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Identification of linear systems with delay via a learning model

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IDENTIFICATION OF LINEAR SYSTEMS WITH DELAY VIA A LEARNING MODEL

BY

HUGH FRANCIS SPENCE, 1942

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IDENTIFICATION OF LINEAR SYSTEMS WITH DELAY VIA A LEARNING MODEL

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Hugh F. Spence

ABSTRACT

The effects of input delay on an identification scheme using a learning model are investigated. The parameter adjustment laws for the learning model are derived through Lyapunov methods similar to those used for the model reference adaptive control systems of Parks. For no measurement noise or delay mismatch between the learning model and system, the parameters are adjusted to bring the error between model and plant to zero.

When there is delay mismatch between the inputs of the learning model and the unknown system, the convergence of the parameters of the learning model to those of the unknown system is no longer guaranteed. However, the error between the learning model and the unknown system is guaranteed to enter and stay within a region close to the origin.
Several methods of reducing the region are investigated. These methods involve deriving additional adaptive laws for controlling an adjustable delay in the learning model. Asymptotic stability is assured when the initial parameter misalignment vector lies in some region close to the origin. The identification schemes are demonstrated with examples.
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I. INTRODUCTION

Many of the early methods of the identification of linear systems centered around the estimation of system impulse response from input and output measurements. Any delay which existed in the system was included as part of the system impulse response. The effect of a delay on the impulse response of a system was a translation along the time axis.

Actually in early identification methods, the sampled approximation of the impulse response was estimated. A system with finite memory can be approximated by using a finite number of sample points of its impulse response. Then the convolution equation for the system

\[ y(t) = \int_{0}^{\infty} h(\tau) x(t-\tau) d\tau \]

can be approximated by the summation

\[ \hat{y}(k) = \sum_{j=1}^{n} \hat{h}(j) x(k + 1 - j) \]

Estimation of the sampled impulse response was largely done by the statistical methods. Lindenlaub and Cooper[1] explored noise limitations of methods of estimation of the impulse response.
The estimation of the continuous impulse response has been proposed by Papoulis[2]. Papoulis used correlation between the system output and its time scaled input. Tomescu and Tomescu[3] used matched filters to estimate the impulse response.

The impulse response of a system can also be identified by other than statistical techniques. The statistical approaches are usually used when the variables involved are of a random nature.

A system can be effectively identified by "learning" the sampled impulse response as was done in the paper by Naguma and Noda[4]. In this paper, the impulse response is approximated by a finite set of samples. The value of each sample point of the impulse response is incremented by a vector error correction method. The method was shown to be stable for a range of gains in the adjustment law. The method was extended by Roy and Sherman[5] to nonlinear, controllable systems which could be represented by a functional power series. Later Belanger[6] commented that the basic parameter adjustment scheme had been proposed by Robert[7] in England. Other methods for identification of the impulse response have included the use of Fast Fourier Transforms (FFT).
Often the problem of linear system identification is simplified to the problem of parameter estimation. The parameter estimation problem assumes that the form of the system transfer function is known. Kalman[8] recognized that the identification of a linear system by estimation of its impulse response was not the most efficient method. He assumed that the input and output of a linear system could be represented by a linear difference equation

\[ \sum_{i=1}^{n} b_i c_{k-i} = \sum_{j=1}^{\ell} a_j m_{k-j} \]

where \( c_i \) and \( m_i \) represent the sampled output and input respectively.

Identification of the system represented in this form consisted of estimating the \( a_i \)'s and \( b_i \)'s of the system.

The number of \( a_i \)'s and \( b_i \)'s needed to represent a system is much less than the number of points of the impulse response. A delay in the system is produced when

\[ a_i = 0, \text{ for } i = 1, \ldots, p < \ell. \]

Other authors have used different methods to estimate the coefficients of the linear difference equation. Smith[9] attempted to compute the Laplace transfer function of a system from the coefficients estimated by Levin's[10] Maximum Likelihood Method.

The identification of special systems which consist only of delay can be treated by methods explained by Lindgren et al[13] and Spilker[14] in their papers on Delay Locked Loops. Faure and Evans[15] also propose a method for the identification of delay times. Both of these methods use correlation in their explanation.

Hsia[16] recently proposed a method of parameter estimation for sampled data systems with delay. He used a least squares technique to estimate the parameters of the system. The form of the system was assumed known. After each estimate of the parameters, the input measurements were shifted by one sample interval.

The best estimate of the system parameters and delay are those which yield the smallest mean square error. The system cannot be used for identification of time varying systems. The unknown delay cannot be accurately estimated since it is assumed to be some integer number of sample intervals. Parameter estimates also suffer.

Robinson and Saudak[17] presented a method for the continuous identification of time delays and parameters in linear systems. The unknown system was described by the general differential-difference equation
The parameters of another differential-difference equation were adjusted to minimize the filtered "error-in-the-satisfaction-of-the-system-equation" or equation error. Adjustment laws were obtained for the parameters and delays by use of the steepest descent law. The system was assumed to have a single input and output. In the authors examples, derivative filters were used to obtain the derivatives of the input and output. The adjustable delays were approximated with a filter obtained through realizations of the truncated series expansion of $\exp[-Tp]$, where $T$ is the delay time and $p \triangleq \frac{d}{dt}$.

A Lyapunov-like proof was shown when the parameter errors are small. The adjustment laws for the variable delays contain a parameter which also is being adjusted. Simulations show that these delay adjustment laws can produce slow convergence.

Many of the parameter adjustment methods used in model reference adaptive control systems are applicable to system identification schemes. The identification
problem can be represented as an inverse of those model reference adaptive control systems with memory. In the model reference adaptive control systems, feedback and gain parameters on the controlled system are adjusted such that the output of the controlled system is forced toward that of a reference system or model.

In the identification scheme, the reference model is replaced by the unknown system and the adaptively controlled plant is replaced by a learning model. (See Figures 1 and 2)

Various schemes have been presented for deriving the laws used to adjust the parameters. Whitaker and Osburn[18] used minimum integral square error as a criterion for adjusting parameters and developed what is often referred to as the "MIT' rule.

Dresseler[19] introduced the idea of sensitivity functions to the development of the adjustment laws. Hsia[20] proposed an identification scheme based on Dresseler's work. Another parameter adjustment method which has been used makes use of the steepest descent law one form of which is

\[ \dot{\rho} = -K|e|^m \text{sgn}(e) \frac{\partial e}{\partial \rho}, \]

where \( \rho \) is an unknown parameter, \( e \) is some error to be minimized and \( K \) is a positive gain. The stability of the
systems produced by these adjustment laws was often not assured. These and other identification methods have been examined by various authors[33 - 69].

The stability theorems of Lyapunov, introduced in western countries by Kalman and Bertram[21] in 1960, were used by several authors to develop parameter adjustment laws which also gave stable systems. Parameter adjustment laws for stable model reference adaptive control systems were discussed by Butchart and Shackcloth[22], Parks[23] and Winsor and Roy[24]. Recently Landau[25] showed stability of these systems by another approach and suggested that other stable parameter adjustment laws could be found by using Popov's results in the field of hyperstability. Many extensions have been made on these laws to improve convergence or to simplify their mechanization. Phillipson[26] showed that feed forward in the adjustment laws could improve the dynamics. Graham[27] also adjusted the dynamics by compensation of a linear approximation of the system. Epstein[28] used saturation or sigum function in place of many of the multipliers normally used in the adjustment laws. Schooley[29] explored the effect of measurement noise and inaccessible state variables.

Pazdera and Pottinger[30] proposed an identification method using Lyapunov techniques to obtain the parameter adjustment laws. The laws found are similar to those used for previously mentioned model reference systems.
The effects of time delay on the parameter adjustment laws derived through Lyapunov techniques has not received attention. The Lyapunov methods hold promise for obtaining stable laws for identification of systems with delay. In many interesting cases, the system which is to be identified contains a transport lag.

It is the purpose of this paper to examine the problems of delay mismatch on continuous system identification and to propose methods of identifying the delay mismatch. Part II of this paper discusses the effect of delay mismatch in an identification scheme derived by Lyapunov techniques. Part III will develop adjustment laws for variable delays. Parts IV and V present implementations of the method.
II. EFFECT OF DELAY MISMATCH ON PARAMETER ADJUSTMENT LAWS

Consider first the identification scheme where the object is to identify an unknown system with delay by using a learning model without delay. The general form of the system is shown in Figure 3. Both the learning model and unknown system are in state representation. Each input of the unknown system is assumed to be delayed. The undelayed inputs and state variables of the unknown system are assumed accessible. All variables of the learning model are accessible. The parameters of the learning model are to be adjusted such that the parameter misalignment and output error between the learning model and unknown system is reduced.

A. System Equations

The unknown systems and the learning model will be described by the following equations

UNKNOWN SYSTEM  \( \dot{x} = Ax + Bu(t,T) \)  \( (1) \)

LEARNING MODEL \( \dot{z} = \hat{Ax} + \hat{Bu} + v \)  \( (2) \)

where \( x, \hat{x}, \) and \( v \) are n-dimensional vectors, \( u \) is m-dimensional and

\[ u(t,T) = (u_i(t - T_i)) \ i = 1,2,\ldots,m \]
\[ A = (a_{ij}) \]
\[ \hat{A} = (a_{ij}(t)) \]
\[ B = (b_{ij}) \]
\[ \hat{B} = (b_{ij}(t)) \]
\[ T = (T_j) \]

The additional input to the learning model, \( v(t) \), is added to assure stability. \( v(t) \) will come about during the development of the parameter adjustment laws.

The output error between the unknown system and the learning model is defined in the natural way

\[ e \overset{\triangle}{=} x - \hat{x}. \]  

Equations (1) and (2) with the derivative of equation (3) produces

\[ \Delta e = Ax - \hat{A}x + Bu(t,T) - \hat{B}u - v = \Delta Ax + \Delta Bu + B(u(t,T) - u) - v \]

where

\[ \Delta A \overset{\triangle}{=} \hat{A} - \hat{A}(t) \]
\[ \Delta B \overset{\triangle}{=} B - \hat{B}(t) \]
B. Derivation of Parameter Adjustment Laws

Since the identification of the unknown system requires that the parameter misalignment be reduced so as to reduce the error between model and system, the Lyapunov function is chosen to be positive definite in error and parameter misalignment. The Lyapunov function is

\[ 2V = e^T K_1 e + \text{Tr} [\Delta A^T K_2 \Delta A] + \text{Tr} [\Delta B^T K_3 \Delta B] \]  (5)

where the superscript \( T \) indicates the transpose and \( \text{Tr}(\quad) \) indicates the trace of the matrix. The matrices \( K_1, K_2, \) and \( K_3 \) are positive definite, real symmetric. All of the \( K \) matrices are \( n \times n \) except \( K_3 \) which is \( m \times m \).

If \( V \) is differentiated, the following result is obtained.

\[ \dot{V} = \text{Tr} (-\dot{\Delta}^T K_2 \dot{\hat{A}}) + \text{Tr} (-\dot{\Delta}^T K_3 \dot{\hat{B}}) + e^T K_1 \dot{\hat{e}} \]  (6)

When equation (4) is substituted in equation (6), the result is

\[ \dot{V} = e^T K_1 (\Delta x + \Delta u + B(u(t,T) - u) - \nu) \]

- \( \text{Tr} [ (\Delta A)^T K_2 \dot{\hat{A}}] - \text{Tr} [\Delta B^T K_3 \dot{\hat{B}}] \)  (7)

If the relationship between vector products is now used i.e. \( \gamma^T \beta = \text{Tr}[\gamma \beta^T] \)
the equation for $V$ can be rewritten
\[
\dot{V} = -\dot{e}^T K_1 e + \text{TR}[\Delta A^T(K_1 e x^T - K_2 \dot{A})]
\]
\[
+ \text{TR}[\Delta B^T(K_1 e u^T - K_3 \dot{B})] + e^T K_1 B(u(t,T) - u)
\]  
(8)

Now the second and third terms of (8) are indefinite but can be eliminated if the parameter adjustment laws are chosen to be
\[
\dot{A} = K_2^{-1} K_1 e x^T
\]  
(9a)
\[
\dot{B} = K_3^{-1} K_1 e u^T
\]  
(9b)

The first term can be made negative definite in $e$ if $v$ is chosen to be
\[
v = K_f e
\]  
(10)

where $K_f$ is a matrix chosen such that the matrix
\[
Q = K_1 K_f
\]
is positive definite.

It has been shown[31] that $Q$ exists if $-K_f$ has eigenvalues with only negative real parts.

The equation for $V$ is now
\[
\dot{V} = -\dot{e}^T Q e + e^T K_1 B(u(t,T) - u)
\]  
(11)
The approach to the derivation of the parameter adjustment laws thus far follows closely that used by Pazdera (see Appendix A).

Consider now the case where the delay term is not present. Equation (11) would be negative definite in error but not in parameter misalignment. The Lyapunov function will decrease as long as any error remains, thus the error is guaranteed to approach zero. However, since terms involving $\delta A$ and $\delta B$ are not present, the convergence of parameter misalignment to the origin must be further argued.

A relationship between the error and parameter misalignment is given by equation (5). If use is made of equation (10), this relationship is

$$
\dot{e} = - K_f \mathbf{e} + \Delta A \mathbf{x} + \Delta B \mathbf{u} + B(u(t,T) - u). 
$$

(12)

Again, if the system is considered first without delay the equation becomes

$$
\dot{e} = - K_f \mathbf{e} + \Delta A \mathbf{x} + \Delta B \mathbf{u}
$$

(13)

It can be seen that the error can only remain at zero if

$$
\Delta A \mathbf{x} + \Delta B \mathbf{u} = 0
$$

(14)
If all elements of the vectors are linearly independent, then the equation (14) can only be satisfied if

\[ \Delta A = [0] \]

and

\[ \Delta B = [0]. \]

Conditions can be placed on the inputs and process which will guarantee that the elements of \( x \) and \( u \) are linearly independent. These conditions are explained in Appendix B.

In a different approach, Robinson[17] has remarked that if \( K \) parameters are to be identified, then at least \( K/2 \) non-harmonically related frequencies must be in the inputs.

C. Effect of Delay on Parameter Estimation

Now consider the effect of the indefinite terms in equation (11) which are due to delay mismatches. Assume that the input is bounded such that

\[ \| B(u(t,T) - u) \| < \Gamma. \]  \hspace{1cm} (15)

For bounded inputs, \( \dot{V} \) will be negative when

\[ \| e \| > \frac{\lambda(K_{1})_{\text{max}}}{\lambda(Q)_{\text{min}}} \Gamma = \gamma \] \hspace{1cm} (16)

where \( \lambda(\cdot)_{\text{max(min)}} \) indicates the maximum (minimum) eigenvalue of the matrix. The derivation of equation 16 is by Schooley and is repeated in a shortened form in Appendix C.
Since the minimum eigenvalue of $Q = K_f K_f$ can be chosen by adjustment of $K_f$, $||e||$ can be forced toward an arbitrary small value. However, simulation has shown that when this is done, parameter adjustment becomes very slow.

Again the possibility exists of $\Delta A$ or $\Delta B$ becoming unbounded while $||e|| < \gamma$. Under the same restriction placed on the input and process as previously, the elements of $x$ and $u$ are linearly independent. Therefore, by equation 12, $\Delta A$ or $\Delta B$ cannot become unbounded without $||e||$ also becoming unbounded. Therefore $\Delta A$ or $\Delta B$ cannot become unbounded for $||e|| < \gamma$.

D. Example

The above identification method was used to identify a first order system. The first order system was chosen because of multiplier and other hardware limitation of the analog computer. The equations of the unknown system and learning model are

$$\dot{x} = -2x + 6u(t - .100)$$

$$\dot{z} = \hat{A}(t)x + \hat{B}(t)u(t) - 10.0e$$
where
\[ \hat{A}(0) = 4.0 \]
\[ \hat{B}(0) = 4.0 \]
and
\[ e = x - z \]

The parameter adjustment laws are found to be
\[ \dot{\hat{A}}(t) = -\frac{K_2}{K_1} e \]
and
\[ \dot{\hat{B}}(t) = \frac{K_3}{K_1} e \]

Figures 5a and 5b show \( \hat{B}(t) \), and \( \hat{A}(t) \) and \( e \) for \( \frac{K_2}{K_1} = \frac{K_3}{K_1} = 10 \).

Figure 4a and 4b show the trajectories for a zero delay difference for comparison.

The input was taken from a random noise generator with bandwidth of 1.5 Hz and rms value of .316 volts with peak of approximately 1.0 volt. \( \hat{A}(t) \) and \( \hat{B}(t) \) approach values other than \( A \) and \( B \) but do not show a tendency to become unbounded.

The error does approach and stay within the region indicated by equation 16.

i.e. \[ ||e|| < 0.6 \text{ volt} \]

however, it can be seen that \( \hat{A}(t) \) and \( \hat{B}(t) \) do not approach \( A \) and \( B \).
The effect of a negative \( \Delta T \) is demonstrated in Figures 6 and 8 where \( T = -0.10 \) seconds and \(-0.192\) seconds. Again the error approaches and stays in the region stated above. \( \hat{A}(t) \) and \( \hat{B}(t) \) approach different values than in the previous example. On some figures, the trajectories have not reached their final region in the period of time shown. All trajectories eventually approach some average value. Additional simulation results are presented in the Appendix C.

Simulations have shown that the average value that \( \hat{A}(t) \) or \( \hat{B}(t) \) approaches is dependent both on the delay mismatch and the bandwidth of the input. Figure 8 shows \( \hat{B}(t) \) for a pseudorandom input with bandwidth of 1.5 Hz. Figure 8 and the following figures were made by setting the initial conditions of \( \hat{A} \) and \( \hat{B} \) within the region known to contain the eventual trajectories of \( \hat{A}(t) \) and \( \hat{B}(t) \) for a known delay mismatch, and then changing the delay mismatch at an extremely slow rate \( (6.4 \times 10^{-4} \text{ sec/sec}) \). The final average value of \( \hat{B}(t) \) peaks for some negative delay mismatch. For even more negative delay mismatches, the average value of \( \hat{B}(t) \) drops rapidly and passes through zero. For positive delay mismatches, the average value of \( \hat{B}(t) \) drops to zero and remains around zero for increasing delay mismatch. Figure 9 shows a similar plot of \( \hat{B}(t) \) for an input signal with bandwidth of 5 Hz. Again the peak and zero crossing are noticed for negative delay mismatches; however, both the peak and zero crossing occur
closer to zero delay mismatch. In general, these characteristics seem to be decreasing functions of input bandwidth. Although Figures 8 and 9 do not actually demonstrate the behavior of $\hat{B}(t)$ as a function of constant $\Delta T$, the slow sweep of $\Delta T$ makes it a close approximation.
III. DERIVATION OF DELAY ADJUSTMENT LAWS

The presence of the delay terms in the unknown system prevents the parameter matrices of the learning model from converging to the correct matrices. Even though the error between the learning system and the unknown system could be reduced to some region, the parameters of the learning model would attempt to compensate for the lag or lead introduced by the delay mismatch. The error between the state vectors would not approach and eventually remain at zero unless the delay difference between the unknown system and the learning model was equal to zero.

Now consider a new model as shown in Figure 10. This model differs from the previous one in that each input contains an adjustable delay. The use of this model in an identification scheme is an approach different than any previous approach. The adjustment law for controlling each delay is to be generated from the measurable state vectors and inputs of the learning model and the unknown system.

Again equation 1 describes the unknown system and the learning model is described by

\[ \dot{z} = \hat{A}x + \hat{B}u(t, \hat{T}) + v \]

(17)

where \( A \) and \( B \) were previously defined and

\[ u(t, \hat{T}) = (u_j(t - \hat{T}_j(t))). \]
The equation for error is now

$$\dot{e} = \Delta Ax + \Delta Bu(t,\hat{T}) + B(u(t,T) - u(t,\hat{T}))- v$$  \hspace{1cm} (18)

A. Derivation of Adjustment Laws for Adjustable Delays

Since identification is achieved when error, parameter mismatch and delay mismatch are zero, the following Lyapunov function is chosen.

$$2V = e^T K_1 e + \text{TR}[\Delta A^T K_2 \Delta A] + \text{TR}[\Delta B^T K_3 \Delta B]$$

$$+ [\Delta T^T K_4 \Delta T]$$  \hspace{1cm} (19)

The vector $\Delta T$ is defined

$$\Delta T = (T_j - \hat{T}_j(t))$$

and $K_4$ is a symmetric and positive definite matrix. The Lyapunov function can be seen to be positive definite in parameter and delay misalignment and error.

If the derivative of $V$ is taken and equations (1), (9), (10) and (18) are used, the result is

$$\dot{V} = - e^T Qe - \Delta T^T K_4 \dot{T} + e^T K_1 B[u(T,\hat{T}) - u(t,\hat{T})]$$  \hspace{1cm} (20)

Since $u(t)$ is assumed continuous and differentiable, the differences between the delayed inputs can be written

$$u(t,T) - u(t,\hat{T}) = G(t) \Delta T$$  \hspace{1cm} (21)
where $G(t)$ is the diagonal matrix obtained from a Taylor series expansion.

Again using the relationship between the products of vectors, $\dot{V}$ can be rewritten

$$\dot{V} = -e^T Q e - [\Delta_T^T(\hat{K}_4 \hat{T} - G B^T K_1 e)].$$  \hspace{1cm} (22)

The second term of $\dot{V}$ is indefinite but may be set to zero if the adjustment law for the delay vector is taken to be

$$\hat{T} = K_4^{-1} G B^T K_1 e$$ \hspace{1cm} (23)

B. Stability

The equation for $\dot{V}$ is now

$$\dot{V} = -e^T Q e$$

$\dot{V}$ is negative as long as there is any error and thus $||e||$ is guaranteed to approach and eventually remain at zero. However, the behavior of $\Delta A$, $\Delta B$ and $\Delta T$ must now be examined.

$V$ is non-increasing. The possibility of $||e||$ approaching and remaining at zero while a misalignment exists must be explored.

The parameter and delay misalignments are related to the error vector by equation 18. If the inputs are sufficiently random such that the elements of $x$, $u$ and $G(t)$
are linearly independent, then $|e|$ cannot remain at zero unless

$$\Delta A = [0]$$
$$\Delta B = [0]$$

and

$$\Delta T = 0$$

The matrix $G(t)$ cannot be generated since that would require some inaccessible variables. However, in a region where delay mismatch is sufficiently small, $G(t)$ may be replaced with another matrix. Sections IV and V will explore several approximations.

1 The inputs are restricted such that the elements of $u(t,T)$ and $u(t,\hat{T})$ are linearly independent for $T \neq \hat{T}$. $\Delta T$ is slowly varying.
IV. USE OF THE MODEL INPUT DERIVATIVES IN THE DELAY ADJUSTMENT LAWS

It is immediately apparent that the control law for the delay derived in Section II cannot be implemented as is. The matrix $G(t)$ requires that $\Delta T$ and $u(t, T)$ be available. It has been assumed that $u(t, T)$ is not available. However, $G(t)$ can be approximated and, under certain conditions, stability of the identification scheme will still exist. This section will examine the use of the input derivatives as an approximation to $G(t)$.

A. Approximation of the Inaccessible Variables by the Model Input Derivatives

Consider the Taylor's series expansion of the difference between delayed inputs. When each row of the vector is expanded around $T_1 = \hat{T}_1$ of the row, the result is

$$u_1(t - T_1) - u_1(t - \hat{T}_1) = -\Delta T_1 \dot{u}_1(t - \hat{T}_1)$$

$$+ \frac{\Delta T_1^2}{2!} \ddot{u}_1(t - \hat{T}_1) - \ldots$$

(24)

or

$$G(t) \Delta T = u(t, T) - u(t, \hat{T}) = \dot{U}(t, \hat{T}) \Delta T + R$$

(25)
where $R$ is the summation of all terms of second and higher order.

Under the conditions that the elements of $R$ are small enough such that they can be neglected, the derivatives of the input can be used in the delay adjustment law. The adjustment law for the delays becomes

$$\dot{\hat{T}} = -k_4^{-1} \dot{U}(t, \hat{T}) B^T K_1 e$$

(26)

Some knowledge of the form of the $B$ matrix is needed now to simplify the expression further.

Obviously, if $B$ is known to be non-singular which is either positive or negative definite, then $K_1$ can be chosen such that

$$K_1 = \pm (B^{-1})^T$$

Where the sign is chosen to make $K_1$ positive definite. The matrix $K_4$ is a positive definite symmetric matrix and can be chosen arbitrarily. The delay adjustment law for this case becomes

$$\dot{\hat{T}} = \pm k_4^{-1} \begin{bmatrix} \dot{u}_1 (t - \hat{T}_1) e_1 \\ \vdots \\ \dot{u}_n (t - \hat{T}_n) e_n \end{bmatrix}$$

A simplification of equation (26) can also be made for systems where the $B$ matrix has a single non-zero element in each column. The $B$ matrix does not need to be square. If the $K_1$ matrix is chosen to be the identity matrix then
where the $b_{ik}$ above are the non-zero elements. The delay adjustment law for system of this type is

$$B^T K_1 e = \begin{bmatrix} b_{i1} & e_i \\ b_{j2} & e_j \\ b_{lm} & e_{\lambda} \end{bmatrix}$$

where $K_4$ is an arbitrary positive definite matrix. The sign of the elements of $B$ must be known.

A third simplification of equation (26) can be made if it is known that the mismatch between the $\hat{B}(t)$ matrix and $B$ are small and will remain negligible. Then the $B$ matrix in the adjustment law may be replaced by $\hat{B}(t)$. Simulations presented in Appendix C show that the degree of success of this substitution depends on the amount of mismatch between model and unknown system.

The above cases are not meant to be exhaustive. Other simplifications may exist.
B. Stability

The derivative for the system with the delay adjustment law of equation (26) becomes

$$\dot{V} = -e^T Q e + e^T K_{\perp} B R.$$  \hspace{1cm} (27)

When $R$ is sufficiently small enough to be neglected, $|e|$ is guaranteed to approach zero.

The same argument that was used to show that $\Delta A$ and $\Delta B$ approached the null matrices can now be used to show that $\Delta A$, $\Delta B$ and $\Delta T$ approach the null matrices.

C. Example

The problem of Section II was set up with the unknown system and model equations as follows.

$$x = 2.0x + 6 u(t - T)$$

$$z = \hat{A}(t)x + \hat{B}(t) u(t - \hat{T}) + K_f e$$

The initial values were

$$\hat{A}(0) = 4.0$$

$$\hat{B}(0) = 4.0$$

$$\Delta T(0) = .100 \text{ sec.}$$

The adjustment laws were selected to be

$$\begin{align*}
\dot{\hat{A}}(t) &= \mu_1 e x \\
\dot{\hat{B}}(t) &= \mu_2 e u(t - \hat{T}) \\
\dot{\hat{T}}(t) &= -\mu_3 e u(t - \hat{T})
\end{align*}$$
The input u(t) was taken from a random noise generator with an rms value of .316v, a peak of approximately 1.0v, and a bandwidth of 1.5 Hz.

Figure 11 shows the trajectories of \( \hat{\mu}, \hat{\Lambda}, e \) and \( \Delta T \) for \( \mu_1 = 10, \mu_2 = 10, \mu_3 = 0.1 \) and \( K_f = 10.0 \). The simulation was repeated for a delay mismatch of \( T = -0.100 \) sec. The results of this simulation are shown in Figure 12.

Other simulation results are shown in Appendix C.

It has been found from simulations that the identification scheme will converge for larger negative initial delay mismatches than positive. The Taylor's series remainder becomes important as delay mismatch is increased. The arguments for convergence do not hold when the Taylor's series remainder is no longer negligible.
The input $u(t)$ was taken from a random noise generator with an rms value of $0.316\text{v}$, a peak of approximately 1.0v, and a bandwidth of 1.5 Hz.

Figure 11 shows the trajectories of $\hat{B}, \hat{A}, e$ and $\Delta T$ for $\mu_1 = 10$, $\mu_2 = 10$, $\mu_3 = 0.1$ and $K_f = 10.0$. The simulation was repeated for a delay mismatch of $T = -0.100$ sec. The results of this simulation are shown in Figure 12.

Other simulation results are shown in Appendix C.

It has been found from simulations that the identification scheme will converge for larger negative initial delay mismatches than positive. The Taylor's series remainder becomes important as delay mismatch is increased. The arguments for convergence do not hold when the Taylor's series remainder is no longer negligible.
B. Stability

When the misalignment between delays is small enough such that the matrix $S$ can be used in place of $G(t)$, the Lyapunov function derivative becomes

$$\dot{V} = -e^T Q e$$

Therefore $||e||$ is guaranteed to approach and eventually remain at zero. By again using previous arguments, $\Delta A$, $\Delta B$ and $\Delta T$ must approach the null matrices.

C. Effects of Positive and Negative Initial Delay Mismatch

When using the approximate derivatives in the delay adjustment laws, a tendency exists for the total system to operate better for delay mismatches of one sign. This can be explained by looking at the elements of $G(t)$ and $-S(t)$.

$$g_1 \Delta T_i = u_1(t - T_i) - u_1(t - \hat{T}_i)$$

$$s_1(t - \hat{T}_i) = K(-s_1(t - \hat{T}_i) + u_1(t - \hat{T}_i))$$

The difference between $g_1(t)$ and its approximate is

$$E_1 = g_1(t) + \dot{s}_1(t - T_i) = (K - \frac{1}{\Delta T_i}) u_1(t - \hat{T}_i)$$

$$+ \frac{1}{\Delta T_i} u_1(t - T_i) - K s_1(t - \hat{T}_i)$$

(31)
The variable \( s_i(t - \hat{T}_i) \) lags \( u_i(t - \hat{T}_i) \). If \( u_i(t - T_i) \) lags \( u_i(t - \hat{T}_i) \) by a similar amount then it can be seen that a minimum of \( |E_i| \) exist for some value of \( K \). However, when \( u_i(t - T_i) \) leads \( u_i(t - \hat{T}_i) \), then no such minimum can be inferred. A minimum approximation error does exist for positive delay misalignment. Simulations will show that this will somewhat balance the region of initial delay mismatch for convergence. In previous discussions, the region extends more into the negative.

D. Examples

The unknown system used in the previous section was again used for simulation of the identification scheme using the approximate derivatives. The parameter adjustment laws were chosen to be

\[
\begin{align*}
\dot{A} &= \mu_1 \text{ex} \\
\dot{B} &= \mu_2 \text{eu}(t - \hat{T}).
\end{align*}
\]

The delay adjustment law was

\[
\dot{\hat{T}} = -K_T \cdot \dot{S}(t - T)
\]

where

\[
\dot{S}(t - \hat{T}) = K_1 (-S(t - \hat{T}) + u(t - \hat{T}))
\]

and \( K_1 = 10.0 \).
The gains were again chosen to be $\mu_1 = 10$, $\mu_2 = 10$, $K_f = 10$, and $K_T = .1$ and the initial conditions were $\hat{B}(0) = 4$, $\hat{A}(0) = 4$. Figure 13 shows the trajectories of $\hat{A}$, $\hat{B}$, $e$ and $\Delta T$ for $\Delta T(0) = .100$. Figure 14 shows the same variables for a delay mismatch of $\Delta T(0) = -.100$. The trajectories for larger delay mismatches are shown in Figures 15 and 16. Additional trajectories are in Appendix C.
VI. CONCLUSIONS

Schemes for the identification of an unknown linear system with input delay have been presented. The effect of a delay mismatch on a learning model which possessed only adjustable input and feedback gains was investigated. It was shown that although asymptotic stability held only for zero mismatch delay, the error vector eventually entered and stayed well within a predicted region. It was further shown by simulations that when the delay mismatch was in some region around zero the adjustable gains moved into areas around values that reduced the error. This region of initial delay mismatch tended to be centered around some negative value of delay mismatch. The width of the region and its center value seem to be inverse functions of bandwidth. Outside the region the gains varied around zero.

A method of adjusting delays was developed similar to previous methods of adjusting parameters. The initial method developed guaranteed reductions of error to zero and, when suitable conditions on the input were present, also guaranteed the convergence of the models parameters to those of the unknown system. Certain variables necessary for the implementations of the delay adjustment laws were inaccessible and approximations to the laws needed to be made.
The derivatives of the inputs were considered as candidates for use in the delay adjustment laws. The justification for use of the input derivatives comes from a Taylor's series expansion of a difference term in the original delay adjustment laws. When the terms of higher order than the first derivative can be neglected, the laws obtained cause the adjustable delays and thus the adjustable parameters to converge to their correct values.

The approximate derivatives of the input were obtained by use of filters. These signals were considered for use when the input derivatives were not available. The use of the approximate derivative filter enlarges the region of delay mismatch where the system will converge over what was observed when the input derivative was used.

The maximum delay mismatch for which the above method performed well seemed to be dependent on input bandwidth. The effects of bandwidth on these systems should be further investigated. Further work should be done to minimize the effects of signal bandwidth. They may include other laws to control the adjustable delays.

Finally, only a special type of unknown system with delay was examined. The system examined contained only input delays. The extension of the methods presented in this paper to other types of systems with delay should be explored.
REFERENCES


FIGURE NUMBERS AND CAPTIONS

Figure 1  Model-Reference Adaptive Control System
Figure 2  Identification with a Learning Model
Figure 3  Identification with a Learning Model of a System Containing Delay
Figure 4  Parameter and Error Trajectories When Delay Mismatch is Zero
Figure 5  Parameter and Error Trajectories When Delay Mismatch is 0.100 Seconds
Figure 6  Parameter and Error Trajectories When Delay Mismatch is -0.192 Seconds
Figure 7  Parameter and Error Trajectories When Delay Mismatch is -0.100 Seconds
Figure 8  Behavior of One Parameter as Delay Mismatch is Slowly Swept. (Bandwidth of 1.5 Hz)
Figure 9  Behavior of One Parameter as Delay Mismatch is Slowly Swept. (Bandwidth of 5 Hz)
Figure 10 Identification with a Learning Model Containing Adjustable Delays
Figure 11 Trajectories for the Identification System Using the Input Derivative Delay Adjustment Laws, Initial Delay Mismatch of $\Delta T = +0.1$ Second.
Figure 12 Trajectories for the Identification System Using the Input Derivative Delay Adjustment Laws, Initial Delay Mismatch of $\Delta T = -0.1$ Seconds
Figure 13 Trajectories for the Identification System Using the Approximate Input Derivative Delay Adjustment Laws, Initial Delay Mismatch of $\Delta T = 0.1$ second
Figure 14 Trajectories for the Identification System Using the Approximate Input Derivative Delay Adjustment Laws, Initial Delay Mismatch of $\Delta T = -0.1$ second
Figure 15  Trajectories for the Identification System Using the Approximate Input Derivative Delay Adjustment Laws, Initial Delay Mismatch of \( \Delta T = 0.192 \) Second

Figure 16  Trajectories for the Identification System Using the Approximate Input Derivative Delay Adjustment Laws, Initial Delay Mismatch of \( \Delta T = -0.192 \) Second

Figure A-1  \( \hat{A}(t), \hat{B}(t) \) and \( e(t) \) for Identification System.

Figure A-2  \( \hat{A}(t), \hat{B}(t) \) and \( e(t) \) for Identification System with Feedforward Around Parameter Loops.

Figure C-1  \( \hat{A}(t), \hat{B}(t), e(t) \) and \( \Delta T(t) \) for Derivative Delay Correction Law, \( \Delta T(0) = -0.192 \) Seconds.

Figure C-2  \( \hat{A}(t), \hat{B}(t), e(t) \) and \( \Delta T(t) \) for Derivative Delay Correction Law, \( \Delta T(0) = -0.32 \) Seconds.

Figure C-3  \( \hat{A}(t), \hat{B}(t), e(t) \) and \( \Delta T(t) \) for Derivative Delay Correction Law, \( \Delta T(0) = 0.192 \).

Figure C-4  \( \hat{A}(t), \hat{B}(t), e(t) \) and \( \Delta T(t) \) for Derivative Delay Correction Law, \( \Delta T(0) = -1.28 \).

Figure C-5  \( \hat{A}(t) \) and \( \hat{B}(t) \) for Derivative Correction Law, \( \Delta T(0) = +.192 \) Seconds, Bandwidth = 0.5Hz.

Figure C-6  \( e(t) \) and \( \Delta T(t) \) for Derivative Correction Law, \( \Delta T(0) = +.192 \) Seconds, Bandwidth = 0.5Hz.

Figure C-7  \( \hat{A}(t), \hat{B}(t), e(t) \) and \( \Delta T(t) \) for Derivative Delay Correction Law With Feedforward Around Parameter Loops, \( \Delta T(0) = -0.192 \) Seconds.

Figure C-8  \( \hat{A}(t), \hat{B}(t), e(t) \) and \( \Delta T(t) \) for Derivative Delay Correction Law with Feedforward Around Parameter Loops, \( \Delta T(0) = +0.192 \) Seconds.

Figure C-9  \( \hat{A}(t), \hat{B}(t), e(t) \) and \( \Delta T(t) \) for Approximate Derivative Delay Correction Laws Using \( B(t) \), \( \Delta T(0) = 0.304 \) Seconds.
Figure C-10 \( \hat{A}(t), \hat{B}(t), e(t) \) and \( \Delta T(t) \) for Approximate Derivative Delay Correction Law Using \( B(t) \), \( \Delta T(0) = -0.32 \) Seconds.

Figure C-11 \( \hat{A}(t), \hat{B}(t), e(t) \) and \( \Delta T(t) \) for Approximate Derivative Delay Correction Law \( \Delta T(0) = -0.384 \) Seconds.

Figure C-12 \( \hat{A}(t), \hat{B}(t), e(t) \) and \( \Delta T(t) \) for Approximate Derivative Delay Correction Law, \( \Delta T(0) = +1.28 \) Seconds.

Figure C-13 \( \hat{A}(t) \) and \( \hat{B}(t) \) for Approximate Derivative Delay Correction Law, with Feedforward in Parameter Loops, \( \Delta T(0) = +0.384 \) Seconds.

Figure C-14 \( \hat{A}(t), \hat{B}(t), e(t), \) and \( \Delta T(t) \) for Approximate Derivative Delay Correction Law, \( \Delta T(0) = -0.448 \) Seconds, Derivative Filter Gain = 100.

Figure C-15 \( \hat{A}(t), \hat{B}(t), e(t), \) and \( \Delta T(t) \) for Approximate Derivative Delay Correction Law, \( \Delta T(0) = 0.192 \) Seconds, Derivative Filter Gain = 100.

Figure C-16 \( \hat{A}(t), \hat{B}(t), e(t), \) and \( \Delta T(t) \) for Approximate Derivative Delay Correction Law, \( \Delta T(0) = 0.100 \) Seconds.

Figure C-17 \( \hat{A}(t), \hat{B}(t), e(t), \) and \( \Delta T(t) \) for Approximate Derivative Delay Correction Law, \( \Delta T(0) = 0.32 \) Seconds.

Figure D-1 Sample Time Delays.

Figure D-2 Time Expansion and Compression.
Figure 1
Figure 3
Figure 14
Figure 15
APPENDIX A

SUMMARY OF THE PAZDERA-POTTINGER IDENTIFICATION SCHEME

A-1. General Development

The linear system to be identified is assumed to be described by

\[ \dot{x} = Ax + Bu \]  \hspace{1cm} (A-1)

where \( A \) and \( B \) are unknown matrices, and \( x \) and \( u \) are the state and input vectors.

The unknown system will be identified by a learning model described by

\[ \dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u + v \]  \hspace{1cm} (A-2)

where \( \hat{x} \) is the state vector of the model and \( \hat{A} \) and \( \hat{B} \) are adjustable. The additional input \( v \) is added to the model to aid stability. The parameter matrices are to be adjusted so that \( \hat{A} = A, \hat{B} = B \) and \( v = 0 \). When this condition is reached, the unknown system is considered identified.

The error between the state vector and the unknown system is defined in the natural way

\[ e = x - \hat{x} \]  \hspace{1cm} (A-3)
The derivative of the error can be written

\[ \dot{e} = \Delta Ax + \Delta Bu - v \]  

(A-4)

where

\[ \Delta A = A - \hat{A}(t) \]

and

\[ \Delta B = B - \hat{B}(t) \]

The matrices A and B should be adjusted as to reduce parameter misalignment and error between the state vectors. Therefore the Lyapunov function is chosen to be

\[ 2V = \text{TR}[\Delta A^T E \Delta A] + \text{TR}[\Delta B^T F \Delta B] + e^T Pe \]  

(A-5)

where \( \text{TR}() \) indicates the trace of the matrix and the superscript 'T' indicates the transpose of a matrix or vector.

The matrices E, F, and P are constant real symmetric positive definite matrices.

It can be seen that V is positive definite in parameter misalignment and error. The derivative of V is

\[ \dot{V} = \text{TR}[- \Delta A^T E \dot{A}] + \text{TR}[-\Delta B^T F \dot{B}] + e^T Pe \]  

(A-6)

Substitution of (A-4) into equation (A-6) and using the relationship between products of vector, i.e.

\[ x^T y = \text{TR}(xy^T) \]
the equation for \( V \) becomes

\[
\dot{V} = \text{TR}[\Delta A^T (-E \hat{A} + Pe x^T)] + \text{TR}[\Delta B^T (-F \hat{B} + Pe u^T)]
- e^T P \bar{V}.
\]  \( \text{(A-7)} \)

The first two terms of \( \text{(A-7)} \) are indefinite but can be removed if the adjustment laws are chosen to be

\[
\hat{A} = E^{-1} Pe x^T 
\]  \( \text{(A-8)} \)

and

\[
\hat{B} = F^{-1} Pe u^T 
\]  \( \text{(A-9)} \)

The last remaining term in \( \text{(A-7)} \) can be made negative definite in \( \bar{e} \) if

\[
\bar{e} = -De 
\]  \( \text{(A-10)} \)

where \( D \) is a matrix whose eigenvalues all have negative real parts.

Then

\[
\dot{V} = e^T \left( PD + \frac{D^T P}{2} \right) e = - e^T Q e 
\]  \( \text{(A-11)} \)

where \( Q \) is a positive definite symmetric matrix satisfying the relationship

\[
PD + D^T P = -2Q
\]  \( \text{(A-12)} \)
Equation (A-11) guarantees that the error vector will approach and eventually stay at zero. It must be further argued that the parameter misalignments will also go to zero.

Equation (A-10) with equation (A-4) show the relationship between parameter misalignment and the error vector. The relationship is

\[ e = -Qe + \Delta Ax + \Delta Bu \]  

(A-13)

The error vector can remain at zero only if

\[ \Delta Ax + \Delta Bu = 0 \]  

(A-14)

for all time.

If the inputs are random such that the elements of \( x \) and \( u \) are linearly independent functions of time, then equation (A-14) can be satisfied over an interval only if

\[ \Delta A = [0] \]

and

\[ \Delta B = [0] \]

A-2. Example A-1

The system to be identified was chosen to be

\[ \dot{x} = -2x + .6u \]
The input is a pseudorandom signal of 3.16 volts rms and bandwidth of 1.5 Hz.

The adjustment laws were found to be

\[ A = \mu_A \text{ex} \]
\[ B = \mu_B \text{eu} \]

and

\[ v = K_f e \]

where the gains were chosen to be

\[ \mu_A = E^{-1}p = 10.0 \]
\[ \mu_B = F^{-1}p = 1.00 \]
\[ K_f = PD = 10.0 \]

The results of the simulation are shown in Figure A-1.

A-3. Improvement of the Dynamics of the Method Through Feedforward

It has been found that increasing the parameter adjustment gains, \( \mu_A \) and \( \mu_B \), produces a faster and more oscillatory action in the dynamics of the system. An increase in \( K_f \), however, shows the action and dampens the oscillations.
Phillipson[26] has suggested feed-forward in the parameter adjustment laws to improve the dynamics of a model reference adaptive system. His method can be applied to this adjustment scheme. Pazdera[31] also discusses the use of feedforward to improve response in a different paper.

Consider a new model

\[ \dot{\hat{z}} = (\hat{A} + \gamma_A)\dot{x} + (\hat{B} + \gamma_B)\dot{u} + v \]

If the Lyapunov function is chosen as previously and a similar procedure is followed in equation (A-7), the following equation is produced.

\[ \dot{V} = \text{TR}[\Delta A(-EA + P_e x^T)] + \text{TR}[\Delta B^T(-FB + P_e u^T)] \]

\[ - e^T P_v - e^T P \gamma_A x - e^T P \gamma_B u. \]

If again the adjustment laws are chosen to be

\[ \dot{\hat{A}} = E^{-1} P e x^T \]

and

\[ \dot{\hat{B}} = F^{-1} P e u^T \]

the expression for \( V \) becomes

\[ \dot{V} = -e^T P \gamma_A x - e^T P \gamma_B u - e^T P v \]

If \( v \) is chosen to be

\[ v = -De \]

where \( D \) was previously defined, and if
\[ \gamma_A = R \hat{A} \]

and

\[ \gamma_B = SB \]

where \( R \) and \( S \) are positive definite symmetric matrices, then

\[ \dot{\mathbf{v}} = -e^T \text{PRE}^{-1} Pe \mathbf{x}^T \mathbf{x} - e^T \text{PSF}^{-1} Pe \mathbf{u}^T \mathbf{u} - e^T \mathbf{Q} \mathbf{e} \]

where \( Q \) was previously defined.

Since the matrices \( E, F, \) and \( P \) are positive definite symmetric forms then the products between these matrices are also positive definite symmetric forms.

The expression for \( \mathbf{v} \) becomes

\[ \dot{\mathbf{v}} = -e^T \mathbf{M} e \mathbf{x}^T \mathbf{x} - e^T \mathbf{N} e \mathbf{u}^T \mathbf{u} - e^T \mathbf{Q} \mathbf{e} \]

where

\[ \mathbf{M} = \text{PRE}^{-1} \mathbf{P} \]

and

\[ \mathbf{N} = \text{PSF}^{-1} \mathbf{P}. \]

The expression for \( \mathbf{v} \) contain additional terms which contribute to the magnitude of \( \mathbf{v} \) and thus the rate of convergence of \( \mathbf{v} \).
A-4. Example A-2

The previous example was again simulated using

\[ E^{-1}p = \gamma_A = 1.00 \]

\[ F^{-1}p = \gamma_B = 1.00 \]

\[ PD = K_f = 10.0 \]

\[ R = 1.0 \]

\[ S = 1.0 \]

The results of the simulation are shown in Figure A-2 and Figure B-2c. It can be seen from a comparison of Figures A-1, and A-2 that the feedforward in the parameter adjustment loops caused a decrease in the oscillation of the system.
Figure A-1
APPENDIX B

CONDITIONS FOR THE LINEAR INDEPENDENCE OF THE ELEMENTS OF x(t) AND u(t)

If $K_x^T x(t) + K_u^T u(t) = 0$ for all times implies $K_x^T = 0$ and $K_u^T = 0$, then the elements of x(t) and u(t) are linearly independent functions of time.

Theorem: If for the system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

(a) the elements of u(t) are linearly independent functions of time,

(b) the elements of u(t) are random functions of time and

(c) the system is controllable,

then the elements of x(t) and u(t) are linearly independent functions of time with probability one.

Proof: The elements of x(t) and u(t) cannot be linearly independent functions of time without the elements of u(t) being linearly independent functions of time.

In the absence of initial conditions, the convolution equation for the system is
\[ x(t) = \int_{0}^{t} e^{A(t - \tau)} B_u(\tau) d\tau \]

\[ = P \int_{0}^{t} e^{J(t - \tau)} Z(\tau) d\tau \]

where

\[ Z(\tau) = P^{-1} B_u(\tau), \]

\[ A = PJP^{-1} \]

and \( P \) is non-singular

If the system is controllable, then no element of

\[ w(t) = \int_{0}^{t} e^{J(t - \tau)} Z(\tau) d\tau \]

can remain at zero for all arbitrary inputs.

In addition, the elements of \( w(t) \) will be linearly independent if the inputs are sufficiently random. The input should have more components than just the solutions to the integral equation.

\[ K_3^T \int_{0}^{t} e^{J(t - \tau)} Z(\tau) d\tau = 0 \]

where the elements of \( K_3^T \) are not all zero.
Linear independence is preserved under a nonsingular linear transformation and, therefore, under the above conditions the elements of \( \mathbf{x}(t) \) are linearly independent functions of time.

i.e. \( K^T \mathbf{x} = 0 \) for all time only if \( K^T = [0, 0 \ldots 0] \)

If the inputs are further restricted to contain more components than the solutions to the previous and also the following integral equation

\[
K_4^T \mathbf{u}(t) = K_5^T \int_0^t e^{A(t - \tau)} B \mathbf{u}(\tau) d\tau.
\]

where the elements of \( K_4 \) and the elements of \( K_5 \) are not all zero, then

\[
K_1^T \mathbf{u} = K_2^T \int_0^t e^{A(t - \tau)} B \mathbf{u}(\tau) d\tau \quad \text{for all time}
\]

only if

\[
K_1^T = 0
\]

and

\[
K_2^T = 0.
\]

The probability of any particular time function in a random signal is zero. Therefore, the probability of the inputs having time functions other than the solutions to the integral equations is 1. The elements of \( \mathbf{u} \) and \( \mathbf{x} \) are linearly independent with probability 1. Q.E.D.
Under the above conditions, the probability of
\[ \Delta A x(t) + \Delta B u(t) = 0 \]
for all time with matrices other than
\[ \Delta A = [0] \]
and
\[ \Delta B = [0] \]
is zero.

The above proof has shown that convergence will occur under the stated conditions. However, the identification system may still converge if the conditions are not satisfied.

Under some additional restrictions, the above proof can be applied to the cases where delay is involved. The elements of \( u(t,T) \) and \( \hat{u}(t,T) \) are restricted to be linearly independent for all \( T \neq \hat{T} \). \( \Delta T \) is slowly varying.

To help show this, the elements of \( G(t) \) will be arranged to form a vector \( \underline{\alpha}(t) \)

\[
\underline{\alpha}(t) = \begin{bmatrix}
g_1(t) \\
g_2(t) \\
\vdots \\
g_m(t)
\end{bmatrix}
\]

If the elements of \( u(t,T) \) and \( \hat{u}(t,T) \) are linearly independent for all \( T \neq \hat{T} \) and if \( \Delta T \) is slowly varying then the elements of \( \underline{u}(t,T) \) and \( \underline{\alpha}(t) \) are linearly independent functions of time since
The linear independence with probability 1 of the elements of \( x(t) \) can be shown as previously. Since the elements of \( x(t), u(t, T) \) and \( a(t) \) are linearly independent functions of time,

\[
\alpha(t) = \begin{bmatrix}
\frac{u_1(t - T_1) - u_1(t - T)}{\Delta T_1} \\
\vdots \\
\frac{u_m(t - T_m) - u_m(t - T)}{\Delta T_m}
\end{bmatrix}
\]

The condition that the elements of \( u(t, T) \) and \( u(t, \hat{T}) \) be linearly independent for all \( T \neq \hat{T} \) also rules out the case of a non-unique solution for \( u_1(t - T_1) = u_i(t - T_i), i \neq 1 \).
APPENDIX C

DETAILED MATHEMATICAL DEVELOPMENTS AND ADDITIONAL SIMULATION RESULTS

C-1. Derivation of equation (16)

Equation 11 repeated here is

\[ \dot{V} = -e^T Q e + e^T K_1 B(u(t,T) - u(t,^\hat{T})) \]

Let's define

\[ \Delta = B(u(t,T) - u(t,^\hat{T})) \]

then

\[ \dot{V} = -e^T Q e + e^T K_1 \gamma \]

\[ \leq -e^T Q e + ||e^T K_1 \gamma|| \leq -e^T Q e \]

\[ + ||e^T K_1|| ||\gamma|| \]

\[ \leq -e^T Q e + ||e^T K_1|| \Gamma \]

where

\[ ||\gamma|| < \Gamma \]

But

\[ -e^T Q e + ||e^T K_1|| \Gamma \leq \lambda(Q)_{\text{min}} ||e||^2 \]

\[ + \lambda(K_1)_{\text{max}} ||e||\Gamma \]
therefore if 

\[ \| e \| > \frac{\lambda(K_1)^{\max}}{\lambda(Q)^{\min}} \gamma \]

V will be negative.

C-2. Derivative Adjustment Law - Additional Simulations

a. Effects of Initial Delay Mismatches

Figures C-1 through C-4 show the effects of different initial delay mismatches on the example used in Section III. Figures C-1 and C-2 show stable convergence for initial delay mismatches of \( \Delta T = -0.192 \) seconds and \( \Delta T = -0.32 \) seconds. In previous examples it was shown that the system converges for delay mismatches of 0.1 second and -0.1 second. Figure C-3 and C-4 show that the system is unstable and does not converge for initial delay mismatches of 0.192 second and -1.28 second. The range of initial delay mismatches for which this particular system will converge lies between \( \Delta T = -1.28 \) second and \( \Delta T = 0.192 \) second. The input for all cases was pseudorandom noise with 1.5Hz. bandwidth and 0.316 volt RMS magnitude.

b. Effect of Bandwidth

It was noticed during the simulation of the system in this paper, that the bandwidth of the input signal strongly affected the characteristics of the trajectories.
and even the stability of the system. Figures C-5 and C-6 show the effects of reducing input bandwidth on a system which was previously unstable. These trajectories can be compared with those of Figure C-3. The initial delay mismatch for both sets of trajectories is $\Delta T = -0.64$ seconds. The bandwidth of the input signal for Figures C-6 and C-5 is 0.5Hz. It was previously stated that the input bandwidth for Figure C-3 was 1.5Hz. Although the system now converges for the bandwidth of 0.5Hz., it does so at a slower rate.

c. Effect of Feedforward

Feedforward in the parameter adjustment loop was added as discussed in Appendix A (A-3). By comparison of Figures C-7a, b and c with Figures C-1a, b and c it can be seen that the addition of feedforward causes a reduction in the oscillations of the trajectories. A comparison of Figures C-8a, b and c with Figures C-3a, b and c shows that feedforward aids stability and increases the range of initial delay mismatches that will converge.
C-3. Approximate Derivative Adjustment Law - Additional Simulations

a. Effects of Initial Delay Mismatch

As the region of initial delay mismatch for the example of Section IV was expanded, eventually values were reached for which the identification scheme failed to work. Figures C-9 and C-10 show that the identification scheme converges for initial delays of $\Delta T(0) = +0.384$ seconds and for $\Delta T(0) = -0.32$ seconds. Simulations were repeated for $\Delta T(0) = -.640$ seconds and $\Delta T(0) = 0.768$ seconds and the identification system did not converge. The range of initial delay mismatches for which this particular system will converge lies from about $\Delta T(0) = -0.384$ to about $\Delta T(0) = 1.28$ seconds. Figures C-11 and C-12 show that the system converges slowly for the large positive delay mismatch.

Comparison with Figures C-3 and C-4 shows that the region of delay mismatch has been enlarged and now lies centered in the positive area.

The input for all cases was pseudorandom noise with 1.5Hz bandwidth and .316 volt rms magnitude.

b. Effects of Feedforward

Feedforward in parameter loops was tried as discussed in Appendix A. Figure C-13 shows $\hat{\theta}(t)$ and $\hat{A}(t)$ for an
initial delay mismatch of $\Delta T(0) = 0.384$ seconds. A comparison of these trajectories with those for no feedforward shown in Figure C-9, shows that feedforward decreased the oscillation of the trajectories. However, the addition of feedforward seems to have decreased the rate of convergence. For this simulation the feedforward gains were $R = S = 1.0$.

c. Effects of Derivative Filter Gain

The filter gain on the derivative filter was varied between $K = 1$ and $K = 100$. For the filter gain of $K = 1$ the system was unstable and never settled down. The derivative filter gain of $K = 10$ was used for most of the simulations of this paper. As the derivative gain is increased further, the system begins to take some of the characteristics of the system where the actual input derivative was used. Since the approximate derivative becomes more and more accurate as the filter gain is increased, this is not difficult to understand. Figures C-14 and C-15 show the system trajectories for delay mismatches of $\Delta T(0) = -0.448$ seconds and $\Delta T(0) = +0.192$ seconds. The derivative filter gain here was $K = 100$. The range of initial delay mismatches extended from about 0.448 seconds to about +0.192 seconds. This range was centered on the negative side as it was for the system using the actual input derivative.
d. Use of the \( \hat{B}(t) \) Matrix in Delay Adjustment Law

The delay adjustment laws developed by steepest descent method contain an adjustable parameter matrix\([17]\). This was done for the system used in previous examples. Figure C-16 shows the effect of this law for a delay mismatch of \(+.1\) second. Again the approximate derivative of \( \hat{B}u(t - T) \) is used due to the difficulty of obtaining \( \hat{B}u(t - T) \).

The constant in the derivative filter was chosen to be 10.0. For the delay mismatch of .1 second the system behaves much like the system where a constant was used in the adjustment law. However, for a larger delay mismatch, the system's rate of convergence becomes slow. Figure C-17 shows the trajectories for \( \Delta T(0) = +.32 \) seconds. Better behavior might have been obtained if \( \hat{B} \) multiplied the approximate derivative of \( u \) in the control law.
Figure C-10
Figure C-11
Figure C-13
APPENDIX D

TRANSPORT DELAY PROGRAM

The program described herein is constructed to provide two independently delayed versions of a single input signal. The program is written for the SCC 650 digital computer and uses the A-D (Analog to Digital) and D-A (Digital to Analog) converter.

The two delayed signals are sampled (stairstep) approximations to the input signal. The lengths of the delays can be controlled by two additional input voltages. The delay times are also affected by the sample rate which is set by the switch register of the SCC 650.

The delay is accomplished by sampling the signal to be delayed at fixed intervals, storing the samples in the computer, and returning the samples to the analog outputs at later times.

Use of the Program

The program is loaded with the Absolute Loader. The starting address for the program is 00008. Before running the program, the sample rate should be set at its maximum (Switch Register - 77778). The sample rate can be reduced to the desired value while the program is running. If the Switch Register is initially set at 00008, the sample...
rate will not change for several minutes. If this mistake is made, stop the program, load 7777\(_8\) in locations 0063\(_8\), 0064\(_8\) and 0065\(_8\), and restart the program with 7777\(_8\) in the Switch Register. If faster sample rates than provided by the existing program are desired, 2102\(_8\) should be loaded at storage location 0054\(_8\).

A-D channel 1 is the input for the signal to be delayed. D-A channels 1 and 2 are the delayed signal outputs. The delay on D-A channel 1 is controlled by the voltage on A-D channel 2. D-A channel 2 delay is controlled by the voltage on A-D channel 3. Zero delay is achieved by supplying +10 volts to A-D channels 2 and 3. Maximum delay occurs when -10 volts is supplied. The A-D and D-A channels have terminals on the hybrid interface.

The switches on the Multiplexer Adapter should be set as follows:

- **POWER switch**: ON
- **FULL SCALE switch**: +10 volts
- **MODE switch**: EXT
- **SAMPLE RATE switch**: 

Operation of the Program

The signal on A-D channel 1 is sampled at fixed intervals and the sample is stored in two's complement form at one end of a storage block. Before the channel is resampled, all samples in the storage area are shifted one
location toward the opposite end of the storage block. Samples are discarded after they reach the opposite end.

The signals on A-D channels 2 and 3 are sampled and converted into addresses of samples in the storage area. The samples at these locations in memory are converted to voltages which appear on D-A channels 1 and 2.

The sample rate is controlled with a time consuming routine. The amount of time consumed is selected with the Switch Register.

The program which does the above steps is in memory locations 0000\textsubscript{8} to 0077\textsubscript{8}. The storage block used for samples of the input signal is from 4000\textsubscript{8} to 7777\textsubscript{8}. The instructions in the program are explained as follows (see listing on page ).

Locations 0000\textsubscript{8} to 0006\textsubscript{8} A-D converter is selected and channel 1 is sampled. The sample is stored at 7777\textsubscript{8}.

Locations 0007\textsubscript{8} to 0016\textsubscript{8} A-D channel 2 is sampled and the sample is converted into an address in the storage block (ADRS 1).

Locations 0017\textsubscript{8} to 0026\textsubscript{8} A-D channel 3 is sampled and the sample is converted into an address in the storage block (ADRS 2).

Locations 0027\textsubscript{8} to 0030\textsubscript{8} Sample rate control word is removed from switches and stored.
Locations 0031_8 to 0037_8  The sample in the storage block at ADRS 1 is converted to a voltage which appears on D-A channel 1.

Locations 0040_8 to 0044_8  The sample in the storage block at ADRS 2 is converted to a voltage which appears on D-A channel 2.

Locations 0045_8 to 0053_8  All samples in the storage block are moved by one location.

    i.e. The sample at 4001_8 is moved to 4000_8,
        the sample at 4002_8 is moved to 4001_8, etc.

Locations 0054_8 to 0065_8  This routine consumes time before the program repeats from 0000_8.
** TRANSPORT DELAY **

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TRANSPORT DELAY
SELECT A-D
CHOOSE CHANNEL L
LOAD INPUT S(T)
STORE AT "LOC"
CHOOSE CHANNEL 2
LOAD OUTPUT ADRS 1 (D1)
ADD TO MID ADDRESS
CHOOSE CHANNEL 3
LOAD OUTPUT ADRS 2 (D2)
ADD TO MID ADDRESS
CHOOSE D-A CHANNEL 1
OUTPUT S(T-D1)
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0000 ERRORS IN PASS 2
EXAMPLE 1

Simple Delays
A-D Channel 1
signal from noise generator
A-D Channel 2
+5 volts
A-D Channel 3
0 volts
Switch Register set at $777_8$

EXAMPLE 2

Time Expansion and Compression
A-D Channel 1
signal from noise generator
A-D Channel 2
10 - .1t volts
A-D Channel 3
0 + .1t volts
Switch register set at $777_8$

Example 1 is demonstrated in Figure D-1. The top curve is the input from the pseudorandom noise generator. The middle curve is from D-A Channel 1 and is the delayed signal controlled by A-D Channel 2. The output of D-A Channel 2 is shown at the bottom. The delay of this channel is controlled by A-D channel 2.

Time compression and expansion is shown in Figure D-2. Again the top is the input signal. The output of D-A channel 1 controlled by A-D channel 1 is in the middle and the output of D-A channel 2 controlled by A-D channel 2 is on bottom.
Figure D-1
VITA

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