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Optimal Diversity Combining Based on Linear Estimation of Rician Fading Channels

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***Abstract*—Optimal receiver diversity combining employing linear channel estimation is examined. Based on the statistical properties of pilot-assisted least-squares (LS) and minimum mean square error (MMSE) channel estimation, an optimal diversity receiver for wireless systems employing practical linear channel estimation on Rician fading channels is proposed. Exact analytical expressions for the symbol error rates of LS and MMSE channel estimation aided optimal diversity combining are derived. It is shown that an MPSK wireless system with MMSE channel estimation has the same SER when the MMSE channel estimation is replaced by LS estimation. This is an interesting counter-example to the common perception that channel estimation with smaller mean square error leads to smaller SER. Extensive simulation results validate the theoretical results.**

I. Introduction

Diversity reception is a classical method used in wireless communication systems for combating the deleterious effects of multipath fading. Most previous performance analyses of coherent diversity systems assume that the receiver has perfect knowledge of the fading channels. However, this assumption is too idealistic for practical wireless systems. In order to maximize the efficacy of practical diversity system design, it is highly desirable to have analytical models for systems operating with practical channel estimation. Recently, considerable attention has been paid to the study of non-ideal systems [1]-[9]. In [1], the effect of Gaussian error in maximal ratio combining (MRC) was studied, but digital modulations and error probability were not considered. The bit error rate of conventional MRC receiver in system with channel estimation error is discussed in [2].

Modified MRC receivers with improved performances were developed in [6], [7] by taking into consideration the statistics of channel estimation errors. The receivers in [6], [7] outperform the conventional MRC receiver owing to the use of additional information from channel estimation. The analysis in all the aforementioned works is based on an assumption of noisy channel estimation, where the channel estimation is conveniently modeled as a sum of true channel gain and independent, Gaussian distributed estimation noises.

In this paper, error probability performance is analyzed for optimal coherent diversity receivers operating in independent and identically distributed (i.i.d.) Rician fading channels, with practical pilot assisted linear channel estimation schemes. The properties of least-squares (LS) and minimum mean square error (MMSE) channel estimation are investigated, and analytical expressions are provided to describe the statistical relationship between channel estimation error and pilot symbol power. It is shown that, under certain system configurations, the conventional MRC receivers is no longer optimum at the presence of channel estimation error. A new optimal decision rule for coherent diversity receivers is proposed by taking into account the effect of channel estimation errors. Exact error probability expressions for the proposed optimal coherent diversity receivers employing both LS channel estimation and MMSE channel estimation in M-ary phase shift keying (MPSK) systems are derived. Due to the presence of channel estimation error, the classical moment generating function (MGF) and characteristic function (CHF) methods cannot be directly applied in the error performance analysis. Instead, a complex Gaussian distribution-based functional equivalency is employed for the evaluation of error probabilities.

Interestingly, both analytical and simulation results show that the wireless MPSK system with LS channel estimation has the same error probability as the system with MMSE channel estimation replacing LS channel estimation, even though MMSE channel estimation outperforms LS channel estimation in terms of mean square error.

The rest of this paper is organized as follows. The statistics of pilot assisted linear channel estimation are investigated in Section II. Section III derives an optimal decision rule for diversity receivers operating with linear estimation of i.i.d. fading channels. The error probabilities of the receivers in Rician fading channels are derived in Section IV. Numerical examples are given in Section V, and Section VI concludes the paper.

II. Pilot Assisted Linear Channel Estimation

Consider a wireless communication system with one transmitter and N diversity receivers, which employs pilot assisted linear channel estimators. The equivalent discrete-time baseband system can be represented in matrix form as

$$\mathbf{r} = \mathbf{h} \cdot s + \mathbf{z} \tag{1}$$

where $\mathbf{r} = [r_1, r_2, \cdots, r_N]^T \in \mathbb{C}^{N \times 1}$ are the discrete-time signal samples at the receivers, with $\mathbf{A}^T$ representing the transpose of matrix $\mathbf{A}$, $\mathbf{h} = [h_1, h_2, \cdots, h_N]^T \in \mathbb{C}^{N\times 1}$ is the equivalent discrete-time channel gain (CG) vector of the physical fading, s is the MPSK modulated data or pilot symbol, and $\mathbf{z} = [z_1, z_2, \cdots, z_N]^T \in \mathbb{C}^{N\times 1}$ is a zero-mean additive white Gaussian noise vector with covariance matrix $N_0 \cdot \mathbf{I}_N$, and $\mathbf{I}_N$ is the $N\times N$ identity matrix. For Rayleigh and Rician fading, the discrete-time CG vector $\mathbf{h}$ contains complex Gaussian random variables (CGRVs) with mean vector $\mathbf{u}$ and covariance matrix $\mathbf{\Phi}_{hh}$, *i.e.*, $\mathbf{h} \sim \mathcal{N}(\mathbf{u}, \mathbf{\Phi}_{hh})$. For i.i.d channels, the m-th branch fading h_m and the n-th branch fading h_n have the same statistical properties. The mean value u, variance σ_h^2, and average power Ω of h_n have the following relationship

$$|u| = \sqrt{K\sigma_h^2} = \sqrt{\frac{K\Omega}{K+1}} \tag{2}$$

where K is the Rice factor defined as the ratio of the powers of the specular component and the scattering components of the fading. For Rayleigh fading channels, one has $K = 0$ and $u = 0$.

In a coherent receiver, the data symbols are detected based on the received samples and the estimated CG vector $\hat{\mathbf{h}} = [\hat{h}_1, \hat{h}_2, \cdots, \hat{h}_N]^T \in \mathbb{C}^{N\times 1}$. The statistical relationship between $\hat{\mathbf{h}}$ and $\mathbf{h}$ in a system with pilot-assisted LS channel estimation or MMSE channel estimation are described in the next two subsections.

A. Least-Squares Channel Estimation

The estimated CG vector that minimizes the LS cost function is [10]

$$\hat{\mathbf{h}} = \frac{\mathbf{r}_p}{s_p} = \mathbf{h} + \mathbf{e} \tag{3}$$

where s_p is a pilot symbol with energy $E_p = \mathbb{E}\left[|s_p|^2\right]$, $\mathbf{r}_p = \mathbf{h}\cdot s_p + \mathbf{z}$ is the received sample vector of the pilot symbol, and $\|\mathbf{a}\| = \sqrt{\mathbf{a}^H\mathbf{a}}$ is the Euclidean norm of the column vector $\mathbf{a}$. The vector $\mathbf{e} = \hat{\mathbf{h}} - \mathbf{h} = \frac{1}{s_p}\mathbf{z}$ is the estimation error vector, which is zero-mean Gaussian distributed with covariance matrix $\mathbf{\Phi}_{ee} = \frac{\Omega}{\gamma_p}\cdot \mathbf{I}_N$, with $\gamma_p = \frac{E_p\Omega}{N_0}$ being the received signal-to-noise ratio (SNR) of the pilot symbol. The channel estimation error vector $\mathbf{e}$ is independent of the true CG vector $\mathbf{h}$. Therefore, LS channel estimation can be said to fall in the category of noisy channel estimation.

The estimated CG vector $\hat{\mathbf{h}}$ and $\mathbf{h}$ are jointly Gaussian distributed. The conditional mean, $\mathbf{u}_{h|\hat{h}}$, and conditional covariance matrix, $\mathbf{\Phi}_{h|\hat{h}}$, are given by

$$\mathbf{u}_{h|\hat{h}} = \mathbf{u} + \mathbf{\Phi}_{h\hat{h}}\mathbf{\Phi}_{\hat{h}\hat{h}}^{-1}(\hat{\mathbf{h}} - \hat{\mathbf{u}}) \tag{4a}$$

$$\mathbf{\Phi}_{h|\hat{h}} = \mathbf{\Phi}_{hh} - \mathbf{\Phi}_{h\hat{h}}\mathbf{\Phi}_{\hat{h}\hat{h}}^{-1}\mathbf{\Phi}_{\hat{h}h} \tag{4b}$$

where $\hat{\mathbf{u}}$ and $\Phi_{\hat{h}\hat{h}}$ are the mean vector and covariance matrix of the estimated CG vector $\hat{h}$, and $\mathbf{\Phi}_{h\hat{h}} = \mathbf{\Phi}_{\hat{h}h}^H$ are the covariance matrices of $\hat{\mathbf{h}}$ and $\mathbf{h}$. Eqn. (4) is readily obtained from the definition of conditional pdf [12, pp.534-535].

For LS channel estimation, the mean of $\hat{\mathbf{h}}$ is $\hat{\mathbf{u}} = \mathbb{E}(\hat{\mathbf{h}}) = \mathbf{u}$. The covariance matrices $\mathbf{\Phi}_{\hat{h}\hat{h}} = \mathbb{E}\left(\hat{\mathbf{h}}\hat{\mathbf{h}}^H\right) = \mathbf{\Phi}_{hh} + \mathbf{\Phi}_{ee}$ and $\mathbf{\Phi}_{h\hat{h}} = \mathbb{E}\left[(\mathbf{h}-\mathbf{u})(\hat{\mathbf{h}}-\mathbf{u})^H\right] = \mathbf{\Phi}_{hh} + \mathbf{\Phi}_{ee}$. Substituting the above results into (4), one has

$$\mathbf{u}_{h|\hat{h}} = \mathbf{u} + \mathbf{\Phi}_{hh}(\mathbf{\Phi}_{hh} + \mathbf{\Phi}_{ee})^{-1}(\hat{\mathbf{h}} - \mathbf{u}) \tag{5a}$$

$$\mathbf{\Phi}_{h|\hat{h}} = \mathbf{\Phi}_{hh} - \mathbf{\Phi}_{hh}(\mathbf{\Phi}_{hh} + \mathbf{\Phi}_{ee})^{-1}\mathbf{\Phi}_{hh}. \tag{5b}$$

For a system with i.i.d. fading, $\mathbf{u}_{h|\hat{h}}$ and $\mathbf{\Phi}_{h|\hat{h}}$ can be further simplified to

$$\mathbf{u}_{h|\hat{h}} = \mathbf{u} + \rho^2(\hat{\mathbf{h}} - \mathbf{u}) \tag{6a}$$

$$\mathbf{\Phi}_{h|\hat{h}} = \sigma_h^2(1-\rho^2)\cdot \mathbf{I}_N \tag{6b}$$

where ρ is the covariance coefficient between h_n and $\hat{h}_n$

$$\rho \triangleq \frac{\mathbb{E}\left[(h_n - u)(\hat{h}_n - u)^*\right]}{\sqrt{\sigma_h^2\sigma_{\hat{h}}^2}} = \sqrt{\frac{\gamma_p}{\gamma_p + K + 1}}. \tag{7}$$

In (7), a^* denotes the complex conjugate of the complex-valued number a, $\sigma_h^2 = \mathbb{E}\left(|h_n - u|^2\right)$ and $\sigma_{\hat{h}}^2 = \mathbb{E}\left(|\hat{h}_n - u|^2\right)$ are the variance of h_n and $\hat{h}_n$, respectively. The value of ρ is in the interval $[0, 1]$ with $\rho = 1$ (or $\gamma_p = \infty$) corresponding to perfect channel information at the receiver.

B. Minimum Mean Square Error Channel Estimation

The MMSE estimation of the CG vector $\hat{\mathbf{h}}$ can be expressed as [10]

$$\hat{\mathbf{h}} = \mathbf{W}\cdot(\mathbf{r}_p - s_p\cdot\mathbf{u}) + \mathbf{u} \tag{8}$$

where $\mathbf{u}$ is the mean of $\mathbf{h}$, $\mathbf{r}_p$ is the receiver sample vector of the pilot symbol, and $\mathbf{W}$ is the MMSE weighting matrix. Since the pilot symbol s_p is known to both the transmitter and receiver, $\mathbf{r}_p$ is a CGRV vector with $\mathbf{r}_p \sim \mathcal{N}(s_p\cdot\mathbf{u}, E_p\cdot\mathbf{\Phi}_{hh} + N_0\cdot\mathbf{I}_N)$. The estimated CG vector $\hat{\mathbf{h}}$ is a linear combination of CGRV vectors. Therefore, $\hat{\mathbf{h}}$ and $\mathbf{h}$ are jointly Gaussian distributed, and $\hat{\mathbf{u}} = \mathbb{E}(\hat{\mathbf{h}}) = \mathbf{u}$ from (8). The MMSE weighting matrix, $\mathbf{W}$, can be obtained by utilizing the orthogonality principle, and the result is

$$\mathbf{W} = \mathbf{\Phi}_{hh}\left(\mathbf{\Phi}_{hh}E_p + N_0\mathbf{I}_N\right)^{-1}\cdot s_p^*. \tag{9}$$

Since $\mathbf{h}$ and $\hat{\mathbf{h}}$ are jointly Gaussian distributed, the error vector, $\mathbf{e} = \hat{\mathbf{h}} - \mathbf{h}$ is a zero-mean complex Gaussian random vector with covariance matrix given by [10]

$$\mathbf{\Phi}_{ee} = \left(\mathbf{I}_N + \frac{\gamma_p}{\Omega}\cdot\mathbf{\Phi}_{hh}\right)^{-1}\mathbf{\Phi}_{hh}. \tag{10}$$

It is worth noting that, for MMSE channel estimation, the error vector $\mathbf{e}$ is correlated with the true CG vector $\mathbf{h}$. Thus, an independent noisy estimation assumption does not hold for MMSE channel estimation. The covariance matrix $\mathbf{\Phi}_{he} \triangleq \mathbb{E}\left[(\mathbf{h}-\mathbf{u})\mathbf{e}^H\right]$ is calculated by

$$\mathbf{\Phi}_{he} = \mathbb{E}\left[(\mathbf{h}-\mathbf{u})\mathbf{e}^H\right] - \mathbf{W}^H\mathbb{E}\left[(\mathbf{r}_p - s_p\mathbf{u})\mathbf{e}^H\right] = -\mathbf{\Phi}_{ee} \tag{11}$$

where the first equality is based on the orthogonality principle.

From (11), one has $\mathbf{\Phi}_{h\hat{h}} = \mathbf{\Phi}_{\hat{h}\hat{h}} = \mathbf{\Phi}_{hh} - \mathbf{\Phi}_{ee}$. Substituting this result into (4) leads to the conditional mean, $\mathbf{u}_{h|\hat{h}}$, and conditional covariance matrix, $\mathbf{\Phi}_{h|\hat{h}}$, for MMSE channel estimation,

$$\mathbf{u}_{h|\hat{h}} = \hat{\mathbf{h}} \quad (12a)$$
$$\mathbf{\Phi}_{h|\hat{h}} = \mathbf{\Phi}_{ee}. \quad (12b)$$

For i.i.d. channel fading, the conditional covariance matrix can be simplified to $\mathbf{\Phi}_{h|\hat{h}} = \sigma_h^2(1-\rho^2)\mathbf{I}_N$. This expression is the same as eqn. (6b) and the correlation coefficient ρ of MMSE channel estimation has the same form as that of LS channel estimation given in (7). However, for MMSE channel estimation, the variances of h_n and $\hat{h}_n$ are related by $\sigma_{\hat{h}}^2 = \rho^2\sigma_h^2$, whereas in LS channel estimation they are related by $\sigma_h^2 = \rho^2\sigma_{\hat{h}}^2$.

It is well know MMSE is superior to LS in terms of mean square errors between $\mathbf{h}$ and $\hat{\mathbf{h}}$. However, the overall system error performance depends not only on channel estimation, but also on symbol detection. Therefore, a better channel estimation may not be sufficient to guarantee a better system error performance. This is elaborated in the following two sections.

III. Optimal Diversity Receiver with Linear Channel Estimation

In this section, optimal decision rules for coherent diversity reception that minimize the error probability of systems with LS channel estimation and MMSE channel estimation are derived.

For pilot assisted linear channel estimation, $\mathbf{h}$ conditioned on $\hat{\mathbf{h}}$ is Gaussian distributed; it follows from (1) that $\mathbf{r}$ conditioned on both $\hat{\mathbf{h}}$ and transmitted data symbol s_m is also Gaussian distributed, *i.e.*, $\mathbf{r}|(\hat{\mathbf{h}}, s_m) \sim \mathcal{N}(\mathbf{u}_{r|\hat{h},s_m}, \mathbf{\Phi}_{r|\hat{h},s_m})$, with the mean vector, $\mathbf{u}_{r|\hat{h},s_m}$, and covariance matrix, $\mathbf{\Phi}_{r|\hat{h},s_m}$, given by

$$\mathbf{u}_{r|\hat{h},s_m} = \mathbf{u}_{h|\hat{h}} \cdot s_m \quad (13a)$$
$$\mathbf{\Phi}_{r|\hat{h},s_m} = \mathbf{\Phi}_{h|\hat{h}} \cdot E_s + N_0 \cdot \mathbf{I}_N \quad (13b)$$

with $E_s = \mathbb{E}(|s_m|^2)$ being the energy of the data symbol.

Proposition 1: For diversity receivers operating in an i.i.d. fading environment with pilot assisted LS channel estimation or MMSE channel estimation, if the transmitted symbols are equiprobable, then the detection rule that minimizes the system error probability is

$$\hat{s} = \underset{s_m \in \mathcal{S}}{\text{argmin}} \left\{|\alpha - s_m|^2\right\} \quad (14)$$

where $\mathcal{S} = \{s_m = \sqrt{E_s}e^{-j2\pi\frac{m}{M}} | m = 1, 2, \cdots, M\}$ is the modulation alphabet set, and α is a decision variable whose value depends on the channel estimation method. The decision variable for LS channel estimation, α_{LS}, and MMSE channel estimation, α_{MMSE}, are expressed as

$$\alpha_{LS} = [\rho^2\hat{\mathbf{h}}_{LS} + (1-\rho^2)\mathbf{u}]^H\mathbf{r} \quad (15a)$$
$$\alpha_{MMSE} = \hat{\mathbf{h}}_{MMSE}^H\mathbf{r} \quad (15b)$$

where ρ defined in (7) is the covariance coefficient between the estimated CG and true CG.

Proof: If the transmitted data symbols are equiprobable, maximum likelihood detection minimizes the error probability. The optimum decision rule can be written as

$$\hat{s} = \underset{s_m \in \mathcal{S}}{\text{argmin}} \left\{(\mathbf{r} - \mathbf{u}_{r|\hat{h},s_m})^H \mathbf{\Phi}_{r|\hat{h},s_m}^{-1} (\mathbf{r} - \mathbf{u}_{r|\hat{h},s_m})\right\}$$
$$= \underset{s_m \in \mathcal{S}}{\text{argmin}} \left\{\left\|\mathbf{r} - \mathbf{u}_{h|\hat{h}} \cdot s_m\right\|^2\right\} \quad (16)$$

where the second equality is based on the fact that $\mathbf{\Phi}_{r|\hat{h},s_m}$ is a scaled identity matrix independent of s_m for i.i.d. fading.

Expanding the term in (16), and after some straightforward algebraic manipulations, one have

$$\hat{s} = \underset{s_m \in \mathcal{S}}{\text{argmax}} \left\{\Re\left(\mathbf{u}_{h|\hat{h}}^H \mathbf{r} \cdot s_m^*\right)\right\} = \underset{s_m \in \mathcal{S}}{\text{argmin}} \left\{\left\|\mathbf{u}_{h|\hat{h}}^H \mathbf{r} - s_m\right\|^2\right\},$$

where $\Re(x)$ denotes the real part of x. Substituting (6a) for LS estimation, or (12a) for MMSE estimation, one has the decision rules given in (14) and (15). ■

According to Proposition 1, the optimal decision rule for a coherent diversity receiver employing MMSE channel estimation is the same as the conventional MRC decision rule. On the other hand, for receivers with LS channel estimation, the quality of channel estimation, which is embedded in ρ, is taken into consideration during the detection process. Only in the ideal case, *i.e.*, $\rho = 1$, the decision variable for LS specializes to the conventional MRC diversity receiver.

IV. Error Performance Analyses

A. Conditional Error Probability

The conditional error probability (CEP), $P(E|\hat{\mathbf{h}})$, is evaluated in this subsection.

If s_m is transmitted, the detection variable $\alpha = \mathbf{u}_{h|\hat{h}}^H\mathbf{r}$ conditioned on both $\hat{\mathbf{h}}$ and s_m is Gaussian distributed, with the conditional mean, $u_{\alpha|\hat{h},s_m}$, and conditional variance, $\sigma_{\alpha|\hat{h},s_m}^2$, given by

$$u_{\alpha|\hat{h},s_m} = \|\mathbf{u}_{h|\hat{h}}\|^2 s_m \quad (17a)$$
$$\sigma_{\alpha|\hat{h},s_m}^2 = \mathbf{u}_{h|\hat{h}}^H \left(\mathbf{\Phi}_{h|\hat{h}} \cdot E_s + N_0 \cdot \mathbf{I}_N\right) \mathbf{u}_{h|\hat{h}}. \quad (17b)$$

The conditional pdf $p(\alpha|\hat{\mathbf{h}}, s_m)$ is written in a polar coordinate system to simplify the CEP derivation [11]. The corresponding pdf written in a polar coordinate system with origin at $u_{\alpha|\hat{h},s_m} = \|\mathbf{u}_{h|\hat{h}}\|^2 s_m$ is

$$p(r, \theta|\hat{\mathbf{h}}, s_m) = \frac{r}{\pi\sigma_{\alpha|\hat{h},s_m}^2} \exp\left(-\frac{r^2}{\sigma_{\alpha|\hat{h},s_m}^2}\right). \quad (18)$$

Based on the decision rule, the detection region of the MPSK symbol s_m should be a $\frac{2\pi}{M}$ angle sector centered around s_m as shown in Fig. 1. Therefore, the CEP $P(E|\hat{\mathbf{h}})$ can be computed as

$$P(E|\hat{\mathbf{h}}) = 2\sum_{m=1}^{M} P(s_m) \int_0^{\pi-\frac{\pi}{M}} \int_{R(\theta)}^{+\infty} p(r, \theta|\hat{\mathbf{h}}, s_m) dr d\theta$$
$$= \frac{1}{\pi} \int_0^{\pi-\frac{\pi}{M}} \exp\left\{-\frac{\|\mathbf{u}_{h|\hat{h}}\|^4 E_s \sin^2(\frac{\pi}{M})}{\sigma_{\alpha|\hat{h},s_m}^2 \sin^2(\phi)}\right\} d\phi \quad (19)$$

where $R(\theta) = \frac{\|\mathbf{u}_{h|\hat{h}}\|^2 \cdot s_m \cdot \sin(\pi/M)}{\sin(\theta+\pi/M)}$, $P(s_m) = \frac{1}{M}$ for equiprobable transmitted symbols, and we have changed the integration variable to $\phi = \pi - (\theta + \frac{\pi}{M})$ in the second equality.

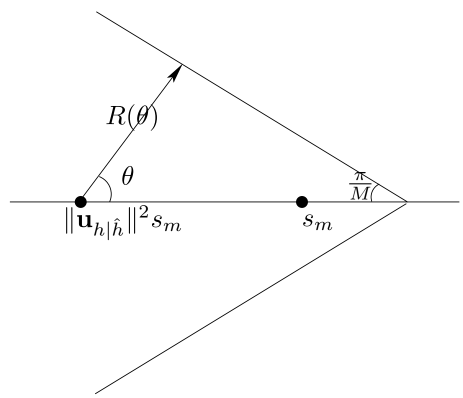

Fig. 1. The decision region for MPSK modulation.

B. Error Probability with LS Channel Estimation

The CEP for a system with LS channel estimation can be obtained by combining (6), (17), and (19). The result is expressed in (20) at the top of the next page.

In (20), $\sigma_{\hat{h}}^2$ is the variance of the estimated CG $\hat{h}_n$, K is the Rice factor, and γ is the average SNR of the data symbol defined as

$$\gamma = \frac{\Omega E_s}{N_0} = \frac{(K+1)\sigma_h^2 E_s}{N_0}. \tag{21}$$

In (20), the presence of channel estimation error prohibits the direct application of the MGF or CHF method for the evaluation of the unconditional error probability. A Gaussian distribution based functional equivalency is employed here for the error probability derivation.

For i.i.d. fading channels, the pdf of the estimated CG $\hat{\mathbf{h}}$ is given by

$$p(\hat{\mathbf{h}}) = \prod_{n=1}^{N} \frac{1}{\pi\sigma_{\hat{h}}^2} \exp\left[-\frac{|\hat{h}_n - u|^2}{\sigma_{\hat{h}}^2}\right]. \tag{22}$$

Combining (20) with (22), we obtain the unconditional error probability $P(E) = \int_{\{\hat{\mathbf{h}}\}} P(E|\hat{\mathbf{h}})p(\hat{\mathbf{h}})d\hat{\mathbf{h}}$ in a Rician fading channel as

$$P(E) = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} [\lambda_{LS}(\phi)]^N \, d\phi \tag{23}$$

where

$$\lambda_{LS}(\phi) = \frac{1}{\pi\sigma_{\hat{h}_n}^2}\int_{\{\hat{h}\}} \exp\left[-\frac{g|\hat{h}_n - au|^2 + |\hat{h}_n - u|^2}{\sigma_{\hat{h}}^2}\right] d\hat{h}_n, \tag{24}$$

with g, a and the equivalent SNR $\tilde{\gamma}$ for LS channel estimation being defined as

$$g = \frac{\tilde{\gamma}\sin^2(\frac{\pi}{M})}{(K+1)\sin^2(\phi)}, \tag{25a}$$

$$a = (1 - \frac{1}{\rho^2}), \tag{25b}$$

$$\tilde{\gamma} = \frac{(K+1)\rho^2}{\gamma(1-\rho^2)+K+1}\gamma. \tag{25c}$$

The equivalent SNR, $\tilde{\gamma}$, is obtained from scaling the average SNR γ by a factor $\beta = \frac{(K+1)\rho^2}{\gamma(1-\rho^2)+K+1}$. Based on the fact that $0 < \rho \leq 1$, it can be easily shown that $\tilde{\gamma} \leq \gamma$, and equality holds when $\rho = 1$.

Since the integrand of (24) is an exponential function of the square of the integration variable $\hat{h}_n$, we can write it as the product of a Gaussian pdf and a constant term. Then, using the properties of Gaussian pdfs, one can get the closed-form solution of $\lambda_{LS}(\phi)$ as

$$\begin{aligned}\lambda_{LS}(\phi) &= \frac{1}{g+1}\exp\left[-\frac{g(a-1)^2}{(g+1)\sigma_{\hat{h}}^2}|u|^2\right] \times \\ &\quad \int_{\{\hat{h}_n\}} \frac{1}{\pi\sigma_{\hat{h}}^2/(g+1)} \exp\left[-\frac{|\hat{h}_n - \frac{ga+1}{g+1}u|^2}{\sigma_{\hat{h}_n}^2/(g+1)}\right] d\hat{h}_n \\ &= \frac{1}{g+1}\exp\left[-\frac{g(a-1)^2}{(g+1)\sigma_{\hat{h}}^2}|u|^2\right]. \end{aligned} \tag{26}$$

Replacing $\lambda_{LS}(\phi)$ in (23) with (26), we obtain the following results.

Proposition 2: For a wireless system with N diversity receivers equipped with LS channel estimators, the symbol error probability of the system over Rician fading channels is given by

$$\begin{aligned} P(E) &= e^{-N\frac{K}{\rho^2}} \int_0^{\pi-\frac{\pi}{M}} \left[1 + \frac{\tilde{\gamma}}{K+1}\frac{\sin^2(\frac{\pi}{M})}{\sin^2(\phi)}\right]^{-N} \times \\ &\quad \exp\left\{\frac{NK}{\rho^2}\left[1 + \frac{\tilde{\gamma}}{K+1}\frac{\sin^2(\frac{\pi}{M})}{\sin^2(\phi)}\right]^{-1}\right\} d\phi \end{aligned} \tag{27}$$

where K is the Rice factor, ρ is the covariance coefficient between the true CG and the estimated CG, and $\tilde{\gamma}$ is defined in (25c).

We conclude this subsection with a few remarks.

Remark 1: For a wireless system with BPSK modulation, $M = 2$, the error probability of the diversity receivers in Rayleigh fading channels can be written in closed-form by changing the integration variable to $z = \cot(\phi)$,

$$P(E) = \frac{\Gamma(N+\frac{1}{2})}{2\sqrt{\pi}N!(\tilde{\gamma}+1)^N} \cdot {}_2F_1(N, \frac{1}{2}; N+1; \frac{1}{\tilde{\gamma}+1}) \tag{28}$$

where $\Gamma(x)$ is the Gamma function, and ${}_2F_1(\cdot)$ is the Gauss hypergeometric function.

Remark 2: When the system has no diversity, *i.e.* $N = 1$, the error probability for MPSK under Rayleigh fading can be

$$P(E|\hat{\mathbf{h}}) = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \prod_{n=1}^{N} \exp\left\{-\frac{\rho^2\gamma|\hat{h}_n - u(1-\frac{1}{\rho^2})|^2 \sin^2(\frac{\pi}{M})}{\sigma_{\hat{h}}^2[\gamma(1-\rho^2)+K+1]\sin^2(\phi)}\right\} d\phi \tag{20}$$

expressed in closed-form by changing the integration variable to $z = \cot(\phi)$,

$$P(E) = \frac{M-1}{M} - \sqrt{\frac{\tilde{\gamma}\sin^2(\frac{\pi}{M})}{1+\tilde{\gamma}\sin^2(\frac{\pi}{M})}} \times \left[\frac{1}{2} + \frac{1}{\pi}\arctan\left(\sqrt{\frac{\tilde{\gamma}\sin^2(\frac{\pi}{M})}{1+\tilde{\gamma}\sin^2(\frac{\pi}{M})}}\cot\left(\frac{\pi}{M}\right)\right)\right] \tag{29}$$

For the special case of perfect channel information, we have $\tilde{\gamma} = \gamma$, and (29) agrees with the result previously obtained in [4, eqn. (36)] through a different approach.

Remark 3: For diversity systems with $M > 2$, the symbol error rate given in (27) must be evaluated numerically. The expression for the SER in (27) contains a single integration with small integration limits, and the integrand is constituted of only elementary functions. Thus, it can be easily evaluated with simple numerical methods.

C. Error Probability with MMSE Channel Estimation

The CEP for a system with MMSE channel estimation can be obtained by combining (12), (17), and (19), and the result is

$$P(E|\hat{\mathbf{h}}) = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \prod_{n=1}^{N} \exp\left\{-\frac{\tilde{\gamma}|\hat{h}_n|^2 \sin^2(\frac{\pi}{M})}{(K+1)\sigma_{\hat{h}}^2 \sin^2(\phi)}\right\} d\phi \tag{30}$$

where the equivalent SNR $\tilde{\gamma}$ is defined in (25c).

Similarly to (23), the unconditional error probability for a system with MMSE channel estimation can be expressed as

$$P(E) = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} [\lambda_{MMSE}(\phi)]^N d\phi \tag{31}$$

with λ_{MMSE} defined as

$$\lambda_{MMSE}(\phi) = \frac{1}{\pi\sigma_{\hat{h}_n}^2}\int_{\{\hat{h}_n\}} \exp\left\{-\frac{g|\hat{h}_n|^2 + |\hat{h}_n - u|^2}{\sigma_{\hat{h}}^2}\right\} d\hat{h}_n. \tag{32}$$

Following the same Gaussian distribution-based function equivalency method described in Sect. IV-B, and noting that $\sigma_{\hat{h}}^2 = \rho^2\sigma_h^2$ for MMSE channel estimation, one can obtain the symbol error probability for MMSE system.

It can be easily shown that $P(E)$ for MMSE has the same expression as (27), which is the error probability for LS channel estimation, given the optimal diversity combining described in Proposition 1 is used. This result indicates that even though the MMSE channel estimation outperforms LS channel estimation in terms of mean square errors of the estimation, the MMSE algorithm is not necessarily better than LS algorithm in terms of error probability. Because the difference between LS channel estimation and MMSE channel estimation is compensated by the optimal diversity combining at the receiver for MPSK modulation.

It should be pointed out that Remarks 1-3 stated in Section IV-B are suitable for the MMSE channel estimation case.

V. Numerical Examples

Numerical examples are given in this section to illustrate the influence of channel estimation on the error performances of diversity receivers in fading channels. Simulation results are also shown to validate our analytical results.

The first example is used to validate the analytical error probability expressions derived for systems with LS or MMSE channel estimation. In Fig. 2, the theoretical symbol error rates (SER) are compared with the results obtained from Monte-Carlo simulation for 8PSK modulated systems. In this example, pilot symbol has the same power as the data symbols. Excellent agreements are observed between analytical results and simulation results for various values of Rice factor K and diversity order N. In addition, the results validate the claim that, if optimal diversity combining is employed at the receiver, a system with LS channel estimation can achieve the same error performance as the system with MMSE channel estimation.

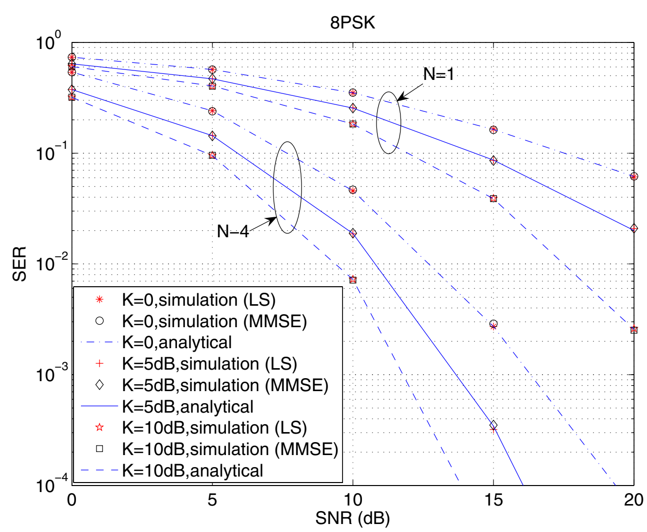

Fig. 2. The SER performance of systems with LS or MMSE channel estimations.

Fig. 3 illustrates the performance difference between the proposed optimal diversity receiver and conventional MRC receiver for a system with LS channel estimation. There are $N = 4$ receive antennas, and the modulation scheme is BPSK. The performance difference between the two receivers increases with the increase of the Rice factor K. At the SER level of 10^{-4}, the proposed optimal diversity receiver outperforms conventional MRC receiver by approximately 1.5

dB and 2 dB for systems with $K = 5$ dB and $K = 10$ dB, respectively.

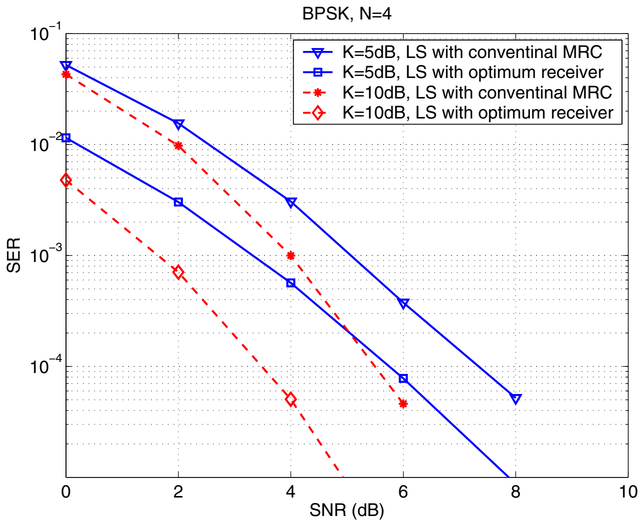

Fig. 3. Comparison of conventional MRC receiver and optimal diversity receiver with LS channel estimation.

Next we investigate the influences of pilot symbol SNR, γ_p, on system performances. Fig. 4 shows the SER curves for different values of γ_p. The horizontal axis of this figure is the average SNR of data symbols. The curve labeled as $\gamma_p = \infty$ corresponds to perfect channel estimation. Error floors resulted from channel estimation errors are observed in this figure for a system with small values of γ_p. Considering data symbol SNR in the range of [0, 10] dB, one can see that $\gamma_p = 25$ dB leads to almost the same performance as a system with perfect channel information, for both $N = 1$ and $N = 4$.

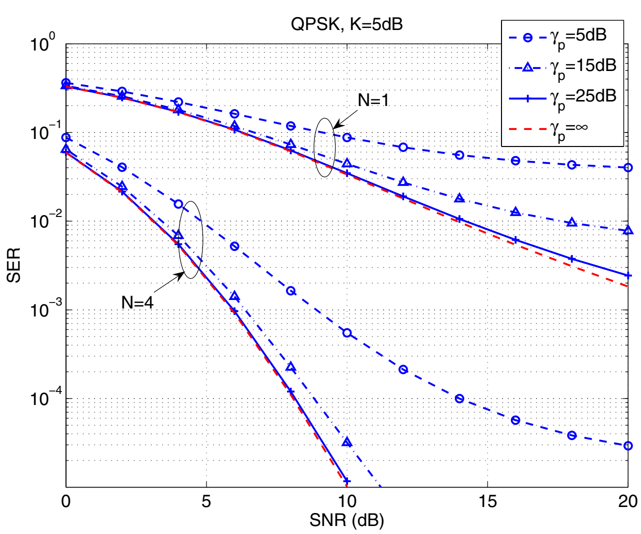

Fig. 4. The SER of QPSK systems under different values of pilot symbol SNR γ_p.

VI. Conclusion

An optimal diversity receiver for system with LS and MMSE channel estimation was derived. Exact error probability expression of the optimal receiver were obtained. One interesting result from our theoretical analysis is that a wireless system with LS channel estimation can have the same symbol error rate as the system with MMSE channel estimation replacing LS channel estimation. Simulation results are in excellent agreement with the theoretical results.

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