The Reflection of Quasi Waves at Free Boundary of a Fibre-Reinforced Medium

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Abstract:

The propagation of plane waves in fibre-reinforced medium is discussed. The expressions of phase velocity of quasi-P (qP) and quasi-SV (qSV) waves propagating in plane symmetry are obtained in terms of propagation vector. We have established a relation from which displacement vector can be obtained in terms of propagation vector. The expressions for the reflection coefficients of qP and qSV waves are obtained. Numerical results of reflection coefficients are obtained and presented graphically. The partition of energy between qP and qSV waves reflected on a free boundary due to incident qP and qSV waves are also obtained and presented graphically.
[Keywords: Reflection, quasi-P, quasi-SV, quasi-SH, fibre-reinforced, reflection coefficients]

1. Introduction

Fibre-reinforced composite materials have become very attractive in many engineering applications recently due to their superiority over other structural materials in applications requiring high strength and stiffness and lightweight components. consequently the characterisation of their mechanical behaviour is of particular importance for structural design using these materials.

Effects of earthquakes on artificial structures are of prime importance to engineers and architects. During an earthquake and similar disturbances a structure is excited into a more or less violent vibration, with resulting oscillatory stresses, which depend both upon the ground vibration and physical properties of the structure [Richter (1958)]. Most concrete construction includes steel reinforcing, at least nominally. So, wave propagation in a reinforced medium plays a very important role in civil engineering and geophysics.

The propagation of body waves in anisotropic media is fundamentally different from their propagation in isotropic media, although the differences may comparatively subtle and difficult to observe [Crampin (1975)]. In general, for any type of anisotropy, there are always three types of body waves propagating with three different velocities. Choosing the three components of displacement adequately, they are called quasi-P, quasi-SV and quasi-SH (qSH) waves. The velocities of these three waves change according to the type of symmetry present in the medium. Because of these properties, anisotropy is detected by observations of change in P-wave velocity along two perpendicular directions and by observations of S-wave splitting. For both effects it is not necessary that the whole medium be anisotropic; only a certain part of it need be [Udias (1999)].Generally, the particle motion is neither purely longitudinal nor purely transverse. For this reason, the three types of body waves in an anisotropic medium are referred as qP,qSV and qSH instead of P,SV and SH.

The problem of reflection and refraction of elastic waves have been discussed by several authors. Without going into the details of the research works in this field we mention the papers by Knott (1899), Gutenberg (1944), Thapliyal (1972), Keith and Crampin (1977,1977a,1977b), Dey and Addy (1979),Tolstoy (1982), Norris (1983), Pal and Chattopadhyay (1984), Achenbach (1976), Henneke (1972) and Chattopadhyay et al (1995).

Crampin and Taylor (1971) studied surface wave propagation in examples of unlayered and multi-layered anisotropic media which is examined numerically with a program using an extension of the Thompson-Haskell matrix formulation. They studied some examples of surface wave propagation in anisotropic media to interpret a possible geophysical structure. Crampin (1975) showed that the surface waves have distinct particle motion when propagating in a structure having a layer of anisotropic material with certain symmetry relations. Chattopadhyay and Saha (1996) have studied the problem of reflection of qSV-wave at free and rigid boundary in a medium of monoclinic type.

The above mentioned authors have not studied the reflection behaviour at a free boundariy of a fibre-reinforced medium. The reflection of qP and qSV waves in a fibre-reinforced medium is discussed. In this paper we have computed the reflection coefficients of qP and qSV waves at the free boundary of a fibre-reinforced medium. It is well known that in an anisotropic medium the direction of particle motion is neither perpendicular nor parallel to the direction of propagation. Considering this fact, a relation has been established to calculate the displacement vector in terms of propagation vector. The expressions for phase velocity of qP and qSV waves are obtained in terms of the propagation vector. The partition of energy between qP and qSV waves reflected for qP and qSV waves incident on a free boundary have been derived and presented graphically. The Seismologists generally conclude in some unknown behaviour of the waves as anomaly. The anomalies may be due to the highly anisotropic nature of the earth. This paper has been solved considering highly anisotropic material and results obtained are totally new and will be helpful to the Seismologists for the prediction of the anomalies in the recorded data of the earth due to earthquakes.

2. Formulation of the problem

The constitutive equations for fibre-reinforced linearly elastic medium whose preferred direction is that of $\vec{a}$ are [Spencer (1972)]

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta(a_k a_m e_{km} a_i a_j) \quad (1)$$

where τ_{ij} are components of stress, e_{ij} are components of infinitesimal strain, a_j are components of $\vec{a}$, all referred to cartesian coordinates. The vector $\vec{a}$ may be a function of position. The coefficients λ, μ_L, μ_T, α and β are elastic constants with dimension of stress.

If $\vec{a}$ is so chosen that its components are (1,0,0). The stress components (1) become

$$\tau_{11} = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)e_{11} + (\lambda+\alpha)e_{22} + (\lambda+\alpha)e_{33}, \tau_{22} = (\lambda+\alpha)e_{11} + (\lambda + 2\mu_T)e_{22} + \lambda e_{33},$$
$$\tau_{33} = (\lambda+\alpha)e_{11} + \lambda e_{22} + (\lambda+2\mu_T)e_{33},$$
$$\tau_{12} = 2\mu_L e_{12},\ \tau_{13} = 2\mu_L e_{13}, \tau_{23} = 2\mu_T e_{23}, \quad (2)$$

where $2e_{ij} = u_{i,j} + u_{j,i}$ and $u_i\ (i = 1,2,3)$ are the displacement components.

We take the plane of symmetry of the fibre-reinforced medium as the x_1x_2-plane and x_2-axis vertically upwards. For the plane wave propagation in x_1x_2-plane, we have

$$\frac{\partial}{\partial x_3} \equiv 0.$$

The non-vanishing equations of motion without body forces are

$$\frac{\partial \tau_{11}}{\partial x_1} + \frac{\partial \tau_{12}}{\partial x_2} = \rho\frac{\partial^2 u_1}{\partial t^2},$$
$$\frac{\partial \tau_{21}}{\partial x_1} + \frac{\partial \tau_{22}}{\partial x_2} = \rho\frac{\partial^2 u_2}{\partial t^2}, \quad (3)$$
$$\frac{\partial \tau_{31}}{\partial x_1} + \frac{\partial \tau_{32}}{\partial x_2} = \rho\frac{\partial^2 u_3}{\partial t^2}.$$

The stress equations of motion (3) using (2) become

$$(\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)\frac{\partial^2 u_1}{\partial x_1^2} + \mu_L\frac{\partial^2 u_1}{\partial x_2^2} + (\lambda + \alpha + \mu_L)\frac{\partial^2 u_2}{\partial x_1 \partial x_2} = \rho\frac{\partial^2 u_1}{\partial t^2}, \quad (4)$$

$$\mu_L \frac{\partial^2 u_2}{\partial x_1^2} + (\lambda + 2\mu_T)\frac{\partial^2 u_2}{\partial x_2^2} +$$

$$(\lambda + \mu_L + \alpha)\frac{\partial^2 u_1}{\partial x_1 \partial x_2} = \rho \frac{\partial^2 u_2}{\partial t^2} \quad (5)$$

$$\mu_L \frac{\partial^2 u_3}{\partial x_1^2} + \mu_T \frac{\partial^2 u_3}{\partial x_2^2} = \rho \frac{\partial^2 u_3}{\partial t^2}. \quad (6)$$

From equations (4) to (6), it is obvious that qSH wave which is represented by u_3 motion in eqn(6) is decoupled from (u_1, u_2) motion representing qP and qSV waves. The phase velocity of qSH wave is

$$\rho c_n^{\ 2} = \mu_L \{p_1^{(n)}\}^2 + \mu_T \{p_2^{(n)}\}^2 \quad (7)$$

where $\vec{p}\,(p_1^{(n)}, p_2^{(n)}, 0)$ denote the unit propagation vector, c_n is the phase velocity and k_n is the wave number of plane waves propagating in the x_1x_2-plane. We consider plane wave solutions of equations (4) and (5) as

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = A \begin{pmatrix} d_1^{(n)} \\ d_2^{(n)} \end{pmatrix} \exp\left[ik_n(\vec{x}.\vec{p} - c_n t)\right] \quad (8)$$

where $\vec{d}\,(d_1^{(n)}, d_2^{(n)}, 0)$ is the unit displacement vector. Using the expressions of eqn (8) for u_1 and u_2 in the equations of motion (4) and (5), we obtain

$$\frac{d_1^{(n)}}{d_2^{(n)}} = \frac{S}{\rho c_n^{\ 2} - R} = \frac{\rho c_n^{\ 2} - T}{S} \quad (9)$$

where

$R = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)\ \{p_1^{(n)}\}^2 + \mu_L\ \{p_2^{(n)}\}^2$,

$S = (\lambda + \alpha + \mu_L)\ p_1^{(n)} p_2^{(n)}$,

$T = \mu_L \{p_1^{(n)}\}^2 + (\lambda + 2\mu_T)\ \{p_2^{(n)}\}^2$. (10)

The equation (9) may be used to find the $\vec{d}$ in terms of $\vec{p}$. From the above equation, we have

$\rho^2 c_n^4 - (R + T)\rho c_n^2 + (RT - S^2) = 0$.

The solutions of the above equation are

$2\rho c_n^2 = (R + T) \pm [(R - T)^2 + 4S^2]^{½}$.

Velocity of qP wave and qSV waves are

$$2\rho c_L^2 = (R+T) + \sqrt{(R-T)^2 + 4S^2} \quad (11)$$

$$2\rho c_T^2 = (R+T) - \sqrt{(R-T)^2 + 4S^2} \quad (12)$$

From the equations (4) and (5), we obtain

$$[(\lambda + 2\alpha + 3\mu_L - 2\mu_T + \beta)\ d_1^{(n)} d_2^{(n)} \{p_1^{(n)}\}^2] + (\mu_L - \lambda - 2\mu_T)\ d_1^{(n)} d_2^{(n)} \{p_2^{(n)}\}^2 + (\lambda + \alpha + \mu_L)\ [\{d_2^{(n)}\}^2 - \{d_1^{(n)}\}^2]\ p_1^{(n)} p_2^{(n)} = 0. \quad (13)$$

Pure longitudinal and shear waves can propagate only in certain specific directions. Longitudinal and transverse specific directions are found by putting $\vec{d} = \vec{p}$ and $\vec{d}$ perpendicular to $\vec{p}$. In anisotropic case no such relations can be considered between the displacement vector and the propagation vector.

We consider a homogeneous fibre-reinforced half-space occupying the region $x_2 \leq 0$ and the plane of symmetry is taken as the x_1x_2-plane. Plane qP wave is incident at the traction-free boundary $x_2=0$ and will generate reflected qP and qSV waves. Let n=0,1,2 be assumed for incident qP, reflected qP and reflected qSV waves respectively. We consider plane strain problem and hence

$u_1 = u_1(x_1,x_2,t),\ u_2 = u_2(x_1,x_2,t),\ u_3 = 0.$

The displacement field may be represented by

$$u_1 = \sum_{j=0}^{2} A_j d_1^{(j)} e^{i\eta_j}, \quad u_2 = \sum_{j=0}^{2} A_j d_2^{(j)} e^{i\eta_j}, \quad (14)$$

where

$$\eta_n = k_n(x_1 p_1^{(n)} + x_2 p_2^{(n)} - c_n t). \quad (15)$$

In the plane $x_2=0$, the displacements and stress components due to incident qP (n=0), reflected qP (n=1) and reflected qSV (n=2) waves may be written as

$$u_1^{(n)} = A_n d_1^{(n)} \exp(i\eta_n),\ u_2^{(n)} = A_n d_2^{(n)} \exp(i\eta_n),$$

$$\tau_{22}^{(n)} = iA_n k_n[(\lambda+\alpha) d_1^{(n)} p_1^{(n)} + (\lambda + 2\mu_T) d_2^{(n)} p_2^{(n)}], \quad (16)$$

$$\tau_{21}^{(n)} = iA_n k_n \mu_L [d_1^{(n)} p_2^{(n)} + d_2^{(n)} p_1^{(n)}]$$

where $\eta_n = k_n(x_1 p_1^{(n)} - c_n t)$.

For incident qP wave, which makes an angle θ_0, we have

$p_1^{(0)} = \sin\theta_0$, $p_2^{(0)} = \cos\theta_0$, $c_0=c_L$. (17)

For reflected qP wave, which makes an angle θ_1, we have

$p_1^{(1)} = \sin\theta_1$, $p_2^{(1)} = -\cos\theta_1$, $c_1=c'_L$. (18)

If the reflected qSV wave makes an angle θ_2, we have

$p_1^{(2)} = \sin\theta_2$, $p_2^{(2)} = -\cos\theta_2$, $c_2=c_T$. (19)

3. Boundary conditions and solution of the problem for incident qP-waves

Case I: Reflection of qP- wave at a free boundary :

When $x_2 = 0$ is a free surface, the sum of the three tractions must vanish at $x_2 = 0$ and we can write the boundary conditions as :

$$\tau_{22}^{(0)} + \tau_{22}^{(1)} + \tau_{22}^{(2)} = 0$$

and

$$\tau_{21}^{(0)} + \tau_{21}^{(1)} + \tau_{21}^{(2)} = 0. \quad (20)$$

Substituting in (20), the values of $\tau_{22}^{(n)}, \tau_{21}^{(n)}$ (for n=0,1,2) from (16), (17), (18) and (19), we obtain after some calculations:

$$\frac{A_1}{A_0} = \frac{a_2 - b_2}{a_1 b_2 - a_2 b_1}, \frac{A_2}{A_0} = -\frac{a_1 - b_1}{a_1 b_2 - a_2 b_1} \quad (21)$$

where

$$a_1 = \frac{P_1}{P_0}, \quad a_2 = \frac{P_2}{P_0}, \quad b_1 = \frac{P_4}{P_3}, \quad b_2 = \frac{P_5}{P_3},$$

$P_0 = k_0\,[(\lambda+\alpha)d_1^{(0)}\sin\theta_0 + (\lambda + 2\mu_T)d_2^{(0)}\cos\theta_0]$,

$P_1 = k_1\,[(\lambda+\alpha)d_1^{(1)}\sin\theta_1 - (\lambda + 2\mu_T)d_2^{(1)}\cos\theta_1]$,

$P_2 = k_2\,[(\lambda+\alpha)d_1^{(2)}\sin\theta_2 - (\lambda + 2\mu_T)d_2^{(2)}\cos\theta_2]$,

$P_3 = k_0\,[\,d_1^{(0)}\cos\theta_0 + d_2^{(0)}\sin\theta_0]$,

$P_4 = k_1\,[-d_1^{(1)}\cos\theta_1 + d_2^{(1)}\sin\theta_1]$,

$P_5 = k_2\,[-d_1^{(2)}\cos\theta_2 + d_2^{(2)}\sin\theta_2]$,

$$\frac{k_1}{k_o} = \frac{c_L}{c'_L} = \frac{\sin\theta_o}{\sin\theta_1},$$

and $$\frac{k_2}{k_0} = \frac{c_L}{c_T} = \frac{\sin\theta_0}{\sin\theta_2}. \quad (22)$$

Now, $\frac{d_1^{(i)}}{d_2^{(i)}}$ (i=0,1,2) can be calculated from (9) and using (17), (18) and (19).

From the equations (11) and (12), the velocities of incident qP, reflected qP and reflected qSV may be defined by

$$2\rho c_L^2 = (R_0 + T_0) + \sqrt{(R_0 - T_0)^2 + 4S_0^2},$$
$$2\rho c_L'^2 = (R_1 + T_1) + \sqrt{(R_1 - T_1)^2 + 4S_1^2},$$
$$2\rho c_T^2 = (R_2 + T_2) - \sqrt{(R_2 - T_2)^2 + 4S_2^2}, \quad (23)$$

where R_0,R_1,R_2, S_0,S_1,S_2,T_0 ,T_1 and T_2 may be obtained from (10) and (17) to (19) using n=0,1,2.

Using the following values of reinforced- free medium

$$\mu_L = \mu_T = \mu, \alpha = \beta = 0,$$

the equations (28) reduce to

$$\frac{A_1}{A_0} = \frac{\sin 2\theta_0 \sin 2\theta_2 - \overline{K}^2 \cos^2 2\theta_2}{\sin 2\theta_0 \sin 2\theta_2 + \overline{K}^2 \cos^2 2\theta_2}, \quad (24)$$

$$\frac{A_2}{A_0} = \frac{2\overline{K}\sin 2\theta_0 \cos 2\theta_2}{\sin 2\theta_0 \sin 2\theta_2 + \overline{K}^2 \cos^2 2\theta_2} \quad (25)$$

where

$$\overline{K} = \sqrt{\frac{\lambda + 2\mu}{\mu}}$$

which are the reflection coefficients of P and SV - waves respectively for free boundary in isotropic case [Achenbach(1976), page –175].

The partition of energy between reflected qP and qSV waves for incident qP wave is given by

$$\left(\frac{A_1}{A_0}\right)^2 \frac{c_L}{c'_L}\frac{m_1\cos\theta_1}{m_0\cos\theta_0} + \left(\frac{A_2}{A_0}\right)^2 \frac{c_L}{c_T}\frac{m_2\cos\theta_2}{m_0\cos\theta_0} = 1 \quad (26)$$

where

$$m_0 = (ap_1^{(0)}d_1^{(0)} + bp_2^{(0)}d_2^{(0)})\sin\theta_0 d_1^{(0)} +$$
$$\mu_L(d_1^{(0)}p_2^{(0)} + d_2^{(0)}p_1^{(0)})(d_1^{(0)}\cos\theta_0 + d_2^{(0)}\sin\theta_0) +$$
$$(bd_1^{(0)}p_1^{(0)} + cd_2^{(0)}p_2^{(0)})d_2^{(0)}\cos\theta_0 ,$$
$$m_1 = (ap_1^{(1)}d_1^{(1)} + bp_2^{(1)}d_2^{(1)})\sin\theta_1 d_1^{(1)} +$$
$$\mu_L(d_1^{(1)}p_2^{(1)} + d_2^{(1)}p_1^{(1)})(-d_1^{(1)}\cos\theta_1 + d_2^{(1)}\sin\theta_1)$$
$$-(bp_1^{(1)}d_1^{(1)} + cp_2^{(1)}d_2^{(1)})d_2^{(1)}\cos\theta_1 ,$$
$$m_2 = (ap_1^{(2)}d_1^{(2)} + bp_2^{(2)}d_2^{(2)})\sin\theta_2 d_1^{(2)} +$$
$$\mu_L(d_1^{(2)}p_2^{(2)} + d_2^{(2)}p_1^{(2)})(-d_1^{(2)}\cos\theta_2 + d_2^{(2)}\sin\theta_2)$$
$$-(bp_1^{(2)}d_1^{(2)} + cp_2^{(2)}d_2^{(2)})d_2^{(2)}\cos\theta_2 ,$$

$a = \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta$, $b = \lambda + \alpha$,
$c = \lambda + 2\mu_T$.

For isotropic case the equation (35) becomes

$$\left(\frac{A_1}{A_0}\right)^2 + \left(\frac{A_2}{A_0}\right)^2 \frac{c_T}{c_L}\frac{\cos\theta_2}{\cos\theta_0} = 1$$

which is same as Achenbach ((1976), page 182).

The velocity of surface wave can be obtained from (28) by equating the denominator to zero. It has been observed that the surface wave velocity at $\theta = 8.8^0$ in case of fibre-reinforced medium is 1.58 times more than the Rayleigh wave in classical case (values of $\lambda,\alpha,\beta,\mu_T$ and μ_L are defined in heading "Numerical discussions") .

4. Reflection of qSV waves at a free boundary

Incident qSV wave will generate reflected qP and qSV waves. Let n=0,1,2 be assumed for incident qSV, reflected qP and reflected qSV waves respectively. For incident qSV wave, which makes an angle θ_0 , we have

$$p_1^{(0)} = \sin\theta_0 , p_2^{(0)} = \cos\theta_0 , c_0 = c_T^{/} . \qquad (27)$$

In the plane x_2=0, the displacement and stress components of incident wave and reflected waves are same as (16), (18) and (19). The equation (17) to be replaced by

$$\eta_0 = k_0 (x_1 p_1^{(o)} - c_T^{/} t) . \qquad (28)$$

5. Boundary conditions and solution of the problem for qSV waves

5.1: Reflection of qSV- wave at a free boundary :

Substituting in (20), the values of $\tau_{22}{}^{(n)}, \tau_{21}{}^{(n)}$ (for n=0,1,2) from (31) and (16), we obtain the expressions same as (21) and (22) except c_L may be replaced by $c_T^{/}$. The ratio $\frac{d_1^{(0)}}{d_2^{(0)}}$ is mentioned in equation (33), other ratio's are same as (24) and (25). We may get the following relations,

$$\frac{k_1}{k_0} = \frac{c_T^{/}}{c_L^{/}} = \frac{\sin\theta_0}{\sin\theta_1} ,$$
$$\frac{k_2}{k_0} = \frac{c_T^{/}}{c_T} = \frac{\sin\theta_0}{\sin\theta_2} . \qquad (29)$$

where ϕ and ω are defined in (24). Solving (21) and (22) we have the same sets of equations as (26) with some changes and are mentioned below:

$\frac{d_1^{(i)}}{d_2^{(i)}}$ (i=0,1,2) may be calculated from (9) and are as under:

$$\frac{d_1^{(0)}}{d_2^{(0)}} = \frac{\rho c_T^{/2} - T}{S} \qquad (30)$$

where R,S and T can be calculated after putting

$p_1 = p_1^{(0)} = \sin\theta_0$ and $p_2 = p_2^{(0)} = \cos\theta_0$ in (10).

$\frac{d_1^{(i)}}{d_2^{(i)}}$ (i=1,2) are defined in (24) and (25).

From the equations (12) and (11), the velocity of incident qSV may be defined by

$$2\rho c_T^{/2} = (R+S) - \sqrt{(R-S)^2 + 4T^2} . \qquad (31)$$

Reflected qP and qSV waves velocities are already defined in (26).

Using the following values of reinforced- free medium (values are mentioned in section 3), we obtain the reflection coefficients for isotropic case as

$$\frac{A_1}{A_0} = -\frac{\overline{K}\sin 4\theta_0}{\sin 2\theta_0 \sin 2\theta_1 + \overline{K}^2 \cos^2 2\theta_0} ,$$
$$\frac{A_2}{A_0} = \frac{\sin 2\theta_0 \sin 2\theta_1 - \overline{K}^2 \cos^2 2\theta_0}{\sin 2\theta_0 \sin 2\theta_1 + \overline{K}^2 \cos^2 2\theta_0} \qquad (32)$$

where $\overline{K} = \left[\frac{\lambda + 2\mu}{\mu}\right]^{1/2}$.

The partition of energy between reflected qP and qSV waves due to incident qSV wave is given by

$$\left(\frac{A_1}{A_0}\right)^2 \frac{c_T^{/}}{c_L^{/}}\frac{m_1\cos\theta_1}{m_0\cos\theta_0} + \left(\frac{A_2}{A_0}\right)^2 \frac{c_T^{/}}{c_T}\frac{m_2\cos\theta_2}{m_0\cos\theta_0} = 1 \qquad (33)$$

where all definitions are defined in (29).

6. Numerical calculations and discussions

The material constants for fibre-reinforced medium have been considered due to Markham (1970)

$\mu_T = 2.46\text{x}10^9\,\text{N/m}^2$, $\mu_L = 5.66\text{x}10^9\,\text{N/m}^2$,
$\lambda = 5.65\text{x}10^9\,\text{N/m}^2$, $\beta = 220.90\text{x}10^9\,\text{N/m}^2$,
$\alpha = -1.28\text{x}10^9\,\text{N/m}^2$, $\rho = 7800\,\text{kg/m}^3$.

6.1 Reflection of qP waves:

In figure 1, curve-II corresponds to reflection coefficient of qP-wave in fibre-reinforced medium. All the values of curve-II are negative except from 0^0 to 10^0 and from 83^0 to 90^0. In isotropic case (curve-I), the values of (A_1/A_0) are all negative except from $\theta_o = 57^o$ to 78^o. Significant differences of values exist from 0^0 to 5^0 and from 80^0 to 90^0 in fibre-reinforced case compared to isotropic case. The values from 21^0 to 80^0 are more in isotropic case compared to fibre-reinforced case.

In figure 2, the reflection coefficients of qSV waves for a free boundary of fibre-reinforced medium at different angle of incidence have plotted along with the curve for isotropic medium. The values of (A_2/A_o) are all positive and equal for curves I and II at $\theta_o = 0^o$, 15^0 and 90^o. The difference in values at $\theta_o = 50^o$ in isotropic case is significantly more compared to fibre-reinforced medium.

Figure 3 shows the comparison of partition of energy between reflected P and qP waves for incident qP waves. In this case $A_1/A_0 = 0$, for angle of incidence at 60^0 and 78^0, and $A_1/A_0 = 1$ for angle of incidence at 0^0 and 90^0 in case of isotropic case. For fibre-reinforced medium (curve-II), $A_1/A_0 = 1$ at 0^0 only.

Figure 4 shows the comparison of partition of energy between reflected SV and qSV waves for incident qP waves. In this case $A_2/A_0 = 0$ at 0^0 for both isotropic and fibre-reinforced (curve-II) media. The critical point exists at 13^0.

6.2 Reflection of qSV waves:

In figure 5, curve-I corresponds to isotropic medium and agrees the result with Achenbach (1976) . Curve-II corresponds to fiber-reinforced medium. The value of A_1/A_0 in a fiber-reinforced medium is more compared to isotropic medium but difference is more at 33^0 .

In figure 6, the reflection coefficient of qSV-wave (curve-II) for reinforced medium for different values of θ_0 ranging from 0^0 to 33^0 have been plotted which are permissible of θ_0 for A_2/A_0 in fiber reinforced medium and compared with isotropic case (curve-I). In this case due to the effect of fiber-reinforced medium the values of A_2/A_0 are more compared to isotropic case.

Figures 7 and 8 show the partition of energy for incident qSV waves due to free boundary.

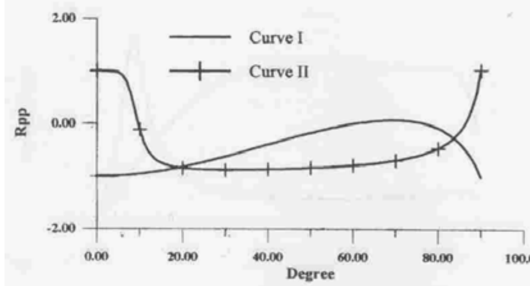

Figure 1. Amplitude ratios of qP waves due to incident qP waves.

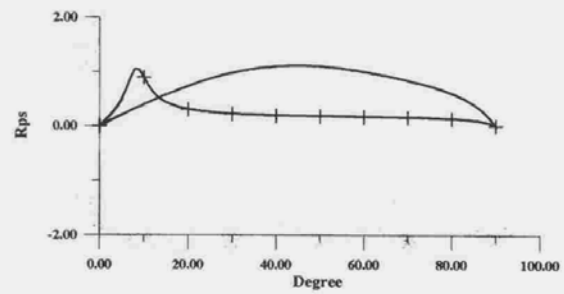

Figure 2. Amplitude ratios of qSV waves due to incident qP waves.

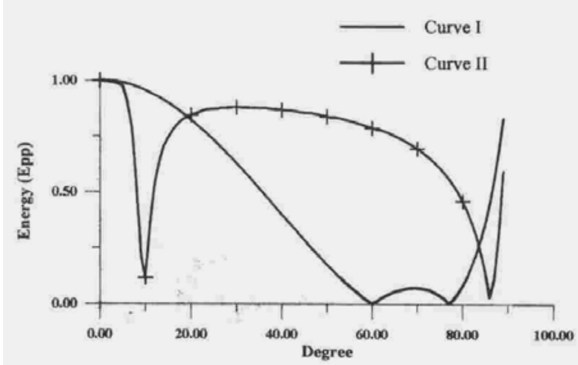

Figure 3. Partition of energy of qP waves due to incident qP waves.

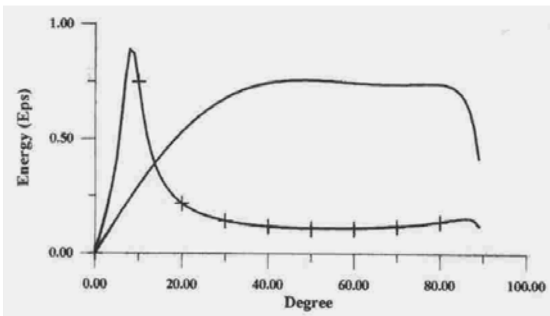

Figure 4. Partition of energy of qSV waves due to incident qP waves.

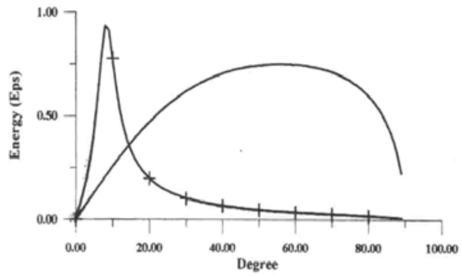

Figure 5: Partition of energy of qSV waves due to incident qP waves in rigid boundary

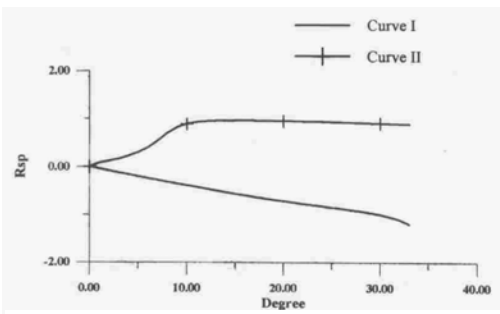

Figure 6: Amplitude ratios of qP waves due to incident qSV waves.

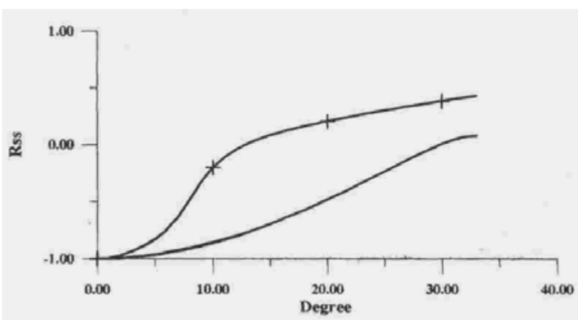

Figure 7: Amplitude ratios of qSV waves due to incident qSV waves

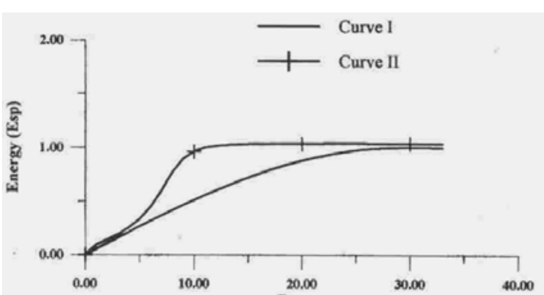

Figure 8: Partition of energy of q Paves due to incident qSV waves

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