 2r) \\
\frac{4(l_{ol})^3}{3(h - 2r)} + l_{ol}(2b - (h - 2r)) & \text{otherwise}
\end{cases}
\]

\( A_s \) Area of steel reinforcement = \( A_{s1} \rho_l \)

\( b \) Short side dimension of a non-circular cross-section

\( b_f \) Width of FRP strip in partial wrapping

\( C_E \) Environmental reduction factor (ACI)

\( D \) Diameter of circular cross-section

Least lateral dimension of the prismatic cross-section (CSA) = \( \sqrt{b^2 + h^2} \)

\( E_{sec} \) Secant modulus of concrete (Concrete Society) = \( \frac{f'_c}{\varepsilon_c'} \)

\( E_{sec,u} \) Secant modulus of elasticity of concrete at ultimate (\( fib \)) = \( E_{sec,u} = \frac{E_c}{(1 + 2\beta \varepsilon_{fu})} \)

\( E_2 \) Slope of linear portion of confined stress-strain curve (Concrete Society) = \( \frac{f''_{cc} - f''_c}{\varepsilon_{ccu}} \)

\( E_c \) Initial modulus of elasticity of concrete

\( E_f \) Tensile modulus of elasticity of FRP

\( f'_c \) Characteristic concrete compressive strength determined from standard cylinder

\( f'_{cc} \) Compressive strength of confined concrete (For experiments: peak load minus the contribution steel and divided by the cross-sectional concrete area)
$f_{\text{co}}$  Compressive strength of unconfined concrete (For experiments: peak load minus the contribution steel and divided by the cross-sectional concrete area)

$f'_{\text{cu}}$  Characteristic concrete compressive strength determined from cube = $f'_c/0.8$

$f_{\text{fu}}$  Ultimate tensile strength of FRP

$f_l$  Confinement pressure due to FRP jacket

$f_y$  Yield strength of longitudinal steel reinforcement

$H$  Height of column

$h$  Long side dimension of a non-circular cross-section

$k_1$  Confinement parameter (CSA) = 6.7$(k_1 f'_l)^{-0.17}$

$k_e$  Confinement effectiveness coefficient accounting for effect of partial wrapping in columns of circular cross-sections (fib) = $(1 - \frac{s'}{2D})^2$

$k_s$  Confinement effectiveness coefficient accounting for the geometry of cross-sections =

\[
\begin{cases} 
1 & \text{Circular (ACI, CSA)} \\
0.25 & \text{Non-Circular (CSA)} \\
1 - \frac{(b - 2r)^2 + (h - 2r)^2}{3A_g (1 - \rho_1)} & \text{Non-Circular (ACI, fib)} \\
\frac{b}{h} - \frac{3A_g}{(1 - \rho_1)} & \text{Non-Circular (Concrete Society)} \\
\end{cases}
\]

$l_{ol}$  Length of overlapping region (Concrete Society) = $\sqrt{\frac{(h - 2r)^2}{4} - \frac{b(h - 2r)}{2}}$

$n$  Number of FRP plies composing the jacket

$P_{\text{theo}}$  Design axial load carrying capacity of FRP-confined RC column

$P_{\text{exp}}$  Experimental axial load carrying capacity of FRP-confined RC column
\( r \)  
Corner radius of non-circular cross-sections

\( s' \)  
Clear spacing between FRP wraps

\( s \)  
Pitch in partial wrapping

\( t_f \)  
FRP nominal ply thickness

\( \beta \)  
Concrete parameter ((\( fib \)) = \( \frac{5700}{\sqrt{f_c'}} \) − 500

\( \varepsilon'^c \)  
Axial compressive strain corresponding to \( f'_c \)

\( \varepsilon_{cu} \)  
Ultimate axial compressive strain of confined concrete

\( \varepsilon_{cu} \)  
Ultimate axial compressive strain of unconfined concrete =

\[
\begin{cases} 
0.003 & \text{(ACI)} \\
0.0035 & \text{(Concrete Society, } fib) 
\end{cases}
\]

\( \varepsilon_{fe} \)  
FRP effective strain (strain level reached at failure)

\( \varepsilon_{fu} \)  
Ultimate tensile strain of the FRP

\( \varepsilon_t \)  
Position of transition region between parabola and straight line (Concrete Society) = \( \frac{2f'_c}{E_c - E_2} \)

\( \phi \)  
Strength reduction factor (ACI)

\( \gamma_c \)  
Partial safety factor for concrete (CSA, Concrete Society, \( fib \))

\( \gamma_e \)  
Partial safety factor for strain of FRP (Concrete Society)

\( \gamma_E \)  
Partial safety factor for modulus of elasticity of FRP (Concrete Society)

\( \gamma_f \)  
Material safety factor for FRP (CSA, \( fib \))

\( \gamma_{mm} \)  
Additional partial safety factor for manufacture of FRP (Concrete Society)

\( \gamma_s \)  
Partial safety factor for steel (CSA, Concrete Society, \( fib \))
Volumetric ratio of FRP reinforcement = \[
\begin{cases}
\frac{4n_t}{D} \left( \frac{b_t}{s} \right) & \text{Circular} \\
\frac{2n_t}{bh} (b + h) & \text{Non-Circular}
\end{cases}
\]

\( \rho_f \)

Ratio of the area of longitudinal steel reinforcement to the cross-sectional area of a compression member = \( A_s / A_g \)

\( \psi_f \)

Additional FRP strength reduction factor (ACI)

9. REFERENCES

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American Concrete Institute, ACI 440.2R-02. (2002). Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening of Concrete Structures. American Concrete Institute, Farmington Hills, MI, USA.


Hognestad, E. (1951). “A Study of Combined Bending and Axial Load in Reinforced Concrete Members.” *Bulletin 399*, Engineering Experiment Station, Univ. of Illinois, Urbana, IL, USA.


Richart, F. E., Brandtzaeg, A., and Brown, R. L. (1928). “A Study of the Failure of Concrete under Combined Compressive Stresses.” Engineering Experimental Station Bulletin No. 185, Univ. of Illinois, Urbana, IL, USA.


Table 1. Strength Reduction and Material Safety Factors for Different Guidelines

<table>
<thead>
<tr>
<th>Guideline</th>
<th>Strength Reduction Factors</th>
<th>Materials Safety Factors</th>
<th>FRP Additional Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI</td>
<td>φ = 0.75 (spiral)(^a)</td>
<td>NA(^b)</td>
<td>(\psi_t = 0.95)</td>
</tr>
<tr>
<td></td>
<td>φ = 0.70 (ties)</td>
<td></td>
<td>(C_E = \text{function of the exposure conditions, fiber and resin type})</td>
</tr>
<tr>
<td>CSA</td>
<td>NA</td>
<td>γ(_c) = 0.60</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td></td>
<td>γ(_s) = 0.85</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>γ(_f) = 0.75</td>
<td></td>
</tr>
<tr>
<td>Concrete Society</td>
<td>NA</td>
<td>γ(_c) = 1.50</td>
<td>(\gamma_{mm} = \text{function of the type of system and method of application or manufacture.})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>γ(_s) = 1.05</td>
<td>For sheets applied by wet lay-up the recommended value is 1.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>γ(_f) = \text{function of type of composite material} = 1.25 for CFRP</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>γ(_E) = \text{function of type of composite material} = 1.1 for CFRP</td>
<td></td>
</tr>
<tr>
<td>fib</td>
<td>NA</td>
<td>γ(_c) = 1.50</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td></td>
<td>γ(_s) = 1.15</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\gamma = \text{function of the FRP type, application system, conditions of applications, quality control})</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) This guideline assigns a less onerous reduction factor in light of the ductile type of failure that spirally reinforced columns undergo, as opposed to columns reinforced with ties.

\(^b\) NA = Not Applicable
### Table 2. Design Guidelines Limitations and Type of Models

<table>
<thead>
<tr>
<th>Guideline</th>
<th>Cross-Section Type</th>
<th>Limitations</th>
<th>Type of Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ACI</strong></td>
<td>Circular</td>
<td>None</td>
<td>Strength and maximum strain only Mander, 1988</td>
</tr>
</tbody>
</table>
|            | Prismatic          | • $b, h \leq 900$ mm  
• $h/b < 1.5$  
• Minimum corner radius $(r): 13$ mm | Strength only  |
| **CSA**    | Circular           | • Concentric axial loading | Strength only Not Specified  |
|            | Prismatic          | • Concentric axial loading  
• $h/b \leq 1.5$  
• Minimum corner radius $(r): 20$ mm | Strength only  |
| **Concrete Society** | Circular | • Concentric axial loading | Stress-strain, and strength and maximum strain Lillistone, 2000  |
|            | Prismatic          | • Concentric axial loading  
• Side dimension $\leq 200$ mm  
• $h/b \leq 1.5$  
• Minimum corner radius $(r): 15$ mm | Strength only Lam and Teng, 2002  
Maalej, 2003  |
| **fib**    | Circular           | • Concentric axial loading | Stress-strain, and strength and maximum strain Spoeletstra and Monti, 1999  |
|            | Prismatic          | • Concentric axial loading  
• Recommended: $15 \leq r \leq 25$ mm or as suggested by manufacturer | Strength and maximum strain  |

**Note:** 1 in = 25.4 mm
Table 3: Summary of Design Guideline Models for Circular Cross-Sections

<table>
<thead>
<tr>
<th>Guideline</th>
<th>Effective Confinement Pressure $f_l$ (MPa)</th>
<th>Confined Concrete Compressive Strength for Purpose $f'_{cc}$ (MPa)</th>
<th>Ultimate Axial Compressive Strain of Confined Concrete $\varepsilon_{ccu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ACI</strong></td>
<td>$f_l = \frac{k_s f_l e_c E_f}{2}$; $k_s = 1$</td>
<td>$f'<em>{cc} = f'</em>{c} \left[ 2.25 \sqrt{1 + \frac{7.9 f_f}{f'<em>{c}}} - 2 \frac{f_i}{f'</em>{c}} - 1.25 \right]$</td>
<td>$\varepsilon_{cc} = \frac{1.71 (5 f'<em>{cc} - 4 f'</em>{c})}{E_c}$</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{ec} = \text{lesser of 0.004 and } 0.75 \varepsilon_{fu}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>CSA</strong></td>
<td>$f'<em>{cc} = 0.85 f'</em>{c} + k_i k_s f_i$</td>
<td></td>
<td>Not Provided</td>
</tr>
<tr>
<td></td>
<td>$k_s = 1$; $k_i = 6.7 (k_s f_i)^{-0.17}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Concrete Society**

A limit is suggested for the applicability of the stress-strain model (Eq. 2).

For cases where the $\varepsilon_{ccu}$ is larger than 0.01, it is recommended to obtain $f'_{cc}$ from the stress-strain curve at the value corresponding to $\varepsilon_{cc} = 0.01$.

Stress-strain curve:

- $f'_{cc} = \varepsilon_{cc} - \frac{(E_c - E_2)^2}{4f'_{c}} \left( \varepsilon_{cc} \right)^2 \rightarrow 0 \leq \varepsilon_{cc} \leq \varepsilon_{t}$
- $f'_{cc} = f'_{c} + E_2 \varepsilon_{cc} \rightarrow \varepsilon_{t} < \varepsilon_{cc} \leq \varepsilon_{ccu}$
- $E_2 = \frac{f_{cc} - f'_{c}}{\varepsilon_{ccu}}$; $\varepsilon_{t} = \frac{2f'_{c}}{E_c - E_2}$
- Design value: $f'_{cc} = \bar{f'}_{cu} + 0.05 \left( \frac{2nt_f}{D} \right) E_f$
- $\bar{f'}_{cu} = \frac{f'_{c}}{0.8}$

$E_{sec} = \frac{f'_{c}}{\varepsilon_{c}}$
Table 3. Summary of Design Guideline Models for Circular Cross-Sections (Cont.)

<table>
<thead>
<tr>
<th>Guideline</th>
<th>Effective Confinement Pressure $f_i$ (MPa)</th>
<th>Confined Concrete Compressive Strength for Purpose of Design $f'_{cc}$ (MPa)</th>
<th>Ultimate Axial Compressive Strain of Confined Concrete $\varepsilon_{ccu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>“Exact” formulas: $f'<em>{cc} = \frac{E_c \varepsilon</em>{ccu}}{1 + 2\beta \varepsilon_{fu}}$</td>
<td>$\varepsilon_{ccu} = \varepsilon_{cc} \left( \frac{2\beta \varepsilon_{fu} E_{cc}}{E_c - E_{cc}} \right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f^*<em>{cc} = f'</em>{cc} \left[ 2.254 \sqrt{1 + 7.94 \frac{f_i}{f'_c} - 2 \frac{f_i}{f'_c} - 1.254} \right]$</td>
<td>$\varepsilon^<em><em>c = \varepsilon'</em>{cc} \left[ 1 + 5 \left( \frac{f^</em><em>{cc}}{f'<em>c} - 1 \right) \right]; \ E</em>{cc} = \frac{f^*</em>{cc}}{\varepsilon^*_c}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta = \frac{5700}{\sqrt{f'_c}} - 500 \neq f'_c$ in MPa</td>
<td>$\varepsilon_{ccu} = \varepsilon_{cc} \left( \frac{2\beta \varepsilon_{fu} E_{cc}}{E_c - E_{cc}} \right)$</td>
</tr>
<tr>
<td><em>fib</em></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_i = \frac{1}{2} k_e \rho_l E_i \varepsilon_{fu}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$k_e = \left( 1 - \frac{s'}{2D} \right)^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f^*<em>{cc} = f'</em>{cc} \left[ 2.254 \sqrt{1 + 7.94 \frac{f_i}{f'_c} - 2 \frac{f_i}{f'_c} - 1.254} \right]$</td>
<td>$\varepsilon^<em><em>c = \varepsilon'</em>{cc} \left[ 1 + 5 \left( \frac{f^</em><em>{cc}}{f'<em>c} - 1 \right) \right]; \ E</em>{cc} = \frac{f^*</em>{cc}}{\varepsilon^*_c}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>*“Practical” formulas: $f'_{cc} = f'_c \left( 0.2 + 3 \sqrt{\frac{f_i}{f'_c}} \right)$</td>
<td>$\varepsilon_{ccu} = \varepsilon_{cc} \left( \frac{2\beta \varepsilon_{fu} E_{cc}}{E_c - E_{cc}} \right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon_{ccu} = \varepsilon_{cc} \left( \frac{2\beta \varepsilon_{fu} E_{cc}}{E_c - E_{cc}} \right)$</td>
<td>$\varepsilon_{ccu} = \varepsilon_{cc} \left( \frac{2\beta \varepsilon_{fu} E_{cc}}{E_c - E_{cc}} \right)$</td>
</tr>
</tbody>
</table>

* $f'_c$ is in MPa

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Table 4. Summary of Design Guideline Models for Non-Circular Cross-Sections

<table>
<thead>
<tr>
<th>Guideline</th>
<th>Effective Confinement Pressure $f_i$ (MPa)</th>
<th>Confined Concrete Compressive Strength for Purpose of Design $f'_{ce}$ (MPa)</th>
<th>Ultimate Axial Compressive Strain of Confined Concrete $\varepsilon_{ceu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI</td>
<td>$f_i = \frac{k_i \rho_i \varepsilon_{fe} E_f}{2}$; $\varepsilon_{fe} =$ lesser of 0.004 and 0.75$\varepsilon_{iu}$</td>
<td>$f'_{ce} = f'<em>c \left[ 2.25 \sqrt{1 + \frac{7.9 f'</em>{ce} - 2 f'_c}{f'_c} - 1.25} \right]$</td>
<td>Not Provided</td>
</tr>
<tr>
<td>CSA</td>
<td>$f_i = \frac{2nt_i f_{fe}}{D}$; $D$ is the lesser of $b$ and $h$</td>
<td>$f'<em>{ce} = 0.85f'</em>{c} + k_s k_i f_i$</td>
<td>Not Provided</td>
</tr>
<tr>
<td>Concrete Society</td>
<td>$f_i = \frac{2f_i nt_i}{\sqrt{b^2 + h^2}}$; $\sqrt{b^2 + h^2} = D$</td>
<td>$f'_{ce} = f'_c + 2k_i f_i$; $k_s = \frac{b A_s}{h A_c}$</td>
<td>Not Provided</td>
</tr>
</tbody>
</table>

*Note: $k_i$, $k_s$, and $\rho_i$ are constants that depend on the specific guideline.*
Table 4. Summary of Design Guideline Models for Non-Circular Cross-Sections (Cont.)

<table>
<thead>
<tr>
<th>Guideline</th>
<th>Effective Confinement Pressure ( f_l ) (MPa)</th>
<th>Confined Concrete Compressive Strength for Purpose of Design ( f'_{cc} ) (MPa)</th>
<th>Ultimate Axial Compressive Strain of Confined Concrete ( \varepsilon_{ccu} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_l = \text{Minimum of } f_{lx} \text{ and } f_{ly} )</td>
<td>( f'<em>{cc} = \frac{E_c \varepsilon</em>{ccu}}{1 + 2\beta \varepsilon_{tu}} ); ( \beta = \frac{5700}{\sqrt{f'_c}} - 500 ) ( f'_c ) (MPa)</td>
<td>( \varepsilon_{ccu} = \varepsilon_{cc} \left( \frac{2\beta \varepsilon_{tu} E_{cc}}{E_c - E_{cc}} \right)^{\frac{E_c}{E_c}} )</td>
</tr>
<tr>
<td></td>
<td>( f_{lx} = \rho_k k_s E_s \varepsilon_{tu} )</td>
<td>( f'_{cc} = f'_c \left[ 2.254 \sqrt{1 + 7.94 \frac{f_l}{f'_c} - 2 \frac{f_l}{f'_c} - 1.254} \right] )</td>
<td>( \varepsilon_{cc} = \varepsilon_{ccu} \left[ 1 + 5 \left( \frac{f'<em>c}{f'<em>c - 1} \right) \right] ; E</em>{cc} = \frac{f'</em>{cc}}{\varepsilon_{cc}} )</td>
</tr>
<tr>
<td></td>
<td>( k_s = 1 - \frac{(b - 2r)^2 + (h - 2r)^2}{3A_s (1 - \rho)} )</td>
<td>( \varepsilon_{cc} = \varepsilon_{ccu} \left[ 2 + 1.25 \frac{E_c}{f'<em>c} f</em>{tu} \sqrt{f'_c} \right] )</td>
<td></td>
</tr>
</tbody>
</table>
### Table 5. Specimens Characteristics

<table>
<thead>
<tr>
<th>Specimen</th>
<th>D (mm)</th>
<th>b (mm)</th>
<th>h (mm)</th>
<th>h/b</th>
<th>H (m)</th>
<th>A_g (cm²)</th>
<th>ρ_l (%)</th>
<th>f_y (MPa)</th>
<th>f_c (MPa)</th>
<th>ρ_r (%)</th>
<th>f'co (MPa)</th>
<th>ε_cu (mm/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>508</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>1.12</td>
<td>20.3</td>
<td>1.53</td>
<td>446</td>
<td>31.7</td>
<td>0.00</td>
<td>26.3</td>
<td>0.003</td>
</tr>
<tr>
<td>C2</td>
<td>446</td>
<td>31.9</td>
<td>0.26</td>
<td>[1.44]</td>
<td>[4.48]</td>
<td>446</td>
<td>31.9</td>
<td>0.26</td>
<td>[1.44]</td>
<td>[4.48]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>NA</td>
<td>458</td>
<td>458</td>
<td>1.0</td>
<td>1.02</td>
<td>21.0</td>
<td>1.48</td>
<td>446</td>
<td>32.1</td>
<td>0.00</td>
<td>26.0</td>
<td>0.002</td>
</tr>
<tr>
<td>S2</td>
<td>446</td>
<td>32.1</td>
<td>0.29</td>
<td>[1.06]</td>
<td>[1.67]</td>
<td>446</td>
<td>32.1</td>
<td>0.29</td>
<td>[1.06]</td>
<td>[1.67]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R1</td>
<td>NA</td>
<td>318</td>
<td>635</td>
<td>2.0</td>
<td>1.37</td>
<td>20.2</td>
<td>1.56</td>
<td>447</td>
<td>30.1</td>
<td>0.00</td>
<td>24.7</td>
<td>0.002</td>
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<tr>
<td>R2</td>
<td>447</td>
<td>30.4</td>
<td>0.32</td>
<td>[1.01]</td>
<td>[3.34]</td>
<td>447</td>
<td>30.4</td>
<td>0.32</td>
<td>[1.01]</td>
<td>[3.34]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** 1 in = 25.4 mm; 1 in² = 6.45 cm²; 1 ksi = 6.9 MPa
Table 6. Theoretical Values of Maximum Concrete Compressive Strength and Ultimate Axial Strain of Confined Concrete - Enhancement Ratios

<table>
<thead>
<tr>
<th>Guideline</th>
<th>$f'_{cc}$ (MPa)</th>
<th>$f'<em>{cc}/f'</em>{co}$</th>
<th>$\varepsilon_{cu}$ (mm/mm)</th>
<th>$\varepsilon_{cu}/\varepsilon_{cu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular Cross-Section</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACI</td>
<td>41.4</td>
<td>1.30</td>
<td>0.005</td>
<td>1.69</td>
</tr>
<tr>
<td>CSA</td>
<td>36.7</td>
<td>1.16</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Concrete Society</td>
<td>47.2</td>
<td>1.49</td>
<td>0.006</td>
<td>1.73</td>
</tr>
<tr>
<td>fib</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>“exact”</td>
<td>47.3</td>
<td>1.49</td>
<td>0.018</td>
<td>5.22</td>
</tr>
<tr>
<td>“practical”</td>
<td>38.1</td>
<td>1.20</td>
<td>0.010</td>
<td>2.96</td>
</tr>
<tr>
<td>Square Cross-Section</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACI</td>
<td>37.5</td>
<td>1.17</td>
<td>0.004</td>
<td>1.26</td>
</tr>
<tr>
<td>CSA</td>
<td>30.5</td>
<td>0.95</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Concrete Society</td>
<td>34.8</td>
<td>1.08</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>fib</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>“exact”</td>
<td>37.8</td>
<td>1.18</td>
<td>0.015</td>
<td>4.15</td>
</tr>
<tr>
<td>“practical”</td>
<td>29.9</td>
<td>0.93</td>
<td>0.009</td>
<td>2.48</td>
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<tr>
<td>Rectangular Cross-Section</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>ACI</td>
<td>34.5</td>
<td>1.14</td>
<td>0.003</td>
<td>1.11</td>
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<tr>
<td>CSA</td>
<td>30.3</td>
<td>1.01</td>
<td>NA</td>
<td>NA</td>
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<tr>
<td>Concrete Society</td>
<td>31.2</td>
<td>1.04</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>fib</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>“exact”</td>
<td>28.5</td>
<td>0.95</td>
<td>0.012</td>
<td>3.37</td>
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<tr>
<td>“practical”</td>
<td>22.1</td>
<td>0.74</td>
<td>0.008</td>
<td>2.13</td>
</tr>
</tbody>
</table>

Note: 1 in = 25.4 mm; 1 ksi = 6.9 MPa. Based on the provisions by each code for the limiting axial strain of unconfined concrete, the theoretical value of $\varepsilon_{cu}$ assumed for ACI was 0.003, and for the Concrete Society and fib was 0.0035.
Table 7. Performance of Guidelines Predictive Equations in Terms of Compressive Strength and Axial Deformation Enhancement

<table>
<thead>
<tr>
<th>Guideline</th>
<th>Circular</th>
<th>Square</th>
<th>Rectangular</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_{cc}'/f_{co}'$</td>
<td>$e_{ccu}/e_{cu}$</td>
<td>$f_{cc}'/f_{co}'$</td>
</tr>
<tr>
<td>ACI</td>
<td>0.90</td>
<td>0.38</td>
<td>1.10</td>
</tr>
<tr>
<td>CSA</td>
<td>0.80</td>
<td>NA</td>
<td>0.89</td>
</tr>
<tr>
<td>Concrete Society</td>
<td>1.03</td>
<td>0.39</td>
<td>1.02</td>
</tr>
<tr>
<td>fib</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>“exact”</td>
<td>1.03</td>
<td>1.16</td>
<td>1.11</td>
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<tr>
<td>“practical”</td>
<td>0.83</td>
<td>0.66</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Note: Based on the provisions by each code for the limiting axial strain of unconfined concrete, the theoretical value of $e_{cu}$ assumed for ACI was 0.003, and for the Concrete Society and fib was 0.0035.

Table 8. Performance of Guidelines Predictive Equations - Ratio of Design Load Capacity to Experimental $P_{theo}/P_{exp}$

<table>
<thead>
<tr>
<th>Guideline</th>
<th>Circular</th>
<th>Square</th>
<th>Rectangular</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI</td>
<td>0.61</td>
<td>0.70</td>
<td>0.71</td>
</tr>
<tr>
<td>CSA</td>
<td>0.52</td>
<td>0.57</td>
<td>0.40</td>
</tr>
<tr>
<td>Concrete Society</td>
<td>0.87</td>
<td>0.75</td>
<td>0.76</td>
</tr>
<tr>
<td>fib</td>
<td>“exact”</td>
<td>0.86</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>“practical”</td>
<td>0.70</td>
<td>0.74</td>
</tr>
</tbody>
</table>
Fig. 1. Schematic Stress-Strain Behavior of Unconfined and Confined RC Columns

Fig. 2. Lam and Teng's Stress-Strain Model for FRP-Confined Concrete Circular Columns (Teng et al. 2002)

Fig. 3. Overlapping Parabolas in Confined Region (Concrete Society 2004)
Fig. 4. Confining Pressure Effect due to Partial Wrapping (fib, 2001)

Fig. 5. Failures of FRP-Wrapped RC Columns: (a) Specimen C2; (b) Specimen S2; (c) Specimen R2
Fig. 6. Stress-Strain Behavior of FRP-Wrapped RC Columns
Fig. 7. Strengthening Ratios vs. Cross-Section Shape

Fig. 8. Guidelines Performance – Ratio of Theoretical Concrete Compressive Strength Enhancement to Experimental
Fig. 9. Guidelines Performance – Ratio of Theoretical Ultimate Axial Deformation Enhancement to Experimental

Fig. 10. Guidelines Performance – Ratio of Design Axial Load Capacity to Experimental
III. INTERACTION DIAGRAM METHODOLOGY FOR DESIGN OF FRP-CONFINED REINFORCED CONCRETE COLUMNS

Silvia Rocca¹ and Antonio Nanni²

Abstract: This paper presents a procedure that allows the construction of a simplified axial load – bending moment interaction diagram for FRP-wrapped Reinforced Concrete (RC) columns of circular and non-circular cross-sections for practical design applications. In the proposed methodology, the analysis of FRP-confined columns is carried out based on principles of equilibrium and strain compatibility equivalent to that of conventional RC columns, the primary difference being the use of the stress-strain model for FRP-confined concrete developed by Lam and Teng. Based on the consideration that the strength enhancement is of significance in members where compression is the controlling failure mode, only the portion of the interaction diagram corresponding to this type of failure is the focus of the methodology. Experimental evidence from RC specimens with a minimum side dimension of 300 mm (12 in.) and subjected to combined axial compression and flexure was collected and compared to the theoretical interaction diagrams. Even though limited experimentation has been conducted in the compression-controlled region for such type of members, data points appear to be consistent with the analytical predictions. A design method for RC members is therefore proposed following the principles of the ACI building code.
1. INTRODUCTION

Columns are structural members subjected to combinations of axial compression and bending moment, rather than pure axial loading. The flexural effect may be induced by different factors, such as unbalanced moments at connecting beams, vertical misalignment, or lateral forces resulting from wind or seismic activity.

Confinement of Reinforced Concrete (RC) columns by means of Fiber Reinforced Polymers (FRP) jackets is a technique being frequently used to seek the increment of load carrying capacity and/or ductility of such compression members. The need for improved strength results from higher load capacity demands because of design/construction errors, change in the facility use, or revisions of code requirements. Improving ductility stems from the need for energy dissipation, which allows the plastic behavior of the element and, ultimately, of the structure. Ductility enhancement is typically required in existing columns that are subjected to a combination of axial load and bending moment because
of reasons similar to those listed for strengthening. Among these reasons, seismic
upgrade and correction of detailing defects (i.e., improper splicing of the longitudinal
reinforcement or lack of transverse ties) are most common.

Several theoretical and experimental studies have been conducted on the behavior
of FRP-wrapped concrete columns subjected to axial force and flexure and have
produced theoretical axial force – moment (P-M) interaction diagrams [1-6]. Other
works have been conducted in the context of concrete-filled FRP tubes [7-9]. Fam et al.
[9] in particular performed experimental and analytical modeling considering the
variability of FRP-confinement effectiveness as a result of the presence of flexural
moments in a column. They proposed an analytical procedure to determine a P-M
diagram for circular cross-sections accounting for the variability of concrete confinement
as a result of the gradual change of the biaxial state of stress developed in the FRP shell
as the eccentricity of the axial force changes. As an overall observation, all the previous
studies conclude that significant enhancement due to FRP-confinement is expected in
compression-controlled RC members.

This paper presents a methodology for the construction of a simplified P-M
diagram following principles of equilibrium and strain compatibility, and it is limited to
the compression-controlled region of the interaction diagram. The procedure is based on
ACI protocols and limiting values (i.e., $P_n$ and $M_n$ multiplied by strength reduction factor
$\phi$; ultimate axial strain of unconfined concrete $\epsilon_{cu}$ equal to 0.003). Additionally, the
paper presents experimental evidence on FRP-wrapped RC columns subjected to
combined compression and flexure and having a size representative of real scale
members. The experimental results are compared to the theoretical simplified P-M
diagrams obtained following the proposed methodology. Data points appear to be consistent with the analytical predictions. The proposed simplified P-M diagram could be used by practitioners as a design tool and for a clear illustration of the procedure; an example application is presented in this paper.

The symbols used throughout the paper are defined in a notation list. Some of these symbols are also defined in the text for clarity.

2. RESEARCH SIGNIFICANCE

The purpose of this study is to provide a methodology for the construction of a simplified axial force – bending moment interaction diagram for FRP-wrapped RC columns for practical design applications. The proposed method is based on principles of equilibrium and strain compatibility. The analysis of the FRP-confined specimens is equivalent to that of conventional RC columns but considers an appropriate stress-strain model for FRP-confined concrete in the compression zone.

3. EXPERIMENTAL BEHAVIOR OF FRP-CONFINED RC COLUMNS

The experimental evidence on real-size type specimens, although not extensive, consists of RC columns of circular and non-circular cross-sections subjected to a combination of axial compressive load and flexure [4, 10–12, 13–18]. The collected experiments from the available literature mainly featured an applied constant axial load and a moment induced by a lateral force. Only one study [4] considered RC columns axially loaded at various eccentricities covering cases of pure compression to pure bending. Additionally, the FRP jacket of this study was characterized by bidirectional
oriented fibers (0°/90°), and it was applied along the entire height of the specimens. The database was assembled under the following restrictions:

- RC columns of circular and non-circular cross-sections with one minimum dimension of the cross-section set at 300 mm (12 in.), with a maximum side-aspect ratio ($h/b$) of 2.0 in the case of non-circular cross-sections.
- FRP-wrapped RC columns with FRP fibers oriented perpendicular to the longitudinal axis of the column.
- Concrete-filled FRP tubes, RC columns part of environmental research studies, and RC columns that were damaged or loaded prior to strengthening were not considered in this study.
- RC columns exhibiting a shear type of failure were not included.

Table 1 presents a total of 13 RC columns of circular cross-section (including control and strengthened units) divided into 3 sets of experiments [11, 13, 16]. The information presented in the table is organized as follows: the first and second columns in the table show the reference to each experimental set and the specimen codes, respectively. The type of FRP jacket, when applicable, is given in the third column. Geometrical and material properties are shown in the columns (4) to (14) in this order: diameter of the cross-section ($D$), height of the specimen ($H$), ratio of the area of longitudinal steel reinforcement to the cross-sectional area of the member ($\rho_g$), volumetric ratio of transverse steel reinforcement to concrete core ($\rho_t$), volumetric ratio of FRP reinforcement ($\rho_f$), unconfined concrete compressive strength ($f'_c$), yield strength of longitudinal steel reinforcement ($f_y$), tensile modulus of elasticity of FRP ($E_f$), ultimate
tensile strength of FRP ($f_{tu}$), ultimate tensile strain of the FRP ($\varepsilon_{tu}$), and FRP nominal ply thickness ($t_f$). The last two columns in the table ((15) and (16)) report the maximum axial load ($P_a$ or $P_{max}$) and maximum moment ($M_{max}$), respectively. The specimens included in this table have unconfined concrete compressive strength varying from 36.5 MPa (5.29 ksi) to 44.8 MPa (6.5 ksi), and the level of constantly applied axial load in terms of $P_a/(A_g f'c)$ varies from 0.12 to 0.65.

Table 2 presents a total of 48 RC columns of non-circular cross-section including control and strengthened units featuring side-aspect ratios ($h/b$) of 1.0, 1.75, and 2.0. These experiments are divided into eight sets [4, 10, 12, 14–18]. The information corresponding to each set of experiments is presented as in Table 1, with the only difference being on the cross-section dimension parameters. Specifically, Table 2 shows the short side dimension ($b$), long side dimension ($h$), and the side-aspect ratio ($h/b$) of the non-circular cross-section instead of diameter ($D$) from Table 1. In the cases of specimens from three sets [15–17], the radius of the chamfered corner of the strengthened specimens was not reported in the source publications. Therefore, a minimum corner radius of 13 mm (0.5 in.) as recommended by the current ACI440.2R-02 [19] was assumed in these columns for the purpose of this study. The specimens included in this table have unconfined concrete compressive strength varying from 17.9 MPa (2.6 ksi) to 44.2 MPa (6.41 ksi), and the level of applied axial load in terms of $P_a/(A_g f'c)$ varies from 0.12 to 0.65.
4. P-M INTERACTION DIAGRAM

The analysis of the FRP-confined specimens is equivalent to that of customary RC columns [20, 21] with the primary difference being the use of a selected stress-strain model for FRP-confined concrete in the compression zone. The following assumptions are considered for the analysis: (a) plane sections remain plane, (b) the tensile strength of concrete is neglected, and (c) complete composite action is assumed between both steel reinforcement-concrete and FRP-concrete.

For ease of calculation, rather than a continuous curve, a conservative P-M diagram was constructed as a series of straight lines joining the axial load and moment values corresponding to five characteristic points (Fig. 1) [20]:

- **Point A:** uniform axial compressive strain of confined concrete $\varepsilon_{ccu}$ (or $\varepsilon_{cu}$ for the case of an unstrengthened cross-section).
- **Point B:** strain distribution corresponding to a maximum compressive strain $\varepsilon_{ccu}$ (or $\varepsilon_{cu}$) and zero strain at the layer of longitudinal steel reinforcement nearest to the tensile face.
- **Point C:** strain distribution corresponding to the balanced failure with a maximum compressive strain $\varepsilon_{ccu}$ (or $\varepsilon_{cu}$) and a yielding tensile strain $\varepsilon_{sy}$ at the layer of longitudinal steel reinforcement nearest to the tensile face.
- **Point D:** strain distribution corresponding to the limiting tension-controlled failure having a maximum compressive strain $\varepsilon_{ccu}$ (or $\varepsilon_{cu}$) and a tensile strain of 0.005 as per ACI318-05 [22] at the layer of longitudinal steel reinforcement nearest to the tensile face.
- **Point E:** point corresponding to the pure bending moment and zero axial force.
In the interaction diagram, point A represents the pure compression case (zero bending moment). For points B, C, and D, the position of the neutral axis \( c \) is directly computed by similar triangles in the strain distribution corresponding to each case (Fig. 1). Point E, as representing the pure bending moment case (zero axial force), the neutral axis position is obtained by following conventional RC beam theory. Assuming no contribution of the FRP-confinement for control and strengthened specimens, the procedure for computing \( M_{\text{max}} \) does not vary [5]. In the case of the experimental set by Chaallal and Shahawy [4] where specimens featured FRP jackets with bidirectional fibers, point E is computed accounting for the FRP in the longitudinal direction and its contribution to the flexural capacity according to ACI440.2R-02 [19].

The nominal axial load \( P_n \) corresponding to point A can be found either by integration of the stresses over the cross-section or by Eq. (1) \( (M_{n(A)} \) equals zero).

\[
P_{n(A)} = \left[ 0.85 f'_c \left( A_g - A_{st} \right) + f'_y A_{st} \right]
\]  

(1)

The nominal axial load \( P_n \) and the nominal bending moment \( M_n \) at points B, C, and D are found by integration of the stresses over the cross-section, and they are given by Eqs. (2) for the case of circular cross-sections and by Eqs. (3) for the case of non-circular cross-sections.

For design purposes, to replace the unconfined concrete stress distribution, ACI318-05 [22] allows the use of an equivalent rectangular stress block distribution with an average stress of \( \alpha_1 f'_c \) (\( \alpha_1 \) equals 0.85) and a depth of \( \beta_1 c \) (\( \beta_1 \) equals 0.65 for \( f'_c \geq 55 \) MPa [8 ksi], 0.85 for concrete with \( f'_c \leq 28 \) MPa [4 ksi], and 0.05 less for each 6.9 MPa)
[1 ksi] of $f'\sigma$ in excess of 28 MPa) for the calculation of the resultant compressive force in the cross-section. However, these values of $\alpha_j$ and $\beta_j$ are no longer valid for FRP-confined concrete stress-strain curve since they were based on experimental observations of unconfined concrete columns and prisms [20, 22].

$$P_{n(B,C,D)} = \int_0^c \left(\frac{D}{2}\right)^2 - \left(y - \left(\frac{D}{2} - d'\right)\right)^2 \int_0^y f'(y) dy + \sum A_{si} f_{si} \tag{2a}$$

$$M_{n(B,C,D)} = \int_0^c \left(\frac{D}{2}\right)^2 - \left(y - \left(\frac{D}{2} - d'\right)\right)^2 \left(\frac{D}{2} - c + y\right) f'(y) dy + \sum A_{si} f_{si} d_{si} \tag{2b}$$

$$P_{n(B,C,D)} = \int_0^c (b) f'(y) dy + \sum A_{si} f_{si} \tag{3a}$$

$$M_{n(B,C,D)} = \int_0^c (b) \left(\frac{h}{2} - c + y\right) f'(y) dy + \sum A_{si} f_{si} d_{si} \tag{3b}$$

In the expressions above, $c$ is the distance from the neutral axis position to the extreme compression fiber in the cross-section. $A_{si}$ and $f_{si}$ are the cross-sectional area and the normal stress, respectively, of the $i^{th}$ layer of longitudinal steel reinforcement, respectively. The parameter $d_{si}$ is the distance from the position of the $i^{th}$ layer of longitudinal steel reinforcement to the geometric centroid of the cross-section.

In Eqs. (2) and (3), $y$ is the variable of integration within the compression zone.

The concrete stress $f_c$ corresponds to the model by Lam and Teng [23, 24]. This model
was selected based on the evaluation of FRP-confined models with a database composed of solely RC columns of a minimum side dimension of 300 mm (12 in.) and subjected to pure axial compressive loading [25]. The model by Lam and Teng as shown in Fig. 2 proved to be the most suitable for predicting the maximum confined compressive strength and strain for RC columns of circular and non-circular cross-sections.

The stress-strain curve of this model is composed of a parabolic first portion followed by a linear second portion (Fig. 2). The curve and the line meet smoothly at a transition strain $\varepsilon_t$. The linear second portion ends at a point where both the maximum compressive strength $f_{cc}'$ and the ultimate axial strain of confined concrete $\varepsilon_{ccu}$ are reached. Based on experimental observations of small-plain concrete specimens with sufficient amounts of FRP confinement, Lam and Teng suggested a minimum ratio of FRP confinement pressure to unconfined concrete compressive strength $f_l/f_{cc}'$ of approximately 0.08 to ensure an ascending second branch in the stress-strain curve. This limitation was later confirmed for circular cross-sections by Spoelstra and Monti [26] using their analytical model.

In the case of unconfined concrete, the model by Lam and Teng reduces to a stress-strain curve composed of an initial parabola followed by a horizontal straight line, which has been adopted by design codes such as BS 8110 and Eurocode 2 [23]. This model is used in this study to compute the stress-strain curve of the FRP-confined concrete. It is given by the following expressions:

\[
 f_c = \begin{cases} 
 E_c \varepsilon_c - \left(\frac{E_c - E_s}{4f_c'}\right)^2(\varepsilon')^2 & 0 \leq \varepsilon_c \leq \varepsilon_t' \\
 f_c' + E_s \varepsilon_c & \varepsilon_t' \leq \varepsilon_c \leq \varepsilon_{ccu} 
\end{cases} 
\]  

\[ (4a) \]
\[ \varepsilon'_t = \frac{2f'_c}{E_c - E_2} \]  

(4b)

\[ E_2 = \frac{f'_c - f'_c}{\varepsilon_{ccu}} \]  

(4c)

Where, \( f_c \) and \( \varepsilon_c \) are the axial stress and the axial strain of confined concrete, respectively. \( E_c \) is the elastic modulus of unconfined concrete, \( \varepsilon'_t \) is the transition strain, \( E_2 \) is the slope of the second linear portion, and \( \varepsilon_{ccu} \) is the ultimate axial strain of confined concrete.

The maximum FRP-confined concrete compressive strength is given by Eq. (5):

\[ f'_{cc} = f'_c + 3.3\kappa_a f_t \]  

(5)

The parameter \( \kappa_a \) is a geometry efficiency factor and is discussed later in the text. The FRP confining pressure \( (f_t) \) in the case of circular cross-sections is determined by equilibrium and strain compatibility. However, in the case of non-circular cross-sections, the diameter \( D \) is replaced by an equivalent diameter, which consists of the diagonal of the rectangular cross-section (Eq. (6)).

\[ f_t = \begin{cases} 
\frac{2nt_f E_f \varepsilon_{fc}}{D} & \text{Circular Cross-section} \\
\frac{2nt_f E_f \varepsilon_{fc}}{\sqrt{b^2 + h^2}} & \text{Non-circular Cross-section}
\end{cases} \]  

(6)
The effective strain $\varepsilon_{fe}$ in Eq. (6) is computed as the product of an efficiency factor $\kappa_e$ and the ultimate tensile strain $\varepsilon_{fu}$. The FRP strain efficiency factor $\kappa_e$ to account for the difference between the actual rupture strain observed in FRP-confined concrete specimens and the FRP material rupture strain determined from tensile coupon testing is considered in the determination of the effective strain in FRP reinforcement attained at failure $\varepsilon_{fe}$ (i.e., $\varepsilon_{fe} = \kappa_e \varepsilon_{fu}$). Based on experimental calibration with CFRP-wrapped, small-scale, plain concrete specimens, an average value of 0.586 was computed for $\kappa_e$ by Lam and Teng [23]. Similarly, Carey and Harries [27] computed a value of 0.58, while experimental tests on medium- and large-scale columns resulted in values of 0.57 and 0.61, respectively [28]. In this study, for practical design applications, the parameter $\kappa_e$ is taken equal to 0.55.

Based on the work by Priestley et al. [29] and as recommended by ACI Committee 440 [19], the effective strain in the FRP at failure $\varepsilon_{fe}$ in members subjected to combined axial compression and bending moment is limited to the minimum value between 0.004 and $\kappa_e \varepsilon_{fu}$ in order to ensure the shear integrity of the confined concrete.

The ultimate axial strain of the FRP-confined concrete compressive stress-strain curve is given as follows:

$$\varepsilon_{ceu} = \varepsilon_c' \left( 1.5 + 12 \kappa_e \frac{f_t}{f_c} \left( \frac{\varepsilon_{fe}}{\varepsilon_c'} \right)^{0.45} \right) \leq 0.01$$

(7)

Where the axial strain at the compressive strength of unconfined concrete $\varepsilon_c'$ is taken equal to 0.002. The model of Lam and Teng [23, 24] originally features a value of
1.75 instead of 1.5, which for the case of unconfined concrete yields an ultimate axial strain $\varepsilon_{cu}$ equal to 0.0035. This change to 1.5 is necessary to limit the axial strain of unconfined concrete $\varepsilon_{cu}$ to 0.003, a value consistent with ACI318-05 [22].

Additionally, based on recommendations from the Concrete Society in the Technical Report 55 [30], a limiting value of ultimate axial strain of 0.01 in the case of confined concrete is introduced only in members under pure compression to prevent excessive cracking and the resulting loss of concrete integrity. When this limit is applicable, the corresponding maximum value of $f'_{cc}$ is recalculated from the stress-strain curve.

The efficiency factors $\kappa_a$ (Eq. (5)) and $\kappa_b$ (Eq. (7)) can be computed according to Eqs. (8) and (9) as they account for the geometry of the cross-section. In the case of circular cross-sections, they are taken as 1.0, and in the case of non-circular cross-sections, they depend on two parameters: the effectively confined area ratio $A_e/A_c$ and the side-aspect ratio $h/b$. These factors are given by the following expressions:

$$\kappa_a = \frac{A_c}{A_e} \left(\frac{b}{h}\right)^2$$  \hspace{1cm} (8)

$$\kappa_b = \frac{A_c}{A_e} \left(\frac{h}{b}\right)^{0.5}$$  \hspace{1cm} (9)
Where:

\[
\frac{A_e}{A_g} = \frac{1 - \left(\frac{b}{h} (h - 2r)^2 + \frac{h}{b} (b - 2r)^2\right)}{1 - \rho_g} \left(\frac{3 A_g}{\rho_g} - \rho_g\right)
\]  \hspace{1cm} (10)

Simplified theoretical interaction diagrams composed of a series of straight lines connecting points A, B, C, D, and E using the equations and procedures described above were built for each set of experiments presented in Tables 1 and 2. These diagrams along with the experimental results, are shown in Figs. 3 and 4 for the cases of circular and non-circular RC columns. No environmental (\(C_e\)) and strength reduction factors (\(\phi, \psi_f\)) were considered in these interaction diagrams.

To express the diagrams independently of the different cross-sectional area sizes (\(A_g\)) and unconfined concrete compressive strength (\(f'_c\)), the diagrams are shown as non-dimensional. This is done by dividing the nominal axial load values \(P_n\) by \((A_g f'_c)\), and dividing the nominal moment values \(M_n\) by \((A_g h f'_c)\). In each of the plots, the legend indicates the percentage quantity corresponding to the FRP volumetric ratio used in each of the specimens.

Every figure shows the analytical curve and experimental data at the same scale: 0 to 0.5 for the x-axis and 0 to 2.0 for the y-axis. All plots are drawn with x-axis equal to \(M_n/(A_g h f'_c)\) and y-axis equal to \(P_n/(A_g f'_c)\).

Due to the difficulty of conducting experiments with real-size cross-sections, Figs. 3 and 4 report a limited number of data points that fall in the compression-controlled region: 2 (from a total of 13 units) and 13 (from a total of 48 units) for the
cases of circular and non-circular specimens, respectively. From this total, for only one data point among the circular specimens [13], its corresponding theoretical P-M diagram appears to be not conservative. However, the original source of publication [13] notes that such specimen experienced a premature failure as a result of the stress concentrations on the jacket induced by one reinforcing bar used to affix measurement instrumentation. For all other cases, the interaction diagrams built on the Lam and Teng [23, 24] model for the FRP-confined concrete stress-strain curve appear to be conservative with respect to the experimental data points.

5. PROPOSED DESIGN METHODOLOGY

Strength enhancement can only be considered when the applied ultimate axial force and bending moment ($P_u$ and $M_u$) are such that a point having these coordinates falls above the line connecting the origin and the balanced point in the interaction diagram for the unconfined member (compression-controlled region) (Fig. 5). This limitation stems from the consideration that the enhancement is only of significance in members where compression failure is the controlling mode [3].

For simplicity, the portion of the unconfined and confined interaction diagrams corresponding to compression-controlled failure can be reduced to two bilinear curves passing through the previously defined points A, B, and C (Fig. 5). Eq. (1) can be used to compute the $P_n$ corresponding to point A of a member with any cross-sectional shape ($M_{n(A)}$ equals zero). The coordinates of points B and C can be computed using Eqs. (2) and (3) for the case of members of circular and non-circular cross-sections, respectively.
To provide a closed form solution, this paper focuses only on the case of members with rectangular cross-sections. Substituting the stress-strain relationship for FRP-confined concrete (Eqs. (4)) in Eqs. (3) and expressing the axial compressive strain \( \varepsilon_c \) at any point in the compression region in terms of the integrating variable \( y \) (similar triangles) (Fig. 6), Eqs. (11) and (12) are obtained for the axial loads and moments at points B and C, respectively:

\[
P_{n(B,C)} = \int_{0}^{y_t} \left[ E_c \left( \frac{\varepsilon_{cu}}{c} y \right) - \frac{(E_c - E_z)^2}{4f'_c} \left( \frac{\varepsilon_{cu}}{c} y \right)^2 \right] b dy + \int_{y_t}^{c} \left[ f'_c + E_z \left( \frac{\varepsilon_{cu}}{c} y \right) \right] b dy + \sum A_n f_{si}
\]

(11)

\[
M_{n(B,C)} = \int_{0}^{y_t} \left[ E_c \left( \frac{\varepsilon_{cu}}{c} y \right) - \frac{(E_c - E_z)^2}{4f'_c} \left( \frac{\varepsilon_{cu}}{c} y \right)^2 \right] b \left( \frac{h}{2} - c + y \right) dy + \int_{y_t}^{c} \left[ f'_c + E_z \left( \frac{\varepsilon_{cu}}{c} y \right) \right] b \left( \frac{h}{2} - c + y \right) dy + \sum A_n f_{si} d_{si}
\]

(12)

In the expressions above, \( c \) represents the distance from the extreme compression fiber to the neutral axis and is given by Eq. (13). The parameter \( y \) represents the vertical coordinate within the compression region measured from the neutral axis position, and it corresponds to the transition strain \( \varepsilon'_t \) (Eq. (14)) (Fig. 6).
\[
\begin{aligned}
    c &= \begin{cases} 
        d & \text{for point B} \\
        d \frac{\varepsilon_{\text{ccu}}}{\varepsilon_{\text{sy}} + \varepsilon_{\text{ccu}}} & \text{for point C}
    \end{cases} \\
    y_i &= c \frac{\varepsilon_i'}{\varepsilon_{\text{ccu}}}
\end{aligned}
\]

After integrating and reordering terms, Eqs. (11) and (12) can be reduced to expressions (15) and (16), respectively, where the coefficients A, B, C, D, E, F, G, H, and I, are given by Eqs. (17).

\[
P_{n(B,C)} = \left[ A(y_i)^3 + B(y_i)^2 + C(y_i) + D \right] + \sum A_{sl} f_{sl}
\]

\[
M_{n(B,C)} = \left[ E(y_i)^4 + F(y_i)^3 + G(y_i)^2 + H(y_i) + I \right] + \sum A_{sl} f_{sl} d_{sl}
\]

\[
A = \frac{-b(E_c - E_2)^2}{12f_c'} \left( \frac{\varepsilon_{\text{ccu}}}{c} \right)^2
\]

\[
B = \frac{b(E_c - E_2)}{2} \left( \frac{\varepsilon_{\text{ccu}}}{c} \right)
\]

\[
C = -bf_c'
\]
\[
D = bc f'_c + \frac{bc E_2}{2} (\varepsilon_{ceu}) \quad (17d)
\]

\[
E = \frac{-b(E_c - E_2)^2}{16 f'_c} \left( \frac{\varepsilon_{ceu}}{c} \right)^2 \quad (17e)
\]

\[
F = b \left( c - \frac{h}{2} \right) \left( \frac{E_c - E_2}{12 f'_c} \right) \left( \frac{\varepsilon_{ceu}}{c} \right)^2 + \frac{b(E_c - E_2)}{3} \left( \frac{\varepsilon_{ceu}}{c} \right) \quad (17f)
\]

\[
G = - \left( \frac{b}{2} f'_c + b \left( c - \frac{h}{2} \right) \frac{E_c - E_2}{2} \left( \frac{\varepsilon_{ceu}}{c} \right) \right) \quad (17g)
\]

\[
H = bf'_c \left( c - \frac{h}{2} \right) \quad (17h)
\]

\[
I = \frac{bc^2}{2} f'_c - bc f'_c \left( c - \frac{h}{2} \right) + \frac{bc^2 E_2}{3} \left( \frac{\varepsilon_{ceu}}{c} \right) - \frac{bc E_2}{2} \left( c - \frac{h}{2} \right) \left( \frac{\varepsilon_{ceu}}{c} \right) \quad (17i)
\]

The values of \( f_{si} \) are calculated by similar triangles from the strain distribution corresponding to points B and C, and depending on the neutral axis position \( c \), the sign of \( f_{si} \) will be positive for compression and negative for tension.

According to ACI design philosophy, the nominal axial load capacity under pure compression loading condition (point A in the interaction diagram) of an RC column featuring ties as steel transverse reinforcement is reduced to 80 percent to account for a
possible accidental eccentricity [21]. In the example application presented in Appendix A and Fig. 5, this point is denoted as A'.

$$\phi P_{n(A')} = \phi 0.8\left[0.85 f_{ce} \left(A_g - A_{st}\right) + f_y A_{st}\right]$$  \hspace{1cm} (18)

For a tied RC column, the strength reduction factors $\phi$ as recommended by ACI318-05 [22] must also be considered. Since the portion of the interaction diagram corresponds to compression-controlled members, the value of $\phi$ is 0.65 (Fig. 5).

The proposed design method consists of closed form equations for the determination of the compression-controlled region of a P-M diagram for the case of a rectangular FRP-wrapped RC column. This portion of the interaction diagram is conservatively represented by two bilinear curves joining the previously defined points A', B, and C. Based on ACI design philosophy, the vertical coordinate (axial force) of point A' can be found with Eq. (18) (bending moment is zero). The coordinates for points B and C can be obtained with Eqs. (15) and (16). The contribution of concrete within the compression region of the cross-section is represented by polynomials of third and fourth order for the case of the nominal axial capacity (Eq. (15)) and the nominal flexural capacity (Eq. (16)), respectively. These algebraic expressions result from mathematical substitution and re-arrangement of terms resulting from the integrals given in Eqs. (11) and (12).

A flow-chart illustrating the application of the proposed methodology for design purposes is shown in Fig. 7, and a practical example application is presented in the appendix.
6. SUMMARY AND CONCLUSIONS

This paper responds to the need for a simple and yet rational design approach to FRP-wrapped RC columns subjected to combined axial force and bending. The proposed design approach that follows ACI principles requires further validation with experimental test results of RC columns having geometry and properties similar to the ones found in practice.

The design method presented in the paper is based on the construction of a simplified interaction diagram for FRP-wrapped RC columns stemming from a process that included the four steps listed below:

(a) Collection of experimental data on FRP-wrapped circular and non-circular RC columns with minimum diameter or side dimension of about 300 mm (12 in.), maximum side-aspect ratio of 2, and featuring FRP fiber orientation perpendicular to the longitudinal axis of the column.

(b) Construction of a theoretical P-M diagram based on principles of equilibrium and strain compatibility, equivalent to that of conventional RC columns but considering the stress-strain model for FRP-confined concrete developed by Lam and Teng.

(c) Validation of the theoretical P-M diagrams with the experimental results.

(d) Simplification of the P-M diagram to a series of straight lines joining characteristic points only in the compression-controlled region of the diagram. This limitation recognizes that capacity enhancement is significant only if the combined external actions are such that the FRP wrapping influences the properties of the concrete.
The paper illustrates the proposed design method for the case of a RC column of rectangular cross-section as an application example (See the appendix).

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APPENDIX: EXAMPLE APPLICATION

Given:

A 610×610 mm (24×24 in.) RC column is subjected to an ultimate axial compressive load $P_u = 8451$ kN (1900 kip) and to an ultimate bending moment $M_u = 515$ kN·m (380 kip·ft) ($e = 0.1h$). It is required to increase the demand by 30 percent at constant eccentricity ($P_u = 10,987$ kN, $M_u = 670$ kN·m). Concrete and steel reinforcement material properties as well as details of the cross-section of the column are shown in Fig. A.1. A method of strengthening the column is sought.

Since the column is located in an interior environment, and CFRP material will be used, according to Table 8.1 in the ACI440.2R-02 guide [19], an environmental-reduction factor $C_E$ equal to 0.95 will be used. Additionally, the application of a reduction factor $\psi_f$ equal to 0.95 as recommended by ACI440.2R-02 [19] is taken into consideration. However, in this example application, instead of applying the latter factor on the resulting confined concrete compressive strength ($f'_{cc}$), it is applied to the sole
contribution of the FRP (i.e., the confinement pressure \( f_i \)) as is the approach that the revised version of the guide is considering.

The properties of the FRP system, as reported by the manufacturer, are shown in Table A. 1.

Solution:

**Step 1:** Determine the simplified curve for the unstrengthened column \((n = 0\) plies).

Points A', B, and C can be obtained by well-known procedures:

\[
\phi P_{n(A')} = 9281\text{kN} \; ; \; \phi M_{n(A')} = 0\text{kN} \cdot \text{m} \\
\phi P_{n(B)} = 8266\text{kN} \; ; \; \phi M_{n(B)} = 874\text{kN} \cdot \text{m} \\
\phi P_{n(C)} = 4127\text{kN} \; ; \; \phi M_{n(C)} = 1198\text{kN} \cdot \text{m}
\]

**Step 2:** Compute the design FRP material properties.

\[
f_{fu} = C_E f_{fu}^* = 0.95 \times 3792\text{MPa} = 3602\text{MPa}
\]

\[
\varepsilon_{fu} = C_E \varepsilon_{fu}^* = 0.95 \times 0.0167\text{mm/mm} = 0.0159\text{mm/mm}
\]

**Step 3:** Determine the simplified curve for the strengthened column.

A wrapping system composed of six plies will be the starting point to construct the curve A-B-C and then be compared with the position of the required \( P_u \) and \( M_u \).

Points A’, B, and C can be computed using Eqs. (18), (15), and (16), where the coefficients A, B, C, D, E, F, G, H, and I are given by Eqs. (17).

**Point A':**
Design axial capacity:

\[ \phi P_{n(A')} = \phi 0.8 \left[ 0.85 f'_c \left( A_g - A_{st} \right) + f_y A_{st} \right] \]

\[ \phi P_{n(A')} = 0.65 \times 0.8 \left[ 0.85 \times 56.96 \text{MPa} \times \left( 37,612 - 9,832 \text{mm}^2 \right) + 414 \text{MPa} \times 9832 \text{mm}^2 \right] \]

\[ \phi P_{n(A')} = 11,224 \text{kN} \]

Where:

\[ f'_c = f'_c + 3.3 \kappa_a f_i = 44.8 \text{MPa} + 3.3 \times 0.425 \times 8.67 \text{MPa} = 56.96 \text{MPa} \]

\[ \kappa_a = \frac{A_c}{A_c} \left( \frac{b}{h} \right)^2 = (0.425)^2 = 0.425 \]

\[ \frac{A_c}{A_c} = \frac{1 - \left[ (b/h)(h-2r)^2 + (h/b)(b-2r)^2 \right] / (3A_g) - \rho_g}{1 - \rho_g} \]

\[ \frac{A_c}{A_c} = \frac{1 - \left[ 1 (610 - 2 \times 25 \text{mm})^2 + (1)(610 - 2 \times 25 \text{mm})^2 \right]}{1 - 2.65 \times 10^{-2}} = 0.425 \]

\[ f_i = \frac{\psi \varepsilon_{te} E_{te} \varepsilon_{te}}{\sqrt{b^2 + h^2}} \]

\[ f_i = \frac{0.95 \times 2 \times 6 \times 0.33 \text{mm} \times 227,527 \text{MPa} \times \left( 0.55 \times 0.0159 \frac{\text{mm}}{\text{mm}} \right)}{\sqrt{2 \times (610 \text{ mm})^2}} = 8.67 \text{MPa} \]

Point B:

Design axial capacity:

\[ \phi P_{n(B)} = \phi \left[ \left( A y_t^3 + B y_t^2 + C y_t + D \right) + \sum A_{si} f_{si} \right] \]
\[ \phi P_{n(B)} = 0.65 \left[ -6.003 \times 10^{-5} \frac{\text{kN}}{\text{mm}^3} (389 \text{mm})^3 + 70.14 \text{MPa} (389 \text{mm})^2 
- 27.32 \frac{\text{kN}}{\text{mm}} (389 \text{mm}) + 16.215 \text{kN} 
+ 3277 \text{mm}^2 (414 \text{MPa}) + 1639 \text{mm}^2 (414 \text{MPa}) + 1639 \text{mm}^2 (257 \text{MPa}) \right] \]

\[ \phi P_{n(B)} = 9829 \text{kN} \]

Where:

\[ A = \frac{-b(E_c - E_2)^2}{12f_c'} \left( \frac{\varepsilon_{\text{ccu}}}{c} \right)^2 \]

\[ A = \frac{-610 \text{mm} (31,685 - 1315 \text{MPa})^2}{12 \times 44.8 \text{MPa}} \left( \frac{4.23 \times 10^{-3} \text{ mm/mm}}{559 \text{mm}} \right)^2 = -6.003 \times 10^{-5} \frac{\text{kN}}{\text{mm}^3} \]

\[ B = \frac{b(E_c - E_2)}{2} \left( \frac{\varepsilon_{\text{ccu}}}{c} \right) \]

\[ B = \frac{610 \text{mm} (31,685 - 1315 \text{MPa})}{2} \left( \frac{4.23 \times 10^{-3} \text{ mm/mm}}{559 \text{mm}} \right) = 70.14 \text{MPa} \]

\[ C = -bf_c' \]

\[ C = -610 \text{mm} \times 44.8 \text{MPa} = -27.32 \text{ kN/mm} \]

\[ D = bcf_c' + \frac{bcE_2}{2} \left( \varepsilon_{\text{ccu}} \right) \]

\[ D = 610 \text{mm} \times 559 \text{mm} \times 44.8 \text{MPa} + \frac{610 \text{mm} \times 559 \text{mm} \times 1,315 \text{MPa}}{2} \left( \frac{4.23 \times 10^{-3} \text{ mm}}{\text{mm}} \right) \]

\[ D = 16,215 \text{kN} \]

For the calculation of the coefficients it is necessary to compute key parameters from the stress-strain model:
\[ y_t = c \frac{\varepsilon'_t}{\varepsilon_{ccu}} = \frac{559 \text{mm} \times 2.95 \times 10^{-3} \text{mm/mm}}{4.23 \times 10^{-3} \text{mm/mm}} = 389 \text{mm} \]

c = 559 \text{mm}

\[ \varepsilon'_t = \frac{2f'_c}{E_c - E_2} = \frac{2 \times 44.8 \text{MPa}}{(31,685 - 1,315 \text{MPa})} = 2.95 \times 10^{-3} \text{mm/mm} \]

\[ E_2 = \frac{f'_{cc} - f'_c}{\varepsilon_{ccu}} = \frac{(50.38 - 44.8 \text{MPa})}{4.23 \times 10^{-3} \text{mm/mm}} = 1315 \text{MPa} \]

\[ f'_{cc} = f'_c + 3.3 \kappa_a f_i \]

\[ f'_{cc} = 44.8 \text{MPa} + 3.3 (0.425)(3.97 \text{MPa}) = 50.38 \text{MPa} \]

\[ \varepsilon_{ccu} = \varepsilon'_c \left( 1.5 + 12 \kappa_b \frac{f_i}{f'_c} \left( \frac{\varepsilon_{fe}}{\varepsilon'_{c}} \right)^{0.45} \right) \]

\[ \varepsilon_{ccu} = 0.002 \text{mm/mm} \left( 1.5 + 12 \times 0.425 \left( \frac{3.97 \text{MPa}}{44.8 \text{MPa}} \right) \left( \frac{0.004 \text{mm/mm}}{0.002 \text{mm/mm}} \right)^{0.45} \right) \]

\[ \varepsilon_{ccu} = 4.23 \times 10^{-3} \text{mm/mm} \]

\[ \kappa_a = 0.425 \]

\[ \kappa_b = \frac{A_c}{A_c} \left( \frac{h}{b} \right)^{0.5} = 0.425 (1)^{0.5} = 0.425 \]

\[ f_i = \frac{\psi f \sqrt{16 t E_f \varepsilon_{fe}}}{\sqrt{b^2 + h^2}} \]

\[ f_i = \frac{0.95 \times 2 \times 6 \times 0.33 \text{mm} \times 227,527 \text{MPa} \times 0.004 \text{mm/mm}}{\sqrt{2 \times (610 \text{ mm})^2}} = 3.97 \text{MPa} \]

\[ \varepsilon_{fe} = \min \left[ 0.004, (\kappa_e \varepsilon_{fu} = 0.55 \times 0.159 \text{mm/mm}) \right] = 0.004 \text{mm/mm} \]

Checking the minimum confinement ratio:
The strains in each layer of steel are determined by similar triangles in the strain distribution (Fig. 7). The corresponding stresses are then given as follows:

\[ f_{s1} = \varepsilon_{s1}E_s = 0.0038 \text{mm/mm} \times 200,000 \text{MPa} = 414 \text{MPa} \]

\[ f_{s2} = \varepsilon_{s2}E_s = 0.0026 \text{mm/mm} \times 200,000 \text{MPa} = 414 \text{MPa} \]

\[ f_{s3} = \varepsilon_{s3}E_s = 0.0013 \text{mm/mm} \times 200,000 \text{MPa} = 257 \text{MPa} \]

\[ f_{s4} = \varepsilon_{s4}E_s = 0 \text{mm/mm} \times 200,000 \text{MPa} = 0 \text{MPa} \]

Design bending moment:

\[ \Phi M_{n(B)} = \Phi \left[ (E(y_i)^4 + F(y_i)^3 + G(y_i)^2 + H(y_i) + I) + \sum A_s f_s d_{si} \right] \]

\[ \Phi M_{n(B)} = 0.65 \left[ -4.5 \times 10^{-5} \frac{\text{kN}}{\text{mm}^3} (389 \text{mm})^4 + 62 \text{MPa} (389 \text{mm})^3 \right. \]
\[ \left. -31.48 \frac{\text{kN}}{\text{mm}} (389 \text{mm})^2 + 6939 \text{kN} (389 \text{mm}) + 500,175 \text{kN.mm} \right] \]
\[ + 3277 \text{mm}^2 (414 \text{MPa}) (254 \text{mm}) + 1639 \text{mm}^2 (414 \text{MPa}) (85 \text{mm}) \]
\[ -1639 \text{mm}^2 (257 \text{MPa}) (85 \text{mm}) \]

\[ \Phi M_{n(B)} = 924 \text{kN.m} \]

Where:

\[ E = \frac{-b (E_c - E_2)^2}{16f'_c} \left( \frac{\epsilon_{	ext{ccu}}}{c} \right)^2 \]

\[ E = \frac{-610 \text{mm} (31,685 - 1315 \text{MPa})^2}{16 \times 44.8 \text{MPa}} \left( \frac{4.23 \times 10^{-3} \text{mm/mm}}{559 \text{mm}} \right)^2 = -4.5 \times 10^{-5} \text{ kN/mm}^3 \]

\[ F = b \left( c - \frac{h}{2} \right) \left( \frac{E_c - E_2}{12f'_c} \right) \left( \frac{\epsilon_{	ext{ccu}}}{c} \right)^2 + b \left( \frac{E_c - E_2}{3} \right) \left( \frac{\epsilon_{	ext{ccu}}}{c} \right) \]
\[
F = 610 \text{mm} \left( 559 - 305 \text{mm} \right) \left( 31,685 - 1315 \text{MPa} \right)^2 \left( \frac{4.23 \times 10^{-3} \text{ mm/mm}}{559 \text{mm}} \right)^2 + \\
\frac{610 \text{mm} \left( 31,685 - 1315 \text{MPa} \right)}{3} \left( \frac{4.23 \times 10^{-3} \text{ mm/mm}}{559 \text{mm}} \right) = 62 \text{MPa}
\]

\[
G = -\left( \frac{b f'}{2} + b \left( c - \frac{h}{2} \right) \frac{\left( E_c - E_2 \right)}{2} \left( \varepsilon_{cu} \right) \right)
\]

\[
G = -\left[ 44.8 \text{MPa} \times 305 \text{mm} + \\
610 \text{mm} \left( 559 - 305 \text{mm} \right) \left( \frac{31,685 - 1315 \text{MPa}}{2} \right) \left( \frac{4.23 \times 10^{-3} \text{ mm/mm}}{559 \text{mm}} \right) \right]
\]

\[
G = -31.48 \text{kN/mm}
\]

\[
H = bf' \left( c - \frac{h}{2} \right)
\]

\[
H = 44.8 \text{MPa} \times 610 \text{mm} \left( 559 - 305 \text{mm} \right) = 6939 \text{kN}
\]

\[
I = \frac{bc^2}{2} f' - bcf' \left( c - \frac{h}{2} \right) + \frac{bc^2 E_2}{3} \left( \varepsilon_{cu} \right) - \frac{bc E_2}{2} \left( c - \frac{h}{2} \right) \left( \varepsilon_{cu} \right)
\]

\[
I = 44.8 \text{MPa} \times 610 \text{mm} \times \frac{\left( 559 \text{mm} \right)^2}{2} - 44.8 \text{MPa} \left( 559 - 305 \text{mm} \right) \left( 559 \text{mm} \right) \left( 610 \text{mm} \right) + \\
1,315 \text{MPa} \times 610 \text{mm} \times \frac{\left( 559 \text{mm} \right)^2}{3} \left( 4.23 \times 10^{-3} \frac{\text{ mm}}{\text{ mm}} \right) - \\
1,315 \text{MPa} \times 610 \text{mm} \times \frac{559 \text{mm}}{2} \left( 559 - 305 \text{mm} \right) \left( 4.23 \times 10^{-3} \frac{\text{ mm}}{\text{ mm}} \right) = 500,175 \text{kN} \cdot \text{mm}
\]

The distances from each layer of steel reinforcement to the geometric centroid of the cross-section are shown below:

\[
d_{s1} = 254 \text{mm}
\]

\[
d_{s2} = d_{s3} = 85 \text{mm}
\]
Point C:

Following the same procedure as for point B, the design axial capacity is as follows:

\[
\phi P_{n(c)} = 5870 \text{kN}
\]

Where:

\[
A = -1.33 \times 10^{-4} \text{kN/mm}^3
\]

\[
B = 104.4 \text{MPa}
\]

\[
C = -27.32 \text{kN/mm}
\]

\[
D = 10,892 \text{kN}
\]

For the calculation of the coefficients it is necessary to compute key parameters from the stress-strain model:

\[
y_t = 262 \text{mm}
\]

\[
c = 375 \text{mm}
\]

The strains in each layer of steel are determined by similar triangles in the strain distribution (Fig. 7). The corresponding stresses are then given:

\[
f_{s1} = \varepsilon_{s1}E_s = 0.0037 \text{ mm/mm} \times 200,000 \text{MPa} = 414 \text{MPa}
\]

\[
f_{s2} = \varepsilon_{s2}E_s = 0.0018 \text{ mm/mm} \times 200,000 \text{MPa} = 350.2 \text{MPa}
\]

\[
f_{s3} = \varepsilon_{s3}E_s = -1.59 \times 10^{-4} \text{ mm/mm} \times 200,000 \text{MPa} = -31.8 \text{MPa}
\]

\[
f_{s4} = \varepsilon_{s4}E_s = -0.0021 \text{ mm/mm} \times 200,000 \text{MPa} = -414 \text{MPa}
\]

The design bending moment: \(\phi M_{n(c)} = 1345 \text{kN} \cdot \text{m}\)

Where:

\[
E = -9.98 \times 10^{-5} \text{kN/mm}^3
\]
F = 79MPa
G = −21.03kN/mm
H = 1,928kN
I = 1,315,459kN · mm

Note:
The designer should bear in mind that for the case of pure compression, the
effective strain in the FRP \( \varepsilon_{fe} \) is limited by \( \kappa_c \varepsilon_{fu} \), and in the case of combined axial and
bending by \( \varepsilon_{fe} = \text{minimum}(0.004, \kappa_c \varepsilon_{fu}) \).

Step 4: Comparison of simplified partial interaction diagram with required \( P_u \) and
\( M_u \).

Table A. 2 summarizes the axial and bending design capacities for the
unstrengthened and strengthened members at points A, A’, B, and C. These points are
plotted in Fig. A.2.

NOTATIONS
The following symbols are used in this paper:

\( A_c \) Cross-sectional area of concrete in column = \( A_g (1 - \rho_g) \)

\( A_e \) Effectively confined area = \( A_g - \left( h - 2r \right)^2 + \left( b - 2r \right)^2 \right) / 3 - \rho_g A_g \)

\( A_g \) Total cross-sectional area

\( A_s \) Area of steel reinforcement = \( A_g \rho_g \)

\( A_{si} \) Cross-sectional area of “\( i \)th” layer of longitudinal steel reinforcement
b  Short side dimension of a non-circular cross-section

C_E  Environmental reduction factor

d_{si}  Distance from position of "i"th layer of longitudinal steel reinforcement to geometric centroid of the cross-section

D  Diameter of circular cross-section

Diameter of equivalent circular column for non-circular cross-sections = \sqrt{b^2 + h^2}

e  Eccentricity of axial load

E_2  Slope of linear portion of confined stress-strain curve = (f'_{cc} - f'_c)/\varepsilon_{cu}

E_c  Initial modulus of elasticity of concrete

E_f  Tensile modulus of elasticity of FRP

f'_c  Characteristic concrete compressive strength determined from standard cylinder

f'_{cc}  Maximum compressive strength of confined concrete

f'_fu  Ultimate tensile strength of FRP

f_{fu}  Design ultimate tensile strength of FRP = C_E f'_fu

f_l  Confinement pressure due to FRP jacket

f_{si}  Stress in "i"th layer of longitudinal steel reinforcement

f_y  Yield strength of longitudinal steel reinforcement

H  Height of column

h  Long side dimension of a non-circular cross-section

M_{max}  Maximum bending moment

M_n  Nominal bending moment capacity

n  Number of FRP plies composing the jacket
\( \text{P}_a \quad \text{Constantly applied axial load} \\
\text{P}_{\text{max}} \quad \text{Maximum applied axial load} \\
\text{P}_n \quad \text{Nominal axial load capacity of a RC column} \\
r \quad \text{Corner radius of non-circular cross-sections} \\
t_f \quad \text{FRP nominal ply thickness} \\
\alpha_1 \quad \text{Factor relating the uniform compressive stress of the equivalent rectangular block} \\
in the compression zone to the compressive strength \( f'_{c} = 0.85 \) \\
\beta_1 \quad \text{Factor relating depth of equivalent rectangular compressive stress block to neutral} \\
axis depth \\
\varepsilon'_c \quad \text{Axial compressive strain corresponding to} \ f'_c = 0.002 \text{ mm/mm} \\
\varepsilon_{\text{ccu}} \quad \text{Ultimate axial compressive strain of confined concrete} \\
\varepsilon_{\text{cu}} \quad \text{Ultimate axial compressive strain of unconfined concrete} = 0.003 \text{ mm/mm} \\
\varepsilon_{\text{fe}} \quad \text{FRP effective strain} \ (\text{strain level reached at failure}) \\
\varepsilon^{*}_{\text{fu}} \quad \text{Ultimate tensile strain of the FRP} \\
\varepsilon_{\text{fu}} \quad \text{Design ultimate tensile strain of the FRP} = C_{\varepsilon}\varepsilon^{*}_{\text{fu}} \\
\varepsilon_{\text{sy}} \quad \text{Strain corresponding to the yield strength of steel reinforcement} \\
\varepsilon'_t \quad \text{Transition strain in stress-strain curve of FRP-confined concrete. It corresponds} \\
to point of change between initial parabola and straight line} = 2f'_c/(E_c - E_2) \\
\phi \quad \text{Strength reduction factor} \\
\kappa_a \quad \text{Efficiency factor for FRP reinforcement in the determination of} \ f'_{\text{cc}} \ (\text{based on the} \\
\text{geometry of the cross-section})
κ₀ Efficiency factor for FRP reinforcement in the determination of $\varepsilon_{ccu}$ (based on the geometry of the cross-section)

κₑ Efficiency factor for FRP strain proposed to account for the difference between the actual rupture strain observed in FRP confined specimens, and the FRP material rupture strain determined from tensile coupon testing

\[ \rho_f \text{ Volumetric ratio of FRP reinforcement} = \begin{cases} 4n_t / D & \text{Circular} \\ 2n_t \left( b + h \right)/(bh) & \text{Non-Circular} \end{cases} \]

\[ \rho_g \text{ Ratio of the area of longitudinal steel reinforcement to the cross-sectional area of a compression member} = A_s / A_g \]

ψ₟ Additional FRP strength reduction factor

REFERENCES


[19] American Concrete Institute, ACI 440.2R, Guide for the Design and Construction ofExternally Bonded FRP Systems for Strengthening of Concrete Structures, American Concrete Institute, 2002, Farmington Hills, MI, USA.


[22] American Concrete Institute, ACI 318-05, Building Code Requirements for Structural Concrete, American Concrete Institute, 2005, Farmington Hills, MI, USA.


### Table 1. RC Columns of Circular Cross-Section

<table>
<thead>
<tr>
<th>Reference</th>
<th>Specimen Code</th>
<th>Fiber Type</th>
<th>D (mm)</th>
<th>H (m)</th>
<th>ρ_e (%)</th>
<th>ρ_f (%)</th>
<th>f'_c (MPa)</th>
<th>f_y (MPa)</th>
<th>E_f (MPa)</th>
<th>f_{fu} (MPa)</th>
<th>ε_{fu} (%)</th>
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<th>P_a or P_{max} (kN)</th>
<th>M_{max} (kN.m)</th>
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<td>500</td>
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**Note:** 1 mm = 0.04 in.; 1 MPa = 0.145 ksi; 1 kN = 0.225 kip; 1 kN·m = 0.738 kip·ft; NA = Not Applicable
Table 2. RC Columns of Non-Circular Cross-Section

| Reference     | Specimen Code | Fibre Type | b (mm) | h (mm) | h/b | H (m) | r (mm) | \(\rho_s\) (%) | \(\rho_c\) (%) | \(f_c\) (MPa) | \(f_t\) (MPa) | \(E_f\) (MPa) | \(f_{fu}\) (MPa) | \(\epsilon_{fu}\) (%) | \(t_f\) (mm) | \(P_a\) or \(P_{max}\) (kN) | \(M_{max}\) (kNm) |
|---------------|---------------|------------|--------|--------|-----|-------|--------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|---------------|----------------|----------------|----------------|
| Nosho, 1996   |               |            |        |        |     |       |        |                |                |                |                |                |                |                |               |                |                |                |
| NO1           | none          | 280        | 280    | 1.00   | 2.36| 13    | 1.02   | 0.25          | 0.00           | 40.6           | 407            | NA             | NA             | NA             | NA            | 1062          | 140            |
| NO2           | CFRP          | 280        | 280    | 1.00   | 2.36| 13    | 1.02   | 0.25          | 0.94           | 41.6           | 407            | 261,484        | 4165           | 1.60           | 0.17          | 1087          | 147            |
| NO3           | CFRP          | 280        | 280    | 1.00   | 2.36| 13    | 1.02   | 0.25          | 0.16           | 41.6           | 407            | 267,358        | 4206           | 1.57           | 0.11          | 1088          | 137            |
| NO4           | CFRP          | 280        | 280    | 1.00   | 2.36| 13    | 1.02   | 0.25          | 0.16           | 41.6           | 407            | 267,358        | 4206           | 1.57           | 0.11          | 1116          | 140            |
| Walkup, 1998  |               |            |        |        |     |       |        |                |                |                |                |                |                |                |               |                |                |                |
| WA1           | none          | 458        | 458    | 1.00   | 3.05| 51    | 1.48   | 0.21          | 0.00           | 24.6           | 460            | NA             | NA             | NA             | NA            | 1299          | 559            |
| WA2           | CFRP          | 458        | 458    | 1.00   | 3.05| 51    | 1.48   | 0.21          | 0.86           | 22.7           | 460            | 230,303        | 3515           | 1.50           | 0.17          | 1399          | 581            |
| WA3           | CFRP          | 458        | 458    | 1.00   | 3.05| 51    | 1.48   | 0.21          | 0.58           | 24.7           | 460            | 230,303        | 3515           | 1.50           | 0.17          | 1352          | 537            |
| WA4           | CFRP          | 458        | 458    | 1.00   | 3.05| 51    | 1.48   | 0.21          | 0.29           | 24.1           | 460            | 230,303        | 3515           | 1.50           | 0.17          | 1334          | 556            |
| Chaallal and  |               |            |        |        |     |       |        |                |                |                |                |                |                |                |               |                |                |                |
| Shahawy, 2000 |               |            |        |        |     |       |        |                |                |                |                |                |                |                |               |                |                |                |
| CS1-a         | none          | 200        | 350    | 1.75   | 2.10| 25    | 1.62   | 1.57          | 0.00           | 25.5           | 414            | NA             | NA             | NA             | NA            | 1878          | 1              |
| CS2-a         | CFRP          | 200        | 350    | 1.75   | 2.10| 25    | 1.62   | 1.57          | 1.57           | 25.5           | 414            | 45,000         | 540            | 1.20           | 0.50          | 2412          | 1              |
| CS1-b         | none          | 200        | 350    | 1.75   | 2.10| 25    | 1.62   | 1.57          | 0.00           | 25.5           | 414            | NA             | NA             | NA             | NA            | 1228          | 105            |
| CS2-b         | CFRP          | 200        | 350    | 1.75   | 2.10| 25    | 1.62   | 1.57          | 1.57           | 25.5           | 414            | 45,000         | 540            | 1.20           | 0.50          | 1335          | 145            |
| CS1-c         | none          | 200        | 350    | 1.75   | 2.10| 25    | 1.62   | 1.57          | 0.00           | 25.5           | 414            | NA             | NA             | NA             | NA            | 712           | 139            |
| CS2-c         | CFRP          | 200        | 350    | 1.75   | 2.10| 25    | 1.62   | 1.57          | 1.57           | 25.5           | 414            | 45,000         | 540            | 1.20           | 0.50          | 828           | 159            |
| CS1-d         | none          | 200        | 350    | 1.75   | 2.10| 25    | 1.62   | 1.57          | 0.00           | 25.5           | 414            | NA             | NA             | NA             | NA            | 310           | 114            |
| CS2-d         | CFRP          | 200        | 350    | 1.75   | 2.10| 25    | 1.62   | 1.57          | 1.57           | 25.5           | 414            | 45,000         | 540            | 1.20           | 0.50          | 441           | 158            |
| CS1-e         | none          | 200        | 350    | 1.75   | 2.10| 25    | 1.62   | 1.57          | 0.00           | 25.5           | 414            | NA             | NA             | NA             | NA            | 240           | 119            |
| CS2-e         | CFRP          | 200        | 350    | 1.75   | 2.10| 25    | 1.62   | 1.57          | 1.57           | 25.5           | 414            | 45,000         | 540            | 1.20           | 0.50          | 356           | 179            |
| CS1-f         | none          | 200        | 350    | 1.75   | 2.10| 25    | 1.62   | 1.57          | 0.00           | 25.5           | 414            | NA             | NA             | NA             | NA            | 0             | 94             |
| CS2-f         | CFRP          | 200        | 350    | 1.75   | 2.10| 25    | 1.62   | 1.57          | 1.57           | 25.5           | 414            | 45,000         | 540            | 1.20           | 0.50          | 0             | 145            |

Note: 1 mm = 0.04 in; 1 MPa = 0.145 ksi; 1 kN = 0.225 kip; 1 kN·m = 0.738 kip·ft; (a) Specimens axially loaded at various eccentricities
### Table 2. RC Columns of Non-Circular Cross-Section (Cont.)

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<th>Reference</th>
<th>Specimen Code</th>
<th>Fiber Type</th>
<th>b (mm)</th>
<th>h (mm)</th>
<th>h/b</th>
<th>H (m)</th>
<th>r (mm)</th>
<th>$\rho_s$ (%)</th>
<th>$\rho_t$ (%)</th>
<th>$f'_c$ (MPa)</th>
<th>$f_y$ (MPa)</th>
<th>$E_f$ (MPa)</th>
<th>$f_{fu}$ (MPa)</th>
<th>$\varepsilon_{fu}$ (%)</th>
<th>$t_f$ (mm)</th>
<th>$P_a$ or $P_{max}$ (kN)</th>
<th>$M_{max}$ (kN.m)</th>
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<td>305</td>
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<td>3.93</td>
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<tr>
<td></td>
<td>IA5-a</td>
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<td>305</td>
<td>1.00</td>
<td>1.47</td>
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<td>1.31</td>
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<td>2.58</td>
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<td>2.62</td>
<td>37.0</td>
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<td>76,350</td>
<td>962</td>
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<tr>
<td>BO1-a</td>
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<td>250</td>
<td>500</td>
<td>2.00</td>
<td>1.60</td>
<td>13</td>
<td>0.83</td>
<td>0.51</td>
<td>0.00</td>
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<td>NA</td>
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</tr>
<tr>
<td>Bousias et al., 2004</td>
<td>BO2-a</td>
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<td>0.83</td>
<td>0.51</td>
<td>0.31</td>
<td>18.1</td>
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<td>3450</td>
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<tr>
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<td>0.83</td>
<td>0.51</td>
<td>0.78</td>
<td>17.9</td>
<td>560</td>
<td>230,000</td>
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<td>1.50</td>
<td>0.13</td>
<td>865</td>
</tr>
<tr>
<td></td>
<td>BO4-a</td>
<td>GFRP</td>
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<td>13</td>
<td>0.83</td>
<td>0.51</td>
<td>1.02</td>
<td>18.7</td>
<td>560</td>
<td>70,000</td>
<td>2170</td>
<td>3.10</td>
<td>0.17</td>
<td>858</td>
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<tr>
<td></td>
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<td>0.51</td>
<td>0.00</td>
<td>17.9</td>
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<tr>
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<td>0.51</td>
<td>0.31</td>
<td>18.1</td>
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<td>230,000</td>
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<td>1.50</td>
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<td>0.78</td>
<td>17.9</td>
<td>560</td>
<td>230,000</td>
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<td></td>
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<td>0.83</td>
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<td>18.7</td>
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<tr>
<td>Elnabelsy and Saatcioglu, 2004</td>
<td>ES1-S</td>
<td>CFRP</td>
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<td>33.0</td>
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<td>0.90</td>
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Note: 1 mm = 0.04 in; 1 MPa = 0.145 ksi; 1 kN = 0.225 kip; 1 kN·m = 0.738 kip·ft.
Table 2. RC Columns of Non-Circular Cross-Section (Cont.)

<table>
<thead>
<tr>
<th>Reference</th>
<th>Specimen Code</th>
<th>Fiber Type</th>
<th>b (mm)</th>
<th>h (mm)</th>
<th>h/b</th>
<th>H (m)</th>
<th>r (mm)</th>
<th>ρ_s (%)</th>
<th>ρ_t (%)</th>
<th>f_c' (MPa)</th>
<th>f_y (MPa)</th>
<th>E_f (MPa)</th>
<th>ϵ_fu (%)</th>
<th>t_f (mm)</th>
<th>P_a or P_max (kN)</th>
<th>M_max (kN.m)</th>
</tr>
</thead>
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<tr>
<td>Harajli</td>
<td>HR1-a</td>
<td>none</td>
<td>150</td>
<td>300</td>
<td>2.00</td>
<td>1.12</td>
<td>13</td>
<td>1.50</td>
<td>0.80</td>
<td>0.00</td>
<td>20.3</td>
<td>534</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>196</td>
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<tr>
<td>and Rteil,</td>
<td>HR2-a</td>
<td>CFRP</td>
<td>150</td>
<td>300</td>
<td>2.00</td>
<td>1.12</td>
<td>13</td>
<td>1.50</td>
<td>0.80</td>
<td>0.26</td>
<td>21.1</td>
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<td>230,000</td>
<td>3500</td>
<td>1.50</td>
<td>0.13</td>
</tr>
<tr>
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<td>HR3-a</td>
<td>CFRP</td>
<td>150</td>
<td>300</td>
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<td>1.50</td>
<td>0.80</td>
<td>0.39</td>
<td>21.7</td>
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<td>230,000</td>
<td>3500</td>
<td>1.50</td>
<td>0.13</td>
</tr>
<tr>
<td>Memon</td>
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<td>305</td>
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<td>42.4</td>
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<td>NA</td>
<td>NA</td>
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</tr>
<tr>
<td>and Sheikh,</td>
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<td>GFRP</td>
<td>305</td>
<td>305</td>
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<td>1.48</td>
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<td>2.58</td>
<td>0.61</td>
<td>3.28</td>
<td>42.5</td>
<td>465</td>
<td>19,754</td>
<td>450</td>
<td>2.28</td>
<td>1.25</td>
</tr>
<tr>
<td>2005</td>
<td>MS3</td>
<td>GFRP</td>
<td>305</td>
<td>305</td>
<td>1.00</td>
<td>1.48</td>
<td>16</td>
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<td>0.61</td>
<td>6.56</td>
<td>42.7</td>
<td>465</td>
<td>19,754</td>
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<td>2.28</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>MS4</td>
<td>GFRP</td>
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<td>305</td>
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<td>2.58</td>
<td>0.61</td>
<td>3.28</td>
<td>43.3</td>
<td>465</td>
<td>19,754</td>
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<td>2.28</td>
<td>1.25</td>
</tr>
<tr>
<td>MS5</td>
<td>GFRP</td>
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<td>305</td>
<td>1.00</td>
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<td>2.58</td>
<td>0.61</td>
<td>1.64</td>
<td>43.7</td>
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<td>19,754</td>
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<td>1.25</td>
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<tr>
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<td>305</td>
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<td>1.48</td>
<td>16</td>
<td>2.58</td>
<td>0.61</td>
<td>4.92</td>
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<td>19,754</td>
<td>450</td>
<td>2.28</td>
<td>1.25</td>
<td>2532</td>
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</tbody>
</table>

Note: 1 mm = 0.04 in.; 1 MPa = 0.145 ksi; 1 kN = 0.225 kip; 1 kN·m = 0.738 kip·ft.
Table A. 1. Manufacturer's Reported FRP Material Properties – Example Application

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness per ply, $t_f$</td>
<td>0.33 mm</td>
<td>0.013 in.</td>
<td></td>
</tr>
<tr>
<td>Ultimate tensile strength, $f_{fu}$</td>
<td>3792 MPa</td>
<td>550,000 psi</td>
<td></td>
</tr>
<tr>
<td>Rupture strain, $\varepsilon_{fu}$</td>
<td>0.0167 mm/mm</td>
<td>0.0167 in/in</td>
<td></td>
</tr>
<tr>
<td>Modulus of elasticity, $E_f$</td>
<td>227,527 MPa</td>
<td>33,000,000 psi</td>
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</tr>
</tbody>
</table>

Table A. 2. Summary of Results – Example Application

<table>
<thead>
<tr>
<th>Point</th>
<th>$n = 0$ plies</th>
<th>$n = 6$ plies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(unstrengthened member)</td>
<td></td>
</tr>
<tr>
<td>$\phi P_n$ (kN)</td>
<td>$\phi M_n$ (kN·m)</td>
<td>$\phi P_n$ (kN)</td>
</tr>
<tr>
<td>A</td>
<td>11,601</td>
<td>0</td>
</tr>
<tr>
<td>A'</td>
<td>9281</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>8266</td>
<td>874</td>
</tr>
<tr>
<td>C</td>
<td>4127</td>
<td>1,198</td>
</tr>
</tbody>
</table>

Note: 1 kN = 0.225 kip; 1 kN·m = 0.738 kip·ft.
Fig. 1. Simplified Interaction Diagram (Circular and Non-Circular Cross-Sections)

Fig. 2. Stress-Strain Model for FRP-Confined Concrete by Lam and Teng (Lam and Teng 2003a)
Fig. 3. Interaction Diagrams and Experimental Results (Circular Cross-Sections)
Fig. 4. Interaction Diagrams and Experimental Results (Non-Circular Cross-Sections)
Fig. 4. Interaction Diagrams and Experimental Results (Non-Circular Cross-Sections) (Cont.)
Fig. 5. Representative Simplified Interaction Diagram

Fig. 6. Strain Distributions of Points B and C for Simplified Interaction Diagram
Given:
Cross-section geometry: b, h, r, A_{si}, \rho_g
Materials properties: f'_{cc}, f_y, \varepsilon'_f, E_f, t_f
Reduction factors: C_E, \psi_f

FRP design properties:
\[ f_{fu} = C_E \varepsilon_{fu} \]
\[ \varepsilon_{fu} = C_E \varepsilon_{fu} \]

\[ \varepsilon_{fe} = \begin{cases} \kappa_f \varepsilon_{fu} & \text{for Point A} \\ \min (0.004, \kappa, \varepsilon_{fu}) & \text{for Points B and C} \end{cases} \]

If \( f_l / f'_c < 0.08 \)
Check

From cross-section geometry:
\( \kappa_a \) (Eq. 8), \( \kappa_b \) (Eq. 9)

If \( \varepsilon_{cu} > 0.01 \)
Recalculate \( f'_{cc} \) using Eqs. (4)

Check

Points B and C

\[ f'_{cc} \] (Eq. 5)
\[ \varepsilon_{cu} \] (Eq. 7)

\[ \phi_P_{n(A)} \] (Eq. 18)
\[ \phi_M_{n(A)} = 0 \]

Point A

\[ c = \begin{cases} d \frac{\varepsilon_{cu}}{\varepsilon_y + \varepsilon_{cu}} & \text{for point B} \\ d \frac{\varepsilon'_{t}}{\varepsilon_{cu}} & \text{for point C} \end{cases} \]
\[ y_1 = c \frac{\varepsilon'_{t}}{\varepsilon_{cu}} \]
where \( \varepsilon'_{t} \) can be found using Eq. (4b)
\[ \varepsilon_{si}, f_{si}, d_{ii} \]

Coefficients A, B, C, D, E, F, G, H, and I (Eqs. 17)

\[ \phi_P_{n(B,C)} \] (Eq. 15)
\[ \phi_M_{n(B,C)} \] (Eq. 16)

---

Fig. 7. Flow Chart for Application of Methodology
Note: The column features steel-ties transverse reinforcement.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value (ksi)</th>
<th>Value (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'_{c}$</td>
<td>6.5</td>
<td>44.8</td>
</tr>
<tr>
<td>$f_{y}$</td>
<td>60</td>
<td>414</td>
</tr>
<tr>
<td>$r$</td>
<td>1 in.</td>
<td>25 mm</td>
</tr>
<tr>
<td>Bars</td>
<td>12#10</td>
<td>12φ32</td>
</tr>
<tr>
<td>$A_{s1} = A_{s4}$</td>
<td>5.08 in.²</td>
<td>3277 mm²</td>
</tr>
<tr>
<td>$A_{s2} = A_{s3}$</td>
<td>2.54 in.²</td>
<td>1639 mm²</td>
</tr>
</tbody>
</table>

ρₙ (%) = 2.65

$\Phi_{P_n}$ without FRP = 9281 kN, 2087 kip
$\Phi_{P_{n(req)}}$ = 11,138 kN, 2504 kip

Fig. A.1. Column Cross-Section Details and Materials' Properties

Fig. A.2. Simplified Interaction Diagram - Example Application
SECTION

3. SUMMARY AND CONCLUSIONS

This dissertation is composed of three technical papers. The first paper deals with the evaluation of experimental results obtained from the testing of 22 RC columns, and a new analytical model to determine the confining pressure in non-circular cross-sections. A second paper is a review of state-of-the-art design methodologies available for the case of FRP-confined RC columns, and their ability to predict the increment of compressive strength and ductility. Finally, a third paper presents a design-oriented methodology for the construction of a simplified interaction (P-M) diagram for FRP-confined RC in the compression-controlled region.

The following general conclusions can be drawn from the work presented in this dissertation:

- Based on the experimental campaign, the increment of concrete compressive strength of FRP-confined non-circular columns is marginal when compared to their circular counterparts. However, in many instances, substantial increment of ductility in terms of axial deformation was observed in spite of the limited strength enhancement. In specimens of circular cross-section, the confinement effectiveness is not affected by the size of the cross-sectional area. On the contrary, in specimens of non-circular cross-section, few indicatives of the possible negative effect of cross-sectional area size in the axial strengthening were noted, however, the scattering and limitation of data-points do not allow at the present time to draw a definite conclusion;
• The level of confinement effectiveness for specimens of different cross-sectional shape (circular, square, and rectangular) featuring similar FRP volumetric ratio (and therefore same cross-sectional area), decreases as the side-aspect ratio increases;

• For CFRP-wrapped non-circular specimens, the lowest observed FRP strain efficiency factor was 0.5, and it is comparable to the one recommended in the literature by Lam and Teng (2003a,b), Matthys et al. (2005), and Carey and Harries (2003);

• An analytical model was developed to correlate the transverse strains on FRP jacket and steel reinforcement to the corresponding confining pressures. The proposed approach is based on the idealization of a portion of the concrete confined area as a two-hinged parabolic arch restrained by a horizontal tie representing the FRP and the transverse steel reinforcement. A strength model was calibrated with the obtained confining pressures, and evaluated with the collected experimental data showing good agreement. Further experimental evidence is needed to confirm the validity of the basic assumptions of the model such us the thickness of the arch (i.e. constant versus variable thickness), the type of loading (i.e. partial uniform load versus uniform load on the entire arch), and the distribution of load between the steel tie and the FRP jacket. These assumptions could be verified by appropriate selection of the instrumentation and the introduction of sensors, such us the NIP sensors, capable of monitoring the pressure at the interface between concrete and FRP jacket.
Part of the collected strain data from the specimens was not suitable for a direct interpretation of certain effects, such as: the instability of the longitudinal steel reinforcement and concrete dilation. Further research is needed to confirm the basic assumptions, and provide relevant and substantial data information to feed and correctly calibrate models. Although a vast experimental campaign on real-size RC columns following the conventional testing methodology is a choice, the current available sensing technology used in a few dimensionally-relevant specimens represents an innovative alternative testing protocol, allowing obtaining accurate information, and most importantly allowing the understanding of the physical phenomena. The measurements should be targeted to the strain distribution along the perimeter of the FRP jacket, the strain distribution of the longitudinal and transverse steel reinforcement, the lateral (outward) deformation of the longitudinal steel bars product of the concrete lateral dilation (bar instability), the concrete dilation, and crack propagation detection. A more meaningful interpretation of the experimental data currently available in the literature would become possible once performance phenomena and controlling parameters fully understood.

The design method was proposed for the construction of a simplified interaction (P-M) diagram that responds to the need for a simple and rational design approach to FRP-wrapped RC columns subjected to combined axial force and bending. The proposed design approach that follows ACI principles requires further validation with experimental test results of RC columns having geometry and properties similar to the ones found in practice.
APPENDIX A

SPECIMENS SPECIFICATIONS AND VIDEOS OF FAILURES OF SPECIMENS
The material included in Appendix A is electronic and it is organized as follows:

Specimens’ specifications: two files in “PDF” format are included, one for the specimens tested at the laboratory at the University of California – San Diego (UCSD), and one related to the specimens tested at the National Institute of Standards and Technology (NIST). Information regarding the fabrication, strengthening, and instrumentation is included in these files. These two documents are accessible when opening the DVD as a unit system.

Videos of specimens failures: for the specimens at UCSD two views are available (overview and platen view). For NIST specimens only an overview is shown.
APPENDIX B
REPORT TO NATIONAL SCIENCE FOUNDATION (NSF)
This appendix contains in an electronic pdf format the report submitted to the National Science Foundation (NSF) on the conducted research program. This document reports the entire set of experimental data and it has been cited in the three technical papers composing the dissertation.
APPENDIX C

ACI 440.2R – SECTION 11.2
This appendix presents the section concerning FRP strengthening of members subjected to axial force and bending moment from ACI Committee 440 design guideline, where the methodology for the construction of a P-M diagram proposed in the third paper has been adopted.

11.2—Combined axial compression and bending


For the purpose of predicting the effect of FRP confinement on strength enhancement, Eq. (11-1) is applicable when the eccentricity present in the member is less or equal than $0.1h$. When the eccentricity is larger than $0.1h$, the methodology and equations presented in Section 11.1 can be used to determine the concrete material properties of the member cross-section under compressive stress. Based on that, the P-M diagram for the FRP confined member can be constructed using well-established procedures (Bank 2006).

The following limitations apply for members subjected to combined axial compression and bending:

- The effective strain in the FRP jacket should be limited to the value given in Eq. (11-12) in order to ensure the shear integrity of the confined concrete:

$$\varepsilon_{fe} = 0.004 \quad (11-12)$$
The strength enhancement can only be considered when the applied ultimate axial force and bending moment ($P_u$ and $M_u$) are such that a point having these coordinates falls above the line connecting the origin and the balanced point in the P-M diagram for the unconfined member (see Fig 11.4). This limitation stems from the fact that strength enhancement is only of significance for members in which compression failure is the controlling mode (Bank 2006).

P-M diagrams may be developed by satisfying strain compatibility and force equilibrium using the model for the stress strain behavior for FRP confined concrete presented in Eqs. (11-2). For simplicity, the portion of the unconfined and confined P-M diagrams corresponding to compression-controlled failure can be reduced to two bilinear curves passing through three points (see Fig 11.4). For values of eccentricity greater than $0.1h$ and up to the point corresponding to the balanced condition, the methodology provided in Appendix A may be used for the computation of a simplified interaction diagram. The values of the $\phi$ factors as established in ACI 318-05 for both circular and noncircular cross-sections apply.

![Fig. 11.1 - Representative interaction diagram](image-url)
Methodology for Computation of Simplified P-M Interaction Diagram for Non-Circular Columns

P-M diagrams may be developed by satisfying strain compatibility and force equilibrium using the model for the stress strain behavior for FRP confined concrete presented in Eq. (11-2). For simplicity, the portion of the unconfined and confined P-M diagrams corresponding to compression-controlled failure can be reduced to two bilinear curves passing through the following three points (see Fig D.1) (the text below only makes reference to the confined case since the unconfined one is analogous):

- Point A (pure compression) at a uniform axial compressive strain of confined concrete \( \varepsilon_{c_{cu}} \);  

- Point B with a strain distribution corresponding to zero strain at the layer of longitudinal steel reinforcement nearest to the tensile face, and a compressive strain \( \varepsilon_{c_{cu}} \) on the compression face; and,

- Point C with a strain distribution corresponding to balanced failure with a maximum compressive strain \( \varepsilon_{c_{cu}} \) and a yielding tensile strain \( \varepsilon_{sy} \) at the layer of longitudinal steel reinforcement nearest to the tensile face

For confined concrete, the value of \( \phi P_{n} \) corresponding to point A (\( \phi M_{n} \) equals zero) is given in Eq. (11-1), while the coordinates of points B and C can be computed as follows:

\[
\phi P_{n(B,C)} = \phi \left[ (A(y_i)^3 + B(y_i)^2 + C(y_i) + D) + \sum A_i f_{si} \right] \tag{D-1}
\]

\[
\phi M_{n(B,C)} = \phi \left[ (E(y_i)^4 + F(y_i)^3 + G(y_i)^2 + H(y_i) + I) + \sum A_i f_{si} d_i \right] \tag{D-2}
\]
Where:

\[
A = \frac{-b(E_c - E_2)}{12f'_c} \left( \frac{\varepsilon_{ceu}}{c} \right)^2 \quad (D-3a)
\]

\[
B = \frac{b(E_c - E_2)}{2} \left( \frac{\varepsilon_{ceu}}{c} \right) \quad (D-3b)
\]

\[
C = -bf'_c \quad (D-3c)
\]

\[
D = bcf'_c + \frac{bcE_2}{2}(\varepsilon_{ceu}) \quad (D-3d)
\]

\[
E = \frac{-b(E_c - E_2)^2}{16f'_c} \left( \frac{\varepsilon_{ceu}}{c} \right)^2 \quad (D-3e)
\]

\[
F = b\left( c - \frac{h}{2} \right) \left( \frac{E_c - E_2}{12f'_c} \right) \left( \frac{\varepsilon_{ceu}}{c} \right)^2 + \frac{b(E_c - E_2)}{3} \left( \frac{\varepsilon_{ceu}}{c} \right) \quad (D-3f)
\]

\[
G = -\left( \frac{b}{2} \right) f'_c + b\left( c - \frac{h}{2} \right) \left( \frac{E_c - E_2}{2} \right) \left( \frac{\varepsilon_{ceu}}{c} \right) \quad (D-3g)
\]

\[
H = bf'_c \left( c - \frac{h}{2} \right) \quad (D-3h)
\]

\[
I = \frac{bc^2}{2} f'_c - bcf'_c \left( c - \frac{h}{2} \right) + bcE_2 \left( \frac{\varepsilon_{ceu}}{c} \right) - \frac{bcE_2}{2} \left( c - \frac{h}{2} \right) \left( \varepsilon_{ceu} \right) \quad (D-3i)
\]

In Eq. (D-3), “c” represents the distance from the extreme compression fiber to the neutral axis (see Fig A.1) and it is given by Eq. (D-4). The parameter “y_t” represents the vertical coordinate within the compression region measured from the neutral axis position (see Figure D.1) and corresponds to the transition strain \( \varepsilon'_t \) (Eq. (D-5) (see Figure D.1)).
in which, $f_{si}$ is the stress in the “ith” layer of longitudinal steel reinforcement. The values are calculated by similar triangles from the strain distribution corresponding to points B and C. Depending on the neutral axis position “c”, the sign of $f_{si}$ will be positive for compression and negative for tension. A flow chart illustrating the application of the proposed methodology is shown in Fig. D.2.

**Fig. D. 1 - Strain Distributions of Points B and C for Simplified Interaction Diagram**
Given:
Cross-section geometry: b, h, d’, r_c, A_u, ρ_g
Materials properties: f_c, f_y, f_{fu}, E_f, t_f
Reduction factors: C_E, ψ_f

FRP design properties:

\[ f_{fu} = C_E f_{\ast fu} \]
\[ \varepsilon_{fu} = C_E \varepsilon_{\ast fu} \]

\[ \varepsilon_{fu} = \begin{cases} \kappa \varepsilon_{fu} & \text{for Point A} \\ \min(0.004, \kappa \varepsilon_{fu}) & \text{for Points B and C} \end{cases} \]

If \( f_{fu} / f_{\ast fu} < 0.08 \)
- Provide “n”
- Check \( f_i \) (Eq. 11-4)

From cross-section geometry:

\[ \kappa_a \] (Eq. 11-9), \[ \kappa_b \] (Eq. 11-10)

If \( \varepsilon_{ccu} > 0.01 \)
- Recalculate \( f_{ccu} \) using Eqs. (11-2)

\[ f_{ccu} \] (Eq. 11-3)

\[ \varepsilon_{ccu} \] (Eq. 11-6)

Points B and C
- Point A

\[ \phi \Phi_{nA} \] (Eq. 11-1)
\[ \phi \Phi_{M_{nA}} = 0 \]

\[ c = \begin{cases} \frac{d}{\varepsilon_{ccu}} & \text{for point B} \\ \frac{d}{\varepsilon_{y_{\ast}} + \varepsilon_{ccu}} & \text{for point C} \end{cases} \]

\[ y_t = c \frac{\varepsilon'}{\varepsilon_{ccu}} \]
where \( \varepsilon' \), can be found using Eq. (11-2c)

\[ \varepsilon_{si}, f_{si}, d_i \]

Coefficients A, B, C, D, E, F, G, H, and I (Eqs. A-3)

\[ \phi \Phi_{n(B,C)} \] (Eq. A-1)
\[ \phi \Phi_{M_{n(B,C)}} \] (Eq. A-2)

Fig. D. 2 - Flow Chart for Application of Methodology
BIBLIOGRAPHY


VITA

Silvia Valentina Rocca Camasca was born in Piura, Peru on December 26, 1977. In December 2001, Silvia obtained her bachelor’s degree in Engineering Sciences with mention in Civil Engineering from the Universidad de Piura (UDEP) in Piura, Peru.

Following graduation, Silvia applied for an internship through IAESTE program (International Association for the Exchange of Students for Technical Experience). She obtained a training position in the Center for Infrastructure Engineering Studies (CIES) at the University of Missouri-Rolla, which started in June of 2002 and finished in December of the same year. In January of 2003, Silvia enrolled at the University of Missouri-Rolla and in May 2004, she completed her Master in Science in Civil Engineering. In December 2007, she received her degree of Ph.D. in Civil Engineering from the University of Missouri-Rolla.

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