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THE MULTIRATE SIMULATION OF FACTS DEVICES IN POWER SYSTEM DYNAMICS

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Abstract

In this paper, the multirate method is applied to the problem of simulating the dynamics of a power system which contains fast components such as induction machine loads and FACTS devices. Results concerning the numerical stability and accuracy of the multirate method are presented. Implementation concerns are also addressed by studying an example power system which contains a wide range of time response behavior.

Key Words: Power system dynamics, computer simulation, multirate methods.

1 Introduction

At the 1994 Summer Meeting of the Power Engineering Society of the Institute of Electrical and Electronics Engineers (IEEE), the Working Group on Dynamic Security Assessment outlined the requirements of an on-line dynamic security assessment application [1]. It is forecasted that a typical dynamic security assessment application will require a time-domain simulation (TDS) of 30 possible contingencies every 20-30 minutes. Each of these contingencies will be simulated for up to 30 seconds each. Thus, real-time or near real-time simulation will be required in the next generation of Energy Management Systems (EMS). Modelling accuracy in these simulations is paramount in order to detect any potential instabilities. Thus, any TDS must include accurate models for industrial loads (which are primarily comprised of induction motors) and any system control devices. Several types of switched thyristor control devices are being introduced into the power network. These devices have become to be known as FACTS (Flexible AC Transmission systems) and encompass a broad spectrum of power system devices, such as Static Var Compensators (SVCs), high-voltage dc (HVDC) lines, and phase shifting transformers (PSTs).

One major impediment to the time domain simulation of a power system which includes FACTS devices and induction machine loads, is the computational complexity involved due to the fast switching action of the thyristor controlled devices. Usually only a small portion of the system is affected by the fast dynamics of these devices, yet the integration time step of the entire system must remain small in order to capture the fast dynamics, thus computational efficiency is lost.

This paper will discuss the use of the multirate method, first proposed in [2], for the simulation of power system dynamics. The potential of multirate methods for analyzing power system dynamics was first reported in [3], and further developments will be discussed in this paper. First, the possibility of n-time scales is explored, rather than simply two time scales. Second, all integration methods are implicit, rather than differentiating between implicit methods for fast time scales and explicit methods for the slow time scales. Lastly, the problem of incorporating systems of differential/algebraic equations is addressed. Specifically, this paper will explore the application of the multirate methods for the simulation of differential/algebraic power system models containing fast switching FACTS devices and induction machine loads.

2 The Multirate Algorithm

Traditionally, variable-step methods have been used for power system simulation [4] [5]. Under certain modelling constraints such as induction machine loads or FACTS devices, these methods may not be the most computationally efficient methods for TDS. For example, consider a variable step method for the simulation of a stiff system. The integration step size is predicted at each time step based on the estimation of the accuracy of the numerical solution. If the system contains rapidly varying states, then in order to insure sufficient numerical accuracy, a small integration step size must be used. The size of the integration time step is usually selected based on an estimation of the error incurred at that step (the local truncation error or LTE). If the system has rapidly decaying transients, the small time steps may be replaced by larger time steps after the transients have decayed, while still maintaining the same level of desired accuracy. However, if the transient effects are only lightly damped, the small time steps will persist for a longer simulation interval. In this case, computational efficiency is lost if a variable step method is used. Power systems containing induction machine loads or FACTS devices are such systems in which the fast transient effects persist.

This problem may be overcome by using the multirate algorithm for integration. A multirate algorithm for integrating ordinary differential equations is one in which

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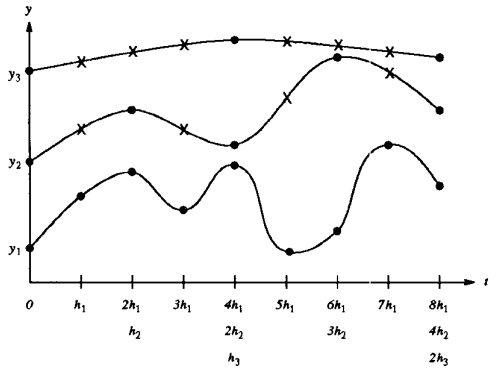


Figure 1: Three time scale example

different equations are integrated by using different step sizes. This method is well suited for stiff systems in which there is sustained oscillatory behavior or systems in which only a few fast modes are frequently perturbed. The basic principle of the multirate method is the integration of each variable with an integration step size which is necessary and sufficient for the required accuracy, which is usually based on some prediction of the local truncation error. The coupling between the different parts of the differential equations is maintained by approximating the slowly varying solution components. The multirate algorithm can best be explained graphically. Consider Figure 1 which gives an example of a three time scale separation. Note that each variable is integrated with the step size which is appropriate for its time response. The fastest varying state $y_1(t)$ is integrated at every time step h_1, the next state $y_2(t)$ is integrated at every time step $h_2 = 2h_1$, and the slowest state $y_3(t)$ is integrated at every $h_3 = 2h_2 = 4h_1$ time steps. Note also that not all states are available at the desired time (those marked by the $\times$ symbol in Figure 1) and must be approximated. The simplest approximation is a linear interpolation between calculated values. Computational efficiency is gained when there is a small number of fast states. Thus, the whole system need only be solved infrequently compared to the solution of the much smaller fast subsystem. Computational accuracy is maintained if the step sizes are chosen with respect to some criteria such as local truncation error, and if the interpolation of the slow subsystem is accurate, within the same error tolerance.

Although multirate methods are conceptually simple, there are still many problems and open questions regarding their theory, formula, and implementation. The numerical properties of consistency, stability, and convergence of traditional variable-step/multi-step methods have been studied and well documented [6]. Although offering far greater computational efficiency, multirate algorithms must be implemented with care, since they do not necessarily share the same numerical robustness of the variable-step/multi-step methods from which they arise. This issue was first discussed in [7] and will be further developed in this paper.

3 Consistency, Stability, and Convergence

The multirate method is based on the well-known multi-step methods. Multi-step methods are chosen for several reasons. Firstly, they are implicit methods which exhibit the desired properties of consistency and stability for a wide range of integration step size. In order to ensure convergence of any integration algorithm, the algorithm must satisfy two constraints. The algorithm must be **consistent** (i.e. the LTE must go to 0 as the integration time step goes to 0) and the algorithm must be **stable** (which essentially insures that any errors incurred do not propagate throughout the solution). These properties are well understood for multi-step methods up to order 6.

The *convergence* of an integration method is defined as the property by which the computed solution (of a Lipschitz function) can be made arbitrarily close to the true solution by picking an integration step size h small enough. Rather than directly proving the convergence of an integration method, it is sufficient to prove that the method is both consistent and stable, which when taken together, yield a convergent solution.

Consistency is not sufficient to guarantee that a numerical integration method is convergent. Consistency only insures that the local errors are small, but does not indicate anything about how the errors propagate from one time step to the next. To insure convergence, it must be verified that the numerical integration method is stable. Stability means that a small perturbation in the computed value will cause only a bounded change in the solution within the time frame of interest as h is reduced to zero. More formally:

Definition *A numerical integration method is* **stable** *for first order equations if, for any first order equation satisfying a Lipschitz condition, there exist constants* $K \leq \infty$ *and* h_0 *such that*

$$\|\hat{x}(\tau_M) - \hat{x}'(\tau_M)\| \leq K\|x_0 - x_0'\|$$

for all $0 \leq t \leq b$ *and all* $h = \left(\frac{\tau_M}{n}\right) \in (0, h_0)$, *where* $\hat{x}(\tau_M)$ *and* $\hat{x}'(\tau_M)$ *are two numerical solutions.* □

The trapezoidal-multirate method applied to linear systems satisfies this criterion. Consider the 2×2 system:

$$\dot{x}(t) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} x(t),$$

where x_1 may be considered to be "fast" and x_2 is "slow." Applying the multirate algorithm, the states at any time $t + m \cdot h$ may be related by the companion matrix $M(m, h)$ to the states at time t:

$$x(t + m \cdot h) = M(m, h)x(t)$$

where m is the ratio of the slow time step to the fast time step h. Note that in this formulation, m must be an integer. The companion matrix is given by:

$$M(m, h) = \frac{1}{\Delta} \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad (1)$$

where

$$\Delta = 1 - \frac{h^2 a_{12} a_{21}}{(2 - h a_{11})(2 - mh a_{22})} \sum_{i=1}^{m} \beta_1^{m-i} (2i - 1)$$

$$M_{11} = \beta_1^m + \frac{h^2 a_{12} a_{21}}{(2 - h a_{11})(2 - mh a_{22})} \sum_{i=1}^{m} \beta_1^{m-i} (2i - 1)$$

$$M_{12} = \frac{h a_{12}}{m(2 - h a_{11})} \sum_{i=1}^{m} \beta_1^{m-i} (2m + (2i - 1)(\beta_2 - 1))$$

$$M_{21} = \frac{mh a_{21}}{(2 - mh a_{22})} (\beta_1^m + 1)$$

$$M_{22} = \beta_2 + \frac{h^2 a_{12} a_{21}}{(2 - h a_{11})(2 - mh a_{22})} \sum_{i=1}^{m} \beta_1^{m-i} (2(m - i) + 1)$$

with $\beta_1 = \frac{2 + h a_{11}}{2 - h a_{11}}$ and $\beta_2 = \frac{2 + mh a_{22}}{2 - mh a_{22}}$.

The definition of stability is satisfied if $||A|| < \infty$ (which implies that $a_{ij} < \infty \ \forall \ i, j \in [1, \ldots, n]$) and for any $h < h_0 = min[\frac{2}{a_{11}}, \frac{2}{m a_{22}}]$. If h is chosen smaller than h_0 then the companion matrix M is bounded by some finite number K and thus the trapezoidal-multirate method satisfies the definition of stability. This analysis can be generalized to a system containing n timescales, in which case $h < h_0 = min[\frac{2}{C_i a_{ii}}]$, $1 \leq i \leq n$, where $C_i = \frac{h_n}{h_1}$ and h_n is the largest time step and h_1 is the smallest.

Any convergent method may be used to approximate a solution within the desired degree of accuracy by choosing extremely small time steps, but this is very inefficient. Numerical stability as stated previously only guarantees that the errors introduced at each time step remain bounded as the solution progresses through time. A "stronger" form of stability is *absolute stability*. The region of absolute stability for a numerical integration method applied to a linear system of equations is the set of values of h and λ for which a perturbation in a single value will produce a change in subsequent values which does not increase from step to step, where λ is set of eigenvalues of the linear system of equations. An integration method is called *A-stable* if the region of absolute stability includes the entire left-half plane of C. In other words, if an A-stable numerical integration method is applied to a stable linear system of equations, it is numerically stable for all choices of step size h. If a method is A-stable, then it is numerically robust and the integration time step can be safely chosen by considering *only* the local truncation error.

Absolute stability of the integration method is guaranteed if the eigenvalues of the matrix $M(m, h)$ lie on or within the unit circle in the complex plane. In some cases, using the multirate method may cause numerical instabilities that are not present in the underlying trapezoidal method. For example, consider the system $\dot{x} = Ax$ where

$$A = \begin{bmatrix} 4 & -9 \\ 7 & -5 \end{bmatrix} \tag{2}$$

This system is stable with eigenvalues $-0.5 \pm j6.5383$. Thus, the trapezoidal method is stable for any h. If $h_{trap} = 0.8$, the maximum eigenvalue of the corresponding $M(h = 1)$ is $0.9505 < 1$. If the multirate method is used with $m = 2$ and $h = 0.4$, then

$$max\,(eig\,(M(2, 0.4))) = 1.3276 > 1$$

thus this is numerically unstable even though the largest time step $mh = 0.8$ is no larger than the time step h_{trap}. In fact, h must be decreased to 0.1525 for the multirate method to be stable.

Due to the inconsistency in time step behavior, it is desirable to relate the step size h, the ratio m, and the system coefficients a_{11}, a_{12}, a_{21} and a_{22} directly to the eigenvalues of $M(m, h)$ in order to ascertain whether or not a chosen time step and step ratio will result in numerical stability.

The matrix $M(m, h)$ is guaranteed to have eigenvalues within the unit circle if one of the following conditions is satisfied:

1. if $M_{11}^2 - 2M_{11}M_{22} + M_{22}^2 + 4M_{12}M_{21} < 0$ (complex eigenvalues) and $M_{11}^2 + M_{22}^2 + 2M_{12}M_{21} \leq 2\Delta^2$
2. if $M_{11}^2 - 2M_{11}M_{22} + M_{22}^2 + 4M_{12}M_{21} > 0$ (real eigenvalues) and
   (a) if $M_{11} + M_{22} \geq 2\Delta$ and $M_{12}M_{21} - M_{11}M_{22} \geq \Delta(\Delta - (M_{11} + M_{22}))$ or
   (b) if $M_{11} + M_{22} \leq 2\Delta$ and $M_{12}M_{21} - M_{11}M_{22} \leq \Delta(\Delta - (M_{11} + M_{22}))$

Note that if $m = 1$, either condition 1 or 2 above must hold, since the trapezoidal method is stable even as $h \rightarrow \infty$. These conditions are rather arduous to trace back to the original A matrix, but several trends can be noted.

Since the system under study is considered to have a slow subsystem and a fast subsystem, it is reasonable to assume that the slow subsystem is not strongly coupled to the fast subsystem. Therefore, consider the case where $a_{21} = 0$, which would be the case if the slow subsystem were completely decoupled from the fast subsystem. If $a_{21} = 0$, then $M_{21} = 0$ and $\Delta = 1$. Condition 1. stated previously would become

$$M_{11}^2 + M_{22}^2 \leq 2 \quad (M_{21} = 0)$$

Thus, as $h \rightarrow 0$, this condition may be simplified to

$$(\beta_1^m)^2 + \beta_2^2 \leq 2 \tag{3}$$

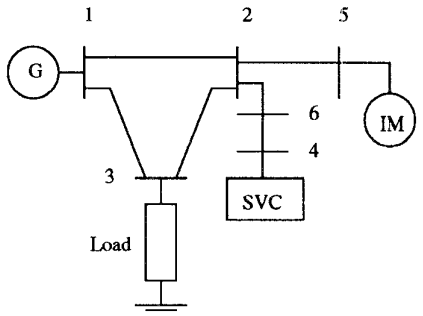
This inequality is obviously satisfied if both $|\beta_1| \leq 1$ and $|\beta_2| \leq 1$ hold. These inequalities will hold if both $a_{11} < 0$ and $a_{22} < 0$. Consider now the case where $a_{11} > 0$, and subsequently $|\beta_1| > 1$. The inequality (3) may still hold, but only for m small and $|\beta_2| \ll 1$. Conversely, however, if $|\beta_2| > 1$ and $|\beta_1| < 1$ this inequality may hold for a large range of m. Considering the matrix A, requiring $a_{11} < 0$ is intuitive. Since the fast subsystem is solved in a decoupled manner from the slow subsystem, the "eigenvalue" of the fast subsystem a_{11} must be negative, otherwise the fast subsystem would immediately exhibit unstable behavior.

As an example of this behavior, consider the following stable linear system where

$$A = \begin{bmatrix} 0.2 & -11 \\ 0.1 & -0.5 \end{bmatrix} \tag{4}$$

in which $a_{11} > 0$ and a_{21} is small compared to a_{12}. For small m, it is expected that the multirate method will be stable, even though $|\beta_1| > 1$, since $|\beta_2| \ll 1$. Indeed this is true. For $h = 1$, the multirate method is stable for $m = 1, \ldots, 4$. If, however, the system is rearranged:

$$A = \begin{bmatrix} -0.5 & 0.1 \\ -11 & 0.2 \end{bmatrix} \tag{5}$$



all lines R+jX = 0.01+j0.01, jB/2 = 0.005
except $X_{2\text{-}6} = X_{SL}$
$X_{4\text{-}6} = X_T - X_{SL}$

Figure 2: Example Power System

For $h = 1$, the multirate method is stable for all m, since $|\beta_1| = 0.6$ and $|\beta_2| = 1.002$, and any m will satisfy (3).

Although the underlying trapezoidal algorithm is A-stable, the multirate method does not satisfy A-stability. However, it is believed that it is possible to find a region of absolute stability, dependent on the step size separations, such that it can be established that if all step sizes fall within that region, the multirate method will be stable. Future work is progressing in this area.

4 Implementation Issues

The issues associated with implementing the multirate algorithm for the simulation of the dynamics of power systems containing FACTS devices will be discussed in context with the test system shown in Figure 2.

This system contains a two-axis model generator including the governor and turbine, an IEEE DC-I type exciter/AVR [8], an IEEE Basic Model I SVC [9], a third-order induction machine load [10], and a constant PQ load. The model of the system is:

Two–Axis Generator Model

$$\dot{\delta} = \omega - \omega_s \tag{6}$$

$$M\dot{\omega} = T_M + \frac{V_1}{x'_d}\left(E'_d \cos(\theta_1 - \delta) + E'_q \sin(\theta_1 - \delta)\right) \tag{7}$$

$$T'_{d0}\dot{E}_q = -\frac{x_d}{x'_d}E'_q + \frac{x_d - x'_d}{x'_d}V_1 \cos(\theta_1 - \delta) + E_{fd} \tag{8}$$

$$T'_{q0}\dot{E}_d = -\frac{x_q}{x'_d}E'_d - \frac{x_q - x'_d}{x'_d}V_1 \sin(\theta_1 - \delta) \tag{9}$$

Assumption: $x'_q = x'_d$ and $R_s = 0$

IEEE Type DC-I Exciter/AVR Model

$$T_E\dot{E}_{fd} = -K_E E_{fd} - S_E\left(E_{fd}\right)E_{fd} + V_R \tag{10}$$

$$T_A\dot{V}_R = -V_R + K_A\left(R_F - \frac{K_F}{T_F}E_{fd} + V_{ref} - V_1\right) \tag{11}$$

$$T_F\dot{R}_F = -R_F + \frac{K_F}{T_F}E_{fd} \tag{12}$$

Turbine/Governor Model

$$T_{RH}\dot{T}_M = -T_M + P_{CH} + \frac{K_{HP}T_{RH}}{T_{CH}}(P_{SV} - P_{CH}) \tag{13}$$

$$T_{CH}\dot{P}_{CH} = -P_{CH} + P_{SV} \tag{14}$$

$$T_{SV}\dot{P}_{SV} = -P_{SV} + P_C - \frac{1}{R}\frac{\omega}{\omega_s} \tag{15}$$

and

$$0 = \frac{V_1}{x'_d}\left(Eq' \sin(\delta - \theta_1) - Ed' \cos(\delta - \theta_1)\right) - V_1 \sum_{j=1}^{n} V_j \left(g_{1j} \cos\theta_{1j} + b_{1j} \sin\theta_{1j}\right) \tag{16}$$

$$0 = \frac{V_1}{x'_d}\left(Eq' \cos(\delta - \theta_1) + Ed' \sin(\delta - \theta_1) - V_1\right) - V_1 \sum_{j=1}^{n} V_j \left(g_{1j} \sin\theta_{1j} - b_{1j} \cos\theta_{1j}\right) \tag{17}$$

Induction Machine Model

$$T_0\dot{E}_R = -\frac{x}{x'}E_R + \frac{x - x'}{x'}V_5 \cos\theta_5 + E_I\omega_5 sT_0 \tag{18}$$

$$T_0\dot{E}_I = -\frac{x}{x'}E_I + \frac{x - x'}{x'}V_5 \sin\theta_5 - E_R\omega_5 sT_0 \tag{19}$$

$$2H\dot{s} = T_L(s) - \frac{V_5}{x'}\left(E_R \sin\theta_5 - E_I \cos\theta_5\right) \tag{20}$$

$T_L(s)$ usually has the form $K(1-s)^\eta$ where K and η are model dependent parameters, and

$$0 = -\frac{V_5}{x'}\left(E_R \sin\theta_5 - E_I \cos\theta_5\right) - V_5 \sum_{j=1}^{n} V_j \left(g_{5j} \cos\theta_{5j} + b_{5j} \sin\theta_{5j}\right) \tag{21}$$

$$0 = -\frac{V_5}{x'}\left(V_5 - E_R \cos\theta_5 - E_I \sin\theta_5\right) - V_5 \sum_{j=1}^{n} V_j \left(g_{5j} \sin\theta_{5j} - b_{5j} \cos\theta_{5j}\right) \tag{22}$$

Static Var Compensator Model - Basic Model I

$$T_R\dot{B}_{ref} = -B_{ref} + K_R\left(V_{ref}^{svc} - V_4\right) \tag{23}$$

$$T_b\dot{B}_{svc} = -B_{svc} + B_{ref} \tag{24}$$

and

$$0 = -V_4 \sum_{k=1}^{n} V_k \left(g_{4k} \cos\theta_{4k} + b_{4k} \sin\theta_{4k}\right) \tag{25}$$

$$0 = V_4^2 B_{ref} - V_4 \sum_{k=1}^{n} V_k \left(g_{4k} \sin\theta_{4k} - b_{4k} \cos\theta_{4k}\right) \tag{26}$$

Network Equations

$$0 = P_{L_i} - V_i \sum_{k=1}^{n} V_k \left(g_{ik} \cos\theta_{ik} + b_{ik} \sin\theta_{ik}\right) \tag{27}$$

$$0 = Q_{L_i} - V_i \sum_{k=1}^{n} V_k \left(g_{ik} \sin\theta_{ik} - b_{ik} \cos\theta_{ik}\right) \tag{28}$$

where $i = 2$, 3, and 6.

4.1 Time Responses

This system is comprised of a wide range of time responses. The eigenvalues of the system at steady-state are shown in Figure 3. There are four sets of complex eigenvalues, giving rise to four modes of oscillatory behavior. These sets of eigenvalues are summarized in Table 1. These sets of eigenvalues give rise to three distinct time frames of interest:

$$0 \leq t \leq 0.05 \quad \text{seconds}$$
$$0.05 \leq t \leq 0.50 \quad \text{seconds}$$
$$0.5 \leq t \leq 100. \quad \text{seconds}$$

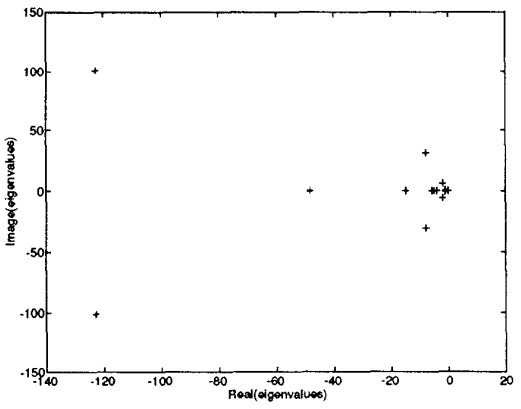


Figure 3: Eigenvalues of the example system

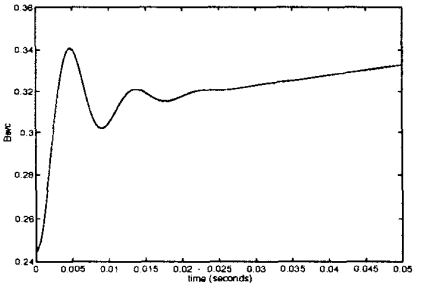
Table 1: Complex Eigenvalues of the example system

-122.86	$\pm$	$j101.05$
-6.72	$\pm$	$j26.68$
-4.27	$\pm$	$j1.01$
-1.04	$\pm$	$j0.53$

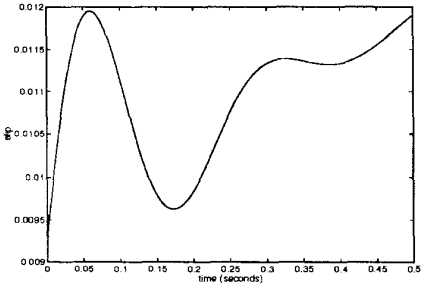
The responses of selected states to a step change in load at bus 3 are shown in Figure 4. The fastest response are the modes associated with the SVC. This response is shown in Figure 4(a) for $0 \leq t \leq 0.05$ seconds. Although highly oscillatory, these modes are also highly damped. The same is true for the induction machine slip s which is shown in Figure 4(b) for $0 \leq t \leq 0.5$ seconds. From Table 1, there are also two sets of eigenvalues which are quite close to the imaginary axis, yielding oscillatory behavior which is considerably slower than the fastest modes. These responses are shown in Figure 4(c) and Figure 4(d). Figure 4(c) shows the induction machine voltage response for $0 \leq t \leq 25$ seconds and Figure 4(d) shows the generator electromechanical speed response which is much slower in scope than any of the previous modes.

These wide variances in time response indicate that a constant step size approach to integration would be extremely inefficient. However, due to the well-damped nature of most of the oscillatory modes, an LTE-based variable-step integration approach would be appropriate. However, the presence of the sustained oscillation in the induction machine variables is well suited for the multirate/LTE method. This system exhibits three very well defined time scales which may be exploited by the multirate method. In the differing time frames, different variables may take on different qualitative behavior. In the fast time frame, the variables B_{ref} and B_{svc} are fast, but they quickly decay to a constant value. Thus, in the second time frame, they now become slow variables, and the slip and exciter variables are fast. This process may be repeated out to any time frame of interest.

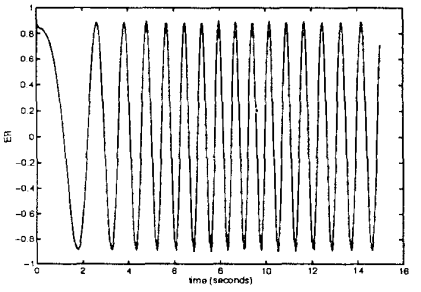
Figure 5 shows the relationship between the various time steps for the first portion of the integration interval of the example system. Note that for $0 \leq t \leq 0.05$ seconds the "fast" time step is very small to account for the extremely fast response of the SVC. For the region $0.05 \leq t \leq 0.5$ seconds, the "fast" time step is much larger as the SVC transients have died out and the SVC variables are now considered to be relatively constant, or "slow." In this time frame, the fast variables are those variables associated with the induction machine, particularly the slip variable. Once again, after about 0.5 seconds, the partitions are once again adaptively altered, to account for the decay of the slip transient. After this point, the two main responses are those of the induction machine voltage (fast) and the remainder of the power system states



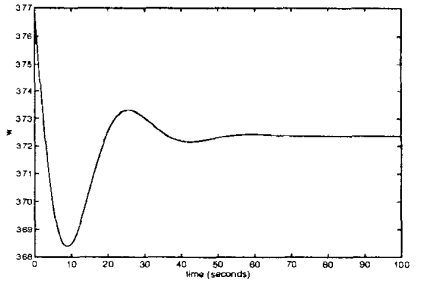
(a): Step Response of Bsvc



(b): Step Response of slip



(c): Step Response of ER



(d): Step Response of w

Figure 4: Step response of the example system

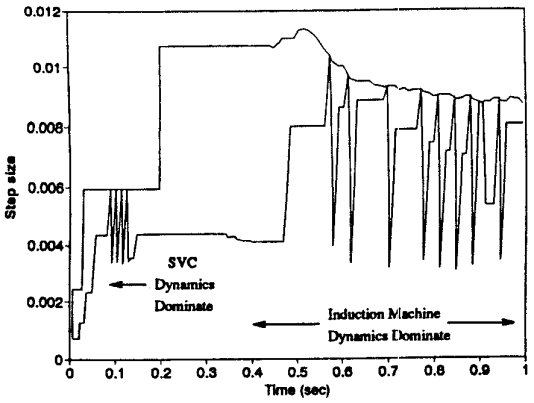


Figure 5: Multirate time step comparison

(slow). Note that in this example, these are the responses to a step change in load. A similar, but more pronounced effect, would be achieved from a contingency study. The multirate method would also be well suited for mid-term dynamic studies in which continual perturbations such as relay operations and tap changes are constantly exciting the fast modes of the SVC and induction machine.

4.2 Step Size Selection

The step size for each time step may be optimized by choosing the largest possible value of h for which the local truncation error remains bounded by a user-specified maximum allowable error. For the trapezoidal integration method, the local truncation error (LTE) is:

$$\text{LTE} = \frac{1}{12}h^3\frac{dx^3(\theta)}{dt^3} \qquad t_n < \theta < t_{n+1}$$

The derivative $\frac{dx^3(\theta)}{dt^3}$ may be approximated by

$$\frac{dx^3(\theta)}{dt^3} \approx (k+1)!\nabla_{k+1}x_{n+1}$$

where k is the order of the trapezoidal method ($k = 2$) and n is the time step interval. Thus, the LTE can be approximated by

$$\text{LTE} = \frac{1}{2}h_{n+1}^3\left|\frac{x_{n+1}-x_n}{h_{n+1}\gamma_{n+1}\delta_{n+1}} - \frac{x_n - x_{n-1}}{h_n\gamma_{n+1}\delta_{n+1}} - \frac{x_n - x_{n-1}}{h_n\gamma_n\delta_{n+1}} + \frac{x_{n-1}-x_{n-2}}{h_{n-1}\gamma_n\delta_{n+1}}\right| \quad (29)$$

where x_{n+1} is the state variable at time $t = t_{n+1}$, $h_{n+1} = t_{n+1} - t_n$, $\gamma_{n+1} = h_{n+1} + h_n$, and $\delta_{n+1} = h_{n+1} + h_n + h_{n-1}$.

For each state, the following test is applied:

1. if $(B_L \leq \text{LTE} \leq B_U)$ then PASS; $h_{\text{new}} = h$
2. else if $(\text{LTE} \leq B_L)$ then PASS; $h_{\text{new}} = \alpha h$
3. else if $(\text{LTE} \geq B_U)$ then FAIL; $h_{\text{new}} = \alpha h$

where

$$\alpha = \left[\frac{B_{\text{avg}}}{\text{LTE}}\right]^{\frac{1}{3}}$$

$$B_{\text{avg}} = \frac{B_L + B_U}{2}$$

and B_L and B_U are the lower and upper bounds on the allowable error, respectively. If even one state has a FAIL, the step size is adjusted accordingly and each state is reevaluated at the new time. For this reason, it is desirable to minimize the number of FAILs.

4.3 Partitioning Strategies

Using normalized LTE values, the partitioning of the state variables of the the system may be performed. One approach to partitioning has been investigated in depth. This is termed the "top down" approach to partitioning.

The top down approach chooses the fastest varying variable (i.e. that state with the largest normalized LTE). This LTE is the upper bound of all LTEs, and is therefore denoted BU. This will then define the grouping of the FAST subsystem. From here the user may define a step size ratio between the FAST subsystem and the next subsystem (this subsystem will be called MEDIUM for illustration purposes, but it is possible to have more than three partitionings). This ratio is denoted by R_{f-m} (Ratio of fast to medium). Similarly, a ratio of the medium to slow subsystems can be R_{m-s}. Then, by scanning the LTEs associated with all states, the partitions become:

1. FAST: all states with an LTE $> \frac{1}{(R_{f-m})^3}BU$
2. MEDIUM: all states not in FAST but with an LTE $> \frac{1}{(R_{m-s})^3}\frac{1}{(R_{f-m})^3}BU$
3. SLOW: all states not in FAST or MEDIUM

For example, if it is defined that a step size ratio of 5:1 is desired between fast and medium (i.e. 5 fast time steps per every 1 medium time step), then the FAST group would contain all states which had an LTE $> \frac{1}{5^3}BU = 0.008BU$. Similarly, if a step size ratio of 10:1 between medium and slow is desired (or 50:1 between fast and slow) then the MEDIUM group would contain all states which had an LTE $> \frac{1}{10^3}\frac{1}{5^3}BU = 0.000008BU$. In a generalized system, this partitioning approach would continue until all states had been assigned a partition. As a rule of thumb, the authors found it best to choose a ratio of at least 4. With any smaller ratio, the overhead computations required for the multirate algorithm outweighs the benefits derived, unless the time scale separation is "sufficiently large."

In the example system, a ratio of $R_{f-m} = 5$ and $R_{m-s} = 5$ yielded the following separation (all given LTEs are $10^6\times$ the actual LTE):

FAST	LTE	MED	LTE	SLOW	LTE
V_R	46.7022	ω	0.0135	δ	0.0000
E_I	19.5245	E'_q	0.0856	R_F	0.0000
s	42.6315	E'_d	0.5579	T_M	0.0000
B_{ref}	1760.1041	E_{fd}	0.0108	P_{CH}	0.0000
B_{svc}	25.3298	E_R	0.0113	P_{SV}	0.0000

which is what would be intuitively expected, given the previous analysis of the system and the natural magnitudes of the LTEs. If a smaller ratio than 5 had been specified (such as 3), then 4 groups would have formed, with B_{ref} in a partition alone.

The next step in the partitioning process is to partition the algebraic variables. The algebraic variables do not have an LTE associated with them. If the LTE formula is applied to the algebraic variables directly, very large LTEs are obtained, due to the fact that the algebraic variables can be assumed to change "instantaneously," which of course would yield a large LTE. For example, the algebraic constraints on the system

$$0 = g(x, y)$$

can be recast as:

$$\epsilon \dot{y} = g(x, y)$$

with ϵ very small. Obviously, with $\epsilon \approx 0$ the time constant of these variables would be extremely large, thus they would all be "fast" variables. Therefore, this is not a good approach. Thus, a "first tier" approach to partitioning the algebraic variables was used. In other words, if a state variable is designated as fast, then any algebraic variable to which this state variable is coupled, is also designated as fast. For the top down partitioning, this gives the following partitioning:

FAST		MEDIUM		SLOW	
V_R	V_1	ω	θ_1	δ	V_2
E_I	V_5	E'_q		R_F	θ_2
s	θ_5	E'_d		T_M	θ_3
B_{ref}	V_4	E_{fd}		P_{CH}	θ_4
B_{svc}		E_R		P_{SV}	V_6
					θ_6

The partitioning of the algebraic variables is a crucial step. An incorrect partitioning can lead to slow convergence of the multirate algorithm due to the difficulty in extrapolating the algebraic variables ahead in time. For shorter times steps (as in FAST or MEDIUM partitionings) the extrapolation error tends to be small. However, for algebraic variables in SLOW, the extrapolation error may be quite large, from which it may take several iterations to rectify.

5 Conclusions

In this paper, the multirate method was further analyzed with respect to its stability properties and implementation issues. The results obtained from a small power system example containing both induction machine loads and a FACTS device indicate the immense potential of the multirate method for simulating power system dynamics. The stability results indicate that care must be taken when implementing the multirate algorithm on partitioned systems in which the fast subsystem is unstable. However, this is seldom the case in power systems, since system instability usually manifests itself through the slow inter-area modes.

Further study is underway on improving techniques necessary for adaptive partitioning, especially in the generalized situation where n time scales may be present. This is necessary in order to fully exploit the time scale separation between states which exhibit decaying transients.

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