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SYNOPSIS Through field tests on clay, soft soil, sand soil, and clay loam etc, we find: 1) Propagating velocity of elastic wave of homogeneous soil is not a constant, but a function of depth. 2) Poisson's ratio is not a constant either. It increases with the increase of depth. 3)The ratio of a lateral compressive stress to vertical compressive stress acted upon an element at different depth is not a constant either. With the increase of depth, the lateral compressive stress tends to be equal to the vertical compressive stress step by step.

Test results of four types of soil are covered in this paper and an analysis is made theoretically as regards to these phenomena. Through tests and analysis, the author is of an opinion that the viewpoint of Poisson's ratio decreasing with the increase of depth by D.D.Barkan is worthwhile being discussed.

INTRODUCTION

Shear modulus G, elastic modulus E, coefficient of subgrade reaction C and Poisson's ratio ν of homogeneous soil are all supposed constant, as a result of restriction of the testing methods ever used before, That is, get a small amount of soil with which to perform the test in laboratory. Such being the case, the initial condition is destroyed. For the stress of soil body caused by the influence of dead-weight at different depth is lost.

To reflect the natural condition of soil body even better, the method of conducting site test is already widely used. A great many field test informations show: Wave velocity of the same soil, due to different laying depths, differs greatly. The wave velocity of soft soil at deep strata might be far greater than that of hard soil at shallow strata. Shear modulus, elastic modulus, Poisson's ratio and propagating velocities of longitudinal and transversal waves etc, for homogeneous soil with little difference over the physical and mechanical properties are not a constant, but a function of depth. This is just one of the basic characteristics over which the soil mechanics differs from other subjects.

The author has found, through field tests on clay, soft soil, sand soil and clay loam etc, Poisson's ratio of homogeneous soil increases with depth of soil. This paper will analize this phenomenon theoretically.

RELATIONSHIP BETWEEN WAVE VELOCITY AND ELASTIC COEFFICIENT

Dynamic stress and strain will appear inside the medium due to the action by dynamic load and be propagated towards outside in the form of waves. Soil of different strata characteristic is of different propagating velocity. In an infinite elastic medium, wave equations are:

$$\left.\begin{aligned} \rho\frac{\partial^2 u}{\partial t^2} &= (\lambda+\mu)\frac{\partial\theta}{\partial x} + \mu\nabla^2 u \\ \rho\frac{\partial^2 v}{\partial t^2} &= (\lambda+\mu)\frac{\partial\theta}{\partial y} + \mu\nabla^2 v \\ \rho\frac{\partial^2 w}{\partial t^2} &= (\lambda+\mu)\frac{\partial\theta}{\partial z} + \mu\nabla^2 w \end{aligned}\right\} \qquad (1)$$

in which

ρ —— mass density of the medium $(\frac{t-s^2}{m^4})$;

u,v,w ——Components of displacement along x, y, z direction (m);

λ,μ —— Lame's constant

$$\lambda = \frac{2\mu\nu}{1-2\nu}, \mu = G\ (t/m^2);$$

ν —— Dynamic Poisson's ratio;

G —— Dynamic shear modulus (t/m^2);

θ —— Relative volume deformation of an infinitesimal element

$$\theta = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

∇^2—— Laplacian.

From the above equations, the following is obtained:

$$\left.\begin{aligned} V_p &= \sqrt{\frac{\lambda+2\mu}{\rho}} = \sqrt{\frac{E}{\rho}\cdot\frac{1-\nu}{(1+\nu)(1-2\nu)}} \\ V_s &= \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{\rho}\cdot\frac{1}{2(1+\nu)}} \\ E &= \frac{\rho V_s^2(3V_p^2-4V_s^2)}{V_p^2-V_s^2} \\ \nu &= \frac{V_p^2-2V_s^2}{2(V_p^2-V_s^2)} \end{aligned}\right\} \qquad (2)$$

in which

V_p —— Longitudinal wave propagating velocity (m/s);

V_s —— Transversal wave propagating velocity (m/s);

E —— Dynamic elastic modulus (t/m^2).

Propagating velocities of both longitudinal and transversal waves are obtained at different depth in field by means of wave speedometer and seismograph. And then elastic modulus, shear modulus and Poisson's ratio are to be worked out according to equation (2). The following figures Nos. 1, 2, 3, 4 are marked with test data Vp, Vs, E, G, ν of clay, soft soil, sand soil and clay loam respectively.

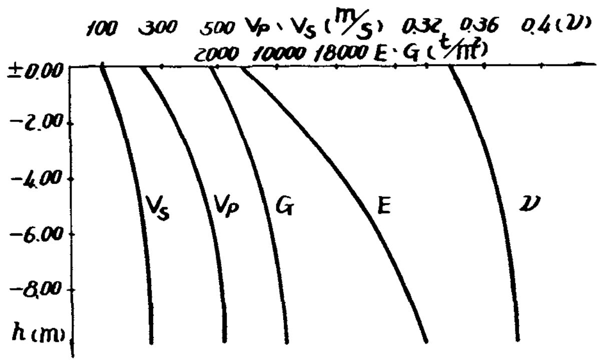

Fig. 1. Relationship Between Vp, Vs, E, G, ν and Depth (sand soil).

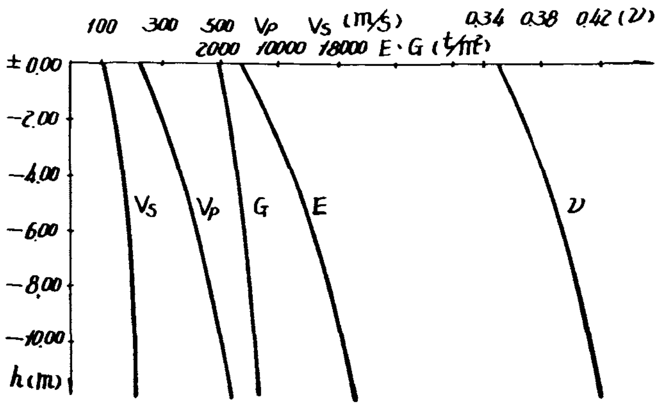

Fig. 2. Relationship Between Vp, Vs, E, G, ν and Depth (clay loam).

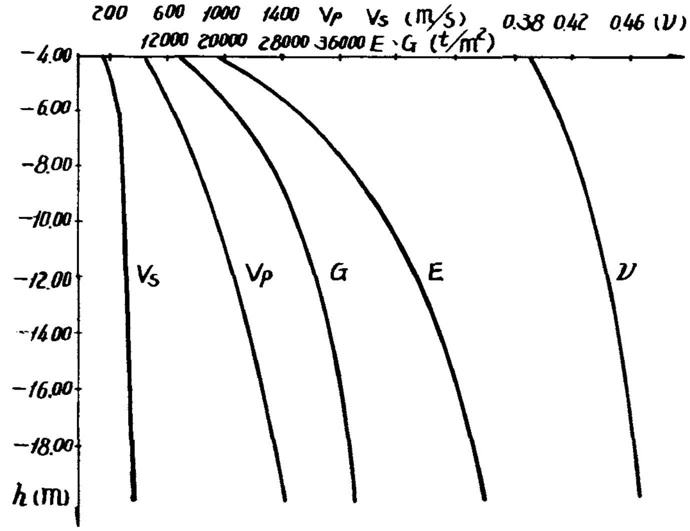

Fig. 3. Relationship Between Vp, Vs, E, G, ν and Depth (clay).

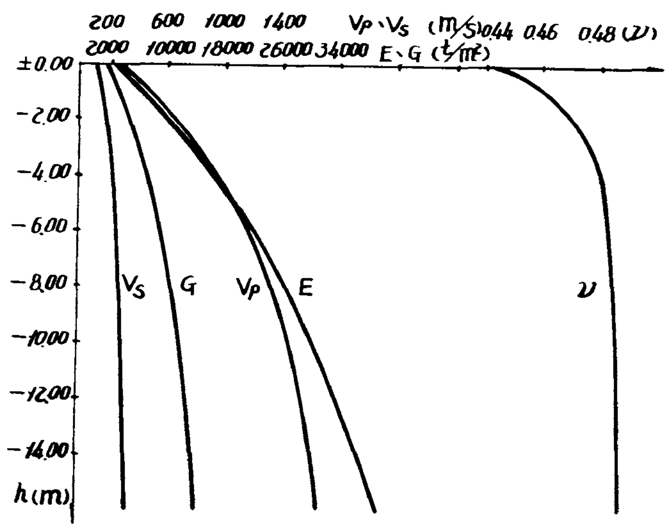

Fig. 4. Relationship Between Vp, Vs, E, G, ν and Depth (soft soil).

RELATIONSHIP BETWEEN POISSON'S RATIO AND DEPTH OF SOIL

According to the studies of parameters of soil properties both at home and abroad, it is known that the action by force is a main exterior factor that cause the change of soil properties. Thus, strength data resulting from the above mentioned factors can be described with the action by exterior force that leads to the formation of soil with different properties. On the other hand, for any given soil body, the interior main factor influencing the strength is density, while the exterior main factor that governs the soil density is three equal forces effecting from X, Y, Z directions.

An infinitesimal element at any depth below the surface of the half space is under the influence of three dimensional stress states due to long duration action by the stress of soil body's dead-weight. An infinitesimal element of soil body at different depths will be subjected to different stresses, in fact it is a "non-homogeneous elastic body", of which the elastic coefficient is a function of coordinates.

To simplify the calculation, it can be supposed that the stratum is formed by numerous infinitesimal thin layers. Due to the fact that each layer is at different depth, the influence by the stress of dead-weight is also different. But the soil body of each infinitesimal thin layer can be taken as homogeneous one. Its stress state is shown as Fig.5(a).

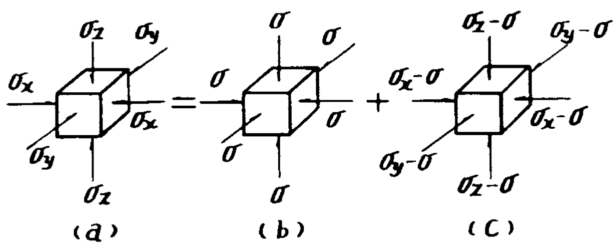

Fig. 5. Stress State of an Element.

Stresses and strains of any element in the soil are correlated as below:

$$\left.\begin{aligned} \varepsilon_x &= \frac{1}{E}[\sigma_x - \nu(\sigma_y+\sigma_z)] \\ \varepsilon_y &= \frac{1}{E}[\sigma_y - \nu(\sigma_x+\sigma_z)] \\ \varepsilon_z &= \frac{1}{E}[\sigma_z - \nu(\sigma_x+\sigma_y)] \\ \varepsilon_{xy} &= \frac{1}{\mu}\tau_{xy} \\ \varepsilon_{yz} &= \frac{1}{\mu}\tau_{yz} \\ \varepsilon_{zx} &= \frac{1}{\mu}\tau_{zx} \end{aligned}\right\} \quad (3)$$

The soil body, under the action by the successive and uniform distributed stress of dead-weight, is confined as regards to the lateral expansion. Therefore, it can be supposed that: $\varepsilon_x=\varepsilon_y=0$, then lateral compressive stresses σ_x, σ_y and vertical compressive stress σ_z are correlated as:

$$\sigma_x = \sigma_y = \frac{\nu}{1-\nu}\sigma_z = \frac{\nu}{1-\nu}\gamma Z \quad (4)$$

in which

$\frac{\nu}{1-\nu}$ —— Coefficient of lateral stress;

γ —— Unit Weight of Soil (t/m³).

The stress components of any element in soil can be divided into stress components related to the change of volume and that related to the change of shape, as shown by Fig. 5(b), (c).

Here, $\sigma = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) = \frac{1}{3}\left(\frac{1+\nu}{1-\nu}\right)\gamma Z \quad (5)$

The element body under the action by the three equal forces, as shown in Fig.5(b), will undergo volume change. The grain of soil body will be cohered with the change of volume. This phenomenon will result in corresponding change of physical and mechanical properties of soil, which can be shown through the change of density. Consequently, under the action of exterior load, the main factor dominating the coherence of soil body is "σ".

The soil body beneath the surface of a semi-infinite body is different over density with different laying dpeth. Whereas shear strength is different for the soils of same type as a result of different densities. The greater the density is, the greater the shear modulus will be as a rule.

Under the action by three equal stresses from different directions, the following equation can be applied to the calculation of relative volume deformation.

$$\begin{aligned} \theta &= \varepsilon_x + \varepsilon_y + \varepsilon_z = 3\varepsilon = \frac{3\sigma}{E}(1-2\nu) \\ &= \frac{1-2\nu}{1-\nu}\cdot\frac{\gamma Z}{2\mu} \end{aligned} \quad (6)$$

Shear modulus and propagating velocity of transversal wave at any depth are correlated as:

$$G(Z) = \rho V_s(Z)^2 \quad (7a)$$

and that at surface of soil is

$$G = \rho V_s^2 \quad (7b)$$

From the tests performed so far, it is known that the factor influencing the propagating velocity of transversal wave rather greatly is porosity ratio or ambient pressure. When porosity ratio is given, the increment of shear modulus is in proportion to the average principal stress σ (Ichiharamatsuhei, 1972).

Namely,
$$\begin{aligned} \Delta G &= \beta_1\sigma = \frac{\beta_1}{3}(\sigma_x+\sigma_y+\sigma_z) \\ &= \frac{\beta_1}{3}\left(\frac{1+\nu}{1-\nu}\right)\gamma Z \\ &= \beta_1\cdot\beta_2\cdot\gamma Z = \beta\gamma Z \end{aligned} \quad (8)$$

in which

β_1, β_2, β are all non-dimensional coefficients.

Considering the influence exerted upon shear modulus by the stress of soil body's dead-weight, shear modulus at any depth is equal to shear modulus at surface of soil G plus the increment ΔG of shear modulus caused by stress of soil body's dead-weight. Namely,

$$G(Z) = G + \Delta G = G + \beta\gamma Z \quad (9)$$

Hence, the relative volume deformation can be known through calculation by using the following equations after consideration is given to the stress of soil body's dead-weight.

$$\theta(Z) = \frac{1-2\nu(Z)}{E(Z)} \cdot 3\sigma(Z) = \frac{1-2\nu(Z)}{1-\nu(Z)} \cdot \frac{\gamma Z}{2\mu(Z)}$$

$$= \frac{1-2\nu(Z)}{1-\nu(Z)} \cdot \frac{\gamma}{2(\frac{\mu}{Z}+\beta\gamma)} \qquad (10)$$

From Eq.(10), it can be seen that: within the region of $[0, \infty]$, Z is a limited continuous function. Therefore, when $Z \to \infty, \theta(Z) \to$ const. That shows: the increment of the relative volume deformation at a sufficient depth tends to be zero when the influence upon shear modulus by stress of soil body's dead-weight is under consideration.

It is supposed that the relative volume deformations of soil bodies located at two adjacent planes Z_1 and Z_2 ($Z_2 > Z_1$) along depth Z are $\theta(Z_1)$ and $\theta(Z_2)$, thus, when Z is sufficient great, we can say:

$$\Delta\theta = \theta(Z_2) - \theta(Z_1) \to 0 \qquad (11)$$

The following equation is obtained by substituting Eq. (10) into Eq. (11):

$$\frac{\frac{1-\nu(Z_1)}{1-2\nu(Z_1)}}{\frac{1-\nu(Z_2)}{1-2\nu(Z_2)}} = \frac{\frac{\mu}{Z_2}+\beta\gamma}{\frac{\mu}{Z_1}+\beta\gamma} \qquad (12)$$

Because $Z_2 > Z_1$, thus $\frac{1-\nu(Z_1)}{1-2\nu(Z_1)} < \frac{1-\nu(Z_2)}{1-2\nu(Z_2)}$ (13)

It can be seen that $\frac{1-\nu(Z)}{1-2\nu(Z)}$ is a monotone increasing function of Z.

Let $f(\nu) = \frac{1-\nu}{1-2\nu}$

then, $f'(\nu) = \frac{1}{(1-2\nu)^2} > 0$

From the above equation it is shown that $f'(\nu)$ is greater than zero despite of any case of ν (as a matter of fact, only $0 \leq \nu \leq 0.5$ deserves discussion). Therefore, $f(\nu)$ is a monotone increasing function of ν also, as $f[\nu(Z_1)] < f[\nu(Z_2)]$, there must be $\nu(Z_1) < \nu(Z_2)$. The above discussion is also shown that Poisson's ratio of soils with same property is not a constant after taking into consideration of stress caused by soil body's dead-weight and increases with depth.

RELATIONSHIP BETWEEN DEPTH AND RATIO OF VERTICAL COMPRESSIVE STRESS TO LATERAL COMPRESSIVE STRESS

It has been proved above that ν is a monotone increasing function of Z. Therefore, when $Z_2 > Z_1$, there must be

$$\frac{\sigma_z(Z_1)}{\sigma_x(Z_1)} > \frac{\sigma_z(Z_2)}{\sigma_x(Z_2)} \qquad (14)$$

Eq.(14) shows: On condition that the dead-weight of soil body influencing shear modulus is under consideration, the ratio of $\sigma_z(Z)$ to $\sigma_x(Z)$ acted upon an element decreases with the increase of depth. Namely, $\sigma_x(Z)$ and $\sigma_y(Z)$ equals to $\sigma_z(Z)$ gradually as the depth increases (see Fig.6).

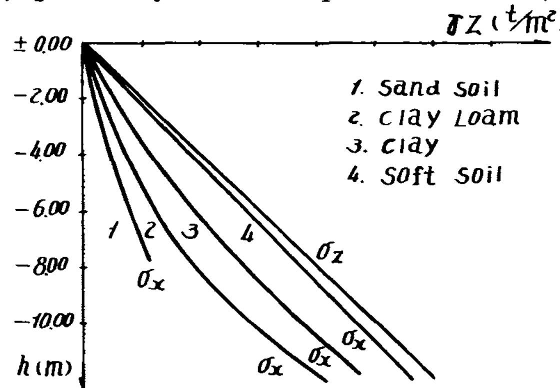

Fig. 6. Relationship Between $\sigma_x(Z)$, $\sigma_z(Z)$, and Depth.

CONCLUSION

Both theory and tests show:

1. Poisson's ratio of homogeneous soil is not a constant, and increases with depth of soil. And furthermore, the function increases differently with the change of property of soil.

2. Ratio of lateral compressive stress to vertical compressive stress acted upon an element is not a constant at any depth. With the increase of depth, lateral compressive stress is approximately equal to vertical compressive stress step by step.

3. The ratio of elastic modulus to shear modulus at different depth is not a constant either due to the fact that Poisson's ratio increases with depth of soil. And what's more, elastic modulus rises faster than shear modulus with the increase of depth of soil.

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