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David Drain
University of Missouri--Rolla

A. M. Gough

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Applications of the Upside-Down Normal Loss Function

David Drain and Andrew M. Gough

Abstract—The Upside-Down Normal Loss Function (UDNFL) is a weighted loss function that has accurately modeled losses in a product engineering context. The function’s scale parameter can be adjusted to account for the actual percentage of material failing to work at specification limits. Use of the function along with process history allows the prediction of expected loss—the average loss one would expect over a long period of stable process operation. Theory has been developed for the multivariate loss function (MUDNFL), which can be applied to optimize a process with many parameters—a situation in which engineering intuition is often ineffective. Computational formulae are presented for expected loss given normally distributed process parameters (correlated or uncorrelated), both in the univariate and multivariate cases.

I. INTRODUCTION

Loss functions quantify the relationship between process performance and manufacturing yield [1]–[8]. When applied along with knowledge of process variation and unit production costs, one can derive the manufacturer’s economic cost of process variation. This paper illustrates applications of the upside-down normal loss function (UDNFL)—a loss function we have found to accurately model losses in a real manufacturing situation, and which has desirable mathematical properties enabling easy prediction of average losses due to typical manufacturing variation.

II. THE UDNFL

A. The UDNFL Defined

The UDNFL is one minus a scaled normal probability density function, with mean \( \tau \) and variance \( \lambda \), defined by the following formula

\[
L_{UDN}(x | \tau, \lambda) = 1 - e^{-\frac{(x-\tau)^2}{2\lambda^2}}
\]

where

\( \tau = \) process parameter target

\( \lambda = \) scale factor.

The UDNFL is zero at the target, and asymptotically approaches one. It thus avoids a disadvantage of quadratic loss functions: unrealistic values far from the target.

The scale factor adjusts the penalty for deviation from the target: a large \( \lambda \) indicates that the process can tolerate relatively greater deviation from the target. \( \lambda \) can be empirically determined, or it can be set to some predetermined fraction of the parameter specification range. Lacking better information, a pragmatic choice is to set \( \lambda \) to 42.5% of the specification range. In this case, the loss when a process parameter is at a specification limit is about 50% (corresponding to step function loss in the same situation).

B. Example: Loss Due to Equipment Variation from Target

The etch rate of a polysilicon etcher has a target of 25 \( \AA \)/s and specification limits of 22 and 28 \( \AA \)/sec. Analysis of historical data established that about 50% of etched die will fail when the process is centered at either specification limit; the scale factor \( \lambda \) is therefore chosen as 0.425 times the specification range of 6 \( \AA \), so \( \lambda = 2.55 \).

C. Example: Symmetric Fit to Yield Data

A new microcontroller product exhibited low yield at hot temperature in its initial manufacturing runs. Examination of the failing die uncovered a speed path in a subcircuit of the device that would cause functional failures if slow transistors were manufactured. The length \( (L) \) of the MOSFETs’ polysilicon gates was suspected to have the greatest effect on transistor speed; this was verified in an experiment which allowed polysilicon CD’s to vary about their target (1.60 \( \mu \)m) from 1.15 \( \mu \)m to 1.90 \( \mu \)m.

A nonlinear regression model was used to fit a UDNFL to the resulting losses

\[
L_{UDN}(x | \tau, \lambda) = L_{Min} + (Y_{MAX} - L_{Min})\left[1 - e^{-\frac{(x-\tau)^2}{2\lambda^2}}\right] + \epsilon
\]

where

\( x = \) poly CD variable (\( \mu \)m)

\( \tau = \) Process target poly CD (\( \mu \)m)

\( \lambda = \) fitted shape parameter (\( \mu \)m)

\( L_{Min} = \) minimum expected loss (constant, die/wafer)

\( Y_{MAX} = \) maximum possible product wafer yield (constant, die/wafer)

\( \epsilon = \) error term of regression.

\( \tau \) was fixed at the process target and \( \lambda \) was fit by the regression. For the microcontroller data, the function

\[
L_{UDN}(x | 1.60, \lambda) = 38 + (538 - 38)\left[1 - e^{-\frac{(x-1.60)^2}{2\lambda^2}}\right]
\]

was fit using nonlinear regression software. The regression determined the best fit with \( \lambda = 0.1851 \), resulting in the function

\[
L_{UDN}(x | 1.60, 0.1851) = 38 + (538 - 38)\left[1 - e^{-\frac{(x-1.60)^2}{2(0.1851)^2}}\right]
\]

This function is shown graphically in Fig. 1.

D. Expected Loss

Expected loss is the average loss one would observe from a stable process over a long period of operation, so it can be a critical piece of information when making process change decisions affecting process targets or variability.

To compute expected loss, one also needs the probability density function of the manufacturing variable \( (x) \) under study. Expected loss is then computed by evaluating the integral of the product of the loss function and the probability density function.

If the actual process parameter distribution and a realistic loss function are given, expected loss can be determined by numerical
integration. However, it is usually reasonable to assume the process parameter is normally distributed with mean $\mu$ and standard deviation $\sigma$, and with this simplifying assumption, expected loss can be determined analytically as follows:

$$E_{\text{UDNLF}}(\mu, \sigma, \tau, \lambda) = 1 - \frac{\lambda}{\sqrt{\sigma^2 + \lambda^2}} e^{-\frac{(\mu - \tau)^2}{2(\sigma^2 + \lambda^2)}} \tag{5}$$

This formula can predict the loss due to typical manufacturing variation, assess the damage caused by a drift from the target, estimate losses due to an increase in variance, and quantify the economic consequences of process changes.

Example: In the case of the microcontroller above, substituting the scale factor and Poly CD target (1.60) and standard deviation (0.0835) into (5) results in an expected loss of 0.0885. This value is then transformed with the same linear transformation used in fitting the UDNLF to the yield data, resulting in an expected loss of 82.2 die/wafer. These results agree with actual losses, which had a median value of 81 die/wafer over one quarter’s production.

III. THE MULTIVARIATE UDNLF

The UDNLF can also be applied to processes with more than one important process parameter.

A. The Multivariate Upside-Down Normal Loss Function Defined

The Multivariate Upside-Down Normal Loss Function (MUDNLF) for $n$ parameters is defined as follows:

$$L_{\text{MUDN}}(x | \tau, L) = 1 - e^{-\frac{1}{2}(x - \tau)^T L^{-1} (x - \tau)} \tag{6}$$

where $x$ and $\tau$ are $n \times 1$ column vectors, and $L$ is an $n \times n$ scaling matrix relating deviation from target to loss for all $n$ parameters. As defined here, $L$ must be symmetric and positive definite: $L$ is symmetric if it remains the same when its rows and columns are interchanged ($L = L^T$); $L$ is positive definite if $x^T L x > 0$ for all nonzero column vectors $x$. These requirements may not actually be necessary for the definition of a reasonable loss function; they were chosen because they give the function desirable mathematical properties.

Off-diagonal elements of $L$ are used to account for interaction effects of process parameters—those cases where the deviation of two factors simultaneously produces a different effect than would be expected from the individual factor effects alone.

B. The Multivariate Normal Distribution

As in the one-parameter case, it is often reasonable to assume process parameters have a normal distribution; however, since multiple parameters are involved, one must apply the multivariate normal distribution. The multivariate normal probability density function has the following form:

$$f(x) = \frac{e^{-\frac{1}{2}(x - \mu)^T M^{-1} (x - \mu)}}{(2\pi)^{\frac{n}{2}} |M|^\frac{1}{2}} \tag{9}$$
where
\[ x = a \times n \times 1 \text{ column vector of variables} \]
\[ M = a \text{ (positive definite, symmetric) covariance matrix} \]
\[ \mu = a \times n \times 1 \text{ column vector of means} \]
\[ n = \text{the number of variables} \]

C. Expected Loss With MUDNLF

The expected loss from a MUDNLF defined by \( L \), and a multivariate normal process parameter distribution defined by \( \mu \) and \( M \), with target \( r \), is given by

\[
E_{\text{MUDNL}}(\mu, M, r, L) = 1 - \frac{(L^{-1} + M^{-1})^{-\frac{1}{2}}}{|M|^{\frac{1}{2}}} e^{-\frac{1}{2} \left( (\mu^T M^{-1} \mu + r^T L^{-1} r) \right)} \times e^{\frac{1}{2} \left( (\mu^T M^{-1} + r^T L^{-1} - r^T L^{-1} - 1 + M^{-1} \mu + L^{-1} r) \right)}.
\]  

This closed-form solution for expected loss has even greater utility in the multivariate case than it does for univariate loss functions because engineering intuition is often ineffective for multivariate problems.

IV. CONCLUSION

We found that actual losses can be predicted by the UDNLF. The loss function is easily adaptable to multivariate cases, even when process variables are correlated and losses are the result of synergy or antagonism between those variables. Loss functions can be used in process design and optimization by aligning losses with process parameter distributions in a way which minimizes expected loss.

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REFERENCES