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Channel error recovery for transform image coding

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Abstract—A method is presented to automatically inspect the block boundaries of a reconstructed two-dimensional transform coded image, to locate blocks which are most likely to contain errors, to approximate the size and type of error in the block, and to eliminate this estimated error from the picture. This method uses redundancy in the source data to provide channel error correction. No additional channel error protection bits or changes to the transmitter are required. It can be used when channel errors are unexpected prior to reception.

I. INTRODUCTION

Two-dimensional block transform coding methods provide very effective image compression. At image data rates of 0.5 to 1.5 bits/pixel, we have found these methods to be judged subjectively superior to other methods at low channel error rates [1]–[3]. The adaptive two-dimensional block transform coding method described by Chen and Smith [4] has produced some of the best results. In this method, the two-dimensional fast discrete cosine transform [5], [6] is performed over subblocks (typically size 16 × 16) of the original image. The blocks are sorted into classes based on one or more features. (Chen and Smith use the total ac energy in the block as the feature.) Then bits are assigned to each class based on coefficient sample variances within each class.

A typical adaptive multiclass zone method is diagrammed in Fig. 1. An original image is shown in Fig. 2. The classification and bit allocation maps used for a compression to 1.0 bits/pixel are shown in Figs. 3 and 4, respectively. The reconstructed image is shown in Fig. 5.

When uncorrected channel errors are present, the effects on the reconstructed picture are much different than for PCM image transmission. Some bits in the compressed data are critical and require that channel errors be detected and corrected to prevent total loss of the image. This critical information is indicated in Fig. 1 and includes the classification map, bit assignment tables, and scale factor.

As an example, a 512 × 512 pixel image coded using 16 × 16 blocks and four classes would require 2048 bits for the classification map, 3072 bits for the bit assignment tables, and 16 bits for the scale factor. These 5136 bits can be protected using a (127, 71) BCH code [7] which will reduce an unprotected bit error rate of $10^{-2}$ to one error every 21000 images. This requires 9187 bits or 0.0350 bits/pixel for the critical overhead information. The bulk of the data that is transmitted consists of the coefficient values themselves. If channel error protection is required, the quality of the reconstruction must be sacrificed to achieve the same data rate.

II. CHANNEL ERROR RECOVERY ALGORITHM

The basic approach is to check the four boundaries around each reconstructed image block. If a discontinuity exists along these boundaries that is consistent with a single dominant transform coefficient error, the coefficient location and amplitude are estimated and a basis picture corresponding to the estimated error is subtracted from the block. A block diagram indicating the order of calculations performed is shown in Fig. 7.

Assume that one error has occurred at the $k$th row and $l$th column of a transmitted block of transform coefficients. Hence, the decoded block at the receiver, denoted by matrix
Fig. 1. Adaptive multiclass zone block transform coding transmitter.

Fig. 2. Airport original, 512 x 512 pixels, 256 gray levels (8 bits).

Fig. 3. Classification map for each 16 x 16 block in Fig. 2 using four energy classes with an equal number of blocks in each class. Class 1 represents the highest ac energy and Class 4 represents the lowest.

Fig. 4. Bit allocation maps for the four classes shown in Fig. 3. Bits are assigned proportional to the sample variance of each coefficient of each class and so that the average data rate including overhead is 1.0 bits/pixel.

Fig. 5. Reconstructed imagery using the classes and bit assignments shown in Figs. 3 and 4, respectively. This represents Fig. 2 compressed to 1.0 bits/pixel with no channel errors. The mean-square error is 27.1.
Fig. 6. Effects of random channel errors with $P_E = 10^{-2}$ on the compression system operating at 1.0 bit/pixel. The mean-square error is 346.8.

Fig. 7. Block diagram of error detection and correction process.
\[ B = \begin{bmatrix} b_{ij} \end{bmatrix}_{N \times N}, i, j = 0, 1, \ldots, N - 1, \text{can be written as} \]
\[ B = A + Q + S \]
(1)
where
\[ A = \begin{bmatrix} a_{ij} \end{bmatrix}_{N \times N} = \text{block to be transmitted} \]
\[ Q = \begin{bmatrix} q_{ij} \end{bmatrix}_{N \times N} = \text{error due to transform coefficient quantization} \]
\[ S = \text{two-dimensional inverse discrete cosine transform of } E \]
\[ e_{ij} = e \quad i = k, j = l \]
\[ E = \begin{bmatrix} e_{ij} \end{bmatrix}_{N \times N} \]
\[ e_{ij} = 0 \quad \text{otherwise} \]
\[ = \text{error matrix due to the noisy channel.} \]

The block size \( N \times N \) is 16 \times 16 for the examples we have shown. If the transmitted block is not on the boundary of the picture, then it is surrounded by four blocks represented, respectively, by matrices \( U, D, L, \) and \( R, \) as shown in Fig. 8.

Form the four difference vectors denoted by \( X_i, i = 0, 1, 2, 3 \) of size \( N \) as follows.
\[ X_i = \begin{bmatrix} x_{i0}, x_{i1}, \ldots, x_{iN-1} \end{bmatrix} \]
\[ x_{0,k} = a_{0,k} - u_{N-1,k} \]
\[ x_{1,k} = a_{N-1,k} - a_{0,k} \]
\[ x_{2,k} = a_{k,0} - l_{k,N-1} \]
\[ x_{3,k} = a_{k,N-1} - r_{k,0} \]
\[ k = 0, \ldots, N - 1. \]

This represents block \( A \) in Fig. 7.

Denote the discrete cosine transform \([5]\) of vectors \( X_i \) by \( Y_i \).
\[ Y_i = \text{DCT} \begin{bmatrix} X_i \end{bmatrix} \quad i = 0, 1, 2, 3. \]
(3)

We claim that, due to the inherent continuity of most image data, the two vectors \( Y_0, Y_1 \) will approximate the first and the last rows of the matrix \( H \) that is obtained by taking a one-dimensional inverse discrete cosine transform along each column of the error matrix \( E \). The basis picture \(2-D\) inverse cosine transform) associated with the error matrix \( E \) has the form
\[ S = \begin{bmatrix} s_{m,n} \end{bmatrix}_{N \times N} \]
where
\[ s_{m,n} = e^{(2m + 1)k \pi / 2N} e^{(2n + 1)l \pi / 2N} \]
\[ s_{m,n} = \frac{1}{N} \quad k,l = 0 \]
\[ = \frac{\sqrt{2}}{N} e^{(2n + 1)l \pi / 2N} \quad k = 0, l \neq 0 \]
\[ = \frac{\sqrt{2}}{N} e^{(2m + 1)k \pi / 2N} \quad k \neq 0, l = 0 \]
\[ m = 0, \ldots, N - 1 \]
\[ n = 0, \ldots, N - 1. \]

Let us denote the first row of the above matrix by
\[ S_0 = \frac{2e}{N} \cos \frac{k \pi}{2N} \left[ \cos \frac{l \pi}{2N}, \ldots, \cos \frac{(2N - 1)l \pi}{2N} \right] \]
where we have implicitly assumed the case \( k, l \neq 0 \). The one-dimensional DCT of \( S_0 \) will yield
\[ T_0 = \begin{bmatrix} t_{0,0}, t_{0,1}, \ldots, t_{0,N-1} \end{bmatrix} \]
(5)
where
\[ t_{0,j} = \sqrt{\frac{2}{N}} e^{\frac{k \pi}{2N}} \quad j = l \]
\[ t_{0,j} = 0 \quad j \neq l \]
\[ T_0 \]

The vector \( T_0 \) is the first row of matrix \( H \) discussed earlier. The vector \( Y_0 \) is our estimate of vector \( T_0 \). It also includes variations due to original image row differences and quantization noise. We shall assume that the channel error component dominates the other sources of variation in vector \( Y_0 \). (If it does not dominate, the error cannot be detected and corrected.)

Thus, a channel error will be manifested as a single large element in vector \( Y_0 \). The location of this element is \( l \) which is the column of the nonzero element in matrix \( E \). The magnitude of the large element in \( Y_0 \) is related to \( e \) as shown in (5). Similarly, vector \( T_1 \) is the last row of matrix \( H \), and error location and size can be estimated from vector \( Y_1 \) which is our estimate of vector \( T_1 \). Similar to (5), we find
\[ t_{1,j} = (-1)^k \sqrt{\frac{2}{N}} e^{\frac{k \pi}{2N}} \quad j = l \]
\[ t_{1,j} = 0 \quad j \neq l. \]
Note from (5) and (6) that the magnitude of \( t_{2j,l} \) is the same as that of \( t_{2j+1,l} \). This fact will be used in the algorithm described later [see (8)-(10)].

Similarly, the two vectors \( Y_2 \) and \( Y_3 \) will approximate the first and last columns of the matrix \( G \), which is obtained by taking a one-dimensional inverse cosine transform along each row of the error matrix \( E \). Vectors \( Y_2 \) and \( Y_3 \) can be used to find the row location of the coefficient in error and its magnitude.

Once the transform of the difference vectors has been obtained as in (3), the most likely row and column locations for an error to have occurred can be determined. Since a single channel error will be manifested as a single large magnitude component in each of the four \( Y_k \)'s, the dominant magnitude coefficient in each \( Y_i \) is investigated. This dominant coefficient is labeled \( \alpha_i \).

Let

\[
|\alpha_i| = \max_{k} \left( |y_{i,k}| \right) = |y_{i,n_i}|	ag{7}
\]

\( i = 0, 1, 2, 3 \)

\( k = 0, 1, \ldots, N-1 \)

where \( n_i \) is the value of \( k \) at which the maximum occurs.

\[
u_i = \max \left( \left| \frac{\alpha_i}{y_{i+1,n_i}} \right|, \left| \frac{\alpha_i}{y_{i,n_i}} \right| \right) \quad i = 0, 2 \tag{8}
\]

\[
u_i = \max \left( \left| \frac{\alpha_i}{y_{i-1,n_i}} \right|, \left| \frac{\alpha_i}{y_{i,n_i}} \right| \right) \quad i = 1, 3 \tag{9}
\]

\[
z_i = \min (v_{2i}, v_{2i+1}) \quad i = 0, 1 \tag{10}
\]

Equations (7)-(10) locate the maximum component in each \( Y_i \) and compare the magnitude of this with the magnitude of the component having the same location but on the opposite side of the matrix (see Fig. 9). Equations (5) and (6) indicate that these magnitudes should be identical if they are caused by a single error. The smaller the values of \( z_0 \) and \( z_1 \) in (9), the closer the amplitude match. This corresponds to block \( B \) in Fig. 7.

A threshold value \( \lambda = 1.6 \) has been empirically chosen as a measure of closeness of two values. Now, if neither of the \( z_i \)'s, \( i = 0, 1 \) satisfy the following condition:

\[
z_i \leq \lambda \quad i = 0, 1 \tag{10}
\]

then we conclude that no correctable error has occurred in the block. This corresponds to block \( C \) in Fig. 7.

On the other hand, if both \( z_i \)'s satisfy the condition of (10), then we choose the values of \( \alpha_k, \alpha_m \) such that

\[
u_k = z_0 \quad k = 0, 1
\]

\[
u_m = z_1 \quad m = 2, 3 \tag{11}
\]

That is, the row and column locations having the best amplitude similarity are chosen as the potential error location.

In addition to the above, an "outlier" test [9]-[11] is also performed to check whether either of the values \( \alpha_k, \alpha_m \) can be regarded as a dominant coefficient in their respective vectors \( Y_k, Y_m \). The test performed is the following.

\[
l |\alpha_i - \hat{m}_j| > 3\hat{o}_j \quad j = k, m \tag{12}
\]

where

\[
\hat{m}_j = \text{sample mean of } y_{i,i} \quad i = 0, 1, \ldots, N-1
\]

\[
\hat{o}_j = \text{sample standard deviation of } y_{i,i} \quad i = 0, 1, \ldots, N-1
\]

Finally, from the theory of discrete cosine transforms, the relationships between the signs of \( \alpha_k, \alpha_m \) can be established, in case an error has occurred in the block. Hence,

\[
\alpha_k\alpha_m > 0 \quad k = 0, m = 2
\]

\[
\alpha_k\gamma_{m-1,n_m} > 0 \quad k = 0, m = 3
\]

\[
\gamma_{k-1,n_k}\alpha_m > 0 \quad k = 1, m = 2
\]

\[
\gamma_{k-1,n_k}\gamma_{m-1,n_m} > 0 \quad k = 1, m = 3 \tag{13}
\]

in case of a detected error.

Therefore, if (10), (12), and (13) are satisfied, we conclude that an error has occurred at the \( n_m \)th row and \( n_k \)th column of the block of received coefficients and we proceed to block \( E \) of Fig. 7.

The error matrix \( E \) can be estimated by using the theory of the discrete cosine transform:

\[
\hat{E} = [\hat{e}_{i,j}]_{N \times N} \quad i, j = 0, 1, \ldots, N-1
\]

where

\[
\hat{e}_{i,j} = 0 \quad i \neq n_m, \quad j \neq n_k
\]
\[ \hat{e}_{n,m,n_k} = \frac{|\alpha_k| + |y_{1-k,n_k}|}{4 \sqrt{\frac{2}{N}} \cos \frac{n_k \pi}{2N}} \sgn(\beta_k) \]
\[ + \frac{|\alpha_m| + |y_{s-m,n_m}|}{4 \sqrt{\frac{2}{N}} \cos \frac{n_m \pi}{2N}} \sgn(\beta_m) \quad n_m, n_k \neq 0 \]
\[ = \frac{|\alpha_k| + |y_{1-k,n_k}|}{4 \sqrt{\frac{2}{N}} \cos \frac{n_k \pi}{2N}} \sgn(\beta_k) \]
\[ + \frac{|\alpha_m| + |y_{s-m,n_m}|}{4 \sqrt{\frac{2}{N}} \cos \frac{n_m \pi}{2N}} \sgn(\beta_m) \quad n_m = 0 \quad n_k \neq 0 \]
\[ = \frac{\sqrt{N}}{4} (\alpha_k + \alpha_m + y_{1-k,n_k} + y_{s-m,n_m}) \quad n_m, n_k = 0 \]

where
\[ \beta_0 = \alpha_0 \]
\[ \beta_1 = y_{0,n_1} \]
\[ \beta_2 = \alpha_2 \]
\[ \beta_3 = y_{2,n_3}. \]

The last case to be considered is when only one of the \( z_i \)'s satisfies (10) (see block F in Fig. 7). This case occurs when errors are present in adjacent blocks, causing one of the error vectors of (2) to be unreliable.

For the sake of simplicity let us assume that
\[ z_0 < \lambda \]
\[ z_1 > \lambda. \]

Therefore, we can find \( n_k \) for \( k = 0 \) or 1 such that
\[ \max \left( \frac{|\alpha_k|}{|y_{1-k,n_k}|}, \frac{|y_{1-k,n_k}|}{|\alpha_k|} \right) = z_0. \]

Now if
\[ \max \left( \frac{|\alpha_k|}{|\alpha_2|}, \frac{|\alpha_2|}{|\alpha_k|} \right) < \lambda \]
and
\[ \beta_k \alpha_2 > 0 \]
and either \( \alpha_k \) or \( \alpha_2 \) form a dominant coefficient (12) in their respective vectors \( Y_k, Y_2 \), we conclude that an error has been detected, and the error matrix can be estimated as
\[ \hat{E} = [\hat{e}_{i,j}]_{N \times N} \quad i, j = 0, 1, \ldots, N - 1 \]
\[ \hat{e}_{i,j} = 0 \quad i \neq n_2, \quad j \neq n_k \]
\[ \hat{e}_{n_2,n_k} = \frac{|\alpha_k| + |y_{1-k,n_k}|}{4 \sqrt{\frac{2}{N}} \cos \frac{n_k \pi}{2N}} \sgn(\beta_k) \]
\[ + \frac{\alpha_2}{2 \sqrt{\frac{2}{N}} \cos \frac{n_k \pi}{2N}} \quad n_2, n_k \neq 0 \]
\[ = \frac{|\alpha_k| + |y_{1-k,n_k}|}{4 \sqrt{\frac{2}{N}} \cos \frac{n_k \pi}{2N}} \sgn(\beta_k) \]
\[ + \frac{\alpha_2}{2 \sqrt{\frac{2}{N}} \cos \frac{n_k \pi}{2N}} \quad n_2 = 0 \quad n_k \neq 0 \]
\[ = \frac{\sqrt{N}}{4} (\alpha_k + y_{1-k,n_k} + 2 \alpha_2) \quad n_2, n_k = 0. \]

On the other hand, if (17), (18), and dominance tests (12) are true for \( \alpha_3 \) instead of \( \alpha_2 \), then we simply replace \( \alpha_2 \) by \( \alpha_3(-1)^{n_k} \) in (19).

In case (15) is satisfied for \( z_1 \) instead of \( z_0 \), steps similar to the process discussed above can be taken.

III. RESULTS

Shown in Fig. 10 are the results of applying the preceding algorithm to the image in Fig. 6. Note the improvement in image quality and in mean-square error. At this high error rate \( 10^{-2} \) there is an average of 2.5 channel errors per block. The algorithm assumes that only a single error is present per block. However, when multiple errors are present, the dominant error is still often detected and corrected. When the
block to be tested occurs on the boundary of the image, zeros are used for the nonexistent vectors in (2). Even in this case, the algorithm often corrects errors since only three of the four boundary difference vectors must be consistent with a single transform error.

A lower error rate ($10^{-3}$) is used to produce the image in Fig. 11. The result of applying the error correction algorithm to this image is shown in Fig. 12. In order to make the errors and corrections more visible, the difference in the pictures between the original (Fig. 2) and Figs. 11 and 12 are shown in Figs. 13 and 14, respectively.

Note that many of the errors visible in Fig. 13 are not visible in Fig. 11 due to image variation. Many of these errors were corrected, as can be seen in Fig. 14. In a few situations the process has introduced new errors. On the average, the improvement in subjective quality and mean-square error is remarkable. This improvement has been achieved with no additional information required from the transmitter such as error correction or overhead bits.

Figs. 15-17 show the effects of this processing when coding at 0.5 bits/pixel and with a channel error probability of $10^{-2}$. At this higher compression, most of the energy is concentrated in the lower frequencies. This is indicated by the bit assignment tables in Fig. 15. Thus, the errors are mainly low frequency errors. The algorithm operates on the reconstructed image of Fig. 16 without needing to know the bit assignments or compression rate involved.

Table I shows a comparison of mean-square error perform-
Fig. 14. Difference picture between the original (Fig. 2) and Fig. 12. The corrections made by the algorithm are more noticeable.

Fig. 15. Bit allocation maps to achieve an average data rate of 0.5 bits/pixel.

Table I

<table>
<thead>
<tr>
<th>Compression Rate</th>
<th>2 bits/pixel</th>
<th>1.0 bits/pixel</th>
<th>0.5 bits/pixel</th>
</tr>
</thead>
<tbody>
<tr>
<td>No errors</td>
<td>7.3</td>
<td>27.1</td>
<td>64.2</td>
</tr>
<tr>
<td>PE = 0.01</td>
<td>369.5</td>
<td>346.8</td>
<td>341.4</td>
</tr>
<tr>
<td>Corrected</td>
<td>81.6</td>
<td>92.9</td>
<td>115.4</td>
</tr>
<tr>
<td>PE = 0.001</td>
<td>26.2</td>
<td>59.1</td>
<td>85.3</td>
</tr>
<tr>
<td>Corrected</td>
<td>12.9</td>
<td>38.0</td>
<td>70.4</td>
</tr>
</tbody>
</table>
 ance for the examples shown as well as other data rates and error probabilities.

We have experimented with applying the algorithm several times recursively on the same received picture. Repeated applications result in marginal improvement in mean-square error.

IV. CONCLUSIONS

The algorithm described has been shown to be an effective tool in combating channel errors in block transform coded imagery, especially when no a priori information is available about the nature of the channel noise. The receiver simply uses image continuity assumptions and knowledge of the type of errors introduced by incorrect transform coefficients. The algorithm has shown the ability to significantly decrease the mean-square error introduced into the reconstructed picture by channel errors.

The algorithm can operate equally well under different data compression rates and channel error rates and requires no a priori knowledge of either. This approach could be applied to any coding method which treats image data in blocks, although the error determination algorithm would be dependent on the coding method used. Although no detailed analysis has been made of the time required to perform these calculations, the number of operations required is on the order of that required for the inverse transform image reconstruction.

REFERENCES


O. Robert Mitchell (S'64–M'72) was born in Beaumont, TX, on July 4, 1945. He received the B.S.E.E. degree from Lamar University, Beaumont, in 1967 and the S.M.E.E. and Ph.D. degrees from the Massachusetts Institute of Technology, Cambridge, in 1968 and 1972, respectively.

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